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Abstract

We implement a new framework to mitigate the errors-in-variables (EIV) problem in the estimation of asset pricing models. Considering an international data of portfolio stock returns from 1990 to 2021 widely used in empirical studies, we highlight the importance of the estimation method in time-series regressions. We compare the traditional Ordinary-Least Squares (OLS) method to an alternative estimator based on a Compact Genetic Algorithm (CGA) in the case of the CAPM, three-, and five-factor models. Based on intercepts α, betas β_M, adjusted R^2, and the Gibbons, Ross and Shanken (1989) test, we find that the CGA-based method outperforms overall the OLS for the three asset pricing models. In particular, we obtain less statistically significant intercepts, smoother R^2 across different portfolios and lower GRS test statistics. Specifically, in line with Roll's critique (1977) on the unobservability of the market portfolio, we reduce the attenuation bias in market risk premium estimates. Moreover, our results are robust to alternative methods such as Instrumental Variables estimated with Generalized-Method of Moments (GMM). Our findings have several empirical and managerial implications related to the estimation of asset pricing models as well as their interpretation as a popular tool in terms of corporate financial decision-making.

Keywords: Asset pricing, CAPM, Fama-French three- and five-factor models, Market Portfolio, Time-series regressions, Ordinary-Least Squares (OLS), Errors-in-variables (EIV), GMM with Instrumental Variables, Compact Genetic Algorithms (CGA).
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Résumé

Cette étude apporte une réponse au problème des erreurs dans les variables (errors in variables ou EIV) dans l'estimation des modèles d'évaluation des actifs. Nous mettons en lumière l'importance de la méthode d'estimation dans les régressions en séries chronologiques des rentabilités des actions. Nous comparons la méthode largement utilisée des moindres carrées ordinaires (ordinary least squares ou OLS) à un estimateur alternatif basé sur un algorithme génétique compact (compact genetic algorithms ou CGA) dans le cas du modèle d'équilibre des actifs financiers ou MEDAF et des modèles de trois et cinq facteurs de Fama et French (1993 et 2015). Sur la base d'un échantillon international couvrant cinq zones géographiques et un historique des rentabilités de trente et un ans (1990 – 2021), les résultats des régressions linéaires montrent que la méthode basée sur la CGA surpasse globalement la méthode OLS pour les trois modèles, les cinq zones géographiques et les cent portefeuilles considérés. Plus précisément, les coefficients alphas des régressions sont moins significatifs, les betas sont plus élevés, les coefficients ajustés R^2 sont lisses et les valeurs du test de Gibbons, Ross et Shanken (1989) sont plus faibles. Par ailleurs, nos résultats sont plus robustes que ceux de la méthode alternative des variables instrumentales estimées avec la méthode généralisée des moments (generalized method of moments ou GMM). Quarante-cinq ans après la publication de l'article de Roll (1977), nous apportons une réponse satisfaisante à la critique relative à la non-observabilité du portefeuille de marché. Les implications empiriques et managériales de notre étude sont nombreuses puisqu'elles concernent tous les sujets faisant appel aux modèles d'évaluation des actifs au niveau des entreprises, des investisseurs et des marchés financiers (évaluation, mesure de performance, gestion de portefeuille, coût des capitaux propres, études d'évènement, etc.).

Introduction

The traditional Sharpe-Lintner-Mossin factor modeling framework (Sharpe 1964; Lintner 1965; Mossin 1966) that has later been considerably tuned by several authors including, among others, Fama and French (1992; 1993; 1995) or Carhart (1997), constitutes the backbone of asset price modeling in finance. These models give the researcher a systematic analysis framework to break down the risk-return trade-off of a particular set of assets or portfolios by building testable relationships between the asset returns and a set of common risk factors. An extensive body of research discusses both the theoretical and empirical issues related to these models of the risk-expected return trade-off, see Cochrane (2005) or Ferson (2019).

In this paper, we contribute to the empirical literature by addressing the errors-in-variables (EIV) problem in the estimation of asset pricing models with a focus on the market risk factor that is still at the heart of most modeling efforts despite numerous theoretical and empirical advances made insofar. In line with the earlier remarks initially set forth by Fama and MacBeth (1973) and Roll (1977), the latter being subsequently came to be known as the "Roll's critique", the measurement error problem is *endemic* in any factor model setup that employs the "market risk" in the spirit of the Sharpe-Lintner-Mossin CAPM paradigm.[1] Defined as the incremental return on a hypothetical market portfolio over the yield on a risk-free asset, the market risk factor relies on the observability of a market portfolio, which Fama and French (1997) consider as the "fundamental input to achieve market clearing conditions under complete agreement among investors".

We thus focus on the *market portfolio* M^* as the central right-hand-side variable predicting the variation in asset returns and consider it subject to EIV by its very definition. The true market portfolio is an artefact, one that can never be truly observed, so is the true market risk premium. We then speculate, fairly reasonably, that any proxy used in the existing body of empirical work can at best be seen as a representation of the true variable but is likely to include some measurement error u, which we define as the difference between the true and observed values as $u = M - M^*$. The implications of ignoring the measurement error are not neutral. To start with the simplest case of the market model framework first, all fitted coefficients that ignore the error in the observed values of the proxy will be biased downwards under some mild assumptions; see Hausman (2001), Racicot

[1] See Huang and Litzenberger (1988, ch. 10) for a complete discussion of the conceptual and econometric issues involved in testing the CAPM.

(2015b) or Greene (2018), among others, for a thorough discussion of this attenuation effect. This implies that any output or indicator related to the market risk premium will be erroneous. That a market beta estimate is systematically less than its true value simply means that the market risk itself is also underestimated. Second, the downward bias in β also induces an upward bias of the opposite sign in the model intercept α as long as the mean of the market risk premium is positive, a condition one naturally expects to hold given the definition of the market portfolio. It then turns out that negative alphas happen to appear "less negative" then they truly are while those positive in reality tend to materialize lower than their true levels. It can be readily seen that the implications of such a behavior of the fitted intercepts can be quite pervasive from the perspective of financial management industry.

To implement our study, we employ a data-set widely used in the empirical asset pricing literature. Specifically, we use portfolio return data from Kenneth French's library broken down into five geographic regions: World, North America, Europe, Asia-Pacific and Japan. Time-series are in monthly frequencies and span the period from 1990 to 2021. We then fit time-series regressions by the traditional OLS that ignore the measurement error in the market portfolio first, and then by a new estimator based on a compact genetic algorithm devised to mitigate the EIV in estimates second. We run a total of 3,000 regressions assuming three baseline specifications, namely CAPM, Fama-French three-factor and Fama-French five-factor models. We compare the results obtained from these two methods by reporting the fitted intercepts, market betas, adjusted R^2 values and the Gibbons-Ross-Shanken (1989) (GRS) test statistics.

Our results shed light on several issues not only on empirical asset pricing but also corporate financial decision-making. First, we show that naive OLS estimations that ignore the EIV in market risk premium are unsatisfactory as a tool to run time-series regressions of stock returns. We observe substantial upward bias in fitted slope coefficients as predicted by the classical EIV model especially for single-factor CAPM regressions. On average, OLS without EIV correction underestimate the market beta by 25% as much as lower than the CGA-based estimations, implying substantial error in the market risk premium and other methods depending on the accurate estimation of the market risk factor loading. By alleviating the downward bias in fitted betas, we also show the extent to which model intercepts can be inflated due to the contamination effect. Second, in line with this latter point, we observe significant drop in GRS test statistics regardless of the portfolio type and geography, giving empirical support to the three model specifications under consideration. The reduction in the GRS statistics is particularly compelling for the CAPM and Fama-French three-factor regressions. Almost all $\mathcal{F}$-values whereby we jointly test the significance of model intercepts exhibit substantial draw-down, except only one test portfolio out of 1000 estimated regressions under these two specifications. We conjecture that our findings lead to several managerial implications in connection with portfolio management, asset pricing, and corporate financial as well as capital-budgeting decisions.

The paper is organized as follows: Section 1 reviews the main literature on the two fields of our interest: Asset pricing and EIV frameworks; Section 2 presents a brief overview of the econometric treatment of classical EIV model; Section 3 indicates the methodology used and the data; Section 4 provides the main results; Section 5 shows robustness results and finally, Section 6 concludes the paper and outlines the main issues for future research.

1 Literature review

1.1 Asset pricing framework

The positive and linear relation between the expected return and the systematic risk of a stock is probably the most tested hypothesis in the field of asset pricing. This relationship is the main state given by the Capital Asset Pricing Model (henceforth CAPM) (Sharpe 1964; Lintner 1965; Mossin 1966; Black 1972). Originally, the CAPM is consistent with the mean-variance optimization framework first proposed by Markowitz (1952) and extended by Tobin (1958). However, despite the popularity of the CAPM, theoretical and empirical works point to the failure of this framework in explaining the risk-return trade-off.

Among the earliest and well-known critics is the Roll's critique (Roll 1977) about the unobservability of the market portfolio. In empirical studies, we commonly talk about "anomalies". Amongst the rich collection of these anomalies highlighted in the empirical work on the CAPM, some are more popular than others. For example, Basu (1977) observes significant abnormal returns for portfolios sorted by the book-to-market ratio and controlled for market betas. Other empirical studies confirm this observation named "value effect" (Rosenberg, Reid, and Lanstein 1985; De Bondt and Thaler 1985; Chan, Hamao, and Lakonishok 1991). However, the main issue that remains is how to explain these anomalies and how to settle between the lack of market efficiency and the misspecification of the asset pricing model.

On the other hand, many other studies report that the beta weakly explains the returns of small stocks (Reinganum 1981; Breeden, M. R. Gibbons, and R. H. Litzenberger 1989; Fama and French 1992). This "size effect" is introduced the first time by Banz (1981). Some empirical studies show that the size effect tends to disappear after the publication of Banz's article. For example, Amihud (2002) do not detect an additional risk premium regarding the size on the U.S. market after 1980. Dimson and Marsh (1999) report a size effect only on the period earlier than 1983. The same result about the size and the book-to-market ratio in explaining the cross-sectional difference of average stock returns, is reported about the profitability ratio (measured by the gross profits to assets) by Novy-Marx (2013). Cochrane (2011) speaks of a "zoo" of explanatory factors. Recently, Hou, Xue and Zhang (2015) identify 437 anomalies.

On the basis of these empirical observations, Fama and French (1992; 1993) develop the three-factor model. They demonstrate empirically that adding size and value effects to the market beta explain better the time-series variation in the returns, in the US market first and with an international data after. The beginning of the 90s marks the start of a huge literature about factor models: three, four, five-factor models; see Carhart (1997), Fama and French (1993; 2012; 2015; 2016; 2018b; 2018a; 2018c; 2019).

Even if some of these factors are subject of debate because of different reasons (weak historical records, vary significantly over time, weaken after discovery, concentrated among micro-cap stocks, weak internationally, lack of theoretical explanation, etc.), researchers' and practitioners' interest on this issue does not stop.

1.2 Errors-in-variables framework

Despite the early acknowledgment as well as the persistent presence of the EIV in financial economics (Fama and MacBeth 1973; Shanken 1992), the efforts to address, if not definitely solve, this challenge has attracted less attention compared to other issues related to the estimation of asset pricing models. At first sight, it may be argued that the consequences of ignoring the EIV are mostly mild for there is no change of sign in the coefficients and the attenuation effect is rather limited in high R^2 models. This argument is likely to hold when it comes to several regression-based modeling efforts in finance once we have good reasons to believe that the linear relationship between the left and right-hand-side variables are strong, such as the link between a tracker and its benchmark. Second, coping with the undesirable effects of the EIV is a notoriously difficult endeavor, especially once we pass from one predictor case to the next stage with multiple regressors. As clearly set forth by Cragg (1994; 1997), the results that are shown to hold for the one-variable no longer apply to the multiple regressions. Greene (2018, p. 283-284) shows that even with a single badly measured variable, it is not possible to derive expressions that indicate the magnitude and/or the sign of the bias in the slope. In addition, other coefficients estimates are also biased with unknown directions unless we have extra information about the true data generating process (Carmichael and Coën 2008, p. 779).

Measurement error models are typically treated with instrumental variables techniques as EIV arises after all as an endogeneity issue given the non-zero correlation between the error terms and the regressors. The econometric literature on the IV-estimation of EIV models is extensive, see, among others, Hausman and Watson (1985), Fuller (1987), Leamer (1987) or Greene (2018). Consistent estimates of model parameters can be obtained as long as we come up with valid instruments correlated with the true but unobserved values of the predictor *and* uncorrelated with the measurement error. Ashenfelter and Krueger (1994) who examine the return to schooling is a classic example of IV-based treatment of EIV problem. "Education" is, like many other socio-economic variables, is intrinsically not measurable and any attempt to parameterize the benefits of education has no other choice but using a proxy for "measuring" it, like "years of schooling". The authors' solution consists in contrasting the wage rates of identical twins with different schooling levels by isolating the outcome from the individual's own traits. Andersson and Moen (2016, p. 114) note that the popularity of IV estimators in applied work is probably due to the fact this method is intuitive and easy to implement. From a practical viewpoint, however, the key difficulty associated with the IV estimation of EIV models lies in finding such valid instruments simultaneously correlated with the true regressor *and* uncorrelated with the measurement error (Durbin 1954; Carroll et al. 2006; Carmichael and Coën 2008). In particular, when the set of instruments is only weakly correlated with the original regressors, the problems due to EIV can even be exacerbated (Racicot and Theoret (2015)). IV estimates not only turn out to exhibit much larger standard errors than their least-squares counterparts but they are also asymptotically biased (Wooldridge 2015; Stock, Wright, and Yogo 2002) and consistent estimation of model parameters turns out to be even more difficult as the orthogonality assumption of the instruments also requires additional instruments for which the assumption should hold (Iwata 1992).

As long as we consider the common ground between the two strands of literature on EIV and asset pricing models, an influential approach based on which numerous subsequent studies have been published, is the method developed by Dagenais and Dagenais (DD) (1997). Loosely speaking, the DD estimation consists of a matrix-like combination of Durbin and Pal's estimators (Durbin 1954; Pal 1980). DD suggest various combinations of the left- and right-hand-side variables' own higher and cross-moments as a set of valid instruments. Unlike other IV-based approaches in the literature, the DD estimator requires no extraneous information as the proposed instruments are derived from sample moments of order higher than two (Dagenais and Dagenais (1997, p. 193)). Carmichael and Coen (2008, p. 781) underline that an important feature of the DD estimator is that it is likely to be particularly suitable to financial time-series given the frequently observed non-normality and significant third and fourth moments of such data. As one potential downside related to the DD estimation, however, Cragg (1997) notes that the number of parameters that must be estimated to mitigate the EIV increases much faster than the number of model parameters because of the number of significant higher cross-moments between different right-hand-side variables (Cragg 1997, p. 89-90).

To paraphrase Carmichael and Coen (2008, p. 779), the DD approach turns out to be promising in financial economics as long as the non-normality of the true unobserved variables is verified. This is in turn quite a plausible assumption given the omnipresent non-Gaussianity in asset returns. Accordingly, several studies have so far addressed the EIV in asset pricing models by constructing instruments from the higher-moments of the original data (Coen and Racicot (2007), Carmichael and Coen (2008), Coen, Hubner and Desfleurs (2010)). Implementing the higher-moment estimator for the CAPM, Fama-French three- and four-factor models, Coën and Racicot (2007, p. 449) note that the IV estimation improves significantly the performance of the fitted models, so that the use of higher-moments techniques should be warranted for interpreting Jensen's (1968) α and β coefficients frequently used in applied work. Using Fama and French (1992) monthly returns on twenty-five value-weighted portfolios from January 1963 to February 2006, Carmichael and Coen (2008) argue that estimates of the Jensen's α differ substantially between OLS and DD higher moments approach. More recently, the higher-moments instrumental variables approach to tackle the EIV issue in linear asset pricing models have been extended by Racicot (2015a) who developed an estimation methodology that enables to deal with the weak instruments problem. Several subsequent papers have then used this new "GMMd" estimation method in the context of some mainstream asset pricing models (Racicot and Rentz (2015), (2016), Racicot, Rentz and Theoret (2018), Racicot, Rentz, Kahl and Mesly (2019)). Racicot and Rentz (2015) apply the GMMd approach to address the EIV problem in Pastor and Stambaugh's model (Pástor and Stambaugh 2003), which is basically an augmented version of the Fama-French 5-factor model by an additional risk factor that incorporates a liquidity premium. The authors note that the "liquidity variable at best contains significant measurement errors or at worst is ill-conceived" Racicot and Rentz (2015) [p. 338]. In a related work, Racicot and Rentz (2016) emphasize that the effect of all systematic risk factors tend to vanish when controlled for measurement errors except the traditional market factor (p. 447). Collectively, the results set forth by Racicot and Rentz (2015), (2016) are consistent with Harvey et al. (2016) or Cochrane (2011), who warn against the "potential unreliability" of the extra risk factors developed by Fama & French or Pastor & Stambaugh, that greatly shaped the financial literature over the last decades (Racicot and Theoret (2015, p.338)).

2 Errors-in-variables

2.1 Consequences of EIV

In this section, we provide an outline of the classical errors-in-variables (EIV) framework. Our presentation is limited to the single variable case for sake of simplicity and considerations on the scope of the paper. The central message carried out by the treatment of the single variable case covers to a large extent how EIV leads to puzzling consequences. That is said, it must be noted that some results derived from the single predictor model do not necessarily generalize to the multiple case when more than one variable is subject to measurement error. For detailed treatments of the issue, see, among others, Cragg (1994), Carroll et al. (2006), Buonaccorsi (2010), or Racicot (2015b).

Consider the following population regression model,

$$Y_t^* = \alpha + \beta X_t^* + \epsilon_t \tag{1}$$

where Y^* and X^* are the *true* but unobserved values of the dependent and independent variables respectively, $\epsilon \sim iid(0, \sigma_\epsilon^2)$ the disturbance term assumed to be serially uncorrelated with homogeneous variance, and α and β the parameters to estimate. The (classical) measurement error framework is typically introduced using the following additive relationship,

$$\begin{aligned} Y_t &= Y_t^* + \nu_t \\ X_t &= X_t^* + u_t \end{aligned}$$

that can be read as "observation is the sum of the true value plus a measurement error". In what follows, we delimit the scope to the measurement error to the right-hand-side variable as it can be easily shown that the impact of the error on Y is limited to a lower goodness-of-the-fit without a cost on parameter estimates.[2]

Suppose now that $Y = Y^*$ and $X = X^* + u$ with the additional assumptions $E(u) = 0$, $E(uX^*) = 0$ and $E(uY^*) = 0$. The measurement error in X is thus uncorrelated with the true values X^* as well as with those of the dependent variable.[3] We can then rewrite equation (1) as,

$$Y_t = \alpha + \beta X_t + (\epsilon_t - \beta u_t) \tag{2}$$

so that the disturbance is now a composite one that includes the slope parameter. It then turns out that the slope is no longer uncorrelated with the disturbance term as $Cov\left(X, (\epsilon - \beta u)\right) = Cov\left((X^* + u), (\epsilon - \beta u)\right) = -\beta\sigma_u^2$. Then, the following least-squares estimator of the slope

$$\widehat{\beta}^{LS} = \frac{Cov(X,Y)}{Var(X)}$$

[2] The cost of a badly measured Y is that the variance of ϵ is equal to $Var(\epsilon) + Var(\nu)$.

[3] These assumptions define the *classical errors-in-variables* model.

will be inconsistent as,

$$\text{plim } \widehat{\beta}^{LS} = \beta \left(1 + \frac{Var(u)}{Var(X^*)}\right)^{-1} = \beta \left(\frac{Var(X^*)}{Var(X^*) + Var(u)}\right) \tag{3}$$

Two important observations come out of this simple analysis: First, and foremost, the least-squares estimate of the slope $\widehat{\beta}^{LS}$ is biased downwards given that the inequality $Var(X^*) + Var(u) > Var(X^*)$ will hold as long as the right-hand-side variable contains an additive measurement error. In addition, equation (3) implies that the bias in β gets worse as the variance of the measurement error $Var(u)$ increases relative to $Var(X^*)$. Hausman (2001) calls this simple yet fundamental result as the *iron law of econometrics* – the magnitude of the estimate is usually smaller than expected. Second, the attenuation effect on β generates a bias of the opposite sign on the intercept α if the mean of the predictor X^* is positive (Cragg 1994, p. 780). In other words, negative intercepts will be estimated higher than their true values, while positive ones will be estimated lower than they truly are.[4]

When the predictor is subject to such measurement error, consistent estimation of the model parameters usually requires additional data or information. As discussed previously, there exists an extensive literature regarding the techniques devised to address the EIV, see, among others, Fuller (1987), Cheng and Van Ness (1999), Buonaccorsi (2010) or Racicot (2015b). Theoretically, one would get such a consistent estimate of the slope via *method of moments* if the value of the ratio $\lambda = Var(X^*)/(Var(X^*) + Var(u))$, also called the *reliability ratio*, is known (Fuller 1987). In this case, a generalized-least-squares estimator can be readily obtained by dividing the fitted slope to the reliability ratio. However, as pointed out by Buonaccorsi (2010), the true value of the reliability ratio is never known in practice so that it must also be replaced by its sample counterpart, $\widehat{\lambda} = \left(\widehat{\sigma}_X^2 - \widehat{\sigma}_u^2\right)/\widehat{\sigma}_X^2$.[5]

Matters get worse when there are k predictors subject to measurement error like $X_{i,t} = X_{i,t}^* + u_{i,t}$, $i = 1, ..., k$. In this case, even when there is only one X measured with error, the EIV effect spills

[4]This is a key result when it comes to the relationship between the measurement error and the regression-based estimation of asset pricing models in finance. Consider, for simplicity, the single-factor model where the right-hand-side variable is the market risk premium. By construction, it is positive. It turns out that under errors-in-variables, positive alphas will tend to be underestimated while negative alphas will be overestimated than their true levels. Recognizing this effect and putting it into context would have profound impacts on the fund management industry. We will later turn to this discussion in this paper.

[5]The algebra of $\widehat{\lambda}$ follows from the fact that $Var(X^*) = Var(X - u)$.

over not just on the intercept and the slope of the variable of interest but also on other slope coefficients. In addition, it is not possible to derive exact formulas to express neither the sign nor the magnitude of the bias in each slope β_i because the error in X_i does not only affect the corresponding coefficient but also other coefficients in the model (Greene 2018, p. 281-285), raising a further issue known as the *contamination effect* (Cragg 1994). There are several methods that has been so far developed in the literature to deal with the multiple case, yet no solution insofar has proved reliable enough to put full faith on to eradicate the undesirable consequences of EIV.[6]

2.2 CGA-based estimation of EIV model

In this section, we describe the CGA-based estimation of the EIV model developed by Satman and Diyarbakirlioglu (2015). We start by rewriting the one-variable EIV model given in equation (2):

$$Y_t = \alpha + \beta X_t + (\epsilon_t - \beta u_t) \tag{4}$$

where $\epsilon - \beta u$ is the compound error term given $X_t = X_t^* + u_t$. It has been previously stressed that ignoring the measurement error u yields biased and inconsistent estimates of the slope even in large samples. Our approach consists in using predictive regressions in order to obtain a *filtered* version of the observed values X and then plug it back to the original model to mitigate the EIV problem.

The central piece of the method is the following auxiliary regression of the badly measured variable on $i = 1, ..., d$ dummy variables D_i,

$$X_t = \phi_0 + \phi_1 D_1 + \cdots + \phi_d D_d + \eta_t \tag{5}$$

where $\eta_t \sim iid(0, \sigma_\eta^2)$ is the disturbance series and ϕ_i are model parameters to be estimated. The fitted model is,

$$X_t^{CGA} = \widehat{\phi}_0 + \widehat{\phi}_1 D_1 + \cdots + \widehat{\phi}_d D_d \tag{6}$$

such that $X_t = X_t^{CGA} + \widehat{\eta}_t$.[7] We can read this as "observed values are equal to corrected ones plus error". The intuition behind this auxiliary regression model is to break down the observed

[6] This is maybe the main reason why textbook treatments of measurement error models are mostly limited to the one-variable model.

[7] We deliberately drop the "hat" ˆ over the fitted series for ease of exposition.

values X, assumed to be subject to measurement error, into two components, one deterministic X^{CGA} and the other stochastic $\widehat{\eta}$. In other words, we consider X^{CGA} and $\widehat{\eta}$ respectively as the *fitted equivalents* of the true values of the predictor X^* and its measurement error u.[8] Plugging the values X^{CGA} back into the original regression model, we get,

$$Y_t = \alpha + \beta X_t^{CGA} + \epsilon_t \tag{7}$$

The estimated coefficients $\widehat{\alpha}^{CGA}$ and $\widehat{\beta}^{CGA}$ are solutions to the following discrete optimization problem,

$$\underset{D_1,\ldots,D_d,\widehat{\alpha}^{CGA},\widehat{\beta}^{CGA}}{\operatorname{argmin}} \sum_{t=1}^{T} \left(Y_t - \left(\widehat{\alpha}^{CGA} + \widehat{\beta}^{CGA} X_t^{CGA}\right)\right)^2 \tag{8}$$

so that we perform a joint estimation of the auxiliary regression and the baseline model by substituting the corrected values back into the original regression.[9] Using Monte Carlo simulations, Satman and Diyarbakirlioglu (2015) showed that the CGA-based intercept and slope coefficients have smaller biases with respect to their initial error-prone LS estimates. This comes with some cost in loss of efficiency because $\widehat{\alpha}^{CGA}$ and $\widehat{\beta}^{CGA}$ have also higher variances, which is due to the fact the estimator seeks maximizing the R-square of the regression. The increase in the variance of the estimate is however offset by the bias correction and, overall, the mean-square-error of the CGA-based estimate is smaller than that of the least-squares as,

$$\left(bias\left(\widehat{\beta}^{CGA}\right)\right)^2 + Var\left(\widehat{\beta}^{CGA}\right) = MSE\left(\widehat{\beta}^{CGA}\right) \leq MSE\left(\widehat{\beta}^{LS}\right) \tag{9}$$

There are a number of noticeable features associated with the CGA-based approach. We discuss these in some depth. We also provide in the appendix a pseudo-code of the estimation procedure to accompany our discussion.

