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Design and Comparison of Two Advanced Core Control Systems for Flexible Operation of Pressurized Water Reactors

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Abstract

This paper focuses on the design of advanced core control systems for future generations of pressurized water reactors. The objective is to improve the flexibility of nuclear power plants to cope with the rapid growth of renewable energies. In practice, this means that the average coolant temperature, the axial power distribution of the reactor core and the position of the control rods have to be properly regulated during power variations. In previous work, conducted by the same authors, two promising approaches were investigated: 1) fixed-structure gain-scheduled control and 2) nonlinear model predictive control. Here, both methods are tested according to industry standards in an attempt to determine the best one for our problem. To achieve this, two different controllers are designed using a new multipoint kinetic model of the reactor core, which provides an accurate representation of the axial power distribution. The advantages and drawbacks of both design methodologies are discussed and then compared on PWRSimu, an intermediate complexity pressurized water reactor simulator developed by Framatome.

Keywords: Nuclear Power Plants, Gain-Scheduling, Model Predictive Control

Nomenclature

ACT	Average Coolant Temperature
AO	Axial Offset
GSC	Gain-Scheduled Control
MS	Multiple-Shooting
NMPC	Nonlinear Model Predictive Control
NP	Nominal Power
PWR	Pressurized Water Reactor
SS	Single-Shooting

1. Introduction

In order to address climate change, many countries are seeking to replace fossil fuel power plants by renewable energy sources [1]. This energy transition poses new challenges in terms of management of the electrical network. Since excess electricity cannot currently be stored on a large scale, generation and consumption have to be continuously balanced to ensure grid stability. For this purpose, a number of power plants must be flexible, i.e., capable of adjusting their power output on demand. However, unlike fossil fuel power plants, which are easily controllable, part of renewable energy sources, such as wind turbines or solar

panels, are inherently intermittent. It is therefore essential to enhance the flexibility of conventional power plants to prepare for the upcoming expansion of renewables [2].

A nuclear power plant can be operated either in base-load or in load-following mode [3]. In base-load mode, the power output of the plant remains constant regardless of electricity demand. Most nuclear power plants around the world are still operated in base-load mode because it is simpler and more cost-effective. In load-following mode, the power output of the plant is adjusted depending on the forecasted electricity consumption. Typically, the grid operator sends a daily load profile in advance to the plant operator, who will then set the power target of the turbine accordingly. Whether in base-load or in load-following mode, the nuclear power plant can also provide frequency control to the grid. This is achieved by a dedicated controller that automatically adjusts the power output of the turbine to offset small load imbalances within seconds.

Flexible operation of a nuclear power plant is directly related to the design of the core control system [4]. The main task of the core control system is to maintain the average coolant temperature (ACT) and the axial power distribution of the reactor core, or axial offset (AO), within appropriate limits. These limits are defined upstream to ensure acceptable performance levels and safe operation of the plant. During power variations, ACT regulation is achieved by moving several neutron-absorbing control rods inside the reactor core. However, their movements have a detrimental impact on its axial power distribution. Initially, the movements of the control rods were mitigated by

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43 adjusting the boron concentration of the primary coolant.
 44 This strategy was limiting in terms of flexibility because
 45 the boron concentration of the coolant can only change
 46 slowly and gradually decreases over the fuel burn-up cycle.
 47 For example in mode A, the maximum power variation rate
 48 of the plant is typically restricted to 1-2 %NP/min at the
 49 beginning of the cycle, and 0.2 %NP/min at the end. To
 50 overcome these limitations, advanced core control systems
 51 usually comprise two separate groups, or banks, of control
 52 rods [4]. In this way, one bank of control rods can be used
 53 to perform power variations by, e.g., controlling the ACT,
 54 while the other can be used to reduce axial power distribu-
 55 tion disturbances. Thus, flexibility is increased with power
 56 variation rates of up to 5 %NP/min. After power varia-
 57 tions, the xenon distribution of the reactor core will slowly
 58 change for several hours until steady-state is reached. Dur-
 59 ing this time, the evolution of xenon concentration has
 60 to be counterbalanced to prevent the AO of the reactor
 61 core from drifting away. In highly advanced core control
 62 systems, such as mode T [5], this can be done either by
 63 moving the control rods or by adjusting the boron concen-
 64 tration of the primary coolant. The main motivation for
 65 using the control rods is to minimize the volume of effluent.
 66 Yet, this can decrease the maneuvering capabilities of the
 67 plant, as the control rods may not be properly positioned
 68 for a quick return to nominal power. Maximum flexibility
 69 is achieved when xenon poisoning is compensated by boron
 70 concentration adjustments and when the level of insertion
 71 of the control rods is controlled.

72 In practice, industrial core control systems manage to
 73 meet all the aforementioned requirements using single in-
 74 put single output Proportional-Integral-Derivative control
 75 along with sophisticated logic rules. Even if this approach
 76 gives satisfactory results for now, it may become insuf-
 77 ficient against more stringent grid management criteria.
 78 Fortunately, a wide variety of core control systems have
 79 already been proposed (see [6] and references therein).
 80 Popular [methods](#) include LQG/LTR control, H_∞ control,
 81 sliding-mode control, model predictive control and gain-
 82 scheduled control. Nevertheless, these conceptual core
 83 control systems do not [tackle](#) all the [issues](#) that arise from
 84 the flexible operation of nuclear power plants. Hence,
 85 this paper proposes to bridge the gap between industry
 86 and academia by developing two different advanced core
 87 control systems using modern design techniques. In view
 88 of the strong operational constraints that are placed on
 89 the nuclear industry, two promising approaches have been
 90 identified: [fixed-structure gain-scheduled control](#) [7, 8] and
 91 [nonlinear model predictive control](#) [9].

92 To the best of our knowledge, very few papers have tried
 93 to apply these techniques to the control of a nuclear reac-
 94 tor core. For instance, it seems that [7, 8] are the first
 95 ones that studied fixed-structure gain-scheduled control in
 96 this context. Regarding model predictive control, only a
 97 handful of examples have been reported in the scientific
 98 literature. In [10, 11], a linear model predictive controller
 99 is designed to achieve both ACT and AO regulation of a

100 reactor core that is operated in load-following mode. How-
 101 ever, the boron concentration of the coolant is adjusted by
 102 another module and the nuclear power plant does not pro-
 103 vide frequency control to the grid. In another paper [12],
 104 the power output of a reactor core is controlled by a robust
 105 nonlinear model predictive controller. Yet, this theoretic-
 106 al work is not industrially relevant as it does not even
 107 address AO regulation. By contrast, the main contribu-
 108 tion of this paper is to conduct a realistic and compre-
 109 hensive study on the design of nuclear reactor core con-
 110 trol systems. The objective is to give practical insights
 111 and methodological guidelines that could help industrial
 112 practitioners working in nuclear engineering. Two mod-
 113 ern control techniques, which were seldom tested in this
 114 field, are compared against the technical specifications of
 115 Framatome. In order to ensure consistency with industry
 116 standards, the actuators and control objectives are identi-
 117 cal to those considered in mode T [5].

118 Following on from the work carried out in [8], the first
 119 core control system is designed based on a fixed-structure
 120 H_2/H_∞ gain-scheduled control approach. The main im-
 121 provement is that the controller is tuned using a multipoint
 122 kinetic model of the reactor core rather than a point ki-
 123 netic one. [In addition, the mathematical formulation of the \$H_2/H_\infty\$ synthesis problem has been refined and made more readable.](#) Moreover, the gains are now simultane-
 125 ously tuned at each operating point and smoothly inter-
 126 polated as a function of the scheduling variable [13]. This
 127 greatly simplifies the design of the controller, as it avoids
 128 *a posteriori* interpolation, or post-processing, of the gains.
 129 The second core control system is designed based on a
 130 nonlinear model predictive control approach [14, 15]. This
 131 time, the multipoint kinetic model of the reactor core is
 132 embedded into the controller to predict the future response
 133 of the plant. Thus, the control inputs can be calculated
 134 online by repeatedly solving a constrained finite horizon
 135 optimal control problem. To this end, the model of the
 136 reactor core is reduced using singular perturbation theory
 137 so that it can be efficiently simulated by the controller.
 138

139 The remainder of this paper is organized as follows:
 140 Section II describes the system under study and formal-
 141 izes the control objectives; Section III presents the design
 142 methodology of the fixed-structure H_2/H_∞ gain-scheduled
 143 controller and Section IV presents the design methodol-
 144 ogy of the nonlinear model predictive controller; Then,
 145 the advantages and drawbacks of both techniques are dis-
 146 cussed in Section V based on a typical load-following
 147 MATLAB[®]/Simulink[®] simulation scenario; Finally, con-
 148 clusions and perspectives are drawn in Section VI.

149 2. Problem Formulation

150 2.1. System Modeling

151 To design the core control systems, a [new](#) multipoint
 152 core kinetic model of a pressurized water reactor (PWR),
 153 drawn in Fig. 1, has been developed ([a brief overview](#)

of the model is given in [16] Appendix F). Its main advantage, compared to the former point kinetic model presented in [7, 16], is that the behavior of the AO is more accurately represented since the reactor core is now divided into six meshes. However, increasing the number of meshes also increases the size of the model. As illustrated in Fig. 1, every new mesh requires five additional states: two average neutron n and delayed neutron c densities, two iodine I and xenon X_e concentrations, and one inlet temperature T . For each mesh, neutron dynamics are described by point kinetics equations with one group of delayed neutrons. An adequately chosen constant exchange coefficient has been added to take account of the transfer of neutrons between meshes. In addition, the iodine and xenon concentrations are calculated by a multipoint iodine-xenon estimator. The rise in coolant temperature, from the bottom to the top of each mesh, is given by a first order differential equation and is proportional to the amount of power generated by nuclear fission. In each mesh, the growth rate of the fission chain reaction is characterized by a quantity known as reactivity [17, 18]. Criticality is achieved when the fission chain reaction is stable and self-sustaining, i.e., when the reactivity of the reactor core equals zero. Reactivity depends on many factors, namely the fuel and coolant temperatures, the xenon concentration of the reactor core, the position of the control rods and the boron concentration of the primary coolant. The resulting feedbacks and underlying interactions between all involved variables lead to strong plant nonlinearities and coupling.

