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Hierarchy in the phase space and dark matter astronomy

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We develop a theoretical framework for describing the hierarchical structure of the phase space of cold dark matter haloes, due to gravitationally bound substructures. Because it includes the full hierarchy of the cold dark matter initial conditions and is hence complementary to the halo model, the stable clustering hypothesis is applied for the first time here to the small-scale phase-space structure. As an application, we show that the particle dark matter annihilation signal could be up to 2 orders of magnitude larger than that of the smooth halo within the Galactic virial radius. The local boost is inversely proportional to the smooth halo density, and thus is $\mathcal{O}(1)$ within the solar radius, which could translate into interesting signatures for dark matter direct detection experiments: The temporal correlation of dark matter detection can change by a factor of 2 in the span of 10 years, while there will be significant correlations in the velocity space of dark matter particles. This can introduce $\mathcal{O}(1)$ uncertainty in the direction of local dark matter wind, which was believed to be a benchmark of directional dark matter searches or the annual modulation signal.

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Among the favorite dark matter candidates are the weakly interacting massive particles (WIMP), yet to be detected in particle accelerators. Different experiments look for a signature of WIMPs directly as they pass through Earth and recoil off the atomic nuclei in laboratory detectors, or indirectly through the by-products of their self-annihilation into standard model particles (such as photons, electron/positron pairs, or neutrinos). The cross section for the (*s*-wave) self-annihilation is also fixed by the relic abundance: $\langle\sigma_{\text{ann}}v\rangle\sim 10^{-31}\text{--}10^{-26}\text{ cm}^3/\text{s}$. Therefore, one would expect an astrophysical luminosity of

$$\frac{dL_i}{dE_i} = \frac{\langle\sigma_{\text{ann}}v\rangle E_i}{2m_\chi^2} \frac{dB_i}{dE_i} \Phi; \quad \Phi \equiv \int \rho^2(\mathbf{x}) d^3\mathbf{x}, \quad (1)$$

where m_χ is the WIMP mass, E_i is the energy of the by-products, dB_i/dE_i is the differential branching ratio into i ($=\gamma, e^+e^-,$ or $\nu\bar{\nu}$) particles, and Φ is fixed by the spatial density distribution of dark matter within the emitting region.

All the factors in (1), except for Φ , are fixed by the particle physics model (e.g. [1]). Unfortunately, the gravitational potential of dark matter haloes, which is constrained by astronomical observations, says little about the contribution of small-scale structure to Φ . The cold dark matter (CDM) primordial power spectrum predicts a large range of mass scales, from $10^{12}M_\odot\text{--}10^{14}M_\odot$ down to $10^{-12}M_\odot\text{--}10^{-4}M_\odot$ [2], far below the resolution limit of the present-day simulations at $z=0$, which is at best 10^4M_\odot (but see [3], which claims a much larger minimum mass for

the CDM hierarchy). In fact, simulations already see a nearly constant contribution to Φ per decade in substructure mass [4,5], suggesting a significant contribution from unresolved structures.

Extrapolation of the simulated properties of the subhaloes to below the resolution limit, as well as assumptions about the spatial/mass dependence of the boost due to substructure (e.g. [6–8]) could lead to significant over/underestimations (especially for a non-scale-invariant linear power spectrum such as CDM). Similarly, the assumption of Maxwell-Boltzmann velocity distribution in direct detection experiments of dark matter particles could miss important phenomenological signatures of clustering in the phase space.

