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Chapter 9

Instrumental Genesis in the Theory of MWS: Insight from Didactic Research on Digital artefacts

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Abstract. The central focus of this chapter is the relationship between the notions introduced by Vérillon and Rabardel (1995), ‘artefact’ and ‘instrumental genesis’, and the homonym notions in the theory of MWS. Following a brief overview of the Piagetian and Activity Theory perspectives which inspired Vérillon and Rabardel, we explore how didactic research on digital artefacts developed two viewpoints on the instrumental genesis: psychological and institutional. The idea of instrumental genesis in the theory of MWS is consistent with the concept of instrumented mediation by Vérillon and Rabardel, but is a more focused notion, since MWS include two additional geneses, semiotic and discursive. The theory of MWS moreover does not theorize about the mediation and as such, in a particular situation where a digital artefact is used, both viewpoints will be helpful. Regarding the poles of the instrumental genesis, we propose characterizing a digital artefact by the underlying data representations and algorithmic treatments, and constructions as data-result models of mathematical configurations, taking advantage of the representations and algorithms in the artefact.

9.1 Motivation

The authors of this chapter are two researchers in mathematics education with a long experience in investigating the classroom use of digital artefacts and designing mathematical software environments. Like many other researchers in this field, we are inspired by Vérillon and Rabardel’s (1995) approach of instrumental activity, using the notions of artefact, instrumental genesis and instrumented techniques. In parallel, we have drawn on the theory of MWS for specific research studies (Minh & Lagrange 2016, Richard & al., 2016). This theory also puts forward the notions of artefact and instrumental genesis, furthermore including two other geneses: semiotic and discursive.

The central focus of this chapter is the relationship between notions introduced by Vérillon and Rabardel and the homonym notions in the theory of MWS. Previous presentations of the instrumental genesis in the theory of MWS (Kuzniak, Tanguay & Elia 2016) have briefly drawn on the work of Vérillon & Rabardel, but without taking into account how the two approaches derive from different research perspectives. The theory of MWS comes from didactic research in geometry, and the work of Vérillon and Rabardel from cognitive ergonomics, or in other words, psychology in the workplace. Arguably then, there is no reason why homonym notions should carry the same meaning. We consider that in mathematics education research, as in other scientific domains, it is important to work continuously on basic notions in order to specify the meanings we seek to convey in a specific theory, and to be cautious with regard to possible semantic bias when separate theories use a similar vocabulary.

This book is therefore an opportunity to accurately discuss instrumental genesis in the two theoretical perspectives and we expect this discussion to provide the firm foundations for further research. In order to retain specific focus, a first choice in this chapter is to consider *digital artefacts* rather than a broader category of artefacts.

The notion of digital artefact applies to a great diversity of devices ubiquitous in human activity and especially in cognition, some of which have been investigated in Chapter 8. ‘Digital’ fundamentally denotes a choice of data representation. For instance, to represent a color, a digital code enables the representation of a potentially great but finite number of colors, in contrast with an analog representation, which allows any combination. This choice of data representation has been adopted because it enables algorithmic treatment, i.e., treatment by an automatic device following given rules: in the example of colors, an algorithm triggers a display device to activate suitable light emitting diodes; other algorithms can be used for various processes on a colored picture... In this way, digital artefacts are characterized by data representation and algorithmic treatment. On the one hand, this characterization seems very simple with regard to the multiple roles digital technologies play in all domains and particularly in mathematics education, especially mediation and communication (Clark-Wilson, Robutti & Thomas, 2020). On the other hand, a simple characterization by data representation and algorithms proved powerful enough to enable Turing (1950) to demonstrate the possibility for digital machines to perform processes which seemed restricted to human cognition and which became a reality more than a half century later. More modestly, we can assume that this simplicity will be helpful for an in-depth discussion about the notions of artefact and instrumental genesis.

Another choice made for this chapter is to return to founding texts: first the above-mentioned paper by Vérillon and Rabardel (1995) and also Kuzniak’s plenary address given at CERME 8 (Kuzniak, 2013) on Geometrical Working Spaces. The aim of this is to understand how the constructs of artefact and instrumental genesis were introduced respectively in cognitive ergonomics and in

didactic research about geometry, along with their purpose. We then contrast the two frameworks, furthermore considering how they evolved after their introduction into the two perspectives. Of course, other works, such as those by Rabardel and colleagues in cognitive ergonomics, and by Kuzniak and colleagues in didactics, could be considered, but we prefer to concentrate on fundamental constructs rather than attempt to cover the whole range of aspects in the two frameworks.

9.2 Genesis and Schemes in Instrumented Activity

First of all, we consider the term "genesis" and its use in "mathematics education". Piaget and Szeminska (1941) use "genesis of the number" in their theoretical framework to denote the acquisition or construction of numerical knowledge in a developmental perspective (stages). At the same time, the notion of scheme, is one of the most important in the Piagetian conception of psychological activities. For Piaget, a scheme is the structure or organization of actions such that they transfer or generalize upon repetition of that action in similar or analogous circumstances.