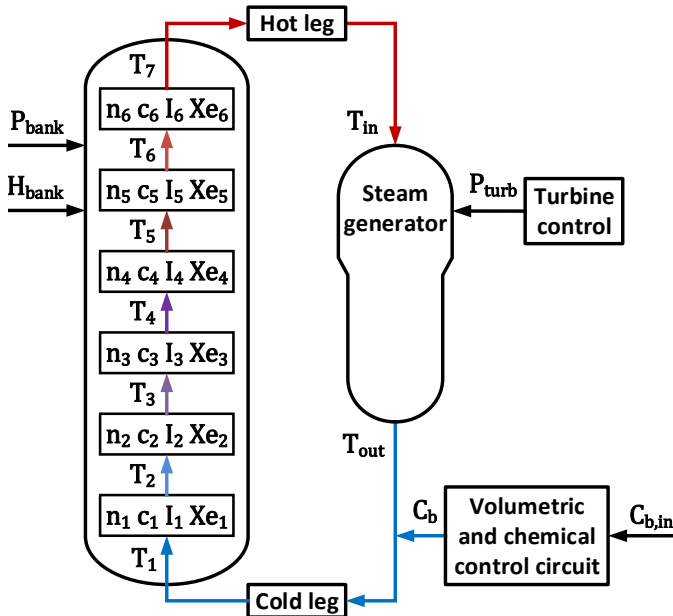


Fig. 1: Simplified diagram of the multipoint PWR core kinetic model

In order to move from reactor core to plant scale, a very simple steam generator is modeled. Once again, it is assumed that the fall in temperature between the inlet and outlet of the steam generator is given by a first order

differential equation and is proportional to the power of the turbine. In fact, whenever the power of the turbine is decreased (resp. increased), the average coolant temperature will increase (resp. decrease). As a result, the nuclear power of the reactor core will also decrease (resp. increase) and ultimately reach steady-state because the PWR was designed so that the fission chain reaction can be stabilized by temperature reactivity feedbacks alone (i.e., Doppler effect and moderation) [17]. However, it is very likely that, without control, the average coolant temperature will deviate from the limiting conditions of operation. Furthermore, xenon-induced spatial power oscillations may appear if the axial power distribution of the reactor core is left uncontrolled. Therefore, the role of the core control system is to ensure that both the ACT and the AO stay within their respective limits before, during and after power variations [4]. This can be achieved by slightly altering the reactivity of the reactor core.

In mode T [5], the nuclear reaction is controlled by moving two separate banks of control rods inside the reactor core and by adjusting the boron concentration of the primary coolant. Reactivity can be increased (resp. decreased) either by withdrawing (resp. inserting) the control rods from (resp. in) the reactor core or by decreasing (resp. increasing) the boron concentration of the coolant. The control rods are mainly used to handle abrupt changes in reactivity but adversely impact the power distribution of the reactor core. In fact, reactivity will mostly decrease in the region of the reactor core where the control rods are being inserted. To alleviate this problem, one bank of control rods, denoted by P_{bank} , is dedicated to ACT regulation whereas the other, denoted by H_{bank} , is dedicated to AO regulation. Conversely, the boron concentration of the primary coolant acts uniformly on the reactor core but takes time to become effective. This is due to the fact that the solutions of boric acid and demineralized water, which are injected to modify the boron concentration, have to flow throughout the volumetric and chemical control circuit before reaching the coolant. Hence, boron concentration adjustments are mostly used to counterbalance axial xenon oscillations after power variations. To summarize:

- ACT regulation is achieved by moving P_{bank}
- AO regulation is achieved by moving H_{bank} during power variations and by adjusting the boron concentration of the primary coolant during xenon oscillations

In addition, the mode T allows the reactor core to quickly return to its nominal power. This is achieved by controlling the level of insertion of P_{bank} in the reactor core. Last but not least, all three controlled variables can be successfully regulated even when the turbine provides frequency control to the grid.

2.2. Control Objectives

As mentioned before, the goal of an advanced core control system is to keep the ACT, the AO and the position

of P_{bank} within appropriate limits during power variations and while frequency control is active. These limits were determined beforehand by nuclear reactor engineers to comply with genuine performance and safety requirements. In mode T, their values are given by:

$$\begin{aligned} |T_a(t) - T_{a,ref}(P_{ref}(t))| &\leq 1.5 \text{ }^\circ\text{C} \\ |AO(t) - AO_{ref}| &\leq 5 \% \\ |P_{bank}(t) - P_{bank,ref}(P_{ref}(t))| &\leq 30 \text{ steps,} \end{aligned} \quad (1)$$

where T_a is the average temperature of the coolant, AO is the axial offset of the reactor core, P_{bank} is the position of the first bank of control rods, and $T_{a,ref}$, AO_{ref} and $P_{bank,ref}$ are their respective reference signals. Note that the reference signals $T_{a,ref}$ and $P_{bank,ref}$ are both piecewise linear functions of the load profile P_{ref} whereas AO_{ref} is a constant. Shorthand notation for the deviations of ACT, AO and P_{bank} are:

$$\begin{aligned} \Delta T_a(t) &= T_a(t) - T_{a,ref} \\ \Delta AO(t) &= AO(t) - AO_{ref} \\ \Delta P_{bank}(t) &= P_{bank}(t) - P_{bank,ref}(P_{ref}(t)). \end{aligned}$$

Actuator saturation should also be considered to make the core control system practically workable. In our case, the reactor core is actuated by changing the speeds of both P_{bank} and H_{bank} , and by adjusting the boron concentration of the coolant. The speeds of P_{bank} and H_{bank} are simply limited by:

$$\begin{aligned} \left| \frac{dP_{bank}(t)}{dt} \right| &\leq 75 \text{ steps/min} \\ \left| \frac{dH_{bank}(t)}{dt} \right| &\leq 75 \text{ steps/min,} \end{aligned} \quad (2)$$

where H_{bank} is the position of the second bank of control rods. Besides, P_{bank} and H_{bank} are saturated as well since the control rods cannot move beyond their maximum insertion and extraction thresholds:

$$\begin{aligned} 36 &\leq P_{bank} \leq 1053 \text{ steps} \\ 9 &\leq H_{bank} \leq 411 \text{ steps.} \end{aligned} \quad (3)$$

The upper and lower bounds of the injected boron concentration variation rate depend on the current boron concentration of the coolant. More precisely, the higher the boron concentration of the coolant, the lower the efficiency of a boric acid injection. Similarly, the lower the boron concentration of the coolant, the lower the efficiency of a demineralized water injection. This translates into the following inequalities:

$$-\frac{10}{M_t} C_b(t) \leq \frac{dC_{b,in}(t)}{dt} \leq \frac{3}{M_t} (C_{b,max} - C_b(t)), \quad (4)$$

where $C_{b,max}$ is the maximum boron concentration of the coolant, M_t is the total mass of water in the primary cooling circuit, $C_{b,in}$ is the injected boron concentration and C_b is the boron concentration of the coolant.

In our model, it is assumed that the turbine control system has already been designed and is functioning perfectly. Thus, the power of the turbine can be expressed as $P_{turb} = P_{ref} + \delta P_{ref}$ where P_{ref} is the known load profile and δP_{ref} is an unknown reference signal that is added when frequency control is active. Specifically, P_{ref} appears as a ramp-like signal during power variations and as a step-like signal otherwise. The additional term δP_{ref} is norm-bounded by 2.5 %NP and changes about every second. From the perspective of the reactor core, P_{turb} is seen as a disturbance signal that has to be rejected by the core control system.

3. Fixed-Structure Gain-Scheduled Controller

The idea of gain scheduling is to break down the nonlinear control problem into a finite number of linear sub-problems [19, 20]. The main advantage of this divide-and-conquer approach is to rely on well-known linear design tools rather than convoluted nonlinear methods. This is particularly interesting given the recent developments in non-smooth optimization for structured robust control [21].

3.1. Linearization of the Nonlinear Model

The first step in designing the fixed-structure H_2/H_∞ gain-scheduled controller is to linearize the nonlinear model of the reactor core around several operating points. All were determined using Framatome's certified three-dimensional core kinetics computer code, named SMART [22], starting from stationary conditions with equilibrium xenon at 100 %NP. The computation of an operating point begins by setting the nuclear power of the reactor core at a constant level. Then, the positions of P_{bank} and H_{bank} are updated so that criticality is achieved with minimal ACT and AO deviations. The boron concentration of the coolant, on the other hand, is fixed according to the fuel burn-up. In total, 21 operating points were computed over an evenly spaced grid of power values between 100 %NP and 50 %NP (i.e., one point every 2.5 %NP). This number of points is sufficient to obtain a fine-grained description of the original system and to ensure a smooth transition of the gains.

