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▶ To cite this version:

Pia Mathias, Laurent D. Johann, Pierre Bomel, Hugo Kerhascoet. Real time heading sensors fusion and fault detection. Chesapeake Sailing Yacht Symposium, Mar 2022, Annapolis, United States. hal-03633619

HAL Id: hal-03633619 https://hal.science/hal-03633619v1

Submitted on 7 Apr 2022

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THE 24TH CHESAPEAKE SAILING YACHT SYMPOSIUM

ANNAPOLIS, MARYLAND, MARCH 2022

Real time heading sensors fusion and fault detection

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ABSTRACT

In modern offshore racing, performance often depends on two main factors: a good autopilot and the right strategy decisions taken by the skipper. Some sensors are crucial to ensure the quality of those two keys of success, among which we can mention the heading sensors. Unfortunately, those sensors, whether magnetometers or GNSS based, are subject to disturbances and faults of various origins: magnetic disturbances from other devices, GPS fix or reception issues, sensor drift, etc... The aforementioned fault on sensors can cause autopilot's solution to diverge which can result in serious damages for the boat or the crew. Assurance of a valid measure is therefore a key point to ensure reliability of autopilot system and skipper's decisions. This paper presents a method to produce consistent values of true heading and yaw rate while detecting fault on sensors.

The proposed solution relies on the hypothesis that sensors using different technologies and placed in different spots inside the boat will not be subject to identical and synchronised disturbances. Thus, by fusing intelligently the information coming from several sources, a continuous and consistent true heading measure can be maintained. A simple dynamic model for the heading and yaw rate is implemented and asynchronous filter update is done depending on available measures. The difference between the estimated state and the measure is used to determine whether a sensor is faulty or valid and the update is done consequently; then the information on sensors status and quality of the estimation can be propagated.

Here, we detail the method able to detect faults on the heading sensors and to provide a substitution value if necessary. The proposed model is validated by test campaigns that were conducted using both data logs and on-board tests. Results show that we can improve and maintain true heading measure quality and detect and isolate faulty sensors.

NOTATION

GNSS	Global Navigation Satellite System	σ_{innov}^2	Inr
KF	Kalman filter	HDT	He
$\hat{\mathbf{x}}_{k k}$	Estimated state at time increment k	COG	Co
\mathbf{z}_k	Measurement vector		
$\mathbf{ ilde{y}}_k$	Innovation of the Kalman filter		
$\mathbf{P}_{k k}$	Estimate covariance matrix		
\mathbf{F}	State transition model matrix		
Н	Measurement model matrix		
\mathbf{Q}	Process noise covariance matrix		
\mathbf{R}	Measurement noise covariance matrix		
\mathbf{S}_k	Innovation covariance matrix at step k		
\mathbf{K}_k	Kalman gain		

novation variance for a single measurement

- eading
 - ourse over ground

INTRODUCTION

When navigating, with or without sails, the main concerns are the position and the direction of the vessel. Since the beginning of the second millennium, sailors have been using compasses to know the direction of their boat. Today, a large panel of technologies are available for seafarers : magnetic compasses, GNSS-based sensors, fiber optic gyrocompasses, etc... If today's technology is way more precise than in the past, heading sensors are still subject to disturbances, errors or failures: a magnetic compass may present a drift and be disturbed by its magnetic environment, even more so on modern high-tech yachts, and a GNSS-based compass rely on the satellite coverage of the area and the reception of the sensor. Moreover, if an autopilot is used on board, other sources of problem when dealing with heading sensors, like the loss of connection between a sensor and the pilot, can appear. To show an example, figure 1 displays the heading measured by three different sensors on board a racing yacht : *sensor1* is a magnetic compass and *sensor2* is a GNSS-based compass, both from the same manufacturer while *sensor3* is a magnetic sensor from a different manufacturer.



