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Evidential Database: a new generalization of databases?

Ahmed Samet, Eric Lefèvre, and Sadok Ben Yahia

Abstract In this paper, we tackle the problem of data representation in several types of databases. A detailed survey of the different support measures in the major existing databases is described. The reminder of the paper aims to prove the importance of using evidential databases in case of handling imperfect information. The evidential database generalizes several ones by the use of specific Basic Belief Assignments. In addition, we show that the precise support, initially introduced on evidential database, generalizes several support measures.

1 Introduction

Data mining is a technique that uses a variety of data analysis tools to discover, hidden but interesting patterns and relationships in data that may be used to make valid predictions. Thanks to its simple formulas, it associates performance and quality in its retrieved results. For this reason, it is used in various fields and attracted interest in different applications [9].

The first studies on data mining relies on a data model under which transactions captured doubtless facts about the items that are contained in each

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transaction. These binary databases have only two scenarios: 1 if an element exists, 0 otherwise. However, in many applications, the existence of an item in a transaction is better captured by likelihood measures. The obvious limits of the binary databases in handling such types of data led the data mining community to adopt imprecise frameworks in order to mine more pertinent knowledge.

In this paper, we present a non exhaustive review of existing data mining databases. The characteristics of binary, probabilistic, fuzzy and evidential databases are detailed. The support measures in the databases are presented. The aim of this paper is to demonstrate the pivotal role of the evidential database, which relies on the evidence theory [5, 12], in representing imprecision and uncertainty. The importance of using an evidential database rather than the other ones is justified. Indeed, we prove that the precise support measure [10] in evidential databases is a generalization of that of the classical ones.

The remainder of the paper is organized as follows: in section 2, the key basic settings of the evidential database are recalled. In section 3, the binary database is studied and its relationship with the evidential database is highlighted. In section 4, probabilistic databases are scrutinized and the correlation between the precise support and the probabilistic support is highlighted. Section 5 stresses on the snugness connection between fuzzy databases with the evidential ones. Finally, we conclude and we describe issues for future work.

2 Evidential database and precise support

In this section, we detail the main concepts of evidential databases as well as as the notion of precise support.

2.1 Evidential Database concept

Introduced by Lee [8], the evidential database was aimed at modelling imperfect information. This type of database is supposed to handle imprecise and uncertain data. An evidential database is a triplet $\mathcal{EDB} = (\mathcal{A}_{\mathcal{EDB}}, \mathcal{O}, R_{\mathcal{EDB}})$. $\mathcal{A}_{\mathcal{EDB}}$ is a set of attributes and \mathcal{O} is a set of d transactions (i.e., lines). Each column A_i ($1 \le i \le n$) has a domain θ_{A_i} of discrete values. $R_{\mathcal{EDB}}$ expresses the relation between the j^{th} line (i.e., transaction T_j) and the i^{th} column (i.e., attribute A_i) by a normalized BBA as follows:

$$m_{ij}: 2^{\theta_{A_i}} \to [0,1]$$
 with

$$\begin{cases} m_{ij}(\emptyset) = 0\\ \sum_{\omega \subseteq \theta_{A_i}} m_{ij}(\omega) = 1. \end{cases}$$
 (1)

Table 1: Evidential transaction database \mathcal{EDB}

Transaction	Attribute A	Attribute B
T1	$m(A_1) = 0.7$	$m(B_1) = 0.4$
	$m(\theta_A) = 0.3$	$m(B_2) = 0.2$ $m(\theta_B) = 0.4$
T2	$m(A_2) = 0.3$	(2 /
	$m(\theta_A) = 0.7$	

Table 1 illustrates an example of an evidential database. An item corresponds to a focal element. An itemset corresponds to a conjunction of focal elements having different domains. The inclusion operator is defined in [3] such that for two itemsets X and Y, we have:

$$X \subseteq Y \iff \forall x_i \in X, x_i \subseteq y_i.$$

where x_i and y_i are the i^{th} element of X and Y. For the same evidential itemsets X and Y, the intersection operator is defined as follows:

$$X \cap Y = Z \iff \forall z_i \in Z, z_i \subseteq x_i \text{ and } z_i \subseteq y_i.$$

An evidential associative rule R is a causal relationship between two itemsets that can be written in the following form $R: X \to Y$ such that $X \cap Y = \emptyset$.

