



HAL
open science

Correcting biases in tropical cyclone intensities in low-resolution datasets using dynamical systems metrics

Davide Faranda, Gabriele Messori, Stella Bourdin, Mathieu Vrac, Soulivanh Thao, Jacopo Riboldi, Sebastien Fromang, Pascal Yiou

► To cite this version:

Davide Faranda, Gabriele Messori, Stella Bourdin, Mathieu Vrac, Soulivanh Thao, et al.. Correcting biases in tropical cyclone intensities in low-resolution datasets using dynamical systems metrics. 2022. hal-03631098v1

HAL Id: hal-03631098

<https://hal.science/hal-03631098v1>

Preprint submitted on 5 Apr 2022 (v1), last revised 17 Apr 2023 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 **Correcting biases in tropical cyclone intensities in low-resolution datasets**
2 **using dynamical systems metrics**

3 Davide Faranda*

4 *Laboratoire des Sciences du Climat et de l'Environnement, CEA Saclay l'Orme des Merisiers,*
5 *UMR 8212 CEA-CNRS-UVSQ, Université Paris-Saclay & IPSL, 91191, Gif-sur-Yvette, France;*
6 *London Mathematical Laboratory, 8 Margravine Gardens, London, W6 8RH, UK; LMD/IPSL,*
7 *Ecole Normale Supérieure, PSL research University, 75005, Paris, France*

8 Gabriele Messori

9 *Department of Earth Sciences and Centre of Natural Hazards and Disaster Science (CNDS),*
10 *Uppsala University, Uppsala, Sweden; Department of Meteorology and Bolin Centre for Climate*
11 *Research, Stockholm University, Stockholm, Sweden.*

12 Stella Bourdin, Mathieu Vrac, Soulivanh Thao

13 *Laboratoire des Sciences du Climat et de l'Environnement, CEA Saclay l'Orme des Merisiers,*
14 *UMR 8212 CEA-CNRS-UVSQ, Université Paris-Saclay & IPSL, 91191, Gif-sur-Yvette, France*

15 Jacopo Riboldi

16 *Department of Earth Sciences, Uppsala University, Uppsala, Sweden.*

17 Sebastien Fromang, Pascal Yiou

¹⁸ *Laboratoire des Sciences du Climat et de l'Environnement, CEA Saclay l'Orme des Merisiers,*
¹⁹ *UMR 8212 CEA-CNRS-UVSQ, Université Paris-Saclay & IPSL, 91191, Gif-sur-Yvette, France*

²⁰ **Corresponding author address: Davide Faranda, Laboratoire des Sciences du Climat et de*
²¹ *l'Environnement, CEA Saclay l'Orme des Merisiers, UMR 8212 CEA-CNRS-UVSQ, Université*
²² *Paris-Saclay & IPSL, 91191, Gif-sur-Yvette, France*
²³ *E-mail: davide.faranda@lsce.ipsl.fr*

ABSTRACT

24 Although the life-cycle of tropical cyclones is relatively well understood,
25 many of the underlying physical processes occur at scales below those re-
26 solved by global climate models (GCMs). Projecting future changes in trop-
27 ical cyclone characteristics thus remains challenging. We propose a method-
28 ology, based on dynamical system metrics, to reconstruct the statistics of cy-
29 clone intensities in coarse-resolution datasets, where maximum wind speed
30 and minimum sea-level pressure may not be accurately represented. We base
31 our analysis on 411 tropical cyclones occurring between 2010 and 2020, using
32 both ERA5 reanalysis data and observations from the HURDAT2 database, as
33 well as a control simulation of the IPSL-CM6A-ATM-ICO-HR model. Using
34 ERA5 data, we compute two dynamical system metrics related to the num-
35 ber of degrees of freedom of the atmospheric flow and to the coupling be-
36 tween different atmospheric variables, namely the local dimension and the
37 co-recurrence ratio. We then use HURDAT2 data to develop a univariate
38 quantile–quantile bias correction conditioned on these two metrics, as well
39 as a multivariate correction method. The conditional approach outperforms a
40 conventional univariate correction of the sea-level pressure data only, pointing
41 to the usefulness of the dynamical systems metrics introduced. We then show
42 that the multivariate approach can be used to recover a realistic distribution of
43 cyclone intensities from comparatively coarse-resolution model data.

44 **1. Introduction**

45 Tropical cyclones are among the most devastating natural disasters, often causing fatalities and
46 extensive economic damage (Smith and Katz 2013; Grinsted et al. 2019). They include several
47 collateral hazards that can have significant impacts on people and property, such as rogue waves,
48 flooding, extreme winds, and tornadoes. Combined, these weather events interact to increase the
49 likelihood of loss of life and extensive property damage. The bulk of costs related to tropical
50 cyclones stems from the few, most intense episodes (Emanuel 2021): thus, it is crucial to model
51 such cyclones correctly. Unfortunately, even state-of-the-art global and regional climate models
52 struggle to reproduce the dynamics of the most severe tropical cyclones (Camargo and Wing 2016;
53 Roberts et al. 2020b). This is mainly due to their insufficient resolution, as a grid spacing of the
54 order of few kilometers is needed to model intense cyclones (Rotunno et al. 2009; Moon et al.
55 2020). Downscaling techniques can be employed to alleviate this issue, but their use is limited by
56 computational costs; furthermore, different downscaling approaches can lead to diverging conclu-
57 sions (e.g., Caron et al. 2011; Lee et al. 2020; Knutson et al. 2020; Emanuel 2021). The situation is
58 complicated by the reduced length of records in available tropical cyclone data sets, which mostly
59 rely on satellite data not available before the '80s (Chang and Guo 2007): this limitation hin-
60 ders the emergence from decadal variability of significant changes in tropical cyclone properties
61 (Knutson et al. 2019).

62 While mid-latitudes synoptic dynamics mostly originate from the chaotic structure of the mo-
63 tions associated with baroclinic instability (Lorenz 1990; Schubert and Lucarini 2015), tropical
64 cyclones are characterized by a rapid organization of convectively unstable flows whose dynamics
65 is turbulent and highly sensitive to boundary conditions (Muller and Roms 2018; Carstens and
66 Wing 2020). Here, we investigate whether it may be possible to exploit this high level of flow

67 organization to obtain reliable statistics of intense tropical cyclones from relatively coarse-gridded
68 atmospheric data. To achieve this, we compute two metrics that describe cyclones as states of a
69 chaotic high-dimensional dynamical system. The first, d , reflects the dimension (i.e. the num-
70 ber of active degrees of freedom) of an instantaneous state of the cyclone. The second, α , is
71 a proxy for the instantaneous coupling between different atmospheric variables. These metrics
72 have recently provided insights on a number of geophysical phenomena, including transitions be-
73 tween transient metastable states of the mid-latitude atmosphere (Faranda et al. 2017; Hochman
74 et al. 2021), drivers and predictability of extreme events (Hochman et al. 2019; De Luca et al.
75 2020; Faranda et al. 2020), palaeoclimate attractors (Brunetti et al. 2019; Messori and Faranda
76 2021) slow earthquake dynamics (Gualandi et al. 2020) and changes in mid-latitude atmospheric
77 predictability under global warming (Faranda et al. 2019). A benefit over previous dynamical sys-
78 tems approaches (e.g Wolf et al. 1985; Cao 1997), is that these metrics can be easily applied to
79 computationally demanding datasets, such as climate reanalyses or climate models.

80 Applications of d and α in the literature have used a Eulerian approach over a fixed spatio-
81 temporal domain, rather than tracking the evolution of specific physical phenomena. Here, we
82 apply for the first time the two metrics in a semi-Lagrangian perspective to characterize the struc-
83 ture of tropical cyclones in the $d - \alpha$ phase-space. A semi-Lagrangian dynamical systems frame-
84 work is particularly convenient to study the complex behavior of convectively unstable flow sys-
85 tems (Crisanti et al. 1991; Vulpiani 2010), such as tropical convection. The signature of intense
86 cyclones propagates through scales due to the presence of inverse cascades as suggested by Levich
87 and Tzvetkov (1985) and, more recently, by Faranda et al. (2018); Dunkerton et al. (2009); Tang
88 et al. (2015). Such inverse cascades are associated with potential–kinetic energy conversions and
89 are particularly notable for intense storms (Bhalachandran et al. 2020). Such energy conversions
90 are related to the deep convection in the eyewall, that redistributes to the upper troposphere the

91 enthalpy gained thanks to surface heat fluxes in the boundary layer: this local process contributes
92 to the generation of the storms' warm core and, ultimately, to wind intensification at the larger,
93 sub-synoptic scale of the storm (Emanuel 1986). We hypothesise that these exchanges have a
94 signature in both the large-scale horizontal velocity and potential vorticity fields, which are rea-
95 sonably well-represented in coarse simulations or reanalyses. If this indeed holds, it would enable
96 a correction conditioned on d and α of cyclone intensity statistics in datasets where we cannot
97 explicitly access the spatial scales underlying the dynamics of intense tropical cyclones.

98 We base our analysis on best track cyclone data from the HURDAT2 database, which we take
99 as ground truth, ERA5 reanalysis data and a high-resolution global climate model simulation. We
100 compute the two dynamical systems metrics using the kinetic energy uv and the potential vorticity
101 field PV at the 850 hPa level. The rationale for using these variables follows the works of Peng
102 et al. (2012) and Fu et al. (2012). The 850 hPa level is chosen because it lies in-between the
103 planetary boundary layer and the middle troposphere, where most of the potential–kinetic energy
104 conversions occur in tropical cyclones (Camargo et al. 2007). The horizontal kinetic energy
105 uv is relevant to the study of tropical cyclones because of its direct connection with the wind
106 speed and with the phases of rapid intensification/decay of the cyclones (Krishnamurti et al. 2005).
107 Lower-tropospheric vorticity is frequently used to detect tropical cyclone seeds and relates to their
108 intensification (Davis et al. 2008; Ikehata and Satoh 2021), and PV integrates this knowledge by
109 taking implicitly into account the strength of the warm core, which is related to the vertical stability
110 of the cyclone eye.

111 Our study is organized as follows: first, we describe the data, variables for cyclone dynamics
112 and the theoretical bases supporting the computation of the dynamical system metrics. Then, we
113 show the general characteristics of the phase space of tropical cyclones in reanalysis data for the
114 different variables, and use these to define a quantile–quantile correction approach for cyclone in-

115 tensities conditioned on the two dynamical systems metrics. The correction is performed on ERA5
116 reanalysis, using sea-level pressure (SLP) from HURDAT2 as ground truth. We then conduct an
117 out-of-sample test comparing this approach to an unconditional quantile–quantile correction. Fi-
118 nally, we test this correction on data from a GCM.

119 **2. Data**

120 *a. HURDAT2 tropical cyclone data*

121 We follow 197 North Atlantic and 214 Eastern North Pacific tropical cyclones that occurred
122 between 2010 and 2020, using the HURDAT2 Best Track Data (Landsea and Franklin 2013). We
123 use six-hourly information on the location and central pressure of the cyclones, as well as their
124 time of landfall.

