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# Impact of magnetic ripple on neoclassical equilibrium in gyrokinetic simulations.

R. Varennes<sup>1</sup>, X. Garbet<sup>1</sup>, L. Vermare<sup>2</sup>, Y. Sarazin<sup>1</sup>, G. Dif-Pradalier<sup>1</sup>,  
V. Grandgirard<sup>1</sup>, P. Ghendrih<sup>1</sup>, P. Donnel<sup>1</sup>, M. Peret<sup>1</sup>, K. Obrejan<sup>1</sup>,  
E. Bourne<sup>1</sup>

<sup>1</sup>CEA, IRFM, F-13108 Saint-Paul-Lez-Durance, France.

<sup>2</sup>LPP, CNRS, Ecole polytechnique, 91128 Palaiseau, France.

**Abstract.** The effect of magnetic field ripple on tokamak plasma without turbulence is studied analytically and numerically. The analytical model includes neoclassical processes in the presence of non-axisymmetric perturbation and stochastic transport. For the numerical study, a magnetic field ripple perturbation has been implemented in the GYSELA gyrokinetic code. Simulations with non-axisymmetric perturbation show good agreement with the neoclassical reduced model in the collisionality range  $\nu^* \in [0.05 - 0.5]$ . In addition, the GYSELA code was successfully benchmarked against the NEO code, that solves the drift kinetic equation. Surprisingly, both the model and GYSELA simulations predict that the thermal drive of the mean poloidal velocity – measured by the  $k_{V_p}$  coefficient – decreases sharply for large yet experimentally relevant magnetic ripple amplitudes, and may even change sign.

*Keywords:* neoclassical transport, stochastic transport, plasma flow, full- $f$  gyrokinetic simulations

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## 1. Introduction

The impact of non-axisymmetric magnetic perturbations on flows is of prime importance to understand confinement. This is particularly true for stellarators. However tokamaks also exhibit such perturbations which can fundamentally modify transport even at small amplitudes [1, 2]. Here special attention is paid to the magnetic field *ripple*, caused by the discrete number of toroidal coils. Magnetic ripple impacts transport by modifying both neoclassic and turbulent transport. The former is studied here. In the absence of turbulence, magnetic ripple modifies the particle trajectories in many ways. First, magnetic ripple causes a loss of axisymmetry of the system and therefore of the associated invariant of motion: the canonical toroidal momentum. Confinement is then breached, leading to non-vanishing radial transport as a result of the local toroidal trapping that occurs between two consecutive toroidal coils. Particles trapped in these magnetic wells undergo vertical magnetic drift that is not compensated anymore by

the motion along the helical magnetic field lines (except in the presence of strong  $E \times B$  drift). Consequently, ripple can lead to transport even in collisionless plasmas. Secondly, neoclassical effects, i.e. collisional processes enhanced by particle trajectories, are substantially changed and modify flows in consequence. In particular, a well established result of the neoclassical theory in an axisymmetric magnetic configuration is the degeneracy between the toroidal velocity  $V_T$  and the radial electric field  $E_r$ . Ripple breaks axisymmetry and forces  $V_T$  and  $E_r$  to converge towards well-defined values. Finally, stochastic transport can arise above a ripple amplitude threshold. Particles then describe chaotic motion resulting in an effective diffusion that lead to transport.

The accurate simulation of these transport properties requires the kinetic approach. To study these effects, magnetic field ripple has been implemented in the gyrokinetic code GYSELA. In this study, this implementation is verified thanks to the toroidal momentum conservation equation as ripple is responsible for an additional toroidal torque, usually called *Neoclassical Toroidal Viscosity*. The ripple implementation is then confronted with a reduced analytical model of neoclassical physics. Good agreement is found on neoclassical transport coefficients used to predict the toroidal velocity, the poloidal velocity and the radial electric field. A benchmark with the drift-kinetic solver NEO is also performed and found in good agreement with GYSELA results. A peculiar observation, shared by GYSELA and the analytical model is the strong effect of ripple on the poloidal velocity even for typical tokamak ripple amplitudes.

The rest of this paper is organised as follows. Neoclassical and stochastic transport mechanisms in the presence of ripple perturbation are recalled in Sec.2 where the model is defined. This model is explored and compared with asymptotic studies in Sec.3. The numerical model of the GYSELA code is presented in Sec.4. In Sec.5, the verification of the implementation of the magnetic perturbation in GYSELA is detailed. Then, neoclassical theoretical predictions are compared with GYSELA and NEO simulations in Sec.6. The modification of the poloidal velocity by neoclassical effects with ripple is discussed in Sec.7. A summary is given in Sec.8.

## 2. Theoretical neoclassical predictions

In this study, a simplified geometry is considered where magnetic surfaces are taken circular and concentric. In toroidal coordinates  $(r, \theta, \varphi)$ , and in the limit of large aspect ratio and small ripple magnitude, the magnetic field amplitude  $B$  can be approximated as follows:

$$B = B_0 \left( 1 - \underbrace{\varepsilon(r) \cos \theta}_{1/R \text{ decay}} + \underbrace{\delta(r, \theta) \cos(N_c \varphi)}_{\text{magnetic ripple}} \right) \quad (1)$$

Here,  $B_0$  is a reference magnetic field amplitude taken on the magnetic axis and in between two coils,  $\varepsilon = r/R_0$  with  $R_0$  the major radius,  $\delta$  is the ripple amplitude and  $N_c$  the number of toroidal coils. The  $1/R$  magnetic field decay is responsible for banana trapping. Ripple is also responsible for so-called local trapping in between two consecutive toroidal coils that create a

magnetic well where particles can bounce back and forth. Without turbulence, particle motion is constrained by the magnetic configuration and collisions. Those constraints account for neoclassical and stochastic transport. The derivation of the neoclassical part heavily relies on the existence of invariants of motion that are not valid with chaos. Consequently, in order to have consistency in the complete model, the neoclassical part is considered to hold only in the adiabatic limit  $N_c \Omega_D \ll \Omega_B$  where  $\Omega_D$  and  $\Omega_B$  are the precession and banana-bounce frequencies. The origin of this limit is explained below. Let us detail the neoclassical and stochastic aspects separately.

First, neoclassical transport is due to the resonant enhancement of collisional processes. At low collisionality, it owes its existence to trapped particles. Indeed, in the vicinity of bounce points, the particle parallel velocity  $v_{\parallel}$  vanishes and the magnetic drift becomes dominant; this corresponds to the resonance. Now, if the plasma is collisionless, transport can only occur if particle trajectories are not confined. It is possible with magnetic field ripple as particle trapped between two coils undergo a vertical magnetic drift which is not compensated by the motion along the magnetic field lines. In the axisymmetric case, there would be no transport at all. Adding collisions to this problem accounts for the so-called neoclassical theory. When the collision frequency becomes comparable to the bounce frequency of the considered trapped population, a random walk with a characteristic step linked to the resonance prevails. Considering banana trapping for example, this step would be roughly the banana width. Subtleties also appear when accounting for the synergies between the different trappings. For example, the ripple causes banana bounce points to drift radially in such a way that they undergo a random walk process. The reduced model is meant to describe all those processes. It relies on a kinetic derivation of the equilibrium distribution function for the considered species. Each population (trapped or passing) tend to relax towards a Maxwellian equilibrium under the effect of collisions. The resulting total distribution function can however be non-Maxwellian and therefore leads to finite fluxes.

This physics is captured by the gyrokinetic code GYSELA. Fig.1 shows the impact of the  $1/R$  magnetic field decay and magnetic ripple on the distribution function in the  $(\varphi, v_{\parallel})$  and  $(\theta, v_{\parallel})$  spaces. Magnetic ripple produces islands in the  $(\varphi, v_{\parallel})$  space corresponding to particles trapped toroidally between coils. The island in the  $(\theta, v_{\parallel})$  space, corresponding to the banana trapping, is also modified with ripple. As discussed previously the effect is maximum at  $v_{\parallel} = 0$  where the resonance takes place. A similar mechanism also exists for particles in ripple wells.

To get the expression of the equilibrium distribution function, one must start with the drift kinetic equation (DKE). The first step of the derivation then relies on solving the DKE for each perturbation, i.e. the  $1/R$  magnetic field decay or ripple, independently. They are labelled ‘primary’ perturbations. Solving the DKE with the  $1/R$  magnetic field decay as primary perturbation leads to the axisymmetric neoclassical theory. Doing the same for the ripple perturbation amounts to consider collisional processes in a tokamak without curvature, i.e. a rippled cylinder, with equally spaced coils leading to local trapping. The next step is to perturb the trapped population by the primary perturbation with the other one, labelled ‘secondary’

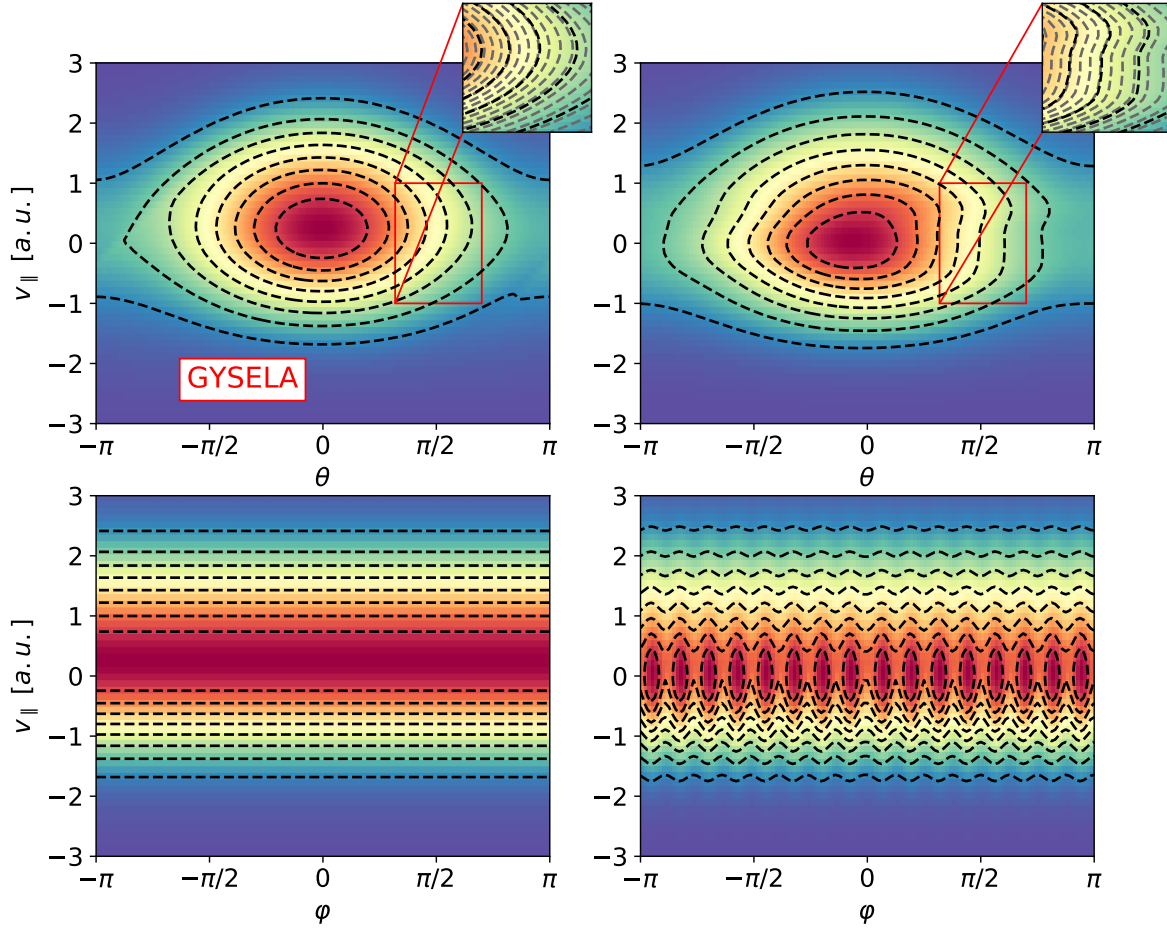


Figure 1: Distribution function of trapped particles without (left) and with (right) ripple, obtained with GYSELA, with  $N_c = 16$ . Ripple accounts for an additional trapping in the toroidal direction and also modifies the island of banana trapped particles near bounce points.