The genesis is thus that of schemes. Vérillon and Rabardel's approach is situated in a Piagetian perspective, drawing on the notions of schemes and genesis. But Vérillon and Rabardel also stress that classical approaches in the psychology of learning overlook situations of knowledge construction where the subject's action is not directed by an epistemic intention but is rather shaped by a practical and functional project. For Vérillon and Rabardel, this pragmatic dimension of learning situations is important, particularly in science teaching and in professional fields. Taking some distance from a pure Piagetian approach of cognition, they refer to an activity theory perspective (Leontiev 1976) considering that "development results from the progressive appropriation of (...) socially formed attainments". For them *artefacts* must be conceived as a component of such socially formed attainments: artefacts can be "all the objects of material culture to which an infant has access during his development". After the notion of artefact, the main construct of the instrumental approach for Vérillon and Rabardel is that of *instrumented activity*. This is illustrated in the SAI diagram (Fig. 9.1 below) where the "instrument" mediates the subject's relationship to the object of their activity. The authors stress that the "instrument" exists as an artefact, in the sense mentioned above, but it also has a psychological dimension due to its appropriation by the subject in the context of their instrumented activity.

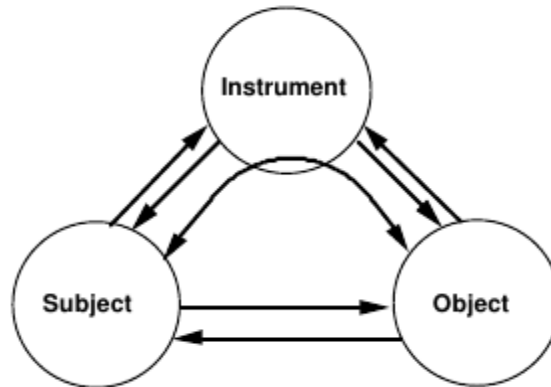


Fig. 9.1. The SAI diagram, taken from Vérillon and Rabardel (1995).

Vérillon and Rabardel describe processes of genesis in the context of the use of machines by students, a robot and a lathe. They observe that the geneses are not reduced to an adaptation (assimilation-accommodation in Piagetian terms) of the subject to the artefact. The student rather simultaneously constructs both an understanding of the functioning of the machine and a conception of the space. They conclude by stressing the consistency of the observations with the SAI diagram.

“The pupils' data acquisition strategies concerning the artefact, and their representative and operational activity during its operation (subject/instrument interaction) always turned out to be interdependent with their conceptions relative to the nature of the transforming process (instrument/object interaction), with the artefact mediating their action on the environment but also, in return, mediating their conceptualization of that environment (subject/object interaction mediated by the instrument).”

In short, Vérillon and Rabardel drew the concepts of scheme and genesis from Piaget and developed their notion of artefact from activity theory.

9.3 Genesis and Digital artefacts

In this subsection, we build upon Vérillon and Rabardel's observations in order to identify characteristics of geneses associated with digital artefacts. We contrast geneses of the lathe and of the robot. The lathe is used to cut a material into a shape of revolution. To create a desired shape, the material is put into rotation and the operator moves a cutting tool along two axes (respectively parallel and perpendicular to the axis of rotation) by way of visible screws. The robot is used to move objects in a 3D workspace. Externally, the robot is made of a mechanical device and a control box. The mechanical device is similar to a human arm: a succession of three mobile segments connect a fixed base to a

clamp through four motorized articulations (joints). The control box is made of three cursors arranged like the three axes of an XYZ orthogonal system. Between the control box and the device there is a digital processor programmed in such a way that each position of the three cursors determines a position of the clamp in a Cartesian representation of the workspace. The consequence of this is that, in contrast with the lathe, there is no direct visible relationship between action on a cursor and movement of a part in the device. Above, we characterized digital artefacts by data representation and algorithmic treatment. According to this characterization, the robot is, for us, a digital artefact: the control box is represented as a triplet of coordinates and the robot is represented by the position of the clamp in the space. Internally, the algorithm implemented on the digital processor uses another representation of the robot made of an angular position for each of the four joints, consistent with the mechanical functioning of the robot: for a given triplet of coordinates, the algorithm computes values of the four angular positions in order to ensure adequate positioning of the clamp.

Investigating this through experiments with students, Vérillon and Rabardel observed, in the case of the lathe, a genesis of a geometrical relationship between the 2D profile created by activating the screws, and the resulting 3D shape of the rotating material. With the robot, the genesis is marked by a progressive awareness by the student of different representations (internal and external) of the 3D space. Vérillon and Rabardel distinguish five microgeneses:

- (1) The cursors are considered to be associated with movements of the different parts of the arm (joints or segments).
- (2) The relationships between the cursors and the parts of the machine vary depending on the zone where the arm is located.
- (3) In some zones, the cursors are also considered to be interacting with each other.
- (4) Pupils become aware of a homomorphism between the geometry of their actions and the geometry of movements of the clamp.
- (5) The relationship between the control space and the working space is no longer conceived in terms of movement but of positions. The action schemes, which were previously movement schemes, become schemes for defining positions inside these two spaces.