125 *b. Representation of tropical cyclones in ERA5 data*

126 We base our analysis on instantaneous¹ $uv = u^2 + v^2$, *SLP* and *PV* at 850hPa from the ERA5
127 reanalysis (Hersbach et al. 2020), sampled every 6h and additionally whenever the HURDAT2
128 database displays a cyclone landfall entry – corresponding to a shift in the cyclone dynamics. The
129 horizontal resolution of the data is 0.25° . We also make use of ERA5 data coarse-gridded to a
130 0.5° horizontal resolution using a nearest neighbour approach. In the vast majority of cases, the
131 SLP difference in the four ERA5 gridpoints around the cyclone core is 1 hPa or less. Using a
132 nearest neighbour approach thus provides similar results to other coarse-graining approaches. The
133 coarse-grained dataset will be hereinafter referred to as ERA5 cg, and will be used to evaluate
134 the sensitivity of our conclusions to the resolution of the dataset. When comparing ERA5 to

¹We follow here ECMWF’s terminology, see: <https://confluence.ecmwf.int/pages/viewpage.action?pageId=82870405ERA5:datadocumentation->■

135 HURDAT2, we search for the SLP minimum in ERA5 within a region of 5° of the HURDAT2
136 cyclone location.

137 There is often a large discrepancy between the minimum SLP reported in HURDAT2 and that in
138 ERA5 (Fig. 1a,c). Especially for the most intense cyclones, ERA5 systematically underestimates
139 minimum SLP. The biases are larger in the Eastern North Pacific than in the North Atlantic basin
140 (cf. Fig. 1a and 1c). This affects the capability of ERA5 to accurately assign each tropical cyclone
141 to the appropriate Saffir-Simpson category (Simpson and Saffir 1974), which we here use relative
142 to SLP rather than maximum sustained wind (see Table 1. Indeed, recent research argues that
143 SLP is more closely related to cyclone damage than maximum sustained winds (Klotzbach et al.
144 2020). In particular, ERA5 has a negative bias in the number of tropical cyclones stronger than
145 category 2, namely $\min(\text{SLP}) < 980$ hPa. While this bias is present in both the North Atlantic and
146 Eastern North Pacific basins, in the latter the biases for the most intense cyclones appear more
147 severe: this might be due to the more intermittent aircraft reconnaissance activity in that basin
148 with respect to the North Atlantic (Knaff et al. 2021). The coarse-graining operation does not
149 notably affect the distribution of cyclone SLP minima (Fig. 1b, d), likely because of the above-
150 mentioned weak SLP variations in the ERA5 dataset in the gridboxes immediately surrounding the
151 cyclone core. We underline that the two ERA5 horizontal resolutions used here are of the same
152 order of magnitude as those of the HighResMIP (High Resolution Model Intercomparison Project,
153 Haarsma et al. 2016) and PRIMAVERA models, which are amongst the current best tools to study
154 climate change impacts on tropical cyclones (Roberts et al. 2020a).

155 *c. Representation of tropical cyclones in IPSL-CM6A-ATM-ICO-HR model data*

156 We use 6-hourly data from the IPSL-CM6A-ATM-ICO model at a horizontal resolution of 0.5° .
157 The simulation we use is a HighResMIP (Haarsma et al. 2016) run lasting for 64 years (1950-

158 2014). We use the TC tracking algorithm from Ullrich et al. (2021). We additionally remove
159 tracks starting at latitudes above 30° , to minimise incorrect classification of extratropical lows as
160 tropical systems.

161 The model produces a total of 327 tropical cyclones in the North Atlantic and 989 in the Eastern
162 North Pacific (defined following Knutson et al. 2020). Among them, 150 are randomly sampled
163 for each basin, which is deemed a sufficiently large statistical sample for our analysis. The total
164 number of cyclones reproduced by the IPSL-CM6A-ATM-ICO-HR model is in line with that of
165 other HighResMIP models (cf. Roberts et al. 2020b).

166 Much like ERA5, the model has a systematic bias in the minimum SLP of the cyclones. A
167 comparison of the distribution of HURDAT2 SLPs with those in the model and in the ERA5 and
168 ERA5 cg data (Fig. 1b, d), suggests that current state-of-the-art GCMs and ERA5 both encounter
169 difficulties when it comes to representing the intensity of tropical cyclones (Kim et al. 2018).
170 Specifically, all three datasets have a sparsely populated left tail of the minimum SLP distribution
171 and, especially for the North Atlantic basin, a different mode when compared to HURDAT2.

172 **3. A dynamical systems view of tropical cyclones**

173 Each instantaneous state of the cyclone, as represented by a given atmospheric variable, cor-
174 responds to a point along a phase-space trajectory representing the evolution of the system. We
175 sample this trajectory at discrete intervals determined by the temporal resolution of our data. In
176 our analysis, for every cyclone we adopt 6-hourly domains of size $\sim 1200 \text{ km} \times 1200 \text{ km}$ (41×41
177 grid points in ERA5 or $\sim 5^\circ$ latitude \times 5° longitude), centred at every timestep on the HURDAT2
178 cyclone location for ERA5 and centered at minimum of sea-level pressure for the simulations. We
179 then consider PV and uv at 850 hPa in this domain. Our aim is to diagnose the dynamical proper-
180 ties of the instantaneous (in time) and local (in phase-space) states of the cyclone, as represented

181 by the chosen atmospheric variable and chosen geographical domain (physical space in Fig. 2).
 182 To do so, we leverage two metrics issuing from the combination of extreme value theory with
 183 Poincaré recurrences (Freitas et al. 2010; Lucarini et al. 2012, 2016).

184 We consider the sequences of PV maps, uv maps and pairs maps of the two variables at all
 185 timesteps i for all tropical cyclones in our dataset, always centred on their location. The maps are
 186 normalised by the respective norms, and we refer to them as PV_i , uv_i and X_i respectively. We thus
 187 construct a semi-Lagrangian framework tracking each cyclone. We take each pair of maps X_i in
 188 turn as the reference state ζ in our calculation. We then define logarithmic returns as:

$$g(X_i, \zeta) = -\log[\text{dist}(X_i, \zeta)] \quad (1)$$

189 Here, dist is the Euclidean distance between the pairs of maps X_i , but it can be any distance
 190 function between two vectors. Since dist tends to zero as pairs of X_i s increasingly resemble each
 191 other, the series of logarithmic returns g_i takes large values for X_i closely resembling ζ .

192 We next define exceedances as $u(\zeta) = \{t, g(X_i, \zeta) > s(q, \zeta)\}$, where $s(q, \zeta)$ is a high thresh-
 193 old corresponding to the q^{th} quantile of $g(X_i, \zeta)$. These are effectively the previously-mentioned
 194 Poincaré recurrences, for the chosen state ζ (phase space in Fig. 2). The Freitas-Freitas-Todd the-
 195 orem (Freitas et al. 2010; Lucarini et al. 2012) states that the cumulative probability distribution
 196 $F(u, \zeta)$ can be approximated by the exponential member of the Generalised Pareto Distribution.
 197 We thus have that:

$$F(u, \zeta) \simeq \exp\left[-\frac{u(\zeta)}{\sigma(\zeta)}\right] \quad (2)$$

198 The parameters u and σ , the scale parameter of the Generalized Pareto Distribution, depend on
 199 the chosen state ζ . From the above, we can define the local dimension d as: $d(\zeta) = 1/\sigma(\zeta)$, with
 200 $0 < d < +\infty$.

201 Next, we introduce logarithmic returns and high thresholds separately for PV and uv as $g(PV_i)$,
 202 $s_{PV}(q)$ and $g(uv_i)$, $s_{uv}(q)$ respectively. These allow us to define the co-recurrence ratio (α) as:

$$\alpha(\zeta) = \frac{v[g(PV_i) > s_{PV}(q) \mid g(uv_i) > s_{uv}(q)]}{v[g(X_i) > s_{PV}(q)]} \quad (3)$$

203 with $0 \leq \alpha \leq 1$. Here, $v[-]$ is the number of events satisfying condition $[-]$, and all other
 204 variables are defined as before. By definition, α is symmetric with respect to the choice of variable
 205 (PV or uv), since $v[g(PV_i) > s_{PV}(q)] \equiv v[g(uv_i) > s_{uv}(q)]$.

206 While the derivation of d and α may seem very abstract, the two metrics can be related
 207 intuitively to the physical properties of the tropical cyclones. d is a proxy for the active number of
 208 degrees of freedom of the cyclones' instantaneous states, while α measures the coupling between
 209 different variables. The relationship between dynamical systems metrics and the structure of
 210 tropical cyclones is elucidated in Figure 3. d and α are anti-correlated: low dimensionality is
 211 generally associated to a high coupling between the PV and uv maps, while the opposite holds
 212 for high dimensionality (Fig. 3a). Most of the considered time steps fall in a regime of relatively
 213 low dimensionality relative to the number of gridpoints in the lagrangian domain (41^2), and of
 214 low coupling. We first investigate the uv and PV atmospheric patterns corresponding to these
 215 timesteps. To do so, we consider all points below the 0.95 quantiles of the d and α distributions,
 216 corresponding to thresholds of $d_H=30$ and $\alpha_H=0.35$. These data show a clear, albeit relatively
 217 weak, cyclonic structure in PV , with values peaking at ~ 2 PVU (Fig. 3b). The uv pattern also
 218 reflects a cyclonic structure, with values peaking at $\sim 12 \text{ m s}^{-1}$ (Fig. 3e). The picture is radically
 219 different for cyclone timesteps with $d > d_H$ or $\alpha > \alpha_H$ (Fig. 3c,d,f,g). High values of d feature

220 a smaller, weaker PV core than the bulk of the data, which is reflected in low values of kinetic
221 energy (Fig. 3c,f). High values of α instead correspond to an intense cyclonic PV core and a
222 correspondingly intense KE structure around it (Figs. 3d,g). The same qualitative features emerge
223 also for the ERA5 cg and HighResMIP data (not shown).

224

225 These results reflect the strong coupling between dynamic and thermodynamic fields in intense
226 cyclones (e.g., Emanuel 1986, 1997; Montgomery and Smith 2017) – the thermodynamics partly
227 encapsulated by the PV structure of the cyclones’ core. Intense tropical cyclones display a
228 quasi-axisymmetric shape, a state where very few degrees of freedom (i.e., low d values, which
229 typically correspond to high α values) are needed to describe their dynamics because of the
230 air parcels all aligning with the cyclones’ global rotation. On the contrary, the cyclones with
231 higher dimensionality likely display asymmetric or multiple PV patches, with a less organised KE
232 landscape.

233

234 The connection between d , α and cyclone intensity holds in general when diagnosing the
235 intensity of the cyclones using the minimum SLP, computed from gridded data (Fig. 4). In
236 general, large values of α (colorscale) correspond to low values of the minimum SLP in both the
237 North Atlantic and Eastern North Pacific basins. Moreover, the most intense tropical cyclones
238 phases are marked by a low dimension d . Similar conclusions hold for the HighResMIP data,
239 as further discussed in Sect. 4b. Remarkably, the coarse-graining operation on ERA5 data does
240 not alter sensibly the dynamical properties d and α and they maintain roughly the same range
241 of values of the original 0.25° reanalysis (Fig. 4b, d). The explanation for this stability follows
242 that given in Faranda et al. (2017) for the SLP data over the North-Atlantic: the dynamical
243 systems metrics are practically insensible to resolution, provided that the resolution is sufficient to

244 represent the underlying dynamics of the data. This is the case here since, as previously discussed,
245 ERA5 cg and ERA5 share near-identical distributions of cyclone SLP minima.