perturbation. If ripple is the secondary perturbation, it then accounts for the radial drift of the banana bounce point induced by ripple. If the  $1/R$  magnetic field decay is the secondary perturbation, it then accounts for the bounce points of the ripple-induced trapped trajectories drifting vertically due to the magnetic curvature drift. Those four contribution branches to neoclassical transport can be summed up in this list:

- the *axisymmetric branch* is the neoclassical contribution of unperturbed banana particles;
- the *helically symmetric branch* is the neoclassical contribution of unperturbed ripple-induced trapping;
- the *banana drift branch* is the neoclassical contribution of ripple on banana trapped particles;
- the *ripple-wells drift branch* is the neoclassical contribution of the  $1/R$  magnetic field decay on particles trapped in ripple-induced magnetic wells.

Let us now talk about stochastic transport. It refers to a collisionless diffusion that can

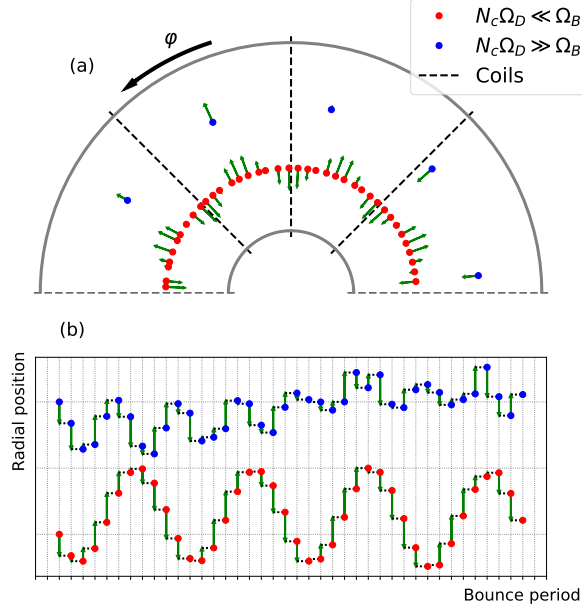


Figure 2: (a) Schematic view in toroidal section of bounce point for trapped particles either below the stochasticity limit (red) or above (blue). Green arrow shows the direction and amplitude of the radial shift  $\Delta r$  exerted on particle by ripple. (b) Successive radial positions of the bounce point for each case.

arise in the presence of ripple for some banana trapped particles. Here is the principle. As mentioned before, ripple causes a radial jump  $\Delta r$  of the banana trapped particles at the bounce point locations. This jump is either inward or outward depending on the relative toroidal angle  $\varphi_B$  of the bounce point relative to the two nearest coils. It can be expressed  $\Delta r = \lambda \rho_i \delta \cos(N_c \varphi_B - \pi/4)$  where  $|\lambda| \sim \sqrt{N_c} (q/\varepsilon)^{3/2}$  [3] with  $q$  the safety factor and  $\varphi_B$  the toroidal angle of bounce points. In most tokamaks an estimate at the edge is  $\sqrt{\langle \Delta r^2 \rangle_\varphi} \sim 2-3 \rho_i$  where  $\langle \cdot \rangle_\varphi$  is the toroidal average. If  $\Delta r > 0$  (outward) in between two coils, then  $\Delta r < 0$  (inward) in the vicinity of the coils. As trapped particles drift toroidally at the precession frequency  $\Omega_D$ , they undergo a different radial shift at each bounce point. After each bounce period  $\Omega_B^{-1}$ , one can define a toroidal phase shift  $\Delta \varphi_B = \varphi_B(t - \Omega_B^{-1}) - \varphi_B(t)$ . If  $\Delta \varphi_B$  is small with respect to the toroidal angle span separating two coils  $\Delta \varphi_C = 2\pi/N_C$ , the consecutive radial shifts exerted on the particle are expected to be as often inward as outward, resulting in a zero mean radial displacement. However, in the other limit, namely  $N_C \Omega_D \gg \Omega_B$ ,  $\Delta \varphi_B$  becomes comparable to  $\Delta \varphi_C$ . In this scenario, consecutive bounce point positions follow no particular order and stochasticity appears. This mechanism is illustrated in Fig.2.

The whole analytical calculation of neoclassical and stochastic transport is a formidable task that will not be detailed in this paper. The interested reader can look at [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] for the neoclassical part and at [3, 15, 16] for the stochasticity part. The model

considered in this paper is based on the review by *Garbet et al.* [17] for which stochastic transport have been added. It leads to a transport matrix that links the the equilibrium fluxes of particles  $\Gamma$  and heat  $Q$  as well as the magnetic drag force  $\mathcal{M}$  with the thermodynamical gradients, the mean toroidal velocity  $V_T$  and the radial electric field  $E_r$ . For a single ion species, it can be expressed in this compact way:

$$\begin{pmatrix} \Gamma_N \\ \Gamma_{V_T} \\ \Gamma_T \end{pmatrix} = \begin{pmatrix} d_0 + \tilde{d}_0 & d_0 & d_1 + \tilde{d}_1 \\ d_0 & d_0 + \hat{d}_0 & d_1 \\ d_1 + \tilde{d}_1 & d_1 & d_2 + \tilde{d}_2 \end{pmatrix} \cdot \begin{pmatrix} A_N \\ A_{V_T} \\ A_T \end{pmatrix} \quad (2)$$

Where  $\Gamma_N = \frac{\Gamma}{n}$ ,  $\Gamma_{V_T} = \frac{\mathcal{M}}{neB_p}$ ,  $\Gamma_T = \frac{Q}{nT}$ ,  $A_N = \frac{1}{n} \frac{dn}{dr} - \frac{eE_r}{T}$ ,  $A_{V_T} = \frac{eB_p}{T} V_T$ ,  $A_T = \frac{1}{T} \frac{dT}{dr}$  with  $n$  the density,  $T$  the temperature,  $e$  the electric charge and  $B_p$  the poloidal component of the magnetic field.  $D_P = \frac{qR_0}{V_{thi}} \left( \frac{T}{eB_0R_0} \right)^2$  is a reference diffusion coefficient, where  $V_{thi}$  is the ion thermal velocity and  $q$  the safety factor.

The transport matrix coefficients are detailed in [Appendix A](#). They only depend on three dimensionless parameters:  $\nu^*$ ,  $\delta/\varepsilon$  and  $N_c q$  (with  $q$  the safety factor). The coefficients  $d_i$  are independent of  $\delta/\varepsilon$ . Without ripple, i.e.  $\tilde{d}_i = 0$  and  $\hat{d}_i = 0$ , one recovers the classic coefficients resulting from the axisymmetric neoclassical theory at lower order in  $\varepsilon$ . In that case, it is clear that the two first lines of the transport matrix are identical. This degeneracy explains why axisymmetric neoclassical theory cannot predict  $A_N$  and  $A_{V_T}$  independently, hence treating the radial electric field and the toroidal velocity as a single unknown.

### 3. Exploring the model

In this section, the previous model is used to compute meaningful quantities and is compared with common asymptotic regimes. Let us first define those "meaningful quantities". First, there are the thermal drives coefficients that link the equilibrium toroidal velocity, poloidal velocity and the radial electric field to the temperature gradient, labelled  $k_{V_T}$ ,  $k_N$  and  $k_{V_P}$  respectively. Their expressions are obtained when considering that both  $\Gamma_N$  and  $\Gamma_{V_T}$  go to zero at equilibrium (without external sources of particles and momentum). Under those conditions, Eq.2 gives the equalities  $A_{V_T} = k_{V_T} A_T$  and  $A_N = -k_N A_T$  such that

$$k_{V_T} = \frac{d_0 \tilde{d}_1 - d_1 \tilde{d}_0}{(d_0 + \tilde{d}_0)(d_0 + \hat{d}_0) - d_0^2} \quad (3)$$

$$k_N = \frac{(d_0 + \hat{d}_0)(d_1 + \tilde{d}_1) - d_0 d_1}{(d_0 + \tilde{d}_0)(d_0 + \hat{d}_0) - d_0^2} \quad (4)$$

Then, one can use the force balance  $A_{V_P} = \frac{eB_T}{T} V_P = A_N + A_{V_T} + A_T$  with  $V_P$  the equilibrium poloidal velocity and  $B_T$  the toroidal component of  $\mathbf{B}$ , to define the thermal drive  $k_{V_P}$  such that  $A_{V_P} = k_{V_P} A_T$  where  $k_{V_P} = 1 + k_{V_T} - k_N$ . Knowing those thermal drive coefficients as well as the temperature gradient, one can make predictions on the final flow.

These coefficients are commonly considered in some asymptotic regimes of collisionality and ripple amplitude. These regimes depend among others on the parameter  $Y(r, \theta) = \frac{\varepsilon |\sin \theta|}{\delta N_c q}$ .

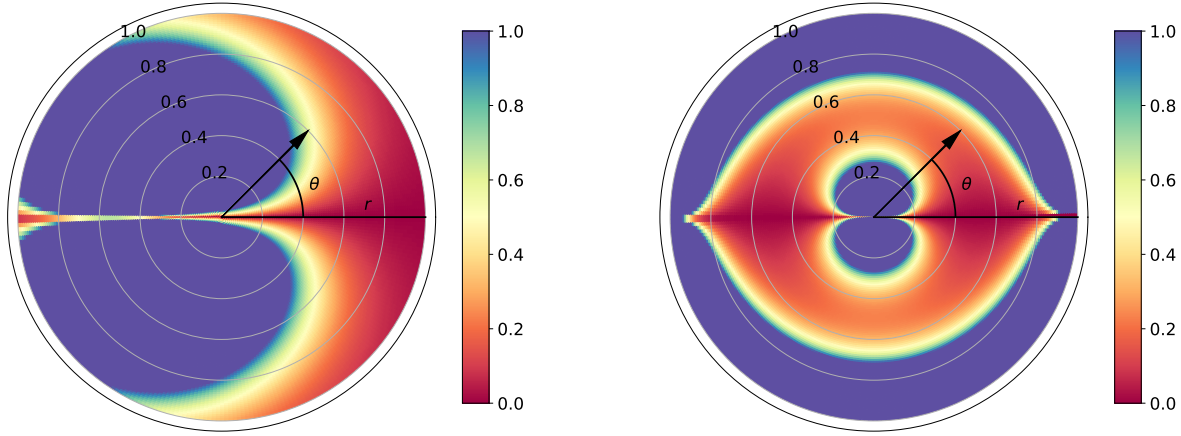


Figure 3: Condition of existence of ripple-induced magnetic wells for the Tore Supra tokamak (left) and a radially gaussian ripple centered at mid-radius with a  $\delta = 3\%$  peak (right). There is no magnetic wells in  $Y > 1$  zone (blue) so no local trapping can occur.

