Generalized velocity-density model based on microscopic traffic simulation

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GENERALIZED VELOCITY-DENSITY MODEL BASED ON MICROSCOPIC TRAFFIC SIMULATION

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Abstract. In case of the IDM-model the actual V(D) Velocity-Density law applied by this dynamic system is not defined, only the dynamic behavior of the vehicles/drivers is determined. So the logical question is whether the related investigations enhance an existing and known law or reveal a new connection. Specifically, which function class/type is enhanced by the Intelligent Driver Model (IDM-model)? The publication presents a model analysis, the goal of which was the exploration by of a feature of the IDM-model which, as yet, "remained hidden". So, in our case the goal was not the validation of the model, but the exploration of a further feature of the validated model. The separate validation of the model was not necessary, since many validated applications for this model have been demonstrated in practice, for example [Kesting et al. (2008)]. In our calculations also the applied model parameter values remained in the range of the model parameters used in the literature.

This paper presents a new approach for Velocity-Density Model (VDM) synthesis. It consists in modelling separately each of the density and the velocity (macroscopic parameter). From this study, safety time headway (microscopic parameter) can be identified from macroscopic data by mean of interpolation method. By combining the density and the velocity models, a generalized new VDM is developed. It is shown that from this one, some literature velocity-density models, as well as their properties, can be derived by fixing some of its parameters.

Keywords: Velocity-Density Model, microscopic traffic simulation, Adaptive Cruise Control

Introduction

Traffic modelling can be classified into the three following classes.

The first class includes the microscopic models where the traffic is viewed as a system of interacting particles (vehicles). This interaction is modelled by differential equations as done by [Bando et al. (1998)], [Helbing & Tilch (1998)], [Treiber et al. (2004)], [Molina (2005)], [Ge et al. (2008)], [Kesting (2008)], [Rakha & Gao (2010)].For interested readers, a general overview of microscopic traffic models is presented by [Derbel, Mourllion & Basset (2012)].

The second class includes the macroscopic models where the traffic is viewed as one group of particles. Macroscopic parameters are traffic density, traffic flow and velocity which are used for graphic representation of the fundamental diagram. Several works are based on this approach such as those ones developed by [Holden & Risebro (1995)], [Herty & Klar (2003)], [Treiber et al. (2004)], [Peter (2012)], [Peter & Szabo (2012)], [Bede & Peter (2013)], [Peter et al. (2013)], [Peter et al. (2013)] [Bede & Peter (2014)].

The third class includes the mesoscopic models, which appears to be an intermediate between the two last classes. Here, the traffic is viewed as clusters of vehicles. Several studies have been realized in this field such as those done by [Prigogine & Herman (1971)], [Paveri-Fontana (1975)] and [Mahnke & Kaupuzs (1999)].

The presented research work deals with microscopic and macroscopic approaches, where the traffic is represented by one cluster. The goal is to predict traffic density and velocity with a minimum of errors. Most of the literature velocity-density models are developed and identified by mean of experimental macroscopic traffic data. The lack of these ones leads to work under simulation. Here, the idea is to generate macroscopic traffic data from microscopic traffic model simulation. Nonetheless, this microscopic model must faithfully reproduce the traffic behaviours and be validated by the fundamental diagram study.

In this paper, the developed Velocity-Density Model (VDM) is based on the Intelligent Driver Model (IDM) ([Kesting (2008)], [treiber et al. (2000)]). Our new method proceeds by the identification of the simulated velocity and density data separately. Among the advantages of this method is the ability to identify each microscopic parameter from macroscopic ones by interpolation method. Then, a new VDM is computed with a mathematical development of the velocity and density models. And it will be shown later that some of existing models in the literature, as well as their properties can be derived from this new VDM by fixing some of its parameters.

In this paper, Section 1 presents a brief state-of-the art of the existing velocity-density models. Section 2 identifies the velocity and the density functions. Section 3 presents the synthesis method of the new VDM. Section 4 presents the generalized VDM and section 5 concludes and gives outlooks.

1. State-of-the-art of the Velocity-Density Models

Two classes of Velocity-Density Models can be distinguished: the stochastic els and the deterministic models. This section presents a brief state-of-the-art of deterministic VDM considering logical order.

1.1GREENSHIELDS MODEL

Up to our knowledge, the first ministic VDM was proposed by [greenshields (1934)]. This affine model is given by the following expression

$$V(D) = V_{Max}(1 - \frac{D}{D_{\max}}) \quad \textbf{(1)}$$

where $D_{\max}$ is the maximum density which is the jam density. This model is identified by linear regression method using seven experimental observations.

Recently, [Wang et al. (2011)] have showed that, with more than seven data, the Greenshields model is not enabling the prediction of velocity and density.

1.2GREENBERG MODEL

By the analogy with fluid flow, [Greenberg (1959)] has developed a logarithmic velocity-density relationship given by

$$V(D) = V_{Max} \log(\frac{D}{D_{\max}}) \quad \textbf{(2)}$$

The main drawback of this model is its inability to predict velocity for low densities. Indeed, the velocity tends to infinity when the density tends to zero, which is unrealistic.

1.3UNDERWOOD MODEL

[Underwood (1961)] has derived an exponential model in order to overcome the drawbacks of the Greenberg

and Greenshields models for the free traffic flow condition. This model is given by

$$V(D) = V_{Max} \exp(-\frac{D}{D_{\max}}) \quad \textbf{(3)}$$

The main drawback of the Underwood model is that velocity becomes zero only when density D reaches infinity and not $D=D_{\max}$. Hence, this model cannot be used for predicting velocities at high densities.

1.4NEWELL MODEL

The VDM of [Newell (1961)] is expressed by

$$V(D) = V_{Max}\left(1-\exp\left(-\frac{\lambda}{V_{\max}}(\frac{1}{D}-\frac{1}{D_{\max}})\right)\right) \quad \textbf{(4)}$$

where $\lambda>0$ is the slope of inter-distance-velocity curve at $V=0$ km/h.

