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Influence of the compensation method of machining errors of Bspline and Cspline

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ABSTRACT

Contemporary advances in manufacturing are moving towards the diversification of interpolation methods in Computer Aided Manufacturing (CAM) softwares thanks to the growing need of free-forms of high quality. New interpolations such as 'Bspline' and 'Cspline' provide accurate toolpaths, but they generate significant machining errors. The nodes insertion technique is one of compensation methods of those errors. This paper presents a study of the influence of this compensation method on the manufacturing process: dynamic behavior of machine-tools, errors of controller, and surface quality. To make this study, Bspline and Cspline interpolations, errors generated, and the compensation method are defined. In addition to that, feed rate, cycle time and roughness before and after compensation are measured in order to examine the impact of the nodes insertion method. The present work aims to deliver a framework that demonstrates how manufacturing companies with such specific parameters optimization can improve smartness and machining efficiency.

KEYWORDS

Bspline and Cspline interpolations; machining errors; nodes insertion; influence of compensation; dynamic behavior; surface quality

1. Introduction

Computer Aided Design and Manufacturing (CAD/CAM) is an essential step in obtaining a part machined by numerically controlled machine-tools. CAD/CAM software calculates tool paths from geometric information, interpolation engineering information, user instructions, or information from specific software (P G and G C 2003). Then, the Numerical Command (NC) file representing the tool paths is transferred to the Numerical Control Unit (NCU) of the machine which follows the instructions imposed by this file. Fidelity to the theoretical model is the main constraint to respect when using CAD/CAM software. The passage between nodes of the tool path causes contour deviations especially in certain critical areas difficult to reach by the tool (sharp angle, connection). The CAM systems provide several interpolation methods in order to obtain a toolpath that respects the fidelity criterion during the machining of complex shapes (Helleno and Schutzer 2006). Thus, the limitations caused by linear and circular interpolations, especially in High Speed Machining

(HSM) of free forms, led to search other ways of path programming such as polynomial interpolations. Our study will give attention to two types of spline interpolation: Bspline and Cspline.

The Bspline (Bezier or Basic spline) can be defined by a polynomial of third degree or more. Programmed positions are not the points of the curve, but only 'control points' of the spline. The curve does not pass directly through these points, but that it 'tends' to them. A 'control polygon' of the spline is created, it links the control points. According to Siemens, the Bspline is ideal for describing tool paths in freeform surfaces machining. The Cspline (Spline Cubic) is an interpolation by a cubic polynomial. It passes exactly through the points of the curve. It has a low curvature variation. However, the Cspline has a strong tendency to oscillations between these points. The modification of the Cspline has an overall effect. The modification of one point influences many blocks.

Recent studies have focused on the spline interpolation and the compensation of machining errors in CNC machine-tools. Lacalle et al. (De Lacalle et al. 2002) have investigated a method of generating

error-free programs for the high-speed milling of molds. Hsu and Wang (Hsu and Wang 2007) and Lei and Hsu (Lei and Hsu 2003) have corrected geometric errors in real time or not by compensation algorithms for 5-axis CNC machine tools. Souza et al. (Souza and Coelho 2007) and Helleno et al. (Helleno and Schutzer 2006) have shown that the type of interpolation can influence the dynamic behavior of the machine. Zuo et al. (Zuo et al. 2013) and Zhu et al. (Zhu et al. 2012) have developed methods of compensation of geometrical errors of machining system NC by the correction of the NC codes. Liang et al. (Liang, Zhao, and Xi 2013) have proposed an interpolator to improve the machining of surfaces modeled by polynomial curves of NURBS type. Msaddek et al. (Msaddek et al. 2014) have tested the influence of different types of interpolation on the machining process in HSM. Zhong et al. (Zhong et al. 2015) have developed a model of identification and compensation of geometric errors of position (5-axis machine) based on servo loops and recursive correction. Poniatowska (Poniatowska 2015) has proposed a method for compensating for systematic errors of free-form surfaces. Lai et al. (Lai et al. 2017) have investigated a novel method for error compensation of free-form surface with critical area using the local refinement of the T-spline surface. Calleja et al. (Calleja et al. 2018, 2014) have proposed a new method for accuracy in 5-axis flank computer numerically controlled (CNC) machining. Ben Makhlouf et al. (Ben Makhlouf et al. 2019) have developed an approach based on the B-Spline formulation for rebuilding the CAD model entities (faces, edges and vertices) from those extracted from the deformed mesh (triangulations, arcs, nodes). The authors (Msaddek et al. 2018, 2017) have studied Bspline and Cspline interpolations, machining errors generated and a compensation method of errors in previous papers without a critical study of the influence of this approach on the manufacturing process.

Despite the smoothing of the tool path, Bspline and Cspline interpolations have presented critical errors penalizing the desired precision. A method based on the insertion of nodes has been developed in order to compensate machining errors. This method has effectively reduced these errors despite some shortcomings. So, it remains to investigate in this paper, after presenting modeling and

simulations, the impact of the compensation method on the dynamics of the machine, on the cycle time, on the NCU errors (following error and contour deviation), and on the surfaces quality of the piece.