Regions with  $Y < 1$  are characterised by local magnetic wells in between two toroidal field coils. Conversely, for  $Y > 1$ , such magnetic wells do not exist so that there is no ripple-induced trapping in those regions.

This subtle point is worth discussing as predictions are significantly different between regions with or without ripple induced magnetic wells. It indicates that a poloidal dependence must be taken into account. Indeed, for the same ripple amplitude but different ripple shapes, the thermal drive predictions can be different. For this reason, and for numerical convenience explained below, the considered ripple perturbation for this academic work is poloidally symmetric. Fig.3 displays the poloidal map of  $Y$  for the Tore Supra tokamak and for a radially gaussian ripple centered at mid-radius. That said, the asymptotic regimes of  $k_{V_T}$ ,  $k_N$  and  $k_{V_P}$  are summarized in Tab.1.

	$Y > 1$		$Y < 1$ (Local mirrors)		$\forall Y$
	$\forall \delta/\varepsilon$		$\delta/\varepsilon < 1$	$\delta/\varepsilon > 1$	$\forall \delta/\varepsilon$
	$\nu^* \ll (N_c q)^{-2}$	$(N_c q)^{-2} \ll \nu^* \ll 1$	$\nu^* < (\delta/\varepsilon)^{3/2}$	$\nu^* < 1$	$\nu^* \gg 1$
$k_N$	3.37	1.5	3.37	3.37	1.5
$k_{V_P}$	1.17	1.17	1.17	-2.37	-0.5
$k_{V_T}$	3.54	1.67	3.54	0.0	0.0

Table 1: Asymptotic values of  $k_{V_T}$ ,  $k_N$  and  $k_{V_P}$  for different ranges of  $\delta/\varepsilon$ ,  $\nu^*$  and  $N_c q$ .

In this table, one can observe that thermal drive coefficients change substantially depending of collisionality regimes. For example,  $k_{V_T}$  can double in regions without magnetic wells under the sole effect of collisionality. It means that for a given temperature gradient, the toroidal velocity can change drastically when the collisionality evolves (due to heating for example). Similar effects can be observed for  $k_N$  and  $k_{V_P}$ . However, this table can easily be



misleading for several reasons. First, the model is poloidally averaged so the resulting thermal drive can be a mix between their predictions in  $Y > 1$  regions and  $Y < 1$  regions. Then, those asymptotic regimes hide that those thermal drives actually depend non-linearly on  $\delta/\varepsilon$  and  $\nu^*$ . Consequently, the transition between those regimes have no reason to be smooth or even monotonous.

Although less accurate than simulation codes, the reduced model has the advantage over simulation codes to provide transport coefficients effortlessly. In other words, it can be used to perform scans on wide ranges of  $\delta/\varepsilon$ ,  $\nu^*$  and  $N_c q$ . Now and for the remaining of the paper, a fixed  $N_c q = 16 \times 1.4$  is considered. The thermal drives  $k_{V_T}$ ,  $k_N$  or  $k_{V_P}$  are scanned for the ranges of  $\delta/\varepsilon$  and  $\nu^*$  as shown in Fig.4.

All asymptotic regimes are recovered. It should be mentioned however that the so-called *Pfirsch–Schlüter* regime is expected to play a role for  $\nu^* \gg 10$ . This regime is not included in the model and is expected to change  $k_{V_P}$  towards even more negative values [18] (indeed the prediction in axisymmetric configuration is  $k_{V_P} \rightarrow -2.1$ ). The transitions between the different regimes are mainly due to the  $(\delta/\varepsilon)^{-1}$  dependence of  $Y$ . Indeed, as  $(\delta/\varepsilon)$  increases, the size of the regions where ripple-induced magnetic wells exist also increases. In the range of the  $(\delta/\varepsilon)$  parameter where the  $Y < 1$  regions become dominant compared with  $Y > 1$  regions, important variations of  $k_{V_T}$ ,  $k_N$  and  $k_{V_P}$  are observed. In particular,  $k_{V_P}$  decreases toward negative values with increasing ripple amplitude. Sec.7 of this document is dedicated to a discussion on this effect. In Sec.6, these observations, obtained with the analytical model, are confronted to the simulation codes GYSELA and NEO.

Another interesting quantity can be defined: the neoclassical toroidal friction  $\nu_\varphi$ . It corresponds to the characteristic relaxation frequency – set by ripple-induced neoclassical processes – of the toroidal flow toward its predicted equilibrium value (governed by  $k_{V_T}$ ). Regarding the analytical model, this quantity appears when a situation out of equilibrium is considered. Subtracting the second line from the first one in Eq.2 reads

$$\Gamma_{V_T} - \Gamma_N = -D_P \left( \hat{d}_0 A_{V_T} - \tilde{d}_0 A_N - \tilde{d}_1 A_T \right) \quad (5)$$

The resulting neoclassical toroidal friction  $\nu_\varphi$  is then the coefficient that multiply  $A_{V_T}$ . Its expression significantly changes regarding the assumptions taken on the fluxes. Indeed, is one considers that neither the particle flux nor the momentum flux vanishes in this out-of-equilibrium situation, i.e.  $\Gamma_N \neq 0$  and  $\Gamma_{V_T} \neq 0$ , the neoclassical friction then reads  $\nu_\varphi = \hat{d}_0$ .

However, keeping the ambipolarity constraint  $\Gamma_N = 0$  but with a finite momentum flux, i.e.  $\Gamma_{V_T} \neq 0$ , Eq.2 gives  $\Gamma_{V_T} = -D_P \nu_\varphi^{\Gamma_N=0} (A_{V_T} - k_{V_T} A_T)$  with

$$\nu_\varphi^{\Gamma_N=0} = \hat{d}_0 + \frac{d_0 \tilde{d}_0}{d_0 + \tilde{d}_0} \quad (6)$$

As  $\hat{d}_0$  and  $\tilde{d}_0$  go to zero when  $\delta/\varepsilon \rightarrow 0$ , for both cases the neoclassical friction vanishes in the axisymmetric case as a consequence of the degeneracy: without ripple the toroidal velocity does not relax toward any prescribed value. Furthermore,  $\hat{d}_0$  and  $\tilde{d}_0$  increase with  $\delta/\varepsilon$  so that the system relaxes faster with higher ripple amplitude, which seems intuitive. Documentation

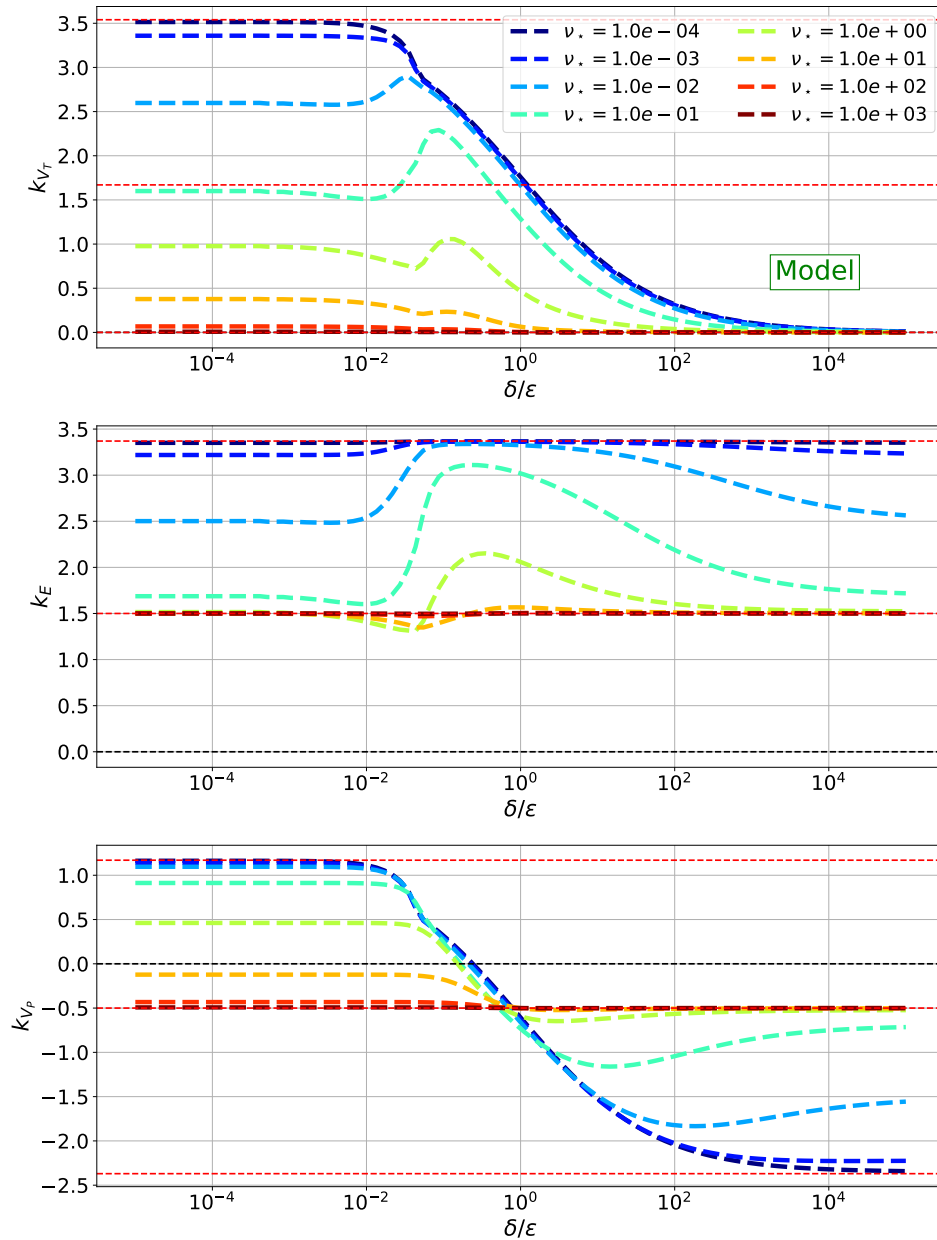


Figure 4: Scan in ripple amplitude of thermal drives  $k_{V_T}$ ,  $k_N$  and  $k_{V_P}$  for a wide range of  $\nu^*$ . Red dashed lines represent the asymptotic values that appear in Tab.1.

on this neoclassical friction can be found in references focused on externally applied magnetic perturbations [19, 20, 21]. However, none of the references known to the authors actually clearly addresses this neoclassical toroidal friction due to magnetic ripple in tokamaks, neither through analytical study nor numerical simulations.