246

247 **4. Bias Corrections of tropical cyclone sea-level pressure minima**

248 *a. Bias Corrections of ERA5 tropical cyclone sea-level pressure minima*

249 We perform two bias corrections for ERA5 minimum sea-level pressure. In both cases we
250 prepare our data as follows. Firstly, we rearrange our set of tropical cyclones by alphabetic
251 order. This mixes tropical cyclones from different years and different parts of the cyclone
252 season in the training and in the verification datasets. We thus avoid intraseasonal effects and
253 possible non-stationarities linked to anthropogenic climate change or interannual variability of the
254 atmospheric circulation. Then we consider the first 4000 datapoints of our rearranged HURDAT2
255 SLP minima for each basin as training data and the remaining 2029 datapoints for the North
256 Atlantic basin and 2313 datapoints for the Eastern North Pacific basin as verification data. This
257 same split is also applied to ERA5 cg.

258

259 We next define objective metrics to determine whether our bias corrections improve the cyclone
260 intensities. We use the original Simpson and Saffir (1974) scale of tropical cyclones category
261 based on the SLP minima and compute them for the verification datasets (Table 1). We define two
262 error metrics: the total count of tropical cyclones time-steps with wrong category Err_T , and the
263 count of major tropical cyclones (intensity ≥ 3) with wrong category Err_C .

264

265 The first bias correction we implement is an unconditional quantile-quantile correction. We
266 begin by subtracting the median of the distributions of SLP minima from the data, which we then
267 add back at the end of the procedure. We then compute the difference $\Delta(q)$ between the empirical
268 cumulative density functions (ECDFs) of SLP minima in HURDAT2 with respect to ERA5 and
269 ERA5 cg evaluated at 100 quantile values q . For each data to correct, the closest quantile q^* in the
270 ECDF(ERA5) or ECDF(ERA5 cg) dataset is found and the value $\Delta(q^*)$ is added to the original
271 value.

272
273 The second bias correction approach is conditioned on the dynamical systems metrics. The
274 results presented in Figures 3 and 4 show that we can discriminate intense tropical cyclones as
275 those having a large value of α and a small value of d . We test several $d_H-\alpha_H$ value pair (using
276 the same convention of Fig.3) to separate the training dataset into intense ($d < d_H, \alpha > \alpha_H$) and
277 non-intense cyclones. We then apply the previously described quantile-quantile correction to the
278 two subsets of cyclones separately. We then use a grid-search approach to select the $d_c-\alpha_c$ value
279 pair minimizing the total error. We scan all combinations of $10 \leq d \leq 38$ and $0.1 \leq \alpha \leq 0.34$, with
280 resolution $\Delta d = 1$ and $\Delta \alpha = 0.1$. Results are displayed in Figures 5-6 for the ERA5 and ERA5 cg
281 datasets respectively. The upper (resp. lower) horizontal planes show the error metrics for the non
282 corrected (resp. corrected without dynamical systems metrics) data. The curved surfaces show the
283 the errors when bias correction is performed using the dynamical systems metrics. Panels (a,b)
284 show Err_T for the North Atlantic and Eastern North Pacific basins respectively, while panels (c,d)
285 show the metric Err_C . In all cases and for both the ERA5 and ERA5 cg datasets the conditional
286 bias correction greatly improves the identification of the tropical cyclones category relative to
287 the uncorrected data. Furthermore, there are large areas of the $d-\alpha$ plane where the conditional
288 bias correction improves the results of the unconditional correction. We select the values d_c and

289 α_c that minimise the total error $Err = Err_T(ATL) + Err_T(ENP) + Err_C(ATL) + Err_C(ENP)$.
290 We additionally impose that the chosen values improve upon the unconditional correction for
291 both basins and both Err_T and Err_C . This analysis yield $d_c = 15$ and $\alpha_c = 0.11$ for ERA5 and
292 $d_c = 18, \alpha_c = 0.13$ for ERA5 cg. As already noted in Fig. 1a,c, the biases for the most intense
293 cyclones are more severe in the Eastern North Pacific. While the bias correction improves the
294 situation, both Err_C and Err_T for the Eastern North Pacific remain larger than for the North
295 Atlantic.

296
297 We illustrate in detail the improvement obtained with the two bias correction approaches in
298 Figures 7-8. Panels (a,e) show that the ECDFs of bias- corrected SLPs are closer to the ver-
299 ification dataset HURDAT2 than the non-corrected ones. Similarly, panels (b,f) show that the
300 correspondence of observed versus modelled SLPs for individual cyclones is improved by the bias
301 correction. However, for many of the tropical cyclone timesteps, the bias correction attributes a
302 higher category than the original data. This is shown in panels (c,g) in terms of Δ between the
303 categories as estimated for ERA5 (cg) data relative to HURDAT2. Finally panels (d,h) show the
304 evident improvements in terms of intensity histograms: the original non corrected data do not
305 have any category 4 or 5 tropical cyclones, whereas with the corrections we are able to retrieve the
306 full intensity spectrum. The difference between ERA5 and ERA5 cg is limited, although panels
307 (d,h) evidence that the coarse-graining does have some effect on the most intense cyclones. As
308 previously noted, a clear difference emerges between the North Atlantic and Eastern North Pacific
309 basins, with the latter showing larger biases for both the uncorrected and corrected data.

310 *b. Bias Corrections of HighResMip tropical cyclone sea-level pressure minima*

311 When looking at the cyclone SLP minima for the HighResMIP data, we remark immediately the
312 virtual lack of minima below 960 hPa and the reduced range of d and α values compared to ERA5
313 or ERA5 cg data (cf. Fig. 9a,c, and 4). These results point to a different dynamical representation
314 of tropical cyclones in HighResMIP. The consequence is that the statistical dependencies between
315 SLP, d and α in HighResMIP are not the same as in the reference data (ERA5 and HURDAT).
316 Up to now, the unconditional and conditional bias correction methods applied to adjust cyclone
317 properties were univariate. Hence, dependencies between the three variables (i.e. SLP minima, d
318 and α) were not corrected by the unconditional approach, and only partly with the conditional one.
319 Thus, the resulting tri-variate dependence structures were likely not appropriately represented by
320 the corrected data. To account for this, we now also make use of a multivariate bias correction
321 (MBC) method to adjust not only the univariate distributions but also the dependence between the
322 three variables of interest. To do so, the “Rank Resampling for Distributions and Dependencies”
323 (R2D2) method is applied to adjust jointly (SLP minima, d , α). R2D2 relies on an analogue-
324 based method applied to the ranks of the time series to be corrected rather than to their “raw” values
325 (Vrac 2018). This MBC method can be easily designed to adjust both inter-variable, inter-site and
326 temporal properties (Vrac and Thao 2020) but is used only in its inter-variable configuration in
327 the present study. Figure 9 shows the scatterplots of minimum SLP vs local dimension d and co-
328 recurrence ratio α without (a,c) and with (b,d) R2D2 correction. The R2D2 correction enables to
329 retrieve SLP minima of order 920 hPa as well as extend the range of values of d and α , reproducing
330 a pattern closer to the one observed in ERA5.

331 We next apply the unconditional and conditional bias-correction methodologies, as well as the
332 R2D2 method to the HighResMIP simulations. The main difference here is that the correction has

333 to be fitted and applied directly to free-running HighResMIP simulations. In such a context, cross-
334 validation techniques for corrections evaluation have been recently heavily criticized (Maraun and
335 Widmann 2018) due to the influence of the model internal variability on the correction results.
336 Hence, in the following, no separation into training and test data is performed: we correct the bulk
337 cyclone statistics for all selected cyclones in the IPSL-CM6A-ATM-ICO-HR year-long simulation.
338 We first estimate the ECDFs and compute the Δ function between HighResMIP and HURDAT2
339 data. We then correct the full dataset using the unconditional, conditional on α, d – using the ERA5
340 cg values d_c and α_c – and R2D2 corrections. The results are shown in Figure 10. For the ECDFs
341 (panels a,c), corrections obtained using or not the dynamical systems metrics seem comparable
342 and improve, once again, the distribution of SLP minima. When looking at the tropical cyclone
343 intensities (panels b,d) we observe that the R2D2 correction of SLP minima allows to recover a
344 distribution of intensities which is very close to that of the HURDAT2 data. This suggests that,
345 when applying R2D2, we are able to take into full account the information provided by the two
346 dynamical systems metrics.

347 **5. Implications of the results for the numerical simulation of tropical cyclones**

348 Starting from the observation that gridded datasets of tropical cyclones have a large bias in
349 the representation of the intensity of extreme cyclones, we have introduced a procedure to bias
350 correct them. We have used: i) an unconditional quantile–quantile correction of the sea-level
351 pressure minima of cyclones timesteps towards the HURDAT2 reference dataset; ii) a correction
352 conditioned on two dynamical systems metrics (dimension and co-recurrence ratio); and iii) the
353 R2D2 correction. When using the dynamical systems metrics, we use the dynamical information
354 from an underlying reduced phase space incorporating horizontal kinetic energy uv and potential
355 vorticity PV at 850 hPa. We have observed that the dynamical systems metrics are able to track

356 intense cyclones as organized states of the dynamics, yielding low dimensions and high dynam-
357 ical/thermodynamic coupling. While all bias corrections improve the bulk statistics of tropical
358 cyclone intensity representation in the ERA5 reanalysis and in the IPSL-CM6A-ATM-ICO-HR
359 HighResMIP model, the categorisation for individual cyclones is not always improved. For the
360 ERA5 reanalyses, we find that the conditional bias-correction improves over its unconditional
361 counterpart. For the HighResMIP model, the best bias correction is the multivariate R2D2
362 correction which takes into account the relationships among SLP, d and α . This suggests that
363 accounting for the multivariate dependence structures associated to the dynamical systems metrics
364 is crucial to correcting the distribution of tropical cyclone intensities.

365
366 These considerations have a number of concrete implications for current research on tropical
367 cyclones. Current GCMs — and even reanalysis products — struggle in reproducing minimum
368 sea-level pressures comparable to those observed. Our study offers a way of mapping intense
369 cyclones in d, α space and a procedure to correct biases in century-long reanalysis products to
370 perform climate change studies of their intensity. This, in turn, may provide a strategy for study-
371 ing changes in tropical cyclone intensity driven by anthropogenic forcing. Indeed, while d and
372 α may not be used to provide a deterministic indication of cyclone intensity, they do provide a
373 robust constraint for statistical correction, especially in reanalysis data. The large range of local
374 dimensions associated with the dynamics of different phases of tropical cyclones may explain why
375 it is so difficult to adequately represent them in numerical models. Follow-up studies will include
376 the use of a larger set of HighResMIP data under different forcing scenarios, to test systematically
377 whether the dynamical systems bias correction may be successfully applied to model data. The
378 bias correction procedures will then be adapted to take into account the non-stationarities intro-
379 duced by anthropogenic forcing, e.g. by replacing the quantile-quantile mapping with a CDF-t

380 correction (Vrac et al. 2012) allowing to account for climate change in the correction procedure.
381 A further avenue for future work will be to move from the pointwise correction of minimum SLP
382 performed here, to correcting the spatial structure of the field, using multivariate bias correction
383 techniques (e.g. Cannon 2018; Vrac 2018; Robin et al. 2019; Vrac and Thao 2020) or tools from
384 machine learning (e.g. François et al. 2021).