Example 1. In Table 1, A_1 is an item and $\theta_A \times B_1$ is an itemset such that $A_1 \subset \theta_A \times B_1$ and $A_1 \cap \theta_A \times B_1 = A_1$. $A_1 \to B_1$ is an evidential associative rule

In the following subsection, we consider the precise support and confidence measures.

2.2 Support and confidence in evidential database

Several definitions for the support's estimation have been proposed for the evidential itemsets such as [3, 6]. Those definitions assess the support based on the belief function Bel(). The based belief support is constructed from the Cartesian product applied to the evidential database. Interested readers may refer to [6]. The support is computed as follows:

$$Support_{\mathcal{EDB}}(X) = Bel_{\mathcal{EDB}}(X) \tag{2}$$

such that:

$$Bel: 2^{\theta} \to [0, 1] \tag{3}$$

$$Bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B).$$
 (4)

In a previous work [10], we introduced a new metric for support estimation. The latter has been shown to provide more accuracy and to overcome several drawbacks of using the belief function. This measure is called Precise support Pr and it is defined by:

$$Pr: 2^{\theta_i} \to [0, 1] \tag{5}$$

$$Pr(x_i) = \sum_{x \subseteq \theta_i} \frac{|x_i \cap x|}{|x|} \times m_{ij}(x) \qquad \forall x_i \in 2^{\theta_i}.$$
 (6)

The evidential support of an itemset $X = \prod_{i \in [1...n]} x_i$ in the transaction T_j (i.e., Pr_{T_j}) is then equal to:

$$Pr_{T_j}(X) = \prod_{x_i \in \theta_i, i \in [1...n]} Pr(x_i). \tag{7}$$

Thus, the evidential support $Support_{\mathcal{EDB}}$ of the itemset X becomes:

$$Support_{\mathcal{EDB}}(X) = \frac{1}{d} \sum_{i=1}^{d} Pr_{T_i}(X).$$
 (8)

Additionally, in [11], we introduced a new measure of confidence for evidential associative rules that we called the *precise confidence measure*. Let us assume an evidential association rule such as $R: R_a \to R_c$, where R_c and R_a respectively, denote the conclusion and the antecedent (premise) part of the rule R. The precise confidence measure can be written as follows:

$$Confidence(R:R_a \to R_c) = \frac{\sum_{j=1}^{d} Pr_{T_j}(R_a) \times Pr_{T_j}(R_c)}{\sum_{j=1}^{d} Pr_{T_j}(R_a)}.$$
 (9)

In the following sections, we highlight the relationships between evidential databases and the main other ones. The link between existing measures and the evidential precise one is also demonstrated.

3 Binary data mining

The first database variants studied from a data mining view are the binary ones. A binary database can be represented by a triplet $\mathcal{BDB} = (\mathcal{A}, \mathcal{O}, R_{\mathcal{BDB}})$. \mathcal{A} represents the set of n binary attributes (i.e., columns). $R_{\mathcal{BDB}}$ is the relation that reflects the existence of an item in a transaction by only the values 0 and 1. $R_{\mathcal{BDB}}(A_i, T_j) = 1$ means that the item A_i exists in the transaction T_j and $R_{\mathcal{BDB}}(A_i, T_j)$ is set equal to 0 otherwise.

