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# Optimization in Wireless Networks

Antoine Oustry, Liding Xu, Sonia Haddad-Vanier, Juan-Antonio Cordero,  
Thomas Clausen

## 1 Introduction

A wireless network is a network of electronic devices (computers, smartphones, sensors, actuators, or other connected objects) that communicate to each other, or to the Internet, through one or several wireless communication channel(s).

Wireless networks may be used in a wide variety of scenarios, and run under various modes of operation: centralized and coordinated by a single authority, decentralized with cooperative nodes, decentralized containing non-coordinated, autonomous, potentially ‘greedy’ nodes, etc. Electronic devices in these wireless networks may present heterogeneous characteristics (memory, computational power), run applications or provide services with different requirements (real-time guarantees, latency, reliability), and connect to each other through heterogeneous wireless communication technologies.

With the emergence of services related to the Internet of Things (IOT), and the pervasiveness of Internet access, both indoors and outdoors, two different types of wireless networks become particularly relevant:

1. Internet access networks, consisting of interconnected Base Stations or Access Points. Electronic devices (such as laptops, smartphones, etc.) within the coverage range of one of these Access Points can connect to the Internet by communicating wirelessly with such AP. Since these networks are deployed to allow users to retrieve contents from the Internet,

traffic in these networks is dominated by down-link (i.e., from the Internet to the devices) traffic.

2. Wireless Sensor Networks (WSNs), i.e. wireless (possibly multi-hop) networks of sensors deployed, for measurement purposes, over a large area, and connected to the Internet through one or several gateways. Traffic in these networks is typically up-link (i.e., from devices/sensors towards destinations in the Internet).

Several optimization problems can be considered for these two types of networks, depending on the specific purposes and conditions of each deployment. This article concentrates on two general optimization problems, related to gateway placement and energy consumption, and explores their corresponding mathematical programming approaches.

In the first case, related to Internet access networks, the article explores the problem of choosing the placement of Internet access points in a certain region. Given a networking coverage target of the region, the objective of such placement is to optimize the quality of the wireless Internet connection achieved by users.

In the second case, this article considers the optimization problems induced by the energy consumption in multi-hop wireless sensor networks (WSN). Given source-to-target traffic demands, interference, and capacity constraints, the goal is to find optimal routing so that overall energy consumption at the device level is minimized. Energy consumption is highly dependent on routing strategies. In multi-hop WSNs, the routing must be unsplitable; each traffic demand should be routed on a single path, subject to interference and resource limitations. Transmission resources are limited by interference and the traffic load can be reduced by the network coding. All these factors lead to very complex combinatorial optimization problems. However, this

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paper presents how arc-path formulations of the multi-commodity flow problems can model these specific technical aspects of WSNs, and how column generation approaches solve them efficiently.

## 1.1 Outline

The rest of this entry is organized as follows. Section 2 introduces basic characteristics of the wireless networks (technologies, interference, energy consumption) addressed in the two optimization cases. Section 3 addresses the optimization problem for placement of wireless Access Points (APs) to provide Internet access in a given (indoors) geographical area. Section 4 presents the optimization approaches, challenges, and strategies for energy consumption in multi-hop WSNs.

## 2 Background: Wireless Networking

The Presented optimization problems for wireless networks take into consideration two key aspects: coverage and interference (section 2.1), and energy consumption (section 2.2). Main underlying technologies for the considered wireless networks are reviewed and discussed in section 2.3.

### 2.1 Coverage and Interference

The **coverage area** of a wireless electronic device  $A$  is the geographical area around  $A$ , in which transmissions from  $A$  can be received and successfully decoded by other electronic devices, in the same wireless medium as  $A$ .

The **interference area** of a wireless electronic device  $A$  is the geographical area around  $A$ , in which electronic devices sharing the same wireless medium as  $A$  are unable to successfully decode transmissions from other devices, if there is an ongoing (concurrent) transmission from  $A$ .

Both coverage and interference areas depend on transmission and reception characteristics (transmission power, modulation, frequency band, receivers sensitivity), as well as on the physical characteristics of the shared wireless channel (noise, obstacles to propagation, reflections, loss, etc.). For a given electronic device  $A$ , the interference area may be larger than the coverage area – i.e., electronic devices in the vicinity of  $A$  that are unable to successfully decode a transmission from  $A$ , may be prevented to decode transmissions from other devices, due to a transmission of  $A$ .