Broadly speaking, the goal of EIV-CGA estimation is to extract from an initial population the *best solution* among all *possible combinations* that could be conceived by the algorithm. The best solution corresponds to the state where the algorithm converges and extracts the filtered values

[8] Constructing such *error-free* estimates of the regressors has been previously suggested within the IV-based methods of EIV models in the literature; see, for example, Dagenais and Dagenais (1997) or Coën and Racicot (2007).

[9] The appendix gives a detailed exposition of the pseudo-code of the algorithm.

X^{CGA}, which we later plug back into the original model.

We initialize the procedure by setting two parameters; namely, (1) the number of dummy variables d that will enter the auxiliary regressions and (2) the *population size*.[10] d is a user-defined parameter to estimate the regression specified in equation 5. Under the current setup, there are no specific guidelines concerning the value d that must be chosen. That is said, Satman and Diyarbakirlioglu (2015, p. 3224-3225) show using simulations involving various configurations that the mean-square-error of the estimator $MSE\left(\widehat{\beta}^{CGA}\right)$ tends to stabilize in the neighborhood of $d = 10$. We also choose this value in our estimations. The *population size* is used to update iteratively the probability vector of the search space until the stability condition for the auxiliary dummy-variables regressions is reached. We set the population size by taking into a consideration the trade-off between the speed with which the algorithm converges and the risk of local optimum trap. Satman and Diyarbakirlioglu (2015) suggest using simulations that for values equal to 20 or above, the population size has negligible effect on the results, with too large values coming at a cost of slowing down the time it takes the CGA to converge.

We turn to the auxiliary dummy-variables regression. First, fitting the model (5) involves several computational problems because of the lack of a closed-form solution, so numerical methods must be used. Genetic Algorithms (GAs) provide a handy toolkit for optimizing the objective function (8) and derive the dummies d_i. In a nutshell, GAs are a family of optimization techniques that mimic the natural selection process, whose mechanics and vocabulary borrow extensively, and unsurprisingly, from the Theory of evolution. GAs perform a parallel search by screening the entire population by randomizing candidates in different areas of the search space. A potential issue associated with GAs is that the optimization of the objective function is likely to consume too much computational memory. Compact Genetic Algorithms (CGAs) are primarily designed to overcome this issue. While it cannot be asserted that CGAs are superior to classical GAs in reaching the global optimum, they represent a number of advantages. Specifically, in a CGA, chromosomes, e.g. candidate solutions, are sampled using a probability vector and the number of iterations is defined with respect to the population size. Therefore, CGAs consume much less computational memory compare to classical GAs. As noted by Baluja and Caruana (1995), CGAs removes "genetics" from

[10]The setup also requires a third parameter n, which is simply the sample size of the mismeasured variable X.

GAs by replacing the term "crossing over" by "sampling".[11]

Second, the decomposition of the observed X's as proposed in equation (5) considers the residuals η_t as the measurement error on X and the remaining portion $\phi_0 + \phi_1 D_1 + \cdots + \phi_d D_d$ as a "clean" version of the true values of the variable. The approach thus requires solving for the appropriate *linear combination* of the dummy variables to construct the estimate X^{CGA}. This is a genuine component of the approach we adopt and constitutes the central block of the algorithm we describe in the appendix. Given d, the iterations starts off with a probability vector whose elements are all equal to 0.5 and iterations continue until all elements of the vector take either the value of 1 or 0. As a simple illustration, consider the following initial state probability vector from which samples, e.g. "chromosomes" of 4-d length will be generated,[12]

$$P = [0.8, 0.1, 0.7, 0.2]$$

which tells that the probability of obtaining $D_1 = 1$ is 0.8, the probability of obtaining $D_2 = 1$ is 0.1, and so on for $P(D_3 = 1)$ and $P(D_4 = 1)$. Given P, sampling a chromosome like $C_1 = [1, 0, 1, 0]$ is much more likely than sampling $C_2 = [0, 1, 0, 1]$. Then, the algorithm initiates by sampling two parents, say C_1 and C_2 using the initial P. The winner is the one who takes the lowest cost function, which is, in our case, the C for which the sum of squared residuals of the dummy regression is lowest. Once C^{winner} is determined, the vector P is updated using the formula,

$$P_i = \begin{cases} P_i + \frac{1}{popSize} & \text{if } C_i^{winner} = 1 \\ P_i - \frac{1}{popSize} & \text{if } C_i^{winner} = 0 \end{cases}$$

With the new P_i obtained, the process moves onward by sampling new parents, generating new off-springs and updating P_i until each element takes either the value of 1 or 0. The final state yields the new series X^{CGA}, which is then plugged back into the baseline model.

[11] Pioneering studies include, among others, Holland (1973; 1975; 1987) and Goldberg (1989). For a review of CGAs, see, among others, Baluja and Caruana (1995) or Harik et al. (1999; 2006).

[12] The term "probability vector" employed here does not correspond to a conventional vector whose elements sum up to 1 but instead refers to a list where each element shows the probability that the given element takes a specific value.

3 Data and Methodology

We consider equity portfolio returns sorted by five equity characteristics: size, book-to-market, operating profitability, investment and momentum[13]. The time-series of monthly excess returns run from November 1990 to February 2021. For presentation purposes, we split our portfolios into four panels as : Panel A (Size vs. Book-to-market); Panel B (Size vs. Operating profitability); Panel C (Size vs. Investment); and Panel D (Size vs. Momentum).

As we consider portfolios sorted by quintiles on each pair of dimensions, we obtain $5 \times 5 = 25$ portfolios under each panel. Therefore, we obtain $25 \times 4 = 100$ portfolios for 5 different geographical regions, namely, World, North America, Europe, Asia-Pacific and Japan. To sum up, we work with 500 different test portfolios broken down by 5 geographies and 4 pairs of equity characteristics.

For each time-series of test portfolios, we fit the following three standard asset pricing models:

CAPM:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \tag{10}$$

Fama-French three-factor model:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{M,t} - r_{f,t}) + s_i(SMB_t) + h_i(HML_t) + \epsilon_{i,t} \tag{11}$$

Fama-French five-factor model:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{M,t} - r_{f,t}) + s_i(SMB_t) + h_i(HML_t) + w_i(RMW_t) + c_i(CMA_t) + \epsilon_{i,t} \tag{12}$$

where $(r_{M,t} - r_{f,t})$ is the market risk factor, SMB size factor, HML value factor, RMW profitability factor, and CMA investment factor. The coefficients α_i and β_i are model parameters of interest, and ϵ_i are regression errors assumed to be serially uncorrelated with constant variance.

As emphasized above, we speculate that the true market risk premium is only observed with error because the *market portfolio* as defined in the underlying theory is simply not observable. The observed market risk premium series contains an additive measurement error such that $M = M^* + u$,

[13] All data are extracted from Kenneth French's website:
`https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

where M^* represents the true market portfolio. Without any further knowledge about the variance of the measurement error in M^*, nor that of the true market portfolio returns, it is not possible to act as if we know the reliability ratio and obtain consistent estimates by generalized least-squares on time-series data.

We suggest comparing the traditional OLS outputs with those one will get by bringing in the EIV-correction to the market risk premium through our CGA-based method. We report for each geographic region and test portfolio one set of fitted coefficients and regression output via least-squares and another set of coefficients and output derived from the CGA-based method. It must be noted in passing that the EIV correction is limited to the market risk factor and we assume no EIV on other variables. Given the current structure of the algorithm, there is no possibility to extend the correction simultaneously to other right-hand-side variables, but the results as it will be shown are quite promising even though we focus exclusively on the market risk premium for reasons discussed previously.

Besides some elementary output from the fitted models, we also report the GRS test statistic devised by Gibbons et al. (1989). The objective of the GRS test is to check the central proposition of the CAPM which holds that if expected returns are linearly related to the market risk factor, then model intercepts should not be systematically different from zero.

Specifically, we calculate the following test statistic separately for OLS and CGA outputs for each panel and each region,

$$GRS = \frac{T-N-K}{N} \times \left(1 + \widehat{\mu}_f^\top \widehat{\Omega}_f^{-1} \widehat{\mu}_f\right)^{-1} \times \left(\widehat{\alpha}^\top \widehat{\Sigma}_\epsilon^{-1} \widehat{\alpha}\right) \sim F_{N,T-N-K} \tag{13}$$

where T is the sample size, N the total number of test assets, K the number of explanatory variables (i.e. systematic risk factors), $\widehat{\mu}_f$ the $K \times 1$ mean vector of explanatory variables, $\widehat{\Omega}_f$ the $K \times K$ co-variance matrix of explanatory variables, $\widehat{\alpha}$ the $N \times 1$ vector of fitted intercepts, and $\widehat{\Sigma}_\epsilon$ the $N \times N$ co-variance matrix of fitted residuals. If the null hypothesis is true, then all intercepts are jointly equal to zero $\mathcal{H}_0 : \alpha_i = 0$, for all test assets $i = 1, ..., N$, and the GRS statistic follows an $\mathcal{F}$ distribution with N and $T-N-K$ degrees of freedom. As long as we analyze the economic significance of a given asset pricing model, comparing the GRS statistics across two competing estimation methods makes sense as one expects a lower value of the statistic as less significant pricing errors among the portfolios' returns.

4 Time series regressions' results

We discuss the results of our estimations. As mentioned above, we run time-series regressions using two methods (OLS and CGA) and three asset pricing models (CAPM, three- and five-factor models) for five different regions (World, North America, Europe, Asia Pacific and Japan). Doing so, we fit a total of 3000 regressions. To set the stage for the discussion of the results, we focus primarily on essential time-series outputs, namely the fitted intercepts $\widehat{\alpha}$ and market risk loadings $\widehat{\beta}_M$ as well as the models' adjusted R^2 scores. We also report the GRS statistics for each pair of "portfolio & geography" and compare the output across OLS and CGA estimations.

4.1 Intercepts (α)

We start the discussion by reporting the intercepts from the models fitted by OLS vs. CGA across Tables 1 to 5. Each table shows the intercepts (in percentage values) of CAPM regressions divided into four panels of test portfolios as defined above. The t statistics are shown below each coefficient and their significance level are marked by an asterisk * next to the coefficients. We expect a lower number of significant intercepts with the CGA estimation as, by construction, it is designed to curb the attenuation bias in market risk coefficients $\widehat{\beta}_M$ and, consequently, the bias in the fitted intercepts $\widehat{\alpha}$ too.

A simple count of the significant intercepts between the OLS and CGA methods is practical. In Table 1, we see that the number of significant intercepts is 77 for the OLS estimations and 72 for the CGA estimations. In the case of North America (Table 2), the improvement is more pronounced with only 46 significant intercepts with CGA versus 79 with OLS. For Europe (Table 3), CGA estimations cut the number of significant intercepts from 80 (OLS) to 67 (CGA). In Table 4 with Asia Pacific markets, CGA method gives the lowest number of significant intercepts with only 43 (compared to 57 with OLS). Finally, in the case of Japan (Table 5), the results of the two methods are similar. The half of the intercepts are not significantly different from zero of all regressions.

Next, we report the output for the three-factor and five-factor models respectively in Tables 6 to 15. Overall the CGA method gives lower number of intercepts significantly different from zero.

More specifically, for the three-factor model and in the case of World (Table 6), we obtain with CGA 40 intercepts at 1%, 21 at 5% and 7 at 10% compared respectively with 72, 8 and 6 with OLS. The results' improvement is similar for the case on North America (Table 7). Out of 100 intercepts, only 54 are significantly different from zero with CGA compared to 82 with OLS. For European markets (Table 8), half of intercepts (51 out of 100) are significantly different from zero with CGA method (88 with OLS). For Asia Pacific (Table 9), we obtain the lowest number of significant αs with CGA (32 intercepts at 1%, 9 at 5% and 6 at 10%). Finally, in the case of Japan (Table 10), results of the two methods are similar (58 significant αs with OLS versus 59 significant αs with CGA).

When it comes to the five-factor model, the difference in terms of the significance of OLS vs. CGA-driven intercepts is minor, if not reversed. Taking individually, there are in some cases more significant intercept terms with the CGA estimation compared to OLS. Specifically, the CGA-based alphas are significantly different from zero for all panels (98 in World Table 6; 96 in North America Table 12; 90 in Europe Table 13; 87 in Asia Pacific Table 14 and 100 in Japan Table 15) compared to OLS results (82 in World Table 11; 80 in North America Table 12 and Europe Table 13; 52 in Asia Pacific Table 14 and 60 in Japan Table 15). We turn to this issue later in the paper when we report the GRS statistics as well as in the discussion.

Overall, if we consider the CAPM as the true specification, we notice that ignoring the measurement error in the right-hand-side variable leads to a considerably higher number of intercepts terms, which, collectively, would tend to cast doubt on the validity of the CAPM equation. Once we consider the EIV correction, however, we obtain a substantially lower number of statistically significant intercepts in line with the central prediction of the model as well as the purpose of the estimation method. Compared to one-factor model, intercepts are also less significant in the case of the three-factor model with the CGA method where we keep our focus on the market risk factor as the potentially mismeasured variable. However, when we add RMW and CMA, we observe no systematic difference between OLS and CGA for intercepts.

4.2 Market risk premium and β_M

We now outline the results about market risk loadings. Box-plots 17 to 21 compare the OLS vs. CGA market betas for the five geographic regions. Each figure displays the fitted coefficients for our four panels (A, B, C and D) of test portfolios for the CAPM, three and five-factor models (from top to bottom). In each figure, the blue color refers to the OLS betas and the red color to the CGA betas. The graphics convey collectively two major observations.

First, the attenuation bias is once more corroborated in our data-set regardless of the specification to describe asset prices or the geography: All OLS market betas are lower than their CGA-driven counterparts. The difference is, unsurprisingly, emphasized for the CAPM regressions where we overlook other possible systematic risk factors. Looking at the market beta estimates for the single factor model, the average of the fitted coefficients across the 500 portfolios (all panels and all regions) is 1.02 while this rises to 1.28 for the CGA-driven market betas. For example, while the average market beta estimate for the North America portfolios is 1.09 when one ignores the EIV, the same average goes up to 1.44 for the same geography as long as we assume that the CAPM is the true specification. Put another way, the measurement error in the market portfolio yields seriously underestimated risk factor loadings in line with the expected consequences of EIV. The systematically lower market betas are thus likely to systematically underestimate the true "market risk" which a given portfolio is exposed to.

Second, the difference between OLS vs. CGA betas is less pronounced with the three and five factor specifications, yet still present, as it can be seen through Box-plots 17 to 21. Numerically, the minimum, average and maximum values of the OLS market betas for the thee-factor specification with all geographies included are, respectively, 0.7872, 1.0064 and 1.3947. These numbers are 0.8561, 1.1305 and 1.8832 when we fit the three-factor model by using the EIV correction to the market risk premium. So, the attenuation in market risk loading is still detectable with some heterogeneity once we divide our test portfolios into our five different geographies. When it comes to the five-factor specification with additional four systematic risk factors added next to the market portfolio, the two set of estimations tend to visually converge as suggested by the blue and red series. The downward bias in market betas does not disappear however with the OLS output that disregard the measurement error in market portfolio while the CGA-based values keep pushing the coefficient estimates upward to some extent. Aggregating over all geographies, the average factor loading on market portfolio is 1.0026 with the OLS while the same average is rounded up to 1.1374 with the CGA.

To sum up, we obtain substantial correction of the downward bias in market betas for all regions with the CGA method compared to the OLS ones in the case of the CAPM. This attenuation bias in β_M is confirmed for all regions with the three- and five-factor models although the downward bias is naturally less pronounced among these multiple regressions framework. Incidentally, we should remember that the CGA-based methodology aim limiting the impact of the measurement error in the observations of the market portfolio proxy by considering a given specification as the true model generating the asset prices. As such, a direct comparison of the extent to which the attenuation in market beta estimates is controlled across the three different specifications is not entirely inward-looking once we consider the design of the algorithm. We turn back to this discussion in the conclusion.

4.3 Adjusted R^2

Figures 1, 2 and 3 show the adjusted R^2's for respectively the CAPM, three and five-factor models. The blue line is for OLS results and the red one is for CGA. From left to right, we give the results for panels A, B, C and D successively for World, North America, Europe, Asia Pacific and Japan (a total of 500 adjusted R^2 in each graph).

First, we can observe that overall the dispersion of the adjusted R^2 is higher for OLS regressions. For example, in the case of World, the minimum (maximum) value of adjusted R^2 is about 66.2% (95%) with OLS versus 74.5% (87.2%) with the CGA estimations. For North America, the 300 different adjusted R^2 values (three asset pricing models) range between 57% and 90.7% for the OLS estimations. The same range collapses to 75.7% and 87.9% with the CGA. Similar results for the adjusted R^2's are observed for other geographies, namely Europe, Asia-Pacific and Japan. For example, while the lowest adjusted R^2 of the CAPM regressions for European portfolios is 63.7%, this lower bound rises to 75% for the same geography when we implement the measurement error correction model. The greater variability in adjusted R^2 is typically attenuated for the one-factor model outputs (compare Figure 1 to Figures 2 and 3). When we look at the results for the Asia-Pacific portfolios, the single factor adjusted R^2 exhibit also less variability with the CGA as the adjusted R^2 values range between 75.8 and 86.5% in contrast to the OLS R^2's between 57.1 and 91.5%.

Second, for the three models and all regions, the average adjusted R^2 for all panels (100 portfolios) are systematically lower with CGA-based estimations compared to OLS. For example, in the case of World, we obtain a mean adjusted R^2 about 82.84% for CAPM, 84.86% for three-factor and 89.98% for five-factor models with CGA versus 83.28% for CAPM, 93.46% for three-factor and 94.17% for five-factor models with OLS. For Europe, the mean adjusted R^2 for all panels (100 portfolios) is 82.59% for CAPM, 85.50% for three-factor and 89.46% for five-factor models with CGA compared to 83.22% for CAPM, 92.55% for three-factor and 93.18% for five-factor models with OLS method.

To sum up, we can retain two main results for the adjusted R^2. Compared to the OLS method, The CGA method reduces the extreme values of the adjusted R^2 for the portfolios (higher values for the minimum and lower values for the maximum). Otherwise, as a consequence of this lower dispersion, the average value of the adjusted R^2 for all regressions is lower with CGA compared to OLS. We turn back to this discussion in the next section to add some other interesting remarks about the adjusted R^2.

4.4 GRS test

For clarity and shortness, we summarize the results of GRS tests in table 16 and figures 4, 5 and 6. In each figure, we plot the GRS test statistics in blue for OLS results and those derived by the CGA estimations in red. We run the test for each panel of portfolio and for each region. We thus obtain 20 (4 panels $\times$ 5 regions) GRS $\mathcal{F}$-statistic for each pair of "portfolio vs. geography". In short, lower GRS test statistics indicate jointly less significant intercepts, thus provide empirical support to the model under inspection. In figures 4 to 6, this is equivalent to a series on average narrower and closer to the center of the plot. In all cases, this is the case of the red series compared to the blue one. Numerical values of these statistics as shown in table 16 also clearly shows the overall difference between the results of the OLS and CGA estimations. Overall, for all regions and all panels, we obtain systematically lower GRS values for the CAPM, three- and five-factor time-series regressions with the CGA methodology.

Specifically, if we consider the CAPM as the true model of the risk-reward trade-off, the message carried out by the figure 4 is inspiring: Regardless of the geography and/or the portfolio, the contour of the GRS statistics delimited by the CGA method considerably shrunks the one drawn by the OLS estimations ignoring the EIV in the market portfolio. The $\mathcal{F}$-statistics are much lower in all cases. This points out to a significant improvement in favor of the CAPM's fundamental prediction that the intercepts must be collectively equal to zero. For some cases like Japan, we even come up with such lower GRS statistics that we cannot reject the null that the intercepts are jointly zero, giving thus empirical support to CAPM. Estimations that overlook the EIV correction in the market risk premium, however, hardly explain the variability in portfolio returns.

If we follow the Fama-French three-factor specification, the picture shown in figure 5 remains qualitatively the same as the previous case with the CAPM: The GRS statistics derived from the OLS with measurement error in market risk premium exhibit a much wider contour compared to the one drawn by the GRS statistics one obtains via CGA. Curbing the attenuation bias even only for one of the right-hand-side variables of the model pays off substantially in terms of almost systematically lower $\mathcal{F}$-statistics and credit in favor of the CGA-based methodology within the three-factor specification too.

Finally, in figure 6 where we compare the results for the five-factor specification, we first notice, relative to the previous two figures, that the GRS statistics derived from the CGA are visually closer to the ones derived from the vanilla OLS. If for some cases the GRS test statistics are even lower with the OLS, the CGA that mitigate the measurement error in the market portfolio still achieves some improvement in most instances like Europe and North America portfolios. Similar observations can be made when we compare the performance of the OLS vs. CGA for the five-factor specification in terms of the adjusted R^2 values as a model selection criteria. We do not consider, however, that this invalidates the performance of the CGA estimation because a horse-race OLS vs. CGA would make sense only in the case of prior information or theoretical guidelines about the true econometric specification. As such, the similar, if not weaker, performance of the CGA within the five-factor specification would at least go through two channels. First, the presence of EIV attached to the market risk factor within the five-factor model would simply be offset by the inclusion of the additional set of explanatory variables that were absent within the Fama-French three-factor and CAPM specifications. This relates the measurement error problem issue to the estimation of asset pricing models to an “omitted variables” problem from an econometric standpoint. Thus, the unresolved question about the *market portfolio* stands no longer as a matter of the market portfolio’s definition itself but a model specification error that neglects additional relevant variables. Second, we restrict our efforts to contain the errors-in-variables issue to the market portfolio and the market risk premium. We are fully aware however that there is no reason, beside the theoretical framework of the Roll's critique, that only the market risk factor would be contaminated with some measurement error. We simply argue that the very definition of the market portfolio makes it a natural lab case to investigate the potential consequences of the EIV as long as it is part of the right-hand-side variables in a given asset pricing model. One should in addition acknowledge the fact that despite the plethora of additional systematic risk factors that had been developed in the existing literature, none of the existing studies ever attempted to drop the market portfolio from the set of common risk factors in explaining asset returns.

4.5 Additional results

After outlining the main outputs of our regressions in the previous section, we highlight here additional interesting results. We focus on three main observations. First, we compare the idiosyncratic volatility between OLS and CGA for the three models (Figure 8). Overall, we can observe that the idiosyncratic volatility of the CGA method has lower disparity than that of OLS method for each model (from top to bottom the CAPM, three and five-factor models) and all regions (World, North America, Europe, Asia Pacific and Japan). More specifically, with the CAPM (three and five factor respectively), we obtain a minimum idiosyncratic volatility of 0.96% (0.76% and 0.69% respectively), a maximum of 5.4% (4.16% and 4.15% respectively) and an average of 2.58% (1.71% and 1.63% respectively) with OLS method versus a minimum of 1.41% (1.42% and 1.09% respectively), a maximum of 3.93% (3.60% and 2.93% respectively) and an average of 2.37% (2.10% and 1.82% respectively) with CGA. In summary, the CGA method reduces the extreme values of the idiosyncratic volatility for the portfolios (higher values for the minimum and lower values for the maximum). Otherwise, as a consequence of this lower dispersion, the average value of the idiosyncratic volatility for all regressions is lower with CGA compared to OLS.

Second, even though our main objective is not to compare the three models but rather to compare the two methods OLS and CGA, we suggest here to see the adjusted R^2 given by the CGA method for the CAPM, three and five-factor models (Figure 7). For the four panels and five regions (500 regressions), the highest value of adjusted R^2 is obtained in the case of the five-factor model and is about 95.37% (North America portfolio Size 1& Book to Market 1). For the three-factor model, it is only about 93.90% (North America portfolio Size 1& Book to Market 3) and for the CAPM 89.47% (Japan portfolio Size 5& Momentum 4). Otherwise, the portfolios' returns that give the lowest adjusted R^2 are Europe Size 4& Momentum 5 for the five-factor model (80.26%), Asia Pacific Size 3 & Investment 5 for the three-factor model (76.10%) and World Size 1 & Investment 4 for the CAPM (74.49%). Overall, with the CGA method, the five-factor model explains better the portfolios' returns with an average adjusted R^2 of 89.56% compared to 86.12% for the three-factor model and 82.65% for the CAPM.

Third, we suggest here to see in depth the results for small portfolios. As mentioned above in the literature review, the size effect is much debated. However, we can state easily that many empirical studies pointed out to the difficulty of asset pricing models to explain returns of small firms. Primary results of our study give an interesting orientation for the future research about the size effect. For shortness and clarity, we focus here on the CAPM model. If we consider all small portfolios of our sample, a total of 100 portfolios (five portfolios of quintile 1 size for each panel and each region), our results show that the CAPM with the CGA method explains on average $82,38\%$ of portfolios' returns (average adjusted R^2) compared to an average of only $69,49\%$ with the OLS method. Moreover, the mean idiosyncratic volatility is about $2,47\%$ with CGA versus $3,29\%$ with OLS. Finally, the attenuation bias of the market beta β_M is reduced through CGA. On average, the market beta for small portfolios is about $0,99$ with OLS regressions and becomes $1,44$ with CGA.