Since the inception of the Apriori algorithm [2], several other approaches have been introduced to reduce the computational complexity. The support of an item A_i in a transaction T_j is defined as follows:

$$Support_{T_i}(A_i) = R_{\mathcal{BDB}}(A_i, T_j). \tag{10}$$

The support of an item A_i in those binary databases is still computed with the same manner:

$$Support(A_i) = \sum_{i=1}^{d} R_{\mathcal{BDB}}(A_i, T_j) = count(A_i).$$
 (11)

The same goes for an itemset $A \cup B$ (or $A \times B$ if we keep the product notation):

$$Support(A \times B) = count(A \cup B). \tag{12}$$

Thus, the support is computed by counting the number of transactions having both A and B. From the support, the confidence measure of a rule $R: R_a \to R_c$ is computed as follows:

$$confidence(R: R_a \to R_c) = \frac{count(R_a \cup R_c)}{count(R_a)}.$$
 (13)

A binary database can be constructed by redefining the $R_{\mathcal{EDB}}$ as a precise BBA. Indeed, each item $A_i \in \mathcal{A}$ can be redefined as an evidential item having the following frame of discernment $\theta_{A_i} = \{\exists, \not\exists\}$. \exists and $\not\exists$ denote respectively the existence and absence of the attribute A_i in the considered transaction. Such a BBA can be written as follows:

$$\begin{cases}
m_{ij}(\{\exists\}) = R_{\mathcal{BDB}}(A_i, T_j) \\
m_{ij}(\{\not\exists\}) = 1 - R_{\mathcal{BDB}}(A_i, T_j)
\end{cases}$$
(14)

where m_{ij} is equivalent to a certain BBA. In that case, the support measure proposed in [10] is equivalent to the binary support equation defined in Equation (10). To demonstrate that equivalence, let us consider a binary database \mathcal{D} and the evidential database \mathcal{EDB} constructed as in the described procedure. Suppose that $R_{\mathcal{BDB}}(A_i, T_j) = 1$ such that $A_i \in \mathcal{A}$, then the corresponding evidential attribute is an $A_i \in \mathcal{A}_{\mathcal{EDB}}$ with $\theta_{A_i} = \{\exists, \not\exists\}$:

$$Pr_{T_{j}}(\exists) = \frac{|\exists \cap \exists|}{|\exists|} m_{ij}(\{\exists\}) + \frac{|\not{\exists} \cap \exists|}{|\not{\exists}|} m_{ij}(\{\not{\exists}\}) = m_{ij}(\{\exists\}) = R_{\mathcal{BDB}}(A_{i}, T_{j}).$$

$$(15)$$

From this point, we deduce that the evidential precise support is a generalization of the binary one. The same goes for the precise confidence given in Equation (9) that generalizes binary confidence since they both rely on the same support fraction.

Example 2. In this example, Table 2 shows how to create an evidential database from a binary one.

Table 2: The evidential transformation of \mathcal{BDB} (Table (a)) to \mathcal{EDB} (Table (b))

		A	В	С
АВС	T_1 m_{11}	$(\{\exists\}) = 0$	$m_{21}(\{\exists\})=1$	$m_{31}(\{\exists\}) = 1$
	$T_2 \ m_{12} \ m_{12} \ T_3 \ m_{13}$	$(\{\exists\}) = 1$ $(\{\exists\}) = 0$ $(\{\exists\}) = 0$	$m_{22}(\{\exists\}) = 1$ $m_{22}(\{\not\exists\}) = 0$ $m_{23}(\{\exists\}) = 1$	$m_{31}(\{ \not\exists \}) = 0$ $m_{32}(\{ \exists \}) = 0$ $m_{32}(\{ \not\exists \}) = 1$ $m_{33}(\{ \exists \}) = 1$ $m_{33}(\{ \not\exists \}) = 0$
			(b)	

The equivalency of the support measure is shown for the itemset $B \times C$.

The support of the itemset $B \times C$ from the transactions of Table 2.a is $Support(B \times C) = \frac{2}{3}$. In the evidential database, it is computed as follows:

$$Support_{\mathcal{EDB}}(B \times C) = \frac{1}{3} \sum_{j=1}^{3} Pr_{T_{j}}(A) \times Pr_{T_{j}}(B)$$

$$Support_{\mathcal{EDB}}(B \times C) = \frac{1}{3} (m_{21}(\{\exists\}) \times m_{31}(\{\exists\}) + m_{22}(\{\exists\}) \times m_{32}(\{\exists\}) + m_{23}(\{\exists\}) \times m_{33}(\{\exists\})) = \frac{2}{3}$$

Thus, the support retrieved from the binary database is the same as the precise support computed from the evidential database.