Then, the operating points are uniquely defined by a set of scheduling variables that covers the whole operating range of the real plant. Initially, two different scheduling variables were considered: the position P_{bank} and the known load profile P_{ref} . However, using both variables is irrelevant here, as the closed-loop dynamic of P_{bank} should be directly linked to P_{ref} . The question that remains is whether to schedule the controller with respect to the internal variable P_{bank} or the exogenous signal P_{ref} . Theoretically speaking, it would be preferable to choose P_{ref} over P_{bank} since the impact of the hidden coupling terms, which appear in the dynamic of the linearized gain-scheduled controller [20], is less pronounced when an external signal is used for scheduling. Yet, in reality, the

337 behavior of the reactor core depends heavily on the position of the control rods. Therefore, just like in [8], P_{bank}
 338 is selected as the only scheduling variable.
 339

340 Subsequently, a collection of 21 linear open-loop models
 341 is numerically computed with the linearization algorithm
 342 of MATLAB[®]/Simulink[®]. For all $i \in \llbracket 1, 21 \rrbracket$, the i -th
 343 linear open-loop model is obtained by linearizing the non-
 344 linear model of the plant around the operating point defined
 345 by $\sigma_i = P_{bank,eq}^{(i)}$. The iodine-xenon estimator has
 346 been discarded from the linearization process because its
 347 dynamics are much slower than that of the controlled out-
 348 puts. Finally, the state-space realization of the i -th linear-
 349 ized open-loop model is given by:

$$\begin{aligned} \dot{x}_r(t) &= A_i x_r(t) + B u(t) + E d(t) \\ y &= C x_r(t), \end{aligned}$$

350
 351 where $x_r \in \mathbb{R}^{25}$ is the truncated state vector (i.e., without
 352 iodine and xenon dynamics), $u = [\dot{P}_{bank}, \dot{H}_{bank}, \dot{C}_{b,in}]^T \in$
 353 \mathbb{R}^3 are the control inputs, $d = P_{turb} \in \mathbb{R}$ is the exogenous
 354 disturbance, and $y = [T_{in}, T_{out}, AO, P_{bank}, P_{core}]^T \in \mathbb{R}^5$
 355 are the measured outputs. Note that the controlled out-
 356 puts $z = [T_a, AO, P_{bank}]^T \in \mathbb{R}^3$ are a subset of the measured
 357 output y in the sense that:

$$z = T y \quad \text{with} \quad T = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

358
 359 Since the control inputs u are the time derivatives of the
 360 state variables P_{bank} , H_{bank} and $C_{b,in}$, the open-loop state
 361 matrix A_i and the control matrix B can be decomposed
 362 as:

$$A_i = \begin{bmatrix} A_{1i} & A_{2i} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad B = \begin{bmatrix} \mathbf{0} \\ I_3 \end{bmatrix}.$$

363
 364 Thus, the poles of the i -th linearized open-loop model are
 365 the union of the eigenvalues of A_{1i} plus three controllable
 366 poles located at $s = 0$. For the chosen set of model param-
 367 eters, it was observed that every matrix A_{1i} is Hurwitz
 368 stable. This numerical assessment is consistent with reality
 369 insofar as the reactor core was designed to be stable
 370 under normal conditions. Hence, it can be concluded that
 371 the linear open-loop models are all stabilizable.

372 3.2. Architecture of the Controller

373 To achieve asymptotic tracking of a class of reference
 374 inputs in the presence of another class of disturbances, a
 375 model of both exogenous signals should be embedded in
 376 the controller. Then, this model should be duplicated as
 377 many times as the number of outputs to be controlled [23].
 378 Here, the reference inputs and the disturbance belong to
 379 the same class of signals that are constant and piecewise
 380 linear. Thus, the controller should comprise at least two
 381 integrators per controlled output channel to achieve the
 382 desired closed-loop performance. However, an integrator
 383 is already included in every input-output transfer func-
 384 tion of the linearized open-loop models. Hence, only one

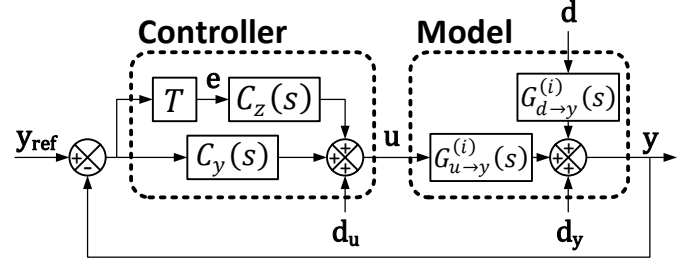


Fig. 2: Block diagram of the linear time-invariant closed-loop system computed at the i -th operating point

385 extra integrator needs to be added per controlled output
 386 channel. This can be realized with the following linear
 387 time-invariant output feedback controller:

$$\begin{aligned} \dot{x}_I(t) &= K_I (z_{ref}(t) - z(t)) \\ u(t) &= K_P (y_{ref}(t) - y(t)) + x_I(t), \end{aligned}$$

388
 389 where $y_{ref} \in \mathbb{R}^5$ are the reference signals of the measured
 390 outputs and $z_{ref} \in \mathbb{R}^3$ are the reference signals of the con-
 391 trolled outputs. This controller can be seen as a multiple-
 392 input multiple-output version of a Proportional-Integral
 393 controller with gain matrices $K_P \in \mathbb{R}^{3 \times 5}$ and $K_I \in \mathbb{R}^{3 \times 3}$.
 394 The proportional part of the controller is then filtered to
 395 provide robustness against high-frequency noise and un-
 396 modeled dynamics:

$$\begin{aligned} \dot{x}_I(t) &= K_I (z_{ref}(t) - z(t)) \\ \dot{x}_P(t) &= \text{diag}\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{b}\right) \left(-x_P(t) + K_P (y_{ref}(t) - y(t))\right) \\ u(t) &= x_P(t) + x_I(t), \end{aligned}$$

397
 398 where $a > 0$ and $b > 0$ are the time constants of the first-
 399 order low-pass filters that are applied to x_P . The time
 400 constants of the first and second filters are chosen equal
 401 since the dynamic of P_{bank} is expected to be similar to that
 402 of H_{bank} . A block diagram of the linear time-invariant
 403 closed-loop system that is computed at the i -th operating
 404 point, $i \in \llbracket 1, 21 \rrbracket$, is shown in Fig. 2, where:

$$\begin{aligned} G_{d \rightarrow y}^{(i)}(s) &= C (sI_{25} - A_i)^{-1} E \\ G_{u \rightarrow y}^{(i)}(s) &= C (sI_{25} - A_i)^{-1} B \\ C_y(s) &= \text{diag}\left(\frac{1}{as+1}, \frac{1}{as+1}, \frac{1}{bs+1}\right) K_P \\ C_z(s) &= \frac{1}{s} K_I, \end{aligned}$$

and $e = z - z_{ref}$ is the tracking error:

$$e = [\Delta T_a, \Delta AO, \Delta P_{bank}]^T. \quad (5)$$

405
 406 The signals $d_u \in \mathbb{R}^3$ and $d_y \in \mathbb{R}^5$ are unknown additive
 407 disturbances that may appear on the actuators and on the
 408 sensors of the plant. The input and the output sensitivity
 409 functions:
 410
 411

$$\begin{aligned} S_u^{(i)}(s) &= \left[I_5 + (C_y(s) + T C_z(s)) G_{u \rightarrow y}^{(i)}(s) \right]^{-1} \\ S_y^{(i)}(s) &= \left[I_3 + G_{u \rightarrow y}^{(i)}(s) (C_y(s) + T C_z(s)) \right]^{-1}, \end{aligned}$$

413 are given by the closed-loop transfer functions from d_u to
414 u and from d_y to y respectively.

415 3.3. Tuning and Gain-Scheduling

416 The conventional approach [19, 20] for scheduling the
417 gains of the controller would be to: 1) synthesize a linear
418 time-invariant controller at each operating point using,
419 e.g., a fixed-structure H_2/H_∞ synthesis algorithm and 2)
420 smoothly interpolate the gains of the controller as a function
421 of the scheduling variable P_{bank} . However, independently
422 tuning the controller at each operating point may
423 lead to undesirable jumps in the values of the gains once
424 they have been interpolated. To prevent this, the gains can
425 be tuned against multiple neighboring models rather than
426 just one [8]. Here, the tuning and scheduling stages are
427 blended together using the Gain Surface Tuning method
428 presented in [13]. First, the gain matrices K_P , K_I and the
429 time constants a , b are written as quadratic polynomials
430 of the scheduling variable $\sigma = P_{bank}$:

$$\begin{aligned} K_P(\sigma) &= K_{P0} + K_{P1}\sigma + K_{P2}\sigma^2 \\ K_I(\sigma) &= K_{I0} + K_{I1}\sigma + K_{I2}\sigma^2 \\ a(\sigma) &= a_0 + a_1\sigma + a_2\sigma^2 \\ b(\sigma) &= b_0 + b_1\sigma + b_2\sigma^2, \end{aligned}$$

432 with coefficients $K_{Pj} \in \mathbb{R}^{3 \times 5}$, $K_{Ij} \in \mathbb{R}^{3 \times 3}$, $a_j \in \mathbb{R}$ and
433 $b_j \in \mathbb{R}$ for all $j \in \{1, 2, 3\}$. Then, these coefficients are
434 tuned to ensure proper closed-loop performance at each
435 operating point. Specifically, let $\mathbf{K} \in \mathbb{R}^{78}$ be the vector of
436 tunable parameters. For each linearized open-loop model,
437 the performance objectives are represented by the following
438 frequency-domain criteria:

$$\begin{aligned} \min_{\mathbf{K}} \quad & \left\| \text{diag}(\beta_1, \beta_2, \beta_3) H_{z_{ref} \rightarrow e}^{(i)}(\mathbf{K}, s) \frac{1}{s^2} \right\|_2 \\ \text{subject to:} \quad & \begin{cases} \left\| W_{11} H_{d \rightarrow e_1}^{(i)}(\mathbf{K}, s) \right\|_\infty \leq 1 \\ \left\| W_{12} H_{d \rightarrow e_2}^{(i)}(\mathbf{K}, s) \right\|_\infty \leq 1 \\ \left\| W_{13} H_{d \rightarrow e_3}^{(i)}(\mathbf{K}, s) \right\|_\infty \leq 1 \\ \left\| W_{21} H_{d \rightarrow u_1}^{(i)}(\mathbf{K}, s) \right\|_\infty \leq 1 \\ \left\| W_{22} H_{d \rightarrow u_2}^{(i)}(\mathbf{K}, s) \right\|_\infty \leq 1 \\ \left\| W_{23} H_{d \rightarrow u_3}^{(i)}(\mathbf{K}, s) \right\|_\infty \leq 1 \\ \left\| \gamma H_{d_u \rightarrow u}^{(i)}(\mathbf{K}, s) \right\|_\infty \leq 1 \\ \text{Re}(p^{(i)}(\mathbf{K})) \leq -\alpha, \end{cases} \end{aligned}$$