Figure 1: Example of faulty sensor on a racing yacht

We can see that in a very short amount of time, the first sensor's measurement changes from acceptable, to degraded, and finally to completely incoherent. Hence, a continuous and perfect heading measurement cannot be guaranteed without redundancy on the sensors. In offshore racing, the precision of the heading measurement is even more crucial due to the speeds reached by the yachts. An error of 10° on the direction of a boat sailing at 25 knots results after one hour in a deviation of 4.3 nautical miles from the projected position, which could cause a serious loss of performance and even safety hazard in certain situations. The problematic is evidently the same whether an autopilot is used or not. As of today, multiple heading sensors are implemented on racing yachts but there is no automatic management of the redundancy whatsoever.

This paper presents a sensor fusion and fault detection method that manages the sensors redundancy and produces a reliable heading value. First we introduce the Kalman filter used for the fusion and its adaptation to circular and asynchronous data such as the heading of a vessel. Then we describe how a fault on a sensor can be detected, from a single outlier to a slow degradation of the compass. Finally, validation on real data is conducted to draw conclusions on the proposed method.

SENSOR FUSION

For statistical and control applications, many algorithms are available when we want to estimate unknown variables from a series of measurements. We choose to use the Kalman filter (KF), also widely applied for sensor fusion applications. The following section presents the functioning of a the basic linear Kalman filter and the adaptations that we made for our application.

Kalman filter

The Kalman filter is a recursive estimator that relies on a discretized linear dynamic system along with process and measurement noise. The state of the filter at time increment k is represented by :

- $\mathbf{x}_{k|k}$ the estimate at time k given past and present observations.
- $\mathbf{P}_{k|k}$ the estimate covariance matrix.

The algorithm then works as a two-steps process, prediction and update:

Prediction

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k \mathbf{x}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k \tag{1}$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_k \tag{2}$$

With \mathbf{F}_k the state transition matrix, \mathbf{B}_k the control input matrix, \mathbf{u}_k the control input and \mathbf{Q}_k the process noise covariance matrix at time increment k.

Update

$$\tilde{\mathbf{y}}_{\mathbf{k}} = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1} \tag{3}$$

$$\mathbf{S}_{k} = \mathbf{H}_{k} \mathbf{P}_{k|k-1} \mathbf{H}_{k}^{T} + \mathbf{R}_{k} \tag{4}$$

$$\mathbf{K}_{k} = \mathbf{P}_{k|k-1} \mathbf{H}_{k}^{T} \mathbf{S}_{k}^{-1}$$
(5)

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \tag{6}$$

$$\mathbf{P}_{k|k} = (I - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \tag{7}$$

With \mathbf{z}_k the current observation, \mathbf{H}_k the measurement model matrix, $\tilde{\mathbf{y}}_k$ the innovation, \mathbf{R}_k the measurement noise covariance matrix, \mathbf{S}_k the innovation covariance and \mathbf{K}_k the optimal Kalman gain.

The system is defined by model matrices $(\mathbf{F}_k, \mathbf{B}_k, \mathbf{Q}_k, \mathbf{H}_k, \mathbf{R}_k)$ and initial conditions $(\mathbf{x}_0, \mathbf{P}_0)$. If the process and measurement errors have a Gaussian distribution, the estimator is optimal. But knowing their covariance is enough to state that the Kalman filter is the best possible linear estimator to minimize the mean squared error (Kalman, 1960).

Variations and extensions of the Kalman filter have been proposed over time to extend its use to different applications, such as the Extended Kalman filter and the Unscented Kalman filter for nonlinear systems (Julier and Uhlmann, 1997). Next subsections will present the system used for application and how we adapted the simple linear Kalman Filter to work with angular variables.

Model chosen

A simple linear model is first chosen for the estimation of heading from the fusion of several sensors. The state of the system is represented by $\mathbf{x} = [hdt, yawRate]$ and the constant state transition matrix is defined as :

$$\mathbf{F} = \begin{pmatrix} 1 & dt \\ 0 & 1 \end{pmatrix}$$

Where dt is the time interval between two increments of the filter. The approximation made is that the *yawRate* is constant, its variations will be included in the process noise matrix \mathbf{Q}_k and corrected in the *update* step of the KF.

