Extending Smith's Rule with Task Mandatory Parts and Release Dates

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1. Introduction

Problem

Note: here and in the following, we will use d_j instead of $\tilde{d}_j$ for deadlines

Long term objective: improve CP integration of $\sum C_j$ objective

- General problem (even without *prec*): $1|r_j; d_j|\sum C_j$
- Polynomial relaxation with a list algorithm: $1|r_j; pmtn|\sum C_j$
- We want a Lower Bound (LB) of good quality and computable quickly

id_j	1	2	3	4	5	6
p_j	14	5	2	3	6	3
r_j	0	0	2	12	16	18
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$$\sum C_j = 138$$

$$(4 + 18 + 24 + 27 + 30 + 35)$$

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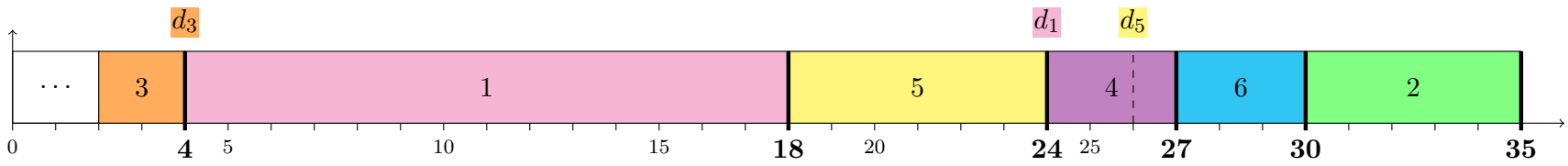

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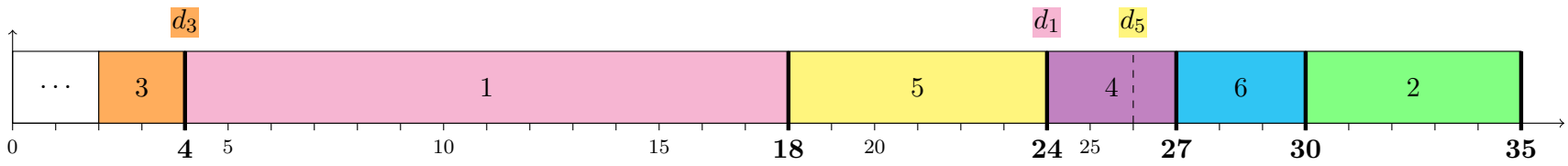

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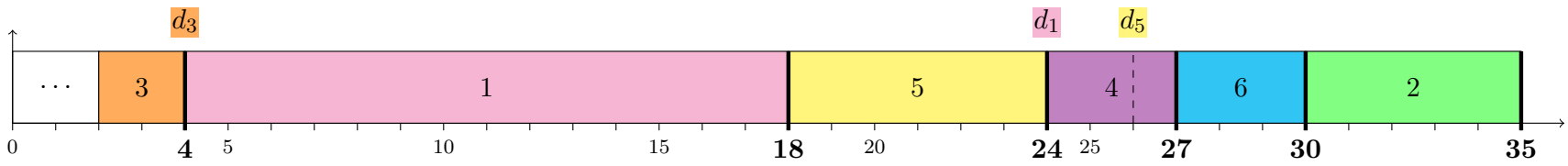

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- $1||\sum C_j$: Smith Rule (or SPT rule) [Smith, 1956]
- $1|r_j; pmtn|\sum C_j$: Modified Smith Rule (MSR) [Baker, 1974 and Brucker, 2006] $\Rightarrow 1^{st}$ Lower Bound (LB)

Objectives

- Extend MSR to enforce the tasks Mandatory Parts (MP)
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Definition: Mandatory Part of a task (MP)

Time interval in which a task **must be executed** under the hypothesis that it is **not late**, **not preempted** and without taking into account the other tasks.

eft_j = earliest finishing time of task j
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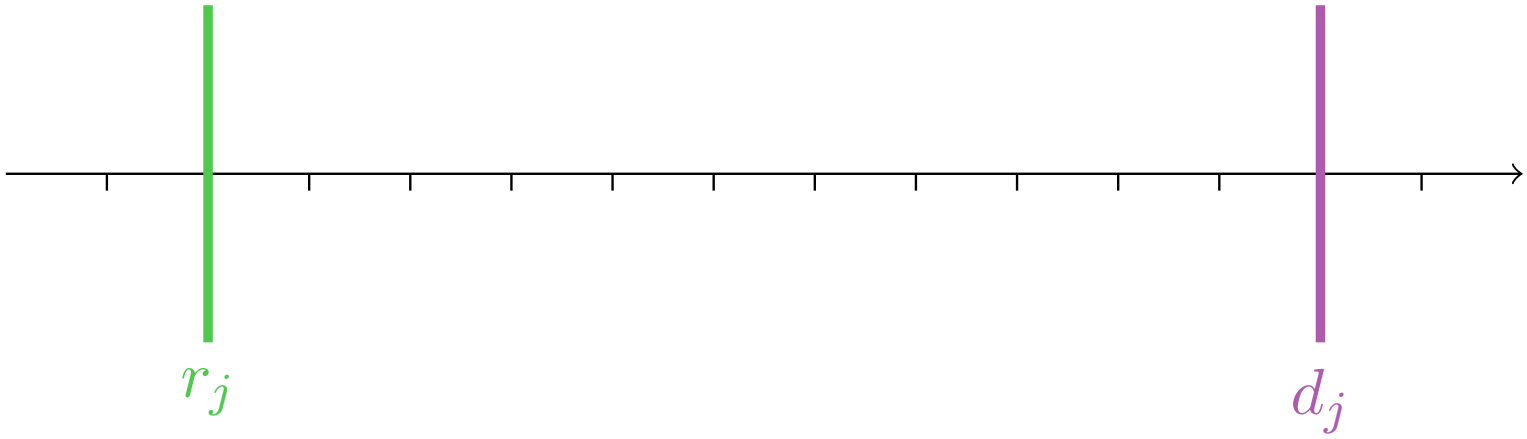

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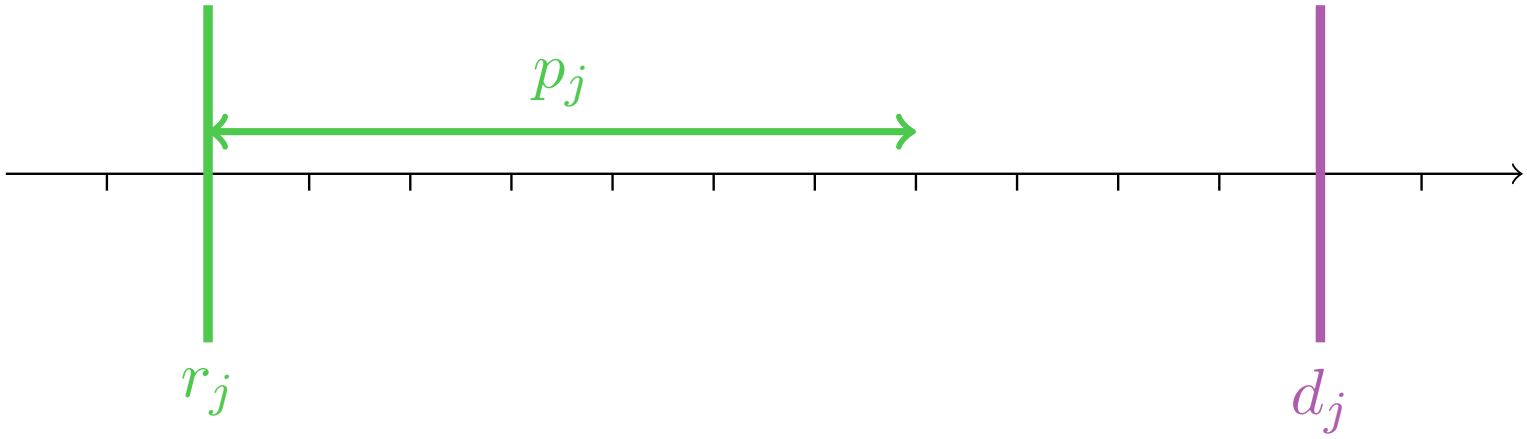

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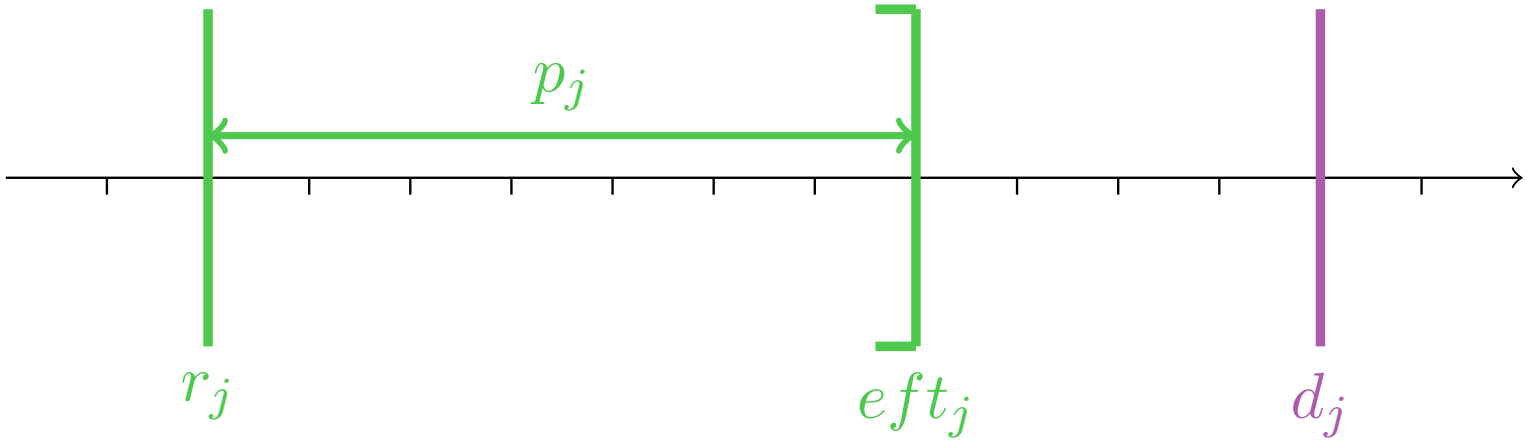

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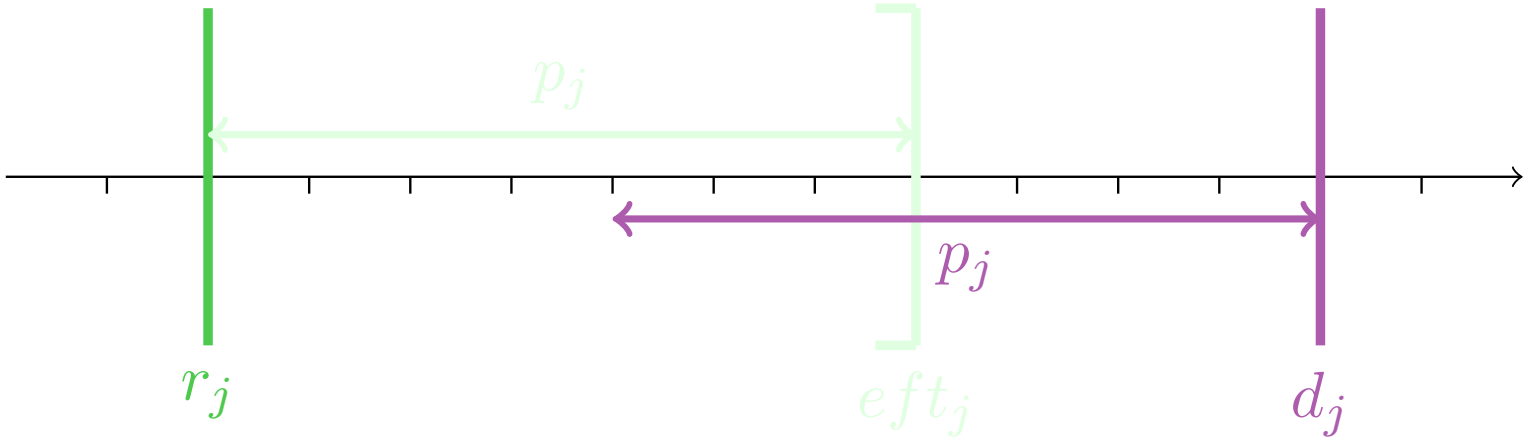

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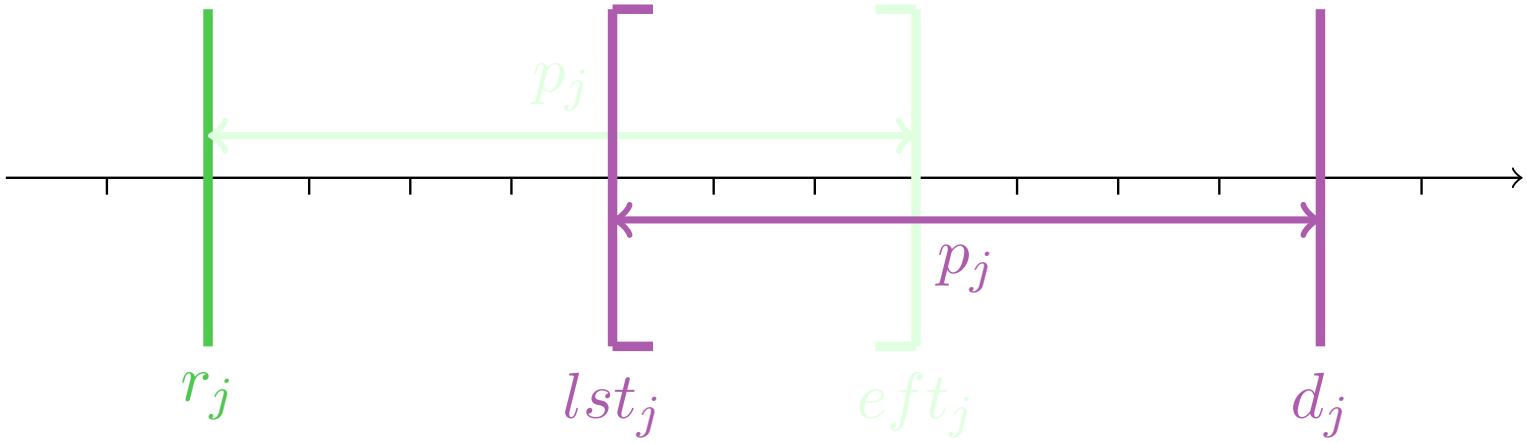

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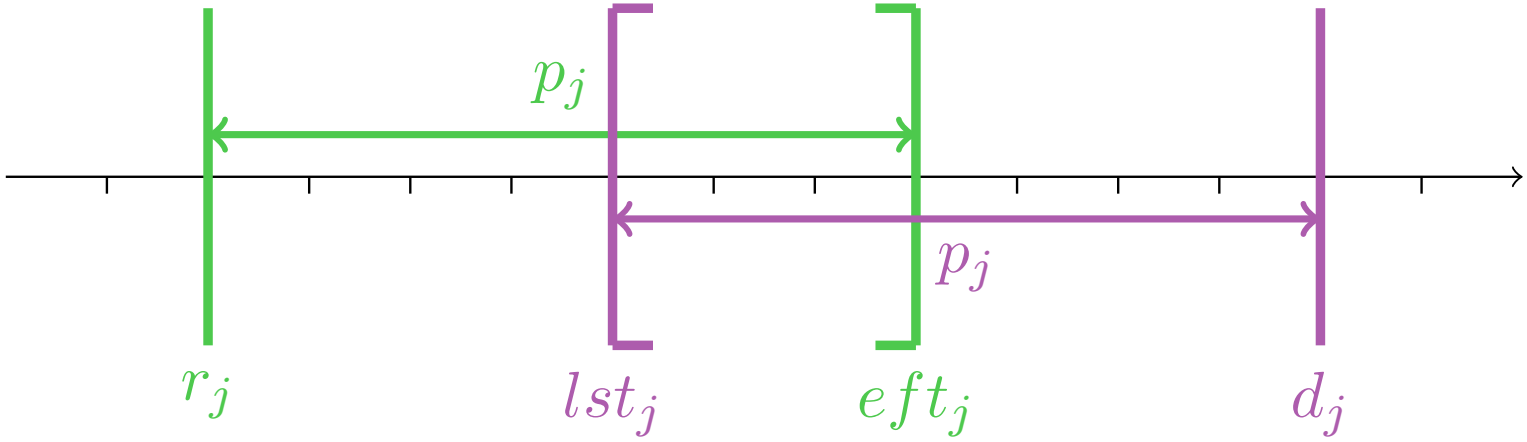

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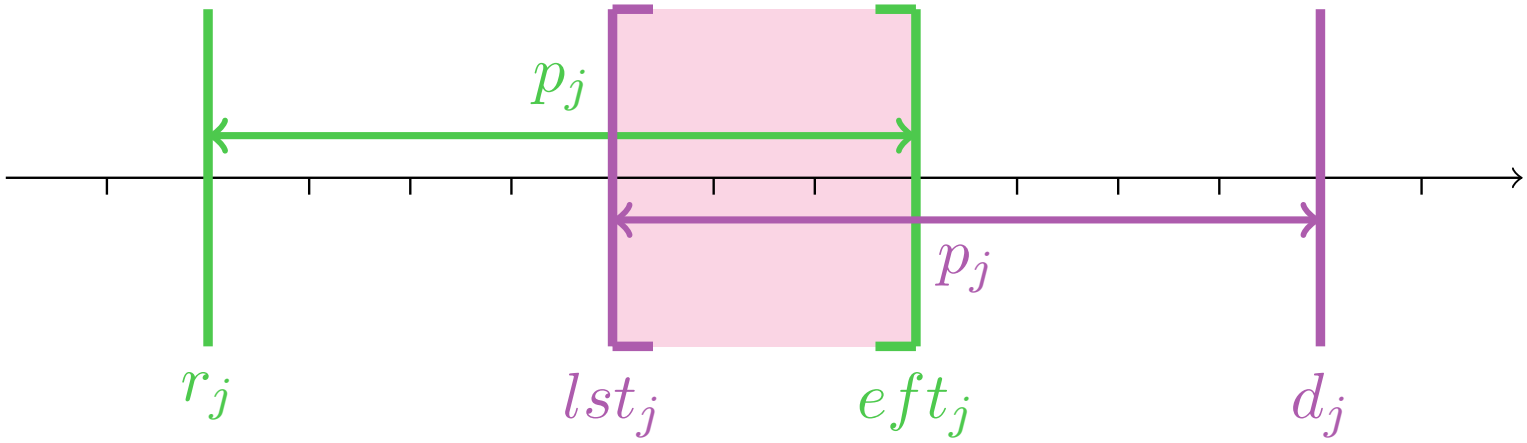

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2. Modified Smith Rule with MPs

Modified Smith Rule (MSR)

Theorem: Modified Smith Rule (MSR) [Brucker, 2006]

At each **release time** or **finishing time** of a task, schedule an unfinished task which is available and has the **smallest remaining processing time** ($\widehat{p}_j$).