5 Robustness tests

In this section, we control for the relevance of our results by running additional estimations based on an instrumental variables method. Specifically, we adopt the GMM_d approach developed by Racicot (2015a) and subsequently implemented by, among others, Racicot and Rentz (2015; 2016) or Racicot et al. (2017; 2019). The GMM_d estimator is basically a robust instruments variables-based extension of Hansen's generalized method of moments estimator (Hansen 1982) and is characterized by the following equation:

$$\arg\min_{\widehat{\beta}} = \left\{ T^{-1} \left[D^\top \left(Y - X\widehat{\beta} \right) \right]^\top W T^{-1} \left[D^\top \left(Y - X\widehat{\beta} \right) \right] \right\} \tag{14}$$

to estimate the $K \times 1$ vector of population parameters in the linear model $Y = X\beta + \epsilon$. T is the sample size and W is the heteroskedasticity and autocorrelation consistent weighting matrix. X is the $T \times K$ matrix of right-hand-side variables. The matrix D has elements $d_{i,t} = x_{i,t} - \widehat{x}_{i,t}$ and is viewed as a "filtered version" of the endogenous variables that should potentially attenuate the measurement errors. $x_{i,t}$ is the mean-centered values of the ith regressor. $\widehat{x}_{i,t}$ in turn are fitted values of the linear model $\widehat{x}_{i,t} = \widehat{\gamma}_0 + z\widehat{\phi}$. The vector of z instruments include $z_0 = i_T$, $z_1 = x \bullet x$, and $z_2 = x \bullet x \bullet x - 3x \text{ diag}\left(\frac{x^\top x}{T}\right)$ where "$\bullet$" is the Hadamard element-wise product operator. Thus, we estimate for all test portfolios the following equations:

$$
\begin{aligned}
r_{i,t} - r_{f,t} &= \alpha_i + \beta_i(iv_{r_{M,t}-r_{f,t}}) + \epsilon_{i,t} && (15)\\
&= \alpha_i + \beta_i(iv_{r_{M,t}-r_{f,t}}) + s_i(SMB_t) + h_i(HML_t) + \epsilon_{i,t} && (16)\\
&= \alpha_i + \beta_i(iv_{r_{M,t}-r_{f,t}}) + s_i(SMB_t) + h_i(HML_t) + w_i(RMW_t) + c_i(CMA_t) + \epsilon_{i,t} && (17)
\end{aligned}
$$

that are respectively the GMM$_d$ equivalents of CAPM, Fama-French three- and five-factor models specified in equations 15, 16 and 17. Given our main focus on the collective significance of intercepts (Jensen's alpha), we collect $\alpha_i^{GMM_d}$ for all test portfolios for every region. Results show that estimations based on GMM$_d$ outperform, on average, OLS but still generate a higher number of significant α than CGA does.

We also follow Motiel-Olea and Pflueger (2013) to verify the robustness as well as the exogeneity of the instruments. We proceed by running the regressions of each explanatory variable in the models to the set of instruments describe above. According to Olea and Pflueger, if at least one of the $\mathcal{F}$-statistics in the regressions of "filtered" x to the instrument set z is above 24, then this is a signal against the potential problem of weak instruments. Finally, we control for the exogeneity of the instruments z by running the following regressions of fitted residuals $\widehat{\epsilon}_{i,t}$ from the three specifications in equations 15, 16 and 17 to the instrument set,

$$\widehat{\epsilon}_{i,t} = c + z\delta + \eta_{i,t} \qquad (18)$$

As pointed out by Racicot and Rentz (2015, p. 335), the coefficients vector δ are analogous to the partial correlation coefficients between the regressors and the instruments. Exogeneity requires these coefficients not to be significantly different from 0 as well as a negligible goodness-of-the-fit measure. We estimate a total of 1500 regressions from (18) for the three specifications across the five geographies and four panels of portfolios, each containing twenty-five series. Given the t-statistics, we conclude that the fitted coefficients are mostly not significantly different from 0. In addition, the goodness-of-the-fit measures, overall, are very low. We obtain an average R^2 by only 0.0181 with a maximum equal to 0.1084 across all of the 1500 estimations. The numbers are consistent across different geographies and specifications under considerations. Thus, the results suggest that the instruments are exogenous.[14]

In tables 17, 18, 19, 20 and 21, we show the market beta obtained for all regions (World, North America, Europe, Asia Pacific and Japan), all panels (A, B, C and D) and the three models (CAPM, FF3F and FF5F) with the three approaches (OLS with blue color, GMM_d with green color and CGA with red color). In each region and each panel, the CGA method corrects the downward bias of $\beta^{r_M - r_f}$. For each region, each panel and each model, the average beta given with the CGA method is higher than the mean betas obtained with the OLS and the GMM_d methods. Moreover, we conduct the GRS test to compare the three methods. In figure 4, we draw the GRS results for all regions and all panels in the case of the CAPM (OLS with blue color, GMM_d with green color and CGA with red color). We observe that the CGA results give the lowest values (closer to the center zero). The same observation is obtained in figures 5 in the case of the three-factor model and figure 6 in the case of the five-factor model. Overall, the CGA estimators outperform the OLS and the GMM_d estimators.

6 Conclusion and future research

We propose a new methodology to mitigate the errors-in-variables (EIV) problem inherent in the estimation of asset pricing models. With the well-known Roll's critique in mind (Roll 1977), we focus on the market portfolio, still at the center of almost all theoretical or empirical efforts deployed to understand the basic risk-reward relationship in finance. Given the definition of the market portfolio and in line with the existing literature, we presume that the true values of the returns on this hypothetical portfolio are at best observed with some measurement error and, consequently, the widespread least-squares regressions of the risk-return relationships ignoring this issue yield to

[14] Detailed results are available upon request.

biased and inconsistent parameters estimates. In the case of CAPM regressions, for example, such measurement errors generate systematically downward-biased coefficients on the market risk factor even in large samples.

We apply an estimation method based on a Compact Genetic Algorithm devised by Satman and Diyarbakirlioglu (2015) to compare the results of time-series regressions on CAPM, Fama-French three and five-factor models using portfolio returns data over the period 1990-2021. We consider five geographic regions and four different panels of portfolios sorted by size, book-to-market, operating profitability and momentum. We verify that the CGA-based estimations reduce the impact of EIV in the market risk factor, an observation shared by the three different model specifications we consider.

Based on fitted intercepts $\widehat{\alpha}$, market betas $\widehat{\beta}_M$, adjusted R^2 and the GRS statistics, we conclude that the CGA-based estimations outperform the traditional OLS for all the three baseline asset pricing models. Collectively, we obtain on average higher adjusted R^2 values and the GRS statistics are substantially lower when we use the CGA estimator. These results provide evidence that naive OLS estimations are unsatisfactory as a tool to run time-series regressions of stock returns in the case of measurement error in at least one of the explanatory variables. We also note that neglecting the measurement error in the market portfolio generates significant attenuation bias in the coefficient estimates as the market risk factor loadings can be underestimated by as much as 25% with CAPM regressions. This downward bias does not disappear with the Fama-French three and five-factor models while the EIV correction we bring to the market portfolio yield more satisfactory results as suggested especially by the GRS statistics (see figures 4 to 6). Finally, the CGA estimators outperform the GMM with IV estimators for all regions, all portfolios and the three models.

We can outline at least three main contributions from our study. First, on the empirical side, we can assert that OLS time-series regressions are unsatisfactory regardless of the specification due to presence of measurement errors not only in the market portfolio M but potentially in other variables. Second, if we consider the CAPM as the *true* model that describes expected returns, OLS regressions exhibit major pitfalls due to attenuation bias in market risk loading. It is essential to take into account the EIV and implement an alternative estimation method to overcome this econometric problem in time-series regressions. OLS regressions that ignore the EIV in market risk premium underestimate substantially the market risk, whereas with the CGA we correct this bias. Third, the improvements of regressions output of the CAPM (attenuation bias of market beta β_M, adjusted R^2, idiosyncratic volatility) with the CGA method compared to the OLS method in the case of small firms give promising explanation for the size effect for future research.

Our findings have several theoretical, empirical and managerial implications. First, on the theoretical side, we revive the critique of Roll (1977) and restart the debate again about the *true* market portfolio. Second, for empirical studies, we show that it is more than necessary to question the OLS method and the GMM with IV in time-series regressions. Third, our findings lead to many managerial implications related to portfolio management (performance measurement, stock picking, etc.), asset pricing (stock valuation, risk-return trade-off, etc.), and corporate financial and investing decisions (cost of capital, valuation, funding).

Finally, our study identifies many issues that are worth-interesting for future research. Based on a well-known and still unresolved debate on the definition of market portfolio and because there is no theoretical basis to suspect EIV on other systematic risk factors, we give in this study an answer to EIV problem limited to M. It is of course interesting to study the case of more than one EIV in other variables and to extend EIV-CGA to multiple risk factors.

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Appendix: Pseudocode of the algorithm

This appendix provides a plain language description of the EIV-CGA approach developed by Satman and Diyarbakirlioglu (2015). All functions and methods necessary to implement the approach are available with the R package `eive` (see Satman and Diyarbakirlioglu (2018)).

We conceive the process in two subsequent parts and a penalty function, which establishes the convergence criterion. The first part sets the initial parameter values and generates the search space within which the CGA will search the solution.

```
# PART I ————————————————————————————————————————
# initializing
set n:= Number of observations
set d:= Number of dummies
set popSize := Population size
# setting up the search space for CGA
# {1 * d} probability vector for {n * 1} chromosomes
chromosomeSize := n x d
set P := [0.5, ..., 0.5]
# initialize the probability vector with P[i] = 0.5
P[i] := generatePopulation(chromosomeSize)
```

Next, we start the iterations until the "best" offspring is obtained among all possible combinations we can extract from the search space. This goes through random sampling of two parent vectors with elements coming from the current state of $P[i]$. Then, we attach the parents, e.g. the samples, a cost function such that the winner to survive the next generation is the one who has the lowest penalty. Next, we update the $P[i]$ using a simple rule in a loop over the entire search space. The algorithm converges when all elements of $P[i]$ are either equal to 0 or 1.

```
# PART II ———————————————————————————————————————
# sample two binary random vectors using P[i] as
    parent1 := sample(P)
    parent2 := sample(P)
# evaluate parent1 and parent2
    costParent1 := costFunction(parent1)
    costParent2 := costFunction(parent2)
# winner with lowest cost function survives as
```

```
    winner := selectBest()
    loser  := selectWorst()
# updating P[i]
For i in 1:chromosomeSize{
    if (winner[i] != loser[i]{
        if (winner[i] == 1){
            P[i] := P[i] + (1 / popSize)
            }
            Else{
            P[i] := P[i] - (1 / popSize)
            }
        }
    }
}
# iterate until each P[i] = 0 or 1
```

The cost function returns the sum of squared residuals from the regression given in equation 5 between two candidate solutions sampled from the search space and declares the "winner" as the one for which this score is lowest. The end-result yields the series X^{CGA} that we consider as a *filtered* version of original observations.

```
# Defining the cost function ──────────────────────────
costFunction := function(chromosome){
    # observed X's = true X's + measurement error
    # generate {n * d} matrix of dummies using P[i]
    candidateDummies[] := extractVariables(chromosome)
    # fit the auxiliary regression:
    # X = phi[0] + phi[1] * D[1] + ... + phi[d] * D[d]
    auxiliaryRegression := regress(X on candidateDummies)
    # extract fitted series X^{CGA}:
    X^{CGA} := predictedValues(auxiliaryRegression)
    # fitting the main regression
    # Y = alpha + beta * X^{CGA} + gamma * otherX + error
    mainRegression := regress(Y on X^{CGA}, otherX)
    # objective: minimize the sum of squared residuals
    min{resid := sumOfSquaredResiduals(mainRegression)}
```

```
    # return sum of squared residuals
    return(resid)
}
```

Table 1: **WORLD: Fitted intercepts from CAPM regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding *t*-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

Panel A: Book-to-Market

	BM1	BM2	BM3	BM4	BM5
Size 1	**-0,16**	**0,14**	**0,38** ***	**0,42** ***	**0,72** ***
	0,91	1,00	2,75	3,70	5,72
Size 2	**-0,09**	**0,18** *	**0,25** ***	**0,34** ***	**0,40** ***
	0,61	1,64	2,59	3,64	3,45
Size 3	**0,02**	**0,12**	**0,23** ***	**0,29** ***	**0,37** ***
	0,13	1,21	2,83	3,30	3,39
Size 4	**0,18**	**0,21** ***	**0,20** ***	**0,28** ***	**0,26** **
	1,52	3,01	3,16	3,37	2,55
Size 5	**0,25** ***	**0,24** ***	**0,19** ***	**0,19** ***	**0,11**
	2,73	4,55	3,85	2,84	0,99

Panel B: Operating Profitability

	OP1	OP2	OP3	OP4	OP5
Size 1	**0,24**	**0,64** ***	**0,59** ***	**0,62** ***	**0,62** ***
	1,62	5,98	5,28	5,39	5,07
Size 2	**-0,05**	**0,34** ***	**0,35** ***	**0,40** ***	**0,52** ***
	0,38	3,71	3,48	3,86	4,77
Size 3	**-0,04**	**0,23** ***	**0,36** ***	**0,36** ***	**0,37** ***
	0,41	2,82	4,52	4,17	3,95
Size 4	**-0,06**	**0,26** ***	**0,33** ***	**0,33** ***	**0,31** ***
	0,60	4,05	4,94	4,60	4,30
Size 5	**-0,35** ***	**0,05**	**0,22** ***	**0,29** ***	**0,36** ***
	4,12	0,86	4,09	5,39	5,64

Panel C: Investment

	Inv1	Inv2	Inv3	Inv4	Inv5
Size 1	**0,54** ***	**0,67** ***	**0,63** ***	**0,56** ***	**0,06**
	3,73	6,31	5,94	5,06	0,40
Size 2	**0,27** **	**0,46** ***	**0,40** ***	**0,35** ***	**-0,06**
	2,41	4,55	4,46	3,55	0,46
Size 3	**0,29** ***	**0,34** ***	**0,39** ***	**0,27** ***	**-0,12**
	3,02	3,96	4,80	3,41	0,99
Size 4	**0,25** ***	**0,37** ***	**0,32** ***	**0,28** ***	**0,00**
	3,08	4,89	5,03	4,33	0,04
Size 5	**0,30** ***	**0,22** ***	**0,21** ***	**0,17** ***	**0,09**
	4,60	3,72	4,13	2,67	1,03

Panel D: Momentum

	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**-0,34** *	**0,36** ***	**0,61** ***	**0,82** ***	**1,10** ***
	1,93	3,36	6,01	7,28	6,72
Size 2	**-0,27**	**0,23** **	**0,36** ***	**0,55** ***	**0,73** ***
	1,62	2,26	3,95	5,58	4,64
Size 3	**-0,19**	**0,19** *	**0,31** ***	**0,38** ***	**0,53** ***
	1,21	1,90	3,73	4,48	3,65
Size 4	**-0,21**	**0,17** *	**0,32** ***	**0,32** ***	**0,54** ***
	1,31	1,91	4,87	4,52	3,78
Size 5	**-0,25**	**0,11**	**0,24** ***	**0,36** ***	**0,42** ***
	1,62	1,35	3,96	4,86	2,85

Intercepts based on Compact Genetic Algorithm (in %)

Panel A: Book-to-Market

	BM1	BM2	BM3	BM4	BM5
Size 1	**-0,45** ***	**-0,06**	**0,21** *	**0,27** ***	**0,51** ***
	3,64	0,55	1,81	2,86	5,09
Size 2	**-0,29** **	**0,06**	**0,15**	**0,25** **	**0,24** **
	2,46	0,55	1,50	2,45	2,45
Size 3	**-0,13**	**0,03**	**0,17**	**0,21** **	**0,24** **
	1,21	0,28	1,54	2,18	2,08
Size 4	**0,06**	**0,16**	**0,16**	**0,21** **	**0,16**
	0,56	1,36	1,55	2,02	1,51
Size 5	**0,16** *	**0,21** **	**0,16**	**0,15**	**0,02**
	1,71	2,25	1,64	1,44	0,13

Panel B: Operating Profitability

	OP1	OP2	OP3	OP4	OP5
Size 1	**0,01**	**0,49** ***	**0,42** ***	**0,47** ***	**0,45** ***
	0,05	5,28	4,42	4,64	4,46
Size 2	**-0,17**	**0,24** **	**0,24** **	**0,29** ***	**0,40** ***
	1,51	2,55	2,56	2,74	3,83
Size 3	**-0,15**	**0,16** *	**0,29** ***	**0,29** ***	**0,28** ***
	1,39	1,67	2,92	2,63	2,72
Size 4	**-0,13**	**0,22** **	**0,29** ***	**0,28** ***	**0,26** ***
	1,03	2,38	3,00	2,78	2,60
Size 5	**-0,42** ***	**0,02**	**0,18** *	**0,26** ***	**0,31** ***
	3,29	0,15	1,92	2,98	3,92

Panel C: Investment

	Inv1	Inv2	Inv3	Inv4	Inv5
Size 1	**0,30** ***	**0,51** ***	**0,49** ***	**0,40** ***	**-0,16**
	2,69	5,97	6,38	3,60	1,34
Size 2	**0,13**	**0,32** ***	**0,29** ***	**0,25** **	**-0,20**
	1,39	3,65	3,21	2,36	1,62
Size 3	**0,20** *	**0,27** ***	**0,31** ***	**0,22** **	**-0,23** *
	1,88	2,90	3,14	2,10	1,79
Size 4	**0,19** *	**0,32** ***	**0,27** ***	**0,25** **	**-0,10**
	1,68	3,43	3,06	2,42	0,83
Size 5	**0,25** ***	**0,17** **	**0,18** **	**0,12**	**0,02**
	2,77	2,29	2,08	1,28	0,18

Panel D: Momentum

	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**-0,57** ***	**0,23** **	**0,48** ***	**0,65** ***	**0,82** ***
	3,98	2,36	5,43	7,78	7,37
Size 2	**-0,49** ***	**0,13**	**0,27** ***	**0,43** ***	**0,48** ***
	3,40	1,33	2,93	4,67	4,47
Size 3	**-0,38** ***	**0,10**	**0,23** **	**0,30** ***	**0,32** ***
	2,82	1,06	2,55	3,50	2,90
Size 4	**-0,41** ***	**0,10**	**0,27** ***	**0,27** ***	**0,34** ***
	2,78	0,90	2,63	2,98	3,02
Size 5	**-0,46** ***	**0,04**	**0,19** **	**0,30** ***	**0,22** **
	3,51	0,44	2,48	3,11	1,96

Table 2: **NORTH AMERICA: Fitted intercepts from CAPM regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding *t*-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**-0,29**	**0,01**	**0,37** *	**0,31** *	**0,59** ***	**0,18**	**0,60** ***	**0,47** ***	**0,51** ***	**0,44** **
	1,00	0,06	1,81	1,88	3,31	0,78	3,72	2,97	2,97	2,33
Size 2	**-0,20**	**0,02**	**0,26** *	**0,27** **	**0,29** *	**-0,23**	**0,33** **	**0,40** ***	**0,54** ***	**0,49** ***
	0,87	0,11	1,81	2,12	1,79	1,19	2,45	2,88	3,52	2,95
Size 3	**0,16**	**0,12**	**0,24** **	**0,25** **	**0,37** ***	**-0,09**	**0,26** **	**0,34** ***	**0,41** ***	**0,40** ***
	0,79	0,83	2,26	2,13	2,61	0,53	2,29	2,88	3,31	2,92
Size 4	**0,26**	**0,17** *	**0,35** ***	**0,26** **	**0,26** *	**-0,11**	**0,31** ***	**0,40** ***	**0,34** ***	**0,47** ***
	1,39	1,79	3,83	2,34	1,95	0,72	3,64	4,23	3,31	4,26
Size 5	**0,28** ***	**0,24** ***	**0,18** **	**0,18** **	**0,05**	**-0,32** ***	**-0,01**	**0,12**	**0,35** ***	**0,35** ***
	2,71	3,50	2,33	2,01	0,35	2,79	0,10	1,61	5,19	4,10

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**0,57** **	**0,61** ***	**0,52** ***	**0,41** ***	**-0,20**	**-0,48** **	**0,34** **	**0,62** ***	**0,81** ***	**0,94** ***
	2,51	3,59	3,27	2,65	0,92	2,02	2,28	4,11	4,70	3,87
Size 2	**0,25**	**0,48** ***	**0,38** ***	**0,28** **	**-0,33** *	**-0,39** *	**0,32** **	**0,35** ***	**0,49** ***	**0,60** **
	1,60	3,29	2,68	1,96	1,72	1,82	2,29	2,62	3,42	2,53
Size 3	**0,31** **	**0,40** ***	**0,41** ***	**0,21** *	**-0,15**	**-0,36** *	**0,21** *	**0,35** ***	**0,40** ***	**0,53** **
	2,54	3,42	3,60	1,83	0,76	1,79	1,72	3,17	3,21	2,45
Size 4	**0,36** ***	**0,48** ***	**0,38** ***	**0,33** ***	**-0,10**	**-0,27**	**0,25** **	**0,37** ***	**0,41** ***	**0,56** ***
	3,44	4,84	4,14	3,53	0,56	1,35	2,28	4,29	4,44	2,69
Size 5	**0,31** ***	**0,25** ***	**0,22** ***	**0,17** *	**-0,02**	**-0,21**	**0,17** *	**0,21** ***	**0,34** ***	**0,48** **
	4,13	3,19	3,25	1,94	0,15	1,14	1,68	2,58	3,78	2,53

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**-1,03** ***	**-0,52** ***	**-0,10**	**-0,03**	**0,16**	**-0,34** **	**0,20**	**0,13**	**0,13**	**0,03**
	6,01	3,00	0,61	0,25	1,16	2,01	1,64	1,26	0,95	0,23
Size 2	**-0,72** ***	**-0,39** ***	**-0,02**	**0,03**	**-0,06**	**-0,59** ***	**0,09**	**0,15**	**0,23** **	**0,18**
	4,10	2,73	0,14	0,22	0,50	3,93	0,75	1,25	2,02	1,32
Size 3	**-0,26**	**-0,10**	**0,10**	**0,04**	**0,12**	**-0,43** ***	**0,10**	**0,15**	**0,19** *	**0,16**
	1,63	0,79	0,83	0,40	1,09	3,08	0,87	1,38	1,68	1,37
Size 4	**-0,04**	**0,05**	**0,25** **	**0,09**	**0,05**	**-0,33** **	**0,22** **	**0,29** **	**0,18** *	**0,28** **
	0,23	0,45	2,39	0,89	0,41	2,26	2,16	2,51	1,71	2,35
Size 5	**0,13**	**0,17** *	**0,08**	**0,05**	**-0,24** **	**-0,46** ***	**-0,09**	**0,04**	**0,28** ***	**0,23** **
	1,30	1,83	0,80	0,58	2,04	3,18	0,86	0,42	3,38	2,35

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**0,04**	**0,22**	**0,19**	**0,07**	**-0,67** ***	**-1,02** ***	**0,04**	**0,28** **	**0,42** ***	**0,29** *
	0,24	1,54	1,59	0,57	4,40	6,00	0,32	2,36	3,35	1,65
Size 2	**-0,06**	**0,15**	**0,06**	**0,04**	**-0,68** ***	**-0,82** ***	**0,08**	**0,10**	**0,19**	**0,07**
	0,46	1,16	0,49	0,30	4,52	4,41	0,70	0,84	1,56	0,44
Size 3	**0,12**	**0,23** **	**0,22** **	**0,04**	**-0,53** ***	**-0,71** ***	**0,03**	**0,18**	**0,19** *	**0,00**
	0,93	2,06	2,03	0,32	3,68	4,17	0,24	1,49	1,67	0,01
Size 4	**0,22** **	**0,32** ***	**0,27** ***	**0,23** **	**-0,36** **	**-0,65** ***	**0,11**	**0,25** ***	**0,29** ***	**0,04**
	2,01	3,22	2,84	2,25	2,18	3,80	1,13	2,59	2,81	0,28
	2,63	1,67	1,65	0,71	1,85	4,59	0,12	1,09	2,66	0,56

Table 3: **EUROPE: Fitted intercepts from CAPM regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding t-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,18**		**0,15**		**0,24**	*	**0,36**	***	**0,51**	***	**-0,07**		**0,45**	***	**0,52**	***	**0,65**	***	**0,53**	***
	1,11		1,08		1,86		3,00		3,96		0,49		3,80		4,18		5,47		4,12	
Size 2	**0,05**		**0,26**	**	**0,26**	**	**0,40**	***	**0,45**	***	**-0,05**		**0,33**	***	**0,35**	***	**0,50**	***	**0,69**	***
	0,33		2,09		2,26		3,36		3,43		0,39		2,89		2,94		4,29		5,45	
Size 3	**0,14**		**0,34**	***	**0,26**	**	**0,27**	**	**0,37**	***	**-0,03**		**0,31**	***	**0,44**	***	**0,33**	***	**0,51**	***
	0,94		3,17		2,52		2,54		3,01		0,30		3,30		4,38		3,24		4,61	
Size 4	**0,28**	**	**0,30**	***	**0,25**	***	**0,24**	**	**0,26**	**	**-0,06**		**0,26**	***	**0,34**	***	**0,44**	***	**0,42**	***
	2,55		3,40		3,22		2,48		2,10		0,61		2,95		3,99		5,20		4,93	
Size 5	**0,18**	*	**0,29**	***	**0,22**	***	**0,27**	***	**0,06**		**-0,24**	**	**0,18**	**	**0,22**	***	**0,18**	***	**0,39**	***
	1,71		3,92		3,55		3,30		0,44		1,99		2,34		3,08		2,68		4,66	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,31**	**	**0,48**	***	**0,49**	***	**0,43**	***	**-0,03**		**-0,66**	***	**0,13**		**0,43**	***	**0,76**	***	**1,37**	***
	2,36		4,29		4,23		3,58		0,21		3,74		1,12		3,79		6,46		8,02	
Size 2	**0,28**	**	**0,47**	***	**0,48**	***	**0,36**	***	**0,10**		**-0,53**	***	**0,11**		**0,42**	***	**0,72**	***	**1,09**	***
	2,18		4,12		4,27		3,15		0,76		2,95		0,87		3,83		6,34		6,91	
Size 3	**0,31**	***	**0,35**	***	**0,41**	***	**0,23**	**	**0,07**		**-0,33**	**	**0,14**		**0,33**	***	**0,59**	***	**0,83**	***
	2,80		3,27		4,17		2,34		0,55		1,97		1,30		3,33		5,60		5,42	
Size 4	**0,29**	***	**0,26**	***	**0,39**	***	**0,34**	***	**0,08**		**-0,27**		**0,20**	**	**0,34**	***	**0,47**	***	**0,75**	***
	2,95		2,95		4,99		4,31		0,74		1,54		2,20		3,92		4,98		5,48	
Size 5	**0,20**	**	**0,31**	***	**0,17**	***	**0,12**		**0,17**	*	**-0,33**	*	**0,09**		**0,30**	***	**0,38**	***	**0,45**	***
	2,42		4,35		2,87		1,58		1,95		1,86		0,88		4,34		4,25		2,97	