Here, when $D=D_{\max}$ then $V=0$ km/h. When $D=0$ then $V=V_{\max}$ km/h. Therefore, the limit conditions are verified by the Newell model.

1.5DRAKE MODEL

[Drake et al. (1967)] have enhanced the Greenberg model by studying various macroscopic traffic models. By estimating the density from velocity and flow data, they propose the new VDM expressed by

$$V(D) = V_{Max} \exp\left(-\frac{1}{2}\left(\frac{D}{D_{\max}}\right)^2\right) \quad \textbf{(5)}$$

According to [Ardekani et al. (2011)], this model presents a better fitting than the above Greenshields, Greenberg and Underwood models for non-congested conditions. In case of congested conditions, the Drake model presents a poor data fitting.

1.6PIPES MODEL

[Pipes (1967)] has generalized the Greenshields model leading to a new velocity-density relationship given by

$$V(D) = V_{Max}\left(1-\left(\frac{D}{D_{\max}}\right)^r\right)^m \quad \textbf{(6)}$$

By varying the values of r and m, a faily of models can be developed. For example, Greenshields model is obtained for $r=1$ and $m=1$.

1.7DREW MODEL

[Drew (1968)] has proposed another model expressed as follows:

$$V(D) = V_{Max}\left(1-\left(\frac{D}{D_{\max}}\right)^{\frac{p+1}{2}}\right) \quad \textbf{(7)}$$

According to [Ardekani et al. (2011)], at free-flow phase, the Drew model presents an underestimated velocity, but in the congested phase, the velocity is overestimated.

1.8DEL CASTILLO MODEL

[Del Castillo & Benitez (1995)] have developed a velocity-density model which is given by the following expression:

$$V(D) = V_{Max}\left(1-\exp\left(\frac{|C_j|}{V_{\max}}(1-\frac{D_{\max}}{D})\right)\right) \quad \textbf{(8)}$$

where C_j is the kinematic wave speed given by

$$C_j = D_{\max} \cdot \left.\frac{dV}{dD}\right|_{D=D_{\max}}$$

According to [MacNicholas & Michael (2008)], the drawback of this model is the large kinematic wave speed range which makes difficult its estimation.

If $D_{max}=1$ and $C_j = \lambda$, then we have the Newell model.

1.9VAN AERDE MODEL

[Aerde (1995)] developed a new velocity-density model which is based on a simple car-following model. This last one is depends on the free and current velocity and a calibrated constants. The minimum desired distance headway is the output of this model. The velocity-density model is given by

$$D(V) = \frac{1}{C_1 + \frac{C_2}{V_{\max}-V} + C_3 V} \quad \textbf{(9)}$$

where C_1, C_2, and C_3 are constants which can be calibrated by nonlinear regression.

1.10 MAC NICHOLAS MODEL

[MacNicholas & Michael (2008)] have proposed the following velocity-density model

$$V(D) = V_{Max}\frac{D_{\max}^q - D^q}{D_{\max}^q + m \cdot D^q} \quad \textbf{(10)}$$

where m and q are real constants. By varying these constants, a family of models can be developed. For example, if $D_{\max}=1$, $m=0$ and $q = \frac{p+1}{2}$ the Drew model expression is found.

1.11 POWER FUNCTION MODEL

The power function proposed by [Del Castillo (2012)] is given by

$$V(D)=\frac{1}{\rho}\left[b+(a-b)\rho-\left[(a\rho)^{\Theta}+(b(1-\rho))^{\Theta}\right]^{\frac{1}{\Theta}}\right] \quad (11)$$

where θ is a shape parameter, $\rho=\frac{D}{D_{\max}}$ and a and b are constants.

1.12 EXPONENTIAL MODEL

The exponential model is given by Del Castillo (2012)as follows

$$V(D)=\frac{1}{\rho}\left[b+(a-b)\rho-\frac{1}{\alpha}\log\left[e^{\alpha\cdot\rho\cdot a}+e^{\alpha\cdot b(1-\rho)}\right]-1\right] \quad (12)$$

where $a=\frac{V_{\max}}{-C_j(1-e^{\alpha b})}, b=\frac{1}{(1-e^{\alpha b})}$ and θ a shape parameter.

1.13 NEGATIVE POWER MODEL

The negative power model is presented by [Del Castillo (2012)] as

$$V(D)=\frac{1}{\rho}\cdot\left[(u_f\cdot\rho)^{-\Theta}+(1-\rho)^{-\Theta}\right]^{\frac{1}{\Theta}} \quad (13)$$

Where w is a shape parameter.

1.14 ENVIRONMENTAL PARAMETERIZATION MODEL

[Peter & Fazekas (2014)] have enhanced the environmental parameterization model by studying various macroscopic traffic models. The classical literature does not deal with the definition of the environmental vector, but the velocity is determined not only by vehicle density, but by other environmental parameterization, as well: this refinement can be implemented with the modification of V_{Max}, or via the modification of the function itself considering the weather, visibility, road quality, width of the road. These environmental, seasonal factors can be represented in the environmental parameter vector $\underline{e}$: $V=v(\rho,\underline{e})$.

$$V(\rho,\underline{e})=\frac{e_4\cdot V_{\max}}{e_3+e_2\cdot\left(\frac{\rho}{1-\rho^{e_5}}\right)^{e_1}} \quad (14)$$

In this case, the parameter vector $\underline{e}$ contains 5 parameters

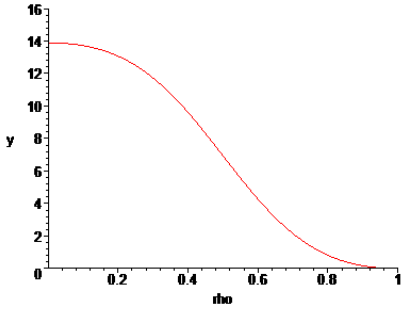

Figure 1 $V(\rho)$ velocity – density function with e_1=2; e_2=1;e_3=1;e_4=1;e_5=1 parameters

The following table demonstrates the favourable and unfavourable parameter domains. The internal domain is located between the two distinct domains, in most of the cases the practical parameter comes from this internal interval. The borders of the intervals are empirical values, in a given case the coordinates of the $\underline{e}$ = [e_1, e_2, e_3, e_4, e_5] parameter-vector are determined via regression analysis after the velocity – density measurement.