#### 4. Ripple implementation in GYSELA

The aim of this section is to recall the basic set of equations of the GYSELA code and explain how magnetic ripple is implemented. A more detailed description of the GYSELA code can be found in *Grandgirard et al.* [22]. In the electrostatic regime considered in this paper, GYSELA solves a kinetic equation per considered species coupled to the quasi-neutrality constraint. Only one ion species and adiabatic electrons are considered in this paper. The kinetic equation is the gyrokinetic Fokker-Planck equation (Eq.7), where the full gyrocenter distribution function  $\bar{F}$  evolves in a 5D phase space  $(\mathbf{X}, v_{\parallel}, \mu)$ .  $\mathbf{X}$  is the gyrocenter position vector in the toroidal coordinates  $(r, \theta, \varphi)$ ,  $v_{\parallel}$  its velocity along the magnetic field lines and  $\mu$  its magnetic moment. It reads

$$\frac{\partial \bar{F}}{\partial t} + v_{\parallel} \nabla_{\parallel} \bar{F} + (\mathbf{v}_{E \times B} + \mathbf{v}_D) \cdot \nabla \bar{F} + \frac{dv_{\parallel}}{dt} \frac{\partial \bar{F}}{\partial v_{\parallel}} = C(\bar{F}) \quad (7)$$

Where  $\mathbf{v}_{E \times B}$  is the electric drift,  $\mathbf{v}_D$  the magnetic drift and  $C$  the collision operator [23]. The quasi-neutrality equation (Eq.8) in the case of adiabatic electrons and in the limit of long wavelengths (with respect to the thermal ion Larmor radius), reads:

$$\frac{ne [\phi - \langle \phi \rangle]}{T_e} - \nabla_{\perp} \cdot \left[ \frac{mn}{eB^2} \nabla_{\perp} \phi \right] = \iint \frac{2\pi B_{\parallel}^*}{m} dv_{\parallel} d\mu \bar{F} - (\bar{F}_{init}) \quad (8)$$

Where  $\phi$  is the electric potential,  $\langle \cdot \rangle$  denotes a flux-surface average,  $T_e$  is the electron temperature,  $m$  is the ion mass,  $B_{\parallel}^*$  is the Jacobian of the coordinate transform and  $\bar{F}_{init}$  is the initial ion gyroaveraged distribution function.

The axisymmetric magnetic field  $\mathbf{B}_{axi}$  is defined as

$$\mathbf{B}_{axi} = \frac{B_0 R_0}{R(r, \theta)} \left[ \frac{r}{q(r) R_0} e_{\theta} + e_{\varphi} \right] \quad (9)$$

where  $e_{\theta}$  and  $e_{\varphi}$  are unit vectors in the poloidal and toroidal direction respectively, and  $R(r, \theta) = R_0 + r \cos \theta$ . The total magnetic field is written  $\mathbf{B} = \mathbf{B}_{axi} + \mathbf{B}_{rip}$  where  $\mathbf{B}_{rip}$  is the magnetic field perturbation due to ripple.

The electrostatic gyrokinetic equations of motion solved in GYSELA are

$$\begin{cases} B_{\parallel}^* \frac{d\mathbf{X}}{dt} = v_{\parallel} \mathbf{B}^* + \frac{\mathbf{b}}{e} \times \nabla \mathcal{H} \\ B_{\parallel}^* \frac{dv_{\parallel}}{dt} = -\mathbf{B}^* \cdot \nabla \mathcal{H} \end{cases} \quad (10)$$

where  $\mathbf{b} = \frac{\mathbf{B}_{axi}}{B_{axi}}$ ,  $\mathbf{B}^* = \mathbf{B}_{axi} + \frac{mv_{\parallel}}{e} \nabla \times \mathbf{b}$  and  $\mathcal{H} = \mathcal{H}_{axi} + \mathcal{H}_{rip}$  with  $\mathcal{H}_{axi} = mv_{\parallel}^2 + \mu B_{axi} + e\bar{\phi}$  (with  $\bar{\phi}$  the gyroaveraged electric potential) and  $\mathcal{H}_{rip} = \mu B_0 \delta \cos(N_c \varphi)$ . In GYSELA, the ripple perturbation is then only kept in the gyro-center Hamiltonian  $\mathcal{H}$ . It can be show that this simplification changes the expression of the magnetic braking, which is the main effect of ripple, from  $\mathcal{M} = - \int d^3v \left\{ (mv_{\parallel}^2/2 + \mu B) \frac{1}{B} \frac{\partial B}{\partial \varphi} F \right\}$  to  $\mathcal{M} = - \int d^3v \left\{ \mu \frac{\partial B}{\partial \varphi} F \right\}$ . At low collisionality, magnetic ripple impacts the trapped particles so the error on  $\mathcal{M}$  can not exceed  $\sqrt{\epsilon}$  for banana trapped particles and  $\sqrt{\delta}$  for locally trapped particles. This reasonable

assumption is detailed in reference [24] that discusses the non-axisymmetric perturbation implementation in the gyrokinetic code GT5D.

## 5. Verification through toroidal momentum conservation

To verify the implementation of the ripple perturbation in GYSELA, a scan in ripple amplitude is performed to check the conservation of toroidal momentum. One of the major effects of a non-axisymmetric magnetic perturbation is an additional toroidal force: the magnetic drag  $\mathcal{M}$ . The flux surface averaged momentum conservation [25] involves this new term:

$$mR_0 \frac{\partial \langle nV_T \rangle}{\partial t} = -\langle \nabla \cdot \Pi_\varphi \rangle - \langle T_\varphi \rangle + \langle \mathcal{J}_D \rangle + \langle \mathcal{J}_E \rangle + \langle \mathcal{M} \rangle \quad (11)$$

Where  $V_T = \frac{1}{n} \int d^3v \{v_{\parallel} (B_\varphi/B) F\}$  is the gyrocenter mean toroidal velocity ( $B_\varphi$  is the toroidal component of the magnetic field),  $\Pi_\varphi$  the toroidal Reynolds stress,  $T_\varphi$  a polarization term,  $\mathcal{J}_D$  and  $\mathcal{J}_E$  the radial currents due to the magnetic and electric drifts respectively.

Boundary physics, inevitably present either in global codes or in experiments, acts as a momentum sink which is not included in the analytical model. For this reason, the considered profile of the ripple amplitude in the following simulations is taken radially gaussian and maximum at mid-radius. Also, no poloidal dependence is accounted for. It reads  $\delta_{sim} = \delta_0 e^{-4(r/a-0.5)^2}$  with  $\delta_0$  the mid-radius ripple amplitude. The usual way to perform simulations for neoclassical studies is to artificially filter out all toroidal Fourier modes of the electric potential  $\phi$  except the axisymmetric component. This cannot be done here as ripple adds toroidal harmonics with  $N_c$  periodicity. The chosen solution to avoid turbulence is then to initiate the simulations below the instability threshold, i.e. with a sufficiently weak temperature gradient. It is also verified that ripple-induced mode amplitudes are low enough such that the inequality  $e\phi_{kN_c} \ll \mu B_{kN_c}$ , with  $k = 1, 2, \dots$ , is satisfied, which ensures that the hamiltonian considered in the reduced model is a relevant approximation for those cases. The parameters of the simulations are summarized in Tab.2.

Species	Deuterium
Aspect ratio $R_0/a$	3.2
Safety factor	$q = 0.854 + 2.184(r/a)^2$
Density gradient	$R_0/L_N = 6$
Temperature gradient	$R_0/L_T = 6$
Collisionality at $r/a = 0.5$	$\nu^* = 0.1$
$\rho_i/a$ at $r/a = 0.5$	$\rho^* = 1/150$
Number of coils	$N_c = 16$

Table 2: Simulation parameters.  $L_N$  and  $L_T$  are the density and temperature gradient's length,  $\rho_i$  is the ion Larmor radius.

Four simulations using different ripple amplitudes are analyzed. The l.h.s of Eq.11, which is a standard output of GYSELA, is compared with the sum of the contributions in

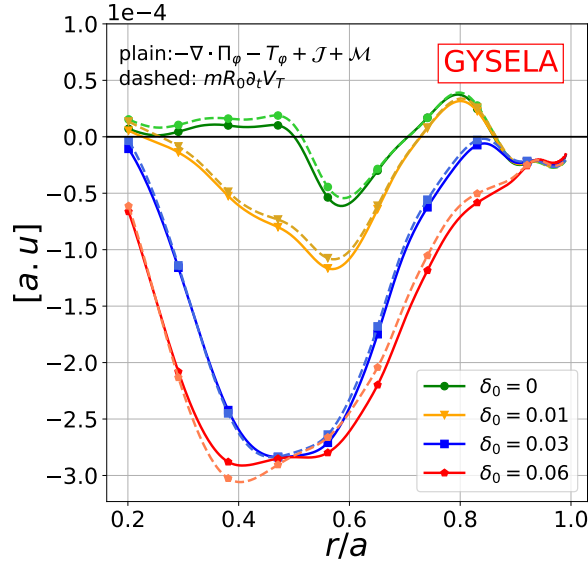


Figure 5: Radial profile of the total toroidal momentum (dashed) and the sum of the expected contributions (plain) for different ripple amplitudes  $\delta_0$ .

the r.h.s for each simulation. The radial profiles of these quantities are plotted in Fig.5 and show a good agreement for all ripple amplitudes attesting good momentum conservation in the code. Without turbulence, magnetic braking quickly becomes the dominant contribution and drives the mean toroidal velocity. The implementation of a ripple magnetic perturbation being successful, the code GYSELA can be confronted to the analytical neoclassical reduced model and NEO.

## 6. Comparison of GYSELA global simulations with neoclassical predictions and NEO

In this section, the ability of GYSELA to assess neoclassical processes is tested against two different tools. The first one is the aforementioned reduced model, directly comparable to GYSELA. The other one is the drift kinetic solver NEO which captures all the neoclassical physics. A perfect benchmark would compare all the transport matrix coefficients from both codes and the model for a wide range of  $\delta/\varepsilon$ ,  $\nu^*$  and  $N_c q$ . However, it would require a too large quantity of simulations and a subsequent post-treatment. Commonly, the neoclassical theory in axisymmetric configurations is benchmarked by comparing the thermal drive coefficient  $k_{V_P}$  of the poloidal velocity  $V_P$  such that its equilibrium value reads  $V_{P,eq} = k_{V_P} \frac{\nabla T}{e B_T}$ . This exercise has already been done to benchmark GYSELA against the axisymmetric neoclassical theory [26]. In the case of non-axisymmetric magnetic perturbation, the transport matrix is not degenerate so this exercise can be extended to the toroidal velocity and also the radial electric field through the thermal drive coefficients  $k_{V_T}$  and  $k_N$  (cf Sec.3). The focus is here on the thermal drive of the toroidal velocity  $k_{V_T}$  and the neoclassical toroidal friction  $\nu_\phi$ . The methodologies to retrieve those coefficients in GYSELA and NEO are described in details in [Appendix B](#).