In a previous paper [9], we studied the phase-space properties of gravitationally unbound substructure, such as tidal streams and caustics (also, see [10,11]). In this paper, we use the *stable clustering hypothesis* in order to predict the clustering of gravitationally bound dark matter substructure in the phase space, and consider the implications for indirect and direct dark matter searches. The stable clustering hypothesis was first introduced by Davis and Peebles [12] as an analytic technique to study the galaxy correlation function in the deeply nonlinear regime, and was subsequently applied into fitting formulas for nonlinear correlation functions (e.g. [13–15]). The hypothesis assumes that the number of neighbors within a fixed physical separation becomes a constant (or pairwise velocity vanishes) on small scales, when the nonlinear structure formation is completed. However, it became clear that this cannot be a good approximation on the scale of virial radius of CDM haloes (~ 1 Mpc today) as halo mergers and the subsequent tidal disruption can dissolve

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the old structures into newly formed haloes (e.g. [16–18]). Nevertheless, the alternative analytic framework, known as the *halo model*, which has been extensively used in the literature over the past decade, misses the small-scale structure of the haloes (i.e. subhaloes, sub-subhaloes, etc.) that do naturally form in hierarchical structure formation, and are ubiquitous in high resolution N -body simulations (e.g. [5]). As we mentioned before, integrating this hierarchy into the halo model requires a host of assumptions about the properties of subⁿ-haloes (often far below the resolution limit of simulations), which are hard to justify independently.

In contrast, the merging and tidal activity should cease at scales much smaller than the virial radius of CDM haloes, suggesting that the stable clustering regime might be achieved on small enough scales. In other words, while subhaloes could lose a large fraction of their mass due to tidal heating/stripping, a small fraction could remain gravitationally bound (e.g. [19]). The reason is that in a hierarchical structure formation scenario, the mean/virial density of haloes drops as $1/\text{time}^2$ (or the cosmic mean density) as a result of major mergers, implying that the gravitationally bound remnant will eventually become resilient to tidal disturbances.

The stable clustering hypothesis can be trivially extended to the phase space, where similar to the real space, it would predict that the number of particles within the physical velocity $\Delta \mathbf{v}$ and physical distance $\Delta \mathbf{r}$ of a given particle does not change with time for small enough $\Delta \mathbf{v}$ and $\Delta \mathbf{r}$. In order to justify the stable clustering formalism in phase space, we start with the collisionless Boltzmann equation at the phase-space coordinates, $\mathbf{r} + \Delta \mathbf{r}$, $\mathbf{v} + \Delta \mathbf{v}$, i.e.

$$\begin{aligned} \frac{df}{dt}(\mathbf{r} + \Delta \mathbf{r}, \mathbf{v} + \Delta \mathbf{v}, t) &= \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{r}} \cdot (\mathbf{v} + \Delta \mathbf{v}) - \frac{\partial f}{\partial \mathbf{v}} \\ &\quad \cdot (\nabla \phi + \nabla \nabla \phi \cdot \Delta \mathbf{r}) \\ &= 0. \end{aligned} \quad (2)$$

The above equation in terms of the new function

$$\tilde{f}_i(\Delta \mathbf{r}, \Delta \mathbf{v}) \equiv f(\mathbf{r}_i + \Delta \mathbf{r}, \mathbf{v}_i + \Delta \mathbf{v}) \quad (3)$$

for particle i can be rewritten as

$$\frac{df}{dt} = \frac{\partial \tilde{f}_i}{\partial t} \Big|_{\Delta \mathbf{r}, \Delta \mathbf{v}} + \frac{\partial \tilde{f}_i}{\partial \Delta \mathbf{r}} \cdot \Delta \mathbf{v} - \frac{\partial \tilde{f}_i}{\partial \Delta \mathbf{v}} \cdot (\nabla \nabla \phi \cdot \Delta \mathbf{r}) = 0, \quad (4)$$

where

$$\frac{\partial \tilde{f}_i}{\partial t} \Big|_{\Delta \mathbf{r}, \Delta \mathbf{v}} = \frac{\partial \tilde{f}_i}{\partial t} + \mathbf{v} \cdot \frac{\partial \tilde{f}_i}{\partial \Delta \mathbf{r}} - \nabla \phi \cdot \frac{\partial \tilde{f}_i}{\partial \Delta \mathbf{v}}. \quad (5)$$

The stable clustering hypothesis assumes that the above expression when averaged over the particles vanishes for small Δr and Δv . If we assume that $\langle \tilde{f}_i \nabla \nabla \phi \rangle_p \approx$