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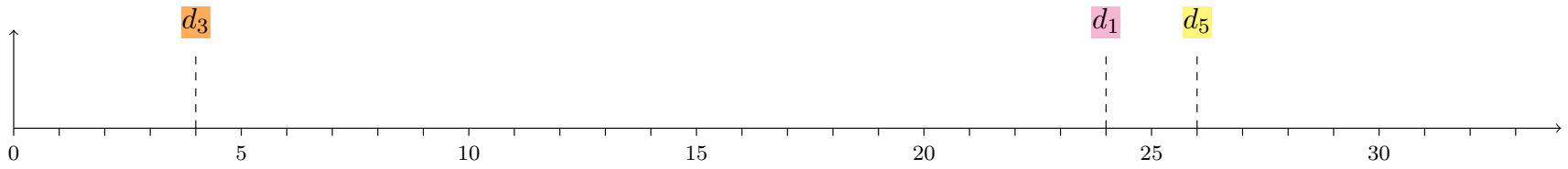

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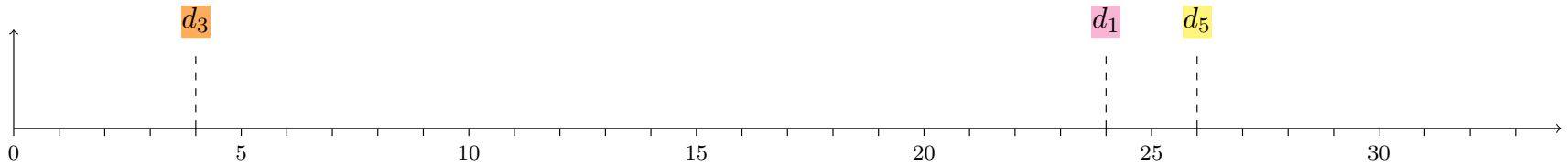

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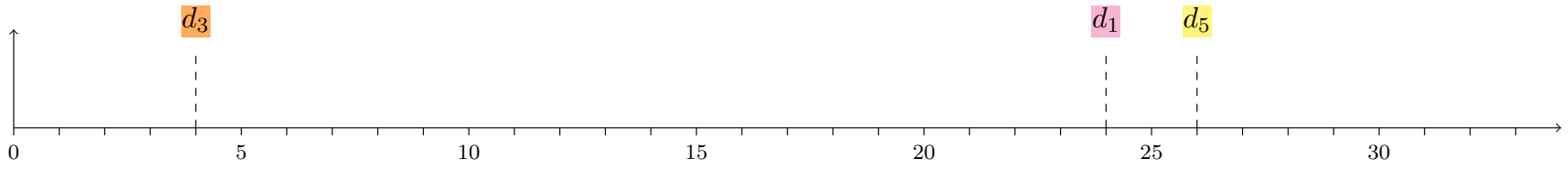

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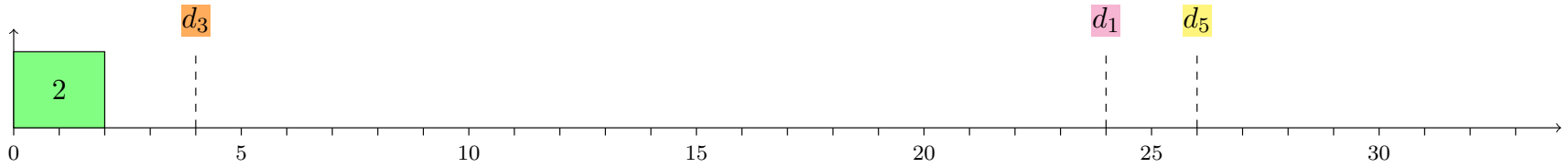

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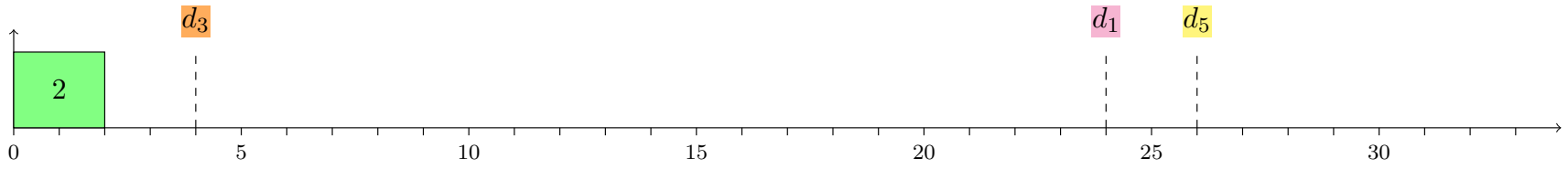

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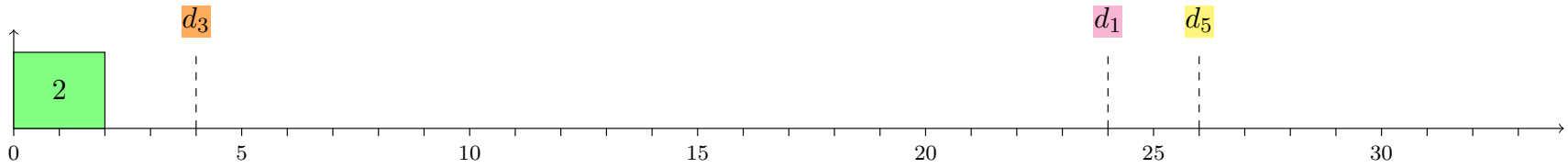

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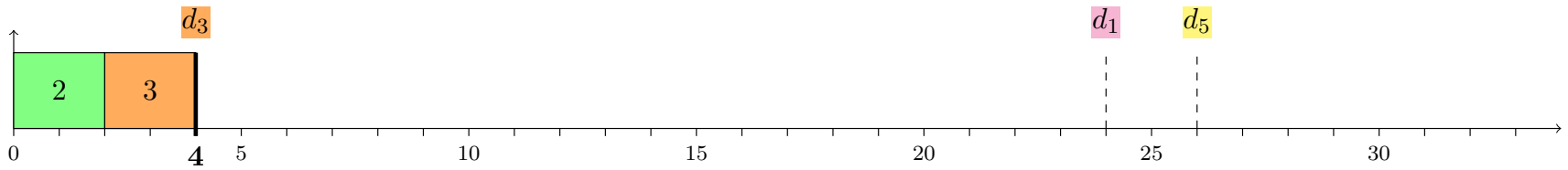

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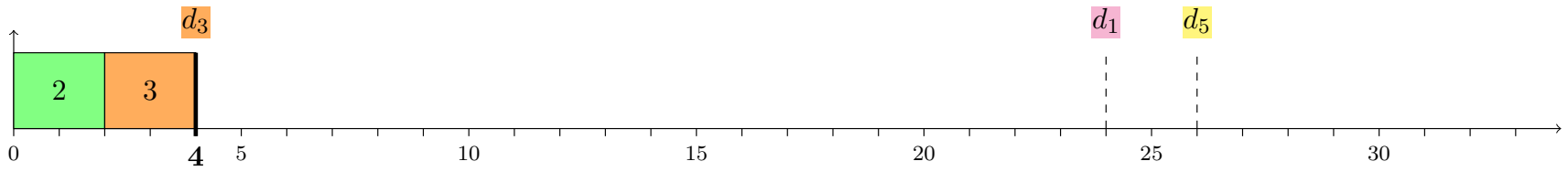

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r_j	**0**	**0**	2	12	16	18
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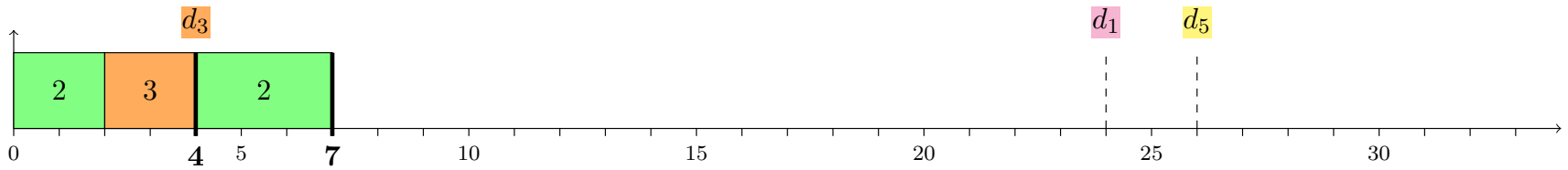

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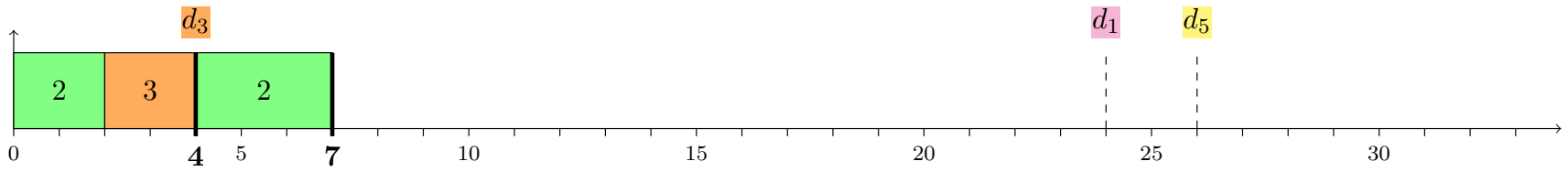

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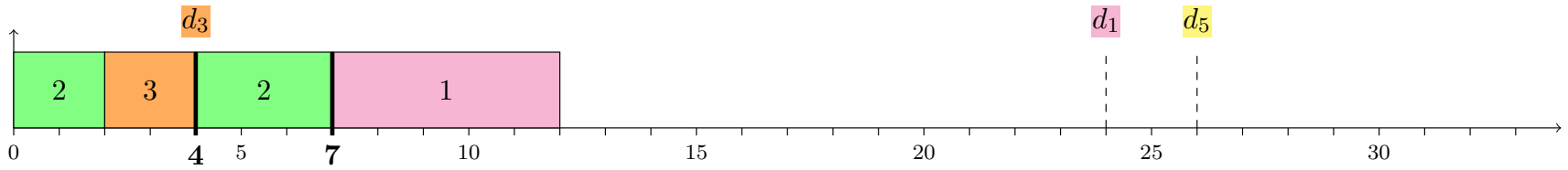

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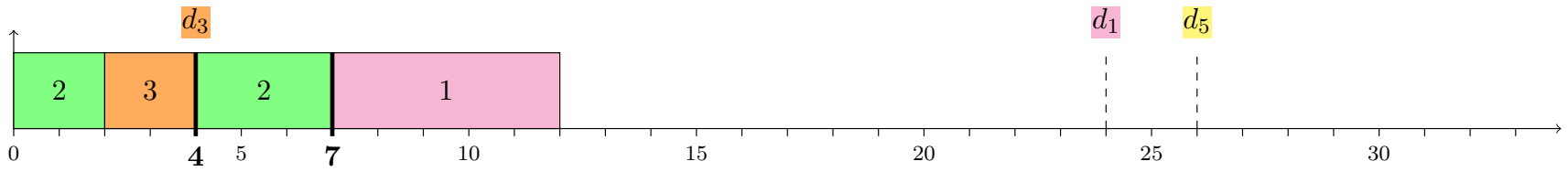

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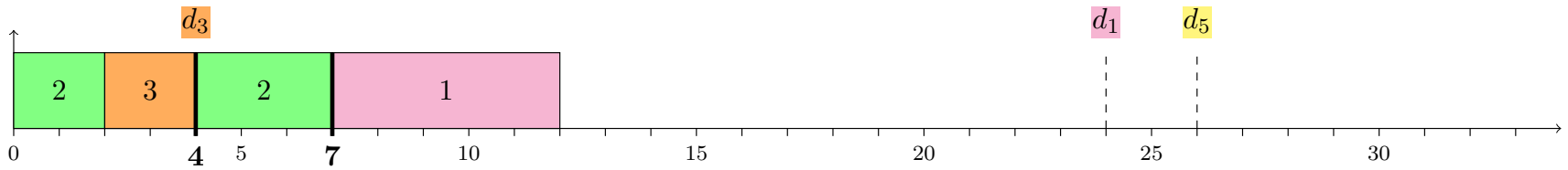

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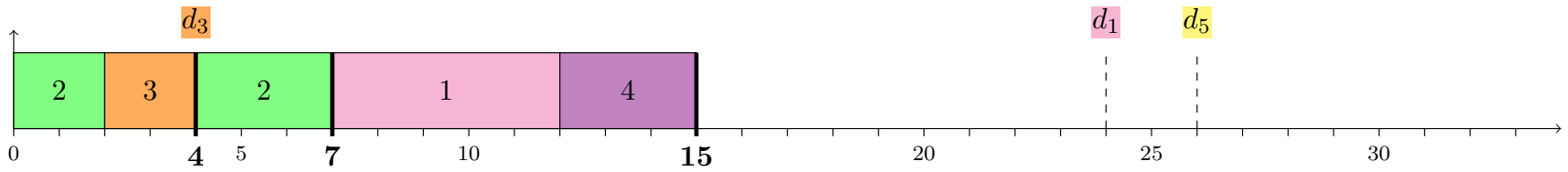

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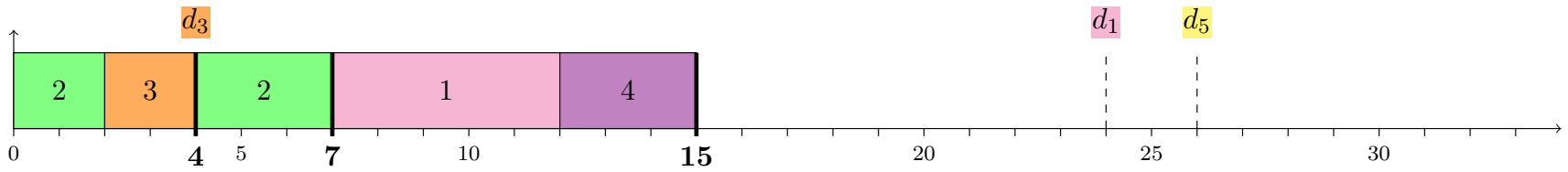