In ideal conditions of *free-space propagation*, coverage and interference areas of  $A$  are circles centered in the geographical position of  $A$ . When the real conditions for propagation are deemed sufficiently close to those of free-space, coverage and interference areas can be approximated to such circles, determined by their coverage and interference range (radius). Whereas this model is sufficient for free areas, it might be not sufficient to predict accurately the signal strength in the context of indoor radio wave propagation [49, 53]. Section 3.3 presents more sophisticated models used for this purpose.

### 2.2 Energy consumption model

Energy consumption in electronic devices depends on device capabilities, usage, modes of operation, architecture, and technologies. In wireless communicating devices, communication (transmission, reception and electronic circuitry for signal processing) takes a substantial share of the overall device energy consumption; other relevant operations may include system background tasks or data processing. In IOT devices, for example, energy consumption can be decomposed into data acquisition (measurements), data processing, system background tasks, and networking [45].

### 2.3 Technologies

Different wireless network technologies support different ranges, achieve different traffic rates, and relay various other characteristics (modulation techniques, medium access, transmission bands, duty-cycling or power restrictions, receivers sensitivity, etc.). For the purpose of the current article, the following technologies are relevant:

- Wireless Local Area Networks (WLANs) allow devices to communicate through an Access Point (AP), over ranges within 50-100 meters. The main family of standards for WLANs is IEEE 802.11, commonly denominated Wi-Fi. Although the specific characteristics change with the version and year of the standard (first Wi-Fi standard was released in 1997, the last one to date is Wi-Fi 6, released in 2021), WLANs based on IEEE 802.11 technologies operate in unlicensed bands of the electromagnetic spectrum, and are designed for high throughput (up to Gbps in the latest releases). Wi-Fi devices communicate over a shared medium, meaning that different devices on the same network may interfere to each other when communicating to their corresponding Access Points.

- Low Power Wide Area Networks (LPWANs) allow communication over very long distances (in the order of kilometers), for constrained devices with very low energy consumption and very low traffic rate (in the order of few Kbps). LPWAN devices transmit over unlicensed bands, and transmissions may be subject to duty-cycling and power transmission limitations. Some of the main LPWAN technologies include Long-Range (LoRa), Sigfox, DASH7, and Weightless.
- Wireless Personal Area Networks (WPANs) allow wireless low-rate (up to 250 Kbps) communication between nearby devices (around 10 meters away). WPANs characteristics have been specified in the standard IEEE 802.15.4, which is used by protocols such as Zigbee (mostly used for home automation), the standard WirelessHART (industrial sensor networks), or the standard 6LoWPAN (for IP-based low-power sensor networks).

	WLAN	WPAN	LPWAN
Throughput	high	low	low
Range	medium	short	long
Consumption	high	low	low

Table 1: Summary of characteristics for wireless networking technologies.

Table 1 provides a qualitative summary of coverage, consumption and throughput in WLANs, WPANs and LPWANs.

Wireless Sensor Networks may rely on WPAN or LPWAN technologies; WLANs may be used to deploy Internet access networks.

### 3 Optimal WLAN deployment

#### 3.1 Problem statement

Since they allow high-quality access to the Internet without requiring static positions for client devices, WLANs are widely used as gateways to the Internet. In airports, hotels, universities, or office buildings, WLAN technologies provide access to the Internet to thousands of people simultaneously. Such large-scale public or private WLANs involve hundreds of Access Points (AP). Hence, deploying such a network is a complex decision problem, which requires taking into account many parameters such as the number and the characteristics of the APs but also the architecture of the building. In

practice, these deployments are still carried out empirically by practitioners. Yet, optimization methodologies have been developed in the past years to streamline the deployments and achieve a better trade-off between Quality-of-Service (QoS), coverage, and costs. When deploying a WLAN in a building, the involved decision variables are mainly the AP positions. Depending on whether the set of possible positions is discretized or not, this results in a discrete or continuous optimization problem. The emission frequency assigned to each AP is also a decision variable that the deployer can consider [28, 21]. For hardware reasons, the set of possible frequencies is discrete. Other AP's characteristics may be played on, like emission power or antenna orientation.