All the sensor measurements are gathered in a single observation vector $\mathbf{z} = [hdt_1, \dots, hdt_N, yawRate_1, \dots, yawRate_M]$ when N measures of heading and M measures of yaw rate are available. The constant observation matrix is then defined as :

$$\mathbf{H} = \begin{pmatrix} 1 & 0\\ \vdots & \vdots\\ 1 & 0\\ 0 & 1\\ \vdots & \vdots\\ 0 & 1 \end{pmatrix}$$

Noise and confidence associated with each sensor are represented by the matrix \mathbf{R}_k .

In our application, we assume that the matrices \mathbf{Q}_k and \mathbf{R}_k are constant and will therefore be written \mathbf{Q} and \mathbf{R} . Implementation of noise matrices is often the trickiest part when using the Kalman filter in practice. Several method have been studied and proposed, such as the data-based auto-covariance least-squares (Rajamani, 2007, Rajamani and Rawlings, 2009) that we used in our applications.

Despite the apparent linearity of our system, special attention has to be drawn on the fact that headings are angular values - on the $0^{\circ}/360^{\circ}$ circle - and therefore intrinsically nonlinear. Thus a linear KF is not adapted to such variables. We propose an adaptation of the simple KF to avoid using more complex variations of the estimator to include non linearities.

Angular Kalman filter

When we want to work with angular data, the occurrence of modulo - whether on the $360^{\circ}/0^{\circ}$ transition or the $-180^{\circ}/180^{\circ}$ transition - introduces non-linearities and discontinuities in the system. Using the Extended Kalman filter (EKF) is not possible either because of the discontinuity on the modulo making the propagation function not differentiable. One way to overcome the problem is to decompose the variables using cosines and sines and adapt the system for the EKF, or to use another version of the Kalman filter. For our application on headings, we adapted the Kalman filter to keep the same transition and observation matrix, thus maintain a pseudo-linear system. The proposition made is to apply modulo at prediction and update and angular difference rather than arithmetical difference when computing the innovation. In practice, this is formalized by applying mod 360 to equation (2) and (6) :

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}\mathbf{x}_{k-1|k-1} \mod 360 \tag{8}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \tag{9}$$

The innovation being the difference of two angles, it can be computed by the following formula, chosen to respect the clockwise rotation convention for headings :

$$diff = (\mathbf{H}\hat{\mathbf{x}}_{k|k-1} - \mathbf{z}_k) \mod 360 \tag{10}$$

$$\tilde{\mathbf{y}}_{\mathbf{k}} = (2 * diff) \mod 360 - diff \tag{11}$$

Similarly, a version for the $-180^{\circ}/180^{\circ}$ scope could be developed. The former equations could be applied separately on each term of the state and innovation vector depending on whether they represent circular variables or not. For our chosen model, the angular transformation will be applied of the first component of the state vector \mathbf{x}_k which is the heading.

Before validating our filter, it is important to verify that our algorithm is suitable for real data. The simple Kalman filter presents an update step with synchronized measurement, which is rarely the case in practice. Our algorithm thus requires a proper adaptation for multi-rate and asynchronous measurements.

Asynchronous update for the KF

The presented Kalman filter and its angular adaption implies that all observations are synchronized and can be grouped in a single measurement vector \mathbf{z}_k . But in practice when performing sensor fusion, measurements are hardly ever received simultaneously nor with the same frequency. Each sensor has its measurement frequency and sensors are usually not synchronized. If the measurements frequencies are multiples of a common frequency, we can easily choose to select this highest common denominator frequency for the application of the filter and artificially synchronize all the observations on this frequency. Using extrapolation, B-spline or other signal processing methods we can virtually synchronize the sensors on any desired frequency. Lastly, an asynchronous update is also possible by running the update step each time a measure is received : we must apply the update step with the matrices \mathbf{H} reduced to its i^{th} row, corresponding to the current measure, and \mathbf{R} reduced to its $r_{(i,i)}$ coefficient. The resulting innovation covariance scalar $S_{k,i}$ and Kalman gain $K_{k,i}$ computed with equations (4) and (5) will only include the effect of the current measurement $z_{k,i}$. For our algorithm, we choose to work with the latter proposition. The measurement frequencies of off-the-shelf compasses, combined with an asynchronous update allow us to get enough measurement for our desired precision. Moreover, in order to run the algorithm at a medium or high frequency, we should minimize the computation time and load.

