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Sensitivity Analysis on a green roof hydrological model

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Context

Urban imperviousness is a major urban issue during rainfall events

Solution → Green Roofs

• lower the peak flow rate in water system by 22% to 93%
• delay the peak flow by 0 to 30 min

Green Roofs hydrologic modeling

• Rainfall data and real water content measured in a green roof of the CEREMA of Nancy
• Simulation of the water infiltration into the layers using Richards equation (1) implemented in Hydrus-1D software combined with Van Genuchten model (2) and Mualem model (3)

\[
\frac{\partial \theta(h)}{\partial t} = \frac{\partial}{\partial x} \left( K(\theta) \frac{\partial h}{\partial x} \right) - a \frac{\partial h}{\partial x}
\]  

\[
\theta(h) = \frac{\theta_s - \theta_r}{(1 + |\alpha h|^m)^n} - \theta_r \quad \text{with} \quad m = 1 - \frac{1}{n}
\]

\[
K(S_i) = K_S S_i^n \left( 1 - (1 - S_i)^{1/m} \right)^m \quad \text{with} \quad S_i(\theta) = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r}
\]

Problem statement

• Input : Rainfall
• Output : Water content \( \theta(t) \) or \( VWC(t) \) in (ii)
• \( k = 6 \) uncertain parameters : \( \theta_s, \theta_r, K_s, \alpha, n, \) and \( \alpha \)

Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Inf</th>
<th>Sup</th>
<th>Inf</th>
<th>Sup</th>
</tr>
</thead>
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<tr>
<td>( \theta_s )</td>
<td>m m^-1</td>
<td>0.072</td>
<td>0.088</td>
<td>0.054</td>
<td>0.066</td>
</tr>
<tr>
<td>( \theta_r )</td>
<td>m m^-1</td>
<td>0.337</td>
<td>0.473</td>
<td>0.27</td>
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<tr>
<td>( \alpha )</td>
<td>m^-1</td>
<td>5.85</td>
<td>7.15</td>
<td>4.5</td>
<td>5.5</td>
</tr>
<tr>
<td>( n )</td>
<td>-</td>
<td>1.422</td>
<td>1.738</td>
<td>1.35</td>
<td>1.65</td>
</tr>
<tr>
<td>( K_s )</td>
<td>( 10^{-3} ) m s^-1</td>
<td>1.95</td>
<td>2.38</td>
<td>3.25</td>
<td>3.972</td>
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<tr>
<td>( l )</td>
<td>-</td>
<td>2.583</td>
<td>3.157</td>
<td>0.09</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Method based on PCE :

\[
Y(t) = \mathcal{G}(X,t) = \sum_{i=1}^{n} a_i(t) \psi_i(X_1,\ldots,X_i)
\]

Method applied at each time instant :
1. Define the distribution of uncertain parameters to compose \( \psi_i \)
   Uniform distribution → Legendre Polynomials
2. Define \( p \), the maximum development order of \( \psi_i \)
3. Estimate the PC coefficients \( a_i(t) \) by solving a least square problem
4. Estimate sensitivity index (SI) with :
First order SI :

\[
S_i(t) \approx \frac{\sum_{k \leq p} \int\int a_i(t)^2 E[Y_i](X_1) \partial X_k}{\sum_{k \leq p} \int\int E[Y]^2(\partial X_k)^2}
\]

where \( I_i \) is polynomial index of \( \psi_i \) which depends only on \( X_i \)

Total SI :

\[
S_L(t) \approx \frac{\sum_{k \leq p} \int\int a_i(t)^2 E[Y_i](X_1) \partial X_k}{\sum_{k \leq p} \int\int E[Y]^2(\partial X_k)^2}
\]

where \( I_{\psi_i} \) is polynomial index of \( \psi_i \) which depends on \( X_i \) and all its interactions.

Results on Green Roofs

How vary the water content \( VWC(t) \) in the substrate when ...

... substrate parameters vary ?

• Variations whatever \( VWC(t) \) evolution
• \( S_j = S_j^n \) : no interaction between parameters
• 3 influential parameters : saturated water content \( \theta_s \), pressure head fitting parameter \( \alpha \) and porosity \( n \)
• \( S_j \) follows \( VWC(t) \) i.e. \( \varphi \) when it rains
• \( S_j \) inversely follows \( VWC(t) \) i.e. \( \varphi \) when it rains
• \( S_j \) varies slightly following \( VWC(t) \)
• The other parameters are not influential

... drainage parameters vary ?

• No variation when \( VWC(t) \) is saturated
As for the variation of substrate parameters, we have :
• \( S_L = S_L^n \)
• Same 3 influential parameters and the other parameters uninfluential
• Same dynamics for \( S_{\alpha}, S_{\theta_s} \) and \( S_n \) but with more intensity

Conclusion and Prospect

• 3 influential parameters, saturated water content \( \theta_s \), pressure head fitting parameter \( \alpha \) and porosity \( n \) and different influences depending on the layer
• Prospects : analyse the outflow and include meteorological uncertainties

References