For us, the main purpose of the genesis is a better awareness of the algorithmic treatment between the control box and the mechanical part, as programmed by the designer. Paradoxically, because the mechanical part is open to observation, the internal representation (angular position of the four joints) is more visible than the external representation required by the user-interface, especially for students whose understanding of the 3D space has yet to be constructed. This is why students initially associate actions on cursors with changes in the angular positions of the joints. In this representation, action on cursors only works locally or when acting on three cursors together (stages 1 to 3). Students only gradually identify the fact that the position of the clamp is the object of

control, implicitly becoming aware of the algorithm at work (microgenesis 4) before then understanding the representation of the robot in the 3D space chosen by the designer (microgenesis 5). In keeping with Vérillon and Rabardel's analysis, we see the robot as an instrument mediating the students' relationship with the 3D space, but we also stress the fact that the artefact becomes an instrument for a subject through their developing awareness of both the different representations and of the algorithm. Awareness is not in the details, but rather in a global understanding of the input and output, and of the nature of the transition between these: in the case of the robot, one must understand that some calculation on positions is made so as to pass from three coordinates to four angles¹.

9.4. Psychological and institutional viewpoints on the instrumental genesis

At the end of the 1990s, researchers in mathematics didactics (Lagrange et al., 2003) stressed the need for a "multidimensional" approach to digital technologies in mathematics teaching. Conducting a review of a significant sample of research papers in this field Lagrange et al. highlighted the fact that a large majority of researchers adopted either an "epistemological and semiotic" perspective, centered on the influence of digital technologies on mathematical knowledge and practices and on the representations used, or a "cognitive" perspective, seeing technologies as opportunities to implement (socio)constructivist approaches. They proposed to break with unidimensional approaches and to consider additional dimensions more directly focused on an analysis of the specific characteristics of the artefacts used. They identified potential new viewpoints² characterized by the indicators of analyses they found in the papers they reviewed. The indicators of the psychological viewpoint were the analysis of the possibilities and constraints of digital artefacts for a given subject, and the evolution of schemes in instrumental geneses. The indicators of the institutional viewpoint were the analysis of the interaction of ICT with tasks and techniques in the culture of a school institution, and the consideration of the role of instrumented techniques in conceptualization of mathematics.

Compared to the majority of research studies on mathematics education about digital technologies currently published, this multidimensional approach has constituted a more nuanced and reflexive approach to the role digital artefacts can play, by showing their potentialities, but also the conditions necessary

¹ It is tempting to refer here to the idea of a "black box" understanding of the algorithm, but it would be misleading since in the mediation, the box is never entirely black, but rather presented in varied "shades of gray".

² Lagrange et al refer to instrumental and institutional dimensions rather than psychological and institutional viewpoints. We adopt the latter formulation to avoid confusions with the use of the words instrumental and dimension in the theory of MWS.

for them to be implemented. This has made it possible to account for the role of "mathematical" digital artefacts (numerical and formal calculation, dynamic geometry, spreadsheets, ad hoc programming, etc.) for students' as well as teachers' conceptualizations, to show the complexity of the phenomena at hand and to develop didactic engineering which takes this complexity into account. The psychological viewpoint derives directly from Vérillon and Rabardel's pioneering work reported above. The link between this work and the institutional viewpoint is less obvious, since it uses the notion of praxeology and associated concepts (tasks, techniques, theorizations) from the Anthropological Theory of Didactics (ATD). Nevertheless, Vérillon and Rabardel stress that

“(the) instrumental relation is the subject of real occultation, producing what Y. Chevallard (1991), in mathematics, called a phenomenon of hypostasiation, through which the artefact is seen as existing in a sort of ideality independently from the concrete practices from which it emerges or which constitute it.”

Indeed, according to Chevallard, knowledge itself is the object of hypostasiation when teaching/learning is reduced to visiting a scientific content as a “monument” overlooking its “*raison d’être*”, that is to say the questions and cultural environments (*institutions* in ATD) which motivated people to develop this content. The idea of praxeology is the theoretical tool helping to position a knowledge content in the development of human culture. Similarly, artefacts, especially digital artefacts, need not be seen as the result of some serendipitous invention, but rather the product of culturally (i.e. institutionally) situated activity. Thus, instruments exist through the praxeologies in which they are set into operation for a given type of task, and the instrumented techniques are not given with the artefact, but rather develop in a culture and in institutional practices. This is the case of the robot: the design with four joints results from mechanical constraints, but for tasks like moving objects under the control of a human operator, moving each joint individually would be a tedious technique. A technique based on a homomorphism between the control space and the workspace is more efficient and is therefore adopted in a workplace praxeology. The theoretical level of the praxeology consists in the 3D representation of the space and in the idea of an algorithm able to control the joints from this representation. Similarly, mathematical instruments exist through the techniques in which they are implemented for a given type of task:

“Taking techniques into account is, along with instrumentation, one of the keys to thinking about the integration of technology in teaching (...) The interest of introducing an instrument can be seen in the development by the students of new techniques that are integrated into renewed praxeologies (Lagrange 2000).”

