



Case study from Université Claude Bernard, Lyon 1

Christian Mercat

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Brief Report

Christian Mercat*

Case study from Université Claude Bernard, Lyon 1

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Abstract: We present the Active Learning tools introduced in basic mathematics courses in the Preparatory Curriculum for Engineering schools at Université Claude Bernard. Additions were introduced in the academic year 2018/2019 in Foundations of Mathematics 2, in the first year of bachelor study program. Its content are Linear Algebra (matrices, vector spaces and linear applications), Calculus (Taylor polynomials, Integration) and Differential Equations (1st and 2nd order). The Active Learning adaptations resided in the introduction of interactive material, mainly Digital Geometry (Geogebra booklets and classes), and more formative assessment, mainly through an exerciser (WIMS) and during classes (using Wooclap and Kahoot).

Keywords: Mathematics courses; Active Learning methods; interactive digital material; ICT.

1 Introduction

The DrIVE-Math project is financed by the European Commission under the Erasmus+ framework [28, 29]. It aims at developing a novel and integrated framework to teach math classes in engineering courses at the university level. Active Learning has been promoted by accreditation boards of engineering curricula such as SEFI, ABET and ENAEE as a good practice to better teach math to the engineers [17, 20, 22, 30].

Previous projects, MetaMath and Math-GeAr TEMPUS projects conducted in Lyon involved international comparisons between German, Finnish, Russian, Georgian and Armenian partners. They identified venues for modernization of the teaching of mathematics for engineers [15], in particular based on the study of the perception of mathematics teaching by students [16]. They pointed out the direction of Active Learning methodologies, especially in their technological aspects,

therefore we engaged in the DrIVE-MATH project to deepen this modernization.

This article presents the adaptations that were conducted in Lyon. We begin with a broad description of engineering studies in Lyon, then of the content of the course used in this project. The real focus of this paper is the detailed description of the adaptation that were conducted and concludes with a discussion of the answer to the lock-down situation we faced in 2020.

It is difficult to define a typical engineering student in France [35]. Indeed, freshmen and sophomores, pregraduation students are trained in preparatory schools that fall within different categories, whether taught by secondary math teachers in secondary schools, university teachers in universities, or by engineering teachers in engineering schools [26]. We are going to focus on the situation in Université Claude Bernard Lyon 1, hosting the *Polytech* engineering school where the preparatory years are taught by university teachers, who are as well researchers. Christian Mercat has taught in first year for the last three years *Foundations of Mathematics 2* where Active Learning has been implemented. The content of this teaching unit is common to all students studying mathematics, whether math, computer science or economics students, as well as the quarter of *Polytech* engineering students. Whereas the actual content is the same, those students have been thoroughly selected and are much better and more motivated than the other three quarters because these courses prepare them for further selection and competitive exams that will impact their path of study in the network of engineering schools.

2 Foundations of Mathematics 2 (FDM2)

This course covers basic linear algebra and calculus up to differential equations.

Matrix operations are introduced, making connection with solving linear systems. Gauß algorithm is treated. Kernel, image, rank and trace are defined.

Vector spaces are defined on rational, real and complex numbers. Examples and special sub-spaces of common

*Corresponding author: Christian Mercat, Université Claude Bernard Lyon 1, Villeurbanne cedex, France, E-mail: christian.mercat@univ-lyon1.fr

vector spaces are discussed, like functions, sequences, polynomials... Free families, generating families, basis are presented in the finite dimensional case. Span of vectors, sums and direct sums, complements.

Linear maps, whether associated with a matrix or not, are defined, together with their image and kernel, characterising injectivity. The image of a free family, a generating family or a basis is discussed in relation with the rank theorem. Changes of basis give rise to the notion of equivalent matrices and similar matrices. Among endomorphisms, the examples of projections, symmetries and rotations are explored.

Properties of **real** and **complex** numbers, especially the sup property in bounded real sets are investigated, giving rise to **calculus** theorems, especially the Mean Value Theorem. Comparison of functions with o , O , \sim Bachman-Landau notations are introduced to give firmer grounds to calculus, higher derivation and continuity. *Complex derivatives* and integration is as well introduced.

Taylor formulae, Taylor-Young and TaylorLagrange allow to get powerful local equivalents of functions.

Integration with step functions and piecewise continuous functions is properly defined through Riemann sums, leading to the fundamental theorem of calculus that differentiation and integration are reciprocal. Change of variable and different integration strategies are discussed, especially for rational fractions because it forms a vector space whose basis are simple elements.

Differential equations are new equations where the unknown is a function. We mainly study the linear, first and second order equations, especially with constant coefficients.

The group of students, between 115 in 2020 and 123 in 2018, attend weekly 2 lectures and 2 tutoring sessions, for 12 weeks, totalling 54h of courses and 86h of exercises. Moreover, they have a 20 min oral exam every two weeks, by stable groups of three, and a written exam every 3 weeks.

3 Interactive mathematics

We believe in giving flesh to mathematical concepts, that swallowing a formula to spit it back at the test is not the way to build competencies, focusing on the *process* of active student's learning rather than on the *content* alone [4]. In this context we proposed four different ways to interact with mathematics: MathaLyon presents puzzles and real experiments, GeoGebra classes and books let students build mental images of concept by interacting with constructions illustrating them [19, 37, 38].

3.1 MathaLyon

Sums of first powers of integers are taught in this course. Students diligently note down that the volume of a tetrahedron of edge a is $V(T_a) = \frac{(\sqrt{2}a)^3}{24}$ that, for an integer n ,

$$\begin{aligned}\sum_{k=0}^n k &= \frac{n(n+1)}{2}, \\ \sum_{k=0}^n k^2 &= \frac{n(n+1)(2n+1)}{6}, \\ \sum_{k=0}^n k^3 &= \left(\frac{n(n+1)}{2}\right)^2,\end{aligned}$$

but what does it *really* mean or even matter to them? MathaLyon is a travelling exhibition that we propose to schools in the Lyon region. With an average of 2 days per month, four math researchers accompany this exhibit made of a set of different puzzles, all with interesting mathematical properties.

In this project, we had a small group of FDM2 students work on the exhibit, and take the position of mathematical guide to the puzzles demonstrated to their fellow students. Peer-instruction is important as an Active Learning strategy [27], promoting engagement and accountability in a meaningful learning [1]. The position of guide on the side instead of sage on the stage is a delicate one for students: in order to be effective, these puzzles have not to be spoiled, but rather fellow students have to be guided along their reasoning, that will lead them not only to the solution of that particular puzzle, but as well to learning from their failures, to persistence [7] and fundamentally to the more critical thinking and metacognition on the solution, that will unravel the theory, the *why* behind the *how* it works. Indeed, the triptych: *manipulate*, *verbalize*, *abstract*, is at the root of this exhibit. We devoted a full hour of course to this exhibit and we could informally ascertain some benefits, in particular a better mastery of these sums of powers of integers, and engagement in the collective phase [20, 21, 44], without an attempt to quantify them.