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,40**	***	**-0,03**		**0,10**		**0,24**	**	**0,36**	***	**-0,21**	*	**0,34**	***	**0,38**	***	**0,53**	***	**0,39**	***
	3,39		0,23		0,82		2,20		3,58		1,75		2,96		3,62		4,31		3,71	
Size 2	**-0,12**		**0,13**		**0,15**		**0,30**	**	**0,32**	**	**-0,17**		**0,22**	*	**0,24**	*	**0,39**	***	**0,57**	***
	1,06		1,22		1,34		2,52		2,52		1,38		1,87		1,94		3,19		4,60	
Size 3	**-0,01**		**0,25**	**	**0,18**		**0,18**		**0,26**	**	**-0,11**		**0,24**	**	**0,37**	***	**0,24**	**	**0,43**	***
	0,07		2,20		1,54		1,39		2,06		0,94		2,25		2,87		2,14		3,47	
Size 4	**0,18**	*	**0,23**	**	**0,20**	*	**0,18**		**0,15**		**-0,15**		**0,19**	*	**0,29**	***	**0,39**	***	**0,36**	***
	1,75		2,18		1,76		1,56		1,23		1,24		1,74		2,81		3,33		3,69	
Size 5	**0,09**		**0,24**	**	**0,20**	*	**0,21**	*	**-0,05**		**-0,34**	***	**0,14**		**0,19**	*	**0,14**		**0,32**	***
	0,87		2,48		1,74		1,71		0,39		2,70		1,15		1,68		1,41		3,33	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,16**		**0,37**	***	**0,37**	***	**0,29**	***	**-0,24**	*	**-0,86**	***	**0,03**		**0,31**	***	**0,63**	***	**1,12**	***
	1,38		3,39		3,72		2,76		1,95		5,79		0,25		2,88		6,64		10,72	
Size 2	**0,14**		**0,37**	***	**0,38**	***	**0,26**	**	**-0,04**		**-0,71**	***	**-0,01**		**0,32**	***	**0,60**	***	**0,88**	***
	1,19		3,15		3,69		2,27		0,27		4,16		0,07		3,13		5,71		6,75	
Size 3	**0,22**	*	**0,26**	**	**0,33**	***	**0,15**		**-0,04**		**-0,49**	***	**0,07**		**0,25**	*	**0,50**	***	**0,63**	***
	1,83		2,51		3,10		1,39		0,27		3,06		0,53		1,87		4,59		5,28	
Size 4	**0,20**	*	**0,21**	*	**0,34**	***	**0,29**	***	**0,00**		**-0,44**	**	**0,15**		**0,27**	**	**0,40**	***	**0,60**	***
	1,78		1,73		3,24		2,60		0,01		2,37		1,08		2,51		3,78		5,56	
Size 5	**0,15**		**0,27**	***	**0,14**		**0,07**		**0,11**		**-0,49**	***	**0,03**		**0,25**	**	**0,30**	***	**0,27**	**
	1,40		2,58		1,42		0,63		0,97		3,30		0,21		2,36		3,01		2,47	

Table 4: **ASIA-PACIFIC: Fitted intercepts from CAPM regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding *t*-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,05**		**-0,13**		**0,16**		**0,46**	**	**0,86**	***	**0,20**		**0,80**	***	**0,65**	***	**0,69**	***	**0,65**	***
	0,19		0,59		0,82		2,31		3,84		0,81		3,82		3,46		3,58		3,66	
Size 2	**-0,53**	***	**-0,25**		**-0,24**		**0,07**		**0,24**		**-0,40**	*	**-0,06**		**0,20**		**0,24**		**0,25**	
	2,88		1,44		1,54		0,46		1,17		1,83		0,33		1,31		1,40		1,58	
Size 3	**-0,36**	*	**-0,11**		**0,10**		**0,15**		**0,17**		**-0,45**	**	**0,19**		**0,02**		**0,38**	***	**0,30**	**
	1,95		0,69		0,68		0,98		0,90		2,55		1,14		0,15		2,82		1,98	
Size 4	**0,19**		**0,28**	**	**0,10**		**0,39**	***	**0,26**		**-0,22**		**0,28**	**	**0,27**	*	**0,39**	***	**0,46**	***
	1,22		2,03		0,79		3,17		1,54		1,33		2,08		1,81		3,07		3,25	
Size 5	**0,15**		**0,28**	***	**0,27**	***	**0,23**	*	**0,29**		**-0,08**		**0,21**		**0,27**	***	**0,40**	***	**0,27**	**
	1,12		3,13		2,80		1,84		1,42		0,47		1,49		2,77		3,68		2,13	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,58**	**	**0,72**	***	**0,77**	***	**0,54**	***	**-0,02**		**-0,34**		**0,29**		**0,74**	***	**1,17**	***	**1,11**	***
	2,49		3,89		3,56		2,69		0,10		1,47		1,53		4,28		5,73		4,58	
Size 2	**-0,01**		**0,33**	**	**0,13**		**0,03**		**-0,53**	***	**-1,17**	***	**0,05**		**0,26**	*	**0,53**	***	**0,63**	***
	0,05		2,03		0,84		0,16		2,69		5,42		0,27		1,76		3,29		2,74	
Size 3	**0,01**		**0,46**	***	**0,10**		**0,17**		**-0,42**	**	**-0,85**	***	**0,03**		**0,23**	*	**0,59**	***	**0,44**	**
	0,09		3,12		0,71		1,16		2,39		4,04		0,25		1,72		4,31		2,06	
Size 4	**0,11**		**0,30**	**	**0,43**	***	**0,32**	**	**-0,05**		**-0,39**	*	**0,05**		**0,30**	**	**0,46**	***	**0,39**	*
	0,89		2,37		3,47		2,08		0,31		1,91		0,36		2,47		3,86		1,92	
Size 5	**0,46**	***	**0,21**	*	**0,24**	**	**0,23**	**	**0,01**		**0,18**		**0,07**		**0,34**	***	**0,37**	***	**0,53**	***
	3,36		1,82		2,57		1,99		0,05		0,77		0,44		3,32		3,11		2,64	

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,59**	***	**-0,44**	***	**-0,12**		**0,13**		**0,47**	***	**-0,22**		**0,43**	***	**0,38**	**	**0,38**	***	**0,40**	***
	2,96		2,79		0,74		0,85		3,07		1,35		3,05		2,57		2,66		2,97	
Size 2	**-0,80**	***	**-0,48**	***	**-0,43**	***	**-0,11**		**-0,05**		**-0,77**	***	**-0,30**	*	**0,02**		**0,03**		**0,06**	
	5,49		2,75		2,94		0,83		0,27		4,31		1,92		0,15		0,17		0,46	
Size 3	**-0,64**	***	**-0,30**	**	**-0,07**		**-0,02**		**-0,04**		**-0,69**	***	**0,00**		**-0,11**		**0,23**	*	**0,14**	
	4,16		1,97		0,47		0,15		0,24		4,25		0,02		0,76		1,81		1,00	
Size 4	**0,01**		**0,15**		**-0,03**		**0,28**	**	**0,11**		**-0,39**	**	**0,18**		**0,09**		**0,26**	*	**0,29**	**
	0,06		1,02		0,22		2,01		0,66		2,49		1,08		0,72		1,83		2,01	
Size 5	**0,00**		**0,20**	*	**0,20**		**0,12**		**0,02**		**-0,22**		**0,06**		**0,21**	*	**0,30**	**	**0,13**	
	0,03		1,77		1,51		0,81		0,11		1,36		0,36		1,65		2,36		1,08	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,15**		**0,41**	***	**0,41**	***	**0,21**		**-0,39**	**	**-0,68**	***	**0,00**		**0,46**	***	**0,83**	***	**0,69**	***
	0,97		3,04		2,70		1,42		2,35		3,92		0,02		3,36		5,83		3,78	
Size 2	**-0,24**		**0,13**		**-0,07**		**-0,23**		**-0,79**	***	**-1,43**	***	**-0,14**		**0,08**		**0,33**	**	**0,25**	
	1,59		1,04		0,49		1,30		4,59		7,19		0,92		0,61		2,46		1,48	
Size 3	**-0,20**		**0,27**	**	**-0,05**		**0,03**		**-0,62**	***	**-1,06**	***	**-0,10**		**0,08**		**0,44**	***	**0,14**	
	1,38		2,01		0,29		0,20		3,97		5,57		0,74		0,69		3,16		0,77	
Size 4	**0,00**		**0,18**		**0,34**	**	**0,17**		**-0,22**		**-0,58**	***	**-0,07**		**0,17**		**0,34**	***	**0,12**	
	0,02		1,31		2,19		1,05		1,38		2,77		0,48		1,29		2,60		0,71	
Size 5	**0,30**	**	**0,10**		**0,17**		**0,13**		**-0,17**		**-0,05**		**-0,05**		**0,25**	**	**0,26**	**	**0,24**	
	2,24		0,81		1,32		0,82		1,14		0,26		0,32		2,10		2,06		1,53	

Table 5: **JAPAN: Fitted intercepts from CAPM regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding *t*-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**0,31**	**0,43** *	**0,48** **	**0,45** **	**0,56** ***	**0,36** *	**0,52** ***	**0,47** **	**0,46** **	**0,73** ***
	1,12	1,93	2,36	2,53	3,00	1,76	2,85	2,53	2,40	3,21
Size 2	**0,26**	**0,06**	**0,21**	**0,36** **	**0,27**	**0,11**	**0,26** *	**0,31** *	**0,32** *	**0,34** *
	1,02	0,31	1,17	2,28	1,61	0,60	1,65	1,82	1,84	1,75
Size 3	**0,01**	**0,08**	**0,16**	**0,19**	**0,33** **	**0,12**	**0,17**	**0,22** *	**0,22**	**0,26**
	0,07	0,53	1,13	1,37	2,14	0,70	1,18	1,67	1,60	1,55
Size 4	**-0,02**	**0,16**	**0,22** **	**0,27** **	**0,26** *	**-0,01**	**0,18**	**0,32** ***	**0,32** ***	**0,17**
	0,13	1,48	2,06	2,32	1,79	0,10	1,54	3,01	2,97	1,45
Size 5	**0,13**	**0,22** **	**0,22** **	**0,33** ***	**0,47** ***	**0,06**	**0,17**	**0,32** ***	**0,22** **	**0,27** ***
	1,06	2,28	2,38	3,02	2,69	0,39	1,55	3,61	2,34	2,73

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**0,42** **	**0,43** **	**0,55** ***	**0,45** **	**0,53** **	**0,47** *	**0,57** ***	**0,54** ***	**0,62** ***	**0,42** *
	1,98	2,34	2,87	2,26	2,39	1,94	3,08	3,10	3,42	1,76
Size 2	**0,27**	**0,25**	**0,28** *	**0,34** *	**0,15**	**0,18**	**0,25**	**0,32** **	**0,30** *	**0,31**
	1,40	1,48	1,72	1,93	0,82	0,80	1,50	1,97	1,82	1,56
Size 3	**0,21**	**0,23**	**0,19**	**0,15**	**0,16**	**0,25**	**0,13**	**0,21**	**0,28** **	**0,24**
	1,29	1,53	1,37	0,96	1,00	1,30	0,86	1,53	2,03	1,29
Size 4	**0,20**	**0,20**	**0,30** ***	**0,11**	**0,19**	**0,26**	**0,25** *	**0,19** *	**0,12**	**0,30** *
	1,51	1,63	2,76	1,05	1,56	1,40	1,93	1,69	1,08	1,76
Size 5	**0,21** *	**0,14**	**0,11**	**0,26** ***	**0,18**	**0,18**	**0,07**	**0,03**	**0,26** **	**0,26**
	1,67	1,33	1,24	2,57	1,46	0,90	0,60	0,35	2,55	1,51

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**0,21**	**0,36** **	**0,42** **	**0,40** ***	**0,50** ***	**0,29** *	**0,46** ***	**0,42** ***	**0,39** ***	**0,65** ***
	1,16	2,47	2,50	2,94	3,70	1,94	3,44	3,27	2,79	4,18
Size 2	**0,17**	**0,01**	**0,16**	**0,32** **	**0,23**	**0,06**	**0,22** *	**0,26** **	**0,27** **	**0,28** **
	0,94	0,07	1,01	2,43	1,62	0,40	1,79	2,04	2,15	2,17
Size 3	**-0,04**	**0,05**	**0,12**	**0,15**	**0,29** **	**0,07**	**0,13**	**0,19**	**0,19**	**0,22**
	0,28	0,36	0,96	1,33	2,45	0,54	1,04	1,35	1,57	1,36
Size 4	**-0,06**	**0,14**	**0,20** *	**0,24** **	**0,23** *	**-0,04**	**0,15**	**0,30** ***	**0,31** **	**0,15**
	0,39	1,15	1,77	2,12	1,75	0,29	1,18	2,86	2,38	1,26
Size 5	**0,11**	**0,20** *	**0,21** *	**0,31** ***	**0,42** ***	**0,03**	**0,15**	**0,31** ***	**0,21** *	**0,26** **
	0,89	1,86	1,92	2,93	2,83	0,22	1,06	2,89	1,89	2,34

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**0,35** **	**0,38** ***	**0,50** ***	**0,39** ***	**0,46** ***	**0,39** **	**0,51** ***	**0,48** ***	**0,57** ***	**0,34** *
	2,18	3,14	3,46	2,68	3,12	2,19	4,13	4,17	4,29	1,88
Size 2	**0,21**	**0,20**	**0,24** *	**0,29** *	**0,10**	**0,11**	**0,20** *	**0,27** **	**0,25** **	**0,24** *
	1,47	1,64	1,91	1,86	0,64	0,67	1,66	2,12	2,15	1,72
Size 3	**0,17**	**0,20**	**0,16**	**0,11**	**0,12**	**0,19**	**0,09**	**0,17**	**0,25** **	**0,18**
	1,29	1,48	1,27	0,98	0,82	1,20	0,70	1,61	2,15	1,33
Size 4	**0,17**	**0,17** *	**0,28** **	**0,09**	**0,17**	**0,21**	**0,22** **	**0,17**	**0,10**	**0,25** *
	1,25	1,72	2,54	0,76	1,40	1,43	1,96	1,41	0,85	1,96
Size 5	**0,18**	**0,12**	**0,10**	**0,24** **	**0,16**	**0,13**	**0,05**	**0,02**	**0,24** ***	**0,21**
	1,45	0,94	0,88	2,14	1,28	0,78	0,36	0,18	2,62	1,61

Table 6: **WORLD: Fitted intercepts from Fama-French three-factor regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding *t*-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,15**	*	**0,10**		**0,30**	***	**0,29**	***	**0,54**	***	**0,15**	**	**0,49**	***	**0,44**	***	**0,48**	***	**0,51**	***
	1,79		1,37		4,45		5,64		9,50		1,97		10,14		8,53		7,88		7,40	
Size 2	**-0,06**		**0,14**	***	**0,15**	***	**0,20**	***	**0,20**	***	**-0,11**	*	**0,21**	***	**0,20**	***	**0,26**	***	**0,40**	***
	1,04		2,80		3,71		4,68		4,74		1,87		4,70		4,60		5,28		7,15	
Size 3	**0,08**		**0,10**	*	**0,13**	**	**0,15**	***	**0,17**	***	**-0,09**		**0,12**	**	**0,25**	***	**0,24**	***	**0,26**	***
	1,32		1,73		2,54		3,04		3,62		1,36		2,24		5,19		4,49		4,42	
Size 4	**0,27**	***	**0,18**	***	**0,12**	**	**0,15**	***	**0,08**	*	**-0,07**		**0,18**	***	**0,25**	***	**0,24**	***	**0,26**	***
	3,99		3,15		2,49		2,80		1,66		0,91		3,65		4,84		4,37		4,47	
Size 5	**0,40**	***	**0,28**	***	**0,17**	***	**0,12**	***	**-0,03**		**-0,35**	***	**0,04**		**0,21**	***	**0,33**	***	**0,44**	***
	8,47		6,49		3,82		2,63		0,51		3,98		0,70		4,69		6,95		8,45	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,40**	***	**0,52**	***	**0,50**	***	**0,43**	***	**0,00**		**-0,49**	***	**0,22**	***	**0,48**	***	**0,71**	***	**1,02**	***
	5,59		11,36		10,26		8,09		0,04		3,73		3,53		8,60		11,04		9,15	
Size 2	**0,12**	**	**0,30**	***	**0,27**	***	**0,24**	***	**-0,09**		**-0,42**	***	**0,09**		**0,24**	***	**0,45**	***	**0,68**	***
	2,36		6,31		6,80		5,40		1,45		3,01		1,46		4,51		8,01		6,86	
Size 3	**0,16**	***	**0,21**	***	**0,27**	***	**0,19**	***	**-0,11**	*	**-0,32**	**	**0,05**		**0,19**	***	**0,29**	***	**0,51**	***
	2,65		4,03		5,60		3,61		1,73		2,30		0,73		3,42		4,83		4,81	
Size 4	**0,13**	**	**0,26**	***	**0,23**	***	**0,23**	***	**0,06**		**-0,31**	**	**0,06**		**0,24**	***	**0,27**	***	**0,56**	***
	2,37		4,91		4,99		4,40		0,77		2,09		0,89		4,55		4,28		4,70	
Size 5	**0,28**	***	**0,19**	***	**0,21**	***	**0,24**	***	**0,20**	***	**-0,28**	*	**0,08**		**0,23**	***	**0,37**	***	**0,51**	***
	4,81		4,26		5,19		4,57		2,98		1,87		1,08		4,45		5,24		3,86	

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,21**	**	**0,04**		**0,25**	***	**0,29**	***	**0,50**	***	**0,08**		**0,47**	***	**0,41**	***	**0,42**	***	**0,46**	***
	2,15		0,44		2,63		3,20		6,53		0,81		5,24		5,80		5,45		5,42	
Size 2	**-0,11**		**0,12**		**0,12**		**0,18**	**	**0,16**	*	**-0,16**		**0,19**	**	**0,19**	**	**0,29**	***	**0,37**	***
	1,00		1,18		1,27		2,13		1,75		1,54		2,21		2,29		3,11		3,95	
Size 3	**0,02**		**0,09**		**0,13**		**0,07**		**0,17**	*	**-0,15**		**0,08**		**0,22**	**	**0,20**	**	**0,25**	**
	0,22		0,94		1,32		0,64		1,81		1,47		0,84		2,41		2,09		2,30	
Size 4	**0,25**	**	**0,20**	**	**0,13**		**0,11**		**0,05**		**-0,09**		**0,16**	*	**0,23**	**	**0,23**	**	**0,24**	**
	2,56		2,13		1,30		1,12		0,51		0,77		1,81		2,25		2,38		2,18	
Size 5	**0,37**	***	**0,28**	***	**0,14**		**0,10**		**-0,03**		**-0,38**	***	**0,02**		**0,20**	*	**0,31**	***	**0,37**	***
	3,82		3,42		1,38		0,98		0,21		3,03		0,21		1,84		3,86		4,69	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,35**	***	**0,50**	***	**0,46**	***	**0,44**	***	**-0,06**		**-0,60**	***	**0,17**	*	**0,46**	***	**0,66**	***	**0,91**	***
	4,11		6,73		5,53		4,79		0,69		5,35		1,81		5,47		8,31		8,52	
Size 2	**0,11**		**0,29**	***	**0,27**	***	**0,24**	**	**-0,10**		**-0,50**	***	**0,07**		**0,22**	***	**0,37**	***	**0,58**	***
	1,15		3,42		3,57		2,46		0,88		3,37		0,86		2,69		4,40		5,05	
Size 3	**0,14**		**0,18**	**	**0,27**	***	**0,17**	**	**-0,13**		**-0,40**	***	**0,01**		**0,17**		**0,25**	***	**0,41**	***
	1,44		2,07		3,49		2,00		1,15		2,84		0,11		1,60		2,80		4,31	
Size 4	**0,13**		**0,26**	***	**0,21**	**	**0,22**	**	**0,05**		**-0,45**	***	**0,03**		**0,24**	***	**0,22**	**	**0,43**	***
	1,29		2,78		2,24		2,34		0,46		3,09		0,33		2,99		2,29		4,19	
Size 5	**0,26**	***	**0,19**	*	**0,20**	**	**0,23**	**	**0,16**	*	**-0,46**	***	**0,00**		**0,21**	**	**0,34**	***	**0,32**	***
	2,66		1,90		2,16		2,44		1,76		3,14		0,02		2,23		3,97		3,14	

Table 7: **NORTH AMERICA: Fitted intercepts from Fama-French three-factor regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding t-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**-0,21**	**0,04**	**0,36** ***	**0,25** ***	**0,47** ***	**0,17**	**0,49** ***	**0,37** ***	**0,41** ***	**0,34** ***
	1,49	0,39	3,86	3,60	6,73	1,51	7,00	5,05	4,70	3,30
Size 2	**-0,09**	**0,06**	**0,22** ***	**0,19** ***	**0,15** ***	**-0,20** **	**0,24** ***	**0,32** ***	**0,45** ***	**0,39** ***
	0,89	0,74	3,47	3,10	2,75	2,37	3,95	4,56	5,46	3,68
Size 3	**0,25** **	**0,13**	**0,19** ***	**0,15** **	**0,24** ***	**-0,05**	**0,20** ***	**0,26** ***	**0,33** ***	**0,33** ***
	2,50	1,50	2,71	2,18	3,54	0,50	2,80	3,21	3,69	3,35
Size 4	**0,38** ***	**0,18** **	**0,31** ***	**0,17** **	**0,13** *	**-0,07**	**0,27** ***	**0,35** ***	**0,27** ***	**0,44** ***
	3,57	2,27	3,97	2,11	1,85	0,55	4,06	4,41	3,34	4,53
Size 5	**0,37** ***	**0,27** ***	**0,15** **	**0,12** *	**-0,08**	**-0,30** ***	**-0,02**	**0,10**	**0,37** ***	**0,39** ***
	6,10	4,32	2,26	1,89	0,91	2,60	0,20	1,58	6,14	5,51

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**0,50** ***	**0,51** ***	**0,44** ***	**0,34** ***	**-0,19** *	**-0,55** ***	**0,25** ***	**0,53** ***	**0,75** ***	**0,94** ***
	4,40	7,01	6,29	4,88	1,86	3,20	3,17	7,38	7,83	5,92
Size 2	**0,17** **	**0,39** ***	**0,29** ***	**0,23** ***	**-0,28** ***	**-0,46** ***	**0,23** ***	**0,27** ***	**0,44** ***	**0,63** ***
	2,46	5,21	4,15	3,62	3,21	2,61	2,76	3,40	5,21	4,14
Size 3	**0,22** ***	**0,31** ***	**0,33** ***	**0,17** **	**-0,08**	**-0,42** **	**0,13**	**0,27** ***	**0,35** ***	**0,56** ***
	2,81	4,28	4,61	2,28	0,70	2,32	1,40	3,53	3,97	3,66
Size 4	**0,28** ***	**0,40** ***	**0,32** ***	**0,32** ***	**-0,02**	**-0,34** *	**0,18** **	**0,32** ***	**0,39** ***	**0,60** ***
	3,70	5,61	4,46	4,10	0,14	1,78	1,97	4,30	4,52	3,52
Size 5	**0,29** ***	**0,21** ***	**0,21** ***	**0,21** ***	**0,09**	**-0,24**	**0,14**	**0,19** ***	**0,34** ***	**0,56** ***
	4,23	3,46	3,88	2,89	0,84	1,34	1,56	2,76	4,00	3,28

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**-0,57** ***	**-0,24** **	**0,14**	**0,10**	**0,32** ***	**-0,01**	**0,39** ***	**0,25** ***	**0,25** ***	**0,12**
	4,96	2,39	1,50	1,27	3,65	0,14	4,02	2,89	2,65	1,06
Size 2	**-0,29** ***	**-0,09**	**0,08**	**0,09**	**0,07**	**-0,33** ***	**0,11**	**0,18** **	**0,31** ***	**0,19** *
	2,62	0,99	0,88	0,99	0,67	2,79	1,10	2,01	3,33	1,93
Size 3	**0,03**	**-0,05**	**0,09**	**0,04**	**0,14**	**-0,30** ***	**0,08**	**0,13**	**0,20** **	**0,16**
	0,27	0,41	0,81	0,39	1,35	2,80	0,84	1,34	2,19	1,53
Size 4	**0,19** *	**0,08**	**0,22** **	**0,06**	**-0,03**	**-0,34** ***	**0,13**	**0,17** *	**0,09**	**0,29** ***
	1,85	0,79	2,46	0,60	0,32	2,96	1,56	1,83	0,93	3,05
Size 5	**0,28** ***	**0,13**	**0,05**	**0,03**	**-0,22** *	**-0,48** ***	**-0,16**	**0,02**	**0,26** ***	**0,29** ***
	3,20	1,55	0,57	0,26	1,94	4,14	1,60	0,15	2,92	3,66