The resulting algorithm, an asynchronous angular Kalman filter, can now be applied on data to validate its properties.

Validation of the sensor fusion

The resulting heading sensor fusion algorithm is applied on real-time data gathered on racing yachts. The boat was equipped with three heading sensors, each with its own precision and sample frequency. The first sensor is the most precise, and *sensor3* is the least precise. In order to validate the $0^{\circ}/360^{\circ}$ modulo management, we have chosen a heading north navigation section.



Figure 2: Result of the filter on three heading sensors

Figure 2 focuses on a section of the results, while extended validation examples are not displayed here. We can see that the transition from 360° to 0° is correctly managed by the filter on this section. Other examples used for validation show the same behaviour around $0^{\circ}/360^{\circ}$, thus validating the modulo management for our angular Kalman filter.

If we focus on the sensor fusion, we can see in the figure 2 that the output of the filter almost coincides with the signal of *sensor1*. This behaviour is expected : the first sensor presents the lowest measurement noise and the dynamic is very low on this section, therefore the contribution of *sensor1* is the most significant one. The contribution of the two other sensors, although smaller, is also taken into account and explains the slight difference between the filter output and *sensor1*. The operation of the filter has been validated on other results, thus, we can conclude that fusion has been correctly applied on the sensors.

The sensor fusion properties of our algorithm have been validated, but such a filter is not robust to deviation or fault on a sensor. If we apply the proposed filter as it is on the example from the introduction (see figure 1), the output will be a fusion of all sensor, including the faulty one, and will therefore be wrong as well. A method to detect fault on sensors is needed to assure robustness of our algorithm.

FAULT DETECTION

Applying the Kalman filter allows us to produce a coherent value for the heading using all the available measurement sensors. But it is crucial for the skipper to know if any of the sensor on board is defective. Moreover, discarding a faulty sensor before the update step of the KF lessens its effect on the fusion solution, thus improving its accuracy.

From equation (4) we can compute the covariance of the innovation, which includes confidence on the *a priori* estimated state $\hat{\mathbf{x}}_{k|k-1}$ and on the current measurement. For an asynchronous update, the matrix becomes a scalar for each measurement, that will be noted σ_{innov}^2 . Assuming the innovation follows a Gaussian distribution, we have the following law :

$$P(|\mathbf{\tilde{y}}_{k,i}| > 4\sigma_{innov}) < 0.1\%$$

As a consequence, we can assume that if the computed innovation (see equation (11)) $\tilde{y}_{k,i}$ is higher than four σ_{innov} , the associated measurement can be considered as an outlier and discarded from the present update step : equations (5), (7) and (11) will not be applied for this measurement.

This process allows us to discard any outlier present in the signal. By following the evolution of the innovation, we can also exclude a faulty sensor : an additional condition has to be set on the number of consecutive outliers for a single sensor. If the threshold is exceeded, the sensor is declared faulty and a warning or error message can be displayed to the skipper to let him know. Faulty sensors can be removed from the measurement process until corrective action is undertaken by the user. That way, the robustness of the filter is increased and a sensor monitoring can be applied.

Fusion and fault detection with the biases

Except for very specific compass, most of the on-the-shell heading sensors have a confidence that depends on whether they are aligned with the longitudinal axis of the boat. A misalignment introduces a bias on the measurement that ultimately can induce an offset between the output of the sensor fusion and the real solution we want to reach. Usually, sensors are placed manually and in different locations on board, so they present different biases from the actually true heading which we try to compute. A calibration, using the course over ground (COG), is often performed to minimize those offsets, but in order to get a robust sensor fusion, our algorithm should incorporate those biases and reduce their negative effect on the Kalman filter solution.