440 where, for all $i \in \llbracket 1, 21 \rrbracket$, $H_{u \rightarrow y}^{(i)}$ denotes the closed-loop
441 transfer function from input signal u to output signal y for
442 the i -th plant model, $\|\cdot\|_2$ and $\|\cdot\|_\infty$ are the H_2 and H_∞
443 norms, and $p^{(i)}$ are the closed-loop poles of the i -th plant
444 model. Having two integrators in the loop transfer function
445 means that $H_{z_{ref} \rightarrow e}^{(i)}$ has two zeros at $s = 0$. Hence, its

H_2 norm can be shaped with a double integrator weighting
446 function which, in this context, is interpreted as a refer-
447 ence signal generator. In fact, minimizing the frequency-
448 weighted H_2 norm of $H_{z_{ref} \rightarrow e}^{(i)}$ ensures both asymptotic
449 tracking and disturbance rejection of step-like and ramp-
450 like exogenous signals. The parameters $(\beta_1, \beta_2, \beta_3) \in \mathbb{R}_{>0}^3$
451 are chosen such that ACT regulation takes precedence over
452 AO regulation which, in turn, takes precedence over P_{bank}
453 regulation. The scalar weights $(W_{11}, W_{12}, W_{13}) \in \mathbb{R}_{>0}^3$ and
454 $(W_{21}, W_{22}, W_{23}) \in \mathbb{R}_{>0}^3$ are used to limit the maximum set-
455 point deviation and the maximum control effort that are
456 induced by a change in the power of the turbine. The reason
457 why the controller is tuned against several single-input,
458 single-output H_∞ constraints, rather than one multiple-
459 input, multiple-output H_∞ constraint, is to make the opti-
460 mization problem more readable. Finally, the parameters
461 $\gamma \in \mathbb{R}_{>0}$ and $\alpha \in \mathbb{R}_{>0}$ are selected to provide a sufficient
462 level of input-robustness and stability to the closed-loop
463 system. 464

465 4. Nonlinear Model Predictive Controller

The principle of model predictive control is to use a
466 model of the plant to predict and optimize its future be-
467 havior by repeatedly solving an on-line optimal control
468 problem over a finite horizon [14, 15]. This control tech-
469 nique, also known as receding horizon control, has proven
470 to be very successful in the process industry because it
471 can handle large scale, multi-input multi-output nonlin-
472 ear systems that are subject to actuator and state con-
473 straints [24]. 474

475 4.1. Model Reduction

The nonlinear state-space representation of the full-
476 order PWR model can be written as:

$$\begin{aligned} \dot{x}(t) &= f_0(x(t), u(t), d(t)) \\ x(t_0) &= x_0, \end{aligned}$$

477 where $x \in \mathbb{R}^{37}$ is the full state vector, $x_0 \in \mathbb{R}^{37}$ is the initial
478 state, $u \in \mathbb{R}^3$ are the control inputs, and $d \in \mathbb{R}$ is the ex-
479 ogenous disturbance. In the sequel, it is assumed that the
480 state is perfectly measured. Since the model is intended
481 to be used in a model predictive control framework, special
482 attention is given to its computational complexity. In
483 practice, many numerical simulation runs are performed
484 to solve the optimal control problem at each time step.
485 However, the multiple time scale behavior of the model,
486 ranging from fast neutron to slow xenon dynamics, ren-
487 ders it numerically stiff [25]. Although the full-order model
488 can be simulated using powerful integration solvers, such
489 as [26] or [27], model reduction is still conducted to make
490 the optimal control problem more tractable. State estima-
491 tion, which is outside the scope of the paper, is another
492 motivation for reducing the order of the model. In the
493 future, a state observer should be designed to implement
494 the NMPC controller [28]. This task would obviously be
495 496

547 easier to achieve with a reduced-order model. Therefore, 537
548 singular perturbation theory [29] is employed to remove 538
549 the dynamics that are not essential regarding the control 539
550 objectives. A well-known approximation in nuclear reactor 540
551 physics is the prompt-jump approximation [17, 18]. 541
552 Whenever a fission reaction occurs, two types of neutrons 542
553 are emitted: prompt neutrons, which are directly emitted 543
554 from fission, and delayed neutrons, which are emitted 544
555 a moment later from the radioactive decay of fission 545
556 products. Under normal operating conditions, the reactor 546
557 core is predominantly governed by the dynamic of delayed 547
558 neutrons. Consequently, prompt neutron dynamics 548
559 can be reasonably approximated by steady-state equations 549
560 without any significant loss of accuracy [30]. The result- 550
561 ing reduced-order model is given by a set of semi-explicit 551
562 differential-algebraic equations: 552

$$513 \begin{aligned} \dot{x}_d(t) &= f(x_d(t), x_a(t), u(t), d(t)) \\ 0 &= g(x_d(t), x_a(t), u(t), d(t)) \\ x_d(0) &= x_{d0}, \end{aligned}$$

514 where the state is divided into a slow differential part $x_d \in$ 537
515 \mathbb{R}^{31} and a fast algebraic part $x_a \in \mathbb{R}^6$. 538

516 4.2. Formulation of the Optimal Control Problem

517 At each time step, the NMPC algorithm yields a se- 537
518 quence of control inputs by solving a finite horizon optimal 538
519 control problem that is initialized with the current state of 539
520 the plant. Usually, only the first component of the control 540
521 input sequence is applied open-loop to the plant until a 541
522 new one is calculated at the next time step. The greatest 542
523 strength of NMPC is that all the previously defined core 543
524 control specifications can be directly incorporated into the 544
525 finite horizon optimal control problem: 545

$$526 \min_{\bar{u}(\cdot)} \int_t^{t+NT_s} \left(\|e(\bar{x}_d(\tau), \bar{x}_a(\tau), \bar{d}(\tau))\|_Q^2 + \|\bar{u}(\tau)\|_R^2 \right) d\tau$$

527 subject to:

$$528 \forall \tau \in [t, t + NT_s] \begin{cases} c(\bar{x}_d(\tau), \bar{x}_a(\tau), \bar{u}(\tau), \bar{d}(\tau)) \leq 0 \\ \dot{\bar{x}}_d(\tau) = f(\bar{x}_d(\tau), \bar{x}_a(\tau), \bar{u}(\tau), \bar{d}(\tau)) \\ 0 = g(\bar{x}_d(\tau), \bar{x}_a(\tau), \bar{u}(\tau), \bar{d}(\tau)) \\ \bar{x}_d(t) = x_d(t), \end{cases} \quad (6)$$

529 where $N \geq 1$ is the length of the prediction horizon, $T_s > 0$ 537
530 is the sampling time of the controller, and $x_d(t)$ is the dif- 538
531 ferential state of the plant that is measured at time $t \geq 0$. 539
532 The core control specifications (1) – (4) are represented by 540
533 the inequality constraints map $c: \mathbb{R}^{31} \times \mathbb{R}^6 \times \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}^{16}$ 541
534 and the tracking error (5) is given by the nonlinear map 542
535 $e: \mathbb{R}^{31} \times \mathbb{R}^6 \times \mathbb{R} \rightarrow \mathbb{R}^3$. The weighted 2-norms $\|e\|_Q$ and 543
536 $\|u\|_R$ are defined by $\|e\|_Q^2 := e^T Q e$ and $\|u\|_R^2 := u^T R u$ 544
537 where $Q \in \mathbb{R}^{3 \times 3}$ and $R \in \mathbb{R}^{3 \times 3}$ are symmetric positive- 545
538 definite matrices. The bar subscript denotes internal 546

537 model variables that are used within the controller to pre- 537
538 dict the behavior of the real plant. In particular, the dis- 538
539 turbance signal is written as $d(t) = \bar{d}(t) + \delta d(t)$ where \bar{d} 539
540 is the known load profile that is sent in advance by the 540
541 grid dispatcher to the plant and $|\delta d| \leq 2.5 \%$ NP is the un- 541
542 known norm-bounded term that appears when frequency 542
543 control is active. 543

544 Transcription of the continuous optimal control problem 544
545 into a finite-dimensional nonlinear programming problem 545
546 is commonly achieved using either direct single-shooting or 546
547 direct multiple-shooting [31, 32]. Here, both options will 547
548 be implemented and tested thereafter. In a direct method, 548
549 the continuous-time control inputs are approximated with 549
550 a finite number of parameters. Most of the time, the con- 550
551 trol input sequence $\bar{\mathbf{u}} = \{\bar{u}[0], \dots, \bar{u}[N-1]\}$ is discretized 551
552 with a piecewise constant parameterization: 552

$$553 \forall k \in \llbracket 0, N-1 \rrbracket, \forall \tau \in [t_k, t_{k+1}], \bar{u}(\tau) = \bar{u}[k],$$

554 where $t_k = t + kT_s$. The key difference between single- 537
555 shooting and multiple-shooting is the parameterization of 538
556 the state trajectory. 539

540 In single-shooting, the state trajectory is computed in 540
541 one shot, from t_0 to t_N , outside of the optimization prob- 541
542 lem. In fact, for a given control input sequence $\bar{\mathbf{u}}$ and a 542
543 fixed initial state $x_d(t)$, the state trajectory can be entirely 543
544 determined by simulating the model of the plant: 544

$$545 \forall k \in \llbracket 0, N-1 \rrbracket, \begin{pmatrix} \bar{x}_d(t_{k+1}) \\ \bar{x}_a(t_{k+1}) \end{pmatrix} = \phi_{ss}(t_{k+1}; x_d(t), \bar{\mathbf{u}}),$$

546 where $\phi_{ss}: \mathbb{R} \times \mathbb{R}^{31} \times \mathbb{R}^{3 \times N} \rightarrow \mathbb{R}^{37}$ is a numerical integra- 537
547 tion operator that returns the solution of the initial value 538
548 problem: 539

$$549 \forall \tau \in [t_0, t_N] \begin{cases} \dot{\bar{x}}_d(\tau) = f(\bar{x}_d(\tau), \bar{x}_a(\tau), \bar{u}(\tau), \bar{d}(\tau)) \\ 0 = g(\bar{x}_d(\tau), \bar{x}_a(\tau), \bar{u}(\tau), \bar{d}(\tau)) \\ \bar{x}_d(t_0) = x_d(t), \end{cases}$$