Table 1 Demonstration of the $\underline{e}$ parameter vector

e_i	Meaning of the parameter	Unfavorable cases	Favorable cases
e_1	Road quality	Bad: e_1=0.1 - 0.3	Good: e_1=3 – 4
e_2	Curly road	Lot of curves: e_2=3 - 4	Few curves: e_2=0.1-0.2
e_3	Slippery road	Bad, slippery: e_3=1.2 - 4	No slippery: e_3 <1
e_4	Safety,visibility	Bad conditions: e_4=0.5 - 0.7	Good conditions: e_4 >1
e_5	Width of road	Narrow: e_5=0.1 - 0.2	Wide: e_5 >4

The specialty of the introduced $V=v(\rho,\underline{e})$ function is that it gives the same results as the linear function of Greenshields, if every parameters' value equals to 1 (Fig. 2.).

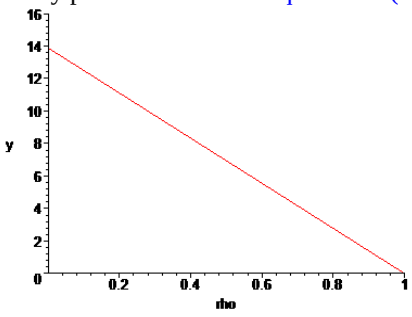

Figure 2 $V(\rho)$ velocity – density function with e_1=1; e_2=1;e_3=1;e_4=1;e_5=1 parameters

This result shows that Greenshields' linear function has parameter values from the mid-range, so that provides really an average $v(\rho)$ velocity-density function relationship in practice.

1.15 SYNTHESIS

Table 2 summarizes the references with the macroscopic traffic models cited in this section. V_{max} is the maximum velocity, D_{max} the jam density, λ the slope of inter-distance-velocity curve at V = 0, C_1, C_2, C_3, m, p, q and r are constants and C_j is the kinematic wave speed at jam density.

Table 2 Velocity Density model summary

Reference	Model
[Greenshields (1934)]	$V(D)=V_{Max}(1-\frac{D}{D_{\max}})$
[Greenberg (1959)]	$V(D)=V_{Max}\log(\frac{D}{D_{\max}})$
[Underwood (1961)]	$V(D)=V_{Max}\exp(-\frac{D}{D_{\max}})$
[Newell (1961)]	$V(D)=V_{Max}\left(1-\exp\left(-\frac{\lambda}{V_{\max}}(\frac{1}{D}-\frac{1}{D_{\max}})\right)\right)$
[Drake et al. (1967)]	$V(D)=V_{Max}\exp\left(-\frac{1}{2}\left(\frac{D}{D_{\max}}\right)^2\right)$
[Pipes (1967)]	$V(D)=V_{Max}\left(1-\left(\frac{D}{D_{\max}}\right)^r\right)^m$
[Drew (1968)]	$V(D)=V_{Max}\left(1-\left(\frac{D}{D_{\max}}\right)^{\frac{p+1}{2}}\right)$
[Del Castillo & Benitez (1995)]	$V(D)=V_{Max}\left(1-\exp\left(\frac{\lvert C_j\rvert}{V_{\max}}(1-\frac{D_{\max}}{D})\right)\right)$
[Aerde (1995)]	$D(V)=\frac{1}{C_1+\frac{C_2}{V_{\max}-V}+C_3V}$
[MacNicholas & Michael (2008)]	$V(D)=V_{Max}\frac{D_{\max}^q-D^q}{D_{\max}^q+m\cdot D^q}$
[Del Castillo (2012)] (power model)	$V(D)=\frac{1}{\rho}\left[b+(a-b)\rho-\left[(a\rho)^\Theta+(b(1-\rho))^\Theta\right]^{\frac{1}{\Theta}}\right]$
[Del Castillo (2012)] (exponential model)	$V(D)=\frac{1}{\rho}\left[b+(a-b)\rho-\frac{1}{\alpha}\log\left[e^{\alpha\rho\cdot a}+e^{\alpha\cdot b(1-\rho)}\right]-1\right]$
[Del Castillo (2012)] (negative power model)	$V(D)=\frac{1}{\rho}\cdot\left[(u_f\cdot\rho)^{-\Theta}+(1-\rho)^{-\Theta}\right]^{\frac{1}{\Theta}}$
[Peter & Fazekas (2014)] environmental parameterization model)	$V(\rho,\underline{e})=\frac{e_4\cdot V_{Max}}{e_3+e_2\cdot\left(\frac{\rho}{1-\rho^{e_5}}\right)^{e_1}}$

It was necessary to review the evolution of the Velocity-density laws in the literature, since these functions have a decisive role in the fundamental equation in case of traffic models. They shall satisfy two conflicting needs: to be as simple as possible, so that the large complex simulation models provide fast numerical calculations, while ing the real practical situations as widely as possible. A common feature of the velocity - density laws is that they result from non-linear regression analysis based on urements and hypotheses (preconceptions). The common flaw of the classical results is that because they do not refer to other environmental impacts, they only examine the effect of speed on the vehicle density, so essentially any of them can be suitable for a given modelling. [Peter & Fazekas (2014)] goes much further and examines also the environmental parameters. In case of the IDM-models the comprehensive examination of the speed-density tions is an interesting problem, because without ception an expanded set of functions can be examined from the point of view of which known function types, or other new models are considered as valid by this model. Of course, neither the classic IDM-models contain the environmental parameters mentioned above, so the studies in this area are far from closed.