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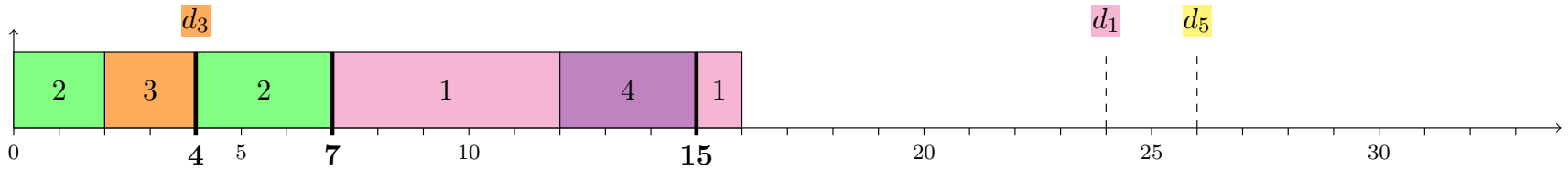

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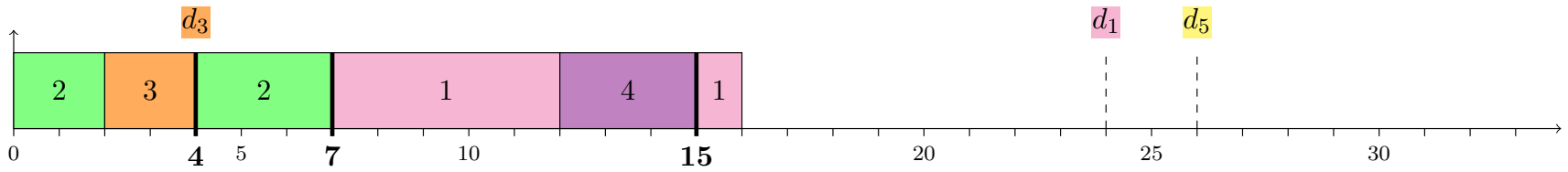

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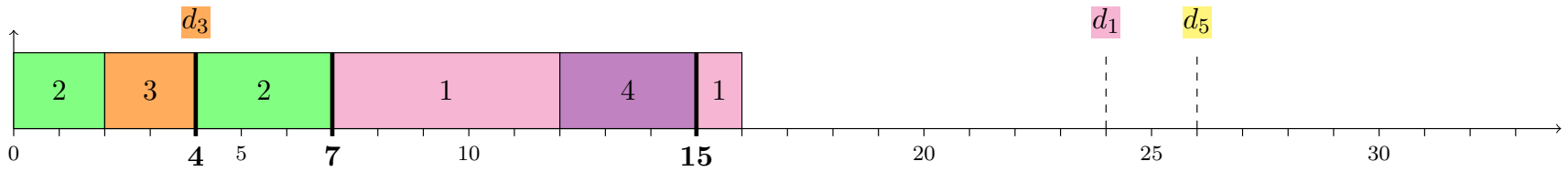

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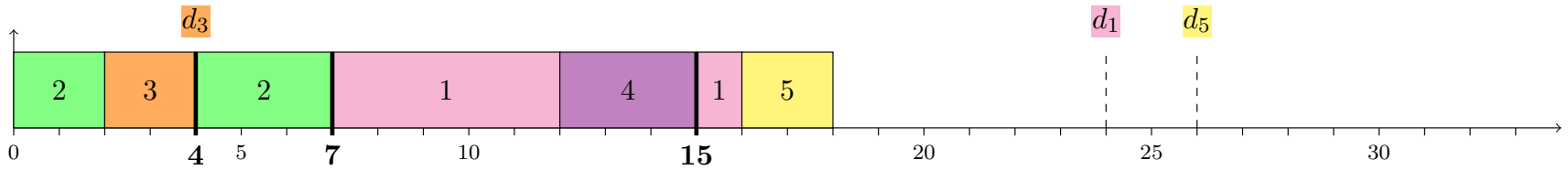

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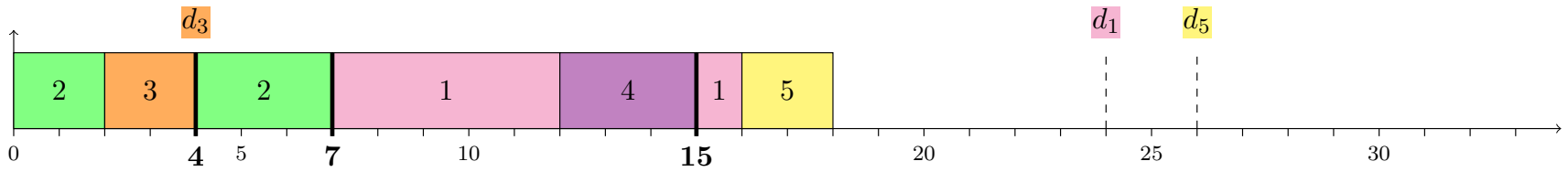

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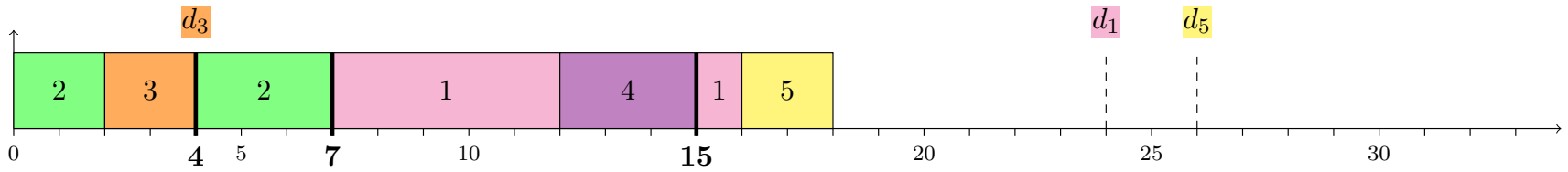

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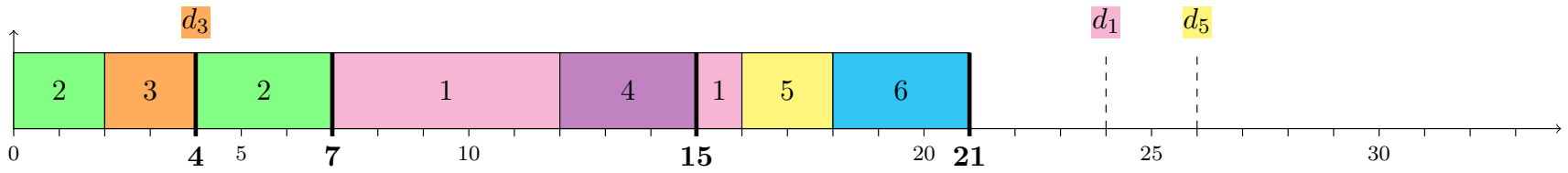

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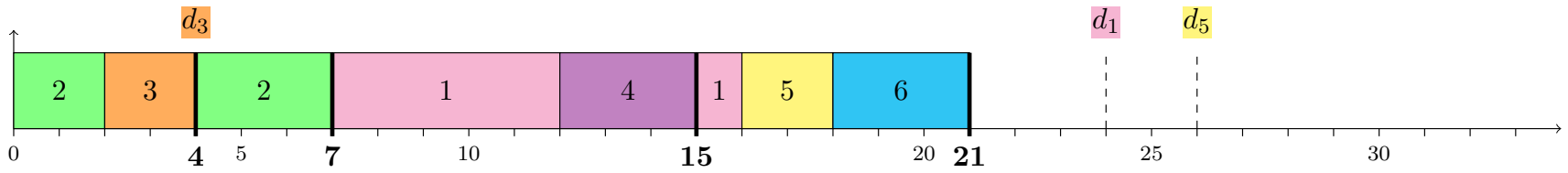

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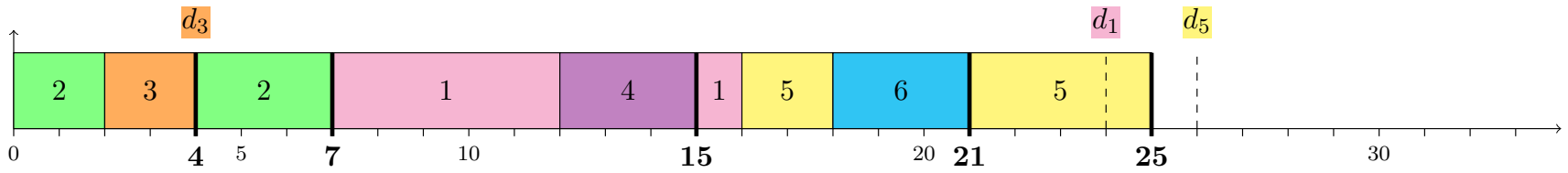

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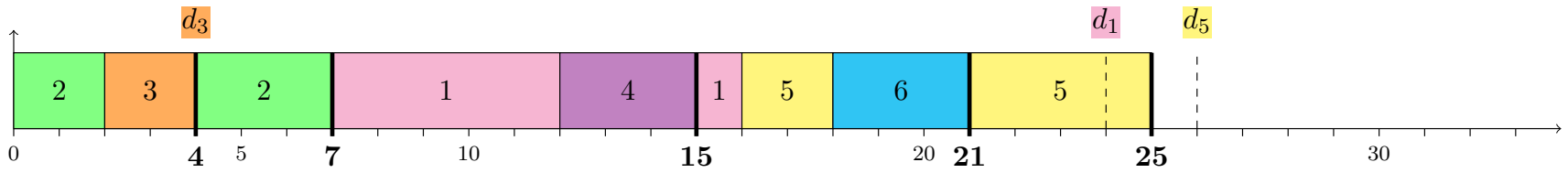

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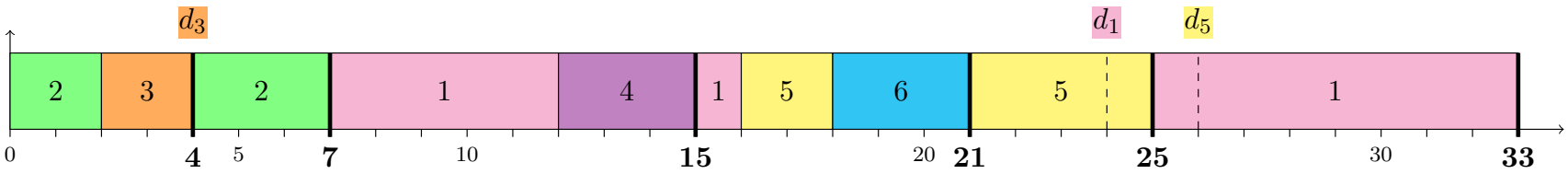

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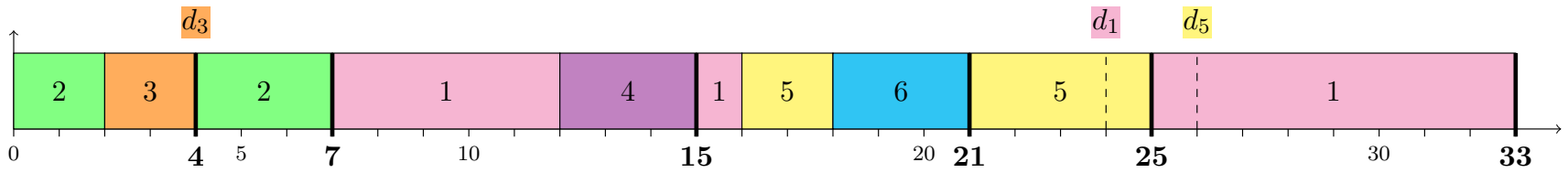

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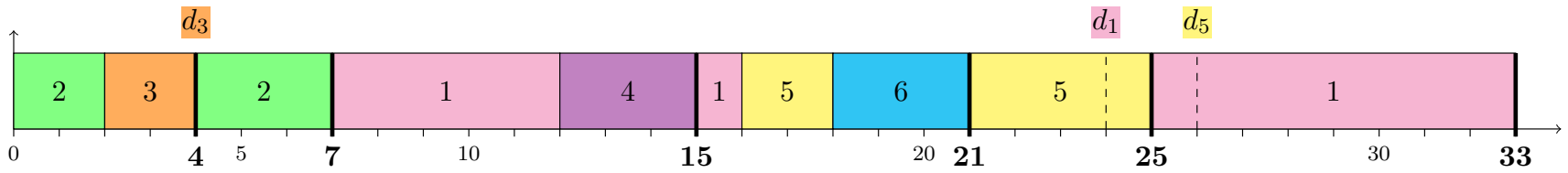

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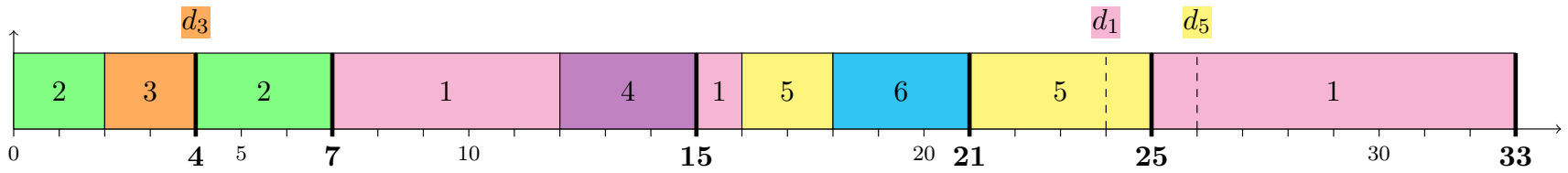

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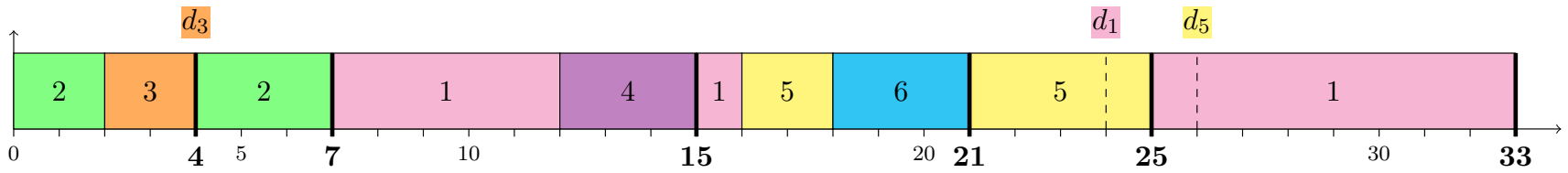

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Theorem: Modified Smith Rule with MPs

1. Schedule **MPs** on their intervals
2. Use **MSR** on the tasks without their MP
   - In case of **equality**, schedule the unfinished task **without MP** or with the **earliest MP**

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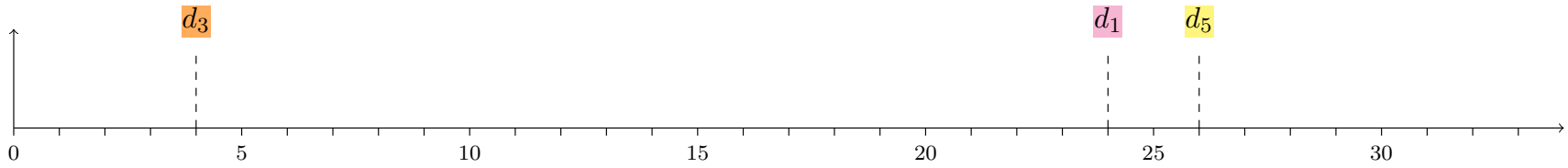

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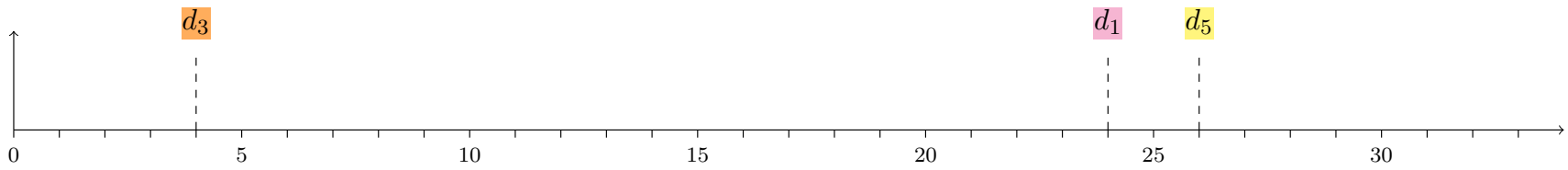

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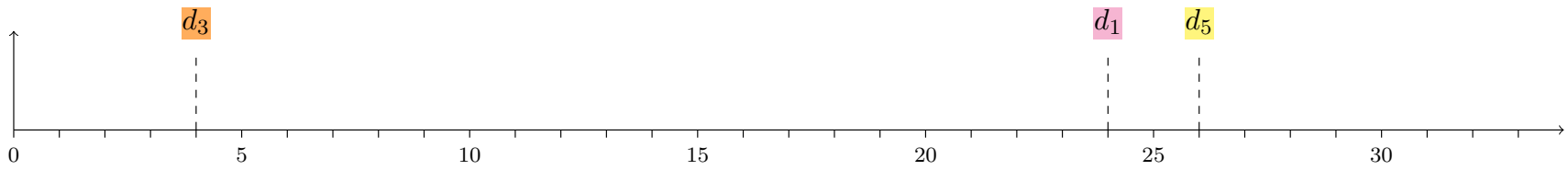