550 at time t_{k+1} . Therefore, the optimization problem can be 537
551 reduced to: 538

$$552 \min_{\bar{\mathbf{u}}} \sum_{k=0}^{N-1} \|e(\phi_{ss}(t_{k+1}; x_d(t), \bar{\mathbf{u}}), \bar{d}(t_{k+1}))\|_Q^2 + \|\bar{u}[k]\|_R^2$$

553 subject to:

$$554 \forall k \in \llbracket 0, N-1 \rrbracket, c(\phi_{ss}(t_{k+1}; x_d(t), \bar{\mathbf{u}}), \bar{u}[k], \bar{d}(t_{k+1})) \leq 0.$$

555 Hence, the advantage of using single-shooting is to remove 537
556 the equality constraints from the optimization problem. 538
557 Moreover, the state trajectory is always guaranteed to 539
558 be consistent with the dynamical equations of the model. 540
559 However, using single-shooting can increase the computa- 541
560 tional complexity of the optimization problem because the 542
561 state equations are repeatedly propagated into both the 543
562 cost function and the constraints. This is especially true 544
563 for nonlinear and/or unstable systems. 545

564 In multiple-shooting, the state trajectory is cut into 537
565 N pieces which are independently computed over each 538

time interval $[t_k, t_{k+1}]$. These pieces are then **reassembled** **within** the optimization problem by imposing equality constraints on the differential and algebraic states:

$$\min_{\bar{\mathbf{x}}_d, \bar{\mathbf{x}}_a, \bar{\mathbf{u}}} \sum_{k=0}^{N-1} \|e(\bar{x}_d[k+1], \bar{x}_a[k], \bar{d}(t_{k+1}))\|_Q^2 + \|\bar{u}[k]\|_R^2$$

subject to:

$$\forall k \in \llbracket 0, N-1 \rrbracket,$$

$$\begin{cases} c(\bar{x}_d[k+1], \bar{x}_a[k], \bar{u}[k], \bar{d}(t_{k+1})) \leq 0 \\ \begin{pmatrix} \bar{x}_d[k+1] \\ \bar{x}_a[k] \end{pmatrix} - \phi_{ms}(t_{k+1}; \bar{x}_d[k], \bar{u}[k]) = 0 \\ \bar{x}_d[0] - x_d(t) = 0, \end{cases}$$

where $\phi_{ms} : \mathbb{R} \times \mathbb{R}^{31} \times \mathbb{R}^6 \rightarrow \mathbb{R}^{37}$ is a numerical integration operator that returns the solution of the initial value problem:

$$\forall \tau \in [t_k, t_{k+1}] \begin{cases} \dot{\bar{x}}_d(\tau) = f(\bar{x}_d(\tau), \bar{x}_a(\tau), \bar{u}(\tau), \bar{d}(\tau)) \\ 0 = g(\bar{x}_d(\tau), \bar{x}_a(\tau), \bar{u}(\tau), \bar{d}(\tau)) \\ \bar{x}_d(t_k) = x_d(t_k), \end{cases}$$

at time t_{k+1} . The drawback of multiple-shooting is that the dimension of the optimization problem is **substantially** increased since the differential and algebraic state sequences $\bar{\mathbf{x}}_d = \{\bar{x}_d[0], \dots, \bar{x}_d[N]\}$ and $\bar{\mathbf{x}}_a = \{\bar{x}_a[0], \dots, \bar{x}_a[N-1]\}$ become decision variables. However, this is not a real issue because the structure of the resulting nonlinear program is block-sparse, provided that the decision variables are arranged as follows:

$$\begin{pmatrix} \bar{x}_d[0]^T, \bar{u}[0]^T, \bar{x}_a[0]^T, \dots, \\ \bar{x}_d[N-1]^T, \bar{u}[N-1]^T, \bar{x}_a[N-1]^T, \bar{x}_d[N]^T \end{pmatrix}^T.$$

In fact, it has often been reported that efficient sparsity exploitation can actually improve the rate of convergence of the Newton-type algorithm which is used to solve the nonlinear program [33]. **Furthermore**, having $\bar{\mathbf{x}}_d$ and $\bar{\mathbf{x}}_a$ as decision variables is beneficial for initializing the problem since *a priori* information about the state trajectory can be supplied to the algorithm. Last but not least, multiple-shooting is better suited for highly nonlinear and/or unstable plants because integration is broken down into short time intervals, meaning that the nonlinearity is equally distributed over the shooting nodes.

4.3. Practical Implementation Issues

The nonlinear program that is transcribed using single shooting is composed of $3N$ decision variables and $16N$ inequality constraints. On the other hand, the nonlinear program that is transcribed using multiple-shooting is composed of $40N+31$ decision variables, $37N+31$ equality constraints and $16N$ inequality constraints. In both cases, it is expected that the resolution time of the optimization problem becomes greater than the duration T_s in between

two sampling instants because the length $N \geq 1$ of the prediction horizon has to be large enough to ensure closed-loop stability. Yet, we still want to recalculate the control inputs at each time step so the plant does not remain in open-loop for too long. Therefore, the optimization routine is deliberately terminated once the resolution time exceeds T_s seconds. Even if the solution is sub-optimal [34], we believe that it should have sufficiently converged to provide an acceptable level of closed-loop performance.

Consequently, a prediction-based delay compensation scheme [35] is incorporated into the NMPC algorithm to account for the non-negligible resolution time $0 < \tau \leq T_s$ of the optimization problem. To illustrate, let $\mu_N(x_d(t))$ be the feedback value given by the (sub-optimal) control input sequence $\bar{\mathbf{u}}^*(x_d(t))$ that (almost) solves the optimal control problem with initial state $x_d(t)$. Because of the computational delay, $\mu_N(x_d(t))$ is applied to the plant at time $t+\tau$ instead of time t . Thus, the greater the mismatch between $x_d(t)$ and $x_d(t+\tau)$, the greater the difference between the expected and the real behavior of the closed-loop system. Knowing the maximum resolution time $\tau_{max} = T_s$ of the optimization problem, the idea of [35] is to solve the optimal control problem in advance, starting from an estimate $\bar{x}_d(t+T_s)$ of the future state in place of the currently measured state $x_d(t)$. The resulting feedback value $\mu_N(\bar{x}_d(t+T_s))$, available at time $t+\tau$, is then applied to the plant accordingly at time $t+T_s$. The next state estimate can be computed straightforwardly by simulating the model of the plant from the newly measured state $x_d(t+T_s)$ with input $\mu_N(\bar{x}_d(t+T_s))$. Note that, since the state is estimated in an open-loop fashion, it is almost certain that the real state $x_d(t+T_s)$ will differ from $\bar{x}_d(t+T_s)$. However, it is still relevant to use the delay-compensated input, as some degree of robustness is expected against estimation errors [36].

5. Simulation Results

5.1. Validation Scenario

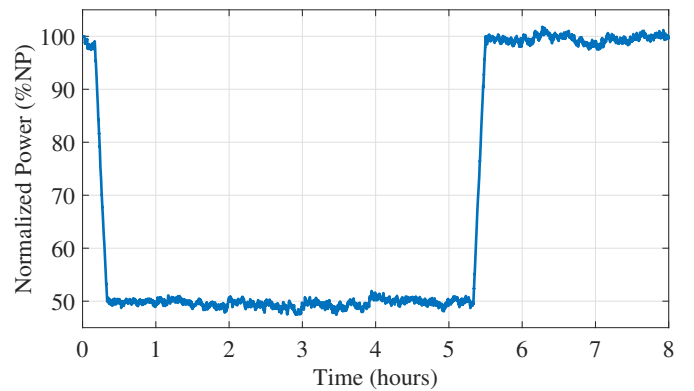


Fig. 3: Normalized power of the turbine with frequency control

To assess and compare the performance of the two core control systems, a typical 8-hour load-following

660 scenario is conducted on PWRSimu, an intermediate
 661 complexity PWR simulator developed by Framatome in
 662 MATLAB[®]/Simulink[®]. As shown in Fig. 3, this sce-
 663 nario involves two power variations (from 100 %NP to
 664 50 %NP and *vice versa*) that are carried out at a rate of
 665 5 %NP/min with frequency control. The full-order non-
 666 linear PWR model is simulated on Simulink with the stiff
 667 adaptive-step integration solver ode15s [27].

668 The fixed-structure gain-scheduled controller has been
 669 tuned with the following set of parameters:

$$\begin{aligned} \beta_1 &= 10/1.5 & \beta_2 &= 5/5 & \beta_3 &= 2/30 \\ W_{11} &= 10/1.5 & W_{12} &= 10/5 & W_{13} &= 10/30 \\ W_{21} &= 10/1.25 & W_{22} &= 10/1.25 & W_{23} &= 10/0.04 \\ \gamma &= 0.5 & \alpha &= 0.0001, \end{aligned}$$

671 using Systune (MATLAB Control System Toolbox[™]) [21].
 672 In order to model the whole closed-loop system in
 673 Simulink, the quadratic polynomials $K_P(\sigma)$, $K_I(\sigma)$
 674 and $b(\sigma)$ are approximated by feeding the scheduling vari-
 675 able $\sigma = P_{bank}$ into 1-D lookup table blocks with linear
 676 interpolation and clip extrapolation options.

