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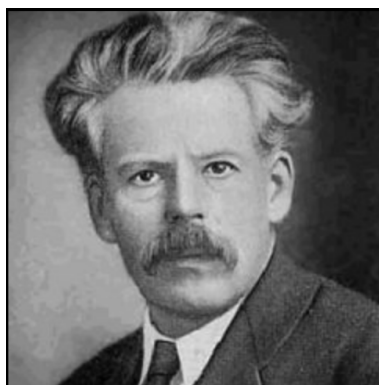
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Eugen (Evgeny Evgenievich) Slutsky (1880-1948)

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Abstract

Eugen Slutsky is well-known to any graduate student in economics for two landmark articles and two operational concepts bearing his name, one in the field of consumer and utility theory ("the Slutsky equation"), the other in the field of the theory of cycles, introducing autonomous and exogenous causes in the analysis of macroeconomic fluctuations ("the Slutsky-Yule effect"). Because of the historical and political circumstances he had to confront in Ukraine, and then in Russia and in the U.S.S.R. in the first half of 20th century, Slutsky was prevented from devoting himself fully to mathematical economics, and he only published a handful more of articles dealing with economics. Over the last twenty years, researchers in Europe, Ukraine and Russia have been involved in making his contributions to mathematics and economics better known. By now, we get a clearer picture of Slutsky's views on economics and we know his network of connections with Western scholars who contributed to draw attention to his work. This essay highlights Slutsky's lasting importance in economics, focusing on the fate of his major and lesser known works.

1 On Slutsky's "influence"

Slutsky's name is famous to any economist; it is associated to several concepts or tools in microeconomics or economic statistics, most notably, to the "Slutsky equation" or "decomposition" of the effect of a price change upon demand into an income and a substitution effect ("à la Slutsky", by way of contrast with a decomposition "à la Hicks-Allen") and the "Slutsky-Yule effect" to denote a random-based correlation in the study of time series, notably of economic cycles. Those two contributions (Slutsky, 1952, 1937b), and some others to the theory of statistics (Seneta, 2001; Bru, 2003), have turned out to be enough to make Slutsky part of the history of microeconomics, econometrics, macroeconomics and statistics.¹ Slutsky's influence through these two articles also stems from the fact that they are the only two articles on economic subjects that were translated into English in the 20th century. "On the theory of the budget of the consumer", originally published in Italian (Slutsky, 1915), was published in English only in 1952—though an English translation circulated privately around 1933-34—and was reissued in 2012 in its original outlet, *Giornale degli economisti*. As for the 1927 article, "The Summation of Random Causes as the Source of Cyclic Processes," (Slutsky, 1927a) it contained an abstract in English and circulated among Russian speaking economists in the West, and an amended version was published in English in *Econometrica* in 1937 (Slutsky, 1937b).

It is to be noted that the ideas and tools introduced by Slutsky in 1915/1952 and 1927/1937 have been accommodated to various kinds of problems and various theoretical settings, manifesting that they are much more than simply a historical landmark to the development of the neoclassical paradigm or any specific theory. Their lasting importance is an invitation to consider Slutsky's classics as well as his other published writings linked to the field of economics as a whole (including his as yet untranslated 1910 master thesis, *Theory of marginal utility* (Slutsky, 2010)). They are testament for Slutsky's strong sense of reflexivity on the foundations of economics. This deeper dimension of Slutsky's thought is already detectable in the two papers that have made his fame, though it was not fully highlighted. In what sense, then, can we say that Slutsky has been influential in the Western world? Certainly not as someone who provided a self-sustained agenda through a set of organised and cumulative works, but rather as someone whose in-

¹The "Slutsky theorem" (also Cramér's theorem), based on the notions of convergence in distribution and convergence in probability, extends some properties of algebraic operations on convergent sequences of real numbers to sequences of random variables (Slutsky, 1925).

vestigations in economics provided both new tools and new methodological questions about the fundamental data of economics, tools that nurtured both macroeconomists and microeconomists throughout 20th century.