4 Probabilistic data mining

Probabilistic data mining [1] was introduced to represent imperfect information thanks to the probability support. It can be represented by a triplet $\mathcal{PDB} = (\mathcal{A}_{\mathcal{PDB}}, \mathcal{O}, R_{\mathcal{PDB}})$. The degree of existence of the item A_i in the transaction T_j is measured through the probability function $p(A_i, T_j) \in [0, 1]$. The support of an itemset $X \in \mathcal{A}_{\mathcal{PDB}}$ in such type of database is defined as follows [4]:

$$p(X, T_j) = \prod_{i \in X} p(i, T_j). \tag{16}$$

Thus, the support of an itemset X in a database is the sum of its expected probability in the transaction:

$$Support_{\mathcal{PDB}}(X) = \sum_{j=1}^{d} p(X, T_j). \tag{17}$$

An equivalent evidential database can be constructed through using Bayesian BBA¹. The BBA can be modeled on a two-member-based frame of discernment $\theta_i = \{\exists, /\exists\}$ where \exists indicates that A_i belongs to the considered transaction, whereas $/\exists$ performs the opposite. Such a BBA can be constructed as follows:

$$\begin{cases}
m_{ij}(\{\exists\}) = p(i, T_j) \\
m_{ij}(\{\not\exists\}) = 1 - p(i, T_j).
\end{cases}$$
(18)

With this construction, the probabilistic support defined in Equation (17) is equivalent to the proposed precise support. Indeed, the assertion can be verified i.e.:

$$Pr_{T_j}(\exists) = \frac{|\exists \cap \exists|}{|\exists|} m_{ij}(\{\exists\}) + \frac{|\not\exists \cap \exists|}{\not\exists} m_{ij}(\{\not\exists\}) = m_{ij}(\{\exists\}) = p(i, T_j).$$
(19)

As is the case for a binary database, the Evidential Data mining Algorithm (EDMA) generalizes the probabilistic version of Apriori: i.e., U-Apriori [4].

 $Example\ 3.$ Table 3 shows how to create an evidential database from a probabilistic one.

Table 3: The evidential transformation of \mathcal{PDB} (Table (a)) to \mathcal{EDB} (Table (b))

	A	В	С
A B C	$T_1 m_{11}(\{\exists\}) = 0$	== (() /	~= (() /
T_1 0.0 0.7 0.8 T_2 0.9 0.7 0.1 T_3 0 0.8 0.7	$T_2 \ m_{12}(\{\exists\}) = 0.9$ $m_{12}(\{\not\exists\}) = 0.1$	$m_{22}(\{\exists\}) = 0.7$ $m_{22}(\{\not\exists\}) = 0.3$	$m_{32}(\{\not\exists\}) = 0.9$
(a)	$T_3 m_{13}(\{\exists\}) = 0$ $m_{13}(\{\not\exists\}) = 1$	$m_{23}(\{\exists\}) = 0.8$ $m_{23}(\{\ \not\exists\}) = 0.2$ (b)	(, , , ,

The equivalency of the support measure is shown for the itemset $B \times C$. The support of the itemset $B \times C$ from the transactions of the Table 3.a is $Support(B \times C) = \frac{(0.7 \times 0.8) + (0.7 \times 0.1) + (0.8 \times 0.7)}{3} = 0.4$. In the evidential

¹ A BBA is called Bayesian only if all its focal sets are singletons.

database, it is computed as follows:

$$Support_{\mathcal{EDB}}(B \times C) = \frac{1}{3} \sum_{j=1}^{3} Pr_{T_{j}}(A) \times Pr_{T_{j}}(B)$$

$$Support_{\mathcal{EDB}}(B \times C) = \frac{1}{3} (m_{21}(\{\exists\}) \times m_{31}(\{\exists\}) + m_{22}(\{\exists\}) \times m_{32}(\{\exists\}) + m_{23}(\{\exists\}) \times m_{33}(\{\exists\})) = \frac{1\cdot2}{3} = 0.4$$

Thus, the support retrieved from the probabilistic database is the same as the precise support computed from the evidential database.