677 Tuning of the NMPC controller has been achieved in two
 678 steps. First, the matrices Q and R of the cost function (6)
 679 are selected diagonal with normalized weights:

$$\begin{aligned} q_1 &= \frac{\varepsilon_1}{(e_{1,max})^2} & q_2 &= \frac{\varepsilon_2}{(e_{2,max})^2} & q_3 &= \frac{\varepsilon_3}{(e_{3,max})^2} \\ r_1 &= \frac{\nu_1}{(u_{1,max})^2} & r_2 &= \frac{\nu_2}{(u_{2,max})^2} & r_3 &= \frac{\nu_3}{(u_{3,max})^2}, \end{aligned}$$

681 where, for all $i \in \{1, 2, 3\}$, $e_{i,max}$ and $u_{i,max}$ denote the
 682 maximum acceptable values of e_i and u_i (see Bryson's
 683 rule [37]). Then, the tunable parameters $(\varepsilon_1, \varepsilon_2, \varepsilon_3) \in \mathbb{R}_{>0}^3$
 684 and $(\nu_1, \nu_2, \nu_3) \in \mathbb{R}_{>0}^3$ are used to penalize, in order of
 685 importance, the deviations of ACT, AO and P_{bank} with-
 686 out unduly restricting the control effort. In the end, the
 687 weighting matrices are given by:

$$688 Q = \text{diag}\left(\frac{30}{1.5^2}, \frac{20}{5^2}, \frac{15}{30^2}\right), R = \text{diag}\left(\frac{0.01}{1.25^2}, \frac{0.01}{1.25^2}, \frac{0.01}{0.04^2}\right).$$

689 The sampling time $T_s = 60$ s and the prediction horizon
 690 $N = 10$ are chosen so as to balance on-line complexity
 691 and closed-loop stability. The NMPC algorithm is im-
 692 plemented in MATLAB R2019a with CasADi v3.5.5 [38].
 693 This open-source software provides several building blocks
 694 which are especially useful for solving large-scale nonlinear
 695 programming problems efficiently. Practically speaking, the
 696 key asset of CasADi is its ability to quickly compute Jaco-
 697 bians and exploit their sparsity pattern using algorithmic
 698 differentiation and graph coloring techniques. Within this
 699 framework, the reduced-order PWR model is simulated
 700 with the stiff adaptive-step integration solver IDAS [26]
 701 and the optimization problem is solved with the primal-
 702 dual interior point solver IPOPT [39] using an Intel[®] Core[™]
 703 i3-6100U processor with 16 GB of RAM. The closed-loop
 704 interactions between the controller and the plant are rep-
 705 resented by the sequential execution of both MATLAB[®]
 706 and Simulink[®] files.

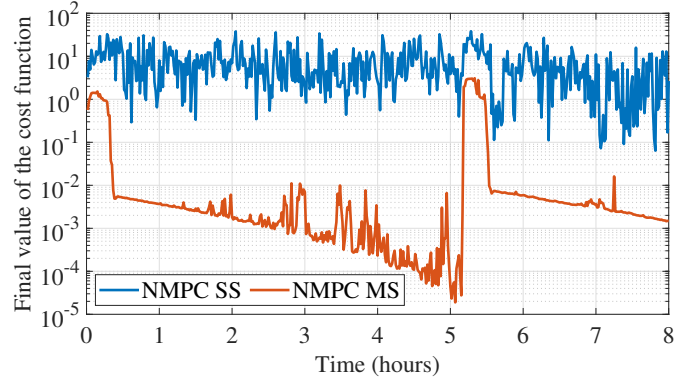


Fig. 4: Semi-log plot of the final value of the cost function (NMPC single-shooting vs NMPC multiple-shooting)

708 In order to choose between single-shooting and multiple-
 709 shooting, both transcription methods were implemented and tested.
 710 As can be seen in Fig. A.7 and A.9, the ACT deviation and the position deviation of P_{bank} are almost identical for both methods. However, the AO deviation, shown in Fig. A.8, is significantly smaller in multiple-shooting because the behavior of H_{bank} , shown in Fig. A.11, is less erratic than in single-shooting. The same phenomenon can be observed, but to a lesser extent, for P_{bank} in Fig. A.10 and for the boron concentration in Fig. A.12. The reason behind this, as Fig. 4 demonstrates, is that the optimization problem converges more quickly to the solution when transcribed in multiple-shooting. In fact, it turned out that further increasing the resolution time in multiple-shooting had no effect on the final value of the cost function. This suggests that the optimal solution can always be found within the allocated time, which is why multiple-shooting is selected over single-shooting.

726 Overall, the performance of the NMPC controller is better than that of the gain-scheduled controller. As can be seen in Fig. A.13, A.14 and A.15, both controllers manage to keep the ACT, the AO and the position of P_{bank} close to their reference signals. Yet, as can be observed in Fig. A.16, A.17 and A.18, the gain-scheduled controller tends to overuse the actuators in comparison with the NMPC controller. This shows that the bandwidth of the controller does not need to be particularly large even when

Table 1: Comparison between the gain-scheduled controller and the nonlinear model predictive controller

GSC	NMPC
Several linear models	One nonlinear model
Frequency domain design	Time domain design
Output-feedback control	State-feedback control
Robust design	Nominal design
Pre-computed gains	Online calculations
Continuous-time control	Discrete-time control
No anticipation	Preventive actions

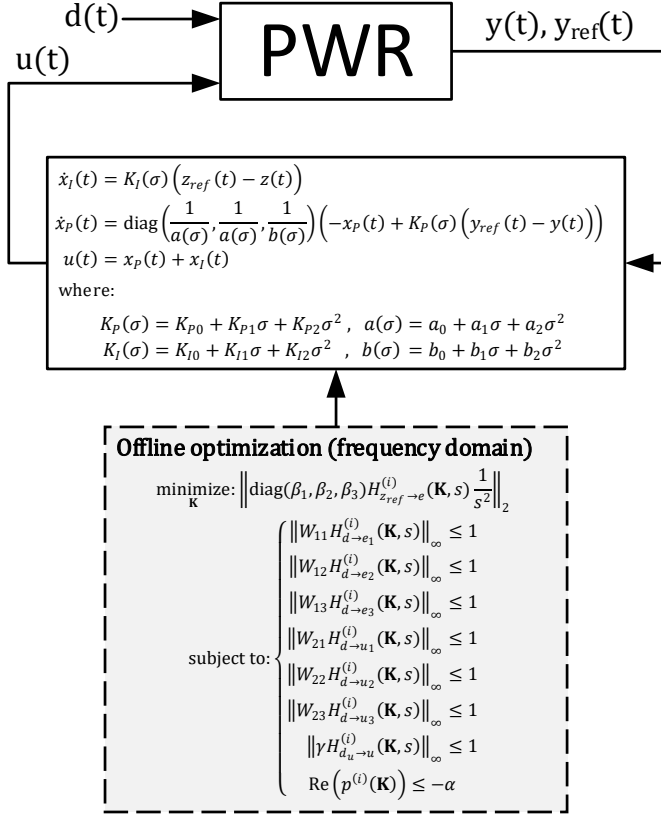


Fig. 5: Fixed-structure gain-scheduled control approach

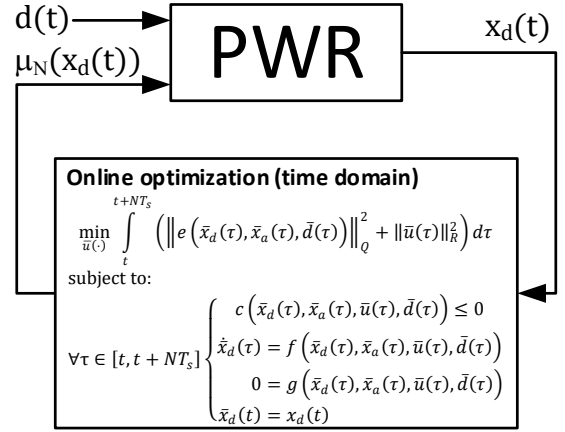


Fig. 6: Nonlinear model predictive control approach

Unfortunately, some state variables, such as the xenon concentration of the reactor core, cannot be physically measured. Thus, the NMPC controller cannot be practically implemented unless a nonlinear state observer is designed. This not only increases the overall complexity of the core control system but also raises additional concerns about its robustness.

6. Conclusion

In this paper, two advanced core control systems have been designed for flexible operation of future PWRs. The first one is a fixed-structure H_2/H_∞ gain-scheduled controller and the second one is an NMPC controller. The design methodologies of both controllers have been detailed and their performances have been compared on the intermediate complexity pressurized water reactor simulator of Framatome, named PWRSimu. The gain-scheduled controller is well adapted to cope with unexpected disturbances and model uncertainties. Moreover, its architecture is readable and easy to implement. However, some improvements should be made to avoid overusing the actuators (a dead-zone nonlinearity could be taken into account in the design of the controller using, e.g., the circle criterion). By contrast, the NMPC controller can handle a wide range of constraints and can easily deal with disturbances that are known in advance. Its predictive capabilities make it possible to achieve good performance while limiting the control effort. However, it is very difficult to analyze its robustness, especially when used together with a state-observer. In future work, the controllers will be combined in a hierarchical manner to overcome the limitations of both design strategies. The NMPC controller could be located at an upper level to provide a feedforward action to the plant as well as reference output trajectories to the gain-scheduled controller. Then, the gain-scheduled controller could be used at a lower level to reject unknown disturbances and to ensure that the closed-loop system stays close to the reference trajectory despite model uncertainties. This hierarchical scheme is also motivated by

frequency control is active. However, the NMPC controller may struggle to reject other sudden and unexpected disturbances, as the control inputs are only updated every minute. To give a concrete example, note that the principal effect of frequency control is to cause ACT fluctuations of about ± 0.8 °C. A closer look at Fig. A.13 reveals that these fluctuations cannot be properly mitigated by the NMPC controller because they are unpredictable and of relatively high frequency. The gain-scheduled controller, on the other hand, gives better results since it provides continuous feedback to the plant and was designed using robust control methods. As a matter of fact, robustness is still an open issue in the field of model predictive control [15]. Therefore, it should be mentioned that the reduced-order PWR model, which is embedded in the NMPC controller, derives from the one that is used for validation. Hence, it is not surprising that the NMPC controller performs better than the gain-scheduled controller, which was designed off-line based on simpler linear models. This is both a strength and a weakness of nonlinear model predictive control. While it is indeed better to use a very detailed and accurate model for prediction, it can also become an issue if the computation time of the NMPC control law gets too long. Worse, using an excessively complex model can even make the optimization problem intractable. Besides, it is also good to recall that the NMPC controller is a state-feedback controller whereas the gain-scheduled controller is an output-feedback one.