In a nutshell, Slutsky could be described as a mathematical statistician and probability theorist with a strong interest in political economy, who inquired on the possibility to build a theory of economic systems on the basis of general behavioral principles, analysed through a theory of human action (praxeology) and a non-deterministic view of the world (probabilistic thinking).² His abilities in mathematics and statistics led him to contribute to the search of the best ways to organize and rationalize empirical data, be there behavioral (price-quantity) or aggregated time series data.

Until recently, to non Russian speaking scholars, Slutsky's known contributions, aside from the 1915/1952 and 1927/1937 articles, were mainly his contributions to statistics and mathematics published in French (Slutsky, 1927b, 1928b,a, 1929b,a, 1938), Italian (Slutsky, 1934, 1937a), German (Slutsky, 1925, 1926a,b), English (Slutsky, 1913a). Among these articles, the article on praxeology (Slutsky, 1926a) was published in a Russian journal. Recent interest in Slutsky's work on the part of historians of economics has made available in English several important articles, notably the two articles edited by John S. Chipman in 2004 (the article on praxeology translated by Claus Wittich and the article on Böhm-Bawerk translated by Roger V. Rosko and John S. Chipman) (Slutsky, 2004a,b), also an article on Sir William Petty (Slutsky, 1914, 2005). Also, we owe to Oscar Sheynin (2010) a translation of several works on statistics and mathematics and of his 1912 booklet on correlation. In addition, Oscar Sheynin also translated two economics-related articles about monetary issues (Slutsky, 1923b,a) and an article on the foundations of probabilities (Slutsky, 1922) and several other pieces of interest (autobiographical notes, biographical notes and tributes, correspondences).³ Vincent Barnett (2011, 196-201), has reissued the substantial five pages English summary of the 1927 article, which was enough to make Slutsky's approach known in the West and he provides us with a translation of the table of contents of Slutsky's master dissertation, *The theory of marginal*

²Barnett (2011, 185) aptly stresses that Slutsky's place within the neoclassical tradition is that of an outlier and that "Slutsky had sought to introduce stochastic concepts into the very heart of the basic elements of economic understanding."

³Curiously enough Barnett (2011, acknowledgments) announces a translation of Slutsky (1923b) which turns out to be merely a step by step presentation of it. He also provides a translation of the probability article titled "On the Question of the Logical Foundations of the Calculus of Probabilities", but Sheynin's translation seems more recommendable since it is based on a revised version published in 1925, after Slutsky had "improved some formulations making them more intellegible" (Sheynin, 2010, 41).

utility (Teoriya Predel'noi poleznosti) (Slutsky, 2010).

The present essay is organised as follows. Section 2 provides biographical elements about Slutsky, pointing out the political circumstances that prevented him from devoting himself fully to economics. Section 3 presents the 1915/1952 article. Section 4 analyses how Slutsky's theory of demand became the backbone of neoclassical demand theory. Section 5 and 6 respectively present the 1927/1937 article and how Slutsky's discovery came to be central within the theory of cycles in 20th century. Section 7 comes back on Slutsky's lesser know articles and attempts to bridge Slutsky's thoughts together.

2 A short bio of Slutsky: economics under political pressure

Eugen (Evgeny Evgenievich) Slutsky (1880-1948) was born in April 1880 in the governorate of Yaroslavl (a hundred miles northeast from Moscow).⁴ His family originated from Ukraine and returned there in 1889. Showing an early interest for physics and mathematics, he joined the faculty of mathematics and physics of the University of Kiev in 1899. A spirited, "impetuous" and "boyish" character, Slutsky was involved in the students protests and meetings that were recurrent in Ukraine and Russia at the turn of the 20th century. After being expelled twice from the University of Kiev, and being furthermore barred from any other Russian higher education institution, Slutsky studied mechanical engineering at the Institute of Technology in Munich (then *Königlich Bayerische Technische Hochschule München*) for three academic years (1902–1905). During those years, his earlier interest for economics strengthened through reading Ricardo and Marx (Slutsky, 1939a, 19-20) and getting acquainted with the marginalist school of economics. He was then able to come back to Russia and Ukraine after the revolutionary events of 1905. There, he went to the Law faculty of the university of Kiev, whose curricula offered courses in political economy. In 1906, he married Yulia Nikolaevna Volodkevich and retreated from revolutionary activities. As he recalls this time span that decided of his professional commitments: "I ... discovered that my visual memory was very weak. Therefore, ..., I could not become a good mechanical engineer. And by the same reason, I very badly memorized people by sight and mistook one person for another

