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## A Hybrid Algorithm Based on Multi-colony Ant Optimization and Lin-Kernighan for solving the Traveling Salesman Problem

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### Abstract

In this article, a hybrid heuristic algorithm is proposed to solve the Traveling Salesman Problem (TSP). This algorithm combines two main metaheuristics: optimization of multi-colony ant colonies (MACO) and Lin-Kernighan-Helsgaun (LKH). The proposed hybrid approach (MACO-LKH) is a so-called insertion and relay hybridization. It brings two major innovations: The first consists in replacing the static visibility function used in the MACO heuristic by the dynamic visibility function used in LKH. This has the consequence of avoiding long paths and favoring the choice of the shortest paths more quickly. Hence the term insertion hybridization. The second innovation consists in modifying the pheromone update strategy of MACO by that of the dynamic  $\lambda$ -opt mechanisms of LKH in order to optimize the solutions generated and save in execution time, hence the relay hybridization. The significance of the hybridization, is examined and validated on benchmark instances including small, medium, and large instance problems taken from the TSPLib library. The results are compared to four other state-of-the-art metaheuristic approaches. It results in that they are significantly outperformed by the proposed algorithm in terms of the quality of solutions obtained and execution time.

### Keywords

Heuristic, Hybridization, multi ant colony , Traveling Salesman Problem.

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## I INTRODUCTION

The field of distribution or collection of goods, logistics face optimization problems like the famous Traveling Salesman Problem (TSP). The TSP is a problem in combinatorial optimization studied in Operations Research, computational mathematics, and artificial intelligence. The Traveling Salesman Problem particularly attracts the attention of many researchers in recent years. Indeed, the search for exact or optimal cost to TSP remains a challenge for scientific

community. To date, we note that several works have already been subject of intense research related to the TSP. Among these works, we note the efficiency of heuristics and metaheuristics for the resolution of this combinatorial optimization problem and other NP-difficult problems. In literature, we find heuristics and metaheuristics such as Ants Colony Optimization (ACO) which are capable of solving small instances of TSP. Despite these advances, the TSP remains difficult to solve when the size of the instances increase. It is a trend to combine ACO with other algorithms to solve very large scale of the traveling salesman problem. With the concept of hybridization that we adopt for this work, the exploitation of several resolution techniques is a new opportunity offered to researchers in the field. In this paper, we present a new hybrid model of ACO with multiple colonies by LKH. Our aim is to use the insertion and parallel hybridization that we explain below, to hybridize the ACO heuristic with the LKH heuristic in order to decrease the computation time of the sequential MACO heuristic and to a certain extent improve the quality of the solutions obtained in the TSP resolution. We opted for insertion hybridization because it is new and efficient. The rest of the work is organized as follows: Section I provides a non-exhaustive state of the art of TSP resolution methods. We also briefly give a formulation of the traveling salesman problem. Section II presents in a detailed manner, the meta-heuristics used in the paper framework. Section III is devoted to the study of hybrid methods. It gives an overview of the different types of hybridization and emphasizes above all the type of hybridization we used: insertion hybridization and parallel hybridization. And finally section IV presents the hybrid approach MACO-LKH the tests and the results obtained. We conclude our work by summarizing the main results obtained and giving new perspectives on the basis of the work carried out.

## II PROBLEM STATEMENT

The Traveling Salesman Problem (TSP) is defined as follows: given  $n$  points (city) and the distances between each point, find a path of minimum total length that passes exactly once through each point and returns to the starting point [1, 10]. The distance can also be seen as the cost in general. This combinatorial optimization problem, therefore, consists in searching for the best solution among several possible choices. However, it is easy to state but difficult to solve [10]. The problem is to determine a turn or Hamiltonian circuit, *i.e.* passing once and only once through the  $n$  city, and that is of minimum cost. It is classified as a NP-difficult problem [1, 2], because there is no known method of resolution that provides exact solutions in reasonable time for large instances (large number of city) of the problem.