The comparison between the reduced model, GYSELA and NEO is performed in the range of collisionalities  $\nu^* \in [0.05 - 0.5]$  and for three ripple amplitudes:  $\delta = 0.5\%$ ,  $\delta = 1\%$  and  $\delta = 3\%$  at  $\varepsilon = 0.16$  and at a fixed  $N_c q = 1.4 \times 16$ . Fig.6 shows a  $\nu^*$  scan of  $k_{V_T}$  for those three ripple amplitudes obtained with GYSELA, NEO and the reduced model. A good agreement is found between GYSELA and NEO, where the average relative error is less than 10%. One of the conclusions is that the model collision operator used in GYSELA is well designed to describe neoclassical processes in the collisionality range  $\nu^* \in [0.05, 0.5]$ . Agreement between both codes and the neoclassical model in non-axisymmetric configurations is also obtained regarding the thermal drives. However the  $\delta = 0.5\%$  case with GYSELA does not seem to undergo the  $1/\nu^*$  trend followed by the model and NEO. One possible explanation is that this is the only case not dominated by  $Y < 1$  regimes (cf Sec.3). In other words, only this case is dominated by the effect of banana trapped particles. The point is that at low  $\nu^*$ , other regimes affecting only banana trapped particles exist [27, 28, 20]. They are sometimes labelled *super-banana* regimes, and correspond to the effect of higher order electric and magnetic drifts at bounce points, negligible at higher collisionalities. These regimes are not included in the present reduced model, nor in the present NEO simulations. However they are treated by GYSELA, which could explain this discrepancy at low collisionality. Note that NEO can actually compute those drifts [29]. They are here not included as the methodology used to retrieve  $k_{V_T}$  in NEO is not compatible if those drifts are activated. However, a few test with reasonable parameters for those drift were performed in NEO and showed a reduction of  $k_{V_T}$  even for collisionality around  $\nu^* \approx 0.1$ . Furthermore, still for the  $\delta = 0.5\%$  case, the discrepancy at high  $\nu^*$  between GYSELA and NEO is higher than for other cases. It remains unexplained, but still in an acceptable agreement with NEO and the reduced model.

Overall, the model which is derived in the large aspect ratio limit  $\varepsilon \ll 1$  is showing a surprisingly good agreement with NEO. Alleviating this latter limit actually already modifies neoclassical predictions in the axisymmetric case quite substantially. In fact, for the axisymmetric neoclassical theory, i.e. without ripple, the finite aspect-ratio corrections [30] can change  $k_{V_p} = 1.17$  to  $k_{V_p} \sim 0.6$  for  $\varepsilon = 0.17$  as observed in GYSELA [26]. To our knowledge, there is no analytical derivation of the aspect ratio correction for the neoclassical theory with ripple. The agreement is then unexpectedly good, especially at high  $\delta/\varepsilon$  where the condition  $Y < 1$  dominates.

The benchmark on the toroidal friction is shown in Fig.7. Both expressions derived in Sec.2, i.e.  $\nu_\varphi$  and  $\nu_\varphi^{\Gamma_N=0}$  are displayed on this figure. It was expected that the expression with the zero particle flux assumption, i.e.  $\nu_\varphi^{\Gamma_N=0}$ , should be the best approximation as the electron are adiabatic in these simulations. However, it appears that the toroidal friction obtained in GYSELA is about 5 to 10 times lower than  $\nu_\varphi^{\Gamma_N=0}$ . The expression of the neoclassical friction obtained by considering a finite particle flux however is in good agreement with GYSELA. The reason for this behaviour is still an open issue. An explanation could be that, rigorously, the particle flux in GYSELA is usually negligible but never strictly zero. Numerical noise always account for a residual  $\Gamma_N$  that could actually play a role for the neoclassical friction.

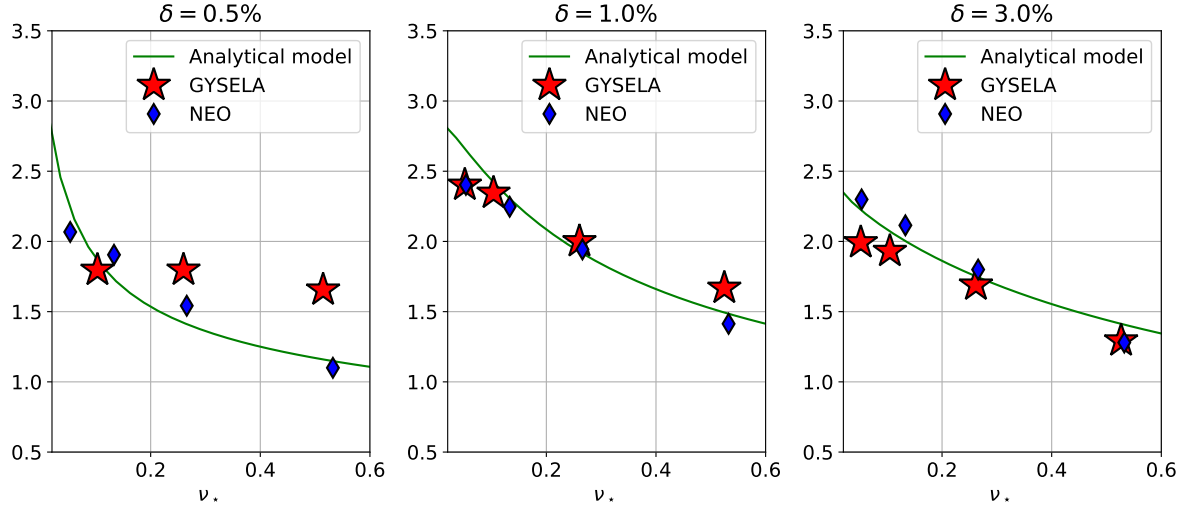


Figure 6: Collisionality scan of  $k_{VT}$  obtained with the reduced model, GYSELA and NEO for  $\delta = 0.5\%$  (left),  $\delta = 1\%$  (middle) and  $\delta = 3\%$  (right) at fixed  $\varepsilon = 0.16$  and  $N_c q = 16 \times 1.4$ .

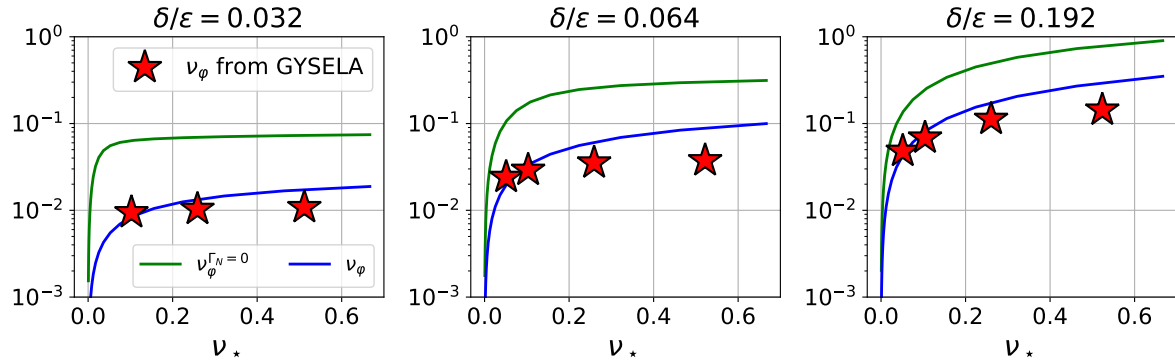


Figure 7: Collisionality scan of  $\nu_\phi$  and  $\nu_\phi^{\Gamma_{N=0}}$  obtained with the reduced model and GYSELA for  $\delta = 0.5\%$  (left),  $\delta = 1\%$  (middle) and  $\delta = 3\%$  (right) at fixed  $\varepsilon = 0.16$  and  $N_c q = 16 \times 1.4$ .

## 7. Impact of ripple on the poloidal velocity

On the basis of the previous section, our reduced model in the presence of non-axisymmetric perturbations captures well the collisional processes in a broad range of collisionalities that roughly encompasses  $\nu^* \in [10^{-2}, 10]$ . Except for the very edge of the plasma, likely dominated by wall interactions and orbit losses, this range is relevant for tokamaks. As mentioned in Sec.3, an unexpected result of the model is the effect of neoclassical processes with ripple on the thermal drive of the poloidal velocity  $k_{VP}$ . Fig.8 shows the  $k_{VP} = \frac{eB_T}{\nabla T} V_{P,eq}$  dependency with  $\delta/\varepsilon$  in the  $\nu^*$  range of interest.

Values of  $k_{VP}$  at  $\delta/\varepsilon = 10^{-2}$  approach the predictions of the axisymmetric neoclassical theory (in particular,  $k_{VP} = 1.17$  in the banana regime at low  $\nu^*$ , cf. Tab.I). At larger  $\delta/\varepsilon$ ,  $k_{VP}$  is found to decrease, with a sharp drop off around  $\delta/\varepsilon \sim 4 \cdot 10^{-2}$ . It can even reverse sign at low collisionality and become negative at  $\delta/\varepsilon \sim 2 \cdot 10^{-1}$ , meaning that the poloidal velocity itself

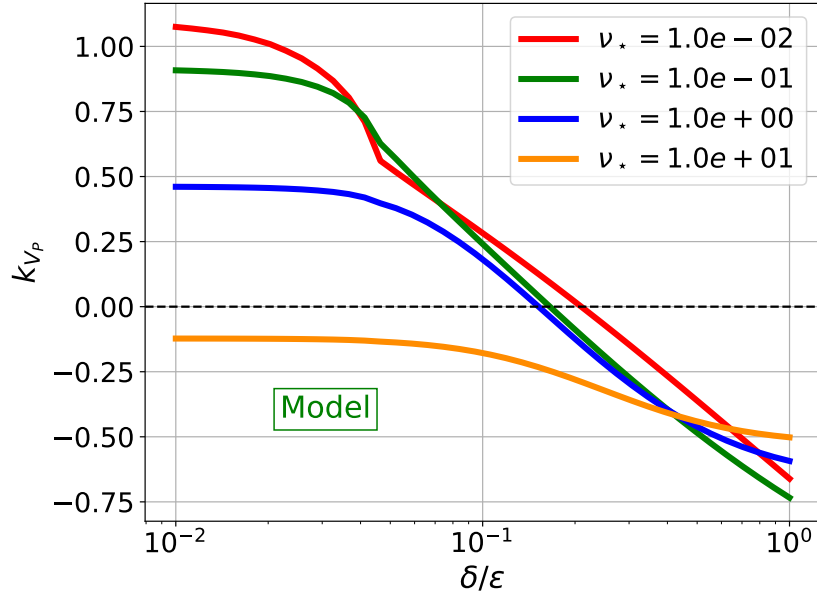


Figure 8: Scan in ripple amplitude of thermal drive  $k_{Vp}$  obtained with the reduced model for different  $\nu_*$  at  $N_c q = 16 \times 1.4$ .

could change sign. This suggests that locally trapped particles have an unexpectedly strong impact even for relatively low ripple amplitudes where local trapping is expected to be weak.