The next section is dedicated to generate microscopic and macroscopic simulated traffic data which are the density and the velocity. In addition, the impact of microscopic parameters to macroscopic ones is investigated.

2. Macroscopic traffic data generation

Based on microscopic traffic simulation, density-time and velocity-time functions will be computed in this section.

Paragraph 2.1 presents the simulation assumptions and the Intelligent Driver Model intended to represent the microscopic longitudinal vehicle motion. Paragraph 2.2 introduces the used mathematical formula to compute macroscopic parameters (velocity and density). Paragraph 2.3 presents the simulation results and discusses it and paragraph 2.4 studies the impact of microscopic parameter on macroscopic ones.

2.1 MICROSCOPIC TRAFFIC MODEL: INTELLIGENT DRIVER MODEL (IDM)

The Intelligent Driver Model (IDM) is an Adaptive Cruise Control system intended for adjusting the driver's longitudinal desired velocity and safety time gap. The IDM model, developed by [Kesting et al. (2008)]), is expressed by

$$\dot{v}_n = a_n\left[1-\left(\frac{v_n}{v_n^0}\right)^4-\left(\frac{s^*(v_n,\Delta v_n)}{s_n}\right)^2\right], \qquad \textbf{(15)}$$

where a_n is the maximum acceleration of the vehicle n (m/s^2), v^0_n the desired velocity of the vehicle n (m/s), and the distance gap (m).

$$s_n = \Delta x_n - l_{n+1}$$

The desired minimum gap of the vehicle n, s^*_n is given by

$$s^*(v_n,\Delta v_n) = s_n^0 + T_n v_n - \frac{v_n \Delta v_n}{2\sqrt{a_n v_n}} \qquad \textbf{(16)}$$

where b_n is the desired deceleration of the vehicle n (m/s^2), s^0_n the jam distance of the vehicle n (m) and T_n the safety time gap of the vehicle n (s).

Compared to the other ACC models in the literature, the IDM shows more advantages in terms of easy implementation, calibration and the intuitive and the availability of its parameters. These ones are the desired velocity v^0_n and the safety time gap T_n which are fixed as system inputs.

The next subsection details the density and velocity formula for computing macroscopic data under microscopic traffic simulation using the IDM model.

2.2DENSITY AND VELOCITY FORMULA

2.2.1 Density formula

As shown in subsection 2.14 in table 1, the quotient between the traffic density and the jam density are used in all models in the literature for density normalization. In this paper, the density is already normalized. Here, the traffic road section is limited by the first and the last vehicle. Then, the density formula can be written as

$$d(t) = \frac{\sum_{k=1}^{N} l_k}{L_{platoon}(t)} \qquad \textbf{(17)}$$

where l_k is the length of the vehicle (k=1,2,…,N), N the number of vehicle in the platoon and $L_{platoon}$ the length of the platoon. And the density d can be normalized in the interval [0,1]. Then, we have

$$d = \frac{D}{D_{\max}}$$

where D_{max} =1 in this case. For this purpose, in the rest of this paper, the density d will be noted as D.

2.2.2 Velocity formula

The platoon velocity v is the average velocity of the vehicles in the platoon which is given by

$$v(t) = \frac{1}{N}\sum_{n=1}^{N} v_n(t) \qquad \textbf{(18)}$$

where v_n is the velocity of the vehicle n.

2.3SIMULATIONS AND RESULTS

2.3.1 The simulation model

The following system of non-linear matrix differential equations shows the structure of the IDM-model under investigation. For a multi vehicle system, the Intelligent Driver Model is given by:

$$\langle \underline{\underline{A}} \rangle^{-1} \ddot{\underline{x}}(t) + \langle \underline{\underline{V}} \rangle^{-1} \underline{f_1}(\dot{\underline{x}}(t)) + \langle \underline{\underline{S}} \rangle \underline{f_2}(x(t)) = \underline{1} \qquad (19)$$

with

$$\langle \underline{\underline{A}} \rangle^{-1} = \left\langle \frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_N} \right\rangle ; \quad \langle \underline{\underline{V}} \rangle^{-1} = \left\langle \frac{1}{v_1^4}, \frac{1}{v_2^4}, \ldots, \frac{1}{v_N^4} \right\rangle ;$$

$$\langle \underline{\underline{S}} \rangle = \langle s_1, s_2, \ldots, s_N \rangle$$

$$s_i = s_{0i} = fixed \text{ or: } s_i = s_i(\dot{x}_{i-1}, \dot{x}_i) \text{ (i=1,2,…,N).}$$

$$\underline{f_1}(\dot{\underline{x}}(t)) = \begin{bmatrix} \dot{x}_1^4 \\ \dot{x}_2^4 \\ \ldots \\ \dot{x}_N^4 \end{bmatrix}, \quad \underline{f_2}(x(t)) = \begin{bmatrix} \frac{1}{(x_0 - x_1)^2} \\ \frac{1}{(x_1 - x_2)^2} \\ \ldots \\ \frac{1}{(x_{n-1} - x_N)^2} \end{bmatrix}, \quad \underline{1} = \begin{bmatrix} 1 \\ 1 \\ \ldots \\ 1 \end{bmatrix}$$

This model examines the longitudinal dynamics in the direction of travelling in case of vehicles travelling in a lane and is specifically capable of analyzing the evolution of the speed-density law. The movement is performed on a line, which is a given lane. We have dealt with the ther generalization of the model in [Derbel, Peter, Mourllion & Basset (2012)] and [Derbel et al. (2013)].

2.3.2 Simulation assumptions

In our simulations, the following assumptions are taken into account:

- The AAC model, which is our microscopic traffic model, is already validated and its parameters are identified based on experimental data ([Kesting et al. (2008)]).
- The traffic is homogeneous. It means that the longitudinal motion of all vehicles is controlled by the same ACC (IDM) with the same parameter values. A heterogeneous traffic includes vehicles with manual and automatic driving modes.
- The road has only one lane.