5 Fuzzy Data mining

Let us assume the triplet $\mathcal{FDB} = (\mathcal{A}_{\mathcal{FDB}}, \mathcal{O}, R_{\mathcal{FDB}})$ that denotes a fuzzy database. $R_{\mathcal{FDB}}$ denotes the fuzzy relationship between an item and a transaction expressed through a membership function. The membership function $\mu_{T_j}(i) = \alpha \ (\alpha \in [0, 1])$ rates the degree of membership of the considered item to the transaction T_j . The support computation in such databases is done by the use of the count() function in the following manner [7]:

$$count(i) = \sum_{j=1}^{d} \mu_{T_j}(i). \tag{20}$$

The support of item i in the fuzzy database is found as follows:

$$Support(i) = \frac{count(i)}{d}.$$
 (21)

Thus, for an itemset X of size q such that $x_i \in X$ and $i \in [1, q]$, the support becomes:

$$support(X) = \frac{\sum_{j=1}^{d} min\{\mu_{T_j}(x_i), i = 1 \dots q\}}{d}.$$
 (22)

The numerator of the support could be seen as the Gödel t-norm (minimum t-norm).

Assuming a fuzzy database is available, it is possible to construct an evidential database. In addition, the precise support sustains fuzzy support in its formulation. Indeed, as can be seen in Equation (8), the precise support is also equal to the sum of the transactional support divided by the database size.

In the following, we show how to obtain analogous evidential support of the fuzzy support. Assuming an attribute $A_i \in \mathcal{A}_{\mathcal{EDB}}$ having a frame of discernment θ_{A_i} such that $\omega_1 \subset \cdots \subset \omega_n \subseteq \theta_{A_i}$, the corresponding BBA for a fuzzy relation $R_{\mathcal{FDB}}(\omega_1, T_j) = \mu_{T_i}(\omega_1)$ is constructed in this form:

$$\begin{cases} m_{ij}(\omega_1) = \mu_{T_j}(\omega_1) \\ \sum m(\cup_k \omega_k) = 1 - \mu_{T_j}(\omega_1). \end{cases}$$
 (23)

We can obviously remark that:

$$T(\mu(A_i), \mu(A_i)) = \min(Bel(A_i), Bel(A_i))$$
(24)

where T is a minimum t-norm. Thus, the fuzzy support can be retrieved in an evidential database as follows:

$$Support_{\mathcal{FDB}}(X) = \frac{\sum_{T_j \in \mathcal{O}} min\{Bel(x_i), x_i \in X\}}{d}.$$
 (25)

Interestingly enough, an equivalent to fuzzy database support in evidential database does exists.

Example 4. Table 4 shows how to create an evidential database from a fuzzy one.

Table 4: The evidential transformation of \mathcal{FDB} (Table (a)) to \mathcal{EDB} (Table (b))

			Support	A		В	
Support	A	В		ω_1	ω_2	ω_1	ω_2
	ω_1 ω_2	ω_1 ω_2	T_1	` '	` '	$m_{31}(\omega_1) = 0.1$	` '
T_1 T_2 T_3	0.5 0.2	0.1 0.8 0.3 0.8 1.0 0.2		$m_{12}(\omega_1) = 0.5$ $m_{12}(\Omega) = 0.5$	$m_{22}(\omega_2) = 0.2$ $m_{22}(\Omega) = 0.8$	$m_{31}(\Omega) = 0.9$ $m_{32}(\omega_1) = 0.3$ $m_{32}(\Omega) = 0.7$ $m_{31}(\omega_1) = 1.0$	$m_{42}(\omega_2) = 0.8$ $m_{42}(\Omega) = 0.2$
	(a)			` '	` '	$m_{31}(\Omega) = 0$	` '
			(b)				