801 the multiple time scales of the plant. In fact, the upper
 802 layer could anticipate future load variations and forecast
 803 the evolution of xenon concentration while the lower layer
 804 could simply focus on disturbance rejection and setpoint
 805 tracking. It would also make sense to leave the **calcula-**
 806 **tion** of boron concentration adjustments to the upper layer
 807 since the **evolution** of **the** boron concentration is **substan-**
 808 **tially** slower than the movements of the control rods.

809 **Conflict of interest**

810 None declared.

811 **Acknowledgment**

812 The authors would like to thank Framatome for the fi-
 813 nancial support.

814 **Appendix A. Simulation Results**

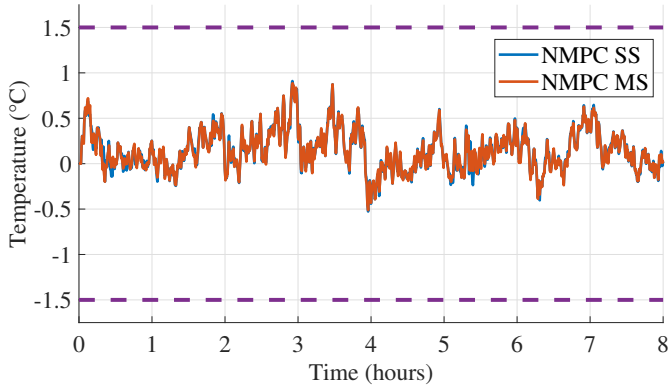


Fig. A.7: Average coolant temperature deviation (NMPC single-shooting vs NMPC multiple-shooting)

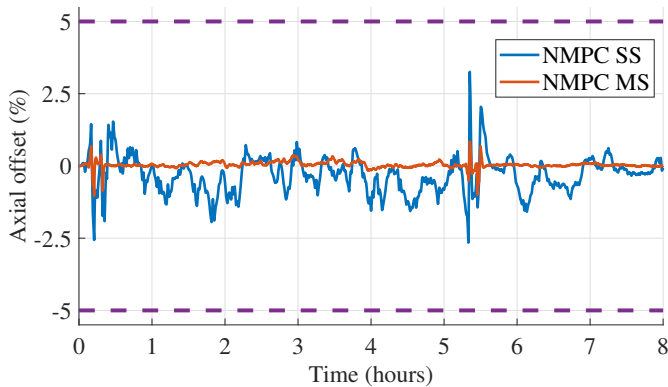


Fig. A.8: Axial offset deviation (NMPC single-shooting vs NMPC multiple-shooting)

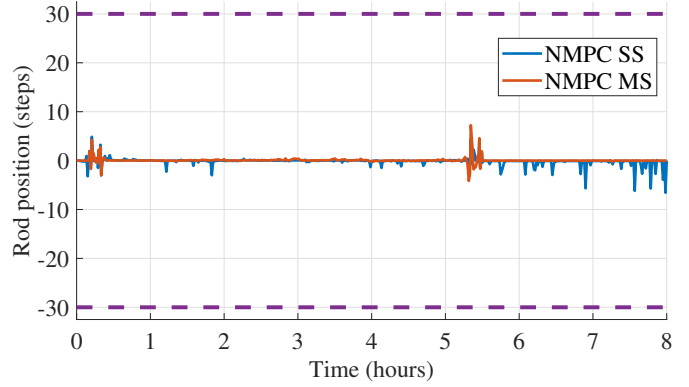


Fig. A.9: Position deviation of P_{bank} (NMPC single-shooting vs NMPC multiple-shooting)

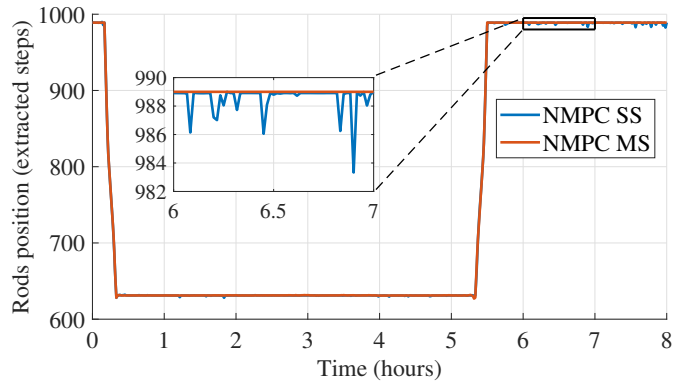


Fig. A.10: Position of P_{bank} (NMPC single-shooting vs NMPC multiple-shooting)

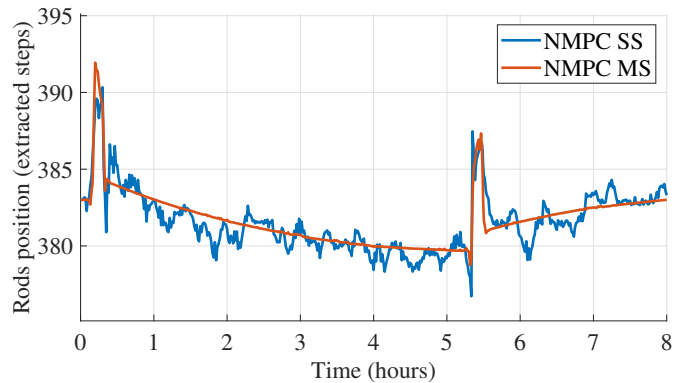


Fig. A.11: Position of H_{bank} (NMPC single-shooting vs NMPC multiple-shooting)

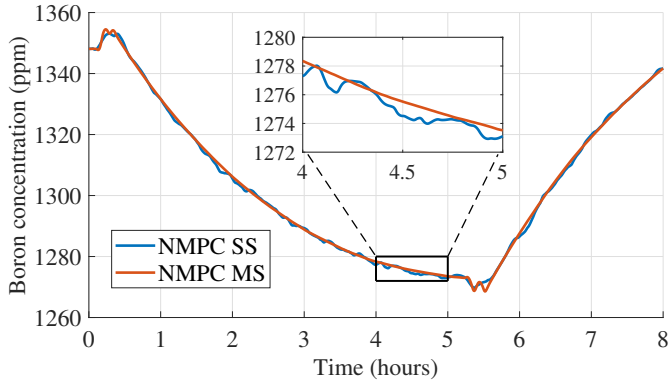


Fig. A.12: Boron concentration of the primary coolant (NMPC single-shooting vs NMPC multiple-shooting)

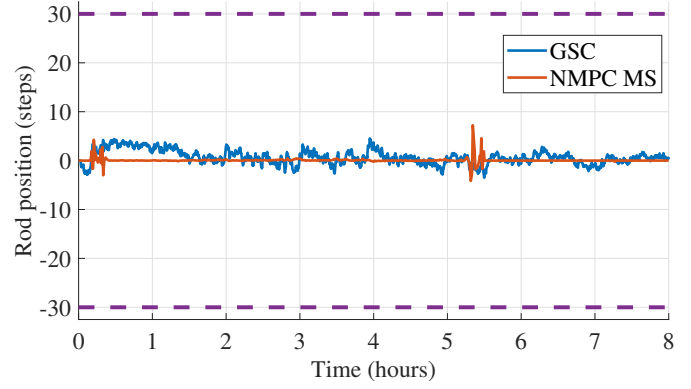


Fig. A.15: Position deviation of P_{bank} (Gain-Scheduled Controller vs NMPC multiple-shooting)

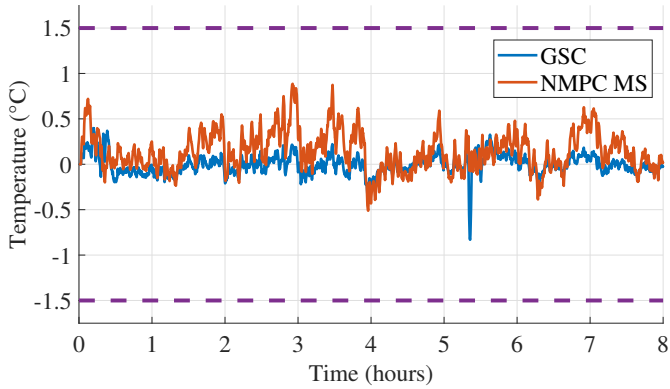


Fig. A.13: Average coolant temperature deviation (Gain-Scheduled Controller vs NMPC multiple-shooting)

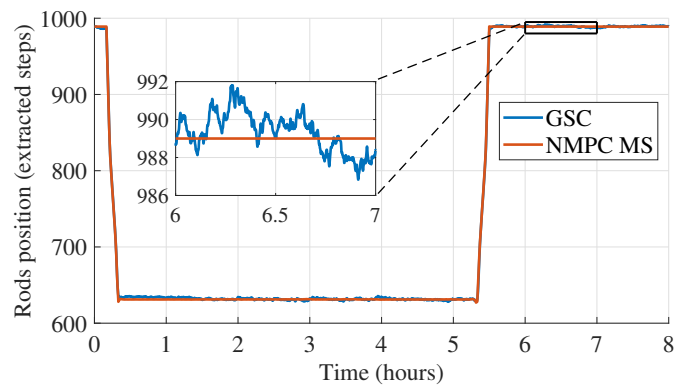


Fig. A.16: Position of P_{bank} (Gain-Scheduled Controller vs NMPC multiple-shooting)