- Only the longitudinal vehicle motion is studied. The lane change and the lateral motion are neglected.

2.3.3 *The simulator*

The development of traffic simulators has received much attention in recent years. Many companies have invested in such projects and have made simulators integrating various dynamic and kinematic models of vehicles, driver and road architectures available. These simulators are known by their complexities in terms of software development. For example, the simulator ARCHISIM adopts multi-agent approach and techniques from artificial intelligence to simulate complex phenomena. Our motivations for the development of a mixed traffic simulator (coexistence of automated and manual driving style in the same traffic section) are justified according to the following two aspects:

- The high cost of traffic simulators in the market.
- The collision management problem. For example, the traffic simulators CarMaker and Microsimulation of Road Traffic Flow (MRTF) do not perfectly manage collisions between vehicles.

Our mixed traffic simulator is developed using the C language and the OpenGL library (Open Graphics brary) for graphical interface. Figure 2 shows the structure of mixed traffic simulator. This simulator has inputs that are used by its different modules to save the output of the simulation in different formats such as video and/or data required for post-processing.

The inputs of the simulator are the following three:

- "XML File" in which some parameters can be set: the simulation number, the initial vehicle number, the road length and the minimum and maximum percentage of automated vehicles.
- "Global Settings": these parameters are fixed throughout the simulation (e.g. road width).
- "OpenGL library": This library is responsible for the GUI management such as scenarios and architectures used in the simulation.

In the simulator body, the following modules are developed:

- "Scenario Development Module": This module cludes the different developed scenarios. They take their input parameters through the "main" function. It selects the script to run from the choice made in the XML file. In this module, there are useful models: the IDM model for longitudinal automated driving style, the Two Velocity Difference Model (TVDM) [Ge et al. (2008)] for longitudinal manual driving style and the Minimizing Overall Braking change Induced by Lane change (MOBIL) model [Kesting et al. (2008)] for the lane change management. These models need to be updated in each simulation step through the numerical integration module (Runge Kutta).
- "GUI Management Module": This module is based on the OpenGL library and the measurements are recorded to generate and update the simulator graphical interface.

The outputs of this simulator are the following three:

- "Screen": Displays the architecture and the scenario.
- "Video": to save the simulation
- "File .txt or .bin": saves the data needed for post-processing

Figure 3 shows the position of the vehicles in a platoon. The IDM simulation model with the parameters given by table 3 is applied to simulate the fully automated traffic using our developed microscopic traffic simulator. The presented parameter values in this table are originated from an identification step using experimental data and used to validate the IDM with the fundamental diagram by [Kesting et al. (2008)]. At each simulation step, the simulated microscopic traffic data such as the position, the velocity and the acceleration of each vehicle are computed. Then, the density and the velocity are computed according to equations (17) and (18) respectively.
Simulation parameters are given in table 4. In this table, the jam distance s^0_n is set to zero in order to have the maximum density equal to 1.

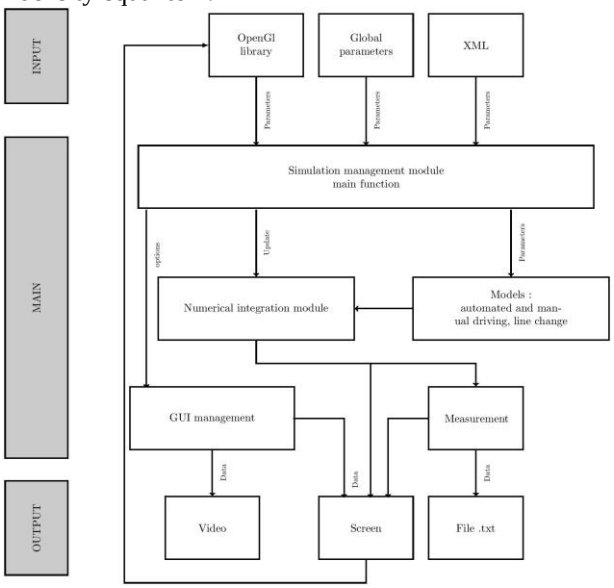

Figure 2 Structure of the developed simulator

To study the sensitivity of macroscopic parameters (velocity and density) according to the microscopic parameters (a_n, b_n, T_n), traffic simulation is performed made for each of the maximum acceleration a_n values (the other microscopic parameters are constants), the desired deceleration b_n (the other microscopic parameters are constants) and the safety time headway T_n (the other microscopic parameters are constants).

2.3.4 *Simulation results*

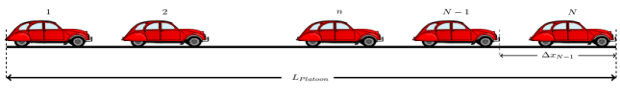

Figure 3 Positions of the vehicles in the platoon

Table 3 IDM simulation parameters

Parameter	Mean value	Unit
Maximum acceleration a_n	3	m/s^2
Desired deceleration b_n	3	m/s^2
Safety time headway T_n	1.5	s

Table 4 Simulation parameters

Parameter	Value	Unit
Vehicle number	20	vehicle
Simulation time	1000	s
Simulation time step	0.01	s
Initial inter distance	0	m
Initial acceleration	0	m/s^2
Desired velocity	50	km/h
Initial velocity	0	km/h
The jam distance	0	m
Vehicle length	5	m

Figure 4 shows the density versus the time (first curve) and the velocity versus the time (second curve) during 1000 s of simulation with the microscopic parameters given by table 3 and the simulation parameters given by Table 4. Here as the density increases, the velocity decreases until the stable state.