2. Bspline and Cspline interpolations and compensation methodology

2.1 Bspline interpolation

According to Šulejic (Šulejic 2011), a n^{th} degree Bspline curve (of order $n + 1$) is defined by:

$$C(t) = \sum_{i=0}^k N_{i,n}(t) P_i \quad (1)$$

The P_i are $k + 1$ control points.

It is considered the vector known as the knot vector be defined $T = (t_0, \dots, t_m)$ where T is a non-decreasing sequence of real numbers with $t_i \leq t_{i+1}$ and $i = 0, \dots, m - 1$. The t_i are called nodes. The i^{th} Bspline basis function of n^{th} degree is defined by Cox-de Boor recursion formula:

$$N_{i,0}(t) = \begin{cases} 1 & \text{if } t_i \leq t < t_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

$$N_{i,n}(t) = \frac{t - t_i}{t_{i+n} - t_i} \cdot N_{i,n-1}(t) + \frac{t_{i+n+1} - t}{t_{i+n+1} - t_{i+1}} \cdot N_{i+1,n-1}(t) \quad (3)$$

The De Boor algorithm provides a method for evaluating a Bspline curve (Šulejic 2011; Sederberg 2005).

The compensation approach of the generated errors using Bspline interpolation adopted inspires from the method of Zhao et al. (Zhao, Zhu, and Ding 2013). Our method allows us to insert two control points around the initial control point of the tool trajectory generated in CAM while respecting the desired tolerance (Figure 1).

$B_0(u)$ is symmetrical with respect to the angular bisector of the two lines $\overline{p_0 p_1}$ and $\overline{p_1 p_2}$. Thus, the maximum deviation between the Bspline and the two lines is the distance between the spline center and P_0 . The approximation of the error can be expressed as:

The estimated error E_r can be expressed as:

$$E_r = \varepsilon = |P_1 Q_2^3| = (d_2 \sin \beta) / 2 \quad (4)$$

With d_2 : distance between P_1 and Q_0^1

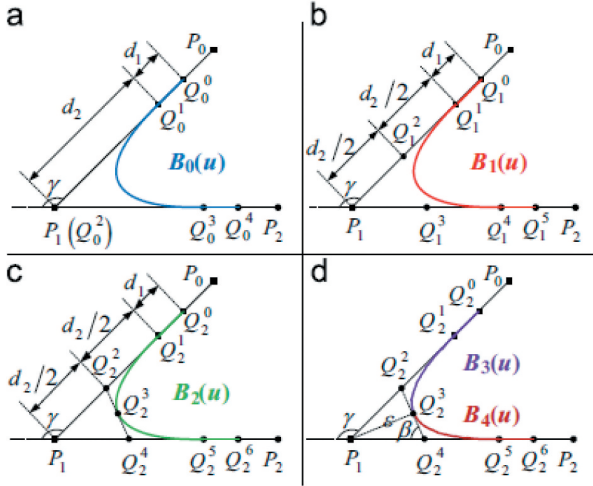


Figure 1. Method steps (Zhao, Zhu, and Ding 2013).

β : angle between $\overline{p_1 p_2}$ $\overline{Q_2^2 Q_2^4}$

To confirm that the estimated error remains within the specified tolerance, the length d_2 shall be calculated as follows:

$$d_2(i) \leq 2Er / \sin \beta(i), \quad i = 0, 1, -2, \quad (5)$$

So the coordinates (X,Y) of the added nodes must be calculated for different cases of $\theta(i)$ and $\theta(i+1)$, in such a way that $d_2(i)$ respects this condition.

The algorithm used is the following:

$$\text{if } \theta(i) < 0 \quad \& \quad \theta(i+1) < 0$$

$$Q_1(i, 1) = B(i, 1) + \cos \theta(i) \times \frac{d_2(i)}{2} \quad (6)$$

$$Q_1(i, 2) = B(i, 2) - |\sin \theta(i)| \times \frac{d_2(i)}{2} \quad (7)$$

$$Q_2(i, 1) = B(i, 1) - \cos \theta(i+1) \times \frac{d_2(i)}{2} \quad (8)$$

$$Q_2(i, 2) = B(i, 2) + |\sin \theta(i+1)| \times \frac{d_2(i)}{2} \quad (9)$$

$$\text{elseif } \theta(i) > 0 \quad \& \quad \theta(i+1) > 0$$

⋮

With $Q_1(i, 1)$: Matrix of the coordinates X of the first inserted nodes.

$Q_1(i, 2)$: Matrix of the coordinates Y of the first inserted nodes.

$Q_2(i, 1)$: Matrix of the coordinates X of the second inserted nodes.

$Q_2(i, 2)$: Matrix of the coordinates Y of the second inserted nodes.

$B(i, j)$: Matrix of the reference nodes.