To ensure the validity of this result, some gyrokinetic simulations were run with different ripple amplitudes. As mentioned in Sec.6 and Appendix B, reaching a neoclassical equilibrium in gyrokinetic simulations is tricky. Indeed, the characteristic time for the system to reach such an equilibrium is of the order of the energy confinement time. Simulations that long would be numerically expensive and all the thermodynamical gradients would evolve significantly. Consequently, no equilibrium would be reached until density and temperature profiles are flat. For this reason, simulations are here used only to get a trend on the poloidal velocity evolution. The simulation parameters are the same as in Tab.2. The initial poloidal velocity is zero. Fig.9a then shows the radial profile of the poloidal velocity at  $\omega_c t = 5 \cdot 10^4$  for the ripple amplitude displayed Fig.9b. For the axisymmetric case, i.e. without ripple,  $V_p$  is essentially diamagnetic and thus follow the pressure gradient. With ripple, it appears that  $V_p$  increases significantly and monotonously with the  $\delta/\epsilon$  shape. This effect is particularly strong for the ‘‘Gaussian 3%’’ case near mid-radius and the Tore Supra case near plasma edge. Again, this figure only shows a trend and not accurate results. Especially, the temperature profile is not exactly the same in each simulation as the neoclassical heat flux (not discussed in this paper) is also modified by ripple and consequently temperature evolves differently for each case. However a clear effect on the poloidal velocity is clearly identified.

To have a more accurate idea of the effect of ripple in a realistic plasma, simulations involving more physics were run. They involve ITG turbulence, a heat source keeping a steady temperature profile and the presence of a limiter [31]. The main simulation parameters at  $r/a = 0.5$  are  $\rho_* = 1/250$ ,  $\nu_* = 0.1$ ,  $R_0/L_T = 8$ ,  $R_0/L_N = 2.2$  and  $q = 1.4$ . Fig.10 shows



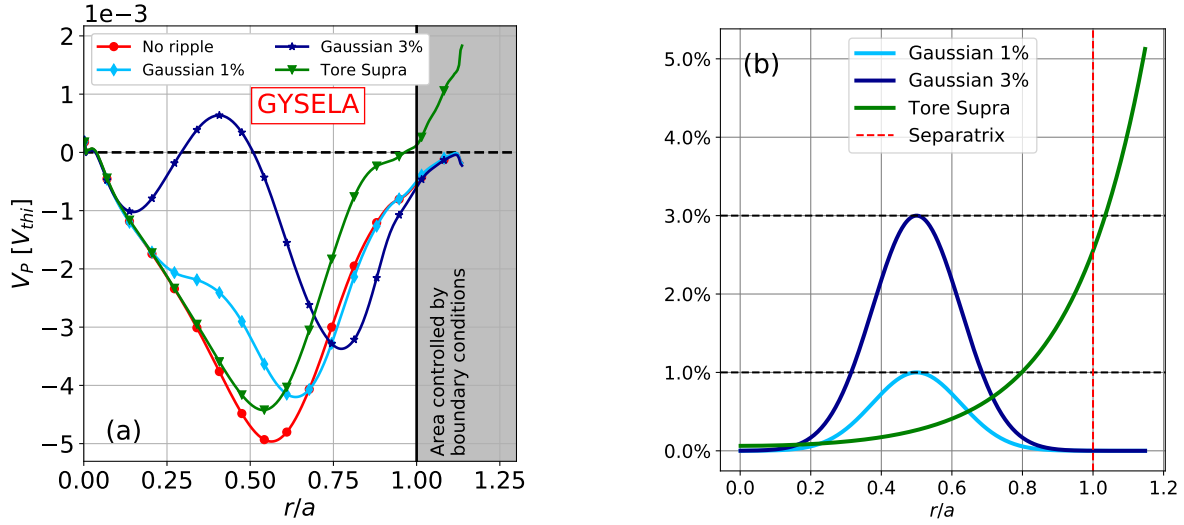


Figure 9: (a) Radial profile of poloidal velocity at  $\omega_c t < 5.10^4$  for the poloidally averaged ripple amplitudes in (b). Velocity is normalized to the ion thermal velocity  $V_{thi}$  in GYSELA.

the radial profiles of the coarse-grained poloidal velocity  $V_P$  for a case without ripple and another with the Tore Supra ripple perturbation. The coarse-graining procedure consists in a temporal averaging between  $6.10^4 < \omega_c t < 9.10^4$  and a radial sliding average with a window of about  $50\rho_i$ . This procedure is necessary to smooth the important fluctuations that appear in the presence of turbulence. As previously, these simulations are not fully converged as reaching a neoclassical equilibrium would require too many numerical resources. However, they provide reliable hints as to the long term behaviour of flows. As for the neoclassical cases discussed just before, the poloidal velocity without ripple is mainly diamagnetic up to an offset due to turbulent contributions. The case with the Tore Supra ripple amplitude displays the same increasing of  $V_P$  seen in Fig.9a. It shows that a realistic magnetic configuration could lead to  $V_P$  being affected strongly at the edge and mildly at mid-radius. The fact that realistic ripple impacts plasma this far from edge is that the  $\delta/\varepsilon$  ratio is not negligible near mid-radius. Ripple contribution to the poloidal velocity is then significant, especially at the plasma edge.

## 8. Conclusion

Magnetic ripple perturbations have been successfully implemented in GYSELA by modifying the effective hamiltonian. Key verification has been achieved through toroidal momentum conservation where a new term, the magnetic braking due to the ripple perturbation, becomes the dominant contribution at large ripple amplitude. GYSELA results have been benchmarked regarding the neoclassical processes occurring in presence of ripple thanks to an analytical neoclassical model and the NEO code. Good agreement with the NEO code is found, meaning GYSELA is able to properly describe ripple-induced neoclassical processes down to relatively low collisionality  $\nu^* \sim 0.05$ . GYSELA results are also found to well agree with the model which is derived in the low collisionality and large aspect ratio limit. One remarkable result

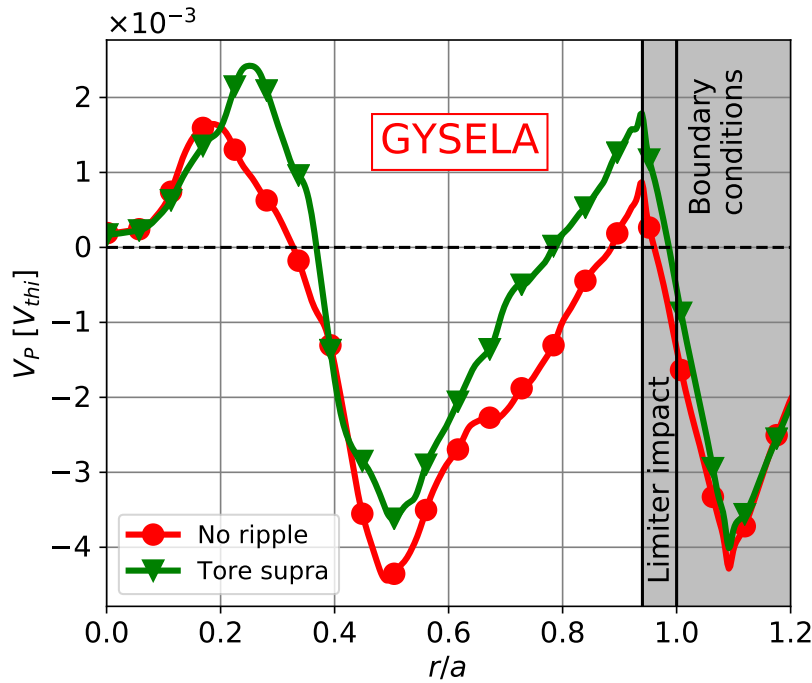


Figure 10: Radial profile of the coarse-gained poloidal velocity obtained in GYSELA simulations of ITG turbulence in the axisymmetric case (red) and for the Tore Supra ripple (green).

of the model, which is recovered in GYSELA simulations, is the change of sign of the poloidal velocity when increasing the ripple amplitude. Such behaviour was expected only for extremely (unrealistic) large values of ripple amplitude. Our findings show that the neoclassical contribution to the poloidal velocity is likely to change direction already at moderate ripple amplitudes. These conditions are expected to be met at the edge of tokamak plasmas like WEST.

## 9. Acknowledgments

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## Appendix A. Explicit expression of the transport matrix coefficients

This section gives the details of the terms that appears in the transport matrix in Sec.2. In the general case the ripple perturbation amplitude  $\delta$  depends on  $(r, \theta)$ . It is useful to separate this amplitude in a poloidal average  $\bar{\delta}(r) = \int \frac{d\theta}{2\pi} \delta(r, \theta)$  and a poloidal modulation  $\tilde{\delta}(r) = \delta(r, \theta) / \bar{\delta}$ .

The transport matrix is symmetrical. Its 6 independent elements are given by the following expressions:

$$d_n = \int_0^{+\infty} du \left(u - \frac{3}{2}\right)^n \mathcal{K}(u) \quad (\text{A.1})$$

$$\tilde{d}_n = \int_0^{+\infty} du \left(u - \frac{3}{2}\right)^n \tilde{\mathcal{K}}(u) \quad (\text{A.2})$$

$$\hat{d}_n = \int_0^{+\infty} du \left(u - \frac{3}{2}\right)^n \hat{\mathcal{K}}(u) \quad (\text{A.3})$$

Where

$$\mathcal{K}(u) = \sqrt{\frac{\pi}{2}} e^{-u} u^2 K_{tor,I}(r, u) \quad (\text{A.4})$$

$$\tilde{\mathcal{K}}(u) = \frac{32}{9} \left(\frac{2}{\pi}\right)^{3/2} \left(\frac{\bar{\delta}}{\varepsilon}\right)^{3/2} \frac{G_1}{v^*} e^{-u} u^{5/2} \frac{1}{\bar{v}(u)} \quad (\text{A.5})$$

$$+ 2 \left(\frac{2}{\pi}\right)^{3/2} \frac{1}{N_b q} \left(\frac{\bar{\delta}}{\varepsilon}\right)^2 \frac{1}{v^*} [1 - H(u - u_c)] e^{-u} u^{5/2} \frac{1}{\bar{v}(u)} K_{rip,II}(r, u) \quad (\text{A.6})$$

$$+ \sqrt{\frac{\pi}{2}} N_b q \left(\frac{\bar{\delta}}{\varepsilon}\right)^2 H(u - u_c) e^{-u} u^2 K_{st}(r, u) \quad (\text{A.7})$$

$$\hat{\mathcal{K}}(u) = \sqrt{\frac{\pi}{2}} (N_b q) \left(\frac{\bar{\delta}}{\varepsilon}\right)^2 e^{-u} u^2 K_{rip,I}(r, u) \quad (\text{A.8})$$

where  $H$  is the Heaviside function.