$\langle \tilde{f}_i \rangle_p \langle \nabla \nabla \phi \rangle_p$, then a solution to (4), averaged over particles, is

$$\langle \tilde{f} \rangle_p \equiv \frac{1}{N} \sum_i \tilde{f}_i = F[\Delta \mathbf{v}^2 + \Delta x_j \Delta x_k \langle \partial_j \partial_k \phi \rangle_p], \quad (6)$$

which is the most general time-independent solution with an isotropic velocity distribution, where F is an arbitrary function, and N is the number of particles in the phase-space volume of interest. We later show that averaging over particles and volume averaging differ only by a constant. Next, we use the approximation that the potential is spherically symmetric, and hence the above solution, using the Poisson equation, can be rewritten as

$$\langle \tilde{f} \rangle_p = \mu \xi_s = F[(\Delta \mathbf{v})^2 + 100H(\xi_s)^2(\Delta \mathbf{r})^2], \quad (7)$$

where ξ_s is the phase-space density at the formation time of the substructure, and we have used the fact that the post-collapse halo density is roughly ~ 200 times the critical density at the formation time [20]. $\mu \sim 1\% - 10\%$ is the mean fraction of bound particles that can survive the tidal disruption period.

In order to find the function F , we use the spherical collapse results

$$\xi_s \sim \frac{10H(\xi_s)}{G^2 M(\xi_s)}, \quad (8)$$

and also that the radius and velocity dispersion of haloes are related by (e.g. [21]): $\sigma_{\text{vir}} \sim 10Hr_{\text{vir}}$, where H is the Hubble constant at the time of the halo's collapse. Hence (8) is roughly the phase-space density of haloes that collapse at Hubble constant $H(\xi_s)$ and mass $M(\xi_s)$. The phase-space volume of the collapsed halo, i.e. the volume of the constant- ξ_s ellipsoid in (7), is M/ξ_s , and using (8) we have

$$\left[\frac{\pi F^{-1}(\mu \xi_s)}{10H(\xi_s)} \right]^3 = \frac{[GM(\xi_s)]^2}{10H(\xi_s)}. \quad (9)$$

Furthermore, the mass scale that collapses at a given cosmological epoch is characterized by

$$\left[\frac{H(\xi_s)}{H_0} \right]^{-2/3} \sigma[M(\xi_s)] \sim \delta_c \simeq 1.7, \quad (10)$$

where δ_c is the linear density threshold for the spherical collapse, while $\sigma[M]$ is the rms top-hat linear overdensity at the mass scale M , and H_0 is the present-day Hubble constant. We remark that the effect of dark energy will be a constant factor that could be absorbed in the definition of μ , since most subhaloes have formed long before the era of dark energy dominance.

Using the above results, the phase-space correlation function takes the form

$$\begin{aligned}
\langle f(\mathbf{r}_1, \mathbf{v}_1) f(\mathbf{r}_2, \mathbf{v}_2) \rangle &\simeq \frac{1}{V_6} \int_{V_6} d^3 r d^3 v f(\mathbf{r}, \mathbf{v}) f(\mathbf{r} + \Delta \mathbf{r}, \mathbf{v} + \Delta \mathbf{v}) \\
&= \frac{1}{V_6} \sum_i f(\mathbf{r}_i + \Delta \mathbf{r}, \mathbf{v}_i + \Delta \mathbf{v}) \\
&= \frac{N}{V_6} \langle \tilde{f} \rangle_p \quad (11)
\end{aligned}$$

$$\simeq \langle f(\mathbf{r}_1, \mathbf{v}_1) \rangle \langle f(\mathbf{r}_2, \mathbf{v}_2) \rangle + \mu \langle f(\bar{\mathbf{r}}, \bar{\mathbf{v}}) \rangle \xi_s(\Delta \mathbf{r}, \Delta \mathbf{v}), \quad (12)$$