$\theta(i)$: Angle between tow linear blocks, $\theta(i) = 180 - \gamma(i)$. (10)

2.2 Cspline interpolation

According to Rabut (Rabut 2010), for a given x , it is first necessary to determine the interval $[x_j \dots x_{j+1}]$ in which x is located, then the cubic polynomial is calculated at which the spline is equal into this interval:

$$p(x) = p(x_j) + (x - x_j) p'(x_j) + \frac{(x - x_j)^2}{2} p''(x_j) + \frac{(x - x_j)^3}{6} p'''(x_j) \quad (11)$$

$p'(x_j)$, $p''(x_j)$ and $p'''(x_j)$ are respectively the first derivative, the second derivative and the third derivative of the polynomial $p(x)$ in x_j .

x and x_j are respectively the abscissa of the spline point and the abscissa of the reference point.

The compensation approach of the generated errors using Cspline interpolation adopted inspires from the method of Liang et al. (Liang, Zhao, and Xi 2013). The authors have proposed to add nodes forming segments of constant lengths in order to keep the trajectory in Cspline interpolation in the desired tolerance ϵ .

Figure 2 shows the original and the compensated Cspline interpolation on one block of trajectory having two nodes P_i and P_{i+1} .

In order to calculate the distance between the added nodes, a block of two reference points P_i and P_{i+1} of coordinates respectively $x(i)$, $y(i)$ and $x(i+1)$, $y(i+1)$ is taken. The points: P_j , P_{j+1} , ... P_{j+n} are the calculated nodes of the Cspline trajectory of coordinates respectively $xx(j)$, $yy(j)$, and $xx(j+1)$, $yy(j+1)$..., and $xx(j+n)$, $yy(j+n)$ (Figure 3).

The error $Er(j)$ is calculated as follows:

For $j=1, \dots, n$

$$xx(j) \leq x(i+1) \quad \text{and} \quad xx(j) > x(i) \quad (12)$$

$$Er(j) = \text{abs}(yy(j) - y(i)) \quad (13)$$

$$\text{If } Er(j) \leq \epsilon \quad (14)$$

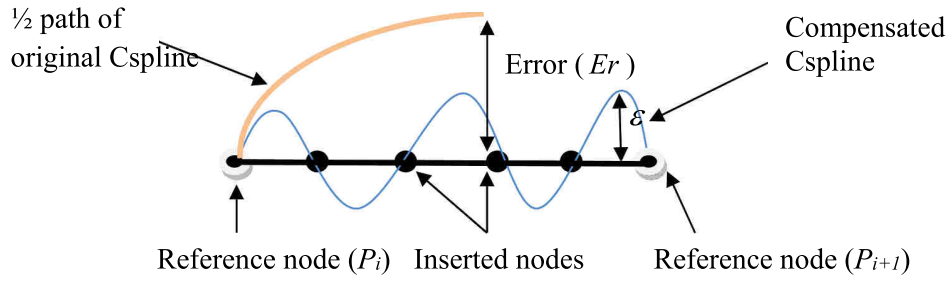


Figure 2. Original and compensated Cspline on one block.

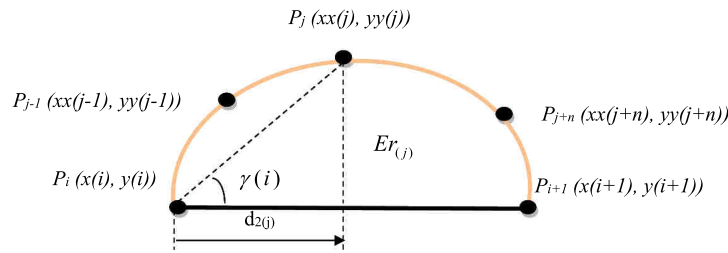


Figure 3. Calculation method of distance d_2 .

$$d_2(j) = \frac{xx(j) - x(i)}{\cos(\gamma(i))} \quad (15)$$

With $\gamma(i)$: angle between $\overline{p_i p_{i+1}}$ and $\overline{p_i p_j}$

So a distance matrix $d_2(i, j)$ is found and the average distance is calculated which respect the tolerance ϵ . Therefore, d_{2avr} is the retained distance between the inserted nodes. Then, the coordinates of the added nodes are calculated by the same algorithm such as the Bspline curve with little difference.

2.3 Simulation and compensation of machining errors of Bspline and Cspline

The method of inserting nodes for Bspline interpolation differs from that applied for Cspline interpolation. The origin of this difference is the mathematical formula of each interpolation.

Figure 4 shows the simulation and the compensation of machining errors of Bspline and Cspline for a machining toolpath of our test piece.

Our theoretical approach is validated with experimental tests on the Huron KX 10 3-axis HSM machine. As a result, major errors are successfully eliminated of the order of 1100 μm , because in this critical area, the control points before insertion are so far. So, this proves the effectiveness of this method in terms of accuracy. This improvement is mainly localized in

critical areas that have major errors. In these areas, the CAM tool generates more spaced reference points which cause the remoteness between the Bspline or Cspline trajectory and the desired one. However, machining by using these two types of interpolations affects very important criteria for machining complex shapes such as cycle time and surface quality.