The normalized collision frequency [32] is defined as:

$$\bar{v}(u) = \frac{3}{4} \sqrt{2\pi} \frac{1}{u^{3/2}} \left( \Phi(u^{1/2}) - G(u^{1/2}) \right) \quad \text{where} \quad \begin{cases} \Phi(u) = \frac{2}{\sqrt{\pi}} \int_0^u dx \exp(-x^2) \\ G(u) = \frac{1}{2u^2} \left( \Phi(u) - u \frac{d\Phi(u)}{du} \right) \end{cases}$$

The functions  $K$  provide smooth transitions between various collision regimes:

$$K_{rip,I}(r, u) = \min \left( G'_0, G''_0 \frac{4}{\pi} \mathcal{I} \frac{v^*}{N_c q} \left( \frac{\varepsilon}{\bar{\delta}} \right)^{3/2} \frac{\bar{v}(u)}{u^{1/2}} \right) \quad (\text{A.9})$$

$$K_{tor,I}(r, u) = \min \left( 1, \frac{4}{\pi} \mathcal{I} v^* \frac{\bar{v}(u)}{u^{1/2}} \right) \quad \text{with } \mathcal{I} = 1.38 \quad (\text{A.10})$$

$$K_{rip,II}(r, u) = 1 + \frac{\pi^2}{8} v^* (N_c q)^2 \frac{\bar{v}(u)}{u^{1/2}} \quad (\text{A.11})$$

$$K_{st}(r, u) = \left( 1 + \frac{1}{\sqrt{\pi}} \frac{mv_{th}}{eB_0} \left( \frac{N_c q}{\varepsilon} \right)^{3/2} \frac{dq}{dr} \bar{\delta} u^{1/2} \right)^{-1} \quad (\text{A.12})$$

Where is defined the continuous functions  $\min(x, y) = \frac{xy}{x+y}$ .

The so-called form factors  $G'_0, G''_0, G_1$ , used to discriminate banana trapped particles and magnetic wells, are given by the relations:

$$\begin{aligned} G'_0(r) &= \int_{Y < 1} \frac{d\theta}{2\pi} \tilde{\delta}^2(r, \theta) \\ G''_0(r) &= \int_{Y < 1} \frac{d\theta}{2\pi} \tilde{\delta}^{1/2}(r, \theta) \\ G_1(r) &= \int_{Y < 1} \frac{d\theta}{\pi} \tilde{\delta}^{3/2}(r, \theta) \sin^2 \theta \end{aligned} \quad (\text{A.13})$$

Where  $Y(r, \theta) = \frac{\varepsilon |\sin \theta|}{N_c q \bar{\delta}} < 1$  is the condition of existence of ripple-induced magnetic wells.

The transition between neoclassic and stochastic regimes is considered through the following limit in energy:

$$u_c = \frac{R_0}{8\pi} \left( \frac{dq}{dr} \right)^{-1} \left( \frac{mv_{th}}{eB_0} \right)^{-2} \frac{q^{-5/2} \varepsilon^3}{N_c^{3/2} \bar{\delta}} \quad (\text{A.14})$$

## Appendix B. Specific methodology to retrieve $k_{V_T}$ and $\hat{v}_\varphi$ in GYSELA and NEO.

As the neoclassical theory gives the transport matrix, the quantity to compare between codes and reduced model are the  $(d_{ij})$  coefficients in GYSELA and NEO which only depend on  $\delta/\varepsilon, v^*$  and  $N_c q$ . Then, given the structure of Eq.2, one could determine those coefficients if enough  $(A_N, A_{V_T}, A_T)$  and  $(\Gamma_N, \Gamma_{V_T}, \Gamma_T)$  sets were known. In theory, only 3 sets would suffice for a given  $(\delta/\varepsilon, v^*, N_c q)$  set. However the least-square method used as linear solver proved to be inefficient and would probably need more statistics. In addition, the benchmark aims at verifying the range of validity of the theory for a wide range of ripple amplitudes and collisionalities. As discussed below, obtaining a set  $(A_N, A_{V_T}, A_T, \Gamma_N, \Gamma_{V_T}, \Gamma_T)$  linked to one combination of  $(\delta/\varepsilon, v^*, N_c q)$  basically amounts to running one simulation. The number of simulations required to retrieve all the matrix coefficient would then be enormous. Instead, only the thermal drives  $k_{V_T}, k_N$  and  $k_{V_p}$  (defined in Sec.3 are benchmarked.

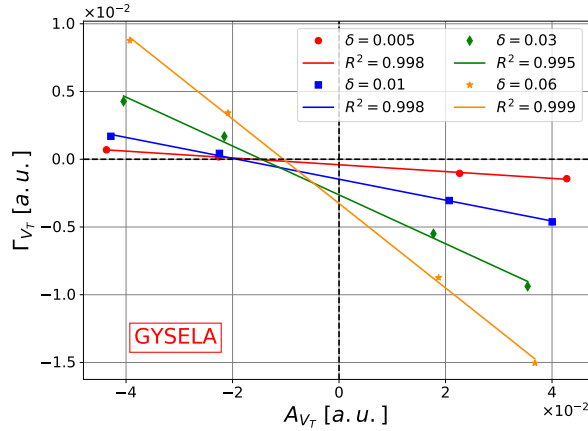


Figure B1: Magnetic braking force  $\Gamma_{V_T}$  versus  $A_{V_T}$  for different ripple amplitudes  $\delta$ . Each point corresponds to a simulation with different initial toroidal velocity profiles and represents the  $(A_{V_T}, \Gamma_{V_T})$  retrieved at  $r/a = 0.5$  after the GAMs at  $\nu^* = 0.1$ .

### Appendix B.1. The GYSELA code

In GYSELA, the main difficulty lies in the boundary conditions that can add extra effects not taken into account in the reduced model like orbit losses, momentum flux carried by waves or scrape-off layer interactions. For this reason, a radially gaussian ripple, as used in Sec.5, is considered. The radial location of interest is then chosen at  $r/a = 0.5$ . In GYSELA,  $(A_{V_T}, A_T)$  are inputs and  $\Gamma_{V_T}$  is an output. The temperature gradient  $A_T$  evolves slowly compared with  $A_{V_T}$  and  $\Gamma_{V_T}$ . In all simulations,  $A_T$  is then fixed at the same value and only the initial  $A_{V_T}$  profile is changed. This makes the numerical resolution even more simple and robust, as a linear regression on  $\Gamma_{V_T}(A_{V_T})$  gives  $\nu_\phi$  and  $k_{V_T}$  which are respectively associated to the slope and intersection with the x-axis. A  $(A_{V_T}, \Gamma_{V_T})$  set is retrieved after the GAM phase which is here longer than a few collision times in each simulation, the characteristic time needed for neoclassical effects to establish. This is illustrated in Fig.B1 obtained thanks to 16 simulations with four different ripple amplitudes, four toroidal velocities, and a single  $\nu^*$  profile.

This exercise has been done for four collisionality profiles for a total of 48 gyrokinetic simulations. All the input profiles are showed in Fig.B2.

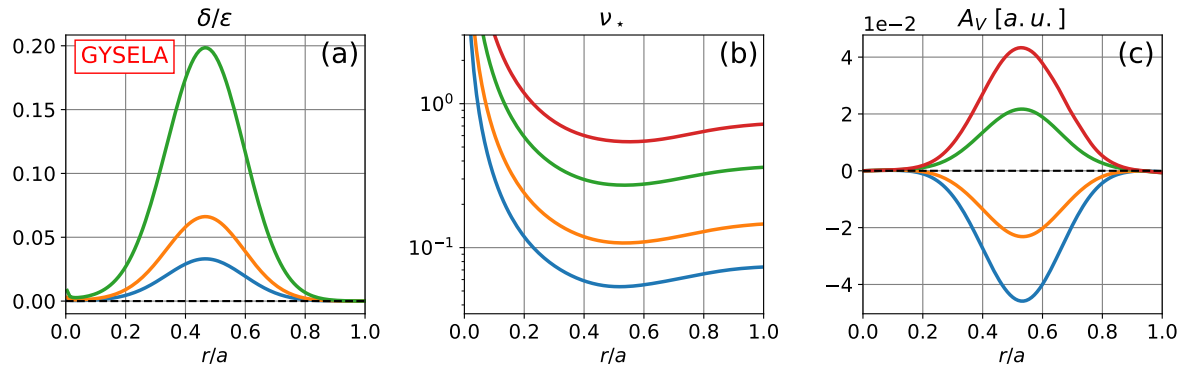


Figure B2: Initial radial profiles of the effective ripple amplitudes  $\delta/\varepsilon$  (a), the collisionalities  $\nu^*$  (b) and the normalized toroidal velocities  $A_{V_T}$  (c) used for the scan.

### Appendix B.2. The NEO code

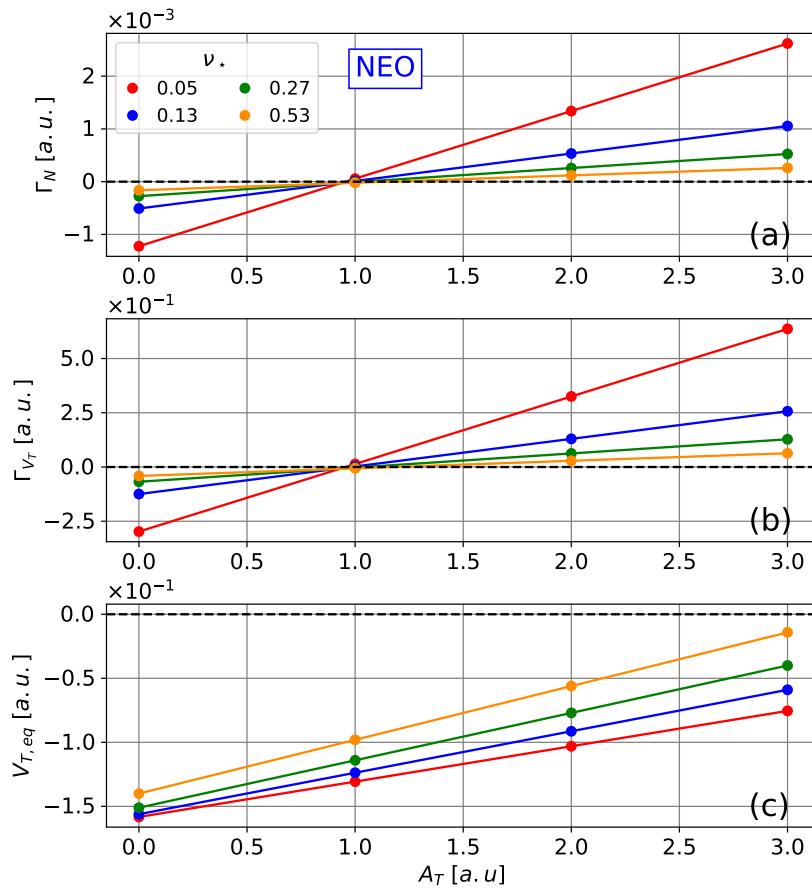


Figure B3: From NEO simulations: Equilibrium particle flux  $\Gamma_{eq}$  (a), magnetic braking force  $\mathcal{M}_{eq}$  (b) and mean toroidal velocity  $V_{T,eq}$  (c) for different collisionalities  $\nu^*$  at fixed  $\delta/\varepsilon = 0.03/0.16$  and  $N_c q = 16 \times 1.4$  over a wide range of  $A_T$ .