A central feature of an optimization problem is the set of *criteria* used, either in the objective to be optimized or in the constraints. Since the problem of WLAN deployment has been extensively studied in the Networking and Operations Research community, there is a large variety of WLAN deployment criteria in the literature. The most intuitive optimization criterion for the deployment of a wireless network is the economic cost, which depends mainly on the number of access points used, the site-specific installation costs and a discounted operating cost. [6]. As it may be expected, there is a trade-off between economic cost and WLAN quality; as required quality is higher (e.g. measured in terms of coverage), optimal cost can be expected to grow. The category of coverage criteria, very common in the literature on WLAN optimization, encompasses metrics characterizing how the network covers the target area in terms of the covered surface or the number of covered clients [2]. The signal intensity received by each client is also used to formulate constraints in [6, 48]. In [2, 33], the authors use the min - max criterion of received power from the best server at the worse client location. Many approaches focus more on the QoS rather than the absolute strength of the received signal as an ultimate criterion to evaluate the deployment. A first QoS-related metric is the throughput, that depends on the signal strength but also on the interference level, this is why the Signal Interference-plus-Noise Ratio (SINR) is often used. These criteria are either involved as a hard QoS constraint to enable data rate guarantee for every or are used as an aggregated criteria (average). Some articles distinguish uplink and downlink SINR [6, 46]. Another QoS-related metric is the Maximum of Channel Utilization, which designates the maximum traffic loads assigned to an AP. This is a practical metric for the wireless network performance because it explains qualitatively its congestion [37]. Sophisticated approaches

use simulation tools to compute QoS metrics depending on the selected networking protocol [32]. As no single criterion may be sufficient to capture every aspects of a satisfying WLAN deployment, some authors are interested in finding deployment solutions balancing different criteria among those mentioned above. This leads to multi-objective optimization models [59], or to heuristically aggregated criteria [2, 33, 32].

### 3.2 Wave propagation, a key aspect of the problem

Understanding radio wave propagation in a building is a crucial aspect for the deployment of the WLAN deployment problem on that building: if an AP transmits from point A, what is the signal received at point B? This data is essential in the optimization process to evaluate whether a deployment solution provides good coverage or not [11]. Since an exhaustive measurement campaign between all possible pairs (A,B) in any large building is costly and time-consuming, may be insufficiently accurate, and is not always feasible, propagation models for estimating propagation are used for optimization purposes.

The simplest models to predict radio-wave propagation are the one-slope or multi-wall models. The one-slope model [49] is a linear relation between the power gain ratio and the distance between points A and B. The multi-wall model takes into account the number of walls and floors between the two points. While simple and fast to compute, the accuracy of these models is limited. The loss of accuracy is mostly due to the lack of architecture details; specifically, they neglect the physical effects of diffraction, self-interference, or corridor effects. Simulation, physics-based methods can be used to integrate these details, which are of major importance in the context of indoor propagation. Ray-tracing techniques are among the most popular methodologies for indoor radio-wave propagation simulation [54, 23]. They describe the physical wave propagation process, based on geometrical optics and the uniform geometrical theory of diffraction. Another family of propagation models consists of simulating Maxwell's equations, in time or frequency-domain. The first indoor radio propagation simulations with finite differences were been proposed in [14, 36], in time-domain. In [29], a specific numerical scheme is proposed to compute steady-state propagation: the Multi-Resolution Frequency Domain ParFlow (MR-FDPF) algorithm allows the efficient solution of the associated linear system of equations. The main advantage of such methods is to naturally model all propagation effects including reflection and diffraction but the computational load is high, especially in a 3D setting. Recently, a frequency-domain

finite-difference simulator in "2.5D" and based on the complex Helmholtz equation was proposed in [46], with an open-source implementation available online.