3.2 GeoGebra classes and books

The software GeoGebra has become the *de facto* Swiss army knife of the European mathematics teacher who wants to quickly illustrate a concept in an interactive way [2, 12, 42]. With sliders and free points, there are many ways to make mathematics come alive. A GeoGebra book

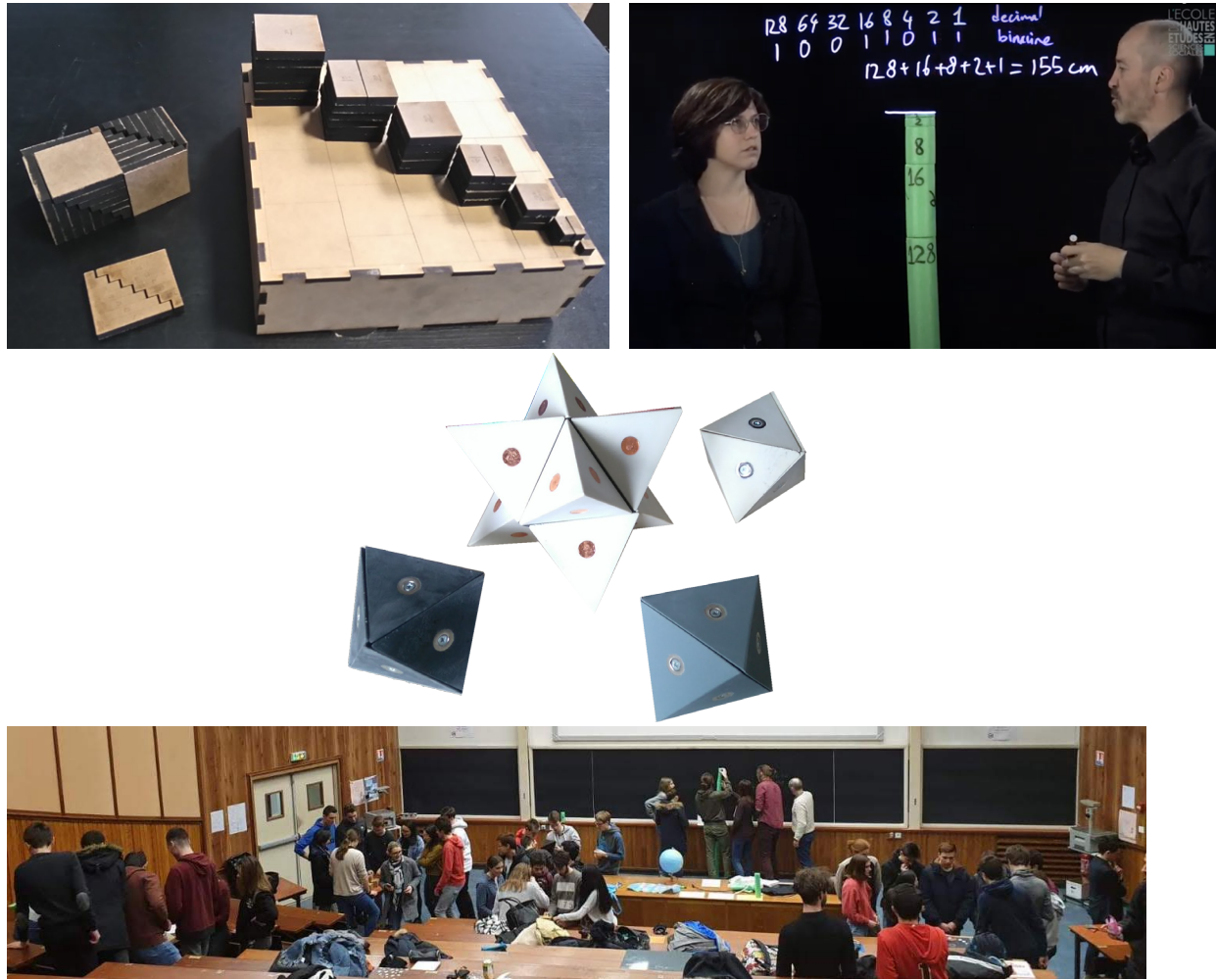


Figure 1: Puzzles demonstrating the formulae for the sums $\sum_{k=0}^n k^p$, binary code with pool noodles divination trick and pieces of a cubic puzzle giving the volume of a tetrahedron.

accompanies the course in order to demonstrate subtle points in an interactive way, like very classical Riemann sums, solutions of second order differential equations with their phase portrait and graphs as shown in Fig. 2, or continuity and comparison of functions in an interactive way like in Fig. 3. Everything can be interactively edited and adjusted: students are not seeing just one fixed example but can experiment, make hypothesis and put them to the trial, and tweak things to their liking. They are recognised as an actor of their own learning by interacting and modifying the learning material to their own point of view [9].

Many concepts, whether in algebra or in calculus are illustrated this way as exemplified in Fig. 4.

Covid pandemic in 2020, unfortunately, allowed us to exercise our newly gained knowledge of differential equations and the effectiveness of logarithmic functions to understand exponential growth and exponential decay.

Grounding the tools we shape in making sense of reality is paramount to epistemic understanding [18, 32]. Real data from the government was analysed live in course and the models described in the news were discussed and understood as a R3-vector valued non linear first order differential equation that we can integrate numerically in Fig. 5:

$$\begin{cases} S' &= -p \times S \times I, \\ I' &= +p \times S \times I - \alpha I, \\ R' &= \alpha I. \end{cases}$$

Whereas some parts of this book are used and commented *during* courses and made available outside the class, GeoGebra classes allows for synchronous interaction with students. An activity containing interactive material, is associated with questions. Students are supposed to

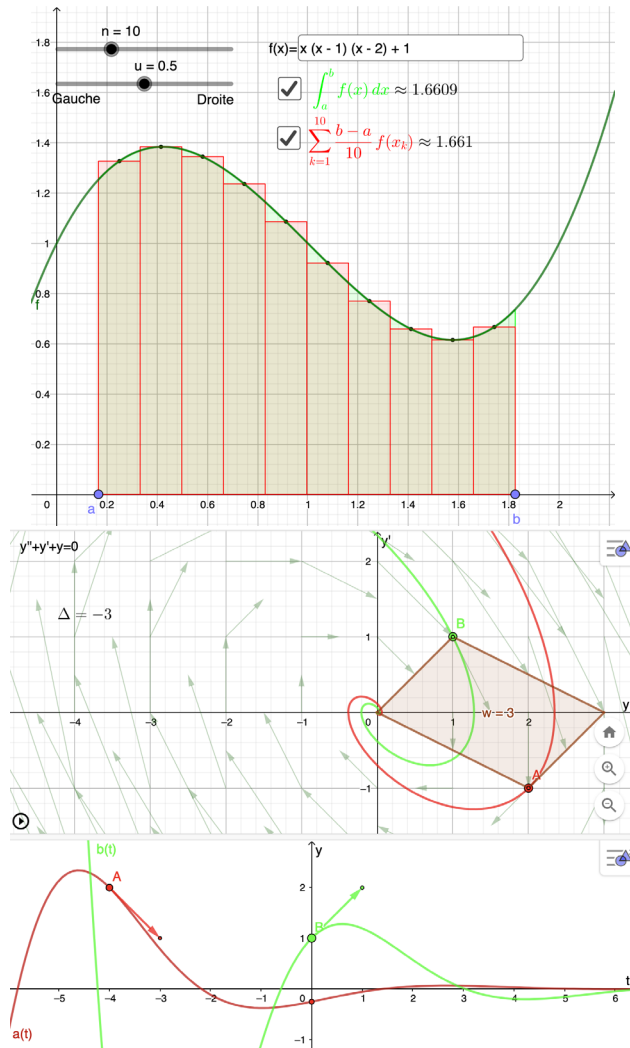


Figure 2: Classical representations of Riemann sums (top) and solutions of 2nd order differential equations (bottom) along with their phase space representation (middle).

use the interactive figure in order to gather information and answer questions [40]. The tutor can synchronously follow what the students are doing, give instant feedback, promote the work of some student (anonymised or not) by sharing it with other students.