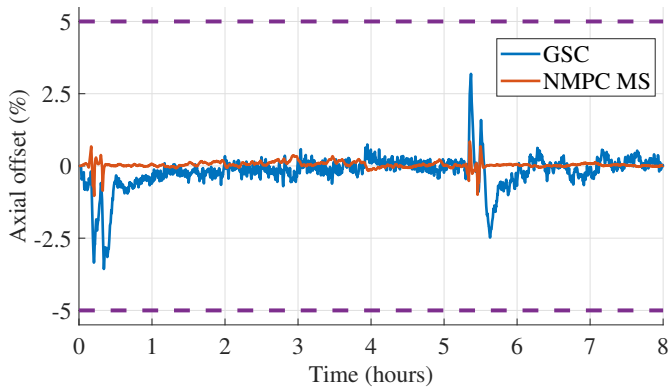


Fig. A.14: Axial offset deviation (Gain-Scheduled Controller vs NMPC multiple-shooting)

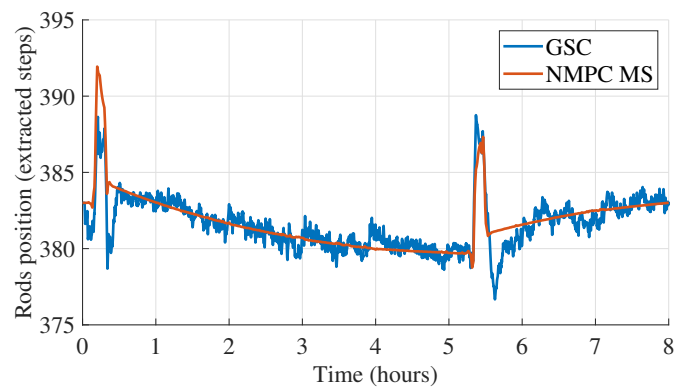


Fig. A.17: Position of H_{bank} (Gain-Scheduled Controller vs NMPC multiple-shooting)

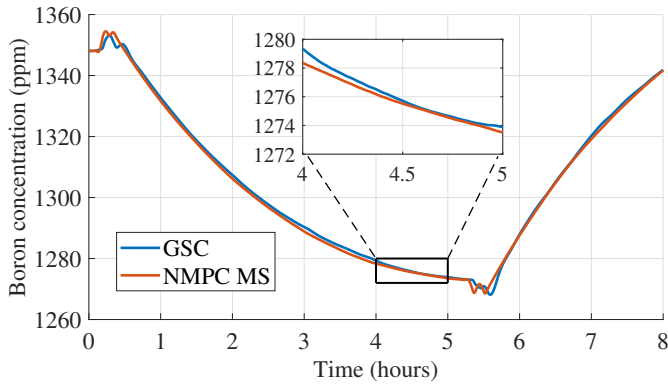


Fig. A.18: Boron concentration of the primary coolant (Gain-Scheduled Controller vs NMPC multiple-shooting)

References

- [1] K. Kosowski, F. Diercks, Quo vadis, grid stability? Challenges increase as generation portfolio changes, *atw International Journal for Nuclear Power* 66 (2) (2021) 16–26.
- [2] P. Morilhat, S. Feutry, C. Lemaitre, J. M. Favennec, Nuclear power plant flexibility at EDF, *atw International Journal for Nuclear Power* 64 (3) (2019) 131–140.
- [3] IAEA, Non-baseload Operation in Nuclear Power Plants: Load Following and Frequency Control Modes of Flexible Operation, no. NP-T-3.23 in Nuclear Energy Series, IAEA Vienna, 2018.
- [4] J.-L. Mourlevat, Évolution des modes de pilotage, *Revue générale nucléaire* (3) (2007) 29–36.
- [5] A. Grossetête, ATMEA1 & EPR™ mode T core control innovative features for high operating flexibility, *Transactions of the American Nuclear Society* 111 (1) (2014) 1095–1098.
- [6] G. Li, X. Wang, B. Liang, X. Li, B. Zhang, Y. Zou, Modeling and control of nuclear reactor cores for electricity generation: A review of advanced technologies, *Renewable and Sustainable Energy Reviews* 60 (2016) 116–128.
- [7] L. Lemazurier, M. Yagoubi, P. Chevrel, A. Grossetête, Multi-objective H_2/H_∞ gain-scheduled nuclear core control design, *IFAC-PapersOnLine* 50 (1) (2017) 3256–3262.
- [8] L. Lemazurier, P. Chevrel, A. Grossetête, M. Yagoubi, An alternative to standard nuclear core control using a multi-objective approach, *Control Engineering Practice* 83 (2019) 98–107.
- [9] G. Dupré, A. Grossetête, P. Chevrel, M. Yagoubi, Enhanced flexibility of PWRs (mode A) using an efficient NMPC-based boration/dilution system, in: 2021 European Control Conference (ECC), IEEE, 2021, pp. 1092–1098.
- [10] M. G. Na, D. W. Jung, S. H. Shin, J. W. Jang, K. B. Lee, Y. J. Lee, A model predictive controller for load-following operation of PWR reactors, *IEEE Transactions on Nuclear Science* 52 (4) (2005) 1009–1020.
- [11] J. H. Kim, S. H. Park, M. G. Na, Design of a model predictive load-following controller by discrete optimization of control rod speed for PWRs, *Annals of Nuclear Energy* 71 (2014) 343–351.
- [12] H. Eliasi, M. Menhaj, H. Davilu, Robust nonlinear model predictive control for a PWR nuclear power plant, *Progress in Nuclear Energy* 54 (1) (2012) 177–185.
- [13] P. Gahinet, P. Apkarian, Automated tuning of gain-scheduled control systems, in: 52nd IEEE Conference on Decision and Control, IEEE, 2013, pp. 2740–2745.
- [14] D. Q. Mayne, J. B. Rawlings, C. V. Rao, P. O. Sokaert, Constrained model predictive control: Stability and optimality, *Automatica* 36 (6) (2000) 789–814.
- [15] D. Q. Mayne, Model predictive control: Recent developments and future promise, *Automatica* 50 (12) (2014) 2967–2986.
- [16] L. Lemazurier, Conception d’un système avancé de réacteur PWR flexible par les apports conjoints de l’ingénierie système et de l’automatique, Ph.D. thesis, Institut Mines-Télécom Atlantique (2018).
- [17] P. Reuss, Précis de neutronique, EDP sciences, 2012.
- [18] W. M. Stacey, Nuclear reactor physics, 3rd Edition, John Wiley & Sons, 2018.
- [19] D. J. Leith, W. E. Leithead, Survey of gain-scheduling analysis and design, *International journal of control* 73 (11) (2000) 1001–1025.
- [20] W. J. Rugh, J. S. Shamma, Research on gain scheduling, *Automatica* 36 (10) (2000) 1401–1425.
- [21] P. Apkarian, D. Noll, A. Rondepierre, Mixed H_2/H_∞ control via nonsmooth optimization, *SIAM Journal on Control and Optimization* 47 (3) (2008) 1516–1546.
- [22] M. Bouffier, L. Daudin, G. Rio, Science, an advanced 3D fuel management code package, in: TopFuel’97, Thomas Telford Publishing, 1997, pp. 5–30.
- [23] B. A. Francis, W. M. Wonham, The internal model principle of control theory, *Automatica* 12 (5) (1976) 457–465.
- [24] S. J. Qin, T. A. Badgwell, A survey of industrial model predictive control technology, *Control engineering practice* 11 (7) (2003) 733–764.
- [25] T. T. Hartley, G. O. Beale, S. P. Chiacelli, Digital simulation of dynamic systems: a control theory approach, Prentice-Hall, Inc., 1994.
- [26] A. C. Hindmarsh, P. N. Brown, K. E. Grant, S. L. Lee, R. Serban, D. E. Shumaker, C. S. Woodward, SUNDIALS: Suite of nonlinear and differential/algebraic equation solvers, *ACM Transactions on Mathematical Software (TOMS)* 31 (3) (2005) 363–396.
- [27] L. F. Shampine, M. W. Reichelt, J. A. Kierzenka, Solving index-1 DAEs in MATLAB and Simulink, *SIAM review* 41 (3) (1999) 538–552.
- [28] R. Findeisen, L. Imsland, F. Allgower, B. A. Foss, State and output feedback nonlinear model predictive control: An overview, *European journal of control* 9 (2-3) (2003) 190–206.
- [29] P. Kokotović, H. K. Khalil, J. O’reilly, Singular Perturbation Methods in Control: Analysis and Design, SIAM, 1999.
- [30] X. Chen, A. Ray, On singular perturbation of neutron point kinetics in the dynamic model of a PWR nuclear power plant, *Sci* 2 (2) (2020) 30.
- [31] L. Grüne, J. Pannek, Nonlinear model predictive control: Theory and Algorithms, 2nd Edition, Springer, 2017.
- [32] J. B. Rawlings, D. Q. Mayne, M. Diehl, Model predictive control: Theory, Computation, and Design, 2nd Edition, Nob Hill Publishing Madison, WI, 2019.
- [33] M. Diehl, H. J. Ferreau, N. Haverbeke, Efficient numerical methods for nonlinear MPC and moving horizon estimation, in: Nonlinear model predictive control, Springer, 2009, pp. 391–417.
- [34] L. Grüne, J. Pannek, Analysis of unconstrained NMPC schemes with incomplete optimization, *IFAC Proceedings Volumes* 43 (14) (2010) 238–243.
- [35] R. Findeisen, F. Allgöwer, Computational delay in nonlinear model predictive control, *IFAC Proceedings Volumes* 37 (1) (2004) 427–432.
- [36] R. Findeisen, L. Grüne, J. Pannek, P. Varutti, Robustness of prediction based delay compensation for nonlinear systems, *IFAC Proceedings Volumes* 44 (1) (2011) 203–208.
- [37] A. E. Bryson, Y.-C. Ho, Applied optimal control: optimization, estimation, and control, Routledge, 1975.
- [38] J. A. Andersson, J. Gillis, G. Horn, J. B. Rawlings, M. Diehl, CasADi: a software framework for nonlinear optimization and optimal control, *Mathematical Programming Computation* 11 (1) (2019) 1–36.
- [39] A. Wächter, L. T. Biegler, On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming, *Mathematical programming* 106 (1) (2006) 25–57.