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The W boson mass weighs in on the non-standard Higgs

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Abstract

We consider the implications of the CDF collaboration high-precision measurement of the W boson mass on models with a non-standard Higgs. We show that this requires an enhancement of more than 5% in the non-standard Higgs coupling to the gauge bosons. This is naturally accommodated in dynamical models such as the dilaton Higgs, the Technicolor and glueball Higgs. The needed composite scale between 2 and 3 TeV can also explain the muon g-2 anomaly, as well as possible violations of lepton flavour universality.

The CDF collaboration measured the W boson mass M_W using data relative to 8.8 inverse femtobarns (fb^{-1}) of integrated luminosity, collected in proton-antiproton collisions at an energy in the center-of-mass of 1.96 TeV, via the CDF II detector at the Fermilab Tevatron collider. With a sample of about 4-million W bosons, they obtained [1]

$$\begin{aligned} M_W|_{\text{CDF}} &= 80,433.5 \pm 6.4_{\text{stat}} \pm 6.9_{\text{syst}} \\ &= 80,433.5 \pm 9.4 \text{ MeV} . \end{aligned} \quad (1)$$

There are two striking results associated to this new measurement.

The first is that the central value is larger than expected, leading to a strong tension with the Standard Model (SM) expectation [2]:

$$M_W|_{\text{SM}} = 80,357 \pm 4_{\text{inputs}} \pm 4_{\text{theory}} \text{ MeV} . \quad (2)$$

The SM result derives from symmetries (mainly the custodial symmetry of the Higgs sector) and a set of high-precision measurements that include the Higgs

and Z boson masses, the top-quark mass, the electromagnetic coupling, the muon lifetime and collider asymmetries, which serve as input to the analytic computations. The estimate of the SM expected value of M_W is affected by uncertainties in the input data and by missing higher-order perturbative computations (theory). All in all, the tension between the CDF measurement and the SM can be quantified to 7 standard deviations (7σ) [1]. However, as pointed out in [3], including the state-of-the-art higher order corrections at N³LL+NNLO accuracy, one order higher than those included in the CDF analysis via the ResBos code [4], leads to a decrease of the central value by at most 10 MeV. Hence, these corrections, which are captured by the data-driven techniques used by CDF, may reduce the disagreement with the SM to 6σ .

The second striking result is that the accuracy of the new CDF measurement exceeds that of all previous measurements combined, coming from the Large Electron Positron collider (LEP) and previous Tevatron analyses [5]:

$$M_W|_{\text{LEP}} = 80,385 \pm 15 \text{ MeV} . \quad (3)$$

More recently, the LHCb collaboration [6] and ATLAS collaboration [7] at the Large Hadron Collider (LHC) presented new results with competitive ac-

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curacy:

$$\begin{aligned} M_W|_{\text{LHCb}} &= 80,354 \pm 32 \text{ MeV}, \\ M_W|_{\text{ATLAS}} &= 80,370 \pm 19 \text{ MeV}. \end{aligned} \quad (4)$$

A simple weighted average of these 4 measurements yields:

$$M_W|_{\text{AVG}} = 80,410 \pm 7 \text{ MeV}. \quad (5)$$

Comparing our naive average with the SM in Eq. 2, the discrepancy is reduced to around 5 standard deviations³.

Taking these measurements at face value, the deviation from the SM can be accounted for by new physics beyond the SM. In particular, the W mass is very sensitive to contributions to the vacuum polarisations of the electroweak bosons, encoded in the oblique parameters. This correction can be expressed in terms of the Altarelli-Barbieri epsilon parameters [9]:

$$\frac{M_W^2}{M_Z^2} = \frac{M_W^2}{M_Z^2} \Big|_{\text{Born}} * (1 + 1.34 \epsilon_1 - 0.86 \epsilon_3), \quad (6)$$

or via the oblique S and T parameters [10, 11]. The latter set only includes the contribution of new physics, and can therefore be defined only once the SM part is known. After the discovery of the Higgs boson [12, 13] and the precise measurement of its mass [14], the new physics contributions to the oblique parameters can be clearly defined:

$$\delta\epsilon_1 = \alpha T, \quad \delta\epsilon_3 = \alpha S, \quad (7)$$

where α is the electromagnetic coupling constant at the Z mass. The correction to the W mass stemming from new physics can therefore be expressed in terms of S and T via the following numerical approximate formula [15]:

$$\Delta M_W \approx 300 \text{ MeV} * (1.43 T - 0.86 S), \quad (8)$$

which we will use in our numerical analysis.