### 2.1 Mathematical formulation of the TSP

Mathematically, the Traveling Salesman Problem can be formulated as follows: Let  $n$  city and  $C_{ij}$ , the cost or distance corresponding to the trip  $i \rightarrow j$ . Let the variable  $X_{ij}$ , which is 1 if the

tour contains the trip  $i \rightarrow j$ , and 0 otherwise. The problem is written as [6]:

$$\left\{ \begin{array}{l} MinZ = \sum_{i=1}^n \sum_{j=1}^n C_{ij} X_{ij} \\ \sum_{j=1}^n X_{ij} = 1 \quad \forall i \\ \sum_{i=1}^n X_{ij} = 1 \quad \forall j \\ \sum_{i \in Q} \sum_{j \in Q} X_{ij} \geq 1 \quad \forall Q \\ X_{ij} \in \{0, 1\} \forall i, \forall j \end{array} \right. \quad (1)$$

- (1) Each vertex  $i$  can only accept one outgoing arc towards another top  $j$ .
- (2) Each vertex  $j$  can only accept one arc entering from another vertex  $i$ .
- (3) In any subset  $Q$  of vertices, select  $|Q|$  bows would allow to form Hamiltonian circuits.

## 2.2 TSP Applications

Globally, the TSP provides an example of a study of an NP-complete problem whose solution methods can be applied to other discrete mathematics problems. It has several real life applications even in its purest formulation, such as planning, manufacture of microchips, transportation, networks, computer wiring, vehicle routing, job-shop scheduling and logistics. For example, to find the shortest route for school buses or, in industry, to find the shortest distance that the mechanical arm of a machine must travel to drill holes in a printed circuit board.

## III STATE OF THE ART

The problem has since attracted many researchers and thousands of different approaches and algorithms have been developed. In this section, we will focus on hybrid methods. Hybridization is a trend observed in much work done on metaheuristics in recent years to solve TSP. It allows to take advantage of the one to fill what is seen as a limit in the other. In literature there are two classes of hybridizations: on the one hand we have hybridization between metaheuristics and exact methods. This is the case of Cotta[14] which proposes a hybridization between a genetic algorithm and the exact Branch and Bound method to replace the recombination operator. Jahira[17] proposes a hybrid algorithm between the genetic algorithm and the Branch and Bound method to solve the traveling salesman problem.

In the same vein, Chabrier et al [16] hybridized a local search with a Branch and Price algorithm to solve the problem of vehicle routes[14]. The execution of algorithms is carried out in parallel while keeping communication between the methods. Mavrovouniotis, Muller, and Yang [23] integrated the memetic ACO algorithm with local search operators to improve solutions in the population. They apply these local search operators to the best solution found in the population in order to possibly improve this solution; A similar idea is used in this article, but instead of local search operators, LKH is used.

On the other hand, we have the hybridization between metaheuristics and metaheuristics. Among the works carried out in this direction, we have that of Martin and Otto [17] who inserted the descent method in a simulated annealing algorithm to solve the traveling salesman problem. This type of hybridization is referred to as low-level relay hybridization in which another algorithm is incorporated to form a new algorithm. Stützle and Hoos [18] incorporate a local search function in an ant colony algorithm to solve the traveling salesman problem. This low-level co-evolutionary hybridization consists of incorporating one or more single solution-based metaheuristics into a solution population metaheuristic. The advantage of this type of hybridization is that it compensates for the operating power of a local search and that exploration of a global search. Fotso Laure et al in 2008 [3] propose two new hybrid heuristics for the TSP. The first between a Genetic (AG) and heuristic (LK) algorithm and the second between the ant colony algorithm (ACS) and heuristics (LK). The heuristics obtained were called respectively AG-LK and ACS-LK [1]. These authors use a single colony. The results of this experiment have sufficiently demonstrated the effectiveness of these hybrid approaches on several instances of TSP [3]. Unfortunately the solution time resulting from this experience remains enormous. To improve the efficiency of the hybrid heuristics proposed by Fotso et al, Nguimeya, et al in 2016 implemented two new hybrid heuristics for the TSP. The first between a Genetic algorithm (AG) and heuristics (LKH) which is an improvement of LK by helsgaun [2] and the second between the ant colony algorithm (ACS) and heuristics (LKH). The heuristics obtained were called respectively "AG-LKH" and "ACS-LKH". The hybridization strategies differ from one author to another [1].

#### **IV PRESENTATION OF SOME HEURISTICS AND META-HEURISTICS USED TO SOLVE THE TSP**

Metaheuristics are a family of stochastic methods which consist in solving optimization problems. One of the advantages of these is their ability to optimize a problem from a minimum amount of information, however they do not offer any guarantee as to the optimality of the best solution found. Metaheuristics are gaining more and more popularity and are constantly evolving. As a result, we can note a large number of metaheuristics that currently exist.