Complexity: $O(n \log n)$ with a sweep line algorithm

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id_j	**1**	2	**3**	4	**5**	6
$\widehat{p}_j$	**14**	5	**2**	3	**6**	3
r_j	**0**	0	**2**	12	**16**	18
d_j	**24**	∞	**4**	∞	**26**	∞

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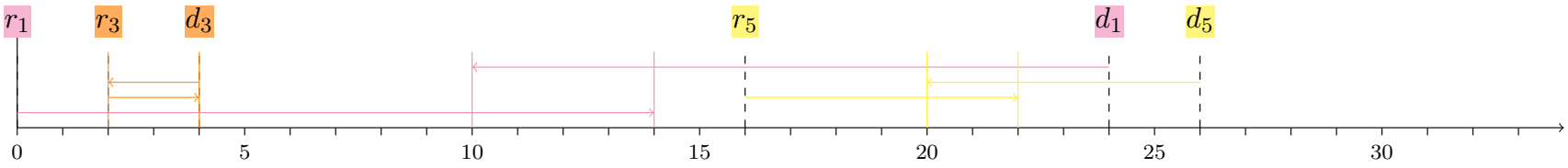

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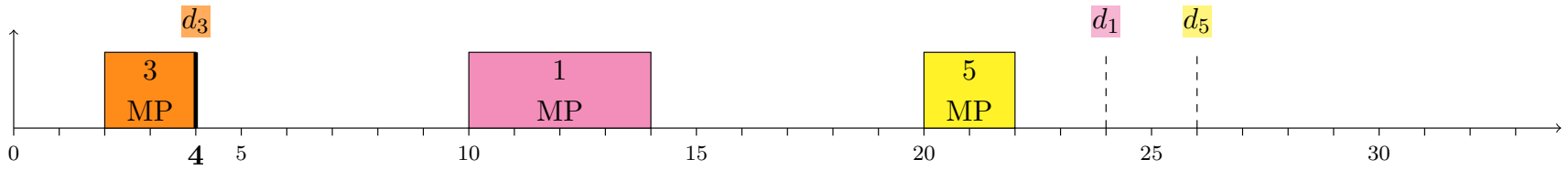

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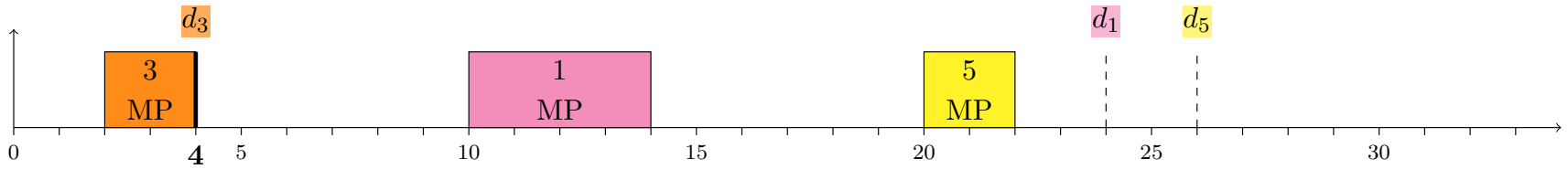

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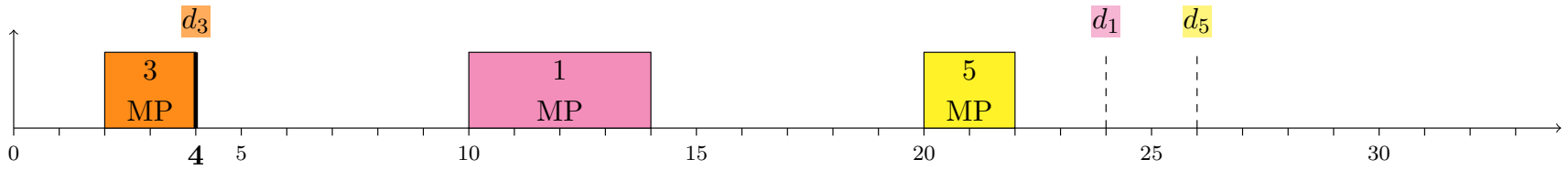

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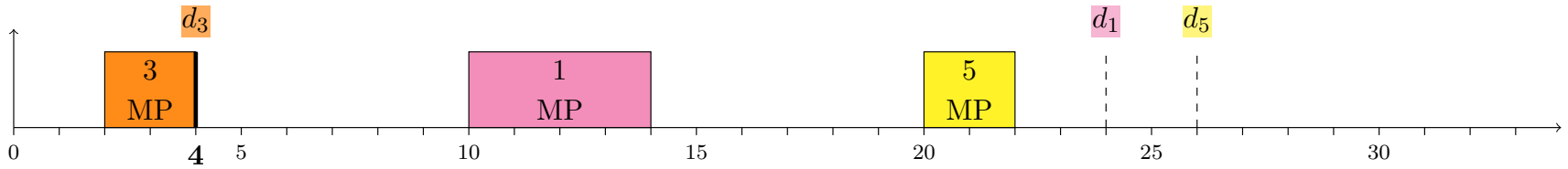

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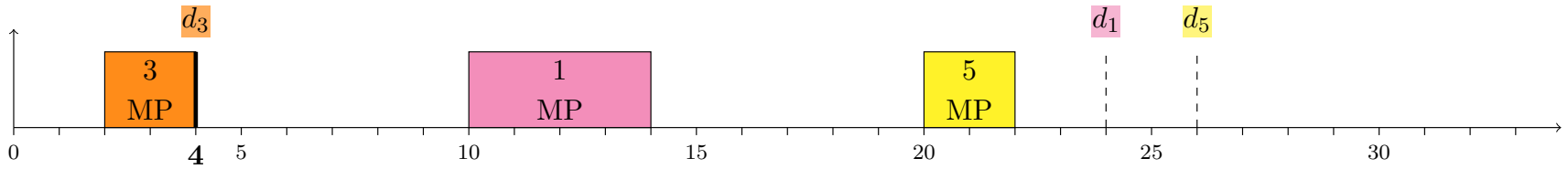

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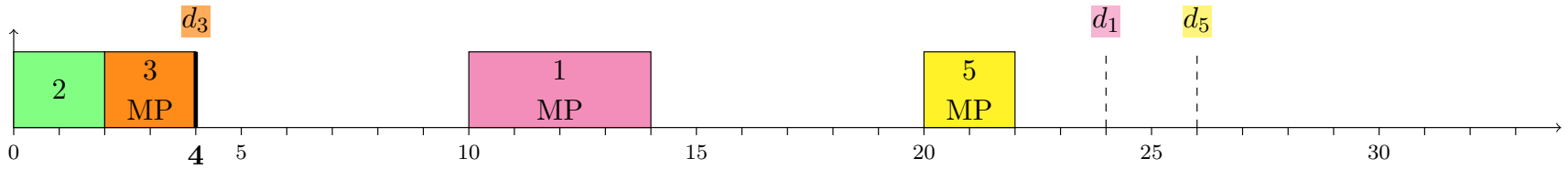

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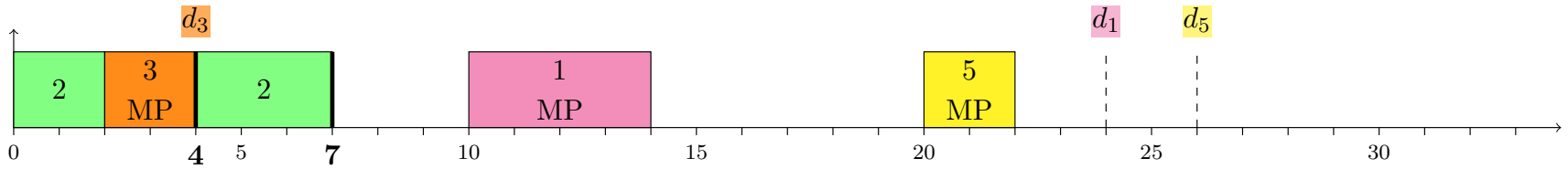

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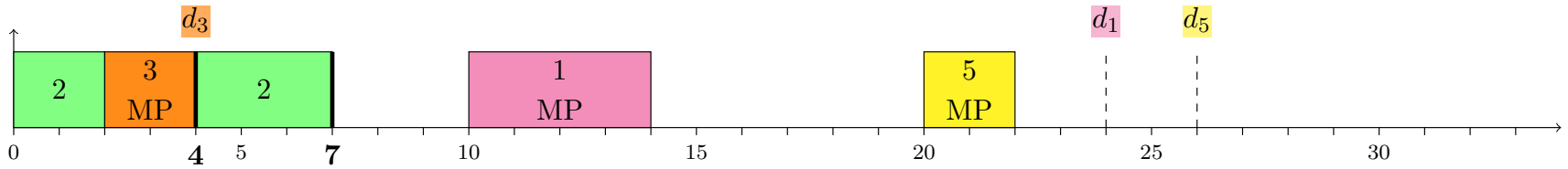

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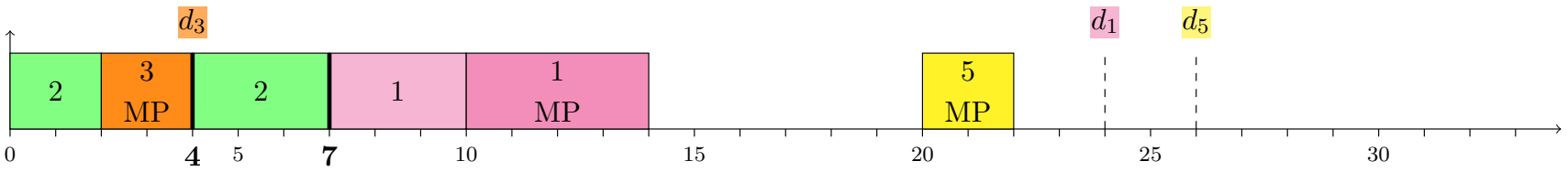

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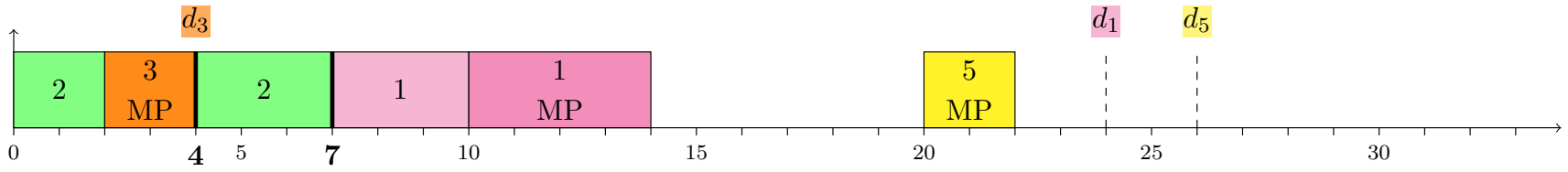

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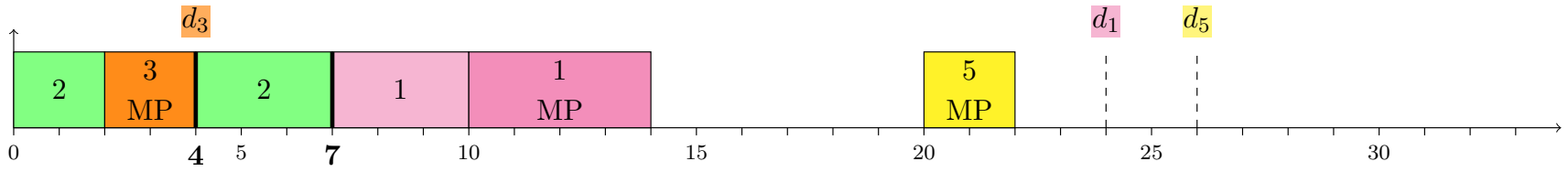

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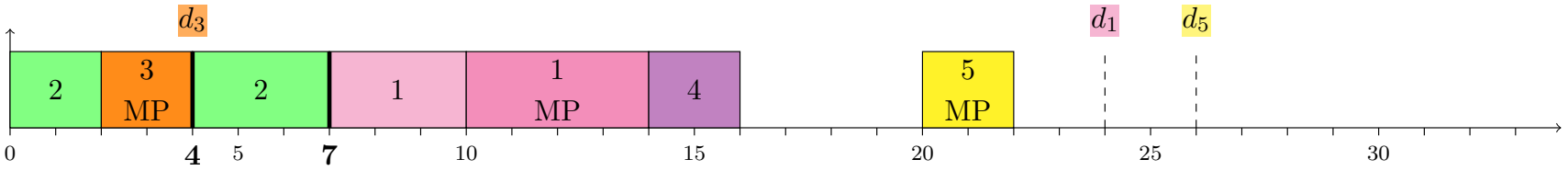

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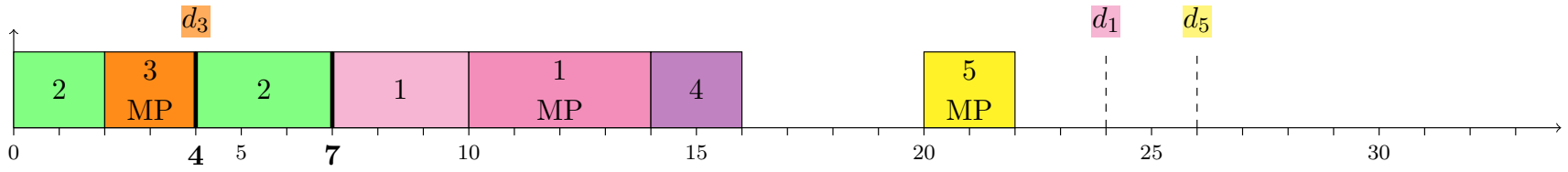

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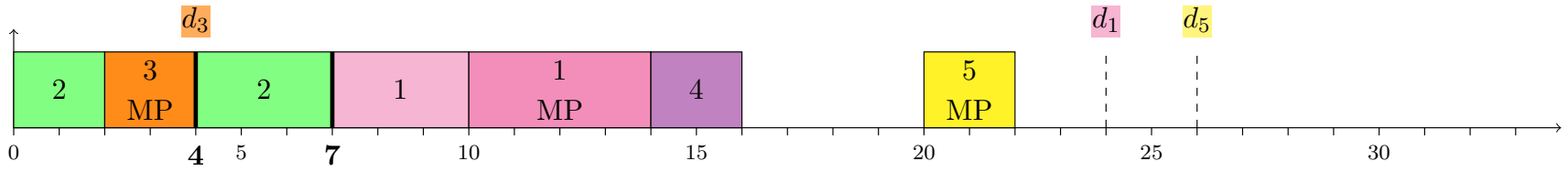

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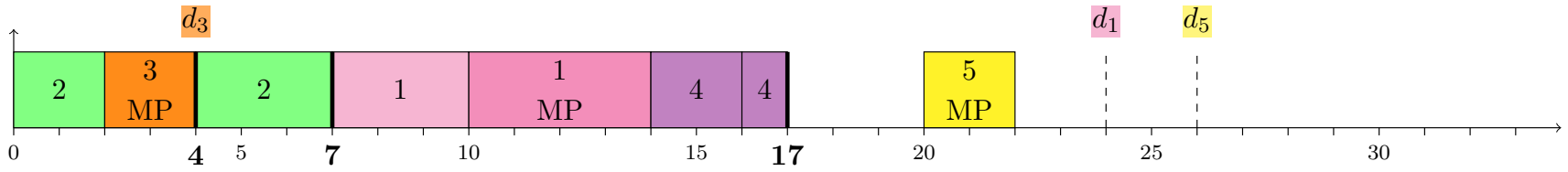

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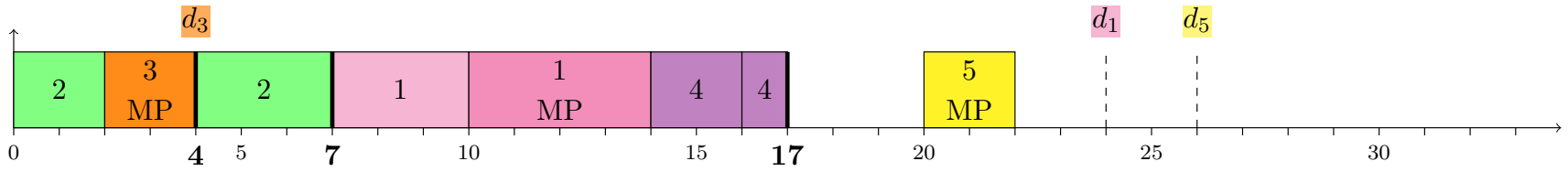