### 3.3 Mathematical programming solution approaches

The plurality of criteria used for WLAN deployment leads to a plurality of optimization problem types: discrete, continuous, convex, and non-convex problems. Hence, various mathematical programming tools are used to solve WLAN deployment models. Approaches with convex criteria and with continuous models for the APs locations use convex optimization techniques, such as the Simplex Algorithm [33], the Bundle Methods [2] the Quasi-Newton BFGS Algorithm or the conjugate gradient search procedure [33, 51] to solve a deployment problem. However, most of the modelling approaches in the literature result with non-convex or discrete mathematical programming formulations. When client-AP or frequency assignment issues arise, the authors use Binary Linear Programming (BLP) or Mixed Integer Linear Programming (MILP) formulations and employ standard branch-and-bound solvers [37, 46], or propose dedicated Dantzig-Wolfe decompositions [4]. The studied optimization problems are often NP-Hard and it is frequent that exact solution approaches do not manage to prove optimality or even find a satisfying solution in a reasonable time. To solve such combinatorial deployment problems, several work recourse to heuristics [46] or meta-heuristics to speed-up the solution process: genetic algorithms [2, 33, 22], Simulated Annealing [33], Large-Neighbourhood and Tabu Search [6], Termite Colony Algorithm [17] or Particle Swarm Optimization [20, 39]. Finally, a few authors use techniques from Constraint Programming [48] and Black-Box Optimization [32].

## 4 Energy optimization in multi-hop WSNs

This section presents a mathematical programming approach to optimize energy consumption in multi-hop Wireless Sensor Networks (WSNs).

WSNs are deployed in several application areas such as healthcare monitoring, military investigations, space exploration, environment monitoring, smart city and smart mobility deployments, and agricultural monitoring. The design and management of WSNs face many challenges, due to the energy limitation of sensor devices, deployment and localization, data routing and data aggregation, data security and limited resources. These questions give rise to new complex problems and

challenges in Operations Research. Indeed, Operation Research is widely used to solve WSNs optimization problems; such as improving network performance and maximizing the lifetime of the system [43,26].

The remainder of this article is devoted to discussing the optimization of energy consumption in multi-hop WSNs.

**Energy consumption** is a crucial parameter in the WSN since the lifetime of the network strongly depends on the energy level of its devices. Each node in the WSN has limited wireless communications capabilities and energy source, usually a battery. Due to the small dimensions of the sensors, the batteries are also small and the available energy is limited. Optimal management of energy is necessary to ensure a long network lifetime. A classification of the main used batteries in the WSN is given in [52].

Three factors, and their impact in energy consumption in WSNs, are taken into consideration:

**Routing** has a major impact on the total energy consumption of networks; it is the most challenging issue and direct concern to energy in multi-hop WSN. In such multi-hop WSN each communication is routed through a single path i.e., unsplitably from its source node to its target node [15]. Hence the intermediate nodes in the path are not in charge of computing, for each data packet, the next hop-node. Unsplittable routing speeds up packet handling processes, minimizes loss rates, allows intermediate nodes to use less memory and enables better quality of service (QoS). However unsplitable routing induces very complex combinatorial optimization problems [34,7,9,57]. For more details on routing protocols in wireless networks, we refer to [16].

**Network coding** [1,12,13,31,40] allows to further reduce energy consumption in WSNs by reducing, through algebraic operations over packets to be forwarded by intermediate nodes, the number of transmissions required in a network in order to carry traffic between the set of sources, and the set of destinations.

**Interference:** Wireless communication relies on shared communication media, that can be accessed by several devices, close to each other, at the same time. Interference between such concurrent transmissions needs to be addressed, since it has a non-negligible impact on energy consumption, networking operation and performance. Several strategies and models are developed in the literature to handle interference in WSNs [58].

While many WSN technologies and deployments rely on single-hop communications, extensions for multi-hop communication in IoT-related technologies, both cellular and non-cellular, are attracting research and industry [50,27]. Multi-hop communication in WSNs, in par-

ticular, would increase network capacity and coverage without requiring new infrastructure

This section focuses on the energy efficiency in multi-hop WSNs, namely the wireless unsplitable multi-commodity flow with network coding (wUMCFC) problem studied in [42].

A mathematical formulation of the (wUMCFC) problem that integrates interference and network coding is presented. A column generation approach and a *branch & price* framework are then described.