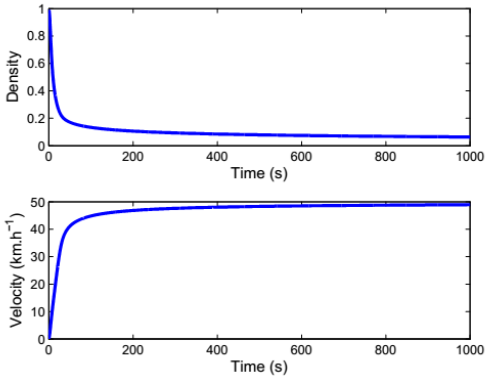

Figure 4 Traffic density and velocity time functions

2.4FROM MACROSCOPIC PARAMETERS TO MICROSCOPIC PARAMETERS

In this section, two goals are fixed: the first one consists in studying the dependency of the shape of the velocity-time and the density-time functions to microscopic parameters. The second goal is to study the ability to identify microscopic parameters from macroscopic parameters. Microscopic parameters are the maximum acceleration a_n, the desired deceleration b_n and the safety time headway T_n. In this step the following assumptions are made: the maximum acceleration $a_n \in [2, 7]$ m/s^2 the desired deceleration $b_n \in [2, 5]$ m/s^2 and the safety time headway $T_n \in [1.5, 4]$ s. The ranges are empirical intervals given by [Kesting et al. (2008)]. In this paragraph, it is noted that all vehicles have the same parameters (homogeneous traffic).

2.4.1 Impact of a_n , b_n and T_n on density

As shown in figure 3, the maximum acceleration and the desired deceleration have no impact on traffic density. Indeed, when the traffic becomes stable,

$\Delta v_n = 0$, $\forall n \in \{1,...,N\}$ and then $\frac{v_n \cdot \Delta v_n}{2\sqrt{a_n v \cdot b_n}} = 0$.

The safety time headway T_n has an impact on traffic density i.e when T_n increases, traffic density decreases. In fact, vehicles tend to increase inter-distance when T_n is high, then the traffic density decreases.

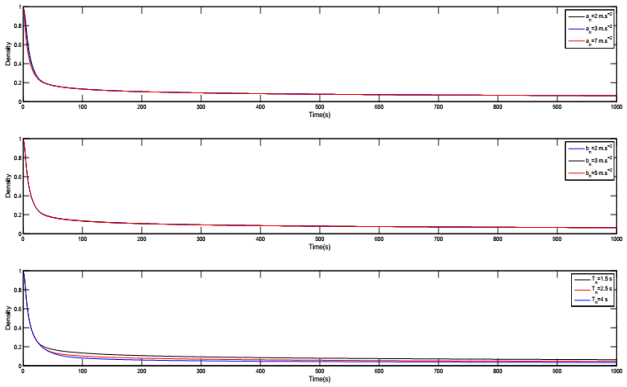

Figure 5 Density for different a_n, b_n and T_n sets

2.4.2 Impact of a_n , b_n and T_n on velocity

As shown in figure 6, the maximum acceleration and the desired deceleration have no impact on the velocity. The same reason cited in 3.4 is the cause of this result: when the traffic becomes stable, $\Delta v_n = 0$ $\forall$ n $\in$ {1...N} and then $\frac{v_n \cdot \Delta v_n}{2\sqrt{a_n v \cdot b_n}} = 0$. The safety time headway T_n has an impact on the velocity i.e. when T_n decreases vehicles velocity increases. Vehicles accelerate to reach the desired velocity until the safety time headway is guaranteed.

Figure 7 shows the impact of the time headway T_n on the macroscopic parameters. Now the curves of this figure will be considered as reference to us. In case of unknown time headway, from the density and the velocity values, we can identify the mean value of time headway between vehicles way by mean of interpolation.

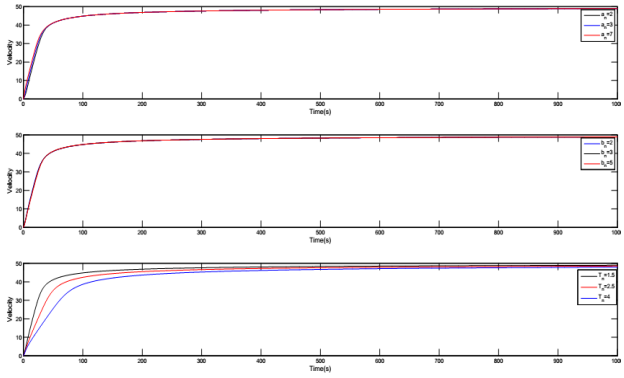

Figure 6 Velocity for different a_n, b_n and T_n sets

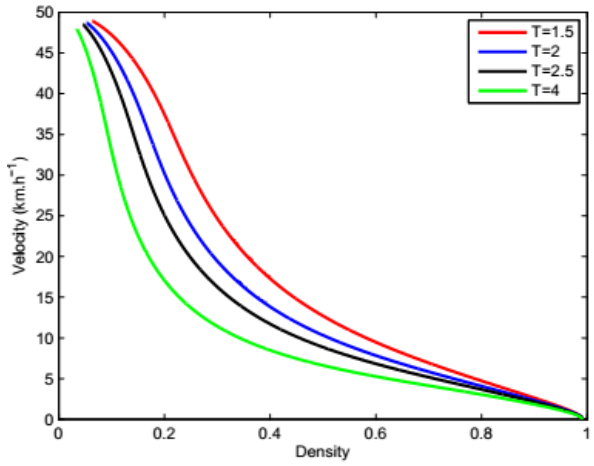

Figure 7 Impact of the time headway on the velocity density curve

3. New velocity-density model synthesis method

The new method consists in studying the variation of each of the two macroscopic parameters (velocity and density given by the figure 2) separately along the simulation time. To carry out this study, a family of functions is computed to approximate the density-time and velocity-time functions. For a given γ, this family of candidate functions is expressed by

$$H_\gamma = \{F / \exists (F_1, F_2, \tau_f) \in \Re^3; F(t) = F_1 \exp\left(-\frac{t^\gamma}{\tau_f}\right) + F_2\}$$