To include the biases, the state is extended with N terms for each heading sensor used for the measurement : $\mathbf{x} = [hdt, yawRate, bias_1, \dots, bias_N]$. Considering the biases are constant, the increased model matrices are now :

	/1	dt	0		0	0)
	0	1	0		0	0
	0	0	1	0		0
$\mathbf{F} =$	0			1		0
	:				·	:
	$\setminus 0$			• • •		1/
	$\binom{1}{1}$	0	1	$\begin{array}{c} 0 \\ 1 \end{array}$		0
	:	0	0		·	:
$\mathbf{H} =$	1	0	0		0	1
	0	1	0			0
						:
	10	1	0			0/

It should be noted that such a system is non observable so a Kalman filter may not converge to a suitable solution (Li et al., 2019, Southall et al., 1998). To overcome this problem, a reference will be chosen among the sensors and its bias will be set artificially to 0 with an additional measure. The observation vector becomes

$$\mathbf{z} = [hdt_1, \ldots, hdt_N, yawRate_1, \ldots, yawRate_M, bias_j]$$

and the observation matrix **H** is extended with the following row : $(0, \ldots, 0, 1, 0, \ldots, 0)$. The new system with the extended observation matrix is now observable.

Several methods to choose the reference sensor are possible to minimize the estimation error and bias. One can obviously choose the sensor which is known to bear no bias at all or the smallest one. To ensure robustness of our algorithm, we prefer to use an automatic selection of the reference. Thereby, if the preferred sensor is declared faulty, the second best sensor might be chosen as reference. To choose automatically the reference, we compute for each sensor the cumulative distance to all other sensors and to both the current estimate and the course over ground (given by the GNSS sensors). In a more formal way, we compute :

$$cumDist_{i} = \sum_{j=1}^{N} |dist(z_{i}, z_{j})| + |dist(\hat{x}_{k}, z_{i})| + |dist(COG, z_{i})|$$
(12)

Weights might be added to convey the confidence given to each sensor. The sensor that presents the smallest cumulative sum is chosen as the reference sensor for the system. Like in any voting decision system, this implies that the faulty sensors cannot be in the majority among the on-board sensors. The fusion solution operates nominally with at least three heading sensors, but thanks to the addition of the distance to the current estimate and to the course over ground, a degraded mode with only two sensors is possible. The goal of our sensor fusion and fault detection algorithm is to detect rapidly a sensor that is not in nominal condition and to separate it from the valid sensors of the system. If the state is transmitted to the user, an action can be taken to ensure the continuity of operations in a nominal mode, that is with three sensors in a valid state. If no action is taken, the faulty sensor is put apart to avoid a "wrong" vote if another sensor came to fail. Lastly, the presented method allows us to minimize the bias that can be present in the output of the Kalman filter but doesn't not guarantee a null bias. A thorough calibration of the sensor is always advised to get the best measurements from the sensors.

The main objective when estimating the biases is to minimize the offset of the filter solution, but it can also be used to detect the deterioration of a sensor.

Secondary fault detection with the bias

Introducing the biases on the compasses improves the solution of the Kalman filter, but it also allows us to perform a second fault detection on the sensors. While outliers can be discarded prior to the update step of the Kalman filter, a slow drift of sensor might not trigger the innovation covariance threshold set for the undermentioned outliers. However for the magnetic compasses, a drift in the value is precisely one of the most common error. This drift can be caused by the magnetic environment of the compass and by the quality of the chosen sensor. If several heading sensors, magnetic-based or not, are present on board, they would not present a common drift. Thus by following the evolution of the bias of each sensor, we can detect the drift or any other degradation of the measurement quality of one or more sensors.

To monitor the bias, methods from statistics and signal processing can be chosen. Among them, we first implement a monitoring on the maximum absolute value, the rolling standard deviation and a *CUSUM* (cumulative sum) method on the mean and the variance of the biases.