Given the network topology, source-target traffic demands, transmissions capacities, and channels' interference, the wUMCFC problem seeks to minimize the energy cost of data transmission in multi-hop WSNs and find an optimal unsplitable traffic routing. The wUMCFC incorporates network coding to reduce data transmission, save energy consumption and improve quality of service (QoS).

The model presented in Section 4.1 is based on an arc-path formulation of the unsplitable multi-commodity flow (UMCF) problem. UMCF is one of the well-known  $\mathcal{NP}$ -hard problems in combinatorial optimization [19,34,35,5].

Multi-commodity flow models are developed for several network optimization problems since they enable modeling complex technical constraints [3,8,10,24,38,57]. In particular, multi-commodity flow models may include unsplitable routing or resource sharing constraints [5,35,30]. Another advantage of arc-path formulations is that they are solved efficiently by decomposition and column generation methods [25,30,47,56].

The wUMCFC problem, considered in this article, generalizes the UMCF problem with introducing additional coding and interference constraints. wUMCFC is proved to be  $\mathcal{NP}$ -hard in [42] by a reduction to the single-source unsplitable flow problem [19,34,44].

#### 4.1 Path-based formulation

This section is dedicated to present the path-based formulation developed in [42]. This formulation leads to better modelling of the unsplitable routing and network coding constraints than the compact edge-based formulations.

##### 4.1.1 Problem notations

The network topology is represented by a bi-directed graph  $G = (V, E)$ , where  $V$  denotes the set of nodes corresponding to wireless transmission devices and  $E$  denotes the set of transmission links that can be used to route the traffic flows.

Unsplittable traffic demands are denoted by a set  $D$  of source-target node pairs  $(s, t)$ .

Interference are modeled using capacity constraints, which are defined over the clique set of an undirected conflict graph.

Let  $G_c = (N, L)$  where  $N = E$  be the undirected conflict graph. The links of  $G_c$  represent interference between the edges of  $G$  which cannot transmit data at the same time. If two edges in  $G$  are in interference, then a link between their corresponding nodes is added to  $G_c$ . Two edges of  $G$  share capacity if they are adjacent in  $G_c$ . Let  $m$  be a subset of edges of  $G$  such that the corresponding nodes in  $G_c$  form a clique, then every edge of  $m$  shares capacity with other edges in  $m$ .

Let's define the following notation:

$M$ : the set of maximal cliques of  $G_c$ .

$C_{ij}$ : the transmission capacity of the edge  $(i, j)$ , it measures the number of data packets that can be sent through the channel  $(i, j)$  per unit of time.

$\beta_{ij}$ : the energy cost parameter for flow transmission along the edge  $(i, j)$ . It measures the energy cost to transmit a unit data packet per unit of time.

$d^{st}$ : the traffic demand from the source node  $s$  to the target node  $t$  for  $st \in D$ .

$\varphi^{st}$ : the set of all simple paths from the source  $s$  to the target  $t$ . We define the following decision variables used by the path-based formulation:

#### Decision variables:

$y_p^{st}$ : binary variable indicating whether demand  $st \in D$  is routed along the path  $p \in \varphi^{st}$ .

$u_{\{k,j\}}^i$ : fractional coding variable denoting the amounts of two opposite flows along  $k \rightarrow i \rightarrow j$  and  $j \rightarrow i \rightarrow k$ , which can be encoded at  $i$ .

The occupancy time ratio (OTR) is an important concept in wireless communication. For each edge  $(i, j) \in E$ , its OTR is defined as the total flow per unit of time divided by its capacity, i.e.,

$$\frac{\sum_{st \in D} \sum_{p \in \varphi^{st}, (i,j) \in p} d^{st} y_p^{st}}{C_{ij}}. \quad (1)$$

The OTR measures the ratio that the channel along  $(i, j)$  is transmitting data per unit of time. Hence, the forwarding node  $i$  consumes energies during the occupancy time.

Without network coding, the energy consumption to send  $u_{\{k,j\}}^i$  separately from node  $i$  to node  $j$  and then to node  $k$  is  $u_{\{k,j\}}^i(\beta_{ij} + \beta_{ik})$ . Let define  $\tau_{\{k,j\}}^i$  as:  $\tau_{\{k,j\}}^i = \beta_{ij} + \beta_{ik} - \beta_{\{k,j\}}^i$ , so the energy cost saved by network coding is  $\tau_{\{k,j\}}^i u_{\{k,j\}}^i$ .