The psychological and institutional viewpoints are arguably complementary and should not be confused. This contrasts with the argument of Drijvers (2020), who writes:

"The observable parts of an instrumentation scheme, the concrete interactions between user and artefact, are called instrumented techniques (...) As such, an instrumentation scheme consists of one or more observable instrumented techniques...".

It is important to carefully distinguish between the subjective dimension of the scheme (How is it formed by the subject? What mental operations are at work?) and the institutional and social dimension of the techniques (In which praxeologies do they fit? In what conditions do these praxeologies develop?) Schemes and techniques each have their own identity, as do, fortunately, subjects and institutions!

9.4 Work, geneses and artefacts in the theory of MWS

We now examine the basic concepts of the theory of MWS in light of the psychological and institutional viewpoints set out above. The first concept is that of mathematical work. Vérillon and Rabardel's approach is primarily concerned with professional and technical domains, as shown by the empirical studies (lathe, robot) cited in the article, and is based on the idea of activity rather than work. Nevertheless, turning an object on a lathe, activating a robot for a given task, are not free activities carried out with the aim of developing abstract geometrical or computational knowledge, but rather activities aiming at the transformation of material entities, and developing in social contexts. In parallel, by emphasizing *work* and by considering mathematics both as a science and as a field of activity, the theory of MWS breaks with the idea of mathematics as an abstract field disconnected from social practices: like the lathe or robot operator, the mathematician works with instruments that have a psychological and an institutional value. Regarding geneses and artefacts, as in the previous sections, we refer to a text which was a precursor to the theory of MWS. In his plenary address to CERME 8, Kuzniak (2013) introduces Geometrical Working Spaces (GWS) and the geneses:

“In our approach, both levels, cognitive and epistemological, need to be articulated in order to ensure a coherent and complete geometric work. This process supposes some transformations (...) Here, in order to insist on the developmental process involved in the constitution of GWS, the notion of genesis is used. For us, a genesis involves not only the origin but also the development of a process.”

The instrumental genesis, on which we particularly focus here, connects *artefacts*, a pole of the epistemological plane, and *construction*, the corresponding pole in the cognitive plane. Three remarks seem important:

– On the MWS classical diagram and at a first glance, the geneses seem objective in character, since they connect components of the geometrical activity in its purely mathematical dimension to observable processes: the instrumental genesis with a drawing square, for example, would be simply the use of the square to draw right angles in a given construction. Nevertheless, the words used - "developmental", "transforms", "provides... status", "gives a meaning" – all suggest that transformations take place within the subject(s) or institutions which use the artefacts. Further texts (Chapter 1 of this book) have explained

how two views of the instrumental genesis can be considered. The bottom-up view describes the actions by which the user appropriates the various techniques of use related to the artefact. The top-down view begins from the user's intended accomplishment, to his or her adequate choice of the artefact with which to perform the required actions. As such, the instrumental genesis changes the way the user acts and thinks, and, in return, also transforms the mathematical knowledge. This is compatible with Vérillon and Rabardel's idea of instrumental mediation (SAI diagram fig. 9.1), but it is more upstream, because it does not theorize about dimensions relating to the subject (acting and thinking) and to institutions (instrumented techniques, impact on praxeologies).

– Vérillon and Rabardel, referring to Vygotsky's psychological instruments, explain in their conclusion that their remarks

“are not only valid for the activities instrumented by material artefacts (...) the instrumental dimension also concerns the mastery of language, writing and numbers which may be considered as technologies intended to communicate, to represent and conserve information and to calculate.”

It is somehow paradoxical to see Vérillon and Rabardel studying psychological processes related to technical artefacts which they recognize regretfully to be overlooked by psychology and then, in their conclusion, to enlarge the scope to geneses of every kind of artefact. There is arguably therefore a risk that the idea of instrumental genesis loses some of its potential by being absorbed into an overly large class of cognitive processes. In contrast, the epistemological plane of the MWS distinguishes three poles, and each can be considered as artefacts in the sense of potential psychological instruments. Consequently, the figurative and semiotic genesis proposed by Kuzniak is an instrumental genesis in accordance with the work of Vérillon and Rabardel. The same observation applies to the discursive genesis of proof, definitions and properties of the corpus of reference being artefacts instrumenting reasoning. It seems that the theory of MWS brings clarification, helping us to consider the specificity of instrumental genesis, among others. We also note that Vérillon and Rabardel distinguish between material and immaterial artefacts, only to then discard this distinction with regard to instrumentation. We can further observe that in the epistemological plane of the MWS, each pole may have a possible material basis, not only artefacts. This is obvious for signs in semiotic systems. As for the corpus of reference, Richard, Venant and Gagnon (2018) provide evidence that this is often “materialized” in varied ways for different types of proofs.