385 *Acknowledgments.* The authors acknowledge the support of the INSU-CNRS-LEFE-MANU
386 grant (project DINCLIC), as well as the grant ANR-19-ERC7-0003 (BOREAS). This work has
387 received support from the European Union’s Horizon 2020 research and innovation programme
388 (Grant agreement No. 101003469, XAIDA) and from the European Research Council (ERC) un-
389 der the European Union’s Horizon 2020 research and innovation programme (Grant agreement
390 No. 948309, CENÆ project). MV was partly supported by the “COESION” project funded
391 by the French National program LEFE (Les Enveloppes Fluides et l’Environnement), as well as
392 the French National “Explore2” project funded by the French Ministry of Ecological Transition
393 (MTE) and the French Office for Biodiversity (OFB).

394 **References**

395 Bhalachandran, S., D. R. Chavas, F. D. M. Jr, S. Dubey, A. Shreevastava, and T. N. Krishnamurti,
396 2020: Characterizing the Energetics of Vortex-Scale and Sub-Vortex-Scale Asymmetries dur-
397 ing Tropical Cyclone Rapid Intensity Changes. *Journal of the Atmospheric Sciences*, **77** (1),
398 315–336, doi:10.1175/JAS-D-19-0067.1, URL [https://journals.ametsoc.org/view/journals/atsc/](https://journals.ametsoc.org/view/journals/atsc/77/1/jas-d-19-0067.1.xml)
399 [77/1/jas-d-19-0067.1.xml](https://journals.ametsoc.org/view/journals/atsc/77/1/jas-d-19-0067.1.xml), publisher: American Meteorological Society Section: Journal of the
400 Atmospheric Sciences.

- 401 Brunetti, M., J. Kasparian, and C. V  rard, 2019: Co-existing climate attractors in a coupled aqua-
402 planet. *Climate Dynamics*, **53** (9-10), 6293–6308.
- 403 Camargo, S. J., K. A. Emanuel, and A. H. Sobel, 2007: Use of a genesis potential index to diagnose
404 ENSO effects on tropical cyclone genesis. *Journal of Climate*, **20** (19), 4819–4834.
- 405 Camargo, S. J., and A. A. Wing, 2016: Tropical cyclones in climate models. *WIREs Clim. Change*,
406 **7**, 211–237, doi:https://doi.org/10.1002/wcc.373.
- 407 Cannon, A. J., 2018: Multivariate quantile mapping bias correction: an n-dimensional probability
408 density function transform for climate model simulations of multiple variables. *Climate dynam-*
409 *ics*, **50** (1), 31–49.
- 410 Cao, L., 1997: Practical method for determining the minimum embedding dimension of a scalar
411 time series. *Physica D: Nonlinear Phenomena*, **110** (1-2), 43–50.
- 412 Caron, L. P., C. Jones, and K. Winger, 2011: Impact of resolution and downscaling technique
413 in simulating recent Atlantic tropical cyclone activity. *Clim. Dyn.*, **37**, 869–892, doi:doi.org/10.
414 1007/s00382-010-0846-7.
- 415 Carstens, J. D., and A. A. Wing, 2020: Tropical cyclogenesis from self-aggregated convection
416 in numerical simulations of rotating radiative-convective equilibrium. *Journal of Advances in*
417 *Modeling Earth Systems*, **12** (5), e2019MS002 020.
- 418 Chang, E. K., and Y. Guo, 2007: Is the number of north atlantic tropical cyclones significantly
419 underestimated prior to the availability of satellite observations? *Geophysical Research Letters*,
420 **34** (14).
- 421 Crisanti, A., M. Falcioni, A. Vulpiani, and G. Paladin, 1991: Lagrangian chaos: transport, mixing
422 and diffusion in fluids. *La Rivista del Nuovo Cimento (1978-1999)*, **14** (12), 1–80.

- 423 Davis, C., C. Snyder, and A. C. Didlake Jr, 2008: A vortex-based perspective of eastern pacific
424 tropical cyclone formation. *Monthly weather review*, **136** (7), 2461–2477.
- 425 De Luca, P., G. Messori, F. M. Pons, and D. Faranda, 2020: Dynamical systems theory sheds new
426 light on compound climate extremes in europe and eastern north america. *Quarterly Journal of*
427 *the Royal Meteorological Society*, **146** (729), 1636–1650.
- 428 Dunkerton, T. J., M. Montgomery, and Z. Wang, 2009: Tropical cyclogenesis in a tropical wave
429 critical layer: Easterly waves. *Atmospheric Chemistry and Physics*, **9** (15), 5587–5646.
- 430 Emanuel, K., 2021: Response of global tropical cyclone activity to increasing CO₂: Results from
431 downscaling CMIP6 models. *J. Climate*, **34**, 57–70, doi:10.1175/JCLI-D-20-0367.1.
- 432 Emanuel, K. A., 1986: An air-sea interaction theory for tropical cyclones. part i: Steady-state
433 maintenance. *Journal of Atmospheric Sciences*, **43** (6), 585–605.
- 434 Emanuel, K. A., 1997: Some aspects of hurricane inner-core dynamics and energetics. *Journal of*
435 *the Atmospheric Sciences*, **54**, 1014–1026, doi:10.1175/1520-0469(1997)054<1014:SAOHIC>
436 2.0.CO;2.
- 437 Faranda, D., M. C. Alvarez-Castro, G. Messori, D. Rodrigues, and P. Yiou, 2019: The hamman
438 effect or how a warm ocean enhances large scale atmospheric predictability. *Nature communi-*
439 *cations*, **10** (1), 1–7.
- 440 Faranda, D., V. Lembo, M. Iyer, D. Kuzay, S. Chibbaro, F. Daviaud, and B. Dubrulle, 2018:
441 Computation and characterization of local subfilter-scale energy transfers in atmospheric flows.
442 *Journal of the Atmospheric Sciences*, **75** (7), 2175–2186.
- 443 Faranda, D., G. Messori, and P. Yiou, 2017: Dynamical proxies of north atlantic predictability and
444 extremes. *Scientific reports*, **7**, 41 278.

- 445 Faranda, D., G. Messori, and P. Yiou, 2020: Diagnosing concurrent drivers of weather extremes:
446 application to warm and cold days in north america. *Climate Dynamics*, **54 (3)**, 2187–2201.
- 447 François, B., S. Thao, and M. Vrac, 2021: Adjusting spatial dependence of climate model outputs
448 with cycle-consistent adversarial networks. *Climate Dynamics*, **57 (11)**, 3323–3353.
- 449 Freitas, A. C. M., J. M. Freitas, and M. Todd, 2010: Hitting time statistics and extreme value
450 theory. *Probability Theory and Related Fields*, **147 (3-4)**, 675–710.
- 451 Fu, B., M. S. Peng, T. Li, and D. E. Stevens, 2012: Developing versus nondeveloping disturbances
452 for tropical cyclone formation. part ii: Western north pacific. *Monthly weather review*, **140 (4)**,
453 1067–1080.
- 454 Grinsted, A., P. Ditlevsen, and J. H. Christensen, 2019: Normalized US hurricane damage es-
455 timates using area of total destruction, 1900- 2018. *Proceedings of the National Academy of*
456 *Sciences*, **116 (48)**, 23 942–23 946.
- 457 Gualandi, A., J.-P. Avouac, S. Michel, and D. Faranda, 2020: The predictable chaos of slow
458 earthquakes. *Science advances*, **6 (27)**, eaaz5548.
- 459 Haarsma, R. J., and Coauthors, 2016: High resolution model intercomparison project (highresmip
460 v1. 0) for cmip6. *Geoscientific Model Development*, **9 (11)**, 4185–4208.
- 461 Hersbach, H., and Coauthors, 2020: The era5 global reanalysis. *Quarterly Journal of the Royal*
462 *Meteorological Society*, **146 (730)**, 1999–2049.
- 463 Hochman, A., P. Alpert, T. Harpaz, H. Saaroni, and G. Messori, 2019: A new dynamical systems
464 perspective on atmospheric predictability: Eastern mediterranean weather regimes as a case
465 study. *Science advances*, **5 (6)**, eaau0936.

466 Hochman, A., G. Messori, J. F. Quinting, J. G. Pinto, and C. M. Grams, 2021: Do
467 atlantic-european weather regimes physically exist? *Geophysical Research Letters*, **48** (20),
468 e2021GL095 574.

469 Ikehata, K., and M. Satoh, 2021: Climatology of tropical cyclone seed frequency and survival rate
470 in tropical cyclones. *Geophysical Research Letters*, **48** (18), e2021GL093 626.

471 Kim, D., and Coauthors, 2018: Process-oriented diagnosis of tropical cyclones in high-resolution
472 gcms. *Journal of Climate*, **31** (5), 1685–1702.

473 Klotzbach, P. J., M. M. Bell, S. G. Bowen, E. J. Gibney, K. R. Knapp, and C. J. Schreck III,
474 2020: Surface pressure a more skillful predictor of normalized hurricane damage than maximum
475 sustained wind. *Bulletin of the American Meteorological Society*, **101** (6), E830–E846.

476 Knaff, J. A., and Coauthors, 2021: Estimating tropical cyclone surface winds: Current status,
477 emerging technologies, historical evolution, and a look to the future. *Tropical Cyclone Research
478 and Review*, **10**, 125–150, doi:<https://doi.org/10.1016/j.tcr.2021.09.002>.

479 Knutson, T., and Coauthors, 2019: Tropical cyclones and climate change assessment: Part
480 I: Detection and Attribution. *Bull. Amer. Meteor. Soc.*, **100**, 1987–2007, doi:10.1175/
481 BAMS-D-18-0189.1.

482 Knutson, T., and Coauthors, 2020: Tropical cyclones and climate change assessment: Part ii:
483 Projected response to anthropogenic warming. *Bulletin of the American Meteorological Society*,
484 **101** (3), E303–E322.

485 Krishnamurti, T., S. Pattnaik, L. Stefanova, T. V. Kumar, B. P. Mackey, A. O’shay, and R. J. Pasch,
486 2005: The hurricane intensity issue. *Monthly weather review*, **133** (7), 1886–1912.

- 487 Landsea, C. W., and J. L. Franklin, 2013: Atlantic hurricane database uncertainty and presentation
488 of a new database format. *Monthly Weather Review*, **141** (10), 3576–3592.
- 489 Lee, C.-Y., S. J. Camargo, A. H. Sobel, and M. K. Tippett, 2020: Statistical–dynamical down-
490 scaling projections of tropical cyclone activity in a warming climate: Two diverging genesis
491 scenarios. *J. Climate*, **33**, 4815–4834, doi:10.1175/JCLI-D-19-0452.1.
- 492 Levich, E., and E. Tzvetkov, 1985: Helical inverse cascade in three-dimensional turbulence as
493 a fundamental dominant mechanism in mesoscale atmospheric phenomena. *Physics reports*,
494 **128** (1), 1–37.
- 495 Lorenz, E. N., 1990: Can chaos and intransitivity lead to interannual variability? *Tellus A*, **42** (3),
496 378–389.
- 497 Lucarini, V., D. Faranda, and J. Wouters, 2012: Universal behaviour of extreme value statistics for
498 selected observables of dynamical systems. *Journal of statistical physics*, **147** (1), 63–73.
- 499 Lucarini, V., and Coauthors, 2016: *Extremes and recurrence in dynamical systems*. John Wiley &
500 Sons.
- 501 Maraun, D., and M. Widmann, 2018: Cross-validation of bias-corrected climate simula-
502 tions is misleading. *Hydrology and Earth System Sciences*, **22** (9), 4867–4873, doi:10.5194/
503 hess-22-4867-2018, URL <https://hess.copernicus.org/articles/22/4867/2018/>.
- 504 Messori, G., and D. Faranda, 2021: Characterising and comparing different palaeoclimates with
505 dynamical systems theory. *Climate of the Past*, **17** (1), 545–563.
- 506 Montgomery, M. T., and R. K. Smith, 2017: Recent developments in the fluid dynamics of tropical
507 cyclones. *Annual Review of Fluid Mechanics*, **49**, 541–574.