#### 4.1.2 Mathematical model

The wUMCF problem aims to find a unique routing path for each demand that minimizes the total energy cost under interference, demand and coding constraints.

The path-based MILP formulation, denoted by wUMCFC-P, follows as:

$$\left\{ \begin{array}{l} \min z_- = \sum_{st \in D} \sum_{p \in \varphi^{st}} \sum_{(i,j) \in p} \beta_{ij} d^{st} y_p^{st} - \sum_{\Lambda_{\{k,j\}}^i \subset E} \tau_{\{k,j\}}^i u_{\{k,j\}}^i \quad (2.0) \\ u_{\{k,j\}}^i - \sum_{st \in D} \sum_{p \in \varphi^{st}} \sum_{(k,i),(i,j) \in p} d^{st} y_p^{st} \leq 0, \\ u_{\{k,j\}}^i - \sum_{st \in D} \sum_{p \in \varphi^{st}} \sum_{(j,i),(i,k) \in p} d^{st} y_p^{st} \leq 0, \quad \forall \Lambda_{\{k,j\}}^i \subset E, \quad (2.1) \\ \sum_{st \in D} \sum_{p \in \varphi^{st}} \sum_{(i,j) \in p \cap m} \frac{d^{st}}{C_{ij}} y_p^{st} - \sum_{\Lambda_{\{k,j\}}^i \subset m} \frac{u_{\{k,j\}}^i}{C_{\{k,j\}}^i} \leq 1, \quad \forall m \in M, \quad (2.2) \\ \sum_{p \in \varphi^{st}} y_p^{st} = 1, \quad \forall st \in D, \\ y_p^{st} \in \{0, 1\}, \quad \forall st \in D, \quad \forall p \in \varphi^{st}, \\ u_{\{k,j\}}^i \in \mathbb{R}_+, \quad \forall \Lambda_{\{k,j\}}^i \subset E. \end{array} \right. \quad (2)$$

*Objective function (2.0)*: energy consumption of data transmissions in the network per unit of time.

*Coding opportunity constraints (2.1)*: every three-hop coding opportunity set induces a pair of constraints, associated with two opposite flows, such that the coding variable  $u_{\{k,j\}}^i$  is at most the aggregated flows along  $k \rightarrow i \rightarrow j$  and  $j \rightarrow i \rightarrow k$ .

*Clique capacity constraints (2.2)* with network coding: OTR within a clique set  $m$  should be at most 1.

*Unsplittable constraints (2.3)*: each demand has to be routed by a single path.

The path-based formulation wUMCFC-P (2) has an exponential number of path variables, but the numerical experiments in [42] show that it can be solved efficiently by the column generation approach.

#### 4.1.3 Column generation

The column generation approaches [18, 25, 41, 55] start by solving the initial linear program (LP) restricted to a small subset of variables and then generate new variables (columns) dynamically. The LP relaxation problem with all columns is called the master problem. At each iteration, a restricted master problem (RMP); limited to a subset of active columns, is solved. The primal and dual solutions of the RMP are used to add new columns (path variables) with a negative reduced cost. Adding these variables to the (RMP) improve the objective function value. Finding new columns with a negative reduced cost is called the pricing problem.

As wUMCFC-P contains binary path variables, it is solved by a *branch&price* algorithm. The pricing algo-

rithm is reduced to a shortest path problem in an extended graph. The following *Algorithm1* and *Algorithm2* summarize the pricing and branching steps, respectively.