##### **4.1 Ant colony algorithm**

This metaheuristic is inspired by collective depositing and tracking behaviors observed in ant colonies [20]. In fact, ants communicate with each other indirectly by depositing chemical substances, called pheromones, on the ground. This type of indirect communication is called stigmergy. Indeed, if an obstacle is introduced on the path of the ants, the latter will, after a search phase, all tend to take the shortest path between the nest and the obstacle. The higher the pheromone level in a given location, the more likely an ant will be attracted to that area. The ants that reached the nest the fastest through the food source were those that took the shortest branch of the route.

This is a new version of ACO that we developed in a previous work [19] and that we use in this work as a starting algorithm for the design of our hybrid algorithm.

##### **Step 1: Construction of the path by each ant (Solution)**

Initially (at time  $t = 0$ ), the algorithm positions  $m$  ants on  $n$  city and the intensity of the trace for all pairs of city  $(ij)$  is set to a small positive value  $T_0$  in the pheromone matrix. A taboo list is maintained to ensure that a city cannot be visited twice during the same round. Each ant  $k$  will

therefore have its own list of  $V_k$ -tabu city which will keep in memory the city already visited. During one iteration of the algorithm, several ants take turns visiting a sequence of city. A cycle ( $N_c$ ) is completed when the last of the  $m$  ants has completed its construction.

After the initialization phase, starting from this sequence of city already visited, within the different colonies, the ants move this time on the different nodes of the graph according to a probability and therefore the equation is that of formula (1). It allows colonies to favor the shortest paths during the different iterations. During a round, each ant  $k$  of a colony  $L$  records in its tabu list (memory) the list of city already visited. The probabilistic formula for the selection of a node by the ant  $k$  of colony  $L$  is defined by the expression of  $p_{ij}^{k,l}$  (2).

$$p_{ij}^{k,l}(t) = \begin{cases} \frac{[\eta_{ij}]^\beta [\tau_{ij}(t)]^\alpha}{\sum_{u \in v_{k_l}} [\eta_{iu}]^\beta [\tau_{iu}(t)]^\alpha} & \text{if } j \in v_{k_l} \\ 0 & \text{else} \end{cases} \quad (2)$$

Where  $v_{k_l}$  represents the set of nodes or city not visited by ants from colony  $L$ .

## Step 2: pheromone deposit

When an ant moves from city  $i$  to city  $j$ , it leaves a certain amount of pheromone (value) on the arc  $(ij)$ . A matrix which is the pheromone matrix records information about the use of the arc  $(ij)$ . At each step of the turn this matrix is updated so that the more this use has been important in the past, the greater the likelihood that these arcs will be used again in the future. The evolution of the update equation is as follows:

$$\tau_{ij}(t+1) = \rho\tau_{ij}(t) + \Delta\tau_{ij} \quad (3)$$

The evaporation equation of the pheromone matrix is described by the formula

$$\tau_{ij}(t+1) = (1 - \rho)\tau_{ij}(t) \quad (4)$$

The inverse of the distance between city  $\eta_{ij} = \frac{1}{C_{ij}}$  called visibility is static information used to guide the choice of ants to nearby city and avoid too many city. distant. The  $\alpha$  and  $\beta$  parameters are used to determine materiality of the intensity of the trace and of the visibility in the construction of a solution. This is based on a compromise between visibility ( $\eta_{ij}$ ) and the quantity of pheromone ( $\tau_{ij}$ ) present between  $i$  and  $j$  at cycle  $t$ . These parameters are the same as those used in OCF [2].

## 4.2 Procedure of Lin and Kernighan (LK)

### 4.2.1 The basic algorithm

The K-opt algorithm is based on the K- Optimality concept:

**Definition:** A visit is said to be  $k$  optimal (or simply  $k$  - Opt) if it is impossible to get a shorter visit by replacing  $k$  links with any other set of  $k$  links [17]. From this definition, it is obvious that every optimal  $k$ -visit is also  $K$  ' optimal for  $1 < k' < k$ .

It's also easy to see that a tour that contains  $n$  city is optimal if and only if it is  $n$  - optimal. Unfortunately, the number of operations to test all  $k$ -exchanges is increasing rapidly. In a naive implementation, testing a  $k$ -link exchange has a time complexity of  $O(n^k)$ . Accordingly, the

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**Algorithm IV.1** MACO Algorithm.