Let's discuss an example. The phase portrait of a derivable function f consists of the set of points $\{(f(t), f'(t)) / t \in \mathbb{R}\}$ in the (y, y') plane. So if you manually sketch the graph of a function, one can sketch its phase portrait and likewise, given a possible phase portrait, one can integrate it into a function, but only if it follows some rules that are the subject of an interactive class illustrated in Fig. 6.

In this setup, students have to actively engage in using the interactive material in order to answer the questions: what is the phase portrait of an increasing exponential, of a decaying exponential, of a constant function, of a

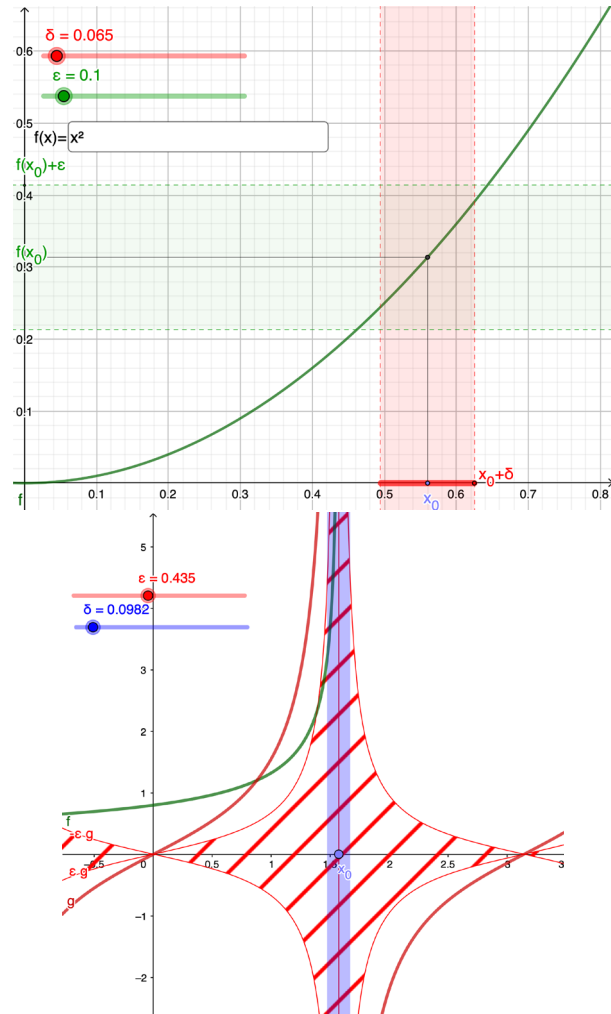


Figure 3: Continuity can be seen as a trading game: the first gamer sets ϵ , the student has to find δ then we check if the horizontal red part of the curve is safely in the green vertical part. Likewise negligibility $f = o(g)$ at the bottom.

linear function, of a periodic function... Playing with the tool, students tend to grasp better the meaning of the phase portrait than simply listening to definitions and examples. Here, they produce examples and have to sort them in different categories themselves.

4 Formative assessment

4.1 WIMS Exerciser

One of the Active Learning strategies that we have been implementing, is to use an online exerciser to let students have more worked out examples and see in a formative way the level of their technical skills. It is important

to understand the scope of such exercisers, because it is difficult to evaluate competencies associated with elaborate higher skills like complex reasoning or writing. Nonetheless such an exerciser provides a very useful complement

to more summative assessment performed by tutors [36]. It is especially the case for rote drilling for technical skills like mechanical derivation of functions, matrix product, application of Taylor-Young formula, computation of the solution of a differential equation and so on. These skills *have* to be exercised and it would be a very bad use of valuable human interaction time to perform these during tutoring sessions, after the first few examples [10]. Therefore we delegate this rote drill and exercise to the online platform.

We chose the *Web Interactive Multipurpose Server* WIMS for its stability (first release in 1997), its power, its open source GNU GPL licence, the large number of already existing free exercises for higher education math, and its easy integration in Single Sign In protocols, allowing for a seamless identification of students. It is fully loaded with powerful tools like matrix, vector, function, statistics calculators and plotters, as well as discrete math, arithmetic or algebraic toolkits for graph manipulation, prime numbers, factorisation and so on [13].

All examples are different, randomly generated, the equivalence check is based on a powerful computer algebra system that allows for very diverse entries. Scores make the exercise a bit more fun but it is a very shallow gamification [43].

But not all exercises are of this useful but boring type, some really are fun, with the right balance between rote fast skills and precise and meticulous application of techniques. The shooting exercises, for example, are of this type, a transformation is given, whether a complex map such as $z \mapsto -\frac{\bar{z}}{|z|}$, a linear transformation in the form of a matrix like

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}$$

or an explicit formula, and a point in the plane, to be understood as whether a complex number z or a vector. Then the student has to click, at the position of its image by the transformation. In order to do so, you have first to understand the concepts and second let go of an exact solution, but rather aim at a fast but reasonably accurate approximation, the way an engineer would first look at a possible solving strategy for a problem [34]. In the first year of implementation, the usage was not seen as very beneficial for students because it was not introduced