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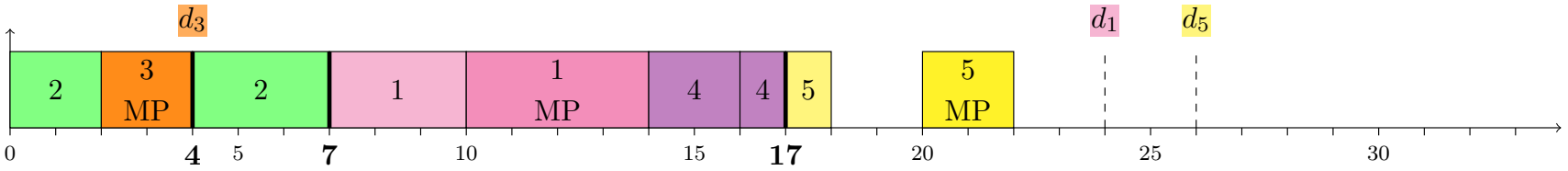

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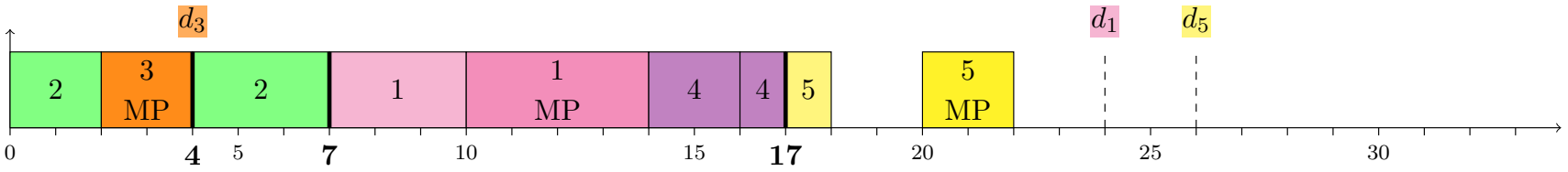

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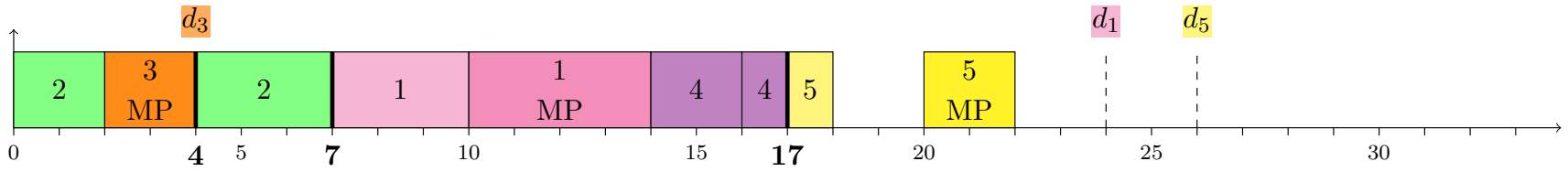

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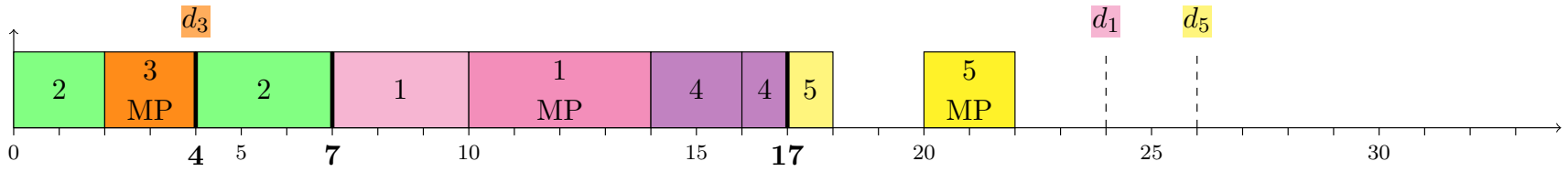

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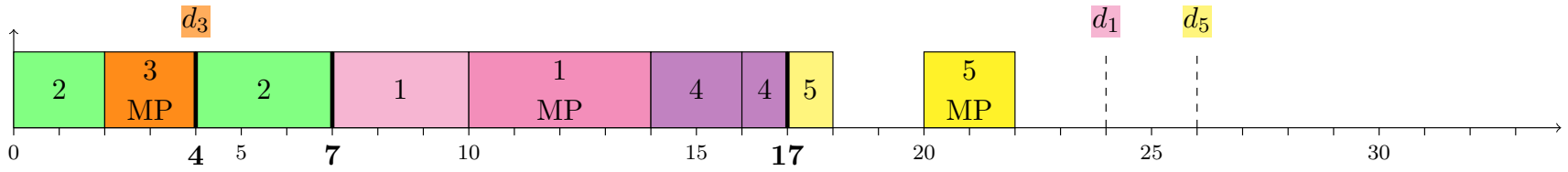

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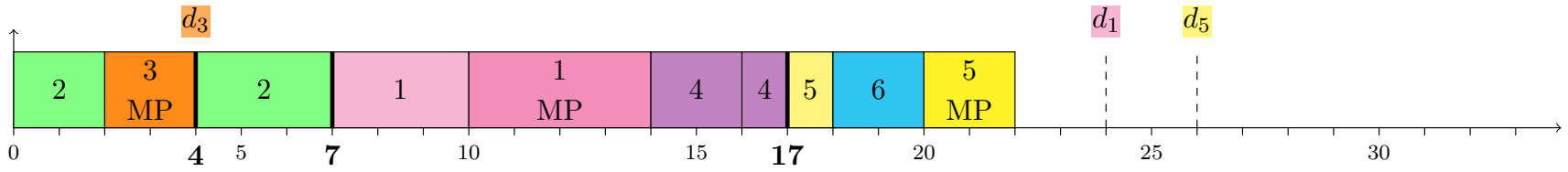

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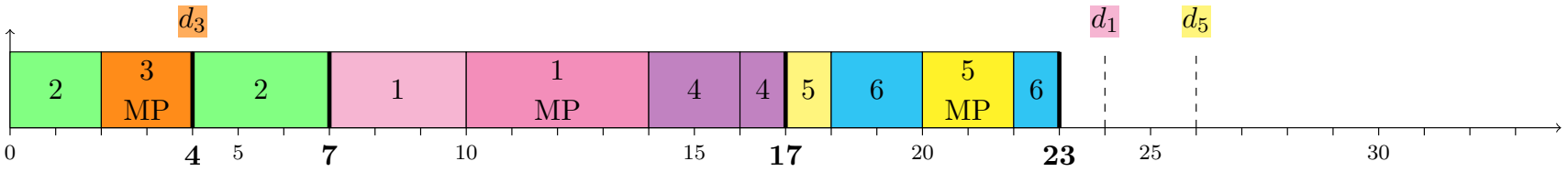

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Modified Smith Rule with MPs

Theorem: Modified Smith Rule with MPs

1. Schedule **MPs** on their intervals
2. Use **MSR** on the tasks without their MP
   - In case of **equality**, schedule the unfinished task **without MP** or with the **earliest MP**

id_j	**1**	2	3	4	**5**	6
$\widehat{p}_j$	**7**	0	0	0	**3**	0
r_j	**0**	0	2	12	**16**	18
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$$\sum C_j = 110$$

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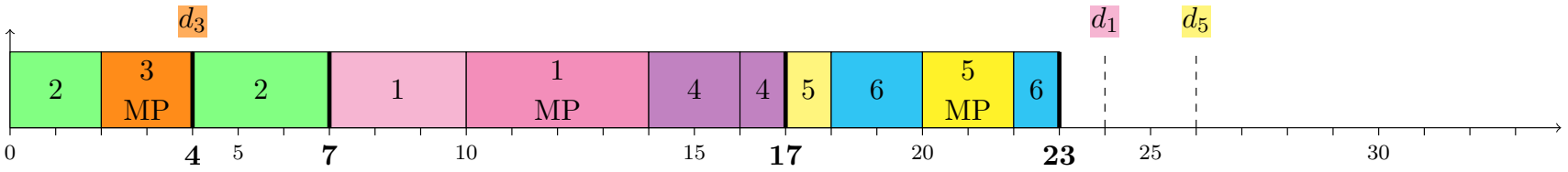

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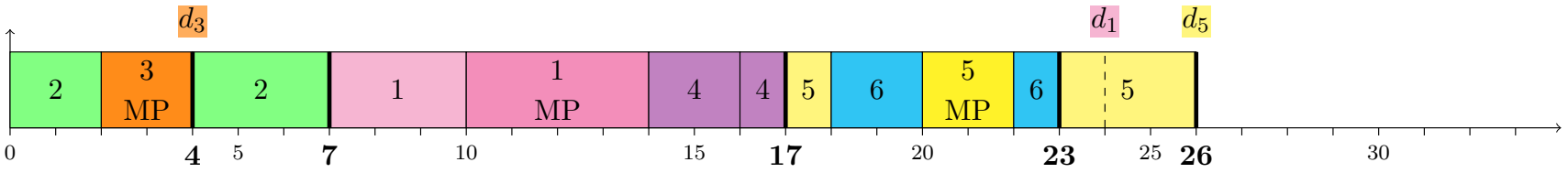

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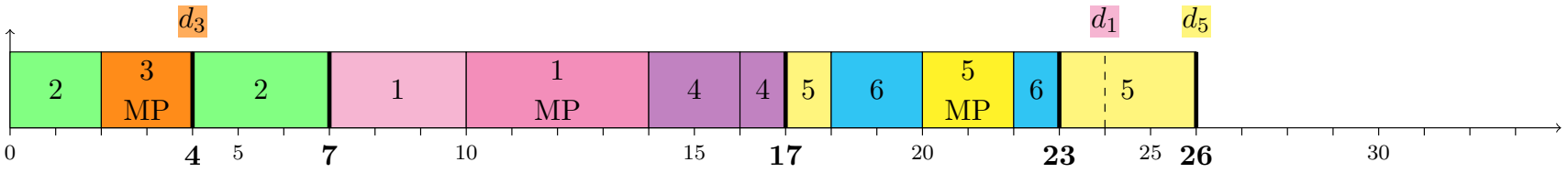

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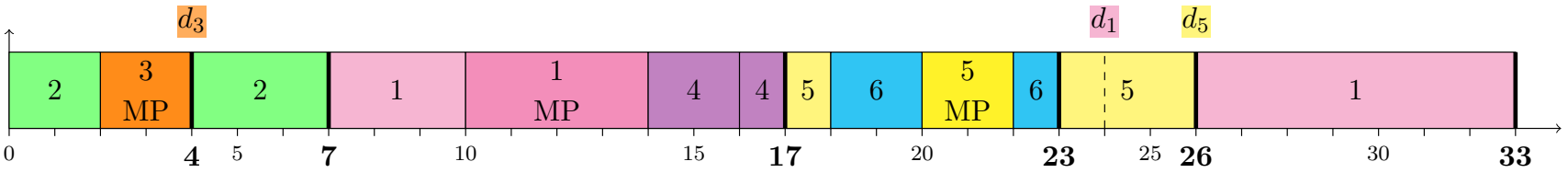

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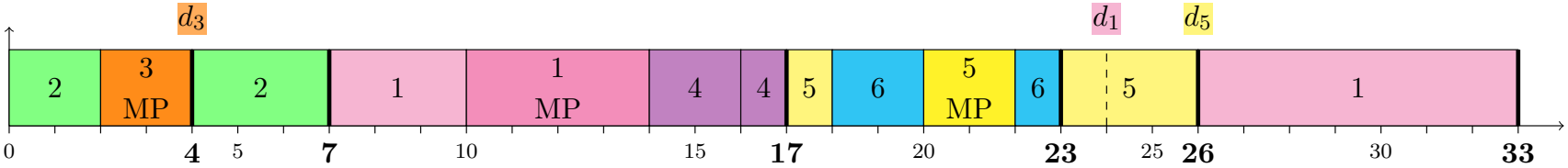

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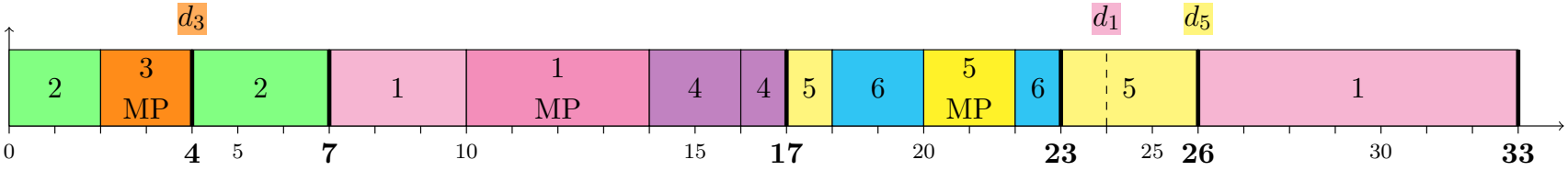

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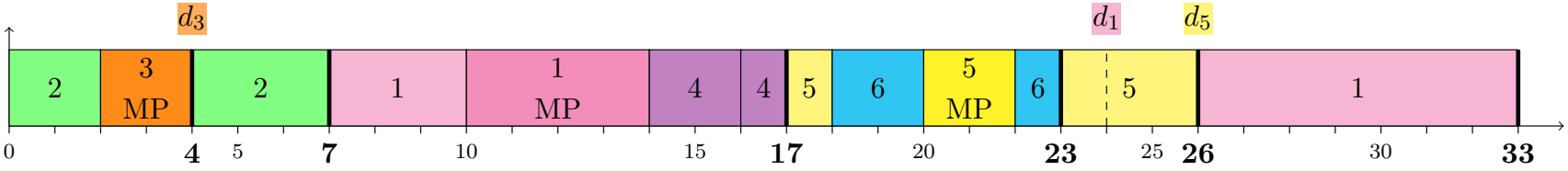

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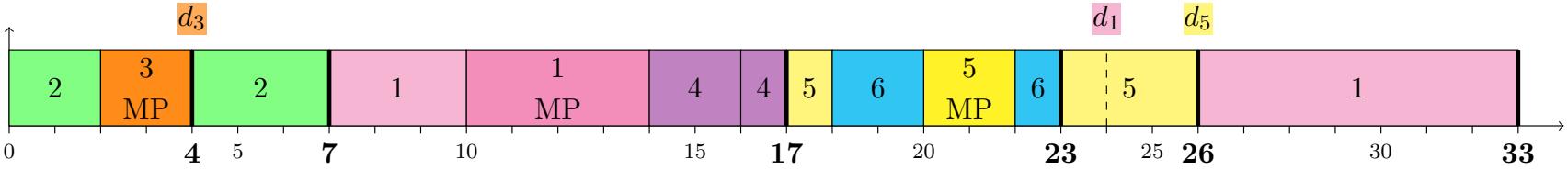

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Optimality of MSR with MPs

Theorem: Optimality of MSR with MPs

MSR with MPs is optimal for $1|r_j; pmtn; MP|\sum C_j$.