VALIDATION ON REAL DATA

A first validation of the sensor fusion has been formerly presented, with the results gathered in figure 2. Let's now take the situation presented in the introduction (figure 1) where a heading sensor drifts abruptly, and see the behaviour of the proposed solution.



Figure 3: Evolution of three heading sensors, one faulty, on a racing yacht

The system comprises three heading sensors, a magnetic-based one and two GNSS-based compasses with the first one

being more precised. After a maneuver that results in a sharp change in the heading, the magnetic based compass (*sensor1*), first oscillates very slowly around the mean heading before dropping abruptly from 250° to 100° in less than 7 minutes.

If we apply the full solution on our system, which results are showed on figure 4, we can see that the filter generates an output which stays around a mean heading of 250° , coherent with the other sensors and the observed dynamic on board.



Figure 4: Solution of the sensor fusion on three heading sensors, one faulty

The loss of *sensor1* is detected 10 min and 35 seconds after the tack, and 45 seconds after the sensor value last matched with the KF output value. This time interval is considered acceptable in light of our requirements, even more so when looking at the heading value produced by the filter during this time interval. The degradation of the sensor being slow compared to the prediction frequency of the Kalman filter, the detection of the fault is triggered with the statistics on the bias rather than the innovation covariance. The example presented here concerns a drift rather than occasional outliers. Other validation campaigns conducted show that the proposed KF-based solution is perfectly robust to single or multiple outliers.

The main requirements for the developed sensor fusion are therefore met. The algorithm produces a continuous value of heading using measurement from different sensors, each with their sampling frequency and their sensibility to noise and errors. It should be noted that the solution is capable of following a maneuver without triggering a fault, which is an obvious requirement for this problem. Secondly, it is robust to one sensor deviating from the correct value of the variable. Moreover, our algorithm is capable of detecting when a sensor does not produce an acceptable value, in comparison with other sensors and its own estimated value, and to exclude it from the measurement vector. Information on sensor status can be conveyed to the user, as well as the confidence given to the estimation, comprised in the covariance matrix **P**.

CONCLUSION

To tackle the problem of heading sensor fault on racing boat, a Kalman filter-based solution has been developed for sensor fusion and fault detection. The classical Kalman filter has been adapted to work with circular data on the $0^{\circ}/360^{\circ}$ circle and with asynchronous and multi-rate measurements. The resulting sensor fusion algorithm is validated on data bearing the modulo transition and a heading value can therefore be produced using different sensors. To improve the robustness of the algorithm to outliers and serious faults on sensors, two fault detection methods have been implemented. The first one relies on the innovation and the innovation covariance, discarding outliers in measurements from the update step of the Kalman filter. The second method extends the state of the KF to incorporate the biases on each sensor and applies statistical and

signal processing methods to detect a sensor fault. To ensure the observability of the system, and guarantee robustness of the solution, an automatic selection of the reference sensor, bearing a null bias, is added. All this results in a robust and modular algorithm that has been validated on real data provided by racing yachts. Validation shows that the algorithm is capable of following maneuvers and producing a heading value which is coherent with the majority of on-board sensors, the observed course over ground and the boat dynamic. It also demonstrates that the proposed solution is able to discard outliers and detect a slow degradation of a sensor that becomes faulty. The operation of the filter is optimal when at least three sensors are available but degraded function with one or two sensors is possible, which makes it acceptable to durably loose sensors. If the presented solution meets the requirements that were fixed, improvements are possible. Other signal processing methods can be applied on the biases to detect a fault sooner for example. The dynamic model of the system can also be made more complex to include the leeway, the course over ground or information from other sensors on board. This would require a non-linear model, thus another version of the Kalman filter, such as the unscented or extended Kalman filter. The proposed fault detection algorithm is able to detect a faulty sensor, and an adapted status or message can be sent to the user to require action. To free from human interaction and thus keep increasing the robustness of the global solution, further developments could focus on automating the correction and repair process after the fault detection. One could also adapt the autopilot mode by considering the analysis of the confidence on the estimation thanks to the covariance matrix P, even more so when all the sensors are lost.

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