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**0,24** **	**0,36** ***	**0,32** ***	**0,25** ***	**-0,39** ***	**-0,91** ***	**0,14** *	**0,38** ***	**0,55** ***	**0,53** ***
	2,55	4,29	3,66	2,87	3,57	6,42	1,76	4,39	6,50	4,72
Size 2	**0,06**	**0,22** **	**0,17** **	**0,13**	**-0,45** ***	**-0,78** ***	**0,08**	**0,11**	**0,27** ***	**0,26** **
	0,65	2,51	2,14	1,24	4,18	5,18	0,80	1,27	3,23	2,26
Size 3	**0,12**	**0,19** **	**0,22** **	**0,01**	**-0,29** **	**-0,77** ***	**-0,01**	**0,16** *	**0,17** *	**0,14**
	1,14	2,26	2,26	0,08	2,50	4,71	0,10	1,71	1,96	1,48
Size 4	**0,19** *	**0,30** ***	**0,23** **	**0,19** **	**-0,28** **	**-0,74** ***	**0,01**	**0,20** **	**0,22** **	**0,12**
	1,85	3,09	2,49	2,19	2,10	5,17	0,11	2,37	2,39	1,05
Size 5	**0,15**	**0,08**	**0,10**	**0,09**	**-0,07**	**-0,68** ***	**-0,03**	**0,05**	**0,14**	**0,04**
	1,58	0,96	1,20	1,00	0,57	4,40	0,25	0,55	1,45	0,37

Table 8: **EUROPE: Fitted intercepts from Fama-French three-factor regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding t-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,23**	**	**0,08**		**0,14**	**	**0,23**	***	**0,34**	***	**-0,17**	**	**0,32**	***	**0,39**	***	**0,54**	***	**0,43**	***
	2,48		1,10		2,28		4,36		6,58		2,50		5,76		7,58		10,12		6,28	
Size 2	**0,03**		**0,19**	***	**0,15**	***	**0,25**	***	**0,27**	***	**-0,16**	**	**0,20**	***	**0,21**	***	**0,38**	***	**0,60**	***
	0,40		3,00		2,76		4,68		4,98		2,42		3,59		3,83		6,45		9,16	
Size 3	**0,15**	*	**0,28**	***	**0,16**	**	**0,14**	**	**0,21**	***	**-0,12**	*	**0,21**	***	**0,35**	***	**0,25**	***	**0,43**	***
	1,81		4,22		2,35		2,22		3,11		1,70		3,51		5,52		3,58		6,02	
Size 4	**0,33**	***	**0,27**	***	**0,19**	***	**0,14**	*	**0,11**		**-0,15**	*	**0,18**	**	**0,27**	***	**0,40**	***	**0,40**	***
	4,15		3,47		2,90		1,91		1,40		1,65		2,49		3,81		5,38		5,63	
Size 5	**0,31**	***	**0,35**	***	**0,24**	***	**0,25**	***	**-0,04**		**-0,28**	**	**0,16**	**	**0,25**	***	**0,22**	***	**0,47**	***
	4,89		5,41		4,14		3,61		0,47		2,53		2,53		3,98		3,35		7,19	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,17**	***	**0,35**	***	**0,37**	***	**0,31**	***	**-0,10**		**-0,81**	***	**0,02**		**0,31**	***	**0,65**	***	**1,27**	***
	2,68		6,26		6,44		5,41		1,28		6,30		0,22		4,88		9,31		10,54	
Size 2	**0,12**	*	**0,34**	***	**0,36**	***	**0,25**	***	**0,03**		**-0,68**	***	**-0,02**		**0,30**	***	**0,62**	***	**1,00**	***
	1,92		5,84		5,94		4,63		0,47		4,76		0,29		4,81		8,63		9,62	
Size 3	**0,19**	***	**0,22**	***	**0,30**	***	**0,15**	**	**0,04**		**-0,47**	***	**0,03**		**0,23**	***	**0,51**	***	**0,77**	***
	2,64		3,42		4,69		2,28		0,48		3,17		0,43		3,41		6,77		6,32	
Size 4	**0,19**	**	**0,17**	**	**0,33**	***	**0,29**	***	**0,09**		**-0,36**	**	**0,14**	*	**0,27**	***	**0,43**	***	**0,72**	***
	2,52		2,51		4,99		4,17		1,06		2,18		1,67		3,68		5,09		5,96	
Size 5	**0,19**	**	**0,31**	***	**0,19**	***	**0,18**	***	**0,22**	***	**-0,36**	**	**0,07**		**0,30**	***	**0,43**	***	**0,51**	***
	2,50		4,87		3,53		2,75		2,80		2,08		0,75		4,93		5,02		3,55	

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,36**	***	**-0,05**		**0,05**		**0,12**		**0,23**	***	**-0,29**	***	**0,23**	***	**0,21**	**	**0,39**	***	**0,32**	***
	3,37		0,42		0,50		1,25		2,60		2,99		2,63		2,46		3,92		2,99	
Size 2	**-0,09**		**0,09**		**0,08**		**0,16**	*	**0,19**	*	**-0,29**	**	**0,08**		**0,10**		**0,29**	***	**0,49**	***
	0,70		0,81		0,76		1,66		1,82		2,56		0,76		0,98		3,06		4,66	
Size 3	**0,03**		**0,12**		**0,01**		**-0,02**		**0,05**		**-0,23**	**	**0,09**		**0,21**	**	**0,08**		**0,29**	***
	0,23		1,10		0,06		0,23		0,49		2,13		0,79		2,32		0,81		2,65	
Size 4	**0,24**	**	**0,18**	*	**0,06**		**-0,01**		**-0,03**		**-0,26**	**	**0,02**		**0,16**		**0,22**	**	**0,28**	***
	2,05		1,88		0,54		0,05		0,30		2,42		0,20		1,30		2,01		2,64	
Size 5	**0,19**	**	**0,24**	**	**0,09**		**0,13**		**-0,16**		**-0,45**	***	**0,00**		**0,09**		**0,13**		**0,36**	***
	2,09		2,56		0,85		1,30		1,29		3,44		0,02		0,85		1,18		3,82	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,05**		**0,26**	***	**0,28**	***	**0,17**	*	**-0,25**	**	**-1,06**	***	**-0,07**		**0,18**	*	**0,53**	***	**1,10**	***
	0,61		2,92		3,27		1,80		2,24		8,45		0,70		1,84		6,04		9,44	
Size 2	**0,02**		**0,21**	**	**0,24**	**	**0,12**		**-0,05**		**-0,87**	***	**-0,18**		**0,18**	*	**0,46**	***	**0,86**	***
	0,21		2,22		2,51		1,19		0,43		5,82		1,51		1,79		4,46		8,36	
Size 3	**0,10**		**0,12**		**0,18**	*	**0,08**		**-0,08**		**-0,66**	***	**-0,10**		**0,08**		**0,36**	***	**0,56**	***
	0,88		1,31		1,87		0,63		0,64		4,36		0,85		0,70		3,75		4,93	
Size 4	**0,07**		**0,02**		**0,20**	**	**0,16**		**-0,05**		**-0,62**	***	**0,01**		**0,15**		**0,27**	***	**0,56**	***
	0,78		0,25		2,08		1,34		0,47		4,01		0,07		1,34		2,65		5,68	
Size 5	**0,05**		**0,18**	*	**0,10**		**0,09**		**0,14**		**-0,74**	***	**-0,09**		**0,19**	**	**0,31**	***	**0,28**	**
	0,59		1,96		0,95		0,70		1,31		4,77		0,71		2,08		3,42		2,06	

Table 9: **ASIA-PACIFIC: Fitted intercepts from Fama-French three-factor regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding t-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**0,32**		**0,15**		**0,36**	***	**0,59**	***	**0,84**	***	**0,41**	***	**0,87**	***	**0,63**	***	**0,71**	***	**0,78**	***
	1,64		1,08		3,17		5,53		8,17		3,08		7,25		6,04		6,44		6,97	
Size 2	**-0,28**	**	**-0,02**		**-0,10**		**0,10**		**0,05**		**-0,40**	***	**-0,16**		**0,23**	**	**0,28**	**	**0,46**	***
	2,11		0,22		0,89		0,99		0,52		3,11		1,49		2,31		2,34		4,21	
Size 3	**-0,05**		**0,10**		**0,25**	**	**0,17**		**-0,05**		**-0,42**	***	**0,07**		**0,07**		**0,48**	***	**0,49**	***
	0,34		0,73		2,03		1,32		0,40		3,36		0,55		0,62		4,03		3,91	
Size 4	**0,40**	***	**0,40**	***	**0,21**	*	**0,37**	***	**-0,04**		**-0,31**	**	**0,26**	*	**0,31**	**	**0,42**	***	**0,61**	***
	2,86		3,03		1,69		3,16		0,31		2,09		1,95		2,08		3,49		4,65	
Size 5	**0,35**	***	**0,32**	***	**0,24**	**	**0,02**		**-0,19**		**-0,35**	**	**-0,04**		**0,21**	**	**0,51**	***	**0,47**	***
	3,33		3,82		2,48		0,18		1,38		2,48		0,34		2,17		4,95		4,58	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,73**	***	**0,81**	***	**0,80**	***	**0,60**	***	**0,16**		**-0,27**	*	**0,35**	***	**0,86**	***	**1,31**	***	**1,35**	***
	5,52		7,31		6,19		5,36		1,22		1,91		2,98		8,05		9,66		8,32	
Size 2	**0,01**		**0,32**	***	**0,11**		**-0,01**		**-0,37**	***	**-1,22**	***	**0,03**		**0,28**	**	**0,64**	***	**0,79**	***
	0,05		2,94		0,97		0,08		3,08		7,72		0,25		2,57		5,87		4,88	
Size 3	**-0,02**		**0,45**	***	**0,09**		**0,23**	*	**-0,24**	*	**-0,87**	***	**0,02**		**0,26**	**	**0,69**	***	**0,64**	***
	0,13		3,54		0,76		1,74		1,91		4,62		0,15		2,28		6,07		3,69	
Size 4	**0,10**		**0,24**	*	**0,44**	***	**0,31**	**	**0,09**		**-0,55**	***	**0,01**		**0,38**	***	**0,52**	***	**0,59**	***
	0,82		1,90		3,66		2,19		0,55		2,88		0,04		3,21		4,44		3,23	
Size 5	**0,39**	***	**0,17**		**0,20**	**	**0,20**	*	**0,09**		**-0,05**		**-0,09**		**0,32**	***	**0,38**	***	**0,63**	***
	2,88		1,49		2,32		1,77		0,53		0,22		0,65		3,08		3,30		3,18	

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,14**		**-0,11**		**0,11**		**0,39**	***	**0,60**	***	**0,15**		**0,73**	***	**0,39**	***	**0,54**	***	**0,59**	***
	0,89		0,84		0,74		2,79		5,31		1,01		6,75		3,02		5,40		4,56	
Size 2	**-0,52**	***	**-0,26**	*	**-0,25**	*	**-0,03**		**-0,08**		**-0,60**	***	**-0,39**	***	**0,09**		**0,04**		**0,34**	**
	4,06		1,93		1,75		0,21		0,54		3,90		2,68		0,69		0,31		2,20	
Size 3	**-0,31**	*	**-0,06**		**-0,01**		**-0,06**		**-0,19**		**-0,59**	***	**-0,20**		**-0,13**		**0,28**	**	**0,24**	*
	1,88		0,35		0,04		0,37		1,29		3,80		1,40		0,94		2,09		1,88	
Size 4	**0,05**		**0,09**		**0,01**		**0,12**		**-0,31**	**	**-0,54**	***	**0,02**		**0,04**		**0,16**		**0,36**	**
	0,40		0,66		0,08		0,90		2,23		3,39		0,12		0,31		1,30		2,48	
Size 5	**0,20**		**0,13**		**0,06**		**-0,10**		**-0,45**	***	**-0,65**	***	**-0,18**		**0,11**		**0,35**	**	**0,30**	**
	1,36		1,00		0,50		0,77		3,01		4,21		1,33		0,76		2,55		2,34	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,46**	***	**0,68**	***	**0,64**	***	**0,35**	***	**-0,15**		**-0,54**	***	**0,19**		**0,68**	***	**1,03**	***	**1,06**	***
	3,52		5,70		4,89		2,60		1,03		3,21		1,64		5,16		8,61		7,78	
Size 2	**-0,10**		**0,10**		**-0,11**		**-0,29**	*	**-0,60**	***	**-1,60**	***	**-0,20**		**0,04**		**0,45**	***	**0,43**	***
	0,66		0,85		1,03		1,94		3,76		9,17		1,47		0,38		3,99		2,76	
Size 3	**-0,29**	**	**0,18**		**-0,20**		**0,02**		**-0,46**	***	**-1,25**	***	**-0,19**		**0,07**		**0,47**	***	**0,41**	**
	2,09		1,48		1,28		0,15		3,21		5,89		1,27		0,63		3,50		2,56	
Size 4	**-0,10**		**0,05**		**0,19**		**0,11**		**-0,05**		**-0,97**	***	**-0,30**	*	**0,13**		**0,25**	**	**0,39**	***
	0,65		0,37		1,50		0,67		0,31		4,82		1,85		1,10		2,10		2,75	
Size 5	**0,11**		**0,05**		**0,05**		**0,03**		**-0,24**		**-0,71**	***	**-0,54**	***	**0,11**		**0,17**		**0,24**	
	1,01		0,38		0,35		0,21		1,45		4,28		3,79		0,91		1,24		1,24	

Table 10: **JAPAN: Fitted intercepts from Fama-French three-factor regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding *t*-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**0,28**	*	**0,37**	***	**0,41**	***	**0,36**	***	**0,41**	***	**0,22**	***	**0,39**	***	**0,36**	***	**0,37**	***	**0,64**	***
	1,80		3,28		3,87		4,83		5,72		2,78		6,06		4,87		4,55		5,82	
Size 2	**0,29**	*	**0,01**		**0,12**		**0,25**	***	**0,09**	*	**-0,04**		**0,14**	**	**0,21**	**	**0,25**	***	**0,27**	***
	1,94		0,17		1,31		3,71		1,81		0,47		2,19		2,25		2,77		2,73	
Size 3	**0,06**		**0,07**		**0,07**		**0,07**		**0,15**	**	**-0,02**		**0,06**		**0,14**	*	**0,16**	*	**0,23**	**
	0,45		0,79		0,89		0,93		2,44		0,19		0,73		1,73		1,68		2,06	
Size 4	**0,06**		**0,16**	*	**0,16**	*	**0,15**	*	**0,08**		**-0,09**		**0,10**		**0,26**	***	**0,27**	***	**0,15**	
	0,55		1,76		1,82		1,84		1,04		0,92		1,03		2,97		2,98		1,57	
Size 5	**0,29**	***	**0,25**	***	**0,18**	**	**0,24**	***	**0,30**	**	**0,02**		**0,15**		**0,35**	***	**0,26**	***	**0,33**	***
	3,63		2,90		2,21		2,69		2,26		0,16		1,46		4,43		3,01		3,63	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,27**	***	**0,30**	***	**0,44**	***	**0,33**	***	**0,48**	***	**0,33**	**	**0,44**	***	**0,43**	***	**0,51**	***	**0,35**	**
	3,26		4,06		4,61		3,77		4,83		2,25		4,63		5,33		5,29		2,18	
Size 2	**0,11**		**0,12**	*	**0,15**	*	**0,24**	***	**0,11**		**0,04**		**0,11**		**0,19**	**	**0,20**	**	**0,27**	**
	1,39		1,65		1,91		2,57		1,27		0,24		1,36		2,32		2,17		1,96	
Size 3	**0,08**		**0,10**		**0,08**		**0,06**		**0,15**		**0,12**		**0,00**		**0,11**		**0,21**	**	**0,23**	
	0,93		1,20		0,99		0,63		1,55		0,81		0,01		1,25		2,28		1,58	
Size 4	**0,11**		**0,10**		**0,23**	***	**0,06**		**0,20**	**	**0,15**		**0,15**		**0,11**		**0,09**		**0,31**	**
	1,04		1,11		2,60		0,67		2,02		0,90		1,44		1,24		0,87		2,02	
Size 5	**0,17**		**0,08**		**0,10**		**0,30**	***	**0,28**	***	**0,11**		**0,04**		**0,01**		**0,27**	***	**0,34**	**
	1,33		0,84		1,21		3,27		2,75		0,56		0,32		0,09		2,73		2,07	

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**0,27**	*	**0,38**	***	**0,45**	***	**0,39**	***	**0,47**	***	**0,29**	***	**0,45**	***	**0,39**	***	**0,38**	***	**0,66**	***
	1,82		3,33		3,43		3,51		4,81		2,62		4,86		3,79		3,37		5,84	
Size 2	**0,32**	**	**0,01**		**0,13**		**0,30**	***	**0,15**		**0,02**		**0,19**	*	**0,24**	**	**0,29**	**	**0,30**	**
	2,39		0,09		1,25		2,78		1,42		0,12		1,71		2,18		2,47		2,55	
Size 3	**0,06**		**0,11**		**0,12**		**0,10**		**0,21**	*	**0,01**		**0,09**		**0,15**		**0,19**	*	**0,27**	**
	0,52		0,89		1,01		0,92		1,85		0,11		0,85		1,30		1,65		2,12	
Size 4	**0,10**		**0,18**		**0,19**	*	**0,21**	**	**0,13**		**-0,06**		**0,11**		**0,30**	***	**0,33**	***	**0,18**	
	0,79		1,40		1,77		2,19		1,12		0,48		0,99		3,07		2,86		1,48	
Size 5	**0,34**	***	**0,26**	**	**0,24**	**	**0,28**	**	**0,37**	***	**0,07**		**0,21**	*	**0,39**	***	**0,29**	***	**0,36**	***
	2,97		2,43		1,99		2,56		2,67		0,43		1,78		3,78		2,82		3,14	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,31**	***	**0,36**	***	**0,46**	***	**0,39**	***	**0,51**	***	**0,34**	**	**0,46**	***	**0,46**	***	**0,52**	***	**0,39**	***
	2,59		3,09		4,42		3,61		4,24		2,42		4,35		4,59		5,16		2,86	
Size 2	**0,19**		**0,17**		**0,18**	*	**0,29**	**	**0,11**		**0,05**		**0,14**		**0,22**	**	**0,23**	**	**0,31**	***
	1,49		1,52		1,81		2,56		0,96		0,37		1,24		1,99		2,04		2,61	
Size 3	**0,09**		**0,18**	*	**0,13**		**0,09**		**0,22**	*	**0,14**		**0,00**		**0,14**		**0,22**	**	**0,22**	**
	0,75		1,84		1,13		0,77		1,71		0,87		0,02		1,30		2,04		2,01	
Size 4	**0,15**		**0,08**		**0,27**	**	**0,08**		**0,25**	**	**0,20**		**0,20**	*	**0,14**		**0,12**		**0,33**	**
	1,12		0,79		2,34		0,75		2,13		1,44		1,70		1,26		1,13		2,47	
Size 5	**0,19**		**0,11**		**0,15**		**0,35**	***	**0,33**	***	**0,15**		**0,07**		**0,03**		**0,29**	***	**0,33**	**
	1,35		1,06		1,33		3,66		3,17		0,89		0,55		0,28		2,79		2,31	

Table 11: **WORLD: Fitted intercepts from Fama-French five-factor regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding *t*-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**-0,01**	**0,22** ***	**0,39** ***	**0,31** ***	**0,55** ***	**0,35** ***	**0,47** ***	**0,35** ***	**0,32** ***	**0,35** ***
	0,17	3,01	5,52	5,63	9,19	4,75	9,31	6,69	5,52	5,28
Size 2	**0,07**	**0,24** ***	**0,17** ***	**0,17** ***	**0,19** ***	**0,17** ***	**0,19** ***	**0,11** **	**0,11** **	**0,23** ***
	1,23	4,68	3,76	3,78	4,20	4,15	4,09	2,48	2,46	4,58
Size 3	**0,25** ***	**0,15** ***	**0,09**	**0,08** *	**0,14** ***	**0,17** ***	**0,11** **	**0,17** ***	**0,10** **	**0,09** *
	4,23	2,63	1,64	1,68	2,78	3,38	2,00	3,44	1,96	1,71
Size 4	**0,45** ***	**0,15** **	**0,05**	**0,02**	**0,05**	**0,22** ***	**0,18** ***	**0,13** **	**0,08**	**0,16** ***
	6,82	2,36	1,07	0,42	0,90	3,51	3,42	2,55	1,46	2,76
Size 5	**0,36** ***	**0,22** ***	**0,13** ***	**0,13** ***	**0,14** **	**0,10** *	**0,20** ***	**0,23** ***	**0,24** ***	**0,24** ***
	7,22	4,94	2,76	2,66	2,14	1,68	3,80	4,86	5,25	5,30

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**0,49** ***	**0,46** ***	**0,47** ***	**0,42** ***	**0,11**	**-0,26** *	**0,16** **	**0,39** ***	**0,64** ***	**1,00** ***
	6,72	9,57	9,44	7,73	1,60	1,94	2,54	6,74	9,41	8,44
Size 2	**0,15** ***	**0,24** ***	**0,23** ***	**0,25** ***	**0,10** **	**-0,13**	**0,07**	**0,15** ***	**0,37** ***	**0,71** ***
	3,75	5,37	5,53	5,41	2,14	0,91	1,00	2,75	6,51	6,70
Size 3	**0,14** **	**0,12** **	**0,20** ***	**0,20** ***	**0,10** *	**-0,09**	**-0,01**	**0,08**	**0,17** ***	**0,51** ***
	2,44	2,51	4,02	3,66	1,72	0,59	0,15	1,46	2,81	4,56
Size 4	**0,07**	**0,13** ***	**0,15** ***	**0,22** ***	**0,28** ***	**-0,02**	**0,01**	**0,14** ***	**0,12** *	**0,54** ***
	1,35	2,70	3,18	4,04	4,18	0,15	0,14	2,64	1,91	4,22
Size 5	**0,23** ***	**0,09** **	**0,19** ***	**0,30** ***	**0,30** ***	**0,00**	**0,08**	**0,13** ***	**0,22** ***	**0,45** ***
	4,76	2,27	4,32	6,31	5,71	0,01	1,00	2,58	3,11	3,21

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**0,35** ***	**0,57** ***	**0,79** ***	**0,70** ***	**0,89** ***	**0,71** ***	**0,80** ***	**0,67** ***	**0,70** ***	**0,80** ***
	3,86	6,39	9,61	9,15	11,26	8,94	13,29	9,59	9,74	10,65
Size 2	**0,50** ***	**0,68** ***	**0,58** ***	**0,64** ***	**0,56** ***	**0,66** ***	**0,63** ***	**0,54** ***	**0,56** ***	**0,67** ***
	5,65	7,87	6,88	8,37	6,73	8,77	8,77	6,93	6,80	7,92
Size 3	**0,70** ***	**0,59** ***	**0,57** ***	**0,53** ***	**0,59** ***	**0,67** ***	**0,55** ***	**0,60** ***	**0,55** ***	**0,60** ***
	8,10	6,64	7,20	7,03	7,92	7,84	6,33	7,27	7,31	6,21
Size 4	**0,91** ***	**0,55** ***	**0,53** ***	**0,51** ***	**0,47** ***	**0,62** ***	**0,66** ***	**0,53** ***	**0,59** ***	**0,64** ***
	10,98	6,85	6,20	5,87	6,28	6,08	8,98	6,15	7,62	8,67
Size 5	**0,68** ***	**0,62** ***	**0,59** ***	**0,60** ***	**0,63** ***	**0,59** ***	**0,69** ***	**0,69** ***	**0,59** ***	**0,67** ***
	9,55	7,89	8,45	7,39	6,97	7,66	8,03	7,81	7,42	9,02

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**0,86** ***	**0,79** ***	**0,80** ***	**0,81** ***	**0,47** ***	**0,07**	**0,52** ***	**0,69** ***	**0,94** ***	**1,25** ***
	10,38	12,63	12,48	10,87	6,11	0,70	7,01	9,77	13,99	11,65
Size 2	**0,60** ***	**0,62** ***	**0,62** ***	**0,65** ***	**0,53** ***	**0,22** **	**0,51** ***	**0,52** ***	**0,78** ***	**1,04** ***
	7,14	8,91	7,80	8,22	6,51	1,96	6,09	7,51	10,68	10,26
Size 3	**0,61** ***	**0,56** ***	**0,60** ***	**0,65** ***	**0,54** ***	**0,28** **	**0,47** ***	**0,49** ***	**0,58** ***	**0,82** ***
	7,39	7,33	8,09	8,81	5,78	2,40	5,29	5,34	7,12	9,73
Size 4	**0,50** ***	**0,57** ***	**0,54** ***	**0,67** ***	**0,78** ***	**0,33** ***	**0,44** ***	**0,57** ***	**0,54** ***	**0,82** ***
	5,57	7,28	6,51	8,71	7,69	2,69	4,33	8,28	6,89	8,94
Size 5	**0,67** ***	**0,52** ***	**0,53** ***	**0,67** ***	**0,71** ***	**0,30** ***	**0,48** ***	**0,53** ***	**0,62** ***	**0,59** ***
	8,27	7,54	5,91	8,80	9,72	2,75	6,83	8,42	7,55	6,44