where we used the assumption of ergodicity to replace the ensemble average $\langle \rangle$ by the volume average, in a given volume of the phase space V_6 , while $(\bar{\mathbf{r}}, \bar{\mathbf{v}})$ are the mean values of $(\mathbf{r}_1, \mathbf{v}_1)$ and $(\mathbf{r}_2, \mathbf{v}_2)$. The second term in Eq. (12) dominates in the stable clustering regime, where $|\Delta \mathbf{v}| = |\mathbf{v}_1 - \mathbf{v}_2| \ll \Delta v_{\text{tid}}$ and $|\Delta \mathbf{r}| = |\mathbf{r}_1 - \mathbf{r}_2| \ll \Delta r_{\text{tid}}$, and Δv_{tid} and Δr_{tid} characterize the tidal truncation radius in the phase space. On the other hand, the first term dominates Eq. (12) for large separations in the phase space, where particles are not correlated. In other words, Eq. (12) is an interpolation between the stable clustering and the smooth halo regimes.

The annihilation signal from the gravitationally bound substructure is found by integrating (12) within the phase-space stable clustering hypothesis:

$$\begin{aligned}
\delta \Phi_{\text{sub.}} &\simeq \int d^3 \mathbf{x} \int d^3 \mathbf{v}_1 d^3 \mathbf{v}_2 \langle f(\mathbf{x}, \mathbf{v}_1) \rangle \mu \xi_s(0, \mathbf{v}_1 - \mathbf{v}_2) \\
&= \int d^3 \mathbf{x} \rho_{\text{halo}}(\mathbf{x}) \int d^3 \Delta \mathbf{v} \cdot \mu \xi_s(0, \Delta \mathbf{v}), \quad (13)
\end{aligned}$$

yielding the local boost factor of

$$\begin{aligned}
B_{\text{sub.}}(\mathbf{x}) &\equiv \frac{\langle \delta \rho_{\text{halo}}(\mathbf{x})^2 \rangle}{\langle \rho_{\text{halo}}(\mathbf{x})^2 \rangle} \simeq \frac{\mu}{\rho_{\text{halo}}(\mathbf{x})} \int d^3 \Delta \mathbf{v} \cdot \xi_s(0, \Delta \mathbf{v}) \\
&\simeq \frac{8\pi^{1/2} \mu}{9\delta_c^3} \left(\frac{\rho_{\text{halo}}(\mathbf{x})}{200\rho_{\text{crit},0}} \right)^{-1} \int_{M_{\text{min}}}^{M_{\text{max}}} M^{-2} d[M^2 \sigma^3(M)], \quad (14)
\end{aligned}$$

where we have used the stable clustering framework developed in (7)–(10) to substitute for ξ_s .

The asymptotic form of the CDM linear power spectrum on small scales is (e.g. [22]) $P(k) \propto k^{n_s-4} \ln^2(k/k_{\text{eq}})$, where n_s is the primordial adiabatic scalar index, and $k_{\text{eq}} \simeq 0.56\Omega_m h^2 \text{ Mpc}^{-1}$ is related to the comoving scale of the horizon at matter-radiation equality. Using this asymptotic form, we can find an analytic approximation for the boost factor:

$$\begin{aligned}
B_{\text{sub.}}(\mathbf{x}) &= \mu \left(\frac{\rho_{\text{halo}}(\mathbf{x})}{200\rho_{\text{crit},0}} \right)^{-1} \left(\frac{\Omega_m h}{0.27 \times 0.7} \right)^{(3/2)(n_{\text{eff},8}+3)} \left(\frac{\sigma_8}{0.8} \right)^3 \\
&\times \left\{ K \left[\ln^{1/2} \left(\frac{M_{\text{eq}}}{M_{\text{min}}} \right) \right] - K \left[\ln^{1/2} \left(\frac{M_{\text{eq}}}{M_{\text{max}}} \right) \right] \right\}, \quad (15)
\end{aligned}$$

where $M_{\text{eq}} = (2.9 \times 10^{14} M_\odot) (\Omega_m h^2)^{-2}$ is the mass associated with the horizon scale at matter-radiation equality,