508 Moon, Y., and Coauthors, 2020: Azimuthally averaged wind and thermodynamic structures of
509 tropical cyclones in global climate models and their sensitivity to horizontal resolution. *J. Cli-*
510 *mate*, **33**, 1575 – 1595, doi:10.1175/JCLI-D-19-0172.1.

511 Muller, C. J., and D. M. Roms, 2018: Acceleration of tropical cyclogenesis by self-aggregation
512 feedbacks. *Proceedings of the National Academy of Sciences*, **115** (12), 2930–2935.

513 Peng, M. S., B. Fu, T. Li, and D. E. Stevens, 2012: Developing versus nondeveloping disturbances
514 for tropical cyclone formation. part i: North atlantic. *Monthly weather review*, **140** (4), 1047–
515 1066.

516 Roberts, M. J., and Coauthors, 2020a: Impact of model resolution on tropical cyclone simulation
517 using the HighResMIP–PRIMAVERA multimodel ensemble. *Journal of Climate*, **33** (7), 2557–
518 2583.

519 Roberts, M. J., and Coauthors, 2020b: Projected future changes in tropical cyclones us-
520 ing the CMIP6 HighResMIP multimodel ensemble. *Geophysical Research Letters*, **47** (14),
521 e2020GL088 662.

522 Robin, Y., M. Vrac, P. Naveau, and P. Yiou, 2019: Multivariate stochastic bias corrections with
523 optimal transport. *Hydrology and Earth System Sciences*, **23** (2), 773–786.

524 Rotunno, R., Y. Chen, W. Wang, C. Davis, J. Dudhia, and G. J. Holland, 2009: Large-eddy
525 simulation of an idealized tropical cyclone. *Bull. Amer. Meteor. Soc.*, **90**, 1783–1788, doi:
526 10.1175/2009BAMS2884.1.

527 Schubert, S., and V. Lucarini, 2015: Covariant lyapunov vectors of a quasi-geostrophic baroclinic
528 model: analysis of instabilities and feedbacks. *Quarterly Journal of the Royal Meteorological*
529 *Society*, **141** (693), 3040–3055.

- 530 Simpson, R. H., and H. Saffir, 1974: The hurricane disaster potential scale. *Weatherwise*, **27 (8)**,
531 169.
- 532 Smith, A. B., and R. W. Katz, 2013: Us billion-dollar weather and climate disasters: data sources,
533 trends, accuracy and biases. *Natural hazards*, **67 (2)**, 387–410.
- 534 Tang, J., D. Byrne, J. A. Zhang, Y. Wang, X.-t. Lei, D. Wu, P.-z. Fang, and B.-k. Zhao, 2015:
535 Horizontal transition of turbulent cascade in the near-surface layer of tropical cyclones. *Journal*
536 *of the Atmospheric Sciences*, **72 (12)**, 4915–4925.
- 537 Ullrich, P. A., C. M. Zarzycki, E. E. McClenny, M. C. Pinheiro, A. M. Stansfield, and K. A.
538 Reed, 2021: Tempestextremes v2. 1: a community framework for feature detection, tracking
539 and analysis in large datasets. *Geoscientific Model Development Discussions*, 1–37.
- 540 Vrac, M., 2018: Multivariate bias adjustment of high-dimensional climate simulations: the rank
541 resampling for distributions and dependences (r 2 d 2) bias correction. *Hydrology and Earth*
542 *System Sciences*, **22 (6)**, 3175–3196.
- 543 Vrac, M., P. Drobinski, A. Merlo, M. Herrmann, C. Lavaysse, L. Li, and S. Somot, 2012: Dynam-
544 ical and statistical downscaling of the french mediterranean climate: uncertainty assessment.
545 *Natural Hazards and Earth System Sciences*, **12 (9)**, 2769–2784.
- 546 Vrac, M., and S. Thao, 2020: R 2 d 2 v2. 0: accounting for temporal dependences in multivariate
547 bias correction via analogue rank resampling. *Geoscientific Model Development*, **13 (11)**, 5367–
548 5387.
- 549 Vulpiani, A., 2010: *Chaos: from simple models to complex systems*, Vol. 17. World Scientific.
- 550 Wolf, A., J. B. Swift, H. L. Swinney, and J. A. Vastano, 1985: Determining lyapunov exponents
551 from a time series. *Physica D: Nonlinear Phenomena*, **16 (3)**, 285–317.

552 **LIST OF TABLES**

553 **Table 1.** Saffir-Simpson tropical cyclone intensity classification. 28

cc

| Category | wind threshold [m/s] | minimum sea-level threshold [hPa] |
|----------|----------------------|-----------------------------------|
| 1 | 33–42 | >980 |
| 2 | 43–49 | 965–979 |
| 3 | 50–58 | 945–964 |
| 4 | 59–69 | 920–944 |
| 5 | <69 | <920 |

TABLE 1. Saffir-Simpson tropical cyclone intensity classification.

554 **LIST OF FIGURES**

555 **Fig. 1.** Scatterplot of minimum SLP for (a) North Atlantic [NATL] and (c) Eastern North Pacific
556 [ENP] tropical cyclones in HURDAT2 versus ERA5 and ERA5 cg. Probability density
557 functions of minimum SLP for (b) North Atlantic and (d) Eastern North Pacific tropical
558 cyclones in HURDAT2, ERA5, ERA5 cg and the IPSL-CM6A-ATM-ICO-HR model. 31

559 **Fig. 2.** Schematic of the computation of the dynamical systems metrics for an instantaneous state
560 of a tropical cyclone. We take a snapshot of the cyclone in physical space (black quadrant),
561 in this example latitude–longitude maps of PV and uv at 850 hPa, which correspond to
562 state ζ in our phase space. The shaded circle is a 2D representation of the hyper-sphere
563 determined by the high threshold $s(q, \zeta)$, which defines recurrences of ζ . The logarithmic
564 distances between measurements defined by $g(X_i, \zeta)$ are marked by double-headed arrows.
565 For all points within the hyper-sphere, $g(X_i, \zeta) > s(q, \zeta)$ holds. In the schematic, only two
566 measurements satisfy this condition (adapted from (Messori and Faranda 2021)). 32

567 **Fig. 3.** The scatter plot displays the values of the instantaneous dimension d and the co-recurrence
568 α computed on ERA5 uv and PV maps during tropical cyclone timesteps in the NATL basin.
569 The black solid lines mark the 0.95 quantiles of the d and α distributions, namely d_H and
570 α_H respectively. The maps show composites of PV (b–d) and uv (e–g) for $d < d_H, \alpha < \alpha_H$
571 (b,e), $d > d_H$ (c,f) and $\alpha > \alpha_H$ (d,g). 33

572 **Fig. 4.** Scatterplots of minimum SLP vs local dimension d and co-recurrence ratio α (colorscale)
573 calculated on uv and PV at 850 hPa for (a,c) ERA5 and (b,d) ERA5 cg, in the (a,b) North
574 Atlantic [NATL] and (c,d) Eastern North Pacific [ENP] basins. 34

575 **Fig. 5.** Cyclone categorisation error for uncorrected ERA5 data, unconditional quantile–quantile
576 correction and quantile–quantile correction conditioned on the dynamical systems metrics.
577 The upper horizontal planes indicate the uncorrected errors, the lower horizontal planes the
578 unconditional correction and the curved surface the dynamical systems corrections. (a,b)
579 show errors for all intensity categories Err_T ; (c,d) show errors for major tropical cyclones
580 only Err_C . Panels refer to the (a,c) North Atlantic [NATL], and (b,d) Eastern North Pacific
581 basin [ENP] basins. The red dots in the four panels show the chosen d_c – α_c values based on
582 minimising the total error Err (see text). The vertical red lines show the points’ projection
583 on the d – α plane. 35

584 **Fig. 6.** As in Figure 5 but for the ERA5 cg data. 36

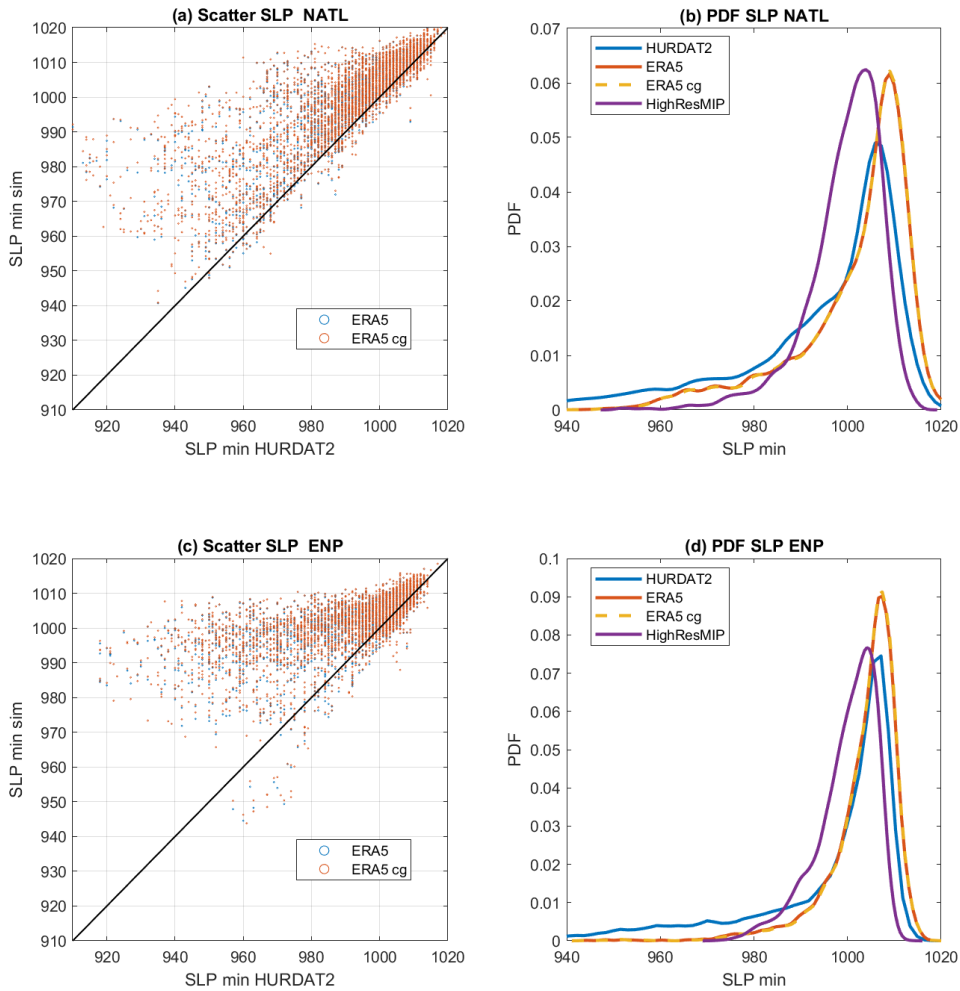
585 **Fig. 7.** Unconditional bias correction and bias correction conditioned on the dynamical systems
586 metrics for the best d – α parameter combination (see Fig. 5, $\alpha_c = 0.11$, $d_c = 15$) for ERA5.
587 (a,e) Empirical cumulative density functions (ECDFs), (b,f) scatter plots, (c,g) error in category
588 intensities (negative values imply underestimation, positive values overestimation),
589 (d,h) histogram of category intensities and ERR_T in the inset. (a–d) North Atlantic [NATL],
590 (e–h) Eastern North Pacific [ENP] basins. See legends for details. 37