---

Inputs m: number of ants per colony, L: Number of colonies, n: number of city

```
1 Nc:=0
2 Dij:=0      // Global matrix
3 dij:=t0     // local matrix of pheromones
4 Initialize t0 // with each ant's home city
5 Place each colony randomly in a starting point (city)
6 For K from 1 to m
7   Construction of a tour by each ant at random
8   Gradual deposition of pheromones in the dij matrix of each colony
9   Evaluation and selection of the best colony
10  Initialize the Dij matrix with the best colony
11  While(Nc<Nmax)
12    For i from 1 to n
13      For j from 1 to L
14        For K from 1 to m
15          Select the city Vi to be added to the current tour with formula(1)
16          Evaluate the solution of the ant K on route(i,j)
17          Update globally the trace according to city pair(i,j) if it's better than
18          the previous ants according to formula 5 and 6 (evaporation)
19          Insert as you go (i,j) in the taboo list so as to construct gradually solution K
20  S:= Best solution (Each colony provides a partial solution)
```

---

values  $k = 2$  and  $k = 3$  are commonly used. It is a disadvantage that  $k$  must be specified in advance because it is difficult to know which  $k$  to use to achieve the best compromise between current time and quality of solution. Lin and Kernighan corrected this drawback by introducing a powerful  $k$  variable algorithm which changes the values of  $k$  during its execution, by deciding on each iteration what the value of  $k$  should be. At each step of the iteration the algorithm examines, for ascending values of  $k$ , whether and exchange of  $k$  links can make it possible to obtain a shorter visit. At each step the algorithm considers an increasing set of potential exchanges (starting with  $k = 2$ ). If the crawl is successful in finding a new, shorter visit, then the actual visit is replaced with this new visit. With a feasible visit, the algorithm performs exchanges that repeatedly reduce the length of the current visit, until a visit is reached and no other exchange can improve it [14].

### 4.3 Lin - Kernighan - Helsgaun algorithm

It is the modified and extended version of LK algorithm. Indeed A central rule in the original algorithm is the heuristic rule which restricts the inclusion of links in the visit to the five nearest neighbors to a given city. This rule directs the search to a shorter visit and reduces the search effort substantially. However, there is a certain risk of not being able to find the optimal cost. Helsgaun amends this rule.

## V STUDY OF HYBRID METHODS

Hybridization is a technique which consists in combining the characteristics of two different methods to derive the advantages of the two methods [4] [10]. In the literature, the hybridization of metaheuristics can be divided into two main parts: hybridization of metaheuristics with metaheuristics and hybridization of metaheuristics with exact methods [15] [10].



## **5.1 Hierarchical classification of metaheuristics**

This classification is characterized by the level and mode of hybridization. The level of hybridization can be low (Low-Level) or high (High-Level) [2]. In the low level, a metaheuristic replaces an operator of another method which encompasses it. On the other hand, in high level hybridization, each metaheuristic keeps its property during hybridization [15, 2]. Each level of hybridization generates two modes of cooperation namely, relay mode and co-evolutionary mode. In relay mode, the methods are executed sequentially, that is to say the result of the first method is the start of the following method [15]. When the different methods work in parallel to explore the search space, we speak of co-evolutionary mode. The combination of modes and levels gives four classes of hybridization which are: low-level relay hybridization, low-level co-evolutionary hybridization, high-level relay hybridization and high-level co-evolutionary hybridization [2].

## **5.2 Low-level relay hybridization**

It encompasses single solution-based metaheuristics in which another method is incorporated to form a new algorithm[2].example: the descent method can be inserted in a simulated annealing algorithm.

## **5.3 Low-level co-evolutionary hybridization**

It consists in incorporating one or more metaheuristics based on a single solution in a metaheuristic with a population of solutions [4] [15]. The advantage of this type of hybridization is to compensate for the exploitation power of a local search and that of the exploration of a global search[2] .

## **5.4 High-level relay hybridization**

It takes place when metaheuristics are used sequentially i.e. the final solution (s) of the first metaheuristic is the initial solution (s) of the following metaheuristic [15]. In this procedure, all the methods keep their integrity. Example introduced Taboo research at the end of a genetic algorithm to improve the solutions obtained [15].

## **5.5 High level coevolutionary hybridization**

In this case, the metaheuristics used work in parallel by exchanging information between them in order to find the optimal solution of the problem posed [2] [15].