3. Influence of the compensation method on the machining process in HSM

3.1 Study methodology

After machining with the compensation method of the test piece, using the interpolations Bspline and Cspline, the major machining errors generated are compensated. To know the impact of this method on the HSM machining process, its influence on the dynamics of the machine (feed rate, cycle time), on the NCU error (following error and contour deviation), and on the surface condition (roughness) will be studied.

3.2 Influence of the compensation method on the dynamics behavior of the machine

The post-processor development principle is to compensate the machining errors by inserting additional

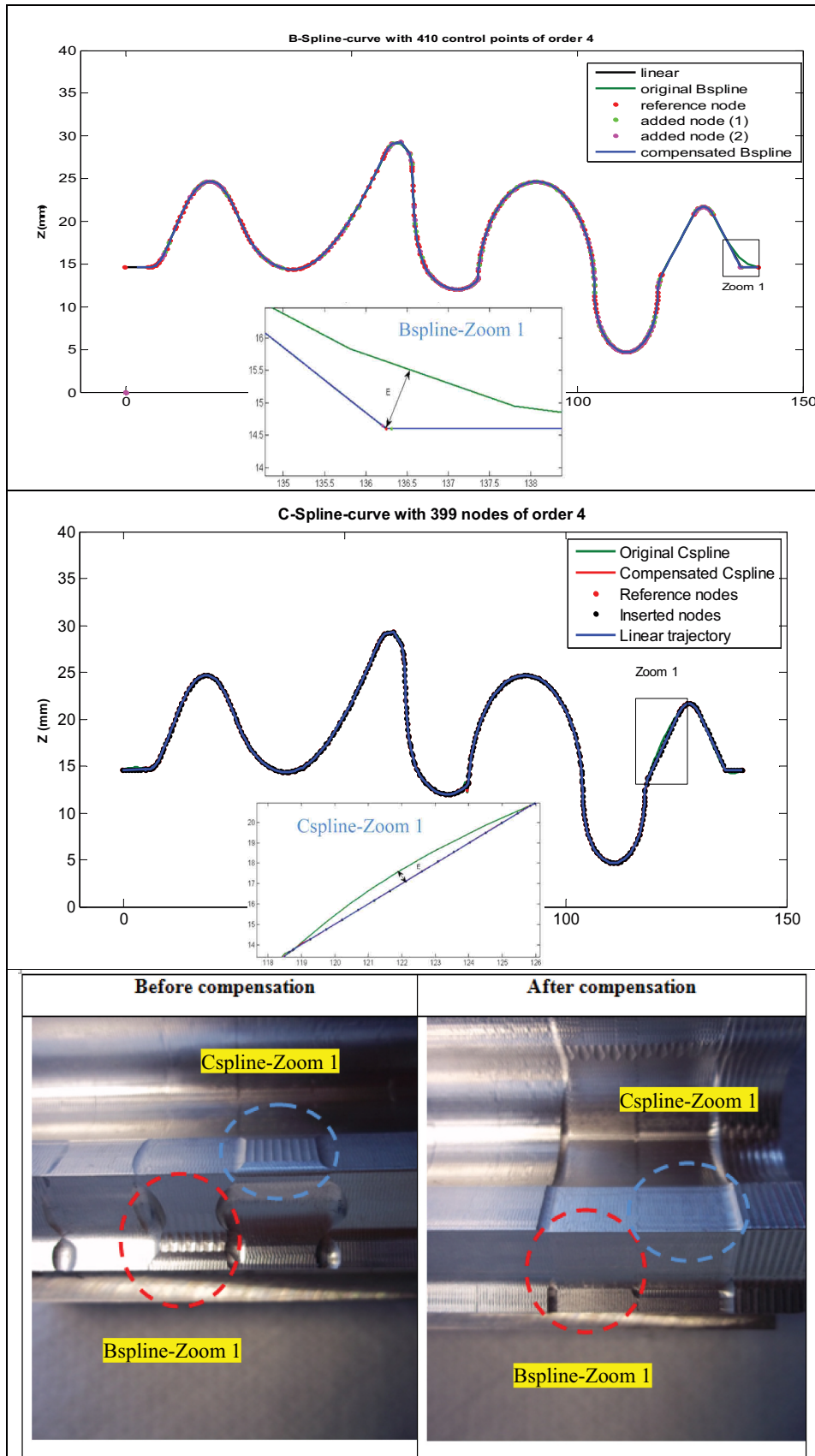


Figure 4. Simulation and compensation of the machining errors in Bspline and Cspline with zooms on critical areas (zoom1 in Bspline and zoom1 in Cspline).

nodes. It is clear that increasing the number of the control points will affect machining accuracy. Consequently, it will affect the machining time and the feed rate with an inversely proportional relationship.

3.2.1 Influence of the compensation method on the feed rate

The feed rate is measured for a single machining pass for Bspline and Cspline interpolations after compensation. Figures 5 and Figure 6 show feed rates with Bspline and Cspline interpolations before and after compensation, with the CAM tolerance $T = 0.025$ mm and the programmed feed rate $F = 4.8$ m/min.