– Kuzniak (2013) explains in detail the semiotic genesis and the discursive genesis. In contrast, instrumental genesis seems to be a blind spot. Consider the example given at the end of the text. The students (10th grade) are faced with a task of constructing a triangle with the same angles as a given triangle. The teacher assumes that the variety of triangles obtained will enable them to infer the proportionality between the dimensions of the initial triangle and those of the triangle constructed. He aims to institutionalize proportionality without demonstration, but he nevertheless wants the students to be aware that it is an

unproven conjecture, the imprecision of measurements enabling this awareness. Kuzniak illustrates these expectations in a first diagram (Fig. 9.2). Although the students are required to complete a construction task, the diagram mentions no instrumental genesis. The artefacts and the construction are connected to the semiotic (figurative) genesis (in blue): they are simply "serving" the visualization, and their connection is not discussed. The construction is also supposed to "trigger" a discursive conjecture-proof genesis (in red).

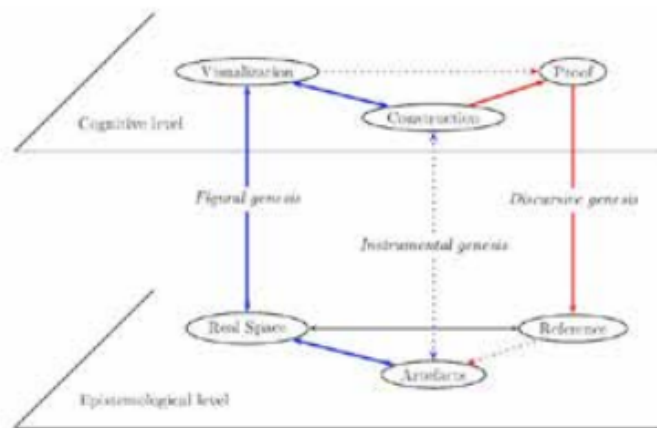


Fig. 9.2

The second diagram (Fig. 9.3) illustrates what actually happens in the classroom after the students have not achieved the construction task or have failed to engage in a process of conjecture: the teacher presents a dynamic geometry (DG) figure on a computer, makes the software calculate the ratios between the measurements of the sides for a variety of configurations, and then asks the students to observe the invariance of the ratios. The dynamic geometry figure enables the students to pass "directly" from the visualization to an empirical "proof" based on the observation of the invariance for several configurations. The precision of the measurements using the software and the plurality of the configurations make the students think that they have carried out a "true demonstration", which was not the teacher's aim. Again, the diagram mentions no instrumental genesis³.

³ The fact that the instrumental genesis is overlooked in this example actually reflects the teacher's lack of attention to the role of instruments in designing and conducting the situation. Subsection 9.5.3 herein will come back to this issue and discuss an alternative design.

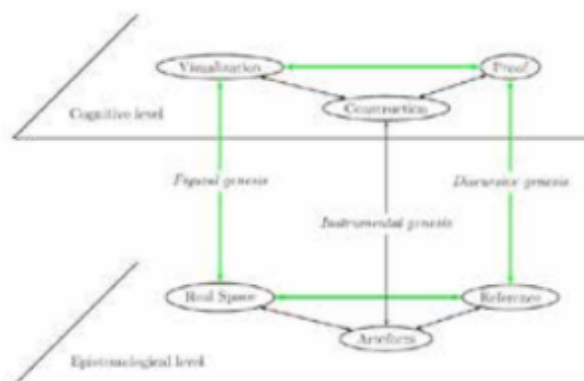


Fig. 9.3

9.5 Reflecting on key issues about the instrumental genesis in MWS

After situating the instrumental genesis among basic constructs of the theory of MWS, we look more closely in this section at this notion from both the "psychological" and "institutional" viewpoints. We start by investigating the two poles of the genesis.

9.5.1 Construction

It is useful to revisit the definition of a construction proposed by Duval (1995): "(a) construction (...) can work like a model in that the actions on the representative and the observed results are related to the mathematical objects which are represented." Construction is thus the elaboration of data-result models of mathematical configurations. By configuration, we mean that the construction models a set of interrelated objects, rather than one or more isolated objects. Models here are not the models of some reality studied in Chapter 11, but they rather share a common operability and then proximity with simulation. This fits well with traditional geometrical figures that model the properties of a set of geometrical objects. Digital geometry software adds dynamicity, enhancing the data-result (simulation) character of the model. In algebra, a digital calculator helps to make a formula work as a model of a function by providing computed values for given inputs. Interpreting Vérillon and Rabardel's robot study in the theory of MWS, we must understand that construction is what the students do when they operate the robot across the five microgenesis. Like a figure in geometry is just one of all possible realizations of a configuration, an action on the cursors is just one of all possible actions. However, it is known that

drawing a figure helps to think of all possible realizations, i.e. it is considered a model of the configuration. Similarly, students' actions on cursors are a way to test an implicit model of the robot and to make this model evolve. This sheds light on one characteristic of the instrumental genesis: it is not a passive application of techniques but rather an active process of selecting actions in order to observe relevant results. Resulting from this active process, the instrumental genesis can foster cognition and contribute to the "work on techniques" (Chevallard, 2002).