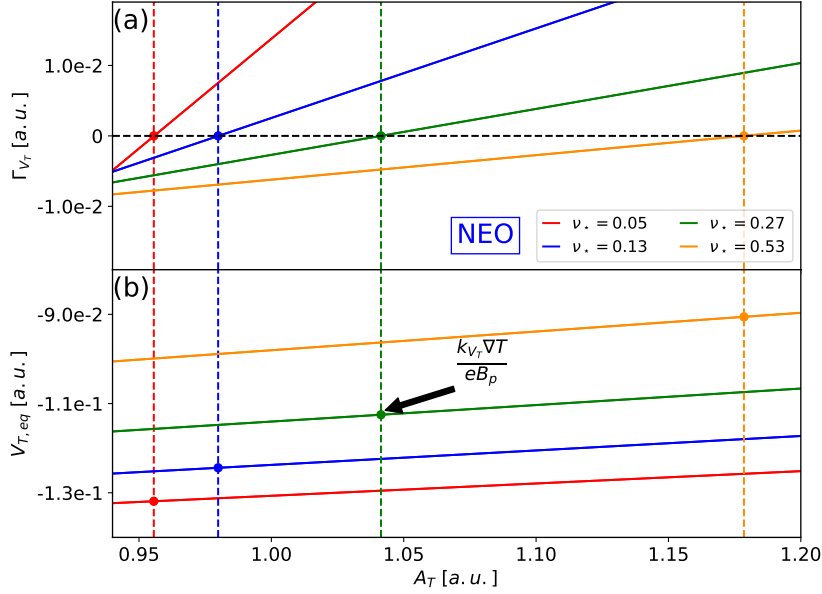


Figure B4: From NEO simulations: Magnetic braking force  $\mathcal{M}$  (a) and the toroidal velocity  $V_T$  (b) for different collisionalities  $\nu^*$  at fixed  $\delta/\varepsilon = 0.03/0.16$  and  $N_c q = 16 \times 1.4$ . The value of  $V_T$  at  $A_T$  for which  $\mathcal{M}$  cancels is directly linked to the neoclassical thermal drive  $k_{V_T}$ .

NEO is an Eulerian local code that solves the drift-kinetic equation with a linearized full Fokker-Planck collision operator [33, 34]. It is considered as a reference code for neoclassical predictions. The only approximation is the so-called *drift-ordering*  $\rho_* \ll 1$ . As NEO is able to handle non-axisymmetric flux surface (see [29] for more details), the following simulations uses the same ripple perturbations used in GYSELA. NEO has to be benchmarked in similar regime to GYSELA for the benchmark. This require zero particle flux  $\Gamma_N = 0$ , as imposed by the adiabatic electron response in GYSELA. However, contrary to the axisymmetric version, here NEO does not enforce ambipolarity so that  $\Gamma_N \neq 0$  in the general case. Notice that, according to Eq.11 at equilibrium, the  $\Gamma_N = 0$  condition also imposes  $\Gamma_{V_T} = 0$ , hence  $V_T = k_{V_T} \nabla T / eB_p$  which is an output in NEO. It follows that, in such regime, this methodology does not allow one to retrieve the neoclassical toroidal friction  $\nu_\phi$  with NEO. To retrieve  $k_{V_T}$ , one must first find an input set of  $(A_N, A_T)$  for which  $\Gamma_N = 0$ . There is no quasi-neutrality constraint in NEO, and consequently no onset of non-linearity, even small, due to the electric potential as in GYSELA. For this reason, the relation between  $(A_N, A_T)$  and  $(\Gamma_N, \Gamma_{V_T}, \Gamma_T)$  is linear in NEO. Taking advantage of this,  $A_N$  is set at a constant value while a scan on  $A_T$  is performed. The resulting outputs of interest, i.e.  $\Gamma_N$ ,  $\Gamma_{V_T}$  and  $A_{V_T}$  then exhibit a linear dependency with respect to  $A_T$ , cf Fig.B3. Note that this method could also have been done by performing a scan on  $A_N$  at fixed  $A_T$ . The  $\Gamma_N = 0$  condition is then obtained by drawing the  $\Gamma_N(A_T)$  line using a few points, each obtained with one simulation, and by retrieving the  $A_T$  value that cancels the particle flux. At this value  $\Gamma_{V_T} = 0$  as expected and already discussed, so the output toroidal velocity is equal to  $A_{V_T} = k_{V_T} A_T$ . This provides the value of  $k_{V_T}$ . Fig.B4 illustrates this procedure, that is repeated for each  $\nu^*$  considered for a given ripple amplitude  $\delta$ .

## References

- [1] Fenzi C, Garbet X, Trier E, Hennequin P, Bourdelle C, Aniel T, Colledani G, Devynck P, Gil C, Gürcan Ö, Manenc L, Schneider M and and J L S 2011 *Nuclear Fusion* **51** 103038 ISSN 0029-5515
- [2] Nave M F F, Johnson T, Eriksson L G, Crombé K, Giroud C, Mayoral M L, Ongena J, Salmi A, Tala T and Tsallas M 2010 *Physical Review Letters* **105** 105005
- [3] Goldston R J, White R B and Boozer A H 1981 *Physical Review Letters* **47** 647–649 ISSN 0031-9007
- [4] Helander P and Sigmar D J 2002 *Cambridge University Press*
- [5] Hinton F L and Hazeltine R D 1976 *Reviews of Modern Physics* **48** 239–308
- [6] Yushmanov P 1982 *Nuclear Fusion* **22** 315–324 ISSN 0029-5515, 1741-4326
- [7] Yushmanov P 1983 *Nuclear Fusion* **23** 1599–1612 ISSN 0029-5515, 1741-4326
- [8] Tsypin V S, Mikhailovskii A B, Galvão R M O, Nascimento I C, Tendler M, de Azevedo C A and de Assis A S 1998 *Physics of Plasmas* **5** 3358–3365 ISSN 1070-664X, 1089-7674
- [9] Shaing K C, Rome J A and Fowler R H 1984 *Physics of Fluids* **27** 1 ISSN 00319171
- [10] Shaing K C, Hirshman S P and Callen J D 1986 *Physics of Fluids* **29** 521 ISSN 00319171
- [11] Shaing K C 1983 *Physics of Fluids* **26** 3315 ISSN 00319171
- [12] Kovrizhnykh L 1984 *Nuclear Fusion* **24** 851–936 ISSN 0029-5515, 1741-4326
- [13] Connor J and Hastie R 1973 *Nuclear Fusion* **13** 221–225 ISSN 0029-5515, 1741-4326
- [14] Mynick H E 2006 *Physics of Plasmas* **13** 058102 ISSN 1070-664X
- [15] Grua P and Roubin J P 1990 *Nuclear Fusion* **30** 1499–1509 ISSN 0029-5515, 1741-4326
- [16] Beidler C D, Kolesnichenko Y I, Marchenko V S, Sidorenko I N and Wobig H 2001 *Physics of Plasmas* **8** 2731–2738 ISSN 1070-664X, 1089-7674
- [17] Garbet X, Abiteboul J, Trier E, Gürcan Ö, Sarazin Y, Smolyakov A, Allfrey S, Bourdelle C, Fenzi C, Grandgirard V, Ghendrih P and Hennequin P 2010 *Physics of Plasmas* **17** 072505 ISSN 1070-664X, 1089-7674
- [18] Hazeltine R D 1974 *The Physics of Fluids* **17** 961–968 ISSN 0031-9171
- [19] Kim K, Park J K, Boozer A H, Menard J E, Gerhardt S P, Logan N C, Wang Z R, Kramer G J, Burrell K H and Garofalo A M 2014 *Nuclear Fusion* **54** 073014 ISSN 0029-5515
- [20] Park J k, Boozer A H and Menard J E 2009 *Physical Review Letters* **102** 065002
- [21] Sun Y, Liang Y, Shaing K C, Koslowski H R, Wiegmann C and Zhang T 2011 *Nuclear Fusion* **51** 053015 ISSN 0029-5515
- [22] Grandgirard V, Abiteboul J, Bigot J, Cartier-Michaud T, Crouseilles N, Dif-Pradalier G, Ehrlacher C, Esteve D, Garbet X, Ghendrih P, Latu G, Mehrenberger M, Norscini C, Passeron C, Rozar F, Sarazin Y, Sonnendrücker E, Strugarek A and Zarzoso D 2016 *Computer Physics Communications* **207** 35–68 ISSN 00104655
- [23] Donnel P, Garbet X, Sarazin Y, Grandgirard V, Asahi Y, Bouzat N, Caschera E, Dif-Pradalier G, Ehrlacher C, Ghendrih P, Gillot C, Latu G and Passeron C 2019 *Computer Physics Communications* **234** 1–13 ISSN 00104655
- [24] Matsuoka S, Idomura Y and Satake S 2017 *Physics of Plasmas* **24** 102522 ISSN 1070-664X
- [25] Abiteboul J, Garbet X, Grandgirard V, Allfrey S J, Ghendrih P, Latu G, Sarazin Y and Strugarek A 2011 *Physics of Plasmas* **18** 082503 ISSN 1070-664X, 1089-7674
- [26] Dif-Pradalier G, Diamond P H, Grandgirard V, Sarazin Y, Abiteboul J, Garbet X, Ghendrih P, Latu G, Strugarek A, Ku S and Chang C S 2011 *Physics of Plasmas* **18** 062309 ISSN 1070-664X
- [27] Shaing K C, Sabbagh S A and Chu M S 2008 *Plasma Physics and Controlled Fusion* **51** 035009 ISSN 0741-3335
- [28] Shaing K C, Sabbagh S A and Chu M S 2009 *Plasma Physics and Controlled Fusion* **51** 055003 ISSN 0741-3335
- [29] Belli E A and Candy J 2015 *Plasma Physics and Controlled Fusion* **57** 054012 ISSN 0741-3335
- [30] Kim Y B, Diamond P H, Biglari H and Callen J D 1991 *Physics of Fluids B: Plasma Physics* **3** 384–394 ISSN 0899-8221
- [31] Caschera E, Dif-Pradalier G, Ghendrih P, Grandgirard V, Asahi Y, Bouzat N, Donnel P, Garbet X, Latu G,



- Passeron C and Sarazin Y 2018 *Journal of Physics: Conference Series* **1125** 012006 ISSN 1742-6596
- [32] Hirshman S P and Sigmar D J 1976 *Physics of Fluids* **19** 1532 ISSN 00319171
- [33] Belli E A and Candy J 2008 *Plasma Physics and Controlled Fusion* **50** 095010 ISSN 0741-3335
- [34] Belli E A and Candy J 2011 *Plasma Physics and Controlled Fusion* **54** 015015 ISSN 0741-3335