$n_{\text{eff},8} \simeq -1.68$ is the logarithmic slope of the linear power spectrum at $\sim 8h^{-1} \text{ Mpc}$, and

$$K[y] \simeq \frac{9}{10^5} \left(\frac{4y^{11}}{11} - y^9 \right) \exp \left[\frac{1}{2} (y^2 - 16)(n_s - 1) \right]. \quad (16)$$

This analytic approximation (15) is within 40% of the exact integral in (14) for $10^{-15} M_\odot < M < 10^{10} M_\odot$ and $0.95 < n_s < 1.05$ (where we compare with the fitting form of [23] for the CDM power spectrum). For supersymmetric dark matter models, the minimum CDM halo mass can range from $10^{-12} M_\odot - 10^{-4} M_\odot$, yielding $K \simeq 10^5$ within a factor of 3. Increasing this minimum mass range by 5–7 orders of magnitude, as suggested by [3] based on an extended Press-Schechter analysis, can decrease K (and hence diminish the boost factor) by 50%–80%. While significant, this will not qualitatively change our conclusions below.

To calibrate the parameter μ , we can compare our results to numerical simulations. Matching the expected boost per mass decade to that of ‘‘Via Lactea’’ mock Milky Way simulation [4], which is around 0.1 for subhalo masses $10^7 M_\odot < M_{\text{sub.}} < 10^{10} M_\odot$, with their assumed cosmology, yields $\mu \simeq 0.026$, which is comparable to the fraction of simulated halo mass found in (resolved) gravitationally bound subhaloes (e.g. 5.3% in [4]).

Figure 1 compares the prediction for the annihilation profile (solid curves) with numerical simulations of [5] (dotted curves), which shows reasonable agreement. To

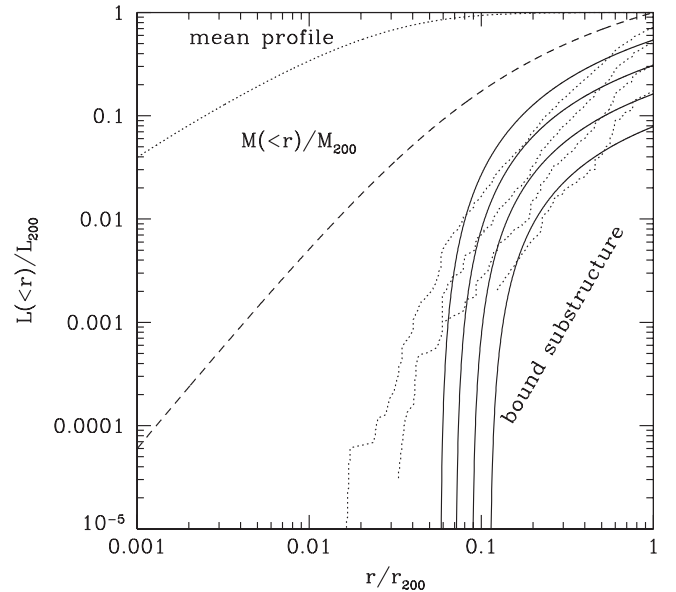


FIG. 1. The cumulative radial profile of DM annihilation: top dotted curve shows the contribution from the mean profile, while the lower dotted curves show the simulated contribution of subhaloes more massive than $10^{5,6,7,8} M_\odot$ [5]. The solid curves show our analytic prediction, with the same cutoffs on M_{min} , and a tidal cutoff at the high mass end. The dashed curve shows the cumulative mass profile for comparison.

do this, we have cut off the integral (14) at high masses by requiring that the formation density of bound subhaloes must be lower than the local halo density. The crudeness of this criterion is most likely responsible for the sharp drop in the signal at small radii, relative to the numerical results. While one can improve the agreement by introducing a smoother (but *ad hoc*) tidal cutoff, we note that (surviving) subsolar mass subhaloes, which dominate the boost for realistic WIMP models, are much less affected by the tidal effects of the host halo (which was the motivation for the stable clustering approximation). Therefore, we expect the stable clustering predictions to be more robust for realistic CDM haloes.