## **5.6 Flat classification of hybrid metaheuristics**

This is another classification of hybrid methods characterized by the type of hybridized methods, their field of application and the nature of their functions [15]. Depending on the type of hybridization, there are homogeneous hybrid methods and heterogeneous hybrid methods. In the homogeneous hybrid methods, the algorithms used are based on the same metaheuristics and in the heterogeneous hybrid methods the metaheuristics used are different[15]. The field of application of hybridized metaheuristics makes it possible to distinguish two main classes of hybridization, global hybridization and partial hybridization. Global hybridization takes place when all the hybridized methods are applied to the whole search space[15].

Partial hybridization, on the other hand, breaks down a problem into sub-problems where each has its own search space.



## VI PROPOSED METHOD: HYBRIDIZATION OF MULTI ANTS COLONY ALGORITHM AND LKH; (MACO-LKH)

### 6.1 Hybridization Aspects to exploit on replace

In this part, we discuss the structure of our algorithm. It is known that the MACO or LKH algorithms are very efficient for difficult problems and TSP in particular [4], but the disadvantage of MACO is the consumption of time [19]. On the other hand, LKH must always start its execution with a good starting solution.

In the MACO algorithm described previously, for each ant, the path of a city  $i$  to a city  $j$  depends on:

- The list of city already visited, which gives the possible choices at each transition, when the ant  $k$  is over the city ;
- The inverse of the distance between city called visibility  $\eta_{ij}$
- The amount of pheromone deposited on the ridge connecting two city, called intensity of the track. This quantity defines the attractiveness of a track, and it is modified after the passage of an ant.

The multi-colony MACO algorithm as described above is made up of three main parts:

- Initialization;
- Construction of paths (Generations of paths) according to the probabilistic formula (2);
- Pheromone deposition and evaporation: Evaluate the solution of each ant in each colony.

To hybridize the two heuristics **MACO** and **LKH**, the visibility function in the MACO heuristic is replaced by the mechanism of the LKH heuristic to be more efficient in restricting the nearest neighbors which results in a gain in execution time. Then the pheromone track update process is replaced by the  $\lambda$ -opt mechanisms used by LKH heuristics in order to accelerate the convergence towards the optimal solution and in another register optimize the optimal solution.

### 6.2 Hybridization of LKH and MACO: (MACO-LKH)

We opted for an insertion hybridization. That is, we hybridize the MACO and LKH heuristics by inserting LKH into MACO. In clear the main characteristics are:

- The visibility equation between city  $i$  and  $j$  disappears. This information is replaced by the mechanisms of the LKH process in order to direct the ant more quickly nearby city and thus avoid too long solutions
- After each round of any ant, there is no longer any direct deposition of pheromone at the end, but rather triggering of the LKH process on this round until the best possible round of ant  $k$  and then pheromone deposition according to MACO equation(3).

In fact, the algorithm is defined in two steps:

#### Step 1: MACO algorithm

- Generation of solutions by ants from different colonies;
- Pheromone deposit.

#### Step 2: Heuristic LKH

- Optimization of solutions;
- Injection of the best ant into the communication matrix.

The algorithm is the following:

---

**Algorithm VI.1 MACO-LKH ALGORITHM**

---

Inputs  $m$ : number of ants per colony,  $L$ : Number of colonies,  $n$ : number of city,  $T$ : the displacement step,

```
1  Nc:=0
2  Tc:=0
3  Dij:=0
4  dij :=0
5  local pheromone matrix
6  Initialize t$ _{0} $ // with each ant's home city
7  Place each colony randomly in a starting point (city)
8  initialization Pheromone = t0;
9  FOR j from 1 to L
10 FOR K from 1 to m}
11 Randomly build a tour by each ant
12 Application of the LKH method after each T transition
13 Pheromone deposit by the best ant
14 Solutions construction
15         While(Nc<Nmax)
16             For i from 1 to n
17                 For j from 1 to L
18                     K:= 1
19                 While( K < m)
20 For each ant K of colony L carry out the transition T by selecting for each transition the citie j to be
21 K = k + 1
22 Application of the LKH Process on each partial solutions obtained
23 Insert progressively (i, j) in tabou list so as to build
24 gradually the solution of K
25 Perform the global update of the trace according to the best ant of colony L according to equations 3
26 and 4 after performing the dynamic k-opt operations of LKH on the partial solution
27 Nc := Nc + 1
28 S := 1 the best solution
```

---

## VII EXPERIMENTATIONS

In this section, we present the numerical results obtained by the proposed algorithm MACO-LKH.