Table 12: **NORTH AMERICA: Fitted intercepts from Fama-French five-factor regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding *t*-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,02**		**0,21**	**	**0,50**	***	**0,30**	***	**0,51**	***	**0,40**	***	**0,48**	***	**0,29**	***	**0,33**	***	**0,25**	**
	0,12		2,00		5,57		4,37		7,12		4,32		6,62		4,05		3,94		2,51	
Size 2	**0,07**		**0,14**	*	**0,21**	***	**0,16**	***	**0,10**	*	**0,07**		**0,19**	***	**0,22**	***	**0,30**	***	**0,15**	*
	0,69		1,82		3,13		2,57		1,79		1,22		3,07		3,21		4,28		1,85	
Size 3	**0,37**	***	**0,17**	**	**0,14**	*	**0,08**		**0,17**	**	**0,27**	***	**0,19**	**	**0,13**	*	**0,15**	**	**0,15**	*
	3,71		1,98		1,91		1,10		2,53		3,24		2,53		1,72		1,98		1,78	
Size 4	**0,54**	***	**0,23**	***	**0,24**	***	**0,08**		**0,09**		**0,24**	**	**0,30**	***	**0,26**	***	**0,10**		**0,29**	***
	5,15		2,80		3,06		1,02		1,29		2,26		4,40		3,30		1,40		3,23	
Size 5	**0,27**	***	**0,22**	***	**0,17**	**	**0,18**	***	**0,04**		**0,11**		**0,16**	**	**0,15**	**	**0,35**	***	**0,19**	***
	4,58		3,49		2,45		2,74		0,47		1,35		2,26		2,31		5,71		3,24	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,57**	***	**0,49**	***	**0,44**	***	**0,42**	***	**0,06**		**-0,29**	*	**0,25**	***	**0,54**	***	**0,78**	***	**1,04**	***
	5,49		6,54		6,13		5,86		0,60		1,71		3,08		7,18		8,02		6,39	
Size 2	**0,12**	**	**0,31**	***	**0,22**	***	**0,22**	***	**-0,01**		**-0,27**		**0,17**	**	**0,18**	**	**0,33**	***	**0,64**	***
	2,07		4,27		3,11		3,53		0,16		1,51		2,05		2,27		3,94		4,12	
Size 3	**0,11**		**0,15**	**	**0,26**	***	**0,20**	***	**0,24**	***	**-0,24**		**0,07**		**0,20**	***	**0,22**	**	**0,56**	***
	1,43		2,27		3,51		2,80		2,77		1,29		0,79		2,64		2,50		3,50	
Size 4	**0,19**	***	**0,26**	***	**0,25**	***	**0,32**	***	**0,25**	**	**-0,15**		**0,12**		**0,24**	***	**0,28**	***	**0,58**	***
	2,59		3,83		3,41		4,07		2,37		0,78		1,33		3,20		3,29		3,27	
Size 5	**0,20**	***	**0,08**		**0,18**	***	**0,30**	***	**0,31**	***	**-0,03**		**0,13**		**0,15**	**	**0,22**	**	**0,56**	***
	3,47		1,36		3,24		4,21		3,82		0,14		1,41		2,13		2,56		3,13	

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,06**		**0,28**	***	**0,48**	***	**0,41**	***	**0,59**	***	**0,49**	***	**0,58**	***	**0,40**	***	**0,37**	***	**0,35**	***
	0,59		2,78		5,33		5,06		7,55		5,61		7,64		4,58		3,82		3,47	
Size 2	**0,16**	*	**0,29**	***	**0,34**	***	**0,35**	***	**0,25**	***	**0,25**	***	**0,33**	***	**0,37**	***	**0,44**	***	**0,28**	***
	1,70		2,98		4,45		4,26		3,00		2,75		4,39		4,53		4,77		3,28	
Size 3	**0,48**	***	**0,26**	***	**0,27**	***	**0,19**	***	**0,40**	***	**0,37**	***	**0,26**	***	**0,23**	***	**0,26**	***	**0,33**	***
	4,89		2,69		2,94		2,64		4,31		4,16		2,73		2,84		3,27		3,04	
Size 4	**0,64**	***	**0,36**	***	**0,39**	***	**0,21**	**	**0,24**	**	**0,34**	***	**0,49**	***	**0,37**	***	**0,26**	**	**0,44**	***
	6,39		3,83		4,48		2,45		2,41		3,38		5,69		3,85		2,48		4,52	
Size 5	**0,44**	***	**0,35**	***	**0,24**	***	**0,31**	***	**0,20**	**	**0,20**	**	**0,30**	***	**0,31**	***	**0,49**	***	**0,34**	***
	5,53		3,94		2,78		3,68		2,26		2,00		3,06		3,70		6,60		3,83	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,56**	***	**0,58**	***	**0,53**	***	**0,53**	***	**0,10**		**-0,41**	***	**0,37**	***	**0,62**	***	**0,79**	***	**0,78**	***
	5,28		8,07		7,35		6,48		1,07		3,15		4,27		8,37		8,15		7,00	
Size 2	**0,36**	***	**0,40**	***	**0,34**	***	**0,38**	***	**0,13**		**-0,31**	**	**0,35**	***	**0,32**	***	**0,43**	***	**0,56**	***
	3,57		4,98		3,92		4,96		1,45		2,30		3,60		3,84		4,87		4,64	
Size 3	**0,30**	***	**0,27**	***	**0,39**	***	**0,33**	***	**0,37**	***	**-0,31**	**	**0,20**	**	**0,32**	***	**0,25**	***	**0,32**	***
	2,90		3,22		4,81		3,56		4,23		2,03		2,24		3,73		2,71		3,05	
Size 4	**0,37**	***	**0,40**	***	**0,38**	***	**0,44**	***	**0,33**	***	**-0,25**	*	**0,23**	**	**0,38**	***	**0,34**	***	**0,29**	**
	4,49		4,36		4,16		5,05		3,04		1,79		2,47		4,72		3,63		2,27	
Size 5	**0,34**	***	**0,16**	**	**0,33**	***	**0,42**	***	**0,44**	***	**-0,16**		**0,21**	**	**0,28**	***	**0,26**	***	**0,28**	***
	4,33		2,19		3,71		4,90		4,81		1,23		2,36		3,32		3,23		2,93	

Table 13: **EUROPE: Fitted intercepts from Fama-French five-factor regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding *t*-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**0,00**		**0,19**	**	**0,26**	***	**0,23**	***	**0,33**	***	**0,05**		**0,34**	***	**0,35**	***	**0,48**	***	**0,34**	***
	0,03		2,55		3,93		3,94		6,06		0,79		5,80		6,29		8,47		4,74	
Size 2	**0,16**	**	**0,25**	***	**0,11**	*	**0,19**	***	**0,25**	***	**0,05**		**0,24**	***	**0,15**	***	**0,24**	***	**0,47**	***
	2,18		3,74		1,95		3,32		4,39		0,82		4,00		2,63		3,94		7,07	
Size 3	**0,32**	***	**0,28**	***	**0,06**		**0,07**		**0,20**	***	**0,09**		**0,22**	***	**0,25**	***	**0,14**	*	**0,29**	***
	3,91		3,87		0,86		0,98		2,75		1,31		3,35		3,81		1,95		3,94	
Size 4	**0,45**	***	**0,25**	***	**0,09**		**0,10**		**0,09**		**0,12**		**0,19**	**	**0,17**	**	**0,30**	***	**0,31**	***
	5,63		3,07		1,34		1,30		1,07		1,36		2,41		2,30		3,87		4,09	
Size 5	**0,32**	***	**0,19**	***	**0,22**	***	**0,26**	***	**0,18**	**	**0,26**	***	**0,37**	***	**0,28**	***	**0,00**		**0,22**	***
	4,64		3,12		3,59		3,56		2,21		3,00		5,87		4,20		0,05		3,72	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,24**	***	**0,32**	***	**0,32**	***	**0,31**	***	**0,07**		**-0,46**	***	**0,05**		**0,26**	***	**0,57**	***	**1,24**	***
	3,80		5,56		5,33		5,13		0,91		3,63		0,67		3,88		7,72		9,62	
Size 2	**0,10**		**0,27**	***	**0,31**	***	**0,20**	***	**0,18**	***	**-0,38**	***	**0,00**		**0,21**	***	**0,50**	***	**0,97**	***
	1,54		4,62		4,80		3,57		3,14		2,58		0,04		3,27		6,78		8,68	
Size 3	**0,14**	*	**0,18**	***	**0,24**	***	**0,16**	**	**0,15**	**	**-0,12**		**0,05**		**0,15**	**	**0,36**	***	**0,65**	***
	1,89		2,66		3,54		2,39		2,10		0,83		0,64		2,12		4,68		5,03	
Size 4	**0,13**	*	**0,14**	*	**0,29**	***	**0,27**	***	**0,21**	***	**0,02**		**0,18**	**	**0,20**	**	**0,28**	***	**0,58**	***
	1,68		1,92		4,16		3,69		2,72		0,10		2,04		2,54		3,17		4,53	
Size 5	**0,15**	**	**0,25**	***	**0,19**	***	**0,20**	***	**0,19**	***	**-0,01**		**0,05**		**0,21**	***	**0,25**	***	**0,34**	**
	2,34		4,06		3,26		3,33		2,73		0,03		0,51		3,30		2,87		2,24	

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market										Panel B: Operating Profitability									
	BM1		BM2		BM3		BM4		BM5		OP1		OP2		OP3		OP4		OP5	
Size 1	**-0,07**		**0,35**	***	**0,38**	***	**0,40**	***	**0,39**	***	**0,18**	**	**0,47**	***	**0,38**	***	**0,57**	***	**0,36**	***
	0,62		3,09		4,21		4,25		4,58		2,18		5,56		4,32		6,29		3,90	
Size 2	**0,30**	***	**0,35**	***	**0,22**	**	**0,22**	***	**0,34**	***	**0,17**	*	**0,33**	***	**0,33**	***	**0,37**	***	**0,66**	***
	3,39		3,46		2,50		2,79		3,50		1,88		3,77		3,32		4,35		6,34	
Size 3	**0,41**	***	**0,40**	***	**0,21**	***	**0,15**		**0,36**	***	**0,26**	**	**0,37**	***	**0,29**	***	**0,30**	***	**0,34**	***
	3,91		3,81		2,59		1,61		3,52		2,55		4,23		3,04		2,83		3,33	
Size 4	**0,46**	***	**0,49**	***	**0,23**	**	**0,32**	***	**0,10**		**0,21**	**	**0,33**	***	**0,26**	**	**0,45**	***	**0,41**	***
	5,46		5,59		2,33		3,51		0,97		2,27		3,71		2,43		4,95		4,21	
Size 5	**0,48**	***	**0,28**	***	**0,35**	***	**0,42**	***	**0,33**	***	**0,32**	***	**0,55**	***	**0,38**	***	**0,08**		**0,39**	***
	5,40		3,02		3,84		4,96		3,54		2,94		6,41		3,73		0,80		3,99	

	Panel C: Investment										Panel D: Momentum									
	Inv1		Inv2		Inv3		Inv4		Inv5		MoM1		MoM2		MoM3		MoM4		MoM5	
Size 1	**0,31**	***	**0,43**	***	**0,39**	***	**0,44**	***	**0,16**		**-0,35**	***	**0,15**	*	**0,32**	***	**0,57**	***	**1,21**	***
	3,14		5,08		4,47		4,67		1,38		3,37		1,68		4,16		6,58		13,18	
Size 2	**0,23**	***	**0,45**	***	**0,35**	***	**0,27**	***	**0,25**	***	**-0,33**	***	**0,07**		**0,32**	***	**0,43**	***	**1,04**	***
	2,57		4,86		4,24		2,71		2,91		2,70		0,77		3,27		3,94		10,51	
Size 3	**0,30**	***	**0,25**	***	**0,35**	***	**0,25**	***	**0,32**	***	**-0,05**		**0,08**		**0,29**	***	**0,27**	***	**0,61**	***
	3,21		3,00		3,88		2,93		3,11		0,36		0,80		2,97		3,07		5,53	
Size 4	**0,22**	**	**0,20**	**	**0,24**	***	**0,46**	***	**0,44**	***	**0,03**		**0,24**	**	**0,34**	***	**0,36**	***	**0,60**	***
	2,18		2,24		2,63		3,96		3,87		0,19		2,15		3,24		3,35		4,55	
Size 5	**0,22**	**	**0,53**	***	**0,32**	***	**0,38**	***	**0,25**	***	**-0,09**		**0,17**	*	**0,31**	***	**0,31**	***	**0,26**	***
	2,24		5,85		3,37		4,23		2,98		0,67		1,81		3,35		3,33		2,65	

Table 14: **ASIA-PACIFIC: Fitted intercepts from Fama-French five-factor regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding *t*-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**0,44** **	**0,30** **	**0,36** ***	**0,62** ***	**0,91** ***	**0,67** ***	**0,90** ***	**0,55** ***	**0,55** ***	**0,61** ***
	2,24	2,19	3,03	5,64	8,54	5,34	7,11	5,00	4,85	5,29
Size 2	**-0,09**	**0,09**	**-0,09**	**0,04**	**0,18** *	**-0,01**	**-0,07**	**0,11**	**0,15**	**0,31** ***
	0,66	0,76	0,81	0,35	1,78	0,09	0,63	1,04	1,19	2,76
Size 3	**0,14**	**0,02**	**0,04**	**0,08**	**0,04**	**-0,12**	**0,07**	**-0,01**	**0,21** *	**0,27** **
	0,90	0,12	0,36	0,57	0,30	0,98	0,49	0,07	1,73	2,28
Size 4	**0,37** **	**0,26** *	**-0,02**	**0,17**	**0,06**	**-0,05**	**0,22**	**0,15**	**0,11**	**0,41** ***
	2,48	1,89	0,12	1,41	0,45	0,31	1,56	0,95	0,95	3,01
Size 5	**0,43** ***	**0,22** **	**0,16**	**0,02**	**0,12**	**0,27** **	**0,32** ***	**0,17**	**0,17** *	**0,24** **
	3,97	2,49	1,60	0,20	0,89	2,45	2,99	1,63	1,85	2,38

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**0,85** ***	**0,84** ***	**0,82** ***	**0,64** ***	**0,22**	**-0,14**	**0,42** ***	**0,93** ***	**1,30** ***	**1,37** ***
	6,75	7,57	6,03	5,37	1,58	0,94	3,40	8,37	9,19	8,52
Size 2	**0,10**	**0,29** **	**0,03**	**0,00**	**-0,14**	**-1,08** ***	**0,04**	**0,24** **	**0,63** ***	**0,95** ***
	0,92	2,57	0,28	0,03	1,17	6,60	0,37	2,09	5,49	5,69
Size 3	**-0,19**	**0,37** ***	**-0,01**	**0,22**	**-0,12**	**-0,75** ***	**-0,05**	**0,20** *	**0,67** ***	**0,72** ***
	1,39	2,78	0,12	1,63	0,95	3,84	0,40	1,70	5,55	3,98
Size 4	**-0,18**	**0,06**	**0,30** **	**0,32** **	**0,23**	**-0,48** **	**-0,13**	**0,19**	**0,40** ***	**0,59** ***
	1,58	0,46	2,38	2,17	1,41	2,35	0,97	1,55	3,26	3,03
Size 5	**0,10**	**0,05**	**0,22** **	**0,31** ***	**0,48** ***	**0,09**	**-0,06**	**0,22** **	**0,24** **	**0,71** ***
	0,88	0,52	2,36	2,89	3,56	0,39	0,39	2,11	2,02	3,36

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**0,62** ***	**0,64** ***	**0,68** ***	**0,91** ***	**1,16** ***	**0,91** ***	**1,16** ***	**0,69** ***	**0,74** ***	**0,88** ***
	4,10	4,42	6,23	7,35	11,47	7,73	10,02	6,23	7,02	6,86
Size 2	**0,13**	**0,43** ***	**0,21** *	**0,30** ***	**0,52** ***	**0,26** **	**0,25** **	**0,33** ***	**0,50** ***	**0,62** ***
	1,19	3,11	1,91	2,80	4,62	2,04	2,41	2,81	3,68	4,60
Size 3	**0,30** ***	**0,29** **	**0,27** **	**0,35** ***	**0,16**	**0,06**	**0,24** *	**0,22** *	**0,45** ***	**0,60** ***
	2,67	2,09	2,06	2,79	1,31	0,56	1,76	1,77	3,95	4,77
Size 4	**0,49** ***	**0,57** ***	**0,26** **	**0,45** ***	**0,41** **	**0,13**	**0,49** ***	**0,33** ***	**0,50** ***	**0,60** ***
	4,11	4,72	2,05	3,18	2,53	0,89	3,13	2,65	4,46	4,86
Size 5	**0,70** ***	**0,53** ***	**0,39** ***	**0,25** **	**0,46** ***	**0,58** ***	**0,70** ***	**0,60** ***	**0,47** ***	**0,60** ***
	6,06	4,60	3,33	1,99	3,90	4,98	6,34	4,54	3,51	6,10

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**1,07** ***	**1,14** ***	**0,99** ***	**0,85** ***	**0,47** ***	**0,00**	**0,63** ***	**1,16** ***	**1,55** ***	**1,67** ***
	9,01	11,32	8,84	8,50	3,44	0,02	5,18	12,28	12,69	10,38
Size 2	**0,42** ***	**0,52** ***	**0,24** **	**0,29** **	**0,19**	**-0,87** ***	**0,28** **	**0,54** ***	**0,91** ***	**1,23** ***
	3,61	4,61	2,26	2,37	1,59	6,36	2,33	4,55	8,14	9,06
Size 3	**0,02**	**0,57** ***	**0,30** **	**0,53** ***	**0,22** *	**-0,53** ***	**0,34** ***	**0,46** ***	**0,86** ***	**0,87** ***
	0,11	4,96	2,45	4,11	1,74	3,67	3,27	4,26	7,11	5,93
Size 4	**0,15**	**0,38** ***	**0,67** ***	**0,60** ***	**0,40** ***	**-0,38** **	**0,26** **	**0,48** ***	**0,63** ***	**0,75** ***
	1,25	2,81	4,94	5,04	3,02	2,32	2,30	4,28	5,02	5,32
Size 5	**0,37** ***	**0,52** ***	**0,51** ***	**0,60** ***	**0,77** ***	**-0,12**	**0,02**	**0,55** ***	**0,57** ***	**0,68** ***
	3,21	4,21	4,54	5,68	6,54	0,70	0,12	4,50	4,62	5,31

Table 15: **JAPAN: Fitted intercepts from Fama-French five-factor regressions**

The table presents, for each portfolio, the intercept in percentage (bold figures) and the corresponding t-statistics. Statistical significance at 1, 5 and 10% is shown by a *, ** and ***, respectively. Portfolios are sorted by firm characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time-series regressions of monthly excess returns run from November 1990 to February 2021 using Ordinary-Least-Squares (top part) and Compact Genetic Algorithm (bottom part).

Intercepts based on Ordinary-Least-Square method (in%)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**0,27** *	**0,38** ***	**0,39** ***	**0,36** ***	**0,40** ***	**0,29** ***	**0,41** ***	**0,34** ***	**0,34** ***	**0,55** ***
	1,71	3,32	3,64	4,86	5,55	3,90	6,27	4,54	4,17	5,24
Size 2	**0,24**	**0,04**	**0,12**	**0,27** ***	**0,12** **	**0,08**	**0,17** ***	**0,22** **	**0,17** **	**0,18** *
	1,60	0,43	1,31	4,10	2,45	1,27	2,67	2,31	2,01	1,93
Size 3	**0,04**	**0,10**	**0,09**	**0,08**	**0,18** ***	**0,09**	**0,10**	**0,13** *	**0,11**	**0,16**
	0,28	1,05	1,02	1,20	2,86	1,16	1,31	1,65	1,17	1,43
Size 4	**0,05**	**0,19** **	**0,18** **	**0,16** *	**0,10**	**0,02**	**0,12**	**0,26** ***	**0,25** ***	**0,10**
	0,46	2,09	2,06	1,95	1,19	0,21	1,33	2,97	2,68	1,05
Size 5	**0,30** ***	**0,26** ***	**0,19** **	**0,24** ***	**0,30** **	**0,29** **	**0,27** ***	**0,34** ***	**0,17** **	**0,19** **
	3,84	3,05	2,22	2,61	2,22	2,45	2,86	4,26	2,15	2,44

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**0,32** ***	**0,34** ***	**0,46** ***	**0,33** ***	**0,45** ***	**0,33** **	**0,45** ***	**0,44** ***	**0,52** ***	**0,32** *
	4,21	4,73	4,79	3,63	4,67	2,22	4,66	5,48	5,32	1,95
Size 2	**0,19** ***	**0,18** ***	**0,20** ***	**0,27** ***	**0,04**	**0,05**	**0,16** **	**0,23** ***	**0,20** **	**0,26** *
	2,87	2,57	2,66	2,85	0,44	0,32	1,97	2,73	2,22	1,81
Size 3	**0,14** *	**0,15** *	**0,11**	**0,09**	**0,12**	**0,16**	**0,05**	**0,13**	**0,22** **	**0,17**
	1,91	1,84	1,37	0,86	1,31	1,09	0,51	1,57	2,39	1,19
Size 4	**0,16**	**0,14**	**0,24** ***	**0,07**	**0,16** *	**0,21**	**0,19** *	**0,13**	**0,07**	**0,23**
	1,63	1,58	2,73	0,78	1,70	1,27	1,85	1,38	0,72	1,51
Size 5	**0,24** ***	**0,10**	**0,11**	**0,25** ***	**0,23** ***	**0,21**	**0,10**	**-0,02**	**0,22** **	**0,26**
	2,97	1,23	1,29	3,18	2,76	1,08	0,84	0,28	2,26	1,60

Intercepts based on Compact Genetic Algorithm (in %)

	Panel A: Book-to-Market					Panel B: Operating Profitability				
	BM1	BM2	BM3	BM4	BM5	OP1	OP2	OP3	OP4	OP5
Size 1	**0,53** ***	**0,65** ***	**0,66** ***	**0,58** ***	**0,72** ***	**0,57** ***	**0,69** ***	**0,62** ***	**0,59** ***	**0,87** ***
	4,00	6,10	6,05	6,27	8,40	5,67	7,85	5,52	5,40	7,74
Size 2	**0,47** ***	**0,31** ***	**0,38** ***	**0,52** ***	**0,35** ***	**0,35** ***	**0,45** ***	**0,44** ***	**0,45** ***	**0,45** ***
	3,73	2,85	3,86	5,43	3,50	3,35	4,52	3,43	4,65	3,84
Size 3	**0,32** **	**0,33** ***	**0,34** ***	**0,34** ***	**0,45** ***	**0,39** ***	**0,30** ***	**0,39** ***	**0,41** ***	**0,40** ***
	2,29	3,37	3,36	3,62	4,02	3,40	3,14	3,59	3,79	3,50
Size 4	**0,35** ***	**0,41** ***	**0,43** ***	**0,45** ***	**0,40** ***	**0,31** ***	**0,39** ***	**0,51** ***	**0,50** ***	**0,41** ***
	3,32	4,03	4,56	4,75	4,11	2,86	4,08	5,58	4,80	4,25
Size 5	**0,55** ***	**0,48** ***	**0,43** ***	**0,47** ***	**0,64** ***	**0,56** ***	**0,51** ***	**0,60** ***	**0,45** ***	**0,43** ***
	5,05	4,11	4,19	4,50	4,62	4,67	4,54	5,94	4,09	4,17

	Panel C: Investment					Panel D: Momentum				
	Inv1	Inv2	Inv3	Inv4	Inv5	MoM1	MoM2	MoM3	MoM4	MoM5
Size 1	**0,59** ***	**0,57** ***	**0,71** ***	**0,57** ***	**0,74** ***	**0,64** ***	**0,68** ***	**0,66** ***	**0,73** ***	**0,54** ***
	4,90	6,20	6,93	5,02	6,89	4,76	6,76	7,98	8,35	4,42
Size 2	**0,50** ***	**0,42** ***	**0,43** ***	**0,56** ***	**0,33** ***	**0,32** **	**0,39** ***	**0,46** ***	**0,45** ***	**0,51** ***
	4,76	3,97	4,35	5,34	3,02	2,57	3,57	5,08	4,71	4,61
Size 3	**0,40** ***	**0,41** ***	**0,37** ***	**0,33** ***	**0,39** ***	**0,44** ***	**0,26** **	**0,33** ***	**0,49** ***	**0,38** ***
	3,28	3,43	3,88	3,24	3,62	3,04	2,43	3,78	5,02	3,07
Size 4	**0,50** ***	**0,34** ***	**0,49** ***	**0,32** ***	**0,47** ***	**0,50** ***	**0,47** ***	**0,33** ***	**0,26** **	**0,44** ***
	4,59	3,65	4,76	3,24	4,27	3,61	4,08	3,36	2,41	3,62
Size 5	**0,52** ***	**0,36** ***	**0,36** ***	**0,49** ***	**0,54** ***	**0,48** ***	**0,31** ***	**0,22** **	**0,51** ***	**0,49** ***
	4,74	3,39	3,35	4,60	4,94	3,23	2,74	2,29	4,78	4,00

Figure 1: **CAPM regressions adjusted R^2 – OLS vs. CGA**

The figure shows the adjusted R^2 values (y-axis) obtained from CAPM regressions for all geographic regions and portfolios sorted by firm characteristics (x-axis). Time-series regressions of portfolio excess returns on systematic risk factor run from November 1990 to February 2021.