Sketch of proof

- S^* = solution of MSR with MPs ;
- i has a higher priority than j
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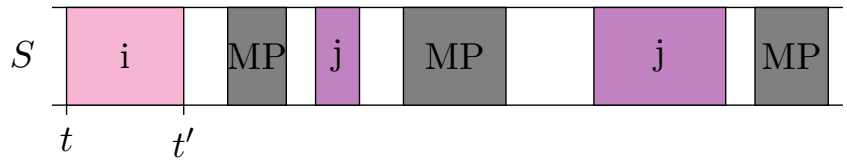

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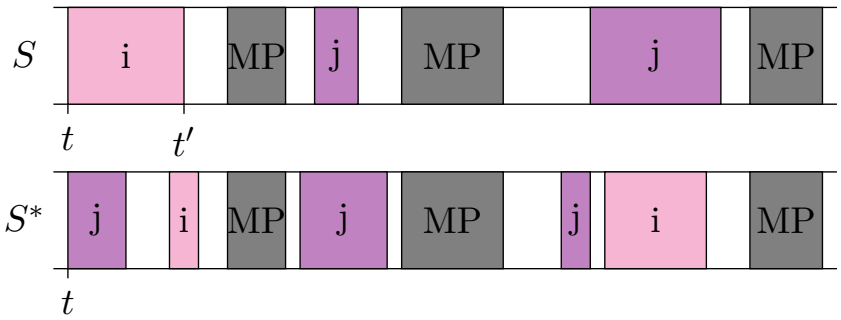

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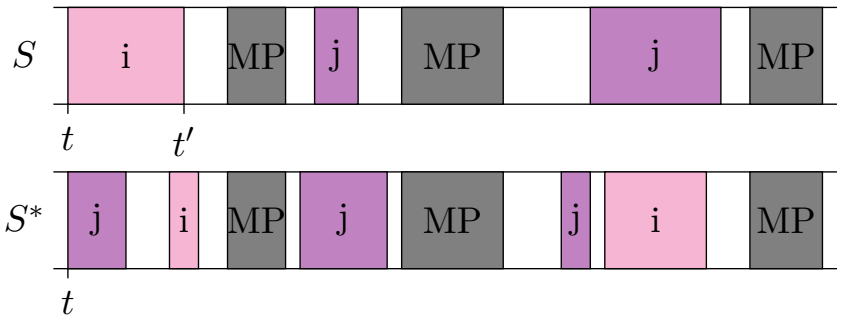

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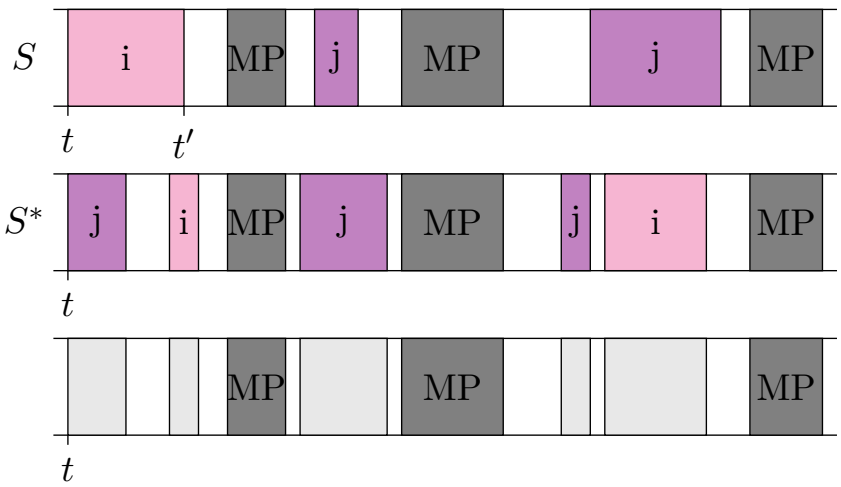

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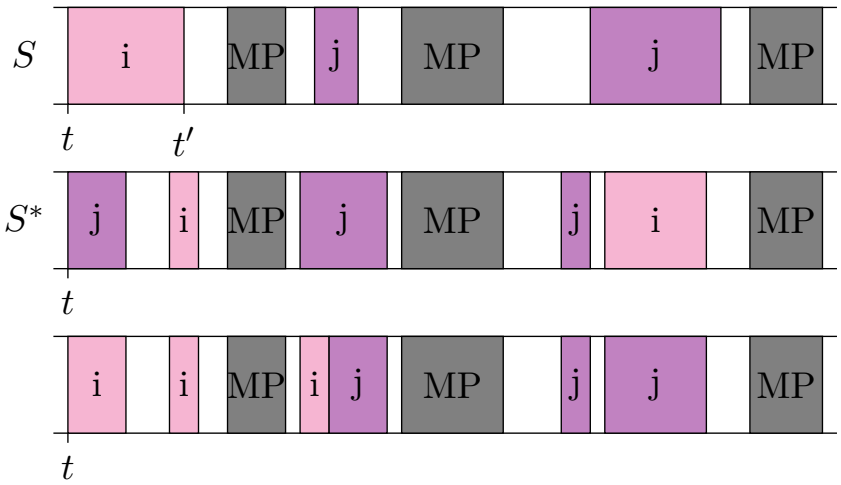

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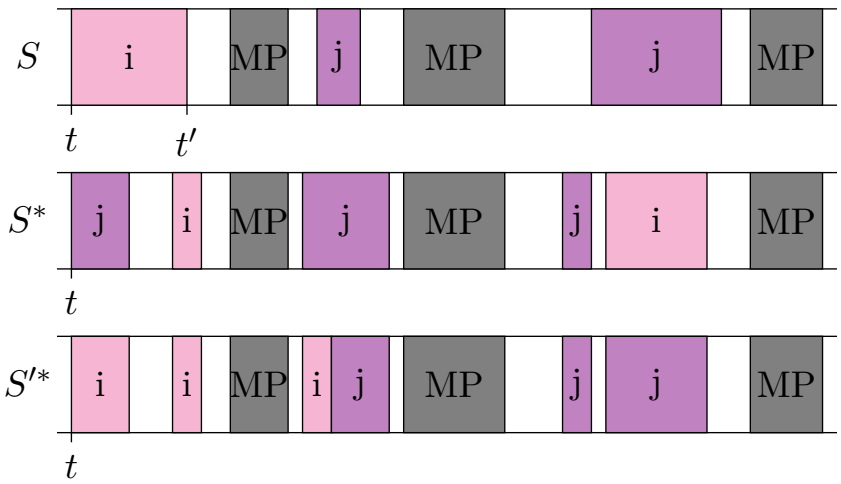

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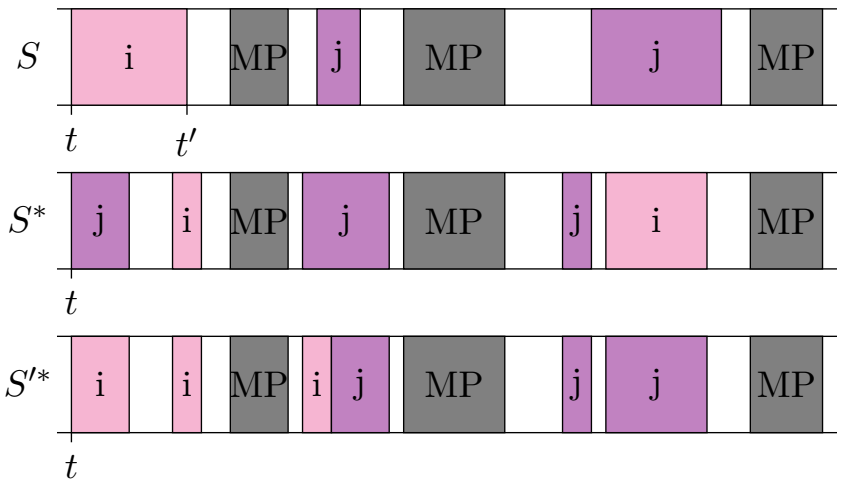

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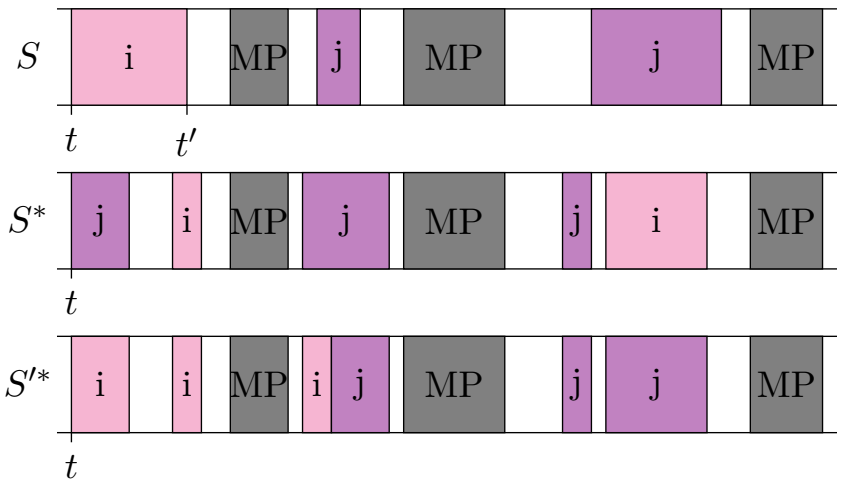

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Lemma: Optimality of MSR with MP

The objective value of S'^* is no greater than those of S^*.

Notations

- $\widehat{p}_k$ = remaining time of k
- C_k (resp. C'_k) = completion time of k in S^* (resp. in S'^*)
- eft_k = earliest finishing time of k (start of the MP)
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3 cases

- Case 1: No MP after $t \Rightarrow$ MSR
- Case 2: MP of i or j in the middle of its execution in S'^*
- Case 3: i or j ends by its MP in S'^*

Goal

- Show that the objective value of S'^* is no greater than those of S^*
- ie. show that $C'_i + C'_j \leq C_i + C_j$

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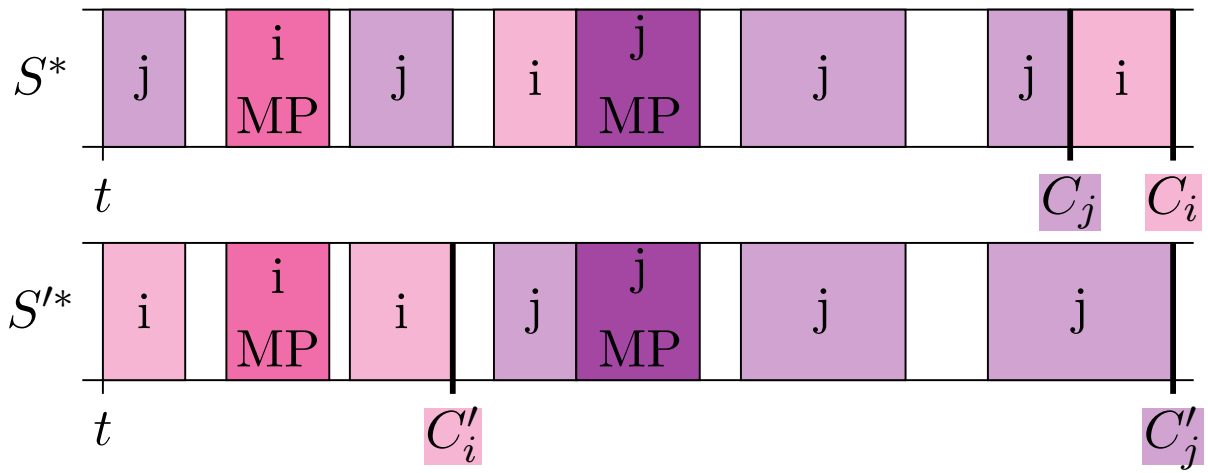

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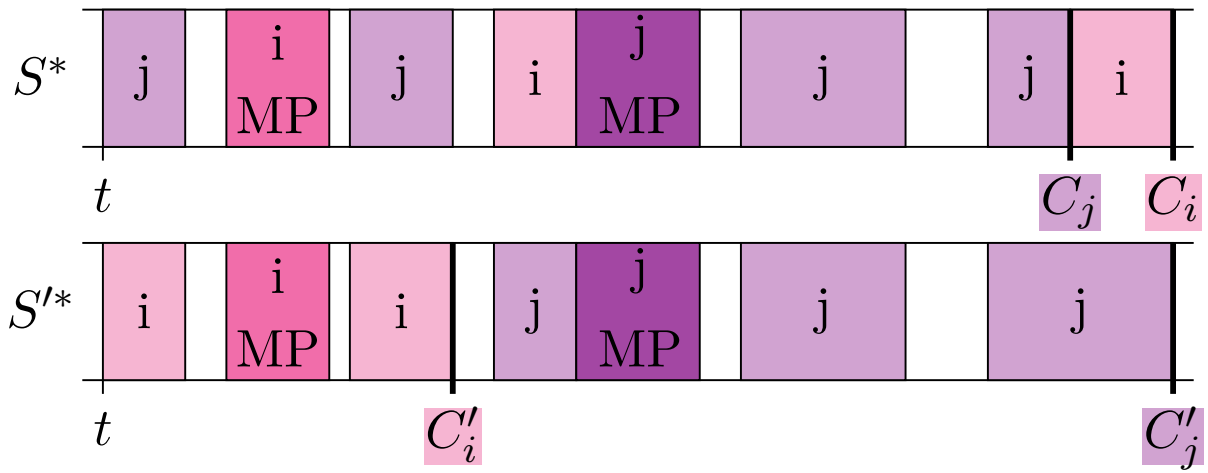

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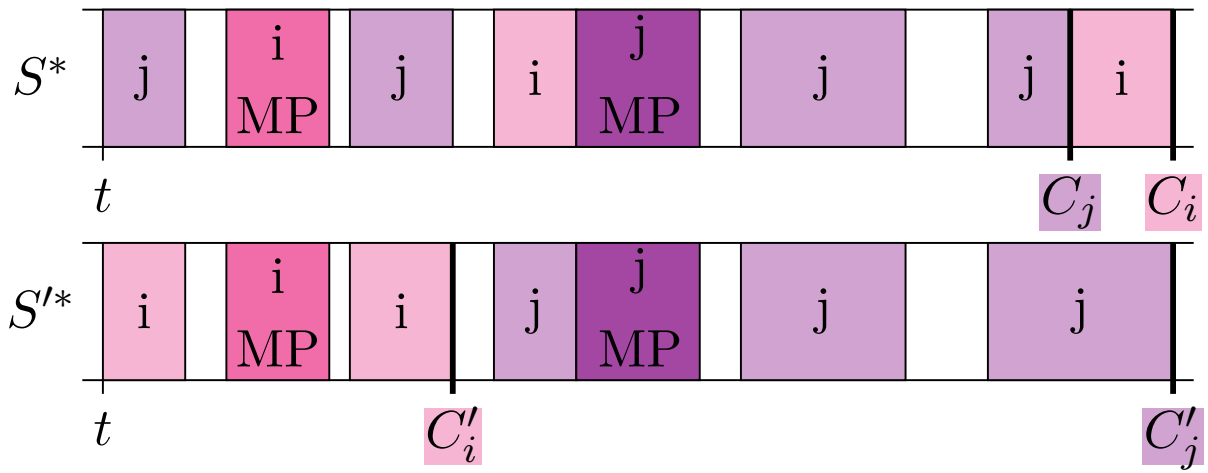

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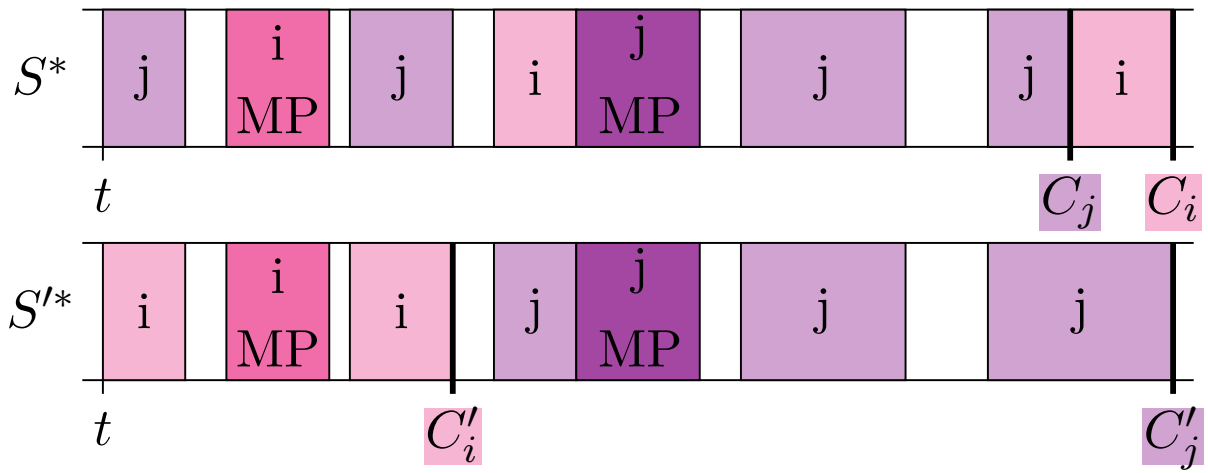

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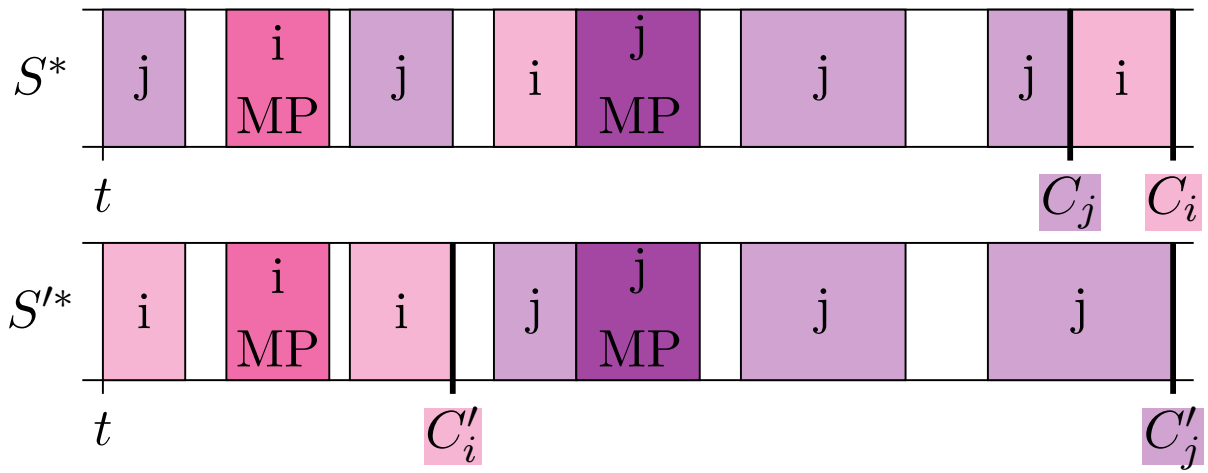