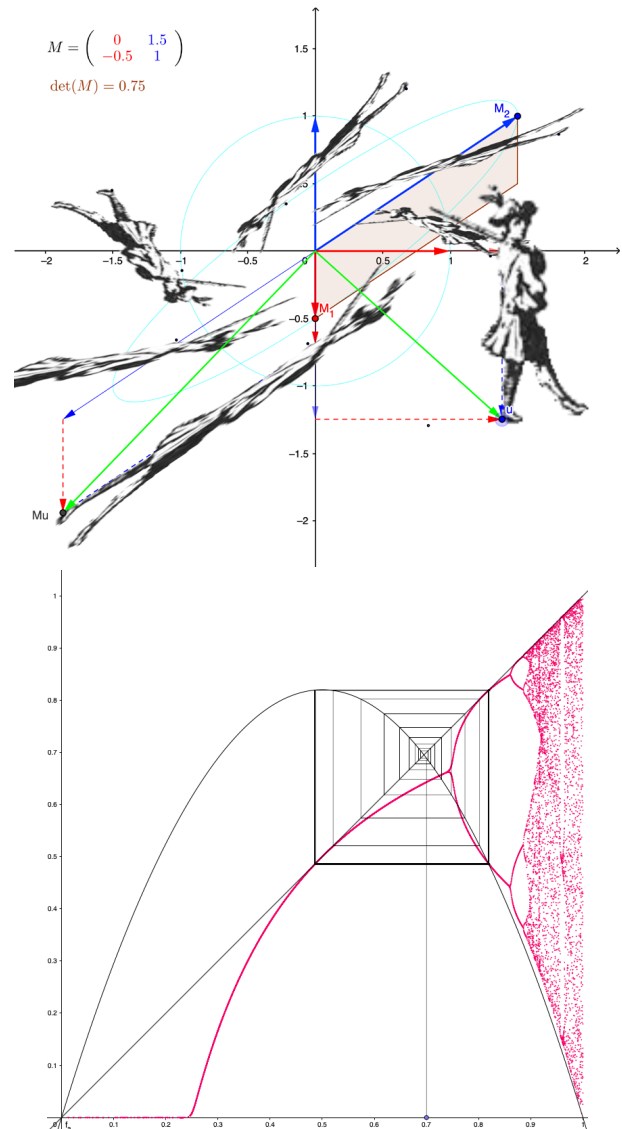


Figure 4: A 2x2 matrix is defined by its columns, image of the basis vectors acting on the whole plane (top), Feigenbaum bifurcation explores chaos as derivative at fixed point grows (bottom).

by tutors and explicitly related to the actual exercises at stake. Therefore students were not enticed to use it and used it in a counter productive way, trying to maximize their scores on already mastered techniques rather than exploring unknown perspectives. Now, tutors explicitly point out relevant exercises to students they find in need of a specific technical drill. In order to be really beneficial, this kind of exerciser has to be taken into account and articulated in the other teaching units [10]. Then, it unburdens tutors of rote exercises and helps secure students in their mastery of techniques without having to rely on tutor's individual answers to fixed questions. Group work is as well beneficial here: since exercises are

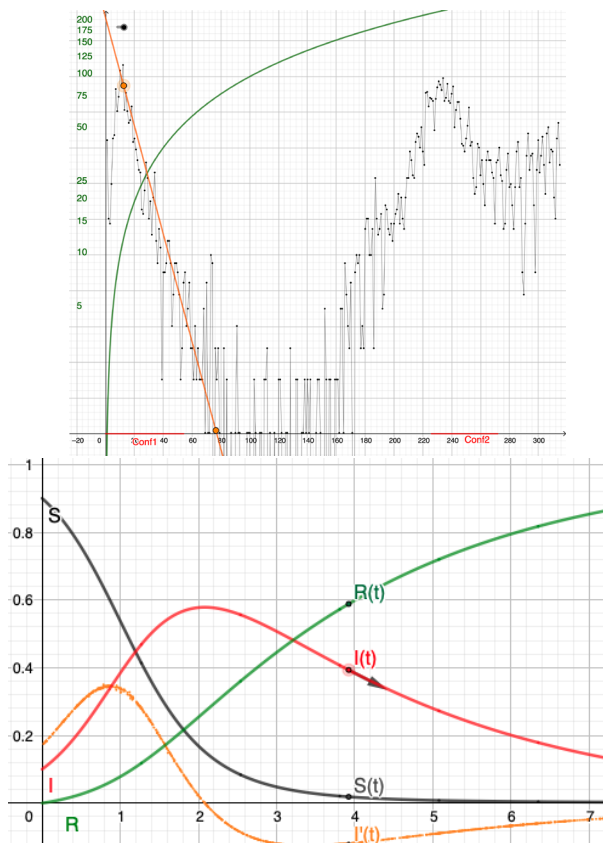


Figure 5: Analysis of real data: lock-down is effective, exponential decrease of admissions in hospitals in Lyon (top). Quadratic ODE: Susceptible-Infected-Recovered model (bottom).

randomly generated, in order to help a colleague stuck with a problem, the only way is to explain the theory behind the technique [27].

4.2 Rapid feedback

Together with the WIMS exerciser, we took time to regularly assess students during classes, with a feedback whether directly during the course using Kahoot and Wooclap, or with a quick multiple choice questionnaire, randomly generated by the Auto-Multiple-Choice software (AMC). It can produce PDF that students fill-in, whether on paper, or electronically when we had to fallback to distance learning in lock-down situations. The software optically reads the answers, assesses and sends back the feedback to students by email.

In class, whether in presence or in distance teaching, formative assessment with immediate feedback allowed direct evaluation, using Kahoot and Wooclap. It revealed to

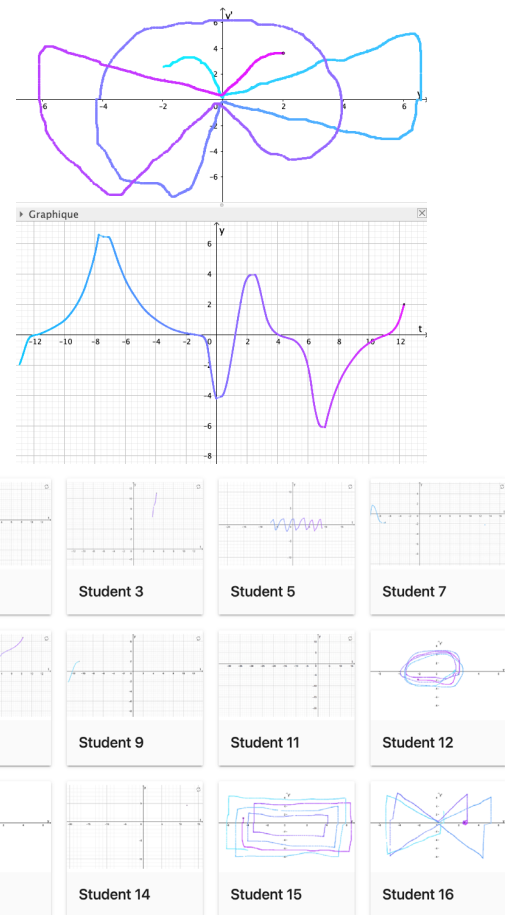


Figure 6: GeoGebra Class around the notion of phase portrait of a function. On top, the production of a particular student, bottom, an overview of the class.

be really helpful to know the state of comprehension of your audience [3]. There exist off-line solutions such as Plickers and QCMCam where students are assigned a QRCode that they have to hold in front of the professor that scans the audience with a smartphone in order to get students' answers. They are reliable for a smaller audience, but with more than a hundred students, with light conditions that are not optimal, it revealed too much of a challenge, and the lock-down situation in 2020 rendered this assessment actually unfeasible. Therefore we favored electronic ways for students to give their answers. This solution requires a computer or a hand-held device, therefore running the risk of diversion to social media and the like [33]. But we took the decision to trust the students and empower them with responsibility. This is a double-edged sword and was detrimental to weaker or lesser motivated students that dwindled in their attention. This appeared to be all the more so comparing Kahoot to Wooclap: the competition for fast answers, while in principle interesting

13/02/2021
Gauss visuel

Gauss visuel

Après addition de 577/101 fois la ligne (4) à la ligne (3), les matrices deviennent à l'étape 16 :