591 **Fig. 8.** As in Fig. 7 but for ERA5 cg data, using $\alpha_c = 0.13$, $d_c = 18$ 38

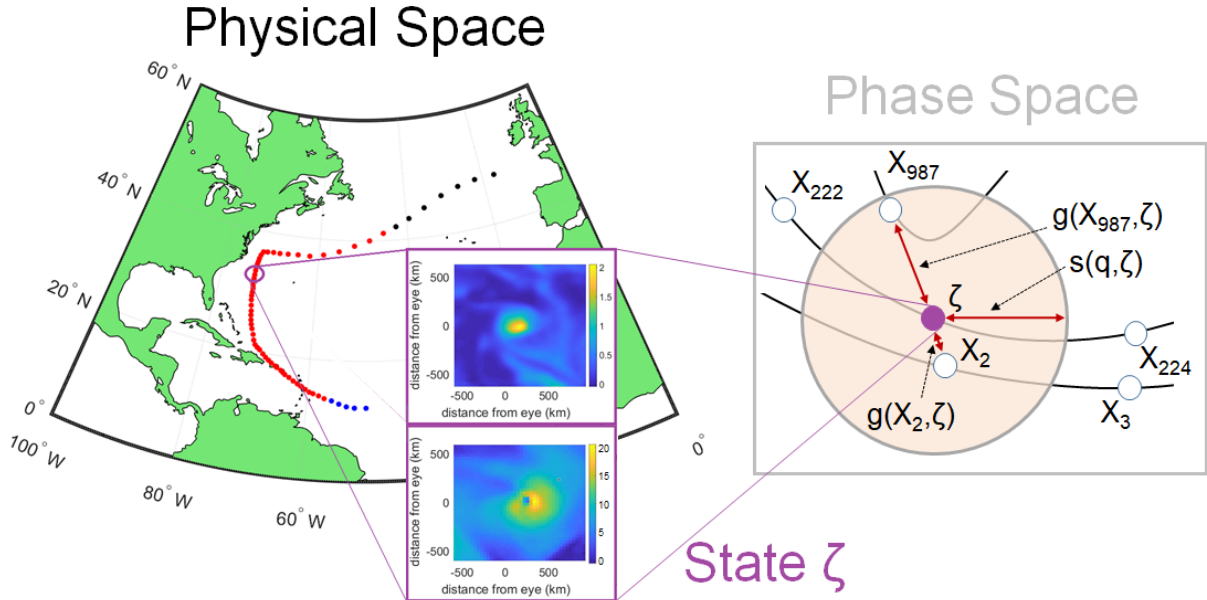
592 **Fig. 9.** Scatterplots of minimum SLP vs local dimension d and corecurrence ratio α (colorscale)
593 calculated on uv and PV at 850 hPa for (a,c) HighResMIP; (b,d) HighResMIP corrected
594 with R2D2, in the (a,b) North Atlantic [NATL] and (c,d) Eastern North Pacific [ENP] basins. 39

595 **Fig. 10.** Unconditional, R2D2, and conditional dynamical systems metrics bias corrections for the
596 IPSL-CM6A-ATM-ICO-HR model. The conditional dynamical systems metrics bias cor-

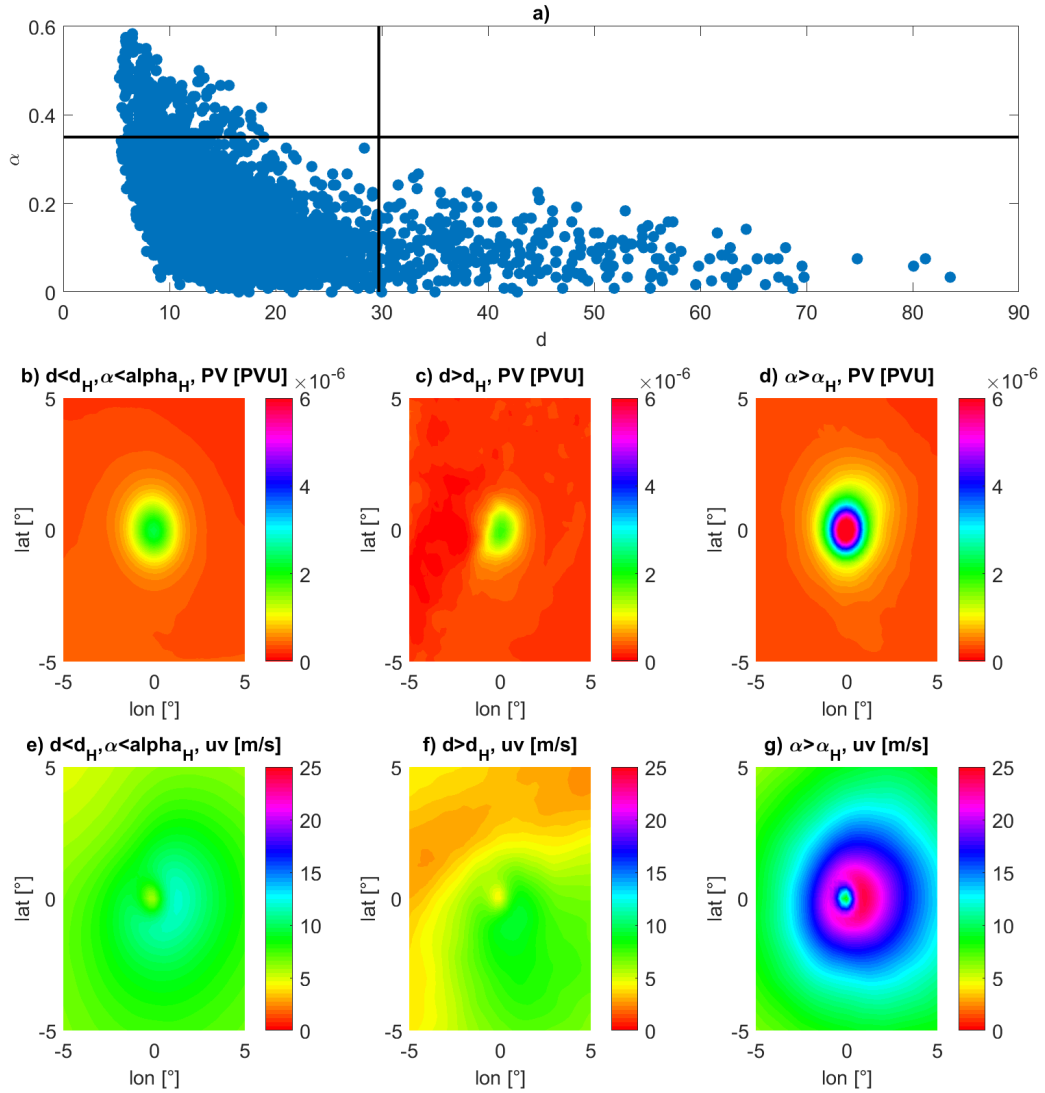
597 rections uses the best d - α parameter combination from ERA5 cg (see Fig. 6: $\alpha_c = 0.13$,
598 $d_c = 18$.) Panels (a,c) show Empirical Cumulative Density Functions (ECDFs) and (b,d)
599 histograms of category intensities for the North Atlantic [NATL] (a,b) and Eastern North
600 Pacific [ENP] (c,d) basins. 40



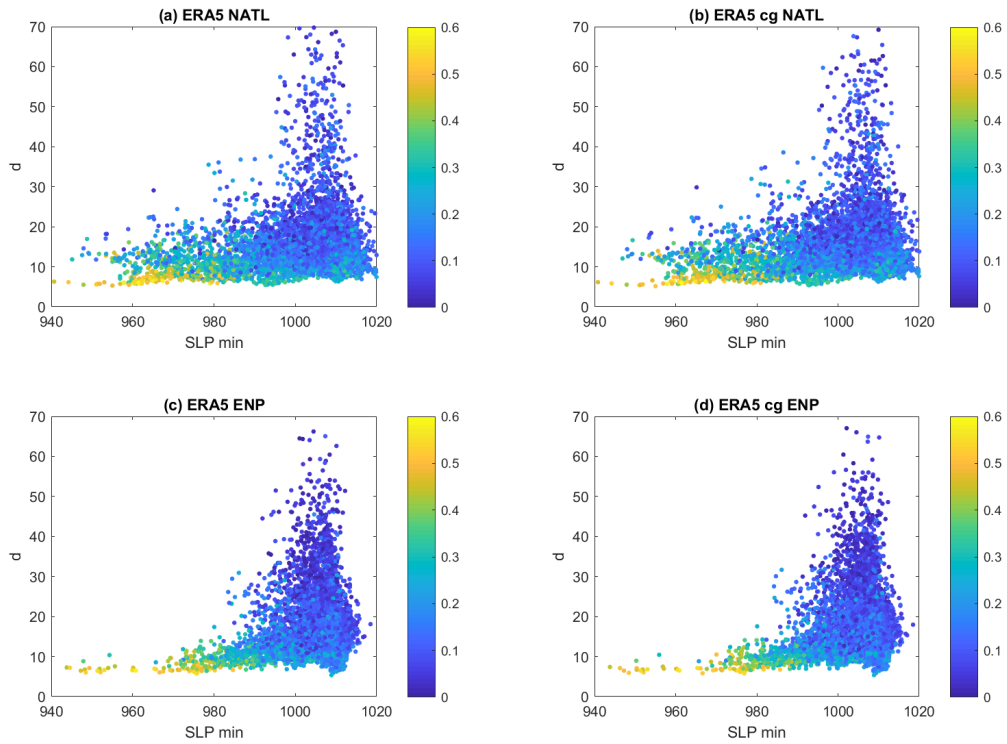
601 FIG. 1. Scatterplot of minimum SLP for (a) North Atlantic [NATL] and (c) Eastern North Pacific [ENP]
 602 tropical cyclones in HURDAT2 versus ERA5 and ERA5 cg. Probability density functions of minimum SLP
 603 for (b) North Atlantic and (d) Eastern North Pacific tropical cyclones in HURDAT2, ERA5, ERA5 cg and the
 604 IPSL-CM6A-ATM-ICO-HR model.