Lemma: proof of case 3 (sub-cases)

Case 3: i or j ends by its MP in S'^*

3 sub-cases:

- Sub-case a): j ends with its MP
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- Sub-case c): j and i end with their MP

Lemma: proof of case 3 a)

Sub-case a): j ends with its MP

- $C_j = C'_j = eft_j$
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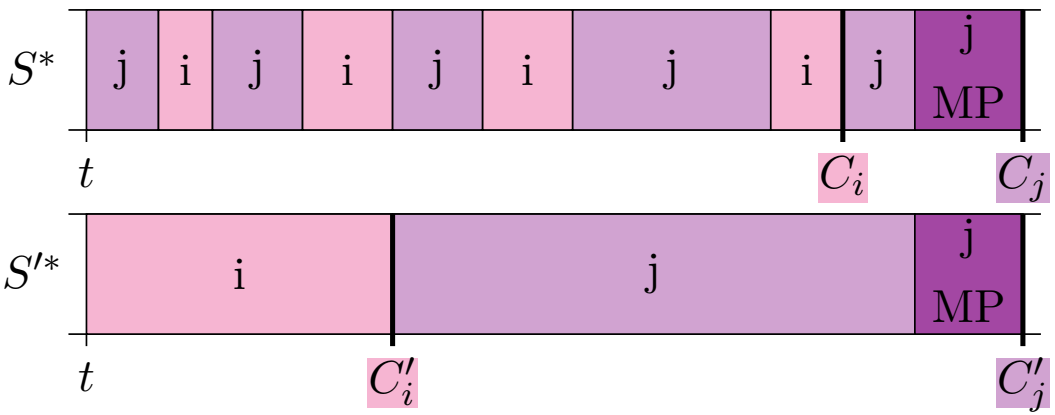

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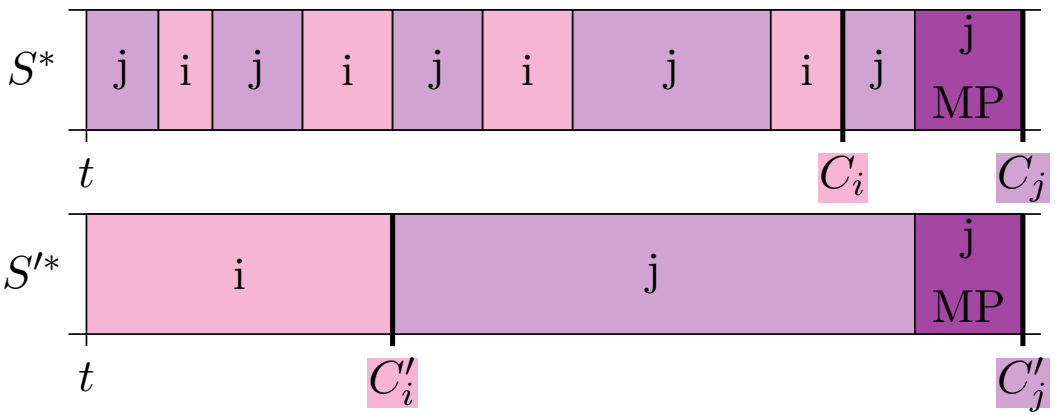

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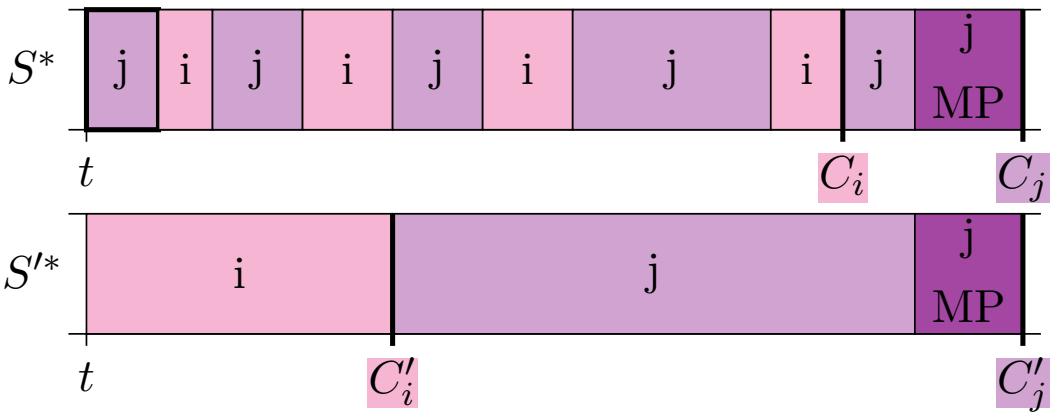

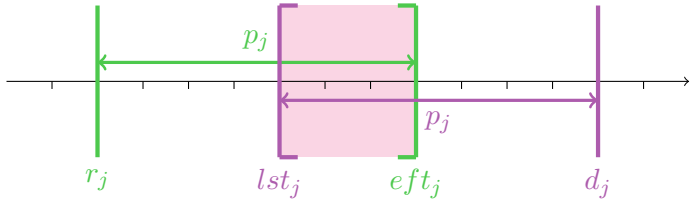

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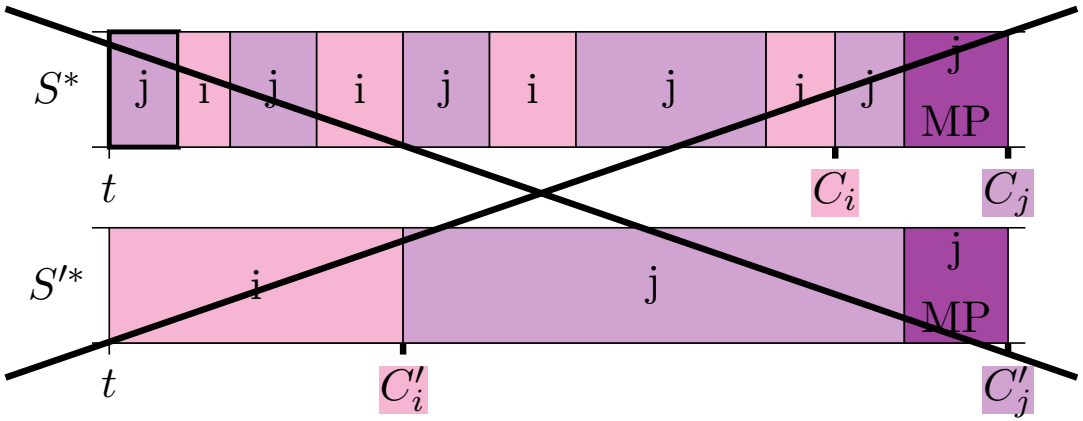

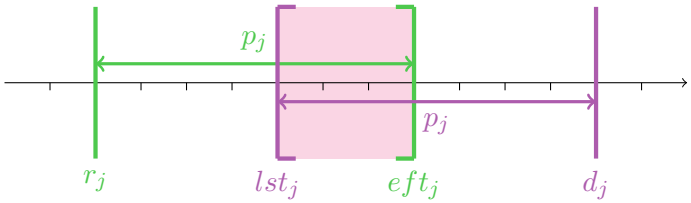

Lemma: proof of case 3 c)

Sub-case c): i and j ends with their MP

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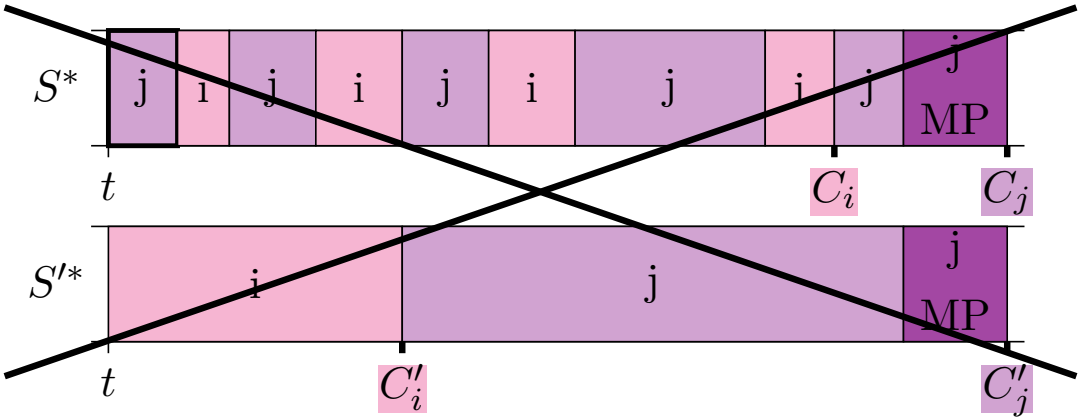

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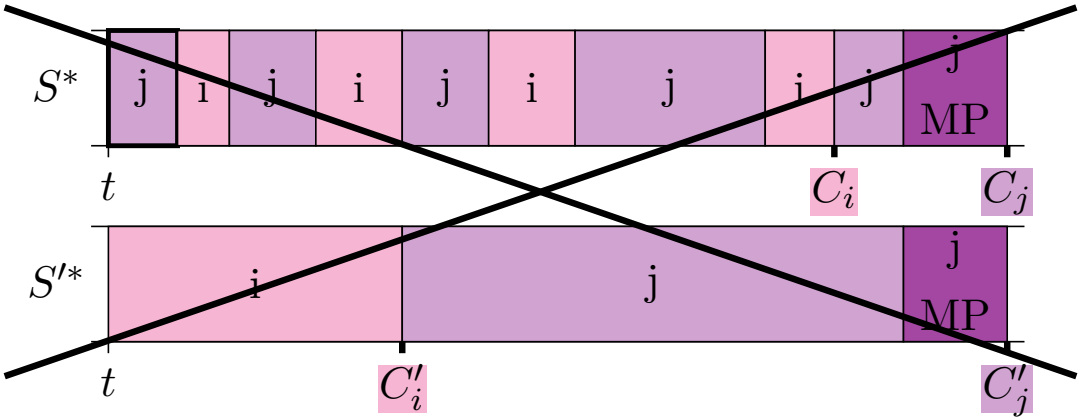

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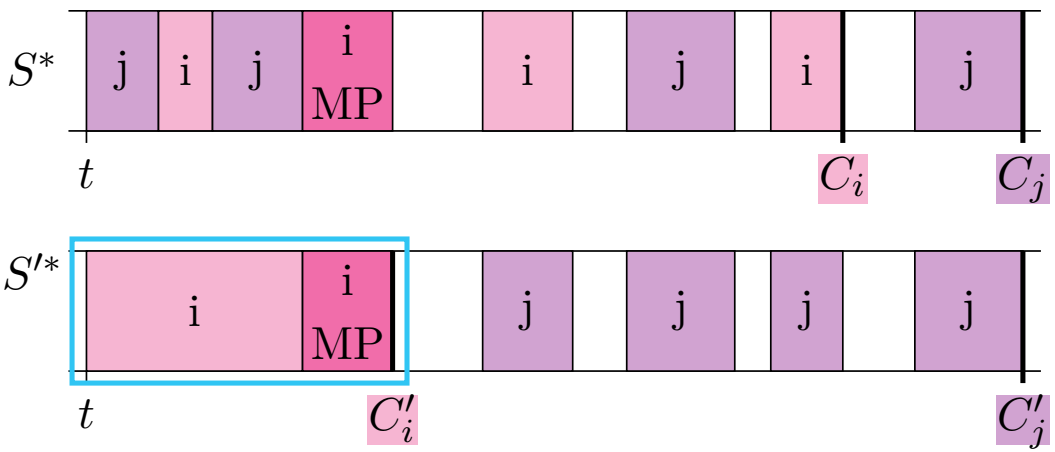

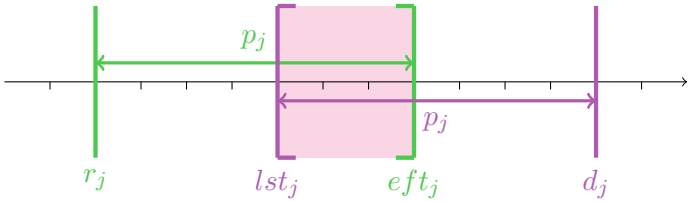

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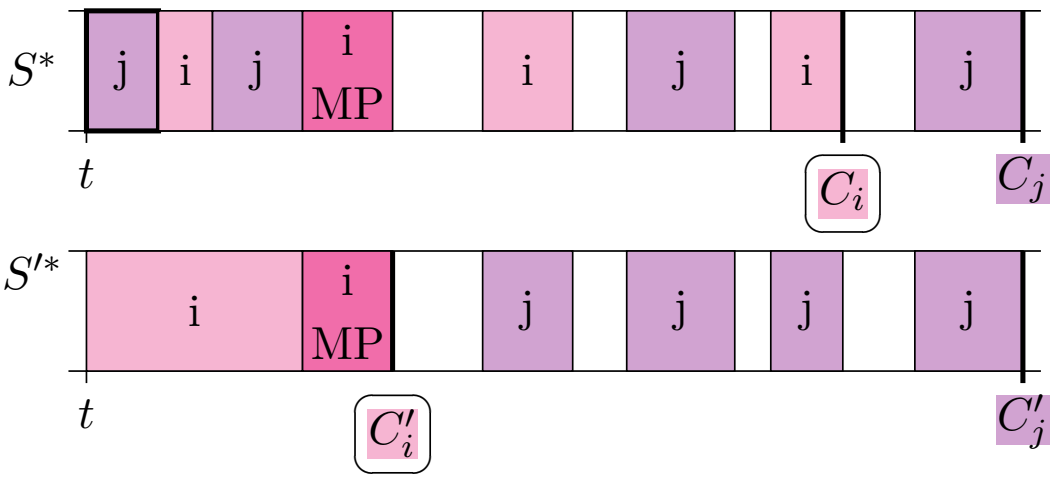

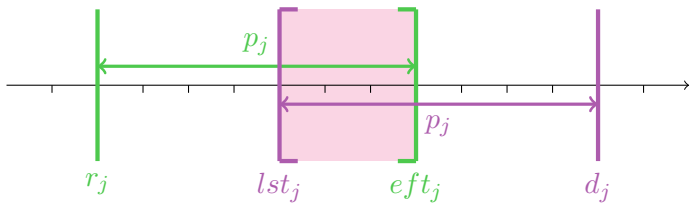

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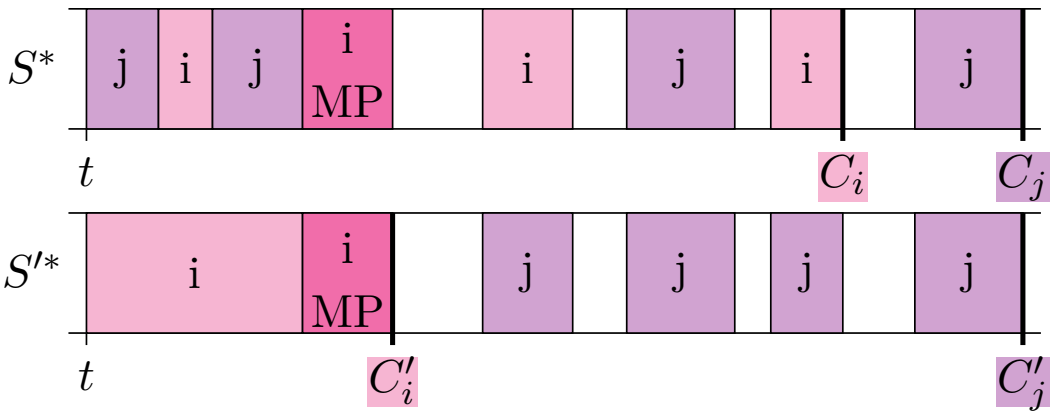

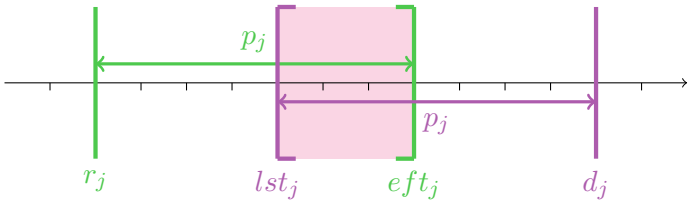

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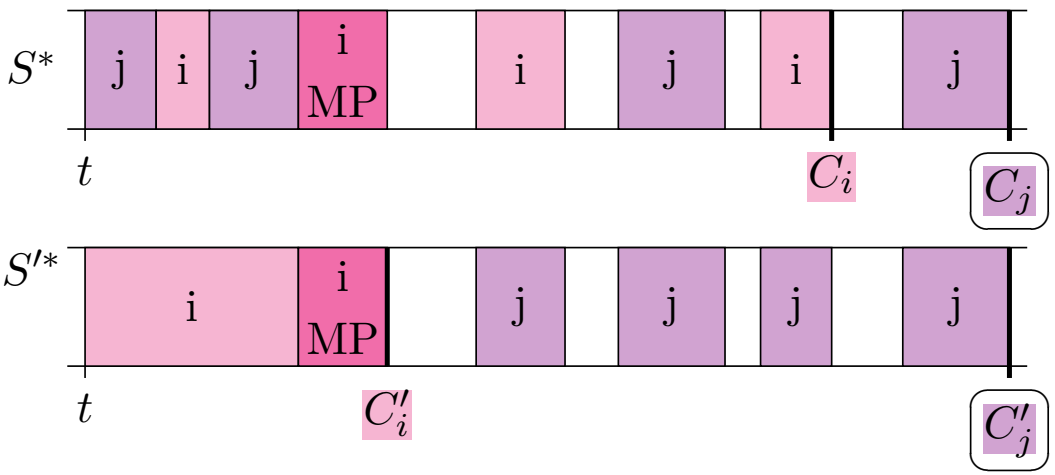