For models featuring a non-standard Higgs bosons, it is the coupling of the Higgs to gauge bosons that is mainly responsible for corrections to

³The naive average tends to underestimate the error, so that this value should be considered ‘optimistic’. A more sophisticated method can be found in [8]. Furthermore, correlations among systematic errors are included in a professional averaging, as done with LEP and previous Tevatron measurements in [5]

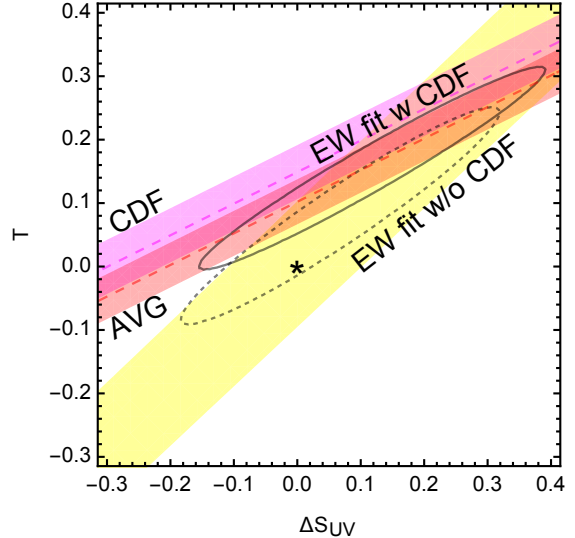


Figure 1: Comparison of the new CDF measurement of the W mass (magenta band) to the precision measurements, represented by the ellipses with (solid) and without (dashed) the CDF result. The yellow band indicates the bound stemming from asymmetries and direct s_W^2 determination. The red band represents our naive average that includes the CDF result. All bounds are shown at 95% confidence level (i.e. 2σ).

the oblique parameters as well as the value of the W mass. This is efficiently encoded in the parameter κ_V , which has the same value for W and Z assuming an underlying custodial model. This parameter can be expressed at tree level in terms of observables as follows:

$$\kappa_V^2 = \frac{\sigma_{\text{VBF}}}{\sigma_{\text{VBF}}^{\text{SM}}} \equiv \frac{\Gamma_{h \rightarrow WW/ZZ}}{\Gamma_{h \rightarrow WW/ZZ}^{\text{SM}}}, \quad (9)$$

hence the SM corresponds to $\kappa_V = 1$. The well-know relation between κ_V and the oblique parameters is:

$$\begin{aligned} T &= -\frac{3}{16\pi c_W^2} (1 - \kappa_V^2) \ln \frac{\Lambda^2}{m_h^2}, \\ S &= \frac{1}{12\pi} (1 - \kappa_V^2) \ln \frac{\Lambda^2}{m_h^2} + \Delta S_{\text{UV}}, \end{aligned} \quad (10)$$

where we have included an unknown contribution to the S parameter stemming from UV physics. Assuming the new UV physics to be custodial, it does not contribute to T . One can express S in terms of T as follows:

$$S = -\frac{4}{9} c_W^2 T + \Delta S_{\text{UV}}. \quad (11)$$

We now compare this correction to the deviation provided by the new CDF measurement and by our naive average:

$$\begin{aligned}\Delta M_W|_{\text{CDF}} &= M_W|_{\text{CDF}} - M_W|_{\text{SM}} \\ &= 76 \pm 11 \text{ MeV}, \\ \Delta M_W|_{\text{AVG}} &= M_W|_{\text{AVG}} - M_W|_{\text{SM}} \\ &= 52 \pm 9 \text{ MeV}.\end{aligned}\tag{12}$$

This is illustrated by the magenta (CDF) and red (AVG) bands at 2σ in Fig. 1, compared to the bound coming from the asymmetries and the direct determination of s_W^2 in the SM [16], represented by the yellow band. The dashed ellipse stems from the fit to the oblique corrections without including the CDF measurement ($S = 0.04 \pm 0.08$ and $T = 0.08 \pm 0.08$ with a correlation of 0.91 [16]) while the solid ellipse shows the updated fit from [17] ($S = 0.065 \pm 0.09$ and $T = 0.155 \pm 0.065$ with a correlation of 0.95). The take-home message is that the independent measurement of the CDF W mass (magenta band) and s_W^2 (yellow band) prefer a small and positive ΔS_{UV} for positive $T > 0.1$.