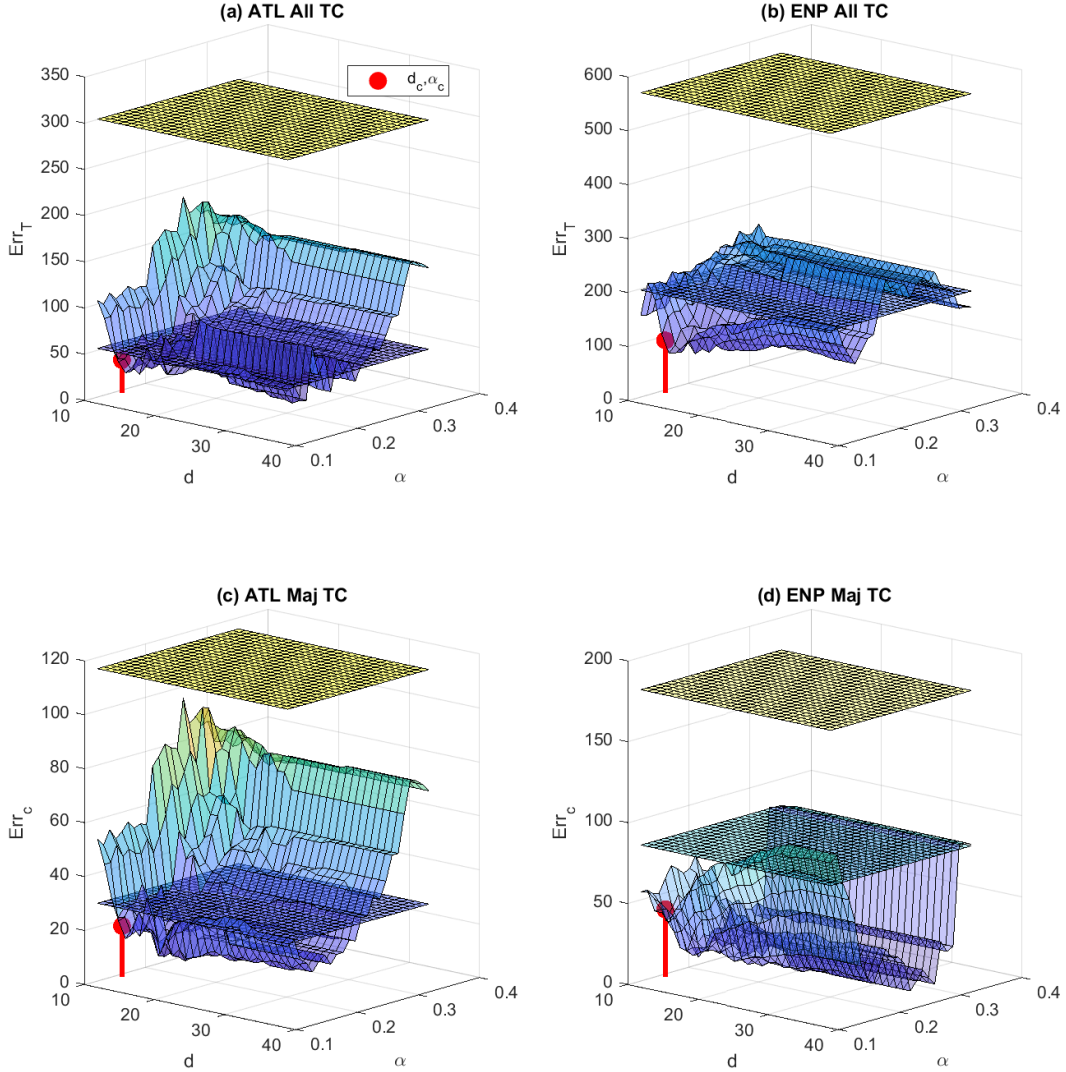
605 FIG. 2. Schematic of the computation of the dynamical systems metrics for an instantaneous state of a tropical
 606 cyclone. We take a snapshot of the cyclone in physical space (black quadrant), in this example latitude–longitude
 607 maps of PV and uv at 850 hPa , which correspond to state ζ in our phase space. The shaded circle is a 2D
 608 representation of the hyper-sphere determined by the high threshold $s(q, \zeta)$, which defines recurrences of ζ .
 609 The logarithmic distances between measurements defined by $g(X_i, \zeta)$ are marked by double-headed arrows. For
 610 all points within the hyper-sphere, $g(X_i, \zeta) > s(q, \zeta)$ holds. In the schematic, only two measurements satisfy this
 611 condition (adapted from (Messori and Faranda 2021)).



612 FIG. 3. The scatter plot displays the values of the instantaneous dimension d and the co-recurrence α com-
 613 puted on ERA5 uv and PV maps during tropical cyclone timesteps in the NATL basin. The black solid lines mark
 614 the 0.95 quantiles of the d and α distributions, namely d_H and α_H respectively. The maps show composites of
 615 PV (b–d) and uv (e–g) for $d < d_H, \alpha < \alpha_H$ (b,e), $d > d_H$ (c,f) and $\alpha > \alpha_H$ (d,g).



616 FIG. 4. Scatterplots of minimum SLP vs local dimension d and co-recurrence ratio α (colorscale) calculated
 617 on uv and PV at 850 hPa for (a,c) ERA5 and (b,d) ERA5 cg, in the (a,b) North Atlantic [NATL] and (c,d) Eastern
 618 North Pacific [ENP] basins.



619 FIG. 5. Cyclone categorisation error for uncorrected ERA5 data, unconditional quantile–quantile correction
 620 and quantile–quantile correction conditioned on the dynamical systems metrics. The upper horizontal planes
 621 indicate the uncorrected errors, the lower horizontal planes the unconditional correction and the curved surface
 622 the dynamical systems corrections. (a,b) show errors for all intensity categories Err_T ; (c,d) show errors for
 623 major tropical cyclones only Err_C . Panels refer to the (a,c) North Atlantic [NATL], and (b,d) Eastern North
 624 Pacific basin [ENP] basins. The red dots in the four panels show the chosen $d_c-\alpha_c$ values based on minimising
 625 the total error Err (see text). The vertical red lines show the points' projection on the $d-\alpha$ plane.

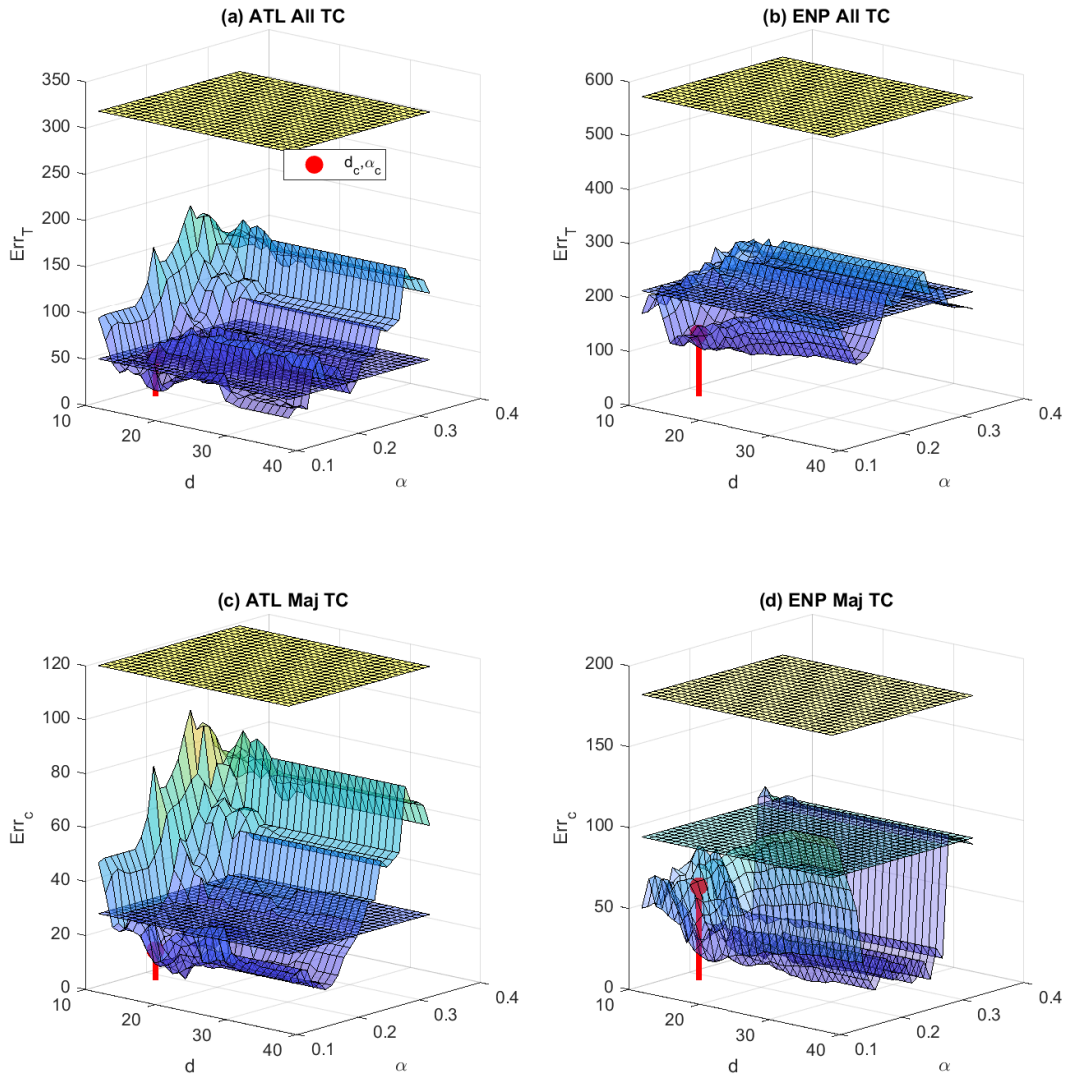
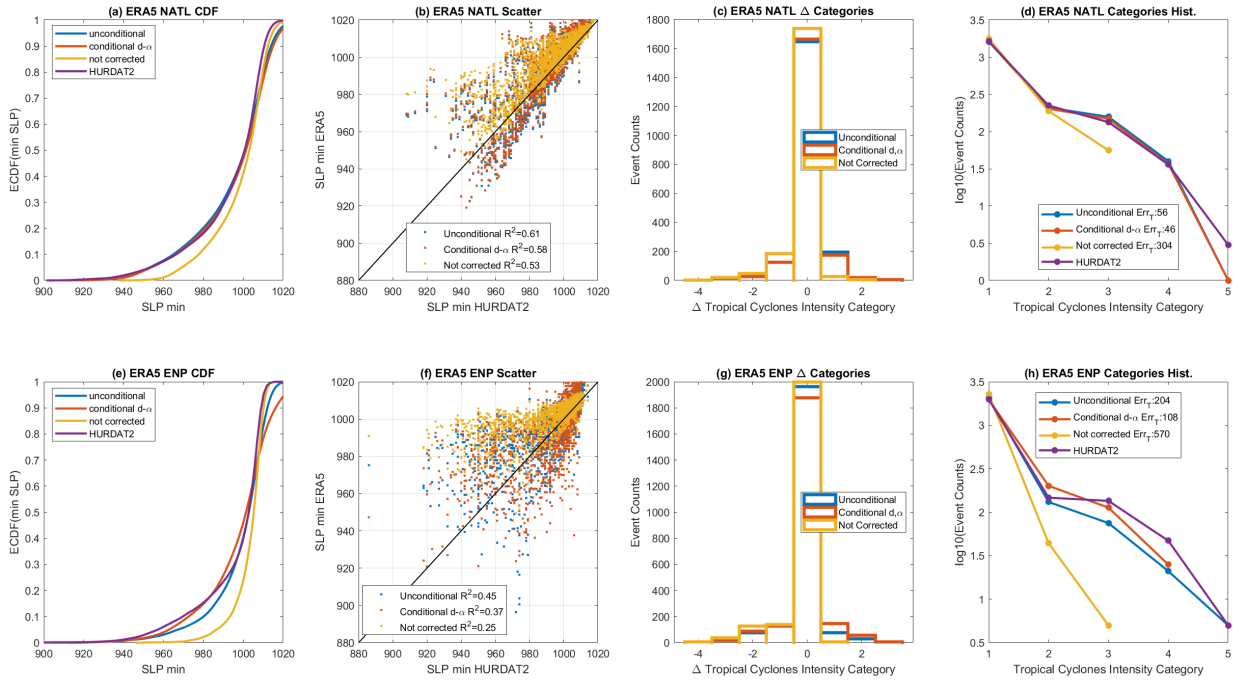


FIG. 6. As in Figure 5 but for the ERA5 cg data.