The feed rates in Bspline and Cspline interpolations are variable, but the increase of oscillations after compensation is noticed. This variation is due to the dynamic behavior of the HSM machine

especially the time IPO (execution time of a block) and thanks to the number increase of small segments and changes of directions. By adding more points, the curvature of the tool-path is changed. Since the curvature is altered, the NC system must re-calculate the machining feed rate to stay within acceleration and jerk limits of the drives. Consequently, it slows down the tool movement speed.

Before compensation, the average feed rate in Bspline was 2.542 m/min. After compensation, the average feed rate decreases to 0.983 m/min. Thus, the same decrease for Cspline is observed. This diminution is due to the increased constraints caused by the inserted nodes (necessary number of nodes to respect the CAM tolerance) such as the slowing down of the machine during the passage of tangency and curvature discontinuities. The most limiting

Table 1. Shows the NC file size and the average feed rate in Bspline and Cspline before and after compensation.

	Initial Bspline	Bspline after compensation	Initial Cspline	Cspline after compensation
NC file size (Ko)	200	431	200	410
Average feed rate (m/min)	2.542	0.983	3.764	2.054

Table 1. Average feed rate in Bspline and Cspline before and after compensation.

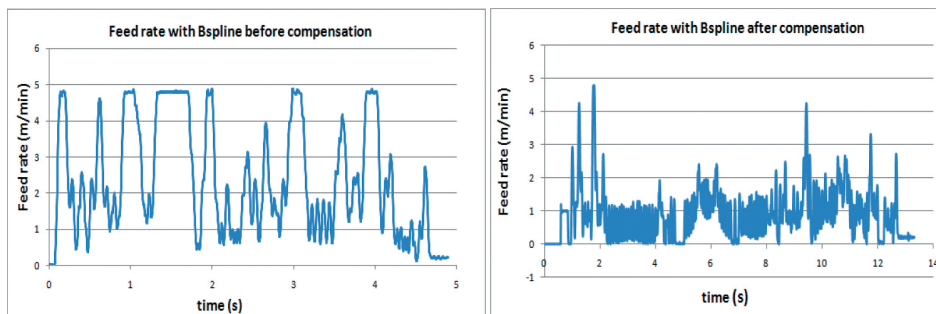


Figure 5. Feed rate with Bspline interpolation before and after compensation.

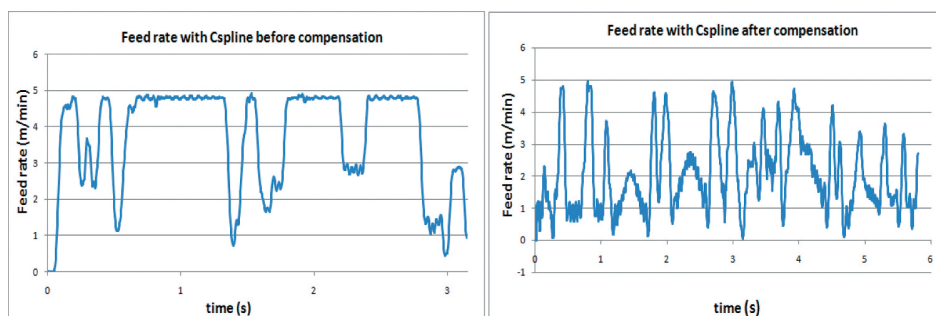


Figure 6. Feed rate with Cspline interpolation before and after compensation.

parameter is the Jerk Max. of the HSM machine (Tapie, Mawussi, and Anselmetti 2007).

3.2.2 Influence of the compensation method on the cycle time

Figure 7 depicts the histogram of the CAM cycle time and the experimental cycle time for interpolations: Bspline and Cspline, before and after compensation.

After compensation, increasing the number of control points from 200 to 410 in Bspline causes elevation 2.5 times of the cycle time from $t_{cy} = 252.324$ s to $t_{cy} = 624.875$ s. So, accuracy requires more time to satisfy the need for quality of the part.

Similarly, for Cspline interpolation, increasing the number of control points from 200 to 399 after compensation, causes elevation 1.5 times of the cycle time from $t_{cy} = 166.094$ s to $t_{cy} = 281.904$ s. Therefore, the Cspline interpolation remains faster than the Bspline interpolation even after compensation.

3.3 Influence of the compensation method on the controller (NCU) errors

The compensation method affects the NCU behavior thanks to the increase of the blocks number in the NC program. Indeed, the generated errors of the controller must be evaluated after adding control points. The main parameters involve the NCU errors available to measure in the controller Siemens 840D are the following error and the contour deviation. The following error is the difference between the theoretical position calculated by the digital control and the real position of the moving part returned by the encoder

or the incremental measurement. The contour deviation is the calculation of the deviation from the programmed path and the output to the controller.

Figure 8 shows curves of the following error and the contour deviation in Bspline interpolation before and after compensation.

Figure 9 shows traces of the following error and the contour deviation in Cspline interpolation before and after compensation.