We can now express the same bounds in terms of the non-standard Higgs coupling κ_V and the scale of new physics Λ , as shown in Fig. 2. The UV contribution to S has been parameterised as

$$\Delta S_{UV} = n_d \frac{1}{6\pi},\tag{13}$$

which roughly counts the number of SU(2) weak doublets n_d present in the UV theory [11]. In Fig. 2 we also showed the limit on κ_V stemming from direct measurements, where the most constraining value is due to the ATLAS collaboration with a combination of the full Run-I data and various integrated luminosities (up to 139 fb^{-1}) for Run-II data [18, 19]:

$$\kappa_V = 1.039_{-0.030}^{+0.031},\tag{14}$$

which is also compatible with the CMS results [20]. This value comes from a 2-parameter fit of the Higgs data, allowing also a common modification of the fermion couplings, $\kappa_F = 0.93 \pm 0.05$ (with a correlation of 43%). We take this as a simple reference limit, keeping in mind that a model-specific fit would be required to extract a more reliable limit. For instance, different modifiers for W and Z could be generated via custodial symmetry breaking, as also hinted in the current LHC data [18, 19, 20]. The naive average points towards $\kappa_V \gtrsim 1.05$, while larger values of κ_V are needed for increasing ΔS_{UV}

(as shown in the right panel), so that a rough upper limit of $n_d \lesssim 4$ can be obtained. We also checked that the CDF measurement alone is incompatible as it requires a too large κ_V in order to generate a sufficient contribution to T , hence it is disfavoured by the direct Higgs coupling measurements at the LHC.

Interestingly, the anomaly in the muon $g - 2$ [21] can be related to a new physics contribution appearing at the scale Λ via the naive relation

$$\Delta a_\mu = \frac{m_\mu^2}{\Lambda^2}.\tag{15}$$

This relation is the right type of estimate for strongly coupled models [22], where no loop factors arise and Λ is directly associated to the scale where new resonances could appear, like a techni-rho. The vertical green band in Fig. 2 indicates the 3σ preferred region, defined by the SM world average for the SM theoretical prediction [23], leading to a 4σ discrepancy. The most crucial part of the SM prediction is the determination of hadronic contribution to the vacuum polarization (HVP), which is determined via e^+e^- data [24, 25, 26, 27, 28, 29, 30], while the recent BMW lattice result is not included [31]. In general, this anomaly is compatible with the new average for the W mass as long as Λ is between 1.5 to 3.6 TeV. If we consider a reduced anomaly, as driven by the lattice results, the scale is pushed to slightly higher values [22], without spoiling the compatibility with the W mass measurement. Interestingly, it has been pointed out in [32] that uncertainties in the SM input parameters cannot explain both the anomalous W mass and the muon $g - 2$. Following [22] one can also address the observed lepton flavour non-universal anomalies [33] via fundamental partial composite models [34], further reviewed in [35], with different couplings of muons with left and right-handed chirality.

The Higgs boson with non-standard interactions, which we analysed so far, arises in many new physics realisations. In this letter we are interested in dynamical models, where a Higgs-like boson emerges as a composite state of a more fundamental interaction. The low energy properties can be described in terms of an effective field theory, which is controllable thanks to some symmetries of the underlying theory. In the following, we will consider four classes of model: a pseudo-dilaton [36], a dynamical glueball [37] or composite Higgs [38], and Goldstone/Holographic composite Higgs