Matrice principale

$$\begin{pmatrix} 1 & -8/7 & 9/7 & -1/7 \\ 0 & 1 & -31/61 & 40/61 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Matrice de référence

$$\begin{pmatrix} -1/7 & 0 & 0 & 0 \\ 5/61 & -7/61 & 0 & 0 \\ -641/4227 & 209/4227 & 572/4227 & -577/4227 \\ -310/4227 & -44/4227 & 547/4227 & -101/4227 \end{pmatrix}$$

Proposez votre modification (Étape 17)

13/02/2021
Calcul Booléen

Calcul Booléen

Exercice.
Reconstituer la négation de l'énoncé

$$\exists y \in \mathbb{N}, \forall z \in \mathbb{Q}^*, \ln(y^2 + 1) < \ln(z^2 + 1) \Rightarrow y \neq z$$

Construisez votre énoncé à partir des réponses suivantes:

☐ $\exists y, z \in \mathbb{N}$
☐ $\forall y, z \in \mathbb{N}$
☒ $\forall y \in \mathbb{N}, \exists z \in \mathbb{Q}^*$
☐ $\exists y \in \mathbb{N}, \forall z \in \mathbb{Q}^*$

☒ $\ln(y^2 + 1) < \ln(z^2 + 1)$
☐ $\ln(y^2 + 1) \geq \ln(z^2 + 1)$
☐ $y \neq z$
☐ $y = z$

☐ $\ln(y^2 + 1) < \ln(z^2 + 1)$
☒ \Rightarrow
☐ $\ln(y^2 + 1) \geq \ln(z^2 + 1)$
☐ $y \neq z$
☒ $y = z$

☐ Je n'ai aucune idée

Envoyer Nouvelle séance.

(Vous devez arriver jusqu'à la fin d'une séance pour avoir une note.)

13/02/2021
ORF Développement limité et Taylor

Tableau 3

Soit f une fonction réelle, dérivable d'ordre 3 sur \mathbb{R} , avec le tableau de dérivées suivant :

x_0	$f(x_0)$	$f'(x_0)$	$f''(x_0)$	$f^{(3)}(x_0)$
-9	-26	-10	-58	156
0	-26	1	14	-24
9	25	25	26	24

Quelle est la partie principale du développement limité de f d'ordre 3 au voisinage de -9, c'est-à-dire le polynôme $P(x)$ dans le développement limité

$$f(x) = P(x) + o((x+9)^3) ?$$

Analyse de votre réponse

$P(x) = -26 - 10(x+9) - 29(x+9)^2 + 156(x+9)^3/6$ bonne réponse

Vous avez obtenu une note de 10 sur 10. Félicitations

Figure 7: A multi-step Gauß algorithm exercise, the construction of the negation of a predicate and a Taylor polynomial exercise.

13/02/2021
Tir aux vecteurs

Tir aux vecteurs

Le dessin ci-dessous contient 3 vecteurs dans le plan, v_1, v_2, v_3 . Soit u une combinaison linéaire d

$$u = \frac{1}{2}(-v_1 + v_2 - v_3)$$

Localisez le vecteur u .

Voici votre réponse, ainsi que les positions réelles des vecteurs.

Résumé des tirs de la séance.

Continuer avec une nouvelle séance

13/02/2021
Tir complexe

Tir complexe

Le dessin ci-dessous représente le plan des nombres complexes et un nombre complexe $z = x + iy$. Veuillez trouver la position du nombre $w = -\frac{z}{|z|}$.

Voici votre réponse, ainsi que les positions réelles des points.

Tir linéaire

13/02/2021
Tir linéaire

Tir linéaire

Le dessin ci-dessous représente le plan cartésien, avec un point $p = \begin{pmatrix} x \\ y \end{pmatrix}$. Veuillez trouver la position du point

$$q = Mp, \text{ où } M \text{ est une transformation linéaire définie par la matrice } \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}.$$

Voici votre réponse, ainsi que les positions réelles des points.

Ecart de la réponse (numéro 2 sur 4): 1.3 mm.

Écart moyen de la séance: 0.9 mm.

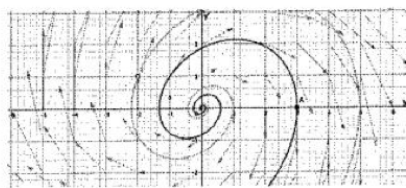
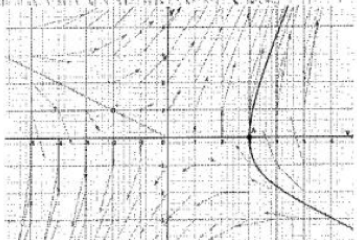
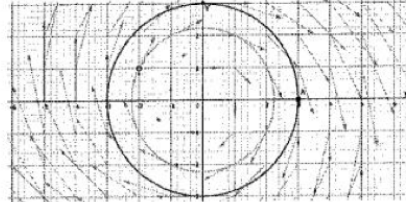
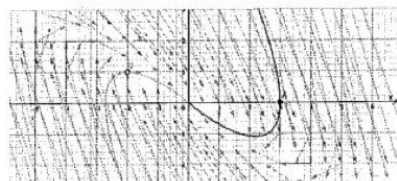
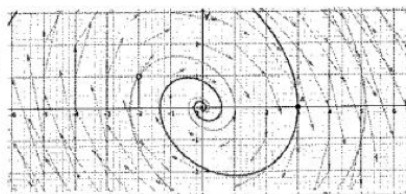
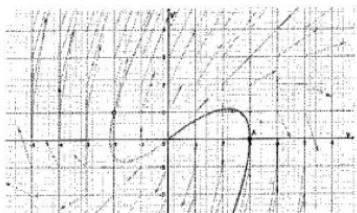
Continuer avec un nouveau tir

Figure 8: Some WIMS shooting exercises, linear combination of vectors, complex or linear transformation.

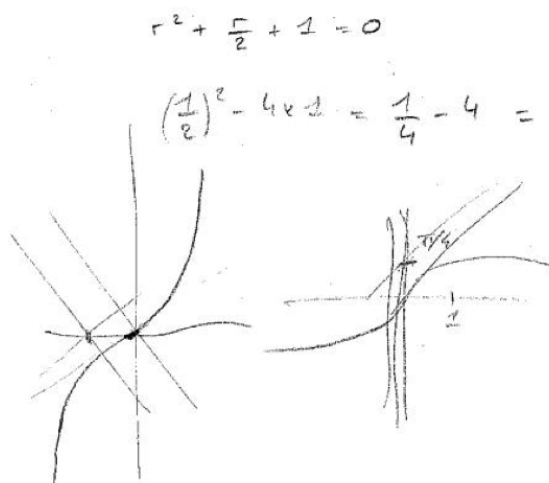


+110/3/20+

Question 4 Trouver parmi les portraits de phase la solution à l'équation différentielle suivante : $y'' + \frac{1}{2}y' + y = 0$.