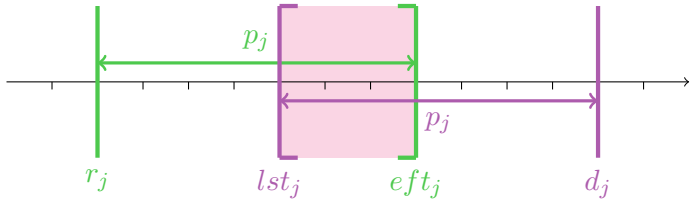

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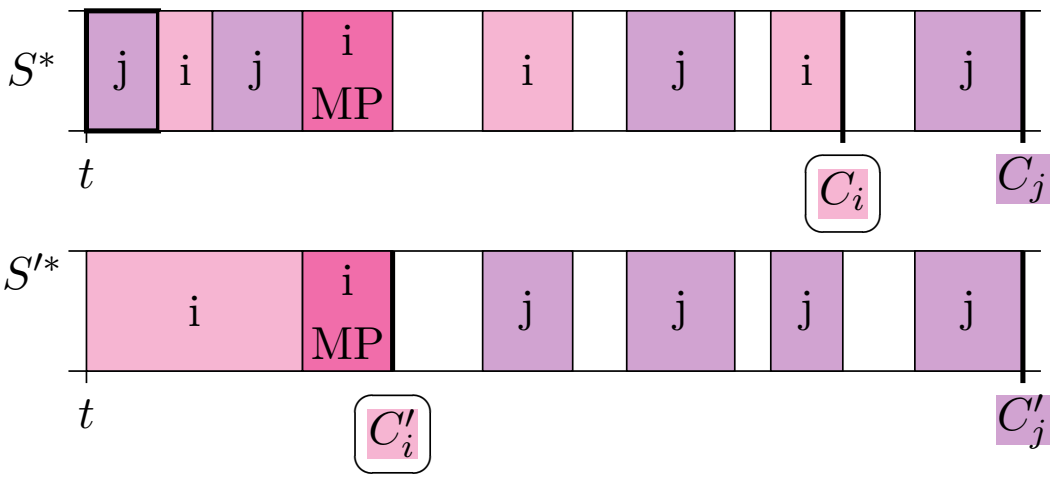

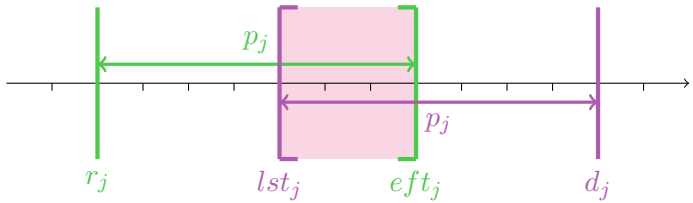

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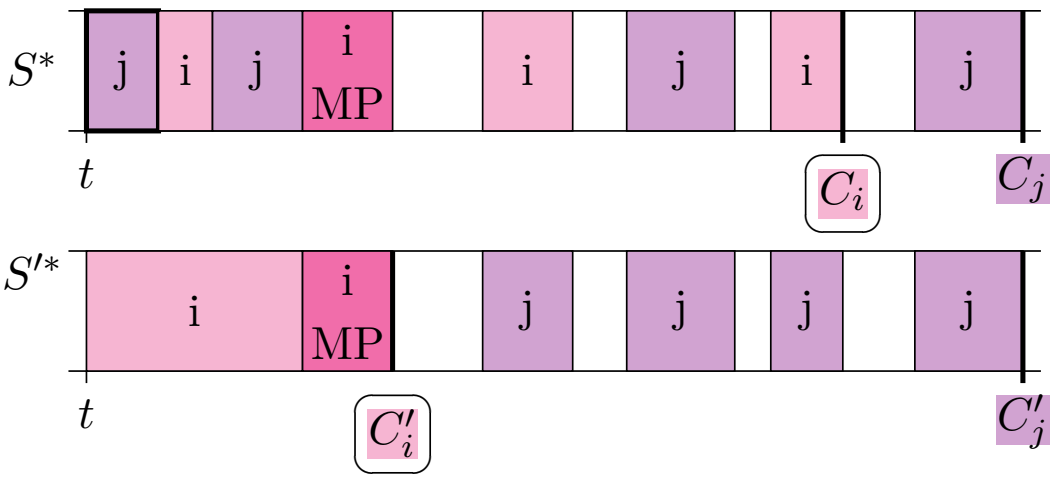

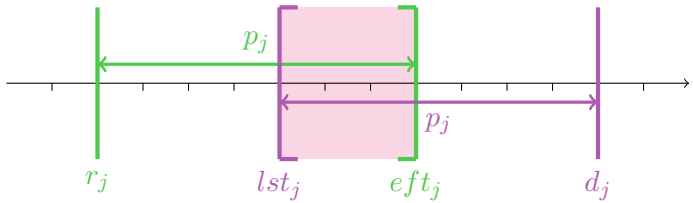

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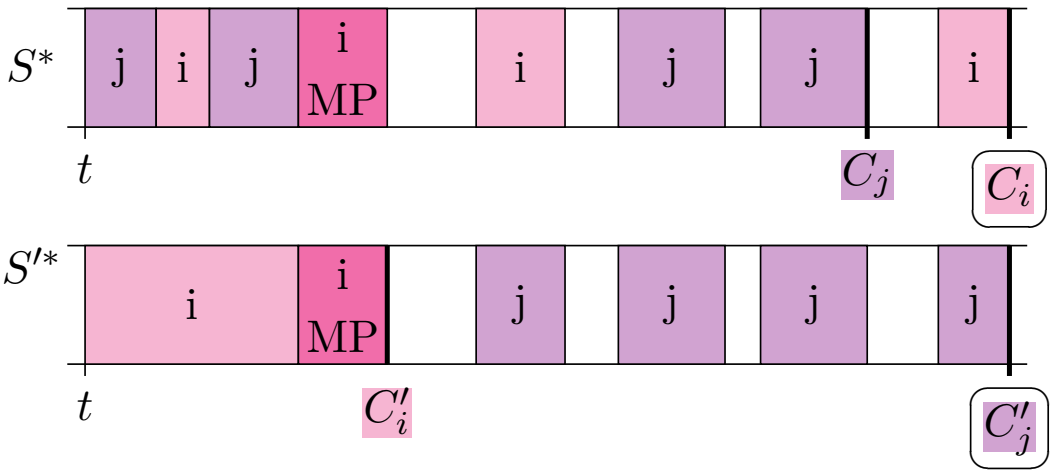

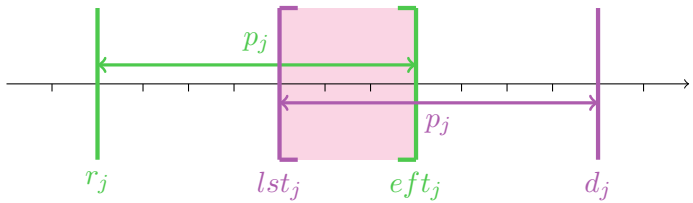

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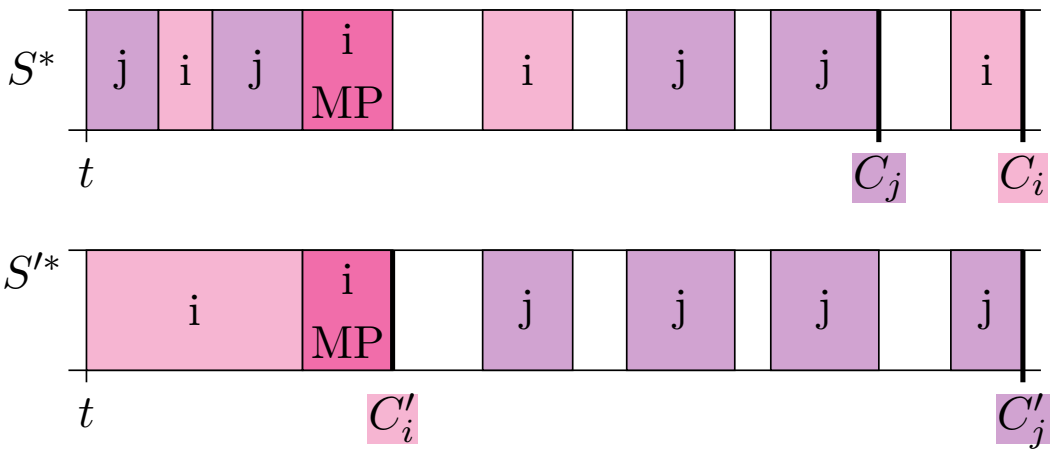

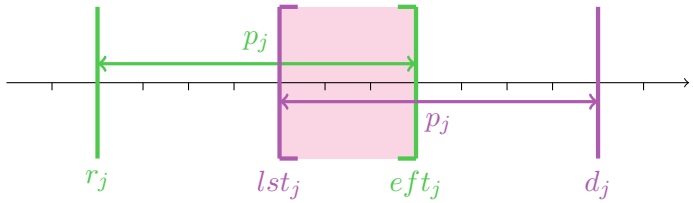

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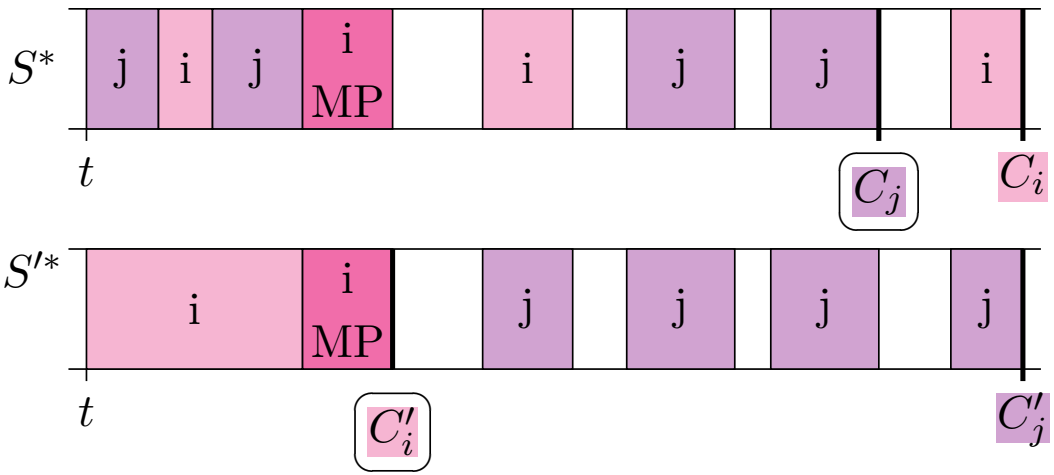

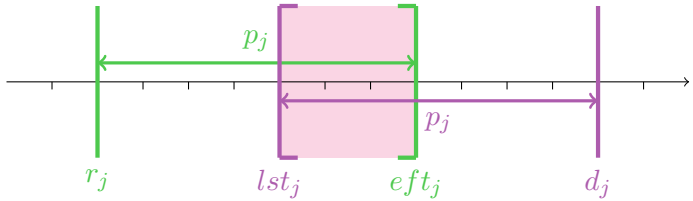

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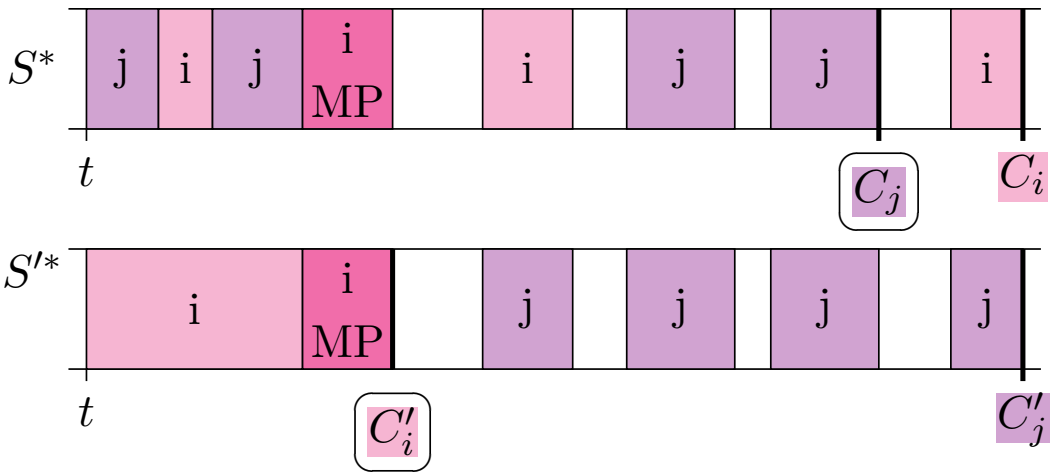

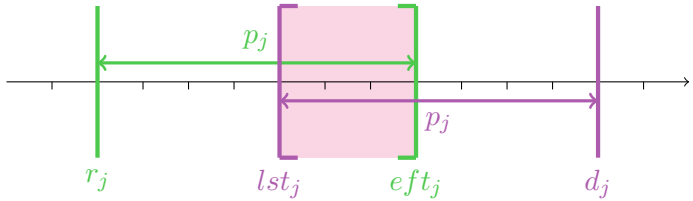

3. Experimentations

Experimental protocol

570 instances generated ...

- with 20, 40, 60, 80 or 100 tasks
- with **durations** in $[1, 10]$ or $[1, 100]$
- with **time windows** $[r_j, d_j]$ more or less close of those of a **possible schedule**

Problems studied

RD-CP	$1\|r_j; d_j\|\sum C_j$	OPT
RDP-MP-IP	$1\|r_j; d_j; pmtn; MP\|\sum C_j$	LB
RDP-IP	$1\|r_j; d_j; pmtn\|\sum C_j$	LB
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Gaps to the optimum analysis: LB ÷ OPT (RD-CP)

Problem	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Gaps
RDP-MP-IP	0.9830	0.9994	0.9999	0.9993	1.0000	1.0000	258
RDP-IP	0.9830	0.9991	0.9998	0.9990	1.0000	1.0000	239
RP-MP-S	0.9575	0.9978	0.9995	0.9980	1.0000	1.0000	403
RP-S	0.9566	0.9967	0.9990	0.9973	0.9998	1.0000	403

Remarks

- RP-S $\leq$ RP-MP-S $\leq$ RDP-IP $\leq$ RDP-MP-IP
- Less results (nb gaps) for IP-solved problems ($\mathcal{NP}$-hard)
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RDP-IP	0.9830	0.9991	0.9998	0.9990	1.0000	1.0000	239
RP-MP-S	0.9575	0.9978	0.9995	0.9980	1.0000	1.0000	403
RP-S	0.9566	0.9967	0.9990	0.9973	0.9998	1.0000	403

Remarks

- RP-S $\leq$ RP-MP-S $\leq$ RDP-IP $\leq$ RDP-MP-IP
- Less results (nb gaps) for IP-solved problems ($\mathcal{NP}$-hard)
- For all problems : the smaller the nb of tasks, the tinier the gap to OPT is
- For all problems : the closer the instance is to a solution, the tinier the gap to OPT is

Experimentations

Gaps to the optimum analysis: LB ÷ OPT (RD-CP)

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4. Conclusion

Conclusion

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- MSR with MP compute LB for $1|r_j; d_j|\sum C_j$ in poly time
- Improve the LB compared to MSR
- Does not complexify too much MSR
- Can be used for improving CP resolution
- MP can be constrained for practical applications : personnel needed for a 2-persons tasks in an hospital
- Possible improving toward value of $1|r_j; d_j; pmtn|\sum C_j$
- No direct integration of d_j

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Prospects

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- Partial integration of d_j
- CP applications
- ...

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Thank you for your attention !

5. Appendix

Sweep line algo VS list algo

Differences

- List: sorts elements with the **same level**
- Sweep line: sorts elements with the **different levels**

- List: all elements have **same actions**
- Sweep line: elements can have **different actions**