Figures 8 and Figure 9 illustrate that the following error decreases after compensation. However, the contour deviation is partially constant depending on the compensation method. The variation is considered negligible.

The average following error of the Bspline after compensation decreases to less than half from $127.57 \mu\text{m}$ to $54.614 \mu\text{m}$. Therefore, the addition of other control points to the tool path helps the servo system to correctly track the CAM tool path with undesirable jerk during machining. As a result, the trajectory executed becomes closer to that of command. Increasing the saturation acceleration number on the critical areas and the speed limitation, set by the interpolation cycle time; favor the machining accuracy of the free-form. The average contour deviation kept a constant value at about $11 \mu\text{m}$ with negligible variations.

Similarly, for Cspline, the average following error after compensation decreases to almost half from $200.47 \mu\text{m}$ to $112.05 \mu\text{m}$. Therefore, adding control points to the Cspline tool path also helps the servo system to correctly track the CAM tool path. So, the tool path executed becomes closer to that of

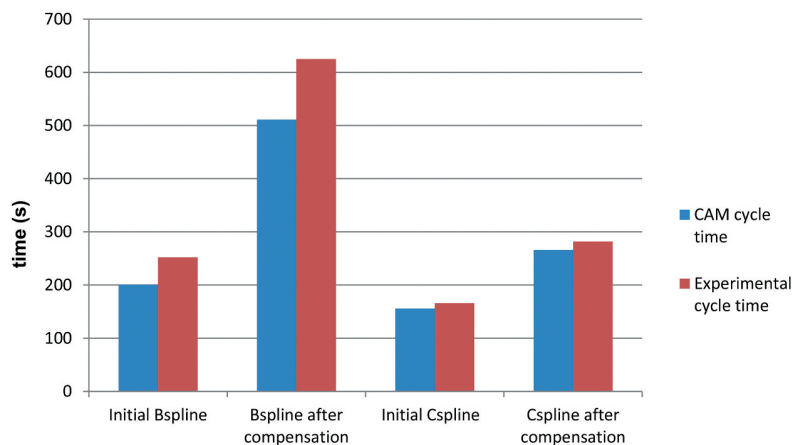


Figure 7. Histogram of the cycle time for Bspline and Cspline interpolations before and after compensation.

Table 2. Shows the average following error and the average contour deviation for Bspline and Cspline interpolations before and after compensation.

	Initial Bspline	Bspline after compensation	Initial Cspline	Cspline after compensation
Average following error (μm)	127.57	54.614	200.47	112.05
Average contour deviation(μm)	10.34	11.785	11.55	12.713

Table 2. Average following error and average contour deviation for Bspline and Cspline interpolations before and after compensation.

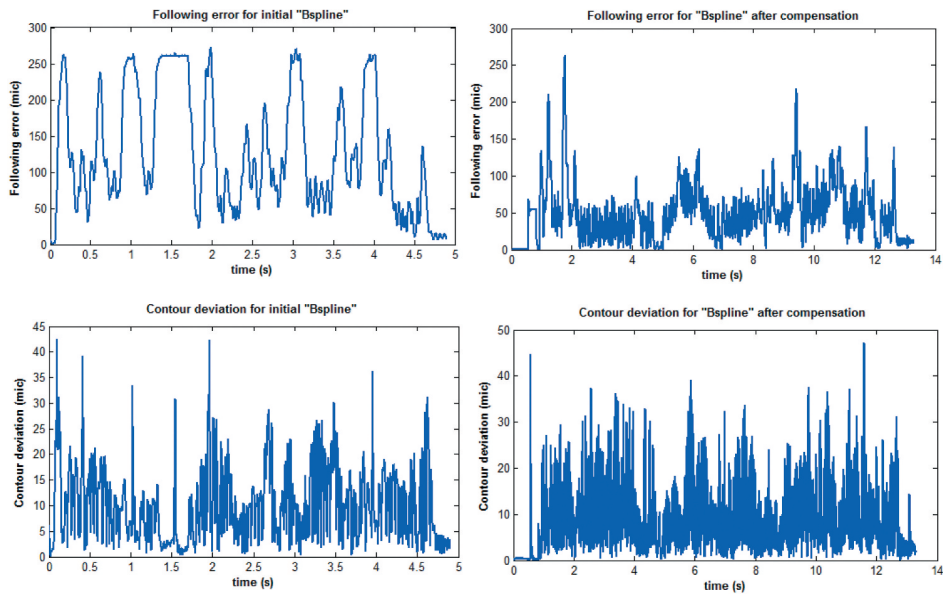


Figure 8. Following error and contour deviation in Bspline before and after compensation.