### 7.1 Implementation environment

The MACO-LKH approach has been implemented in C language on instances of the TSPLIB online instance library for the TSP on a server with the following characteristics: *4 GHZ processor, 08 GHZ RAM, 500 GO DD* after 100 executions. In the absence of the real data of the problem, we were satisfied with the data instances from the TSPLIB library to evaluate the efficiency of algorithm.

## 7.2 Results - Discussions and Analysis

By making a comparative study between the heuristic MACO-LKH with the heuristics ACS-LKH [4], AG [10], MACO [19], AG -LKH [4] which make it from the best heuristics for the TSP of the literature, and under identical test conditions, we obtain the results of table 1,2 and 3 below.

Problems	Algorithms	Mean success /100	Minimum cost	Maximum cost	Average cost	Average time
Pr136 (96772)	MACO	100	96772	96772,2	96772	98,6
	MACO-LKH	100	96772	96772,2	96772	98
	GA-LKH	100	96772,9	96773,5	96772,75	128,3
Pr439 (107217)	MACO	100	107217	107217	107217	97,8
	MACO-LKH	100	107217	107217	107217	93,02
	GA-LKH	80	107297	107301	107298,3	633,2
Rd100 (7910)	MACO	100	7910	7910	79100,1	1,33
	MACO-LKH	100	7910	7910,2	7910,1	0,9
	GA-LKH	100	7910	7910,9	7910,52	11,2
ts225 (126643)	MACO	100	126643	126643	126643	12,5
	MACO-LKH	100	126643	126643	126643	12,03
	GA-LKH	85	126643,01	126644,6	126643,98	64,3
Tsp225 (3916)	MACO	100	3916	3916	3916	3,7
	MACO-LKH	100	3916	3916	3916	3,17
	GA-LKH	88	3916	3916	3916	13,2
U574 (36905)	MACO	100	36905	36905	36905	32,905
	MACO-LKH	100	36905	36905	36905	30,05
	GA-LKH	100	36905,6	36909	36905,9	90,5
U724 (41910)	MACO	100	41910	41910,8	41910,01	43,1
	MACO-LKH	100	41910	41910,6	41910,01	40,5
	GA-LKH	77	41915,5	41911,5	41919,22	191,2
Si535 (48450)	MACO	100	48450	48450	48450	46,5
	MACO-LKH	100	48450	48450	48450	45,3
	GA-LKH	98	48450,01	48452,4	48450,5	84,4
	MACO-LKH	100	294358	294358,9	294358,2	88,9
	GA-LKH	98	294359,6	294368	294366,6	346,5
gr229 (134602)	MACO	100	134602	134602	134602	46,02
	MACO-LKH	100	134602	134602	134602	37,6
	GA-LKH	100	134602,6	134603	134602,69	124,2
Gr431 (1711414)	MACO	100	1711414	1711424,3	1711414,25	141,4
	MACO-LKH	100	1711414	1711414	1711414	114
	GA-LKH	100	1711419,2	1711420	1711419	711

Table 1: Comparison of MACO-LKH calculation cost and calculation time with GA-LKH and MACO Algorithms on (small instances) from TSPLib

Problems	Algorithms	Mean success /100	Minimum cost	Maximum cost	Average cost	Average time
Pcb1173 (56892)	MACO	98	56892,55	56896	56892,2	141,2
	MACO-LKH	97	56892,6	56896	56892,03	99,98
	GA-LKH	100	57892,6	56899,6	56896,6	930,5
Pcb3038 (137694)	MACO	62	137694,24	137696	137694,9	688,5
	MACO-LKH	62	137694,04	137695,84	137694,24	658,5
	GA-LKH	20	147694	147694,9	147694,32	1235,55
Pla7397 (23260728)	MACO	58	23260729	23260736	23260729,3	919,2
	MACO-LKH	66	336557	336557	336557	768,63
	GA-LKH	/	Undefined	Undefined	Undefined	Undefined
Pla85900 (142382641)	MACO	89	142382643	142382661	142382644	788,3
	MACO-LKH	89	142382641	142382641	142382644	778,5
	GA-LKH	/	Undefined	Undefined	Undefined	Undefined
rl1323 (270199)	MACO	88	270199	270199	270199	10,9
	MACO-LKH	86	270199	270199	270199	10,99
	GA-LKH	60	270199,9	270201,9	270199,97	98,25
U1060 (224094)	MACO	100	224092	224094,86	224094,01	124,6
	MACO-LKH	100	224094	224094	224094	114,01
	GA-LKH	60	224098	224168	224160,2	409
U1432 (152970)	MACO	82	152970,2	152975	152970,8	297
	MACO-LKH	82	152970,2	152975	152970,8	270,5
	GA-LKH	/	Undefined	Undefined	Undefined	Undefined
U2319 (234256)	MACO	66	234256,01	234259	234256,8	259,3
	MACO-LKH	66	234256,11	234259	234256,8	246,9
	GA-LKH	54	234366	234380,34	234366,33	663,7
Vm1084 (239297)	MACO	80	239297	239297,6	239297,01	92,2
	MACO-LKH	80	239297	239297,9	239297,1	90,5
	GA-LKH	76	239298	239299	239298,55	216,8
Si1032 (92650)	MACO	100	92650	92651,5	92650,02	65,2
	MACO-LKH	100	92650	92651,5	92650,02	50,02
	GA-LKH	99	92651,2	92651,9	92651,06	92