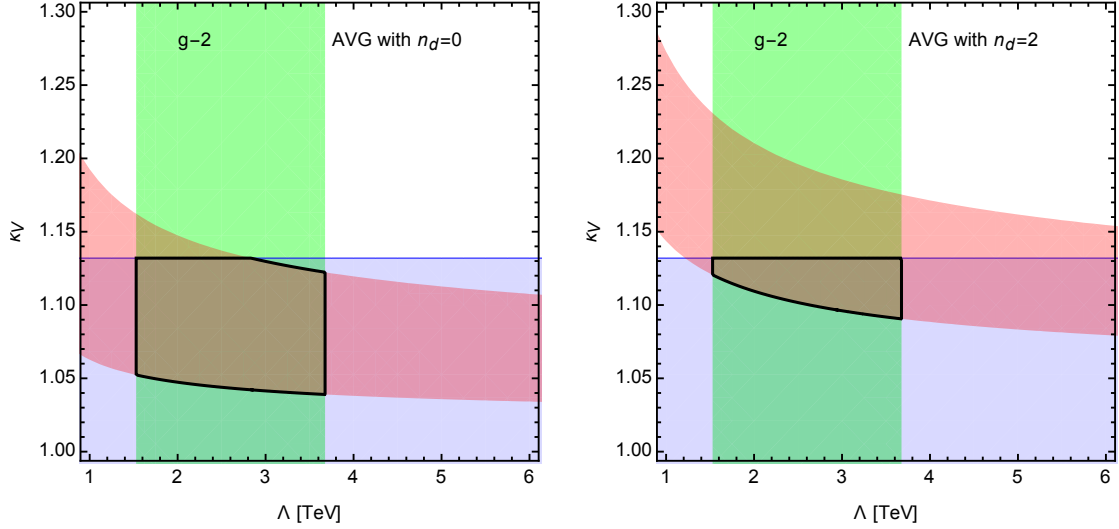


Figure 2: Allowed regions at 99% Confidence Level (3σ) from electroweak precision (red) and the direct κ_V measurement (blue). The green vertical band corresponds to the region preferred by the muon $g-2$ anomaly. The two panels correspond to vanishing UV contribution to S (left) and $n_d = 2$ (right). The regions compatible with all bounds are highlighted by the black contour.

[39, 40]. The non-standard interactions are captured by the following master Lagrangian [37]:

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\overline{\text{SM}}} \\
&+ \xi_V \left(1 + 2\kappa_V \frac{h}{v} + \kappa_{2V} \frac{h^2}{v^2} \right) \times \\
&\quad \frac{v^2}{4} \text{Tr} D_\mu U^\dagger D^\mu U \\
&+ \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{\tilde{m}_h^2}{2} h^2 \left(1 + V_{0,1} \lambda_{3h} \frac{h}{v} \right) \\
&- \frac{\tilde{y}_f v}{\sqrt{2}} \xi_f \left(1 + \kappa_f \frac{h}{v} \right) \left[\bar{f}_L U f_R + \text{h.c.} \right] \\
&+ \dots \tag{16}
\end{aligned}$$

where $U = \exp(i2\pi^a T^a/v)$ is the usual non-linear map of the Goldstones π^a produced by the breaking of the electroweak symmetry.

The difference among the four classes lays in different counting schemes for the couplings in the above Lagrangian. A useful tool consists in relying on a large- N counting, where N is the number of colours in the confining underlying theory [37]. In the glueball case, the Higgs can be envisioned as the lightest glueball state of a new Yang-Mills theory with a new N -dependent string tension proportional to Λ_H , scale not automatically related to the electroweak scale $v = 246$ GeV. A composite

Higgs [38] in Technicolor-like theories [41, 42], instead, is associated to a fermion-antifermion bound meson. The counting now depends on the fermion representation [43]. Here we express the counting with respect to a reference theory based on $\text{SU}(\overline{N})$ gauge with fermions in the fundamental. In the dilaton case, the scalar state is associated to a spontaneously broken conformal dynamics, and its couplings are associated to a single scale $F \equiv N\Lambda_H$. Finally, in the case of the Goldstone/Holographic Higgs [40], the counting is based on the misalignment [39] between the electroweak scale and the compositeness scale f , which can be expressed in terms of an angle [44], $v = fs_\theta$ with $s_\theta \ll 1$.