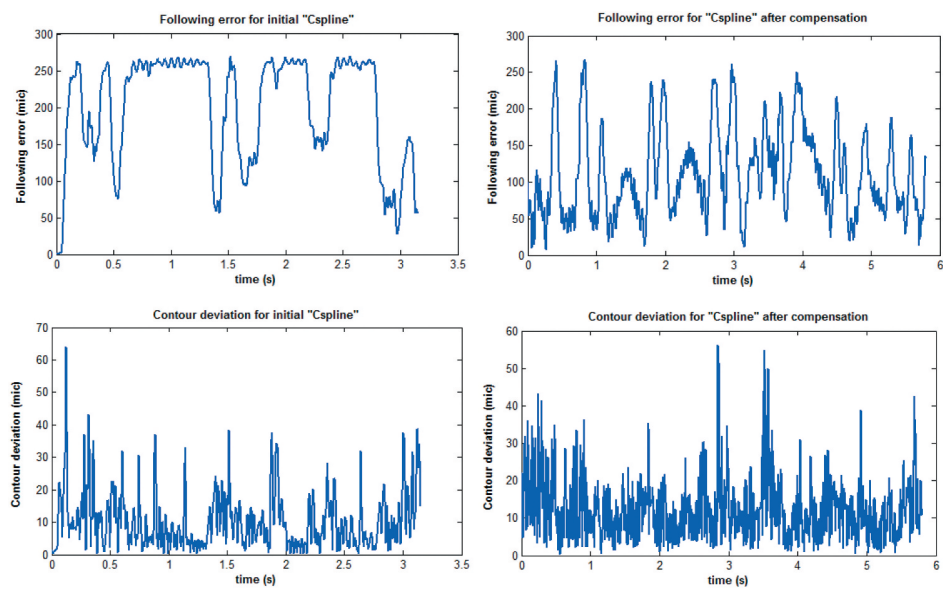


Figure 9. Following error and contour deviation in Cspline before and after compensation.

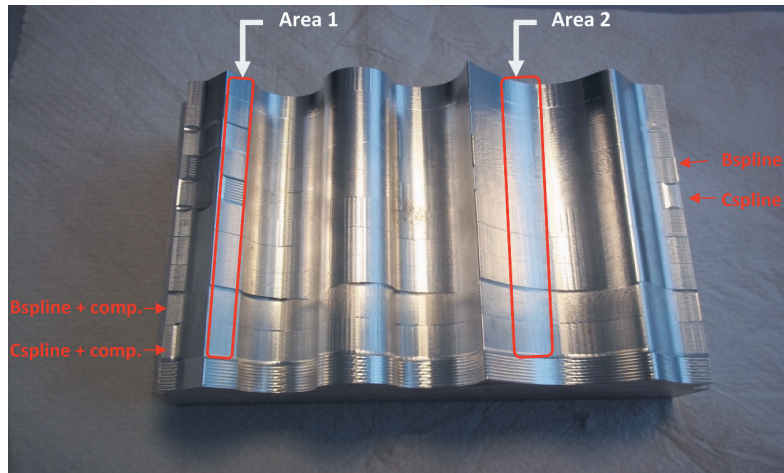


Figure 10. Capture areas: 1 and 2 of the surface condition.

command. Thus, the average contour deviation kept an average value of 12 μm with negligible variations.

Subsequently, the compensation method favors the tracking of the CAM trajectory and serves to reduce the NCU error in general. In fact, the form to machining becomes more respected by reducing the NCU errors.

3.4 Influence of the compensation method on the surface quality

In order to show the influence of the compensation of the machining errors generated by the Bspline and

Cspline interpolations on the surface condition, two test piece areas (Figure 10) were machined by the Bspline and Cspline interpolations before and after compensation.

Figure 11 shows the surface condition of the selected area 1 captured in zoom by a digital microscope for the Bspline and Cspline interpolations before and after compensation, with measurement of the roughness Ra.

Figure 12 shows the surface condition of the selected area 2 captured in zoom by a digital microscope for the Bspline and Cspline interpolations before and after the compensation.

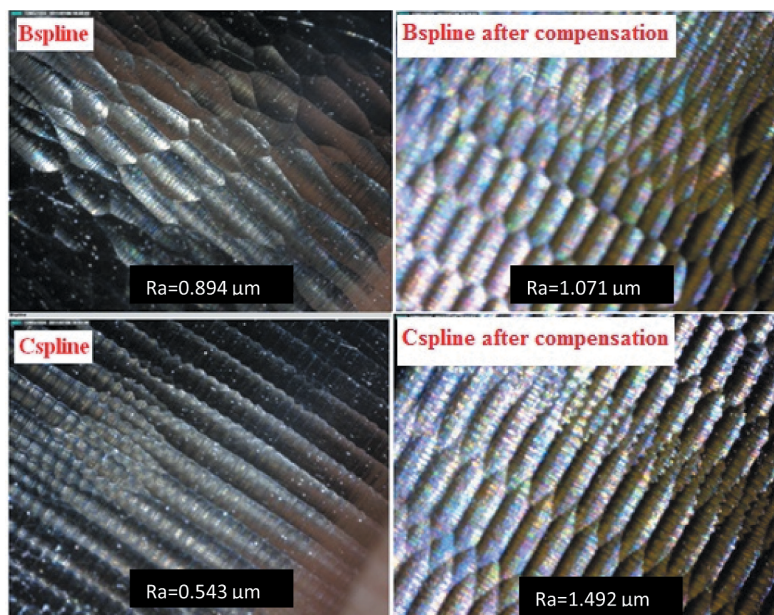


Figure 11. Surface Condition for Bspline and Cspline interpolations before and after compensation (area 1).