The equivalency of the support measure is shown for the itemset $B \times C$. The support of the itemset $A_{\omega_1} \times B_{\omega_2}$ from the Table 4.a is $Support(A_{\omega_1} \times B_{\omega_2}) = \frac{0.3 + 0.5 + 0.2}{3} = 1.0$. In the evidential database, Table 4.b, it is computed as follows:

$$Support_{\mathcal{EDB}}(A_{\omega_{1}} \times B_{\omega_{2}}) = \frac{1}{3} \sum_{j=1}^{3} min(Bel(A_{\omega_{1}}), Bel(A_{\omega_{2}}))$$
$$Support_{\mathcal{EDB}}(A_{\omega_{1}} \times B_{\omega_{2}}) = \frac{1}{3} (Bel_{T_{1}}(A_{\omega_{1}}) + Bel_{T_{2}}(A_{\omega_{1}}) + Bel_{T_{2}}(B_{\omega_{2}})) = 1.0$$

Despite the fact that the precise support is not equivalent to the fuzzy support, it is still possible to recover the same value with the use of the Equation (25).

6 Conclusion

In this paper, we detailed the data mining measures such as the support and the confidence on the several databases such as binary, probabilistic, fuzzy databases. We have proven the generalization relation between precise measures in evidential databases and measures used in other databases. In future works, we aim to study the evidential transformation of other imperfect databases such as fuzzy-possibilistic database [13].

References

- Aggarwal, C.C.: Managing and Mining Uncertain Data. Springer Publishing Company, Incorporated (2009)
- Agrawal, R., Srikant, R.: Fast algorithm for mining association rules. In Proceedings of international conference on Very Large DataBases, VLDB, Santiago de Chile, Chile pp. 487–499 (1994)
- Bach Tobji, M.A., Ben Yaghlane, B., Mellouli, K.: Incremental maintenance of frequent itemsets in evidential databases. In Proceedings of the 10th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty, Verona, Italy pp. 457–468 (2009)
- Chui, C.K., Kao, B., Hung, E.: Mining frequent itemsets from uncertain data. In: Z.H. Zhou, H. Li, Q. Yang (eds.) Advances in Knowledge Discovery and Data Mining, Lecture Notes in Computer Science, vol. 4426, pp. 47–58. Springer Berlin Heidelberg (2007)
- Dempster, A.: Upper and lower probabilities induced by multivalued mapping. AMS-38 (1967)
- Hewawasam, K.K.R., Premaratne, K., Shyu, M.L.: Rule mining and classification in a situation assessment application: A belief-theoretic approach for handling data imperfections. Trans. Sys. Man Cyber. Part B 37(6), 1446–1459 (2007)
- Hong, T.P., Kuo, C.S., Wang, S.L.: A fuzzy AprioriTid mining algorithm with reduced computational time. Applied Soft Computing 5(1), 1–10 (2004)
- Lee, S.: Imprecise and uncertain information in databases: an evidential approach. In Proceedings of Eighth International Conference on Data Engineering, Tempe, AZ pp. 614–621 (1992)
- Liao, S.H., Chu, P.H., Hsiao, P.Y.: Data mining techniques and applications a decade review from 2000 to 2011. Expert Systems with Applications 39(12), 11,303–11,311 (2012)
- Samet, A., Lefevre, E., Ben Yahia, S.: Mining frequent itemsets in evidential database.
 In Proceedings of the fifth International Conference on Knowledge and Systems Engeneering, Hanoi, Vietnam pp. 377–388 (2013)
- 11. Samet, A., Lefèvre, E., Ben Yahia, S.: Classification with evidential associative rules. In Proceedings of 15th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, IPMU, Montpellier, France, to appear (2014)
- 12. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press (1976)
- Weng, C., Chen, Y.: Mining fuzzy association rules from uncertain data, springerverlag new york, inc. new york, ny, usa issn: 0219-1377 doi. knowledge and information systems 23, 129-152 (2010)