2/2



$$r^2 + \frac{r}{2} + 1 = 0$$

$$\left(\frac{1}{2}\right)^2 - 4 \times 1 = \frac{1}{4} - 4 = -\frac{15}{4} \approx -4$$

$$\arctan(1) = \frac{\pi}{4}$$

$$\frac{3}{4}$$

$$r > 0$$

$$1 + r >$$

$$1 - r <$$

$$r > \leq$$

$$1 + \tan^2$$

$$\frac{1}{1 + \tan^2}$$

$$2 \left(1 - \frac{r^2}{2} + \frac{r^4}{4} - \frac{r^6}{6} \right)$$

Figure 9. Automatically optically read MCQ on paper.

as a gamification feature, actually lead students to choose quick random pick rather than thoughtful decisions. The fact that this strategy doesn't last for long doesn't seem to matter. Having some easy bait that can be dismissed at a glance when thinking about it is a good clue to know which proportion of students is actually fooling around and not paying attention. Of course, a thorough discussion of why proposed items are distractors, impossible, plausible but false and the right answer, constitute the heart of the course. When taken seriously by students, it keeps them in a state of awareness and working out examples straightaway helps them stay tuned. But in order to do so, difficulty and time left to answer have to be adjusted: too easy or too long drive students astray, unfocused and day dreaming, too difficult or too short is frustrating and promotes random answers [5, 39].

Therefore when considering this type of formative assessment, benefits and risks have to be weighted: electronic is ok with motivated and mature students. When comparing solutions, regarding optical QR-Code answering, QCMCam was preferred to Plickers, and some other minor technical aspects were as well in favor of Wooclap compared to Kahoot: chiefly spreadsheet import and export of questions, LATEXintegration when painful production of images has to be used in Kahoot for real mathematics rendering, easy navigation in questions and answers when Kahoot hides the question and only shows the answer, answering with an SMS rather than an app and internet connection.

5 Embodied mathematics

Embodied mathematics is a trend that is here to stay: people think with their bodies [8, 31]. When moving around, abstract schemes might be incorporated into kinesthetic ones. Moreover, considering one's body as a tool to apprehend the world can give sense to mathematical situations. That's the hypothesis behind some activities that we conducted in Lyon: Mathematical webcams and Mathematical trails.

5.1 Mathematical webcams

Mathematical webcams let students move their bodies in front of a computer in order to interact with mathematical content. There are several levels of interactivity: first the math, functions and parameters can be changed with the *keyboard* or at the click of the mouse, then *movements* are interpreted by the machine and change parameters of

the activity, and third, there is the *image* of the students themselves: it concerns them because it's their image that is used [24, 41].

Pedro Lealdino Filho's work on the *Function Hero* activity aims at training students to recognize quickly graphs of functions. Presented with a series of functions coming up his way, the student has to move his/her arms and body in order to dance like the function and mimic its graph in a fast and accurate way. Students in teams have first to elaborate a series of functions, that we call a *choreography*, then compete against another team, exchanging choreographies and trying to maximize scores ratio.

This activity was performed as well with students of the INSA engineering school in Lyon as part of Pedro Lealdino Filho's PhD on creativity in mathematics [14]. Students tend to be faster at guessing the graph of a function when used to playing with the activity but it was not the focus of the analysis.

Geometrical sequences are central in calculus: many proofs of convergence of sequences or series are done by comparison with a geometric sequence. And making these sequences *really* geometric is important for building a mental image of the process. That's the topic of a series of webcams and especially the *human tree* where two geometric sequences are intertwined in an *Iterated Function System*. There is a left one, sending the trunk to the left arm, and a right one, sending the trunk to the right arm. These two transformations are then iterated and the proportions of the body control the *reason* of each geometric sequence, tending to create oak or fern designs. These webcams are programmed using the software Cinderella [24, 25, 41].

A still more powerful mathematical webcam that we teach is the complex map webcam to simply let students understand that a derivative is a rate of change. We first manipulate simple linear and similarity transformations $z \mapsto A \times z + B$ where A and B are points that can be interactively modified. This allows to understand how numbers of the plane *operate* on one another through the operations of addition (graphical translation) and multiplication (enlargement, reduction and rotation). Then higher degree polynomials and especially Taylor polynomials are demonstrated, especially their difference with the actual function they are approaching (see Fig. 12).

5.2 Mathematical trails on the campus with MathCityMap

The last activity we want to share from our Active Learning experimentation in Lyon, involving embodied

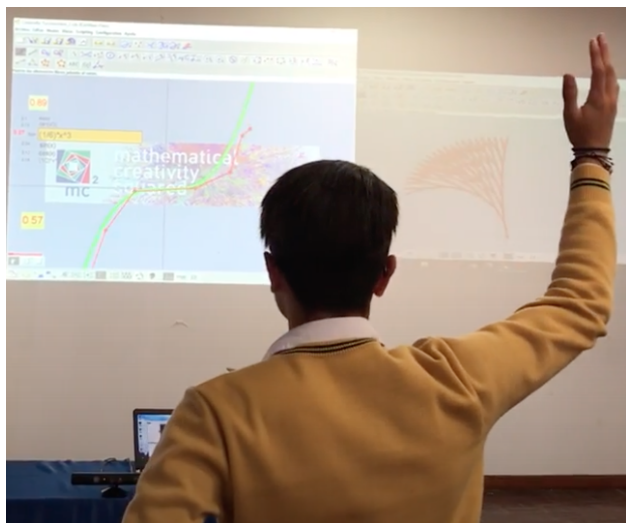


Figure 10: Allow your camera with proper lighting in front of a plain background and... *dance like a function!*.



Figure 11: Two intertwined geometric sequences give rise to an Iterated Function System.

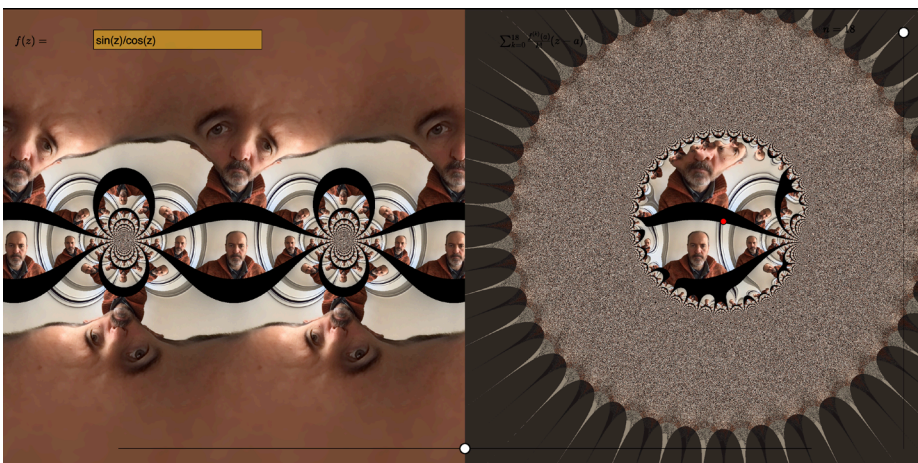


Figure 12: The analytic continuation of the tangent function to the plane and its Taylor polynomial of degree 18 at a certain (red) point. Notice the zeros accumulating on the boundary of the disk of convergence.

mathematics, is the running and construction of *mathematical trails* on the campus [6]. In this optional course (so called *transverse* course), not formally part of Foundations of Math 2, we train sixty students per year for a total of 15h. In the direction of the *realistic mathematics* movement, we try to let students open a scientific eye upon the world around them, to envision the mathematical tools that we teach them as tools to solve problems. In ordinary courses, most problems are exercises to illustrate an abstract notion and have no connection whatsoever with concrete reality. At the other end of the spectrum, real world problems that an engineer solves everyday are usually out of reach technically and contextually. To fill the gap, we setup mathematical trails on the campus that pose, if not *realistic* problems, at least *contextualised* problems. To solve these problems one needs to be on site in order to understand the problematic, gain some information from the environment, like measuring the size of an object or interpreting some signs, modelling the situation and giving an answer to the question at stake. The trails are implemented inside the *MathCityMap* mobile application, developed in the realm of the MoMaTrE and MaSCE3 Erasmus+ European projects [11].