9.5.2 Digital artefacts, data representation and algorithms

Since Kuzniak's 2013 plenary, the notion of 'artefact' has been vastly investigated.

"In the theory of MWS, mathematical artefacts will generally be associated with material objects to avoid confusion with other components of the epistemological plane. But these objects will be intimately linked to rules and techniques of construction or calculation (Euclidean division, "classical" constructions using a ruler and compass, etc.). These techniques are based on algorithms, the validation and theoretical status of which are no longer problematic (This book 1.4.2.1)."

To delve deeper in a reflection on digital artefacts, we return to didactical work about computer algebra systems (CAS). For the sake of simplicity and consistency with the idea of construction as a data-result model, we restrict the focus here to symbolic calculators like Maple or Derive conceived as input/output systems. While these systems presuppose a material basis, this aspect is not decisive. Certainly, the uses of CAS can be different according to whether they are carried out on a calculator or on a computer, but the stakes for the conceptualizations go beyond this material dimension. Computer algebra can be characterized as a set of internal representations of algebraic entities and algorithms, "transparent" to the user, in principle (Elbaz-Vincent 2005). It also offers a user interface with a symbolism which is similar to usual mathematical symbolism. In mathematical work, the algorithms can be seen as playing an analogous role to routinized paper-and-pencil techniques mentioned in the citation above. However, there is a difference concerning one fundamental issue: controlling (*pilotage* in French) a mathematical calculation, without which applying algorithms is useless. In paper-and-pencil techniques, initiation to calculation algorithms is generally carried out in coordination with the learning of this subtle control, whereas with formal calculation it is precisely this control of CAS by the student user that is at stake (Lagrange 2000). How can CAS be used for a given task? What awareness of the underlying representation and algorithm does this imply? What mastery of the semiotic system specific to the software is necessary⁴? How can one control a mathematical calculation as-

⁴ Concerning the necessary awareness of underlying algorithms, consider the example in the work of Elbaz-Vincent (2005) of the treatment of the expression $\cos(a)^2 + \sin(a)^2$ by CAS. This expres-

sisted by CAS? On what set of reference knowledge can this control be based? Questions linked to CAS-instrumented techniques have been investigated theoretically and empirically (Artigue, 2002; Lagrange, 2005) but this seems to have little effect, and in countries which have not ruled out the use of CAS for educational purposes, it is very deceptive (Jankvist, Misfeldt & Aguilar 2019). The theory of MWS has the potential to make us reconsider and "untangle" these questions by differentiating between what is properly instrumental -here the implemented representations and algorithms and the awareness of these by the user, and what belongs to other geneses, semiotic and discursive. The instrumental genesis (in the sense of the MWS) is not isolated, it is rather articulated with other geneses in the working space where the artefact is used (see Chapters 1 and 2).

We note that the idea of digital artefacts based on data representation and algorithm is not present in the research by Vérillon and Rabardel. In the present chapter, we see a difference between the lathe and the robot, because action on the lathe is not mediated via a computerized interface: the genesis is different in the sense that the link between an action on a screw and a movement of the tool is not problematic. Indeed, the real problem for a beginner is the effect of a movement on the shape cut into the material; this effect cannot be thought of as a process of computation on representations. In mathematics education, there is a tendency to overlook the representational and algorithmic character of a digital artefact. Common approaches favor the vision of a digital artefact integrating the mathematical field for which it was designed without modification. For instance, publications for teachers present dynamic geometry (DG) as "embedding" elementary Euclidean geometry. They neglect the representations and algorithms underpinning DG and overlook notions which do not exist in classical geometry, such as the notions of free object and dependent object. These publications suggest that the practice of dynamic geometry could immediately be integrated into a usual working space, presenting extended possibilities of action without transforming its nature. This is a double illusion. On the one hand, the extended possibilities are in no way spontaneous in the absence of an awareness of the data representation and algorithms at work. On the other hand, by depriving the students of a working space which enables this awareness and association with the semiotic and discursive aspects, teaching also deprives students of opportunities to learn. In a suitable working space, attention to the in-

sion it is not *evaluated* to 1, even if a is declared real. In contrast it is *simplified* to 1, The reason is that *evaluation* and *simplification* are different algorithms, The first one performs a standard syntactic treatment in order to get a canonical expression and the second applies heuristically a set of transformations based on mathematical rules and properties, aiming at decreasing the complexity of the expression.

As for symbolism, it is very often only partially similar to the usual mathematical symbolism; the reason for this is that canonical expressions have to be compatible with internal representations and algorithms. For example, summations, limits and integrals must be entered with functional symbolism. In addition to this, output rarely directly conforms to what a student expects, especially at a secondary level: an example of this is $x^{\frac{3}{2}}$ when the user expects $x\sqrt{x}$.

strumental genesis of computer algebra could help students to explicitly learn how to control a calculation. Taking advantage of the functional aspect specific to dynamic geometry (identifying dependencies in a dynamic construction) could support effective approaches to geometry or functions.