Table 2: Comparison of MACO-LKH calculation cost and calculation time with GA-LKH and MACO Algorithms on ( medium instances) from TSPlib

The comparison results table 1,2, 3 shows that the MACO and MACO-LKH multi-colony approaches are better than the other approaches. However, the MACO-LKH approach is better than all the other methods studied in terms of execution time since the optimal solution is quickly reached and the algorithm stops without necessarily reaching the maximum number of cycles. Moreover with MACO -LKH the optimal cost is reached at least for 99% of the cases.

For all tested instances, the average execution time of the MACO-LKH hybrid algorithm is above the other heuristics (GA-LKH, ACO-LKH, MACO). This thanks to a few simple improvements to MACO during hybridization (for example, limiting the search to a certain number of nearest neighbors thanks to the mechanism of LKH). The deviations of the cost between

Problems	Algorithms	Mean success /100	Minimum cost	Maximum cost	Average cost	Average time
Lin 318 (42029)	GA-LK	40	42090.4	42090.4	42090.4	1.6
	ACS-LKH	100	42029	42029	42029	0.8
	GA-LKH	80	42029	42029	42029	1.6
	MACO	100	42029	42029	42029	0.341
	MACO-LKH	<b>100</b>	<b>42029</b>	<b>42029</b>	<b>42029</b>	<b>0.340</b>
att532 (27686)	GA-LK	100	27686	27686	27686	15.4
	ACS-LKH	22	27678.7	27686.7	27686.7	15.6
	GA-LKH	100	27686	27686.0	27686.0	13.74
	MACO	100	276787.0	276786.0	276786.0	11.74
	MACO-LKH	<b>100</b>	<b>276787.7</b>	<b>27686.0</b>	<b>27686.0</b>	<b>11.70</b>
rat783 : (8806)	GA-LK	100	8806	8806	8806	2.00
	ACS-LKH	100	8806	8806.01	8806.01	2.88
	GA-LKH	100	8806	8806	8806	1.209
	MACO	100	8806	8806	8806	2.86
	MACO-LKH	<b>100</b>	<b>8806</b>	<b>8806</b>	<b>8806</b>	<b>1.12</b>
pr2392 : (378032)	GA-LK	88	378034.8	378034.8	378034.8	1114.1
	ACS-LKH	100	378034.0	378034.25	378034.25	114.1
	GA-LKH	100	378034.0	378034.8	378034.8	119.524
	MACO	100	378032.2	378032.2	378032.02	0.972
	MACO-LKH	<b>100</b>	<b>378032.00</b>	<b>378032.00</b>	<b>378032.00</b>	<b>0.944</b>
Usa13509 (19982859)	GA-LK	/	Undefined	Undefined	Undefined	Undefined
	ACS-LKH	2	1998470	1998470	1998470	2113.03
	GA-LKH	/	Undefined	Undefined	Undefined	Undefined
	MACO	22	19984330	19982859.02	19982859.02	2110.39
	MACO-LKH	<b>22</b>	<b>19982843.2</b>	<b>19982859.02</b>	<b>19982859.02</b>	<b>2110.11</b>
Pla33810 (66048945)	GA-LK	/	Undefined	Undefined	Undefined	Undefined
	ACS-LKH	/	Undefined	Undefined	Undefined	Undefined
	GA-LKH	/	Undefined	Undefined	Undefined	Undefined
	MACO	/	Undefined	Undefined	Undefined	Undefined
	MACO-LKH	/	Undefined	Undefined	Undefined	Undefined
vm1748 (336556)	GA-LK	80	336556.2	336559.2	336559.2	119.2
	ACS-LKH	88	336557	336557	336557	71.86
	GA-LKH	80	336556.02	336557.2	336556.2	111.07
	MACO	88	336557.0	336557.86	336557.0	0.972
	MACO-LKH	<b>88</b>	<b>336557.0</b>	<b>336557.9</b>	<b>336557.0</b>	<b>0.942</b>
d15112 (1573084)	GA-LK	<b>77</b>	2579	2587.2	2580.9	12.8
	ACS-LKH	<b>89</b>	2579	2582	2580.00	12.0
	MACO	36	1573090,2	15730904	1573092.22	159,5
	GA-LKH	<b>86</b>	2579	2582	2579,8	12,68
	MACO-LKH	<b>99</b>	2579	2579,65	2579,02	1,04
d18512 (645238)	GA-LK	/	Undefined	Undefined	Undefined	Undefined
	ACS-LKH	<b>89</b>	645298	645299.5	645298.02	5829
	MACO	34	645240	645243	645240,7	2669,5
	GA-LKH	/	Undefined	Undefined	Undefined	Undefined
	MACO-LKH	<b>34</b>	<b>645243</b>	<b>645240</b>	<b>645240,7</b>	<b>2659,2</b>