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**Algorithm 1: Pricing Algorithm**


---

**Data:**  
 $G$ : the original graph representing network,  $G := (V, E)$   
 $\beta$ : unity energy costs on edges  
 $D$ : the set of source and target vertex pairs of demand  
 $LP$ : information of LP solution of current node  
**Result:** Priced Paths  $P$   
 $P \leftarrow \emptyset$   
 $\zeta, \gamma, \eta \leftarrow \text{GetDual}(LP)$   
**if**  $LP$  is infeasible **then**  
  |  $\beta \leftarrow 0$   
 $N \leftarrow E \cup \{(\bar{i}, \bar{j}) \mid (i, j) \in E\}$   
 $A \leftarrow \{((\bar{k}, \bar{i}), (i, j)) \mid (k, i), (i, j) \in E\} \cup \{((i, j), (\bar{i}, \bar{j})) \mid (i, j) \in E\}$   
**for**  $(k, i), (i, j) \in E$  **do**  
  |  $w_{(\bar{k}, \bar{i}), (i, j)} \leftarrow -\gamma_{kij}$   
**for**  $(i, j) \in E$  **do**  
  |  $w_{(i, j), (\bar{i}, \bar{j})} \leftarrow \beta_{ij} + \sum_{m \in M: (i, j) \in m} \frac{\zeta_m}{C_{ij}}$   
 $EG \leftarrow \text{Extended-Graph}(N, A, w)$   
**for**  $st \in D$  **do**  
  |  $p, cost \leftarrow \text{BellmanFord}(EG, s, t)$   
  | **if**  $f^{st} cost + \eta_{st} < 0$  **then**  
  | |  $P \leftarrow P \cup \{\pi^{-1}(p)\}$   
**return**  $P$

---

At every node of the *branch&price* tree, if the column generation converges to a fractional LP solution, then branching rules are used to enforce the integrality of path variables.

The branching rule strategy is explained in the following *Algorithm2*.

To analyse the performance of the proposed column generation approach, a computational study on realistic instances is performed in [42]. The obtained computational results shows that the proposed *branch&price* algorithm outperforms the CPLEX solver both in terms of running time and duality gap. For all the instances, network coding reduces energy cost significantly. For hard instances, it enables more feasible routes with lower objective function values. These results show the efficiency of network coding and provide important information for strategic decisions on routing policies and network management. Developing arc path formulations for complex network optimization problems and solving them with decomposition and column generation methods remains an effective strategy. In general, mathematical programming provides effective decision-making tools for various optimization problems in wireless networks.

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**Algorithm 2: Branching rule**


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**Data:**  
 $G$ : the original graph representing the network,  
 $G = (V, E)$   
 $\chi$ : the set of path variables with non-zero values for each demand.  
 $F$ : the set of forbidden edge sets at the current B&P node.  
**Result:** Updated forbidden edge sets for two child nodes  
 $\bar{st} \leftarrow \arg \max_{st \in D: |\chi^{st}| > 1} d^{st}$   
Construct the sub-graph  $T$  supporting  $\chi^{\bar{st}}$   
Find the first divergence node  $i$  of  $T$   
 $N_i^{\bar{st}} \leftarrow \{(i, j) \mid (i, j) \in T\}$   
 $O_i^{\bar{st}} \leftarrow \{(i, j) \mid (i, j) \in E\} \setminus (F^{\bar{st}} \cup N_i^{\bar{st}})$   
Compute values of  $\bar{st}$ -flow on the edges of  $N_i^{\bar{st}}$  according to  $\chi^{\bar{st}}$   
Sort  $N_i^{\bar{st}}$  in decreasing order by flow values on the edges  
 $F_1^{\bar{st}}, F_2^{\bar{st}} \leftarrow \emptyset$   
**for**  $(i, j) \in N_i^{\bar{st}}$  **do**  
  | **if**  $|F_1^{\bar{st}}| < |F_2^{\bar{st}}|$  **then**  
  | |  $F_1^{\bar{st}} \leftarrow F_1^{\bar{st}} \cup \{(i, j)\}$   
  | **else**  
  | |  $F_2^{\bar{st}} \leftarrow F_2^{\bar{st}} \cup \{(i, j)\}$   
**for**  $(i, j) \in O_i^{\bar{st}}$  **do**  
  | **if**  $|F_1^{\bar{st}}| < |F_2^{\bar{st}}|$  **then**  
  | |  $F_1^{\bar{st}} \leftarrow F_1^{\bar{st}} \cup \{(i, j)\}$   
  | **else**  
  | |  $F_2^{\bar{st}} \leftarrow F_2^{\bar{st}} \cup \{(i, j)\}$   
**return**  $F_1^{\bar{st}}, F_2^{\bar{st}}$

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