Mathematics education is increasingly confronted with diverse and complex artefacts, among which some based on artificial intelligence algorithms: proof support software, virtual or augmented reality, data mining, machine learning, deep learning, large-scale data analysis, etc. (Richard, Vélez Melón & Van Vaerenbergh, in preparation). Designers are working hard to develop increasingly "accessible" and "transparent" interfaces. Even CAS and DG software have been augmented with new features and their analysis is not reducible to that of the preceding paragraph. However, we hypothesize that a similar analysis is possible: the simplicity and efficiency of the theoretical constructs (one working space, three geneses...) of MWS should make it possible to both guard against presupposed epistemic effects of the use of a given software and to ask the right questions: what are the underlying representations and algorithms, what are the new semiotic systems that the student will be confronted with and the coordination between objects which they presuppose, what theoretical frame of reference can be used to guide them, what praxeologies can develop, how can these aspects and the associated geneses be articulated in a (personal, suitable, reference) MWS?

9.5.3 Schemes and techniques in the instrumental genesis of MWS

We noted above that the instrumental genesis in MWS is, in a certain way, upstream of the approach of Vérillon and Rabardel and then further analyzed from a psychological viewpoint (development of schemes allowing the use of an artefact), as well as from an institutional viewpoint (development of instrumented techniques and associated praxeologies) in a given MWS. In this subsection, we illustrate how the two viewpoints can shed light on the example (9.4) proposed by Kuzniak (2013).

In this example, students have not integrated the use of an angle transfer artefact (protractor or tracing paper), or this use does not lead them to the side measurements on which visualization and reasoning would operate. Instrumental genesis is nonexistent, no scheme is developed and no technique emerges. The teacher's use of DG in the second phase denotes the widespread conception that we mentioned above, assuming that simply making students look at DG figures and software calculated ratios will trigger a conceptual reflection. Focusing on digital artefacts, we propose an alternative situation assuming that students have the use of DG software (not only the teacher). A strategy often

proposed in DG research studies is to let students start by performing a *soft construction*⁵.

Starting from a triangle ABC, a segment DE and a free point F, they have to guess one or several positions of F, such that DEF has the same shape as ABC (i.e. corresponding angles have the same value). It can be expected that students will mark angles in the two triangles and drag F to a suitable position. Students could then be advised that their construction is not robust in the sense that it does not resist dragging of one end of segment DE. The ability of DG to provide automatic proof (oracle) can also be a reason to reject this construction, because, in spite of visible angle equalities, the oracle will declare that it is not proven (Richard, Venant and Gagnon 2018). Students could then be asked to *harden* the construction. The assumption is that they will think of transporting angles, but transporting is incommodious in dynamic geometry. Students will then have to infer the equality of ratios from various configurations and transpose the procedure for constructing a triangle into DG, knowing its dimensions.

The path that students are expected to follow includes movements along the instrumental genesis and between the construction pole and the two other cognitive poles (figure).

- A first movement connects the dragging capability in the DG artefact and the soft construction.
- A second movement involving the construction and the proving poles of the cognitive plane consists in becoming aware of the limits of this construction.
- In a third movement, the visualizing and construction poles are involved in order to explore relations between distances.
- The fourth movement connects different capabilities of DG, especially an intersection of circles and a hard construction.

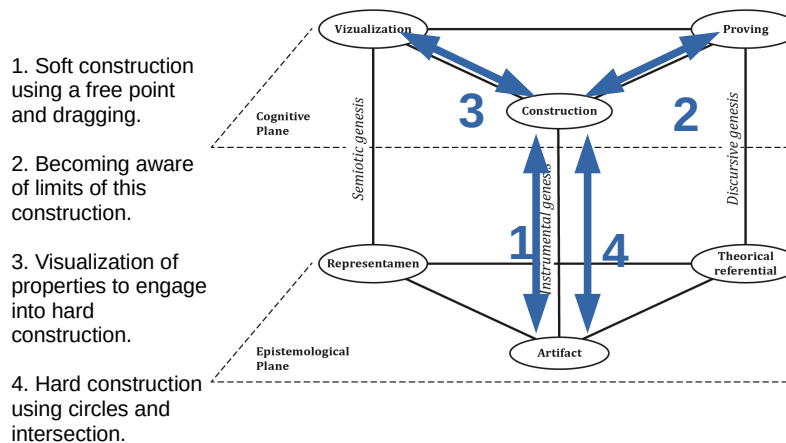


Fig 9.4 The path students are expected to follow in four movements.

⁵ Leung (2014) presents in more depth how "soft constructions" can be productively implemented in relation to conservation properties.

From a psychological viewpoint, the instrumental genesis in the first movement consists in appropriating (i.e. build schemes of) the use of a free point F and the dragging gesture to find an adequate position. In the second movement, understanding why the construction does not resist and/or why the oracle denies proof implies some awareness of DG (algorithmic) functioning. In the third movement, students would have to appropriate DG capabilities related to measures, and in the fourth movement, capabilities related to circles and intersection.