Students are first taught how to calibrate themselves in order to be able to gauge quantities in the field. Time is clearly no longer based on the pulse rate, we have stopwatch in our smartphones, but it is essential for future engineers to understand that a measure is not a number but an interval, or better a distribution, that you can accurately estimate lengths, angles volumes, simply using your body and simple scaling rules [23]. Moreover, modelling a situation can be done in many different ways and it leads to precise or off the charts estimations.

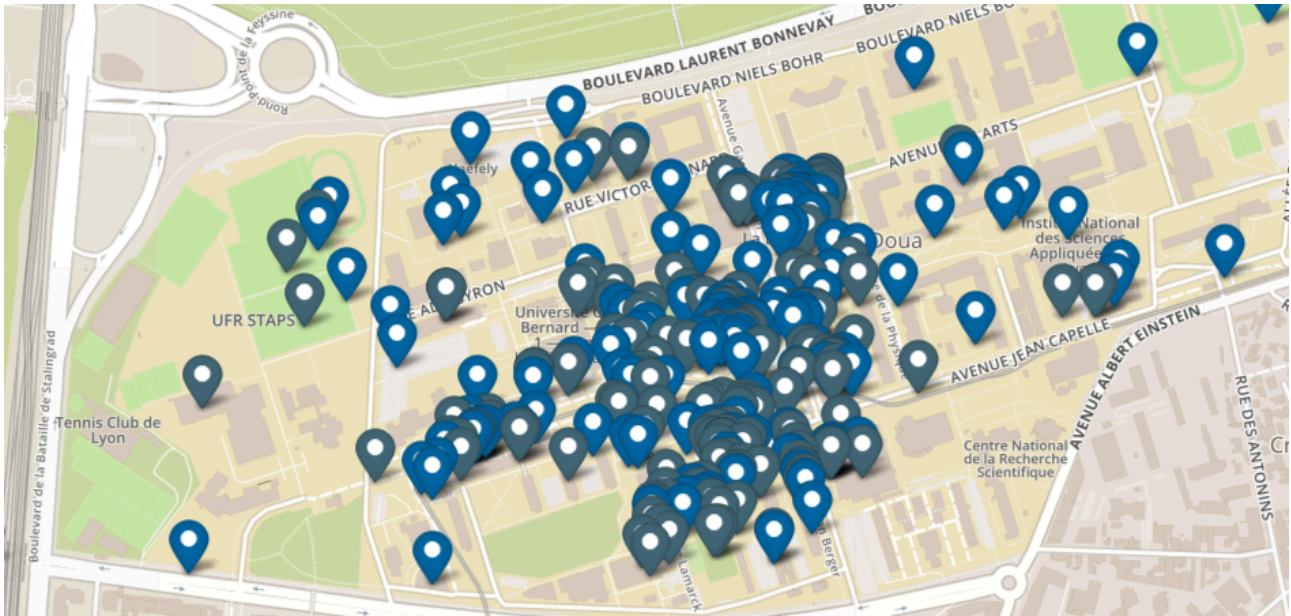
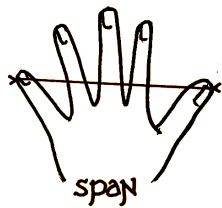
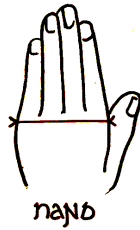


Figure 13: Mathematics tasks and trails are constructed by students, based on objects from their environment on the campus.

Anthropometric ID card of



Index = \pm cm \pm $^{\circ}$

Thumb = \pm cm \pm $^{\circ}$

Hand = \pm cm \pm $^{\circ}$

Span = \pm cm \pm $^{\circ}$

Foot = \pm cm

Step = \pm cm

Gauge = \pm cm

Height = \pm cm

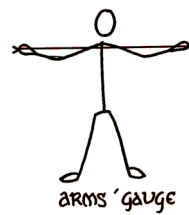


Figure 14: Envision your body as a tool to grasp reality, calibrate, then go into the field.

Then more sophisticated theories can model situations using trigonometry, calculus and statistics. Students run a prescribed trail that we setup for them, gaining points [11] and modelling the world around them, then create their own trail.

6 Distance teaching

On March 16, 2020, the President of the French Republic announced firm measures to curb the spread of the Covid19 which was then raging in France and throughout the world, especially in European partners' countries. These measures included, for example, the closure of



Figure 15: Mathematics trails are run on smartphones with a geolocalized app.

the entire educational system from nursery to university. France was then in a state of total containment.

Therefore, a question that nagged higher education teachers was how to continue teaching activities; remote assessment and courses were scarcely used, lessons were not all in electronic format; which platform to use for videoconferencing, online meeting, chat etc. Here are the testimonies of two tutors, Léon Tine and Louis Dupaigne, faced with ensuring pedagogical continuity, especially in an Active Learning context.

At the time of lock-down, the teaching team quickly made the decision to continue the lessons and tutorials remotely, by videoconference, without any interruption compared to the initial schedule. For us, it was a question of losing as few students as possible and of keeping a strong link with them. The four tutorial groups met by videoconference, twice a week as usual, according to the modalities urgently chosen by each tutor.

The months that followed were very intense and busy in terms of course preparation, electronic format and design of electronic assessments. And all this without counting the many educational, administrative and childcare tasks, knowing that the nurseries were closed.

6.1 Individualized Assessment

For each teaching unit, Léon used the cisco Webex platform made available by our university, to plan the lectures and tutorials. We also used the university LMS, Claroline Connect platform, to deposit the course files and the corrections of exercises after video sessions in a shared space to allow students to come back to certain points at any time.