For reasons which will be later explained, γ is fixed for all candidate functions of the two macroscopic parameters.
The function $D \in H_\gamma$, which is a candidate to approximate the density-time function, is given by

$$D(t) = D_1 \exp\left(-\frac{t^\gamma}{\tau_d}\right) + D_2 \quad \textbf{(20)}$$

where D_1, τ_d and D_2 are constants. The function v $\in H_\gamma$, which is a candidate to approximate the velocity-time function, is given by

$$v(t) = V_1 \exp\left(-\frac{t^\gamma}{\tau_v}\right) + V_2 \quad \textbf{(21)}$$

where $V_1<0$, and $\tau_v >0$, $V_2 >0$ are constants ($V_2 \geq$ïV_1½). γ is fixed for the two candidate functions to have an analytical time independent velocity-density relationship from equations (20) and (21). D and v functions are used to fit separately the density and
the velocity data in the least square sense. Figure 6 shows the fitted curves of the density and the velocity together with the macroscopic data given by IDM simulation. Parameters of these two functions, obtained with setting γ = 0.8, are summarized in table 5. The mean, the maximum and the minimum absolute errors are given by table 6. The absolute error between data given by microscopic simulation and the computed functions D and v is small compared to the maximum values of each one. The IDM is written as

$$\dot{v}_n = a_n \left[1 - \left(\frac{v_n}{v_n^0}\right)^4 - \left(\frac{s^*(v_n, \Delta v_n)}{s_n}\right)^2\right],$$

and

$$s^*(v_n, \Delta v_n) = s_n^0 + T_n v_n - \frac{v_n \Delta v_n}{2\sqrt{a_n v_n}}.$$

The solution for the velocity is given by

$$v(t) = V_1 \exp\left(-\frac{t^\gamma}{\tau_v}\right) + V_2$$

When traffic become stable, we obtain $\dot{v}_n = 0$ and $e^{\frac{-t^\gamma}{\tau_v}} \to 0$. Then,

$$\left(\frac{T_n}{s_n}\right)^2 \cdot \left(v_n^0\right)^4 \cdot V_2^2 + V_2^4 = \left(v_n^0\right)^4 \quad \textbf{(22)}$$

And, as shown in equation (22), V_2 depends on the inter-distance and the safety time headway T_n and not from a_n and b_n.

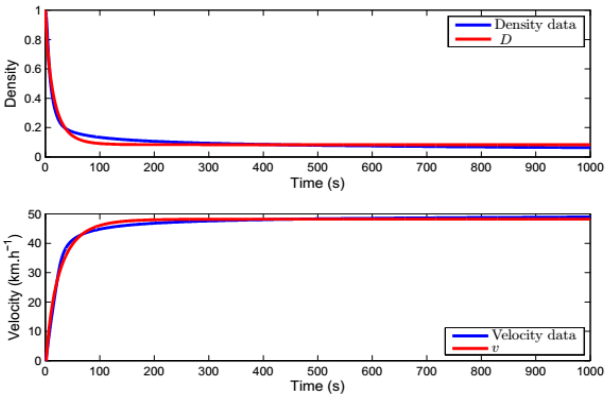

Figure 8 Density-time function with D function and velocity-time function with v function

Table 5 Parameters of the D and v functions with γ = 0.8

Parameter	Mean value	Unit
D_1	0.8971	-
τ_d	0.1205	s
D_2	0.1029	-
V_1	-48.4583	*km/h*
τ_v	0.0813	s
V_2	48.48205	*km/h*
R^2 (Velocity)	0.9788	-
R^2 (Density)	0.9399	-

Table 6 Density and velocity error

Parameter	Mean	Max	Min
Density error	0.0157	0.0786	0
Mean velocity error	0.6402	6.3786	0

4. Velocity-density model synthesis

From the density model defined by equation (20) and the velocity model defined by equation (21), the VDM is developed analytically in this section.

4.1MODEL SYNTHESIS

Equations (20) is given by

$$D(t) = D_1 \exp\left(-\frac{t^{\gamma}}{\tau_d}\right) + D_2 \quad \textbf{(23)}$$

The function v ∈ H_γ, which is a candidate to approximate the velocity-time function, is given by

$$v(t) = V_1 \exp\left(-\frac{t^{\gamma}}{\tau_v}\right) + V_2 \quad \textbf{(24)}$$

From these two equations, we get

$$\begin{cases} t^{\gamma} = \tau_d \cdot \ln\left(\dfrac{D(t) - D_2}{D_1}\right) \\ t^{\gamma} = \tau_v \cdot \ln\left(\dfrac{v(t) - V_2}{V_1}\right) \end{cases} \quad \textbf{(25)}$$

$$v(D(t)) = V_1\left(\frac{D(t) - D_2}{D_1}\right)^{\frac{\tau_d}{\tau_v}} + V_2 \quad \textbf{(26)}$$

Let *V*, the function defined as

$$V : D \to V_1\left(\frac{D(t) - D_2}{D_1}\right)^{\frac{\tau_d}{\tau_v}} + V_2 \quad \textbf{(27)}$$

With slight abuse of notation, we have *v* (*D* (t)) = *V* (*D*), Then, the analytic velocity-density relationship can be written as

$$v(D) = V_1\left(\frac{D - D_2}{D_1}\right)^{\frac{\tau_d}{\tau_v}} + V_2 \quad \textbf{(28)}$$

To calibrate this model, the limit conditions are applied in the next sub-section.