Table 3: Comparison of MACO-LKH calculation cost and calculation time with GA-LKH, ACS-LKH, GA-LKH and MACO Algorithms on (Big instances) from TSPLib

MACO-LKH and the algorithms (GA-LK, ACS-LKH, GA-LKH) can go up to 15% with regard to the quality of the cost and approximately 30% with regard to the execution time.

The figure 1 present a comparison of the performance of our approach MACO-LKH compared to the improved AG -LKH, ACS -LKH, and MACO approaches in terms of runtime.

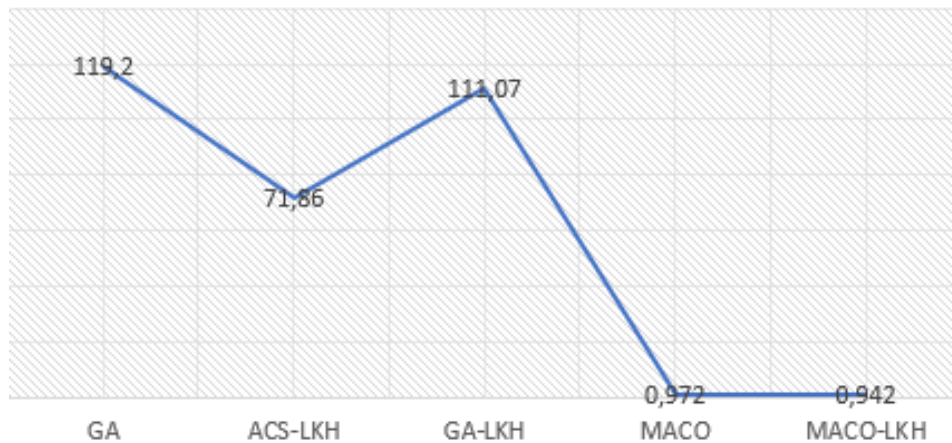


Figure 1: Comparison MACO, AG-LKH, ACS-LKH,MACO-LKH in terms of running times

## VIII CONCLUSION

In this work, we have proposed a novel hybrid multi-colony ant-LKH heuristic (MACO-LKH). It has been applied to solve the TSP. Compared to other heuristics, MACO-LKH is able to provide better cost with shorter execution times than the ACS-LKH, MACO, GA-LKH heuristics. The proposed MACO-LKH algorithm shows improvements both in terms of cost and execution time. The tests and comparisons carried out prove that MACO-LKH is competitive with other best heuristics for the TSP. A limit related to our approach is the use of many more resources (memory and processor). We also find that although hybrid heuristics and metaheuristics provide good quality costs in reasonable time to the TSP, they are still however computationally expensive for some problem instances like (pla33810). Also for most large instances the optimal cost is only guaranteed to be around 60 – 65%.

In perspective, we are thinking of using our algorithm on real data from transport logistics issues and real e-commerce from African companies, we are also counting on parallelizing the MACO heuristic and evaluating its effectiveness on big data linked to transport.



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