The relevant couplings in the four classes are summarised in Table 1, where the Goldstone/Holographic couplings correspond to the minimal model of Ref. [45]. Note that in the Goldstone/Holographic case, the couplings to gauge bosons are always reduced with respect to the SM values, as $\kappa_V = c_\theta \leq 1$. This is a universal property of Goldstone Higgs models [46]. Only couplings to fermions can be enhanced compared to the SM values, for specific choices of the model details [47]. In all the other cases, κ_V is naturally larger than unity when the new scale is somewhat smaller than v , or the coupling $r_\pi > 1$. As the new W mass measurement points towards a positive deviation of κ_V

	ξ_V	κ_V	κ_{2V}	λ_{3h}	ξ_f	κ_f
Glueball	1	$\frac{r_\pi v}{N\Lambda_H}$	$\frac{s_\pi v^2}{N^2\Lambda_H^2}$	$\frac{v}{N\Lambda_H}$	1	$\frac{r_f v}{N\Lambda_H}$
Technicolor-like	$\frac{N}{\bar{N}}$	$r_\pi \sqrt{\frac{\bar{N}}{N}}$	$s_\pi \frac{\bar{N}}{N}$	$\sqrt{\frac{\bar{N}}{N}}$	$\sqrt{\frac{\bar{N}}{N}}$	$r_f \sqrt{\frac{\bar{N}}{N}}$
pseudo-dilaton	1	$\frac{v}{N\Lambda_H}$	$\frac{v^2}{N^2\Lambda_H^2}$	$\frac{v}{N\Lambda_H}$	1	$\frac{v}{N\Lambda_H}$
Goldstone/Holo.	1	c_θ	$c_{2\theta}$	c_θ	1	c_θ

Table 1: Couplings in Eq. 16 in the four classes of models [37].

from unity above 5% this leads to a composite scale $\Lambda \sim 4\pi N\Lambda_H \approx 4\pi v$, which is in agreement with the $g-2$ preferred region. Note that the coupling to fermions is also expected to receive a similar enhancement, considering that different couplings to the various flavour may be generated in more detailed models. The dilaton case is most predictive, as other measurable couplings can be determined once the scale of new physics is fixed, for instance the Higgs trilinear coupling that will be measured at future colliders.

To summarise, in this letter we discussed the impact on non-standard Higgs models of the new W mass measurement from the CDF collaboration, which has not only the highest precision to date, but also a central value significantly higher than previous measurements and than the SM fit result. Our analysis includes composite models, but it is not limited to them. Firstly, we noted that the CDF measurement on its own, which has a 7σ discrepancy to the SM fit, is very hard to accommodate as it would require a large departure in the Higgs couplings to W and Z bosons, unless large custodial violations are present in the UV model. However, a naive average with the LEP, previous Tevatron, LHCb and ATLAS W mass measurements, while reducing the anomaly to 5σ , renders the overall result easier to accommodate.

The main implication stemming from the new average including the CDF result is that the coupling of the non-standard Higgs to W and Z bosons is required to be larger than the SM value by more than 5%. This is rather hard to accommodate in weakly coupled models due to perturbative unitarity sum rules [48], unless Higgses in $SU(2)$ representations larger than doublets are allowed to develop sizeable vacuum expectation values [49, 50], such as in

generalised Georgi-Machacek models [51, 52]. Similarly, in models of Goldstone/Holographic Higgs a reduction of this coupling is usually obtained, hence needing a sizeable source of custodial violation is present in the UV theory to give a positive contribution to T . However, this feature can be naturally achieved in composite models where the Higgs emerges as a resonance, as in Technicolor-like theories or pseudo-dilaton models. Furthermore, a similar enhancement is expected in the fermion couplings of the Higgs. The High-Luminosity LHC and future Higgs factories will be able to establish the non-standard nature of the Higgs couplings, as they are expected to reach a precision of about 2% [53] and $<1\%$ [54], respectively. Furthermore, the anomaly in the muon $g-2$ measurement is also compatible with the dynamical origin of the Higgs, if the new composite scale lies between 1.5 to 3.6 TeV, where new resonances like a spin-1 techni-rho are expected.

In conclusion, it is tantalising that the anomalies that are emerging in the precision measurement of the SM properties conspire towards a coherent picture, where the Higgs boson emerges from a dynamical composite sector at a few TeV scale. This scenario will be discovered or excluded by the end of the LHC program, and most certainly by the next high-precision colliders, whose construction is being discussed.

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