626 FIG. 7. Unconditional bias correction and bias correction conditioned on the dynamical systems metrics for
 627 the best $d-\alpha$ parameter combination (see Fig. 5, $\alpha_c = 0.11$, $d_c = 15$) for ERA5. (a,e) Empirical cumulative
 628 density functions (ECDFs), (b,f) scatter plots, (c,g) error in category intensities (negative values imply under-
 629 estimation, positive values overestimation), (d,h) histogram of category intensities and ERR_T in the inset. (a-d)
 630 North Atlantic [NATL], (e-h) Eastern North Pacific [ENP] basins. See legends for details.

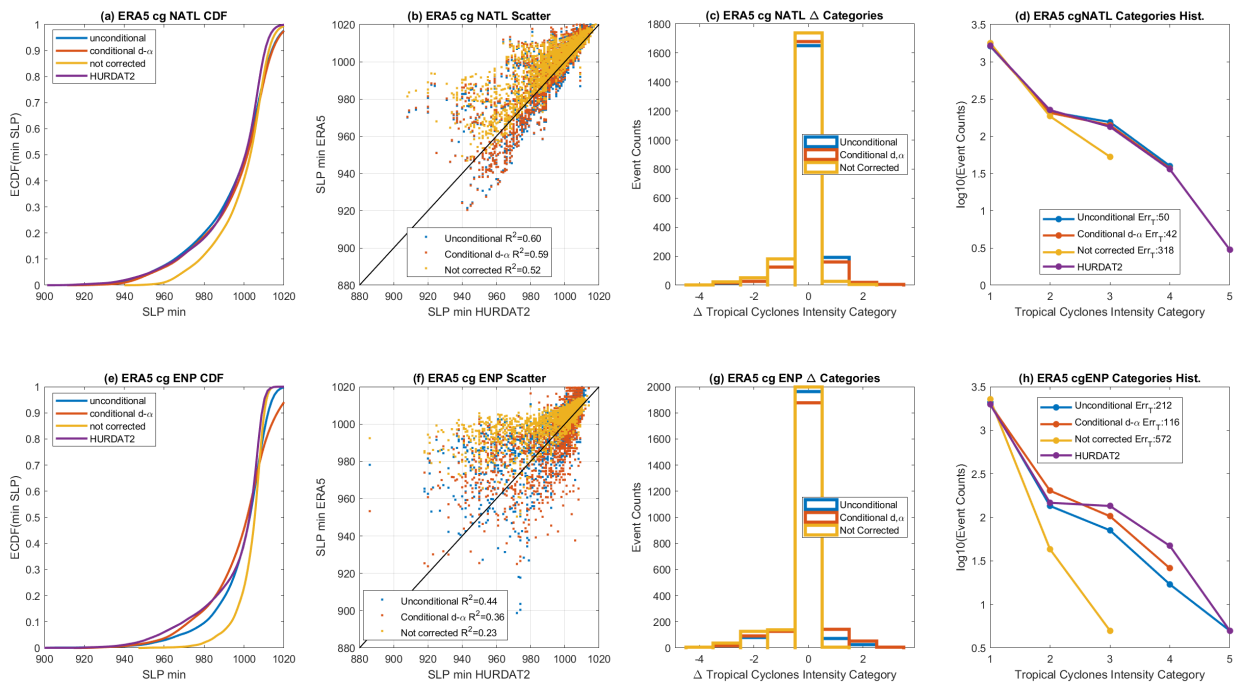
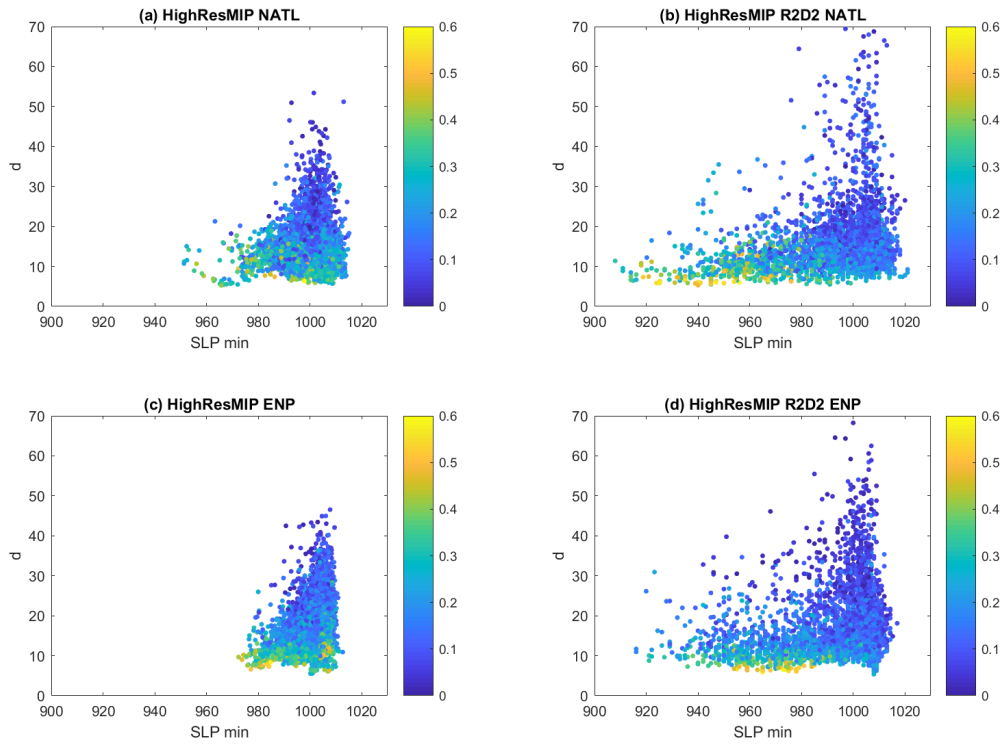
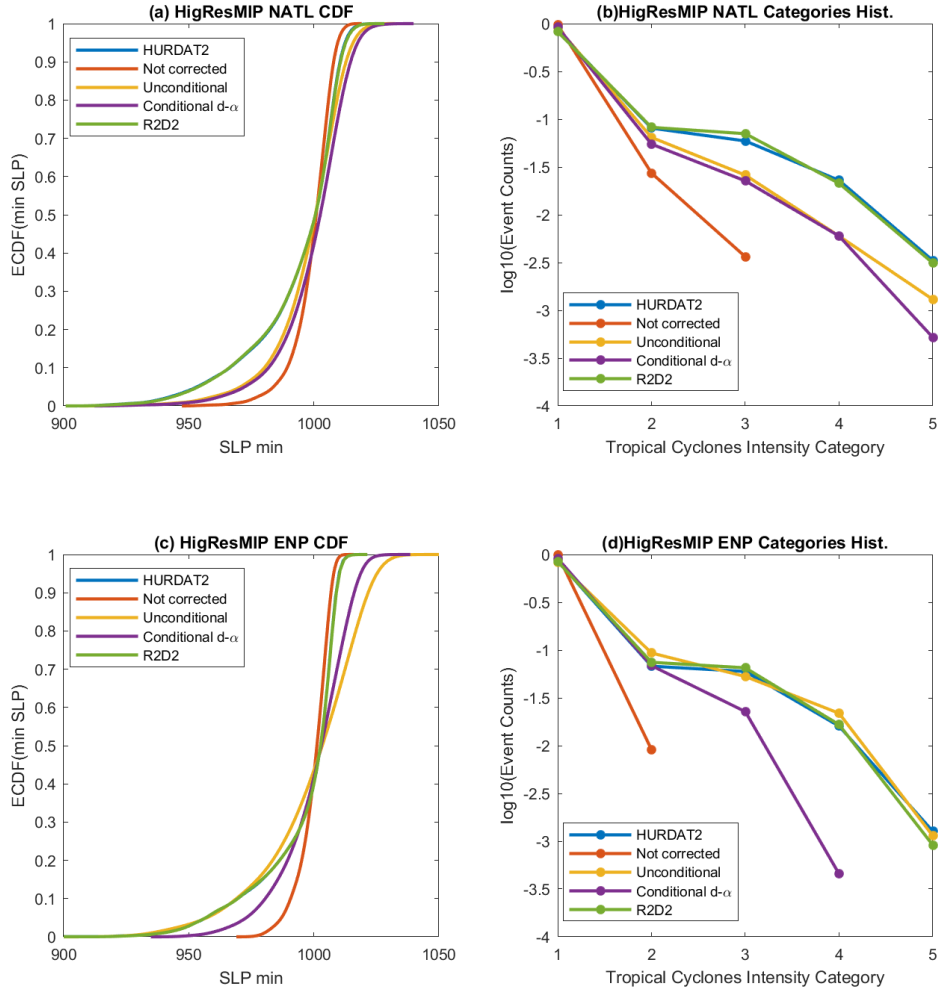


FIG. 8. As in Fig. 7 but for ERA5 cg data, using $\alpha_c = 0.13$, $d_c = 18$.



631 FIG. 9. Scatterplots of minimum SLP vs local dimension d and corecurrence ratio α (colorscale) calculated
 632 on uv and PV at 850 hPa for (a,c) HighResMIP; (b,d) HighResMIP corrected with R2D2, in the (a,b) North
 633 Atlantic [NATL] and (c,d) Eastern North Pacific [ENP] basins.



634 FIG. 10. Unconditional, R2D2, and conditional dynamical systems metrics bias corrections for the IPSL-
 635 CM6A-ATM-ICO-HR model. The conditional dynamical systems metrics bias corrections uses the best $d-\alpha$
 636 parameter combination from ERA5 cg (see Fig. 6: $\alpha_c = 0.13$, $d_c = 18$.) Panels (a,c) show Empirical Cumulative
 637 Density Functions (ECDFs) and (b,d) histograms of category intensities for the North Atlantic [NATL] (a,b) and
 638 Eastern North Pacific [ENP] (c,d) basins.