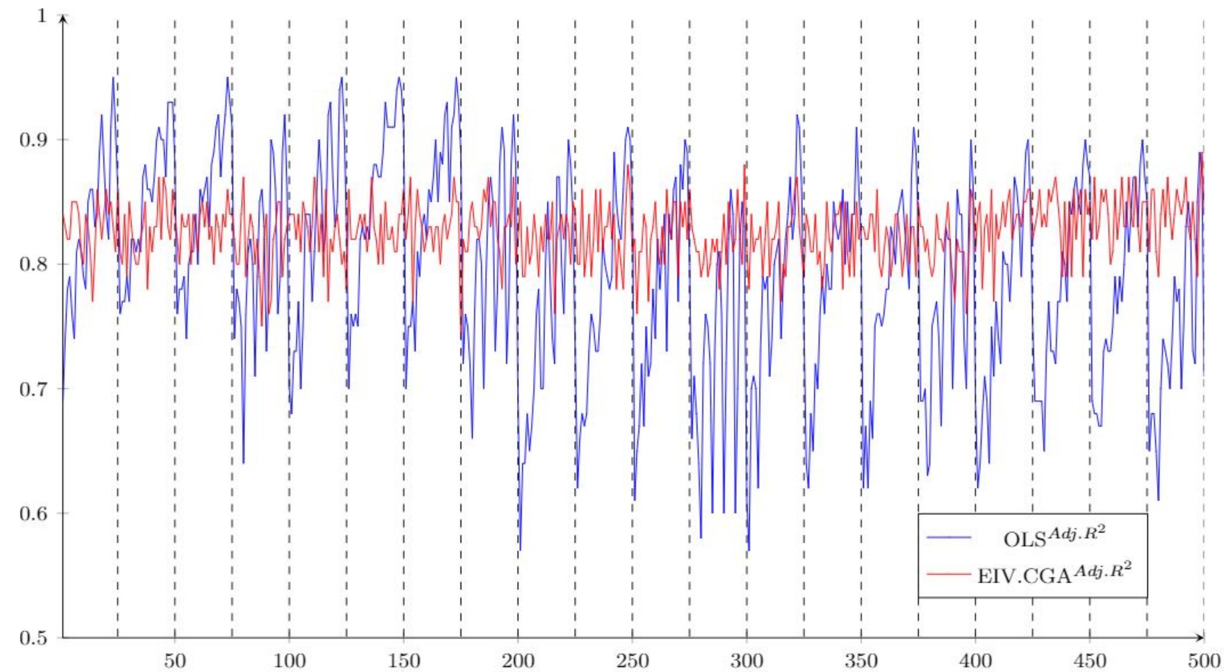


Figure 2: **FF 3-factor regressions adjusted R^2 – OLS vs. CGA**

The figure shows the adjusted R^2 values (y-axis) obtained from Fama-French three-factor regressions for all geographic regions and portfolios sorted by firm characteristics (x-axis). Time-series regressions of portfolio excess returns on systematic risk factors run from November 1990 to February 2021.

Figure 3: **FF 5-factor regressions adjusted R^2 – OLS vs. CGA**

The figure shows the adjusted R^2 values (y-axis) obtained from Fama-French five-factor regressions for all geographic regions and portfolios sorted by firm characteristics (x-axis). Time-series regressions of portfolio excess returns on systematic risk factors run from November 1990 to February 2021.

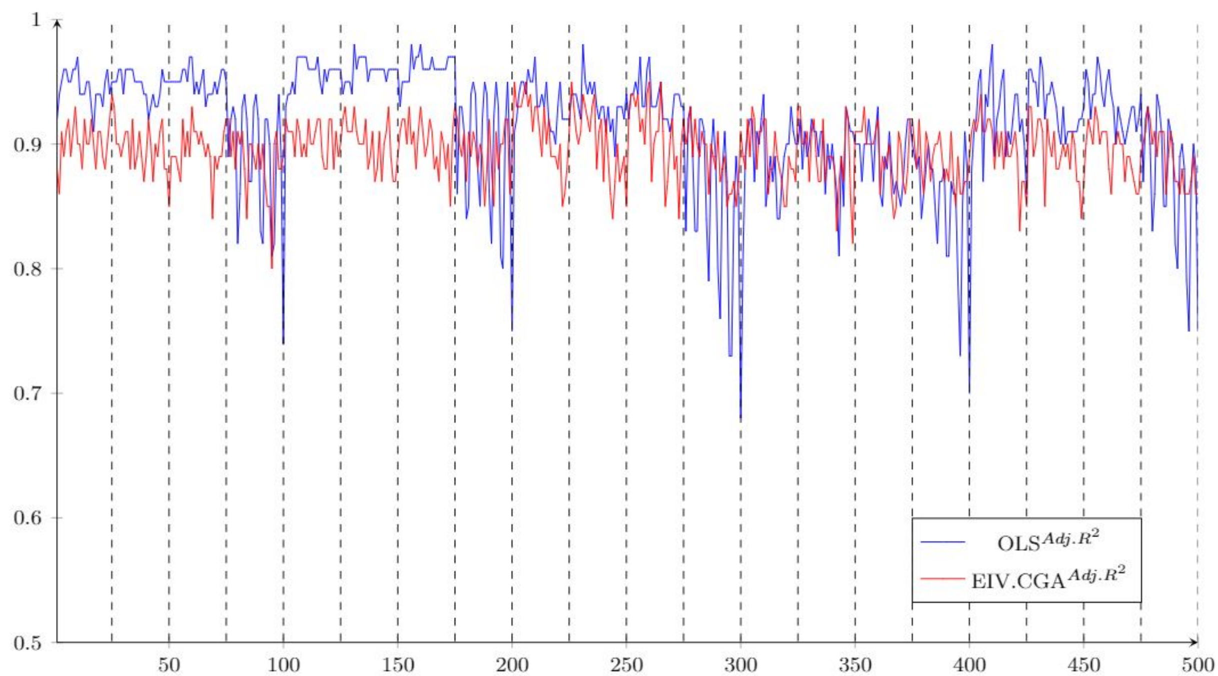


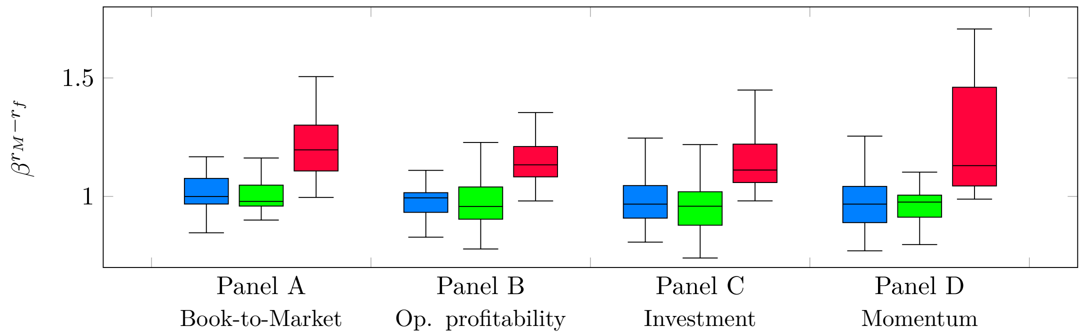
Table 16: **Gibbons, Ross and Shanken statistics per panel per region for each model: November 1990 to February 2021**

		Book-to-Market		Op. Profitability		Investment		Momentum	
		OLS	CGA	OLS	CGA	OLS	CGA	OLS	CGA
World	CAPM	17,99	3,39	19,31	5,23	20,34	4,70	11,42	8,10
	FF3F	17,76	4,32	19,98	5,67	22,70	4,37	15,02	9,52
	FF5F	16,58	12,42	16,90	14,04	18,91	15,12	13,26	16,52
		Book-to-Market		Op. Profitability		Investment		Momentum	
		OLS	CGA	OLS	CGA	OLS	CGA	OLS	CGA
North Am.	CAPM	13,43	3,66	12,55	3,41	12,30	4,17	7,21	5,60
	FF3F	14,33	3,65	13,21	5,50	14,51	5,14	10,50	9,12
	FF5F	13,23	5,25	12,90	4,98	14,01	6,12	9,88	6,41
		Book-to-Market		Op. Profitability		Investment		Momentum	
		OLS	CGA	OLS	CGA	OLS	CGA	OLS	CGA
Europe	CAPM	15,37	2,51	25,34	5,31	16,75	2,84	12,15	11,40
	FF3F	15,21	2,14	26,18	6,28	17,64	2,20	15,81	12,86
	FF5F	13,95	4,10	21,77	4,92	14,18	3,21	12,82	13,41
		Book-to-Market		Op. Profitability		Investment		Momentum	
		OLS	CGA	OLS	CGA	OLS	CGA	OLS	CGA
Asia-Pac.	CAPM	9,97	4,71	13,43	5,48	11,58	5,52	11,27	8,54
	FF3F	9,78	5,16	15,04	8,93	11,33	6,82	12,49	13,34
	FF5F	13,23	8,14	12,90	9,82	14,01	12,63	9,88	18,30
		Book-to-Market		Op. Profitability		Investment		Momentum	
		OLS	CGA	OLS	CGA	OLS	CGA	OLS	CGA
Japan	CAPM	7,35	1,74	12,21	1,96	5,86	1,32	3,16	1,93
	FF3F	8,26	2,25	12,30	3,32	6,45	1,99	4,62	2,67
	FF5F	9,35	6,39	11,83	6,04	6,83	5,01	4,59	6,95

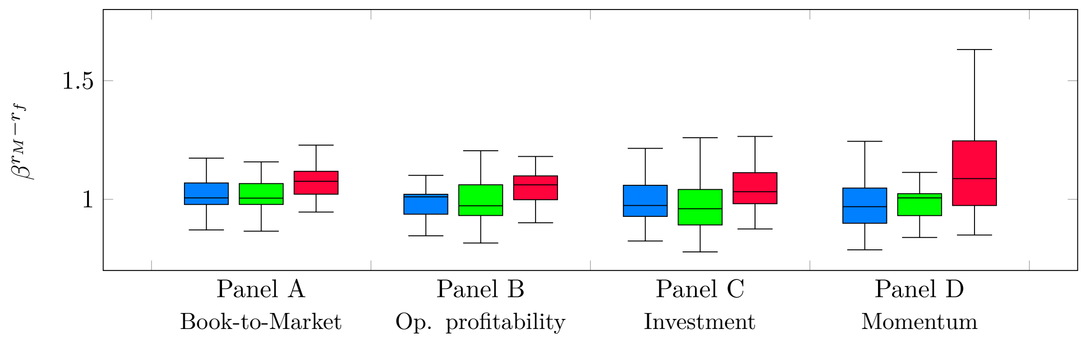
Table 17: **WORLD: Tukey Box Plots of the $\beta^{r_M - r_f}$ for the CAPM, the Fama & French three- and five-factor model for each panel: November 1990 to February 2021**

The figures below present, for each asset pricing model - the CAPM, the Fama & French three- (FF3F) and five-factor model (FF5F) - the Tukey Box Plots of the $\beta^{r_M - r_f}$ for every single panel. Portfolios are sorted by firms' characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time series regressions of monthly excess returns run from November 1990 to February 2021 using 1. Ordinary-Least-Square method (blue box), 2. Hansen's Generalized Method of Moments using Instrumental Variables based on higher moments (green box) and 3. a Compact Genetic Algorithm (red box).

$$\text{(CAPM)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \end{cases}$$



$$\text{(FF3F)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + \epsilon_{i,t} \end{cases}$$



$$\text{(FF5F)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + w_i^{OLS}(RMW_t) + c_i^{OLS}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + w_i(RMW_t) + c_i^{GMM}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + w_i^{CGA}(RMW_t) + c_i^{CGA}(CMA_t) + \epsilon_{i,t} \end{cases}$$

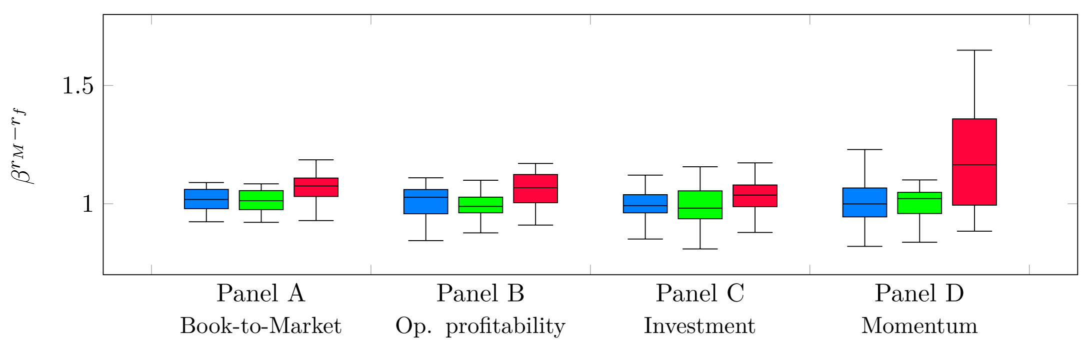
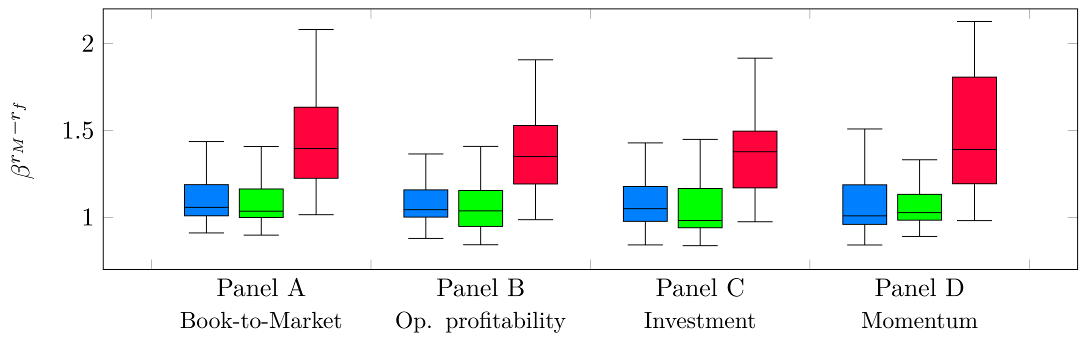


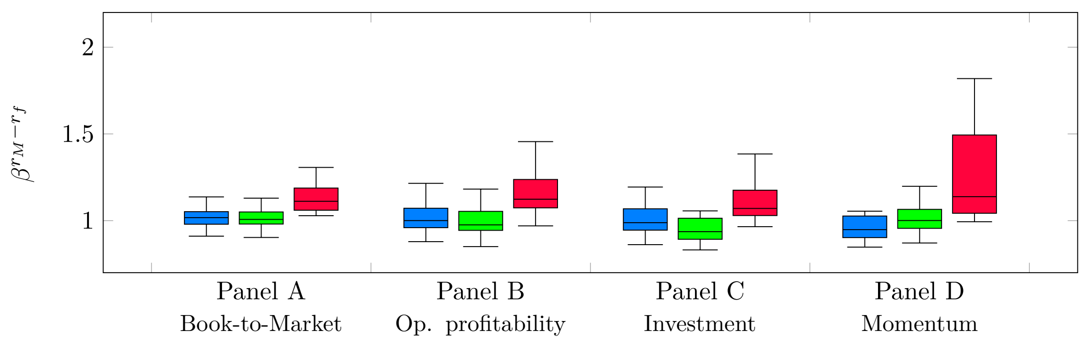
Table 18: **NORTH AMERICA: Tukey Box Plots of the $\beta^{r_M - r_f}$ for the CAPM, the Fama & French three- and five-factor model for each panel: November 1990 to February 2021**

The figures below present, for each asset pricing model - the CAPM, the Fama & French three- (FF3F) and five-factor model (FF5F) - the Tukey Box Plots of the $\beta^{r_M - r_f}$ for every single panel. Portfolios are sorted by firms' characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time series regressions of monthly excess returns run from November 1990 to February 2021 using 1. Ordinary-Least-Square method (blue box), 2. Hansen's Generalized Method of Moments using Instrumental Variables based on higher moments (green box) and 3. a Compact Genetic Algorithm (red box).

$$\text{(CAPM)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \end{cases}$$



$$\text{(FF3F)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + \epsilon_{i,t} \end{cases}$$



$$\text{(FF5F)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + w_i^{OLS}(RMW_t) + c_i^{OLS}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + w_i(RMW_t) + c_i^{GMM}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + w_i^{CGA}(RMW_t) + c_i^{CGA}(CMA_t) + \epsilon_{i,t} \end{cases}$$

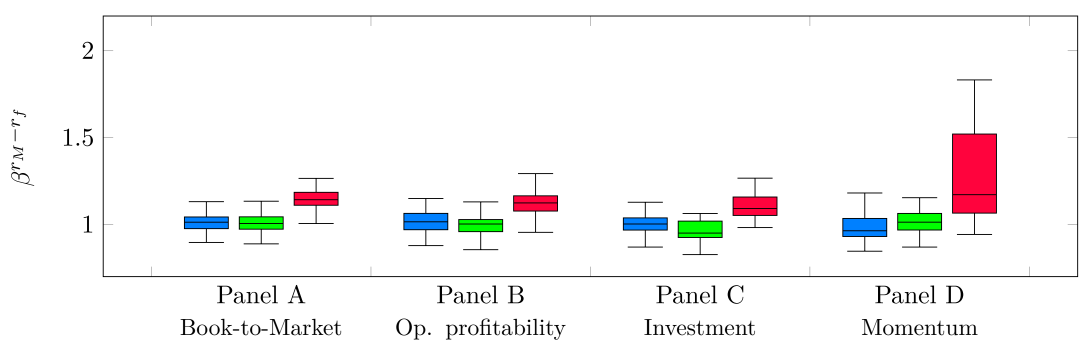
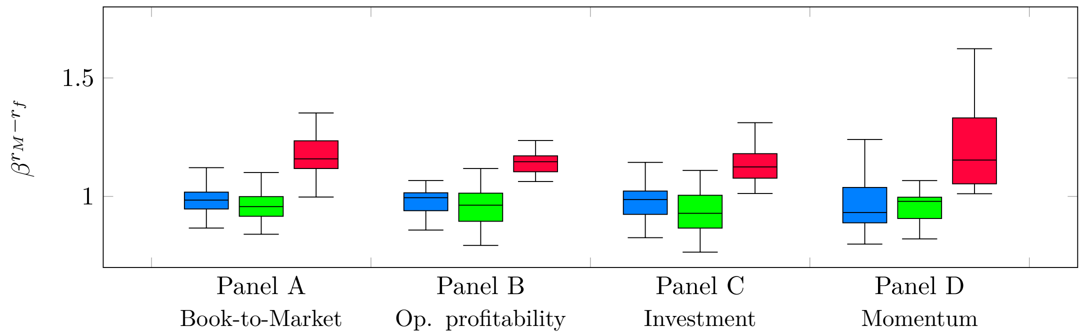


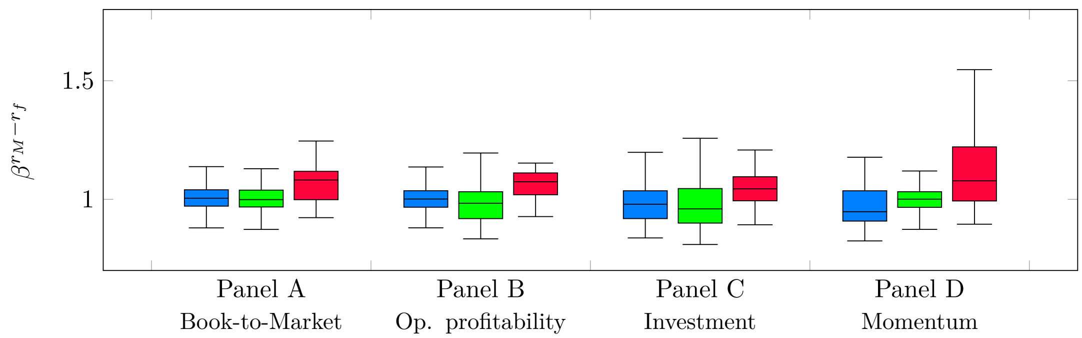
Table 19: **EUROPE: Tukey Box Plots of the $\beta^{r_M - r_f}$ for the CAPM, the Fama & French three- and five-factor model for each panel: November 1990 to February 2021**

The figures below present, for each asset pricing model - the CAPM, the Fama & French three- (FF3F) and five-factor model (FF5F) - the Tukey Box Plots of the $\beta^{r_M - r_f}$ for every single panel. Portfolios are sorted by firms' characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time series regressions of monthly excess returns run from November 1990 to February 2021 using 1. Ordinary-Least-Square method (blue box), 2. Hansen's Generalized Method of Moments using Instrumental Variables based on higher moments (green box) and 3. a Compact Genetic Algorithm (red box).

$$\text{(CAPM)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \end{cases}$$



$$\text{(FF3F)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + \epsilon_{i,t} \end{cases}$$



$$\text{(FF5F)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + w_i^{OLS}(RMW_t) + c_i^{OLS}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + w_i(RMW_t) + c_i^{GMM}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + w_i^{CGA}(RMW_t) + c_i^{CGA}(CMA_t) + \epsilon_{i,t} \end{cases}$$

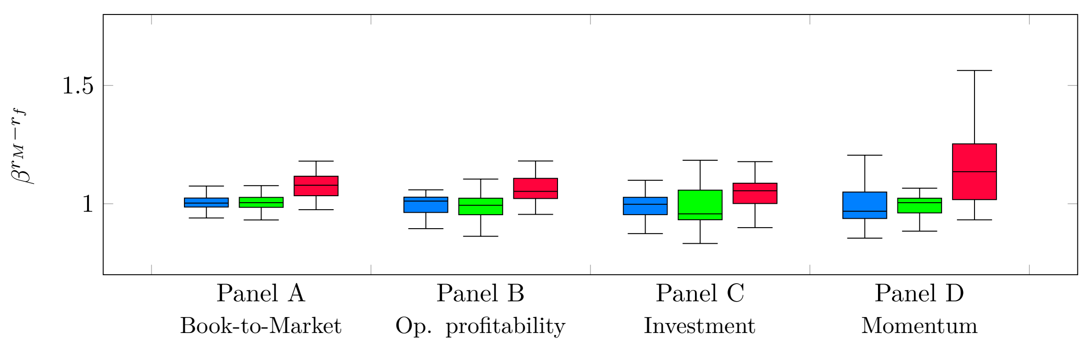
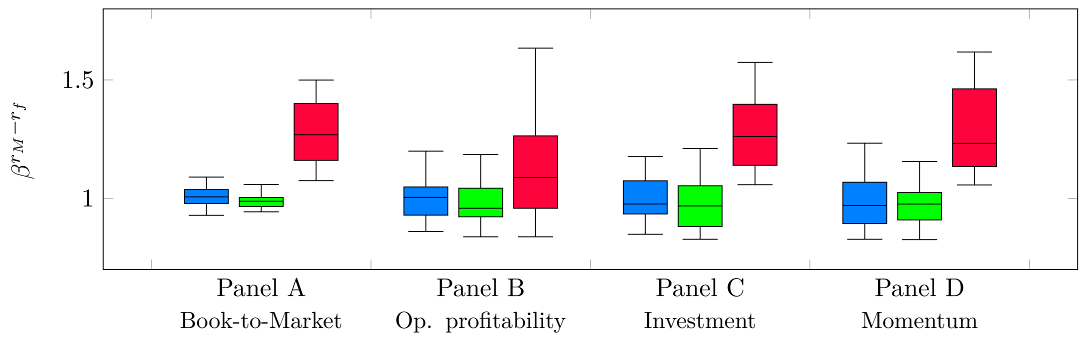


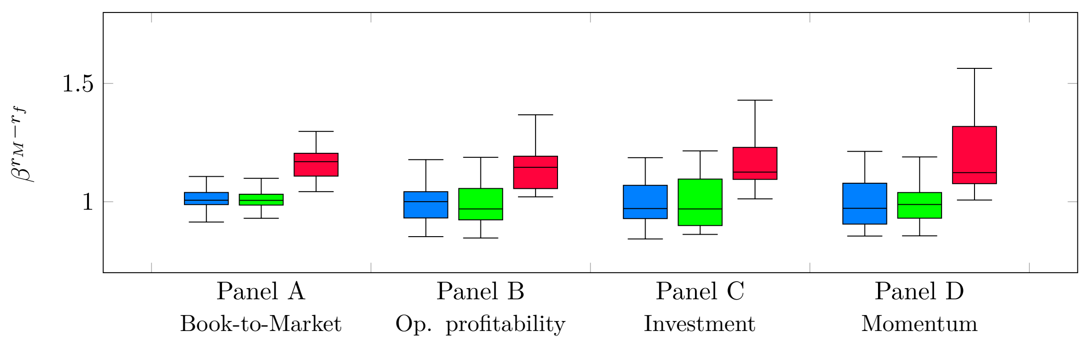
Table 20: **ASIA PACIFIC: Tukey Box Plots of the $\beta^{r_M - r_f}$ for the CAPM, the Fama & French three- and five-factor model for each panel: November 1990 to February 2021**

The figures below present, for each asset pricing model - the CAPM, the Fama & French three- (FF3F) and five-factor model (FF5F) - the Tukey Box Plots of the $\beta^{r_M - r_f}$ for every single panel. Portfolios are sorted by firms' characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time series regressions of monthly excess returns run from November 1990 to February 2021 using 1. Ordinary-Least-Square method (blue box), 2. Hansen's Generalized Method of Moments using Instrumental Variables based on higher moments (green box) and 3. a Compact Genetic Algorithm (red box).