Figure 2 shows that the CDM hierarchy can not only significantly boost the total annihilation signal from the Milky Way (and dark matter haloes in general), they can also affect the local density variance at the solar radius ($B_{\text{sub}} \sim 2-6$), which can have interesting implications for direct dark matter detection searches. We can quantify the latter through the temporal correlation of dark matter detection signal $D(t)$, which traces the local density of the dark matter halo at the solar system. For simplicity, we will assume that the solar system is moving through the CDM hierarchy at $v \approx \sqrt{2.5} \times 250$ km/s (assuming a singular isothermal mean phase-space distribution). The two-point correlation function of $D(t)$ simply measures the two-

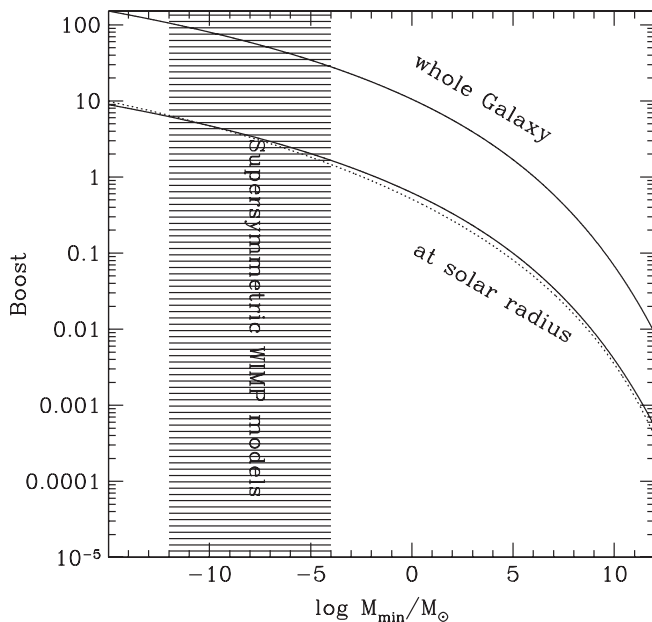


FIG. 2. Boost to the dark matter annihilation signal due to substructure with the formation mass larger than M_{min} . The lower solid curve is the local boost factor at solar radius (assuming $\rho = 10^5 \rho_{\text{crit}}$), while the upper solid curve shows the mean boost estimated for the whole Milky Way halo (out to 16 times NFW scale radius [4]), where we have assumed $n_s = 0.96$, $\sigma_8 = 0.82$, $\Omega_m = 0.28$, and $h = 0.7$. The dotted curve is our analytic approximation to the boost factor (15). The dashed region shows the theoretical expectation for the mass cutoff [2].

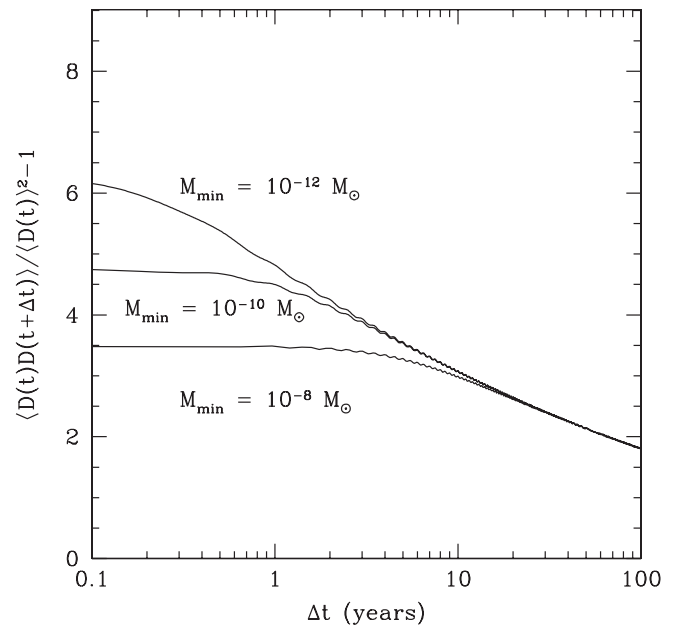


FIG. 3. Temporal correlation function of the dark matter detection signal within the solar system (assuming $\rho_{\text{CDM}} = 10^5 \rho_{\text{crit}}$) for three different cutoffs of the CDM hierarchy.