4.2 GENERALIZED VELOCITY-DENSITY MODEL

Applying the limit conditions given by

$$\begin{cases} D = 1 \Rightarrow V = \varepsilon_V \approx 0; (\varepsilon_V = V_2 + V_1) \\ D = D_2 \Rightarrow V = V_{\max} \end{cases} \quad \textbf{(29)}$$

to equation (28), we have

$$\begin{cases} V_1\left(\dfrac{1 - D_2}{D_1}\right)^{\frac{\tau_d}{\tau_v}} + V_2 = \varepsilon_V \\ V_1\left(\dfrac{D_2 - D_2}{D_1}\right)^{\frac{\tau_d}{\tau_v}} + V_2 = V_{\max} \end{cases} \quad \textbf{(30)}$$

Equations (30) is given by (31) and (32)

$$V_2 = V_{\max} \quad \textbf{(31)}$$

$$V_1 = \frac{-V_{\max}}{\left(\frac{1-D_2}{D_1}\right)^{\frac{\tau_d}{\tau_v}}} + \frac{\varepsilon_V}{\left(\frac{1-D_2}{D_1}\right)^{\frac{\tau_d}{\tau_v}}} \quad \textbf{(32)}$$

using (28), (31) and (32)

$$V(D) = \left[\frac{-V_{\max}}{\left(\frac{1-D_2}{D_1}\right)^{\frac{\tau_d}{\tau_v}}} + \frac{\varepsilon_V}{\left(\frac{1-D_2}{D_1}\right)^{\frac{\tau_d}{\tau_v}}} \right] \cdot \left(\frac{D-D_2}{D_1}\right)^{\frac{\tau_d}{\tau_v}} + V_{\max} \quad \textbf{(33)}$$

Then, the generalized VDM can finally be expressed as

$$V(D) = V_{\max} \cdot \left[1 - \left(\frac{D-D_2}{1-D_2}\right)^{\frac{\tau_d}{\tau_v}} \right] + \varepsilon_V \left(\frac{D-D_2}{1-D_2}\right)^{\frac{\tau_d}{\tau_v}} \quad \textbf{(34)}$$

$$D \in [1, D_2]$$

Where $\tau_d>0$, $\tau_v>0$, $D_1>0$, $D_2>0$ are constants, and $D_1+D_2=1$. From this calibrated model, some of models defined in section 2 can be derived. Next subsections show which of these models can be obtained.

4.3SUB MODELS

From the model defined by equation (34), some of existing models in the literature can be derived by fixing some of its parameters.

4.3.1 If $D_2=0$ and $e_V=0$

In this case,

$$V(D) = V_{\max}(1-D)^{\frac{\tau_d}{\tau_v}} \quad \textbf{(35)}$$

represents the Pipes model given by equation (6) where $r=1$, $e_V=0$, $m = \frac{\tau_d}{\tau_v}$ and $D_{\max}=1$. One of the advantages of the new model is the finding of the physical meaning of the parameter m.

4.3.2 If $D_2=0$, $e_V=0$ and $\tau_d = \tau_v$

In this case,

$$V(D) = V_{\max}(1-D) \quad \textbf{(36)}$$

Equations (20) and (21) can be written as

$$D(t) = D_1 \exp\left(-\frac{t^\gamma}{\tau_d}\right) + D_2 \quad \textbf{(37)}$$

$$V(t) = V_1 \exp\left(-\frac{t^\gamma}{\tau_v}\right) + V_2 \quad \textbf{(38)}$$

Then

$$V(D) = V_1\left(\frac{D}{D_1} - 1\right) + V_2 \quad \textbf{(39)}$$

Here, the relationship between velocity and density remains linear. In case of the Greenshields model, $V_{\max}=V_2$ and $D_{\max}=1$.

4.3.3 If $D_2=0$, $e_V=0$ and $\frac{\tau_d}{\tau_v} = \frac{p+1}{2}$ and $p \geq -1$

In this case,

$$V(D) = V_{\max}(1 - D^{\frac{p+1}{2}}) \quad \textbf{(40)}$$

which is the Drew model given by equation (7), where $D_{\max}=1$. Since τ_d and τ_v are two time constants, then $\frac{\tau_d}{\tau_v} \geq 0$. Therefore, the choice of $\frac{\tau_d}{\tau_v} = \frac{p+1}{2}$ leads to have $p+1 \geq 0$ and then $p \geq -1$. In this study, the density is expressed as

$$D(t) = D_1 \exp\left(-\frac{t^\gamma}{\tau_d}\right) + D_2 \quad \textbf{(41)}$$

It implies that when $t \to \infty$, $D_1 \cdot \exp(-\frac{t^\gamma}{\tau_d}) << D_2$ .If $D_2 \to 0$, then only lower densities can be predicted. Therefore, velocity can be estimated only in free flow condition. Thus, the model cannot predict lower velocities. This result has been proven by[Ardekani et al. (2011)], empirically and proved analytically now by our method of velocity-density synthesis and through our generic model.

5. Conclusions

The Velocity-density functions known from the literature are results of non-linear regressions, which are obtained by the determination of functions that fit the measured coherent speed-density values with the minimum error.

The first step of the analysis we have performed in the field of IDM-models is also a non-linear regression, which was developed based on the speed and density changes over time. Next we have described a direct function relation for the speed-density function using a time parameter. There is no defined speed-density law in the IDM-model, so it was an important achievement to show what speed-density function is followed by the IDM-model.

A separate new interesting result is that the law presented by us has defined a more general class of function compared to what was known earlier. At the same time this has also integrated three known speed-density relationships as special cases (with the proper selection of parameters) [Greenshields (1934)], [Pipes (1967)] and [Drew (1968)].

We emphasize that the analysis - in terms of the development of simulation models - led to another important question. The known speed-density functions are univariate functions of the vehicle densities. The significant formal differences between these types of functions raise the question whether the differences can significantly be predetermined by the environmental parameters, as well. An interesting question is whether more general classes of functions that integrate more known function types can be determined by broader measurement and analysis, and further IDM analysis. These known function types are multivariate functions that depend on the density and environmental parameters, as well, for example the one presented in [Peter & Fazekas (2014)]. The simulator discussed in this article is capable of carrying out these further investigations.

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