$$\text{(CAPM)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \end{cases}$$



$$\text{(FF3F)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + \epsilon_{i,t} \end{cases}$$



$$\text{(FF5F)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + w_i^{OLS}(RMW_t) + c_i^{OLS}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + w_i(RMW_t) + c_i^{GMM}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + w_i^{CGA}(RMW_t) + c_i^{CGA}(CMA_t) + \epsilon_{i,t} \end{cases}$$

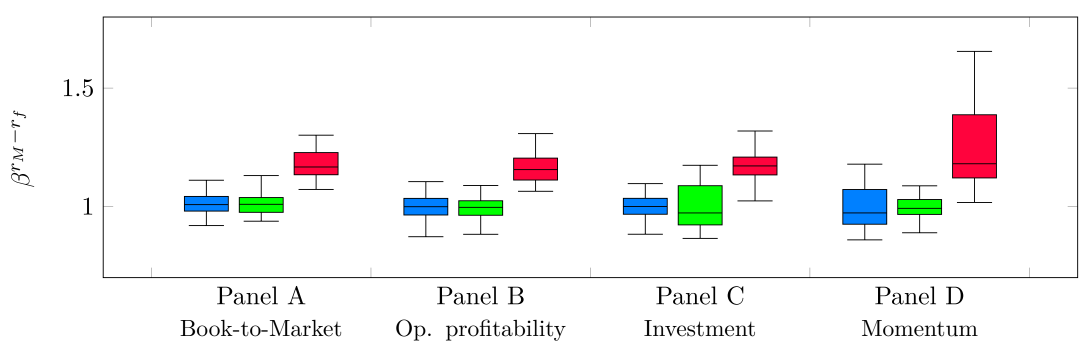
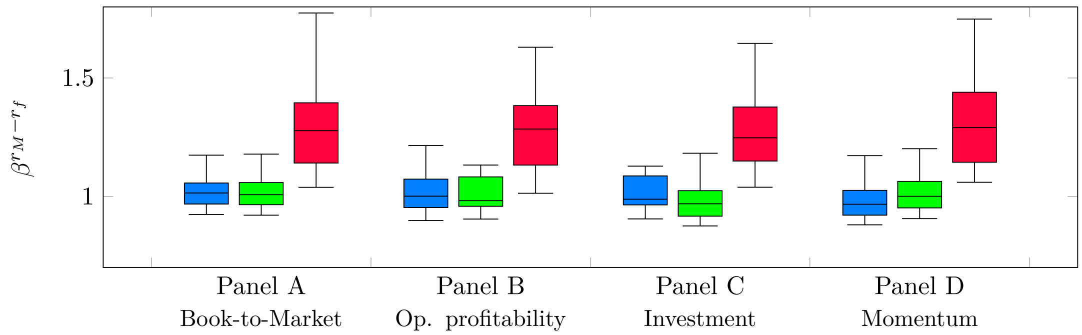


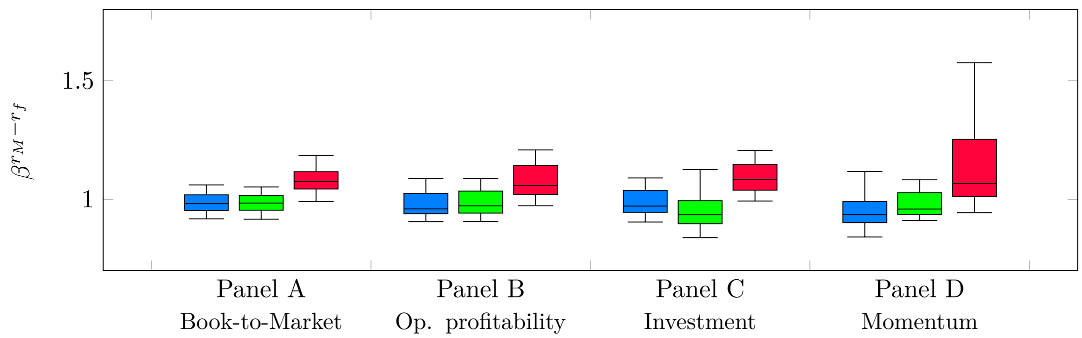
Table 21: **JAPAN: Tukey Box Plots of the $\beta^{r_M - r_f}$ for the CAPM, the Fama & French three- and five-factor model for each panel: November 1990 to February 2021**

The figures below present, for each asset pricing model - the CAPM, the Fama & French three- (FF3F) and five-factor model (FF5F) - the Tukey Box Plots of the $\beta^{r_M - r_f}$ for every single panel. Portfolios are sorted by firms' characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time series regressions of monthly excess returns run from November 1990 to February 2021 using 1. Ordinary-Least-Square method (blue box), 2. Hansen's Generalized Method of Moments using Instrumental Variables based on higher moments (green box) and 3. a Compact Genetic Algorithm (red box).

$$\text{(CAPM)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \end{cases}$$



$$\text{(FF3F)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + \epsilon_{i,t} \end{cases}$$



$$\text{(FF5F)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + w_i^{OLS}(RMW_t) + c_i^{OLS}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t} - r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + w_i(RMW_t) + c_i^{GMM}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + w_i^{CGA}(RMW_t) + c_i^{CGA}(CMA_t) + \epsilon_{i,t} \end{cases}$$

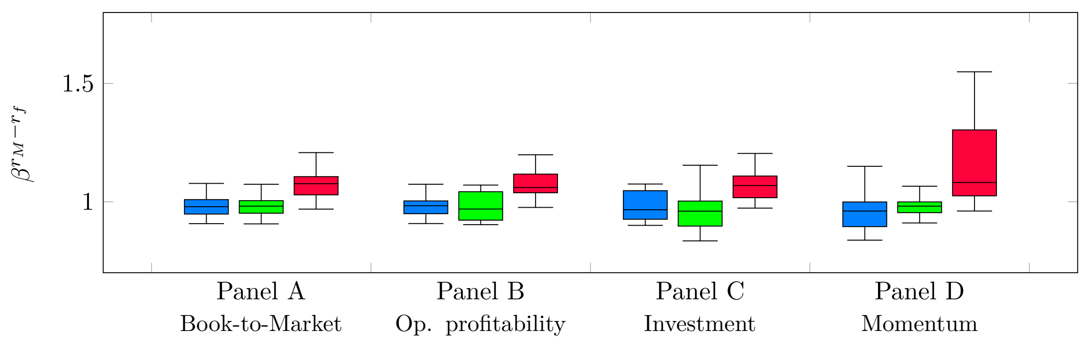


Figure 4: **GRS test statistics: Time series regressions with the CAPM per panel for every region: November 1990 to February 2021**

The figure shows the GRS test statistics were $GRS = \frac{T-N-K}{N} \times \left(1+\widehat{\mu}_f^\top \widehat{\Omega}_f^{-1}\widehat{\mu}_f\right)^{-1} \times \left(\widehat{\alpha}^\top \widehat{\Sigma}_\epsilon^{-1}\widehat{\alpha}\right) \sim \mathcal{F}_{N,T-N-K}$ with T, the number of observations, N the number of dependent variables, K the number of risk factors, $\widehat{\alpha}$ the vector of Jensen's α, $\widehat{\mu}_f$, the mean vector of the explanatory variables, Σ_ϵ the covariance matrix of residuals and Ω_f the covariance matrix of explanatory variables for the CAPM for all portfolios sorted by firms' characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time series regressions of monthly excess returns run from November 1990 to February 2021 using 1. Ordinary-Least-Square method (blue line), 2. Hansen's Generalized Method of Moments using Instrumental Variables based on higher moments (green line) and 3. a Compact Genetic Algorithm (red line) such as:

$$\text{(CAPM)} \quad r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t}-r_{f,t}}) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + \epsilon_{i,t} \end{cases}$$

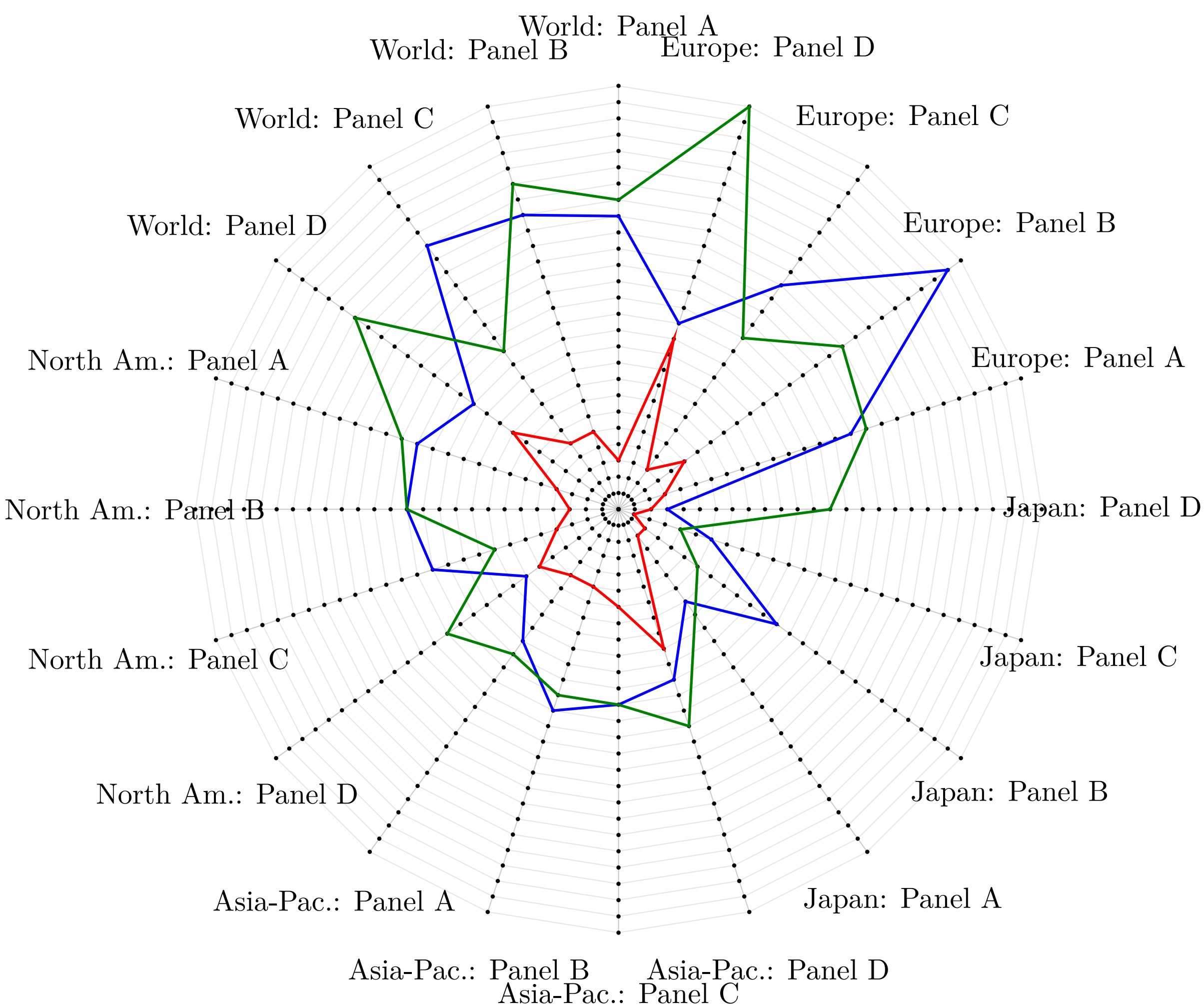


Figure 5: **GRS test statistics: Time series regressions with the Fama and French three-factor model per panel for every region: November 1990 to February 2021**

The figure shows the GRS test statistics were $GRS = \frac{T-N-K}{N} \times \left(1 + \widehat{\mu}_f^{\top} \widehat{\Omega}_f^{-1} \widehat{\mu}_f\right)^{-1} \times \left(\widehat{\alpha}^{\top} \widehat{\Sigma}_{\epsilon}^{-1} \widehat{\alpha}\right) \sim \mathcal{F}_{N,T-N-K}$ with T, the number of observations, N the number of dependent variables, K the number of risk factors, $\widehat{\alpha}$ the vector of Jensen's α, $\widehat{\mu}_f$, the mean vector of the explanatory variables, Σ_ϵ the covariance matrix of residuals and Ω_f the covariance matrix of explanatory variables for the Fama and French three-factor model (FF3F) regressions for all portfolios sorted by firms' characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time series regressions of monthly excess returns run from November 1990 to February 2021 using 1. Ordinary-Least-Square method (blue line), 2. Hansen's Generalized Method of Moments using Instrumental Variables based on higher moments (green line) and 3. a Compact Genetic Algorithm (red line) such as: (FF3F) $r_{i,t} - r_{f,t} = \begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t}-r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + \epsilon_{i,t} \end{cases}$

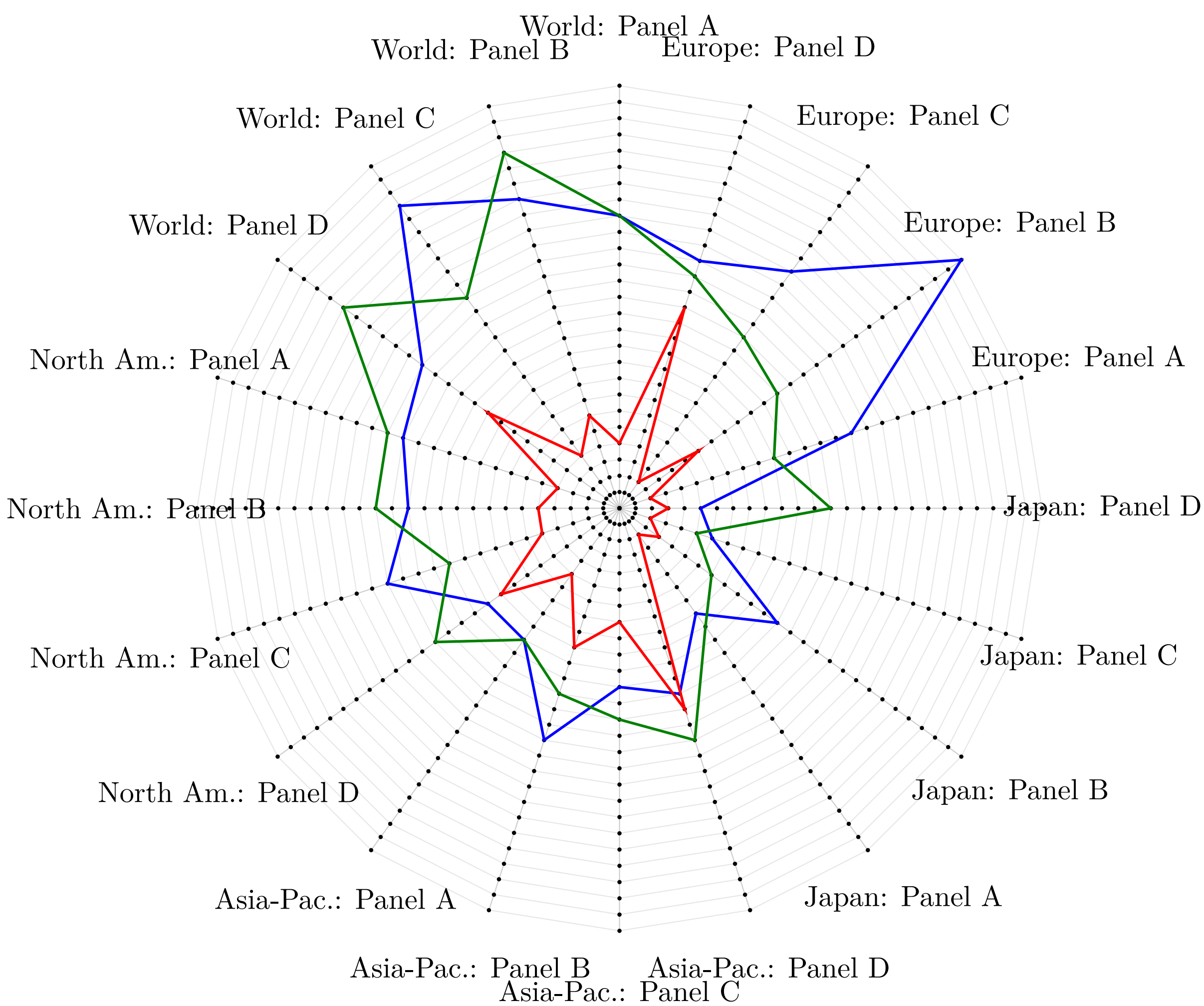


Figure 6: **GRS test statistics: Time series regressions with the Fama and French five-factor model per panel for every region: November 1990 to February 2021**

The figure shows the GRS test statistics were $GRS = \frac{T-N-K}{N} \times \left(1 + \widehat{\mu}_f^{\top}\widehat{\Omega}_f^{-1}\widehat{\mu}_f\right)^{-1} \times \left(\widehat{\alpha}^{\top}\widehat{\Sigma}_{\epsilon}^{-1}\widehat{\alpha}\right) \sim \mathcal{F}_{N,T-N-K}$ with T, the number of observations, N the number of dependent variables, K the number of risk factors, $\widehat{\alpha}$ the vector of Jensen's α, $\widehat{\mu}_f$, the mean vector of the explanatory variables, Σ_{ϵ} the covariance matrix of residuals and Ω_f the covariance matrix of explanatory variables for the Fama and French five-factor model (FF5F) regressions for all portfolios sorted by firms' characteristics: Size-Book to market (Panel A); Size-Operating profitability (Panel B); Size-Investment (Panel C) and Size-Momentum (Panel D). Time series regressions of monthly excess returns run from November 1990 to February 2021 using 1. Ordinary-Least-Square method (blue line), 2. Hansen's Generalized Method of Moments using Instrumental Variables based on higher moments (green line) and 3. a Compact Genetic Algorithm (red line) such as: (FF5F) $r_{i,t} - r_{f,t} =$

$$\begin{cases} \alpha_i + \beta_i^{OLS}(r_{M,t} - r_{f,t}) + s_i^{OLS}(SMB_t) + h_i^{OLS}(HML_t) + w_i^{OLS}(RMW_t) + c_i^{OLS}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{GMM}(iv_{r_{M,t}-r_{f,t}}) + s_i^{GMM}(SMB_t) + h_i^{GMM}(HML_t) + w_i(RMW_t) + c_i^{GMM}(CMA_t) + \epsilon_{i,t} \\ \alpha_i + \beta_i^{CGA}(r_{M,t} - r_{f,t}) + s_i^{CGA}(SMB_t) + h_i^{CGA}(HML_t) + w_i^{CGA}(RMW_t) + c_i^{CGA}(CMA_t) + \epsilon_{i,t} \end{cases}$$

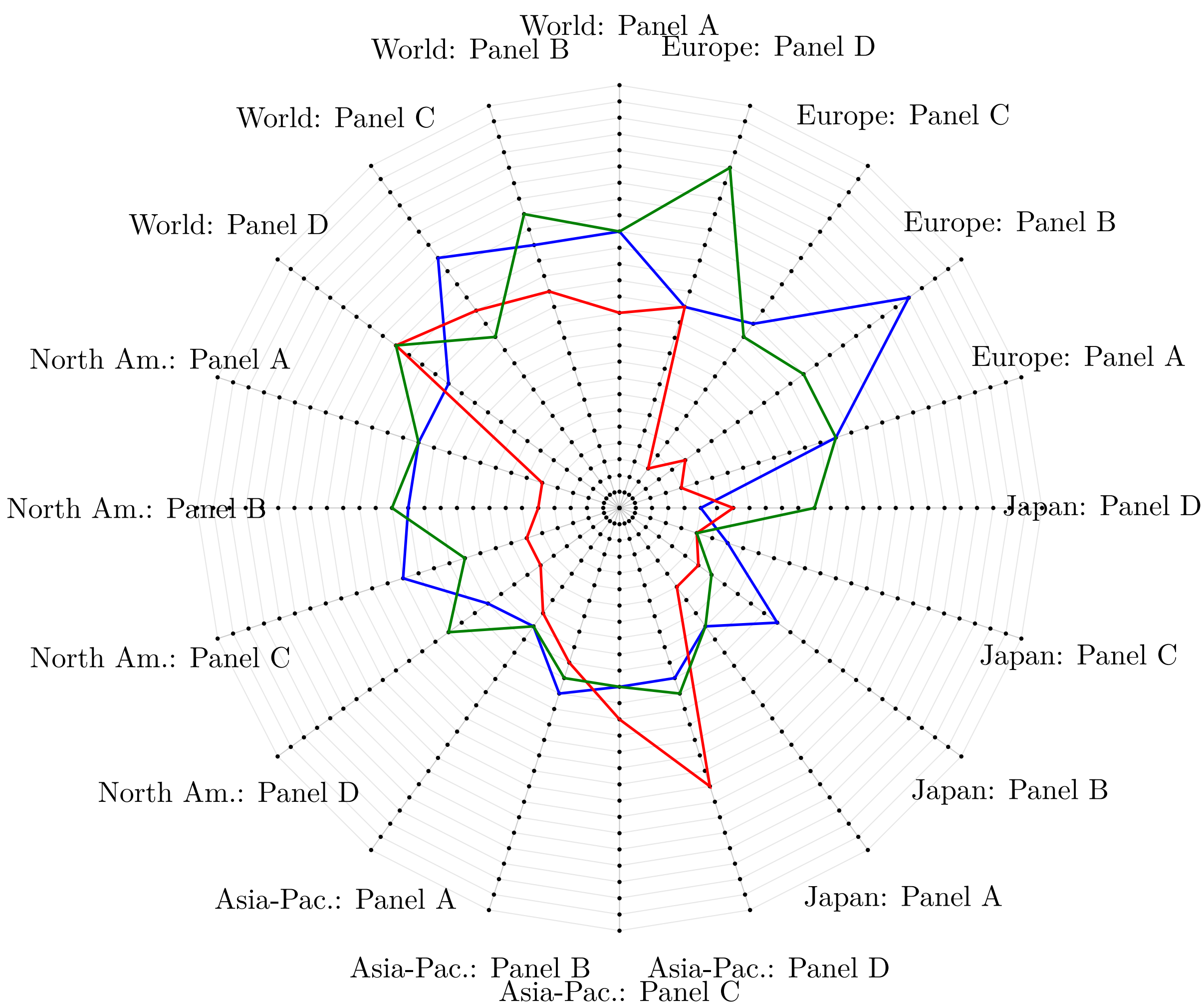


Figure 7: **Adjusted R-square for every single strategy for the CAPM and for the Fama & French three- and five-factor models all estimated with the Compact Genetic Algorithm (CGA)**

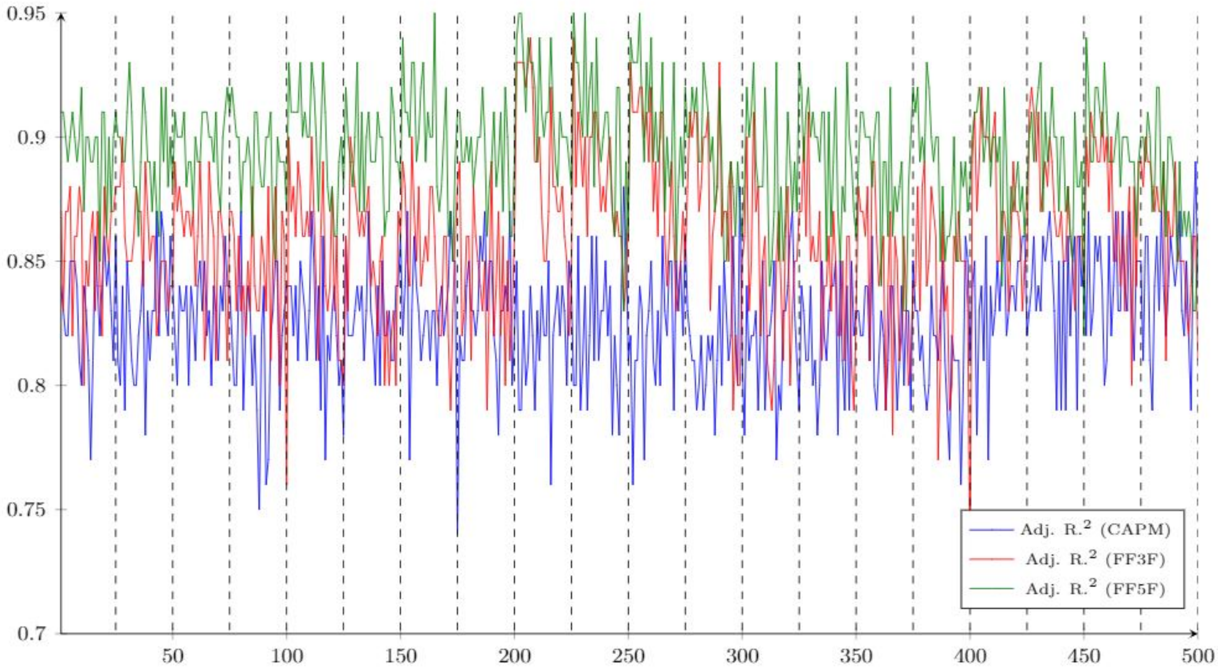


Figure 8: **Distribution of idiosyncratic volatilities (σ_ϵ^2) for each strategy for all regions based on OLS vs CGA for the CAPM and the Fama-French 3- and 5-factor models: November 1990 to February 2021**

The figure shows the distribution with boxplots of the idiosyncratic volatilities of each strategy for the five regions (World, North America, Europe, Asia-Pacific and Japan) estimated with OLS and CGA for the CAPM and the Fama and French three- and five-factor models.