point correlation of CDM density, by projecting (12) into the real space, which is then modulated by the annual motion of Earth around the Sun. This can be done through a simple generalization of (14) to allow for finite separation in real space, and is shown in Fig. 3 for three different cutoffs of the CDM hierarchy, assuming $\rho_{\text{CDM}} = 10^5 \rho_{\text{crit}}$

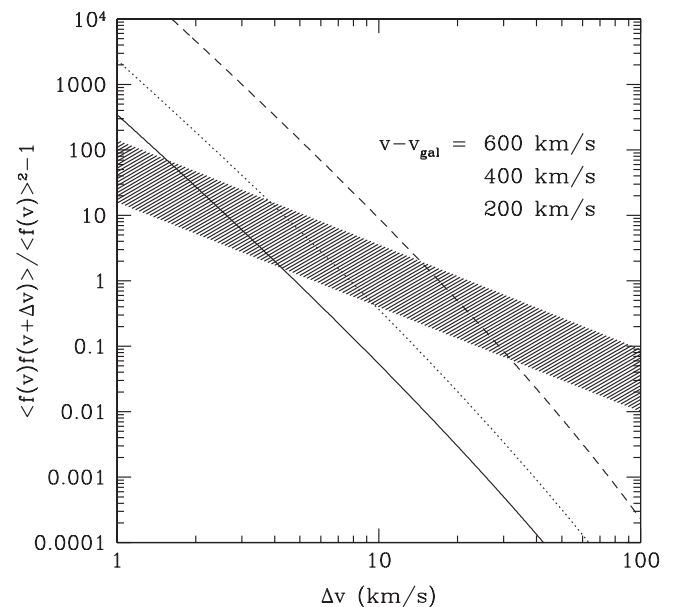


FIG. 4. DM phase-space correlation due to bound subhaloes at solar radius. The three lines show velocities: 200, 400, and 600 km/s in the Galactic frame. The shaded area shows the expected contribution from the unbound substructure [9].

within the solar system. We thus predict that, depending on the cutoff in the CDM hierarchy, the dark matter detection signal could gradually change by up to a factor of 2 within a 10-year period. Potential measurement of this temporal correlation could shed light on the cutoff of the CDM hierarchy, which is directly related to the mass of CDM particles.

To conclude this paper, we will turn to directional dark matter searches, which can directly probe the phase space of the CDM halo. These searches are now underway and can potentially provide the first telescopes for the rich field of *Dark Matter Astronomy* (e.g. [24]). We can use our formalism Eqs. (7)–(10) to predict the velocity space correlation function at the solar radius, which can be directly measured if CDM particles are detected in directional searches. This is shown in Fig. 4. For simplicity, we assumed a singular isothermal sphere for the mean phase-space density at the solar radius. The dimensionless correlation function is more prominent at higher velocities and shows a correlation length of 5–20 km/s in the velocity space. For comparison, the shaded area shows the expected level of correlation due to unbound substructure, which is

10%–30% at $\Delta v \sim 100$ km/s [25], and scales as $|\Delta v|^{-1.6}$ [9].

The fact that $B_{\text{sub}} \sim 2\text{--}6$ implies that the local CDM density may be dominated by small subhaloes with random velocities with respect to the Galaxy. This will introduce an $\mathcal{O}(1)$ uncertainty in the direction of local dark matter wind (or dipole), which was believed to be the benchmark of directional dark matter searches, or the annual modulation signal. Nevertheless, as we argued above, the richness of structure introduced by the CDM hierarchy leads to novel observables/smoking guns for dark matter searches to aim for.