Regarding the assessments, the task was more difficult, between how to set up a remote evaluation and how to avoid cheating [1]. A measure of group work is good and students [27] organized straightaway informal gathering of the sort that was convenient to them, using Discord, WhatsApp, Signal or other social media. Once it became apparent that uniform assessment was of no significance, when students developed great competencies in sharing efficiently answers, but avoiding learning in mathematics, the choice to do individualized evaluations appeared to be more reasonable. So we invested some work in the AMC (Automultiple Choice) software to carry out the preparation of individual subjects composed of Multiple Choice Questions part, marked with

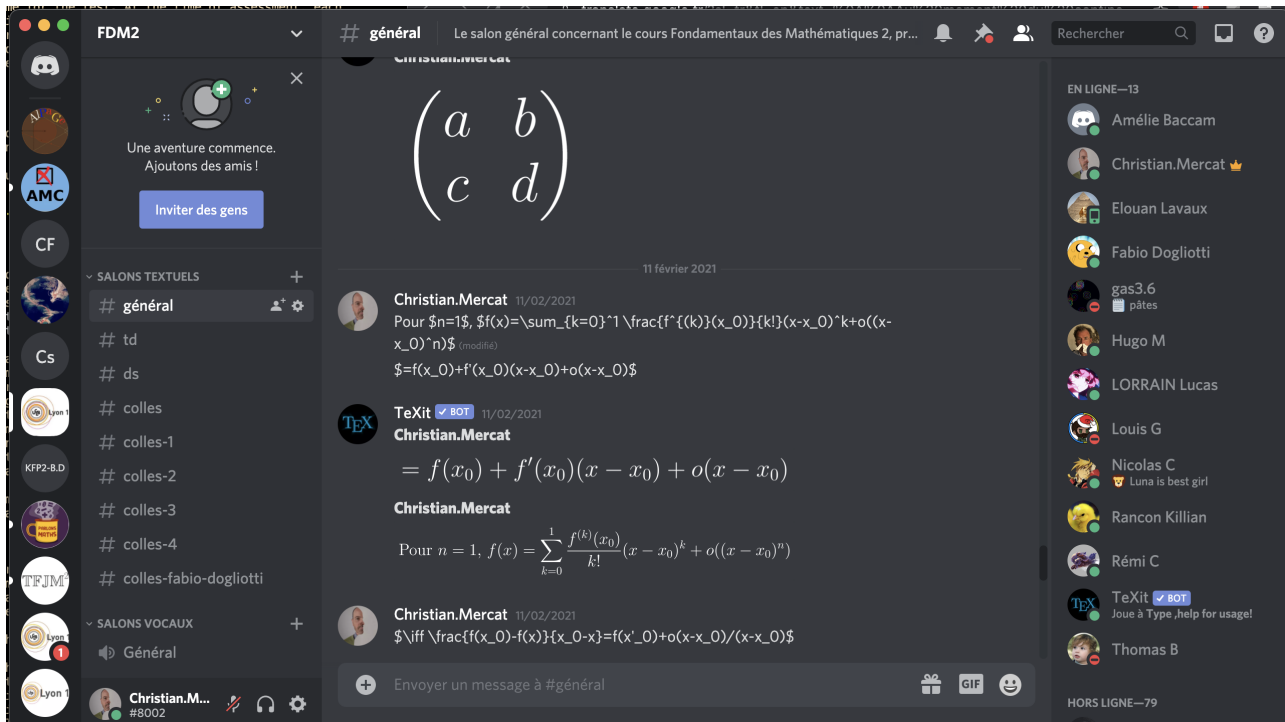


Figure 16: Discord server with the TeXit bot LATEXtypesetting.

half the points, and a written exercise part for the other half. Assessments were uploaded, as editable PDF, to the Lyon 1 University grade management platform Tomuss and hidden from students until the scheduled time for the test. At the time of assessment, each student downloaded their individual test, and uploaded a scanned version of their answers before the end of the allotted time. The MCQ part was corrected automatically by AMC and the other part manually. It worked pretty well despite the enormous amount of time it took.

The confinement period allowed us to acquire a strong experience in distance education, in particular in terms of distance assessment with or without multiple choice questions, making it possible to limit cheating between students as much as possible. Léon mainly used the AMC software which not only allowed to generate individualized subjects but also to automate part of the correction which is quite practical for teaching more than a hundred students.

6.2 Remote Tutoring

For teleconference tutoring, students were asked to divide into groups of three, each group bearing the name of a famous mathematician of their choice, they chose for

example Gauß, Euler, Weierstraß, Hypatie, Agnesi, Villani, Ramanujan or Poincaré. Within each group, students had to take one of the following three roles: *writer*, *speaker*, *contradictor* and the roles rotated at each tutorial session. Each group's mission was to prepare an exercise together: the *writer* types down a correction for the exercise and the group agrees on this writing.

During the tutorial, the *speaker* presents and explains the exercise to the whole class, taking the virtual floor, using the written document prepared by the group as a visual aid. Finally, the *contradictor* of each group has the task of asking at least one question in tutorial to the speaker of each group making their presentation on a given day. Students organized themselves as they wished outside of the tutoring sessions, for example as a WhatsApp group or Discord channel to which we did not have access to. Discord is actually the preferred channel for mathematics since, with the help of a LATEXbot, proper mathematics typesetting can be used.

After some trial and error, we decided (by vote) that the tutorials would take place in a fairly precise format, in two separate halves: At the start of the tutorial, a group is designated to present their work to the class. The contradictors, as well as the tutor, ask questions during the presentation.

The speakers have to be paced down a lot, in order to check after each line of calculation, each element of the demonstration, whether their comrades are understanding. Some questions were asked to certain students, at random, by name, to ensure engagement and check even simply the state of connection to the session. Students responded massively and primarily through chat, which is an extremely useful tool, as well as through other media such as social media, SMS, WhatsApp group, used simultaneously by students to help each other live, especially in the event of a loss of connection, without interrupting the whole tutorial each time. Some also took the microphone for the less shy of them. When an exercise was completely solved, we moved on to the next group. Usually, this occupied us for 1h30 and allowed us to pass 2 groups per session.

During the second half of the tutorial, the floor is taken again by the tutor, asking a particular student at random to establish a dialogue in order to solve a small piece of exercise together. This time, the tutor is usually using his webcam to view his hand writing on a sheet of paper or digitally scribbling with a tablet on the electronic version of the exercise or an electronic whiteboard. Depending on the quality of the equipment and internet connection, this did vary a lot, from poor man's textual chat to full duplex communication, through a single snapshot of written material every minute.

This blind technique had actually interesting effects because it forced each student to take their own notes, do calculations simultaneously on their own and use the instructional writing only as a control.

Usually, the tutor's notes were uploaded to the LMS after each tutorial session, which appeared essential to compensate for the connection problems experienced by everyone in different ways.

This period of lock-down, that we hope will not come back often, has every student, teacher and researcher jump start in remote and electronic teaching and learning. Everybody agrees that presence teaching is more efficient but nevertheless we all acquired competencies that we should not forget.

7 Conclusion

The implementation of Active Learning strategies in Lyon, in the realm of the DrIVE-MATH project, has been a very rich experience. It has faced some challenges, first of all the reluctance of the system to accommodate changes, in a competitive setup of preparatory schools that has a certain

format that can not be easily adapted. This institutional aspect had as well a scientific side, and while pedagogical effects have been fulfilling for teachers and, hopefully, for students, its impact on their actual learning has not been properly assessed. Intimate conviction is not enough incentive for *others* to sustainably change. Although our choices of adaptations were based on a scientific literature review, the scale of the project was under-estimated and we should have implemented a smaller but a more realistic project, because we have clearly missed the opportunity to not only implement but as well fully evaluate the effect of these changes.

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