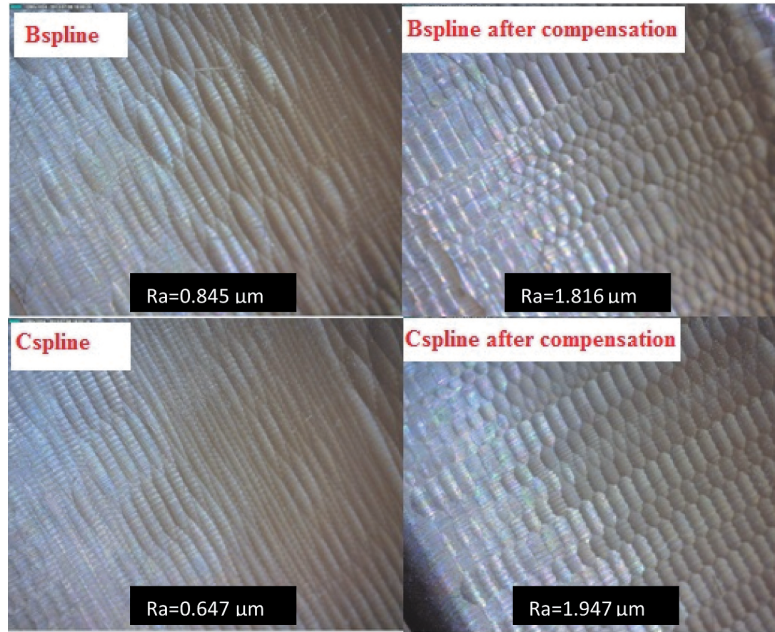


Figure 12. Surface roughness in Bspline and Cspline before and after compensation (area 2).

According to previous results, the compensation method of machining errors damages the surface condition. The protrusions of the surfaces generated after the compensation are larger than those of the initial surfaces. So, polynomial Bspline and Cspline interpolations after compensation did not generate good surface quality.

Figure 13 shows the histogram of roughness values Ra for Bspline and Cspline before and after compensation on two areas: Area 1 and Area 2.

According to the histogram, the roughness Ra of the Bspline and Cspline surfaces increases after the compensation for both interpolations. It is

concluded that our compensation method has a negative effect on the surface quality of the machined part. By adding more control points, the tool path is basically made more accurate. Also, the curvature of the toolpath is changed. Since the curvature is altered, the controller slows down the tool speed. Consequently, the roughness increases. The main causes of this result are the multi-point discretization and the tool geometry (ball-end). By adding nodes, the number of surface peaks increases affecting the machining kinematics and accordingly the surface quality.

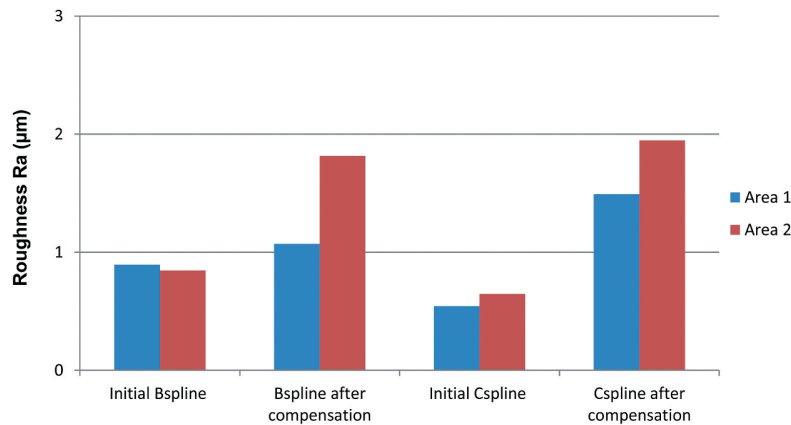


Figure 13. Histogram of roughness values Ra for Bspline and Cspline before and after compensation.

4. Conclusions

The compensation method of the machining errors of Bspline and Cspline consists in the insertion of nodes between theoretical control points in order to modify the tool path geometry. The major machining errors especially in critical areas are minimized. In addition, the controller following error is decreased while keeping the contour deviation unchanged. So, a better precision for the machining of complex parts in HSM is obtained. However, the feed rate is decreased and consequently, the machining time is increased. Nevertheless, the roughness is augmented. Logically, when the machine slows down, the surface quality is improved but it happens the inverse in this case. Feed rate does not control the surface finish. Discretization, surface peaks and machining kinematics represent the underlying causes of this inconvenience. Still the precision of the machining part is ameliorated with the smoothing of the tool path. In future, it is needed to incorporate novel interpolations on the CAD/CAM manufacturing processes.

Disclosure statement

No potential conflict of interest was reported by the authors.

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