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- [1] P. Gondolo *et al.*, *J. Cosmol. Astropart. Phys.* **07** (2004) 008.
 - [2] S. Profumo, K. Sigurdson, and M. Kamionkowski, *Phys. Rev. Lett.* **97**, 031301 (2006).
 - [3] R. E. Angulo and S. D. M. White, [arXiv:0906.1730](https://arxiv.org/abs/0906.1730).
 - [4] J. Diemand, M. Kuhlen, and P. Madau, *Astrophys. J.* **657**, 262 (2007).
 - [5] V. Springel, S. D. M. White, C. S. Frenk, J. F. Navarro, A. Jenkins, M. Vogelsberger, J. Wang, A. Ludlow, and A. Helmi, *Nature (London)* **456**, 73 (2008).
 - [6] L. E. Strigari, S. M. Koushiappas, J. S. Bullock, and M. Kaplinghat, *Phys. Rev. D* **75**, 083526 (2007).
 - [7] L. Pieri, G. Bertone, and E. Branchini, *Mon. Not. R. Astron. Soc.* **384**, 1627 (2008).
 - [8] M. Kuhlen, J. Diemand, and P. Madau, *Astrophys. J.* **686**, 262 (2008).
 - [9] N. Afshordi, R. Mohayaee, and E. Bertschinger, *Phys. Rev. D* **79**, 083526 (2009).
 - [10] M. Vogelsberger, S. D. M. White, R. Mohayaee, and V. Springel, [arXiv:0906.4341](https://arxiv.org/abs/0906.4341).
 - [11] M. Vogelsberger and S. D. M. White, [arXiv:1002.3162](https://arxiv.org/abs/1002.3162).
 - [12] M. Davis and P. J. E. Peebles, *Astrophys. J. Suppl. Ser.* **34**, 425 (1977).
 - [13] A. J. S. Hamilton, A. Matthews, P. Kumar, and E. Lu, *Astrophys. J.* **374**, L1 (1991).
 - [14] B. Jain, H. J. Mo, and S. D. M. White, *Mon. Not. R. Astron. Soc.* **276**, L25 (1995).
 - [15] J. A. Peacock and S. J. Dodds, *Mon. Not. R. Astron. Soc.* **280**, L19 (1996).
 - [16] C.-P. Ma and J. N. Fry, *Astrophys. J.* **538**, L107 (2000).
 - [17] R. E. Smith *et al.* (The Virgo Consortium), *Mon. Not. R. Astron. Soc.* **341**, 1311 (2003).
 - [18] L. M. Widrow, P. J. Elahi, R. J. Thacker, M. Richardson, and E. Scannapieco, [arXiv:0901.4576](https://arxiv.org/abs/0901.4576).
 - [19] T. Goerdt, O. Y. Gnedin, B. Moore, J. Diemand, and J. Stadel, *Mon. Not. R. Astron. Soc.* **375**, 191 (2007).
 - [20] J. E. Gunn and I. Gott, J. Richard, *Astrophys. J.* **176**, 1 (1972).
 - [21] N. Afshordi and R. Cen, *Astrophys. J.* **564**, 669 (2002).
 - [22] J. M. Bardeen, J. R. Bond, N. Kaiser, and A. S. Szalay, *Astrophys. J.* **304**, 15 (1986).
 - [23] D. J. Eisenstein and W. Hu, *Astrophys. J.* **496**, 605 (1998).
 - [24] D. Stiff, L. M. Widrow, and J. Frieman, *Phys. Rev. D* **64**, 083516 (2001).
 - [25] M. Vogelsberger, A. Helmi, V. Springel, S. D. M. White, J. Wang, C. S. Frenk, A. Jenkins, A. Ludlow, and J. F. Navarro, *Mon. Not. R. Astron. Soc.* **395**, 797 (2009).