From an institutional viewpoint, soft and hard constructions are two instrumented techniques for the same task, using different capabilities of DG and belonging to different praxeologies: one based on visualization and approximation, and the second on geometrical properties. It can be expected that students will not easily abandon a soft construction “that works” to adopt a more intellectually demanding hard construction. It is therefore essential that the teacher involves students in a discussion on techniques, stressing the interest of adopting robust procedures consistent with digital software. This discussion is central to the second movement⁶.

9.6 Conclusion

We conclude with a summary of the main points of our contribution. Vérillon and Rabardel’s instrumental genesis is one of schemes anchored in a Piagetian perspective. Their analysis implies a fine-grained identification of microgeneses specific to the type of artefact used. Focusing on digital artefacts, we observed how schemes develop through the subject’s emerging awareness of the representations and algorithms which underpin the artefact. Consistent with Chevallard’s (YEAR) ATD and Leontev’s (YEAR) activity theory, Vérillon & Rabardel also insist on breaking with an idealistic view of artefacts, consider-

⁶ Lagrange and Erdogan (2009) present a similar case illustrating the importance of a discussion on techniques. Students in two classes (11th grade, not scientific majors) have to implement a recurrence relationship on a spreadsheet from a real-life situation. The teachers in both classes expect students to implement the relationship by way of a formula copied down along a column. The goal is that they understand the interest of an algebraic formula as a model of the real-life situation. Contrary to this expectation, students easily compute the requested values by hand, enter these values into the cells, and decline to enter any formula. Using colors and borders, they are happy to use the spreadsheet as a tool to display data. Both teachers are disconcerted. One teacher tries ineffectively to instruct students to make a formula. The other teacher, better prepared for situations where technology is used, starts an improvised whole-class discussion on techniques for calculating values of series, stressing the respective values of by hand and automatic calculation. Students become aware of the actual potentialities of the spreadsheet as a calculation tool and of the power of the algebraic symbolism.

ing them as emerging from socially situated practices and constitutive of these. Building on the theoretical elaboration of Vérillon and Rabardel, and of Chevallard, didactic research introduced two complementary viewpoints or angles from which to consider the instrumental genesis of digital artefacts: the psychological viewpoint deals with the formation of schemes and associated mental operations; the institutional viewpoint focuses on instrumented techniques, the praxeologies in which they fit and the social conditions in which they develop.

As for the instrumental genesis in the theory of MWS, it is first an objective entity, linking tangible artefacts and observable processes of construction. Because it also comprises a conceptual dimension transforming both the user and the mathematical knowledge, it is compatible with Vérillon and Rabardel's idea of instrumental mediation. However, instrumental genesis in the theory of MWS is based on different choices compared to Vérillon and Rabardel's notion: on the one hand, it is part of a theorization of mathematical work in educational settings not limited to the use of instruments, and on the other hand, it does not theorize about the transformation occurring in the instrumental genesis. The two complementary viewpoints, psychological and institutional (one providing insight into the cognitive work, the other into how techniques are understood and implemented), derived from the works of Vérillon and Rabardel, and of Chevallard, can therefore help to “flesh out” an analysis of the instrumental genesis in a particular MWS, on the condition of being cautious not to merge or confuse ideas drawn from different theoretical perspectives.

Looking at the poles of the instrumental genesis in MWS, we see construction as data-result models of mathematical configurations. In this modelling perspective, the instrumental genesis is not a passive application of techniques, but rather an active process of selecting actions and data in order to observe relevant results. Regarding artefacts, we noted that Vérillon and Rabardel enlarged the scope, and we stressed the risk that the idea of instrumental genesis lose some of its potential by being “absorbed” into a too large class of cognitive processes. In contrast, the epistemological plane of MWS distinguishes three poles, and each of these can be considered as artefacts potentially integrated by the subject through a specific genesis. This clarification is very helpful, yet implies that characterization of artefacts in the instrumental genesis of MWS is not straightforward. Indeed, it requires us to precisely identify capabilities which intervene in the modelling process, beyond their material aspect. Regarding digital artefacts, these capabilities derive from the representations and algorithms implemented. We have given an example demonstrating how it is possible to propose an efficient strategy for using a digital artefact, how an a priori analysis in the MWS framework gives a central role to the instrumental genesis and how this analysis can be enriched by considering the genesis from two complementary viewpoints: psychological and institutional.

Our focus on digital artefacts was productive in the sense that it helped to question notions in the theory of MWS, and also because it enabled us to better

characterize these artefacts: a contribution to didactic research on digital environments. As mentioned in this chapter, digital artefacts are ubiquitous, yet their educational use is not often straightforward in real-life teaching contexts, meaning that both empirical and theoretical research is more necessary than ever. We illustrated how a simple characterization of these artefacts, combined with the potential of the theory of MWS helps us to gain a more precise perception of situations of use. Our focus is certainly also a limitation with regard to other artefacts such as the mathematical machines (Bartolini-Bussi & Maschietto, 2006) mentioned in Chapter 8. We nevertheless believe that the ideas presented in this chapter will be useful to analyze other types of artefacts in a MWS framework.

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