⁴This short bio borrows mainly from Barnett (2011) and from first hand and second hand sources (Chetverikov, 1959; Seneta, 2001) and two short autobiographical notes (Slutsky, 1939a, 1942).

one even if having met them several times so that I was unable to be a political figure either." (Slutsky, 1939a, 227). During those hectic years, he worked on his own on the application of mathematics to economics. The output of this immersion into mathematical economics within the context of the Marxist's criticism of bourgeois' economics and the raging debates between agrarian socialists (populists) and advocates of a proletarian revolution (Allison, 2014, 2015) was a master diploma dissertation devoted to pure theoretical issues, *Theory of Marginal Utility* (1910) for which he received a gold medal in 1911. This dissertation is testament that Slutsky's interests went beyond economics and mathematics to include also philosophy and psychology, delving deep into the foundations of economics.

Slutsky felt that his own individuality found its full expression through mathematical inquiry and its application to various subjects, notably economics. Between 1909 and 1915, he became a member of several Kievian scientific societies (Society of Economists, Mathematical Society, Sociological Society) and full member of the Society for the Development of Social Sciences at Moscow University (Chetverikov, 1959, 252). Being barred once more from the university of Kiev, Slutsky eventually obtained a Master of Political Economy and Statistics from Moscow University in 1917. Due to his revolutionary tendencies, Slutsky was not able to get a position at the university of Kiev. In 1912, he accepted his father-in-law proposal to teach in his school at Saint-Petersbourg, and was later hired—probably under Chuprov's recommendation (Sheynin, 1993, 250)—to teach mathematics and statistics at Kiev Commercial Institute (Campbell, 2012)

By that year Slutsky had gained recognition as a statistician. Indeed, towards 1911-1912 he had been discovering theoretical statistics through a book by A. V. Leontovich introducing Gaussian and Pearsonian statistics (Seneta, 2009, 120-121), and this would be the starting point for his lifelong passionate involvement in mathematics, statistics and probability theory. In the field of statistics, Galton's innovations (coefficient of correlation, coefficient of regression, principle of regression toward the mean) and Karl Pearson's work did foster numerous works trying to establish connections between social, biological, natural and economic phenomena. Slutsky deepened his interest in such matters and endeavoured to publish a synthesis on the contributions of the English school of statistics (or biometric school), which came out in 1912 as *The Theory of Correlation and Elements of the Doctrine of the Curves of Distribution* (Slutsky, 1912). The booklet is conceived of as a manual exposing Pearson's theory of correlation, together with some critical assessments of it and additional personal results. From that moment on, Slutsky would become part of the international community of

statisticians and his methods would be discussed by Pearson and Ronald Fischer (Pearson, 1916; Fisher, 1922). In Russia, it ensured Slutsky's reputation as a mathematician and statistician—though his stance was not in line with some features of the Russian school of probability.⁵ Also, the booklet shows Slutsky's interest in applying statistics to social sciences, warning researchers from hasty conclusions regarding the proper influence of some external factors on economic phenomena by use of partial correlation measures (to avoid attributing to one factor the causality on another, such as rain and temperature on the yield of a crop) (Barnett, 2011, 28).

In 1913, Slutsky contributed in the *Journal of the Royal Statistical Society* with an article on the theory of regression, providing a criterion to find the most probable regression curve in a family of curves from a given type ("On the Criterion of Goodness of Fit of the Regression Lines and on the Best Method of Fitting them to the Data") (Slutsky, 1913a). It arouse comments by Yule, Pearson and Fisher and was recognized by Deming (1934, 372) as "an invention in curve fitting". It contained an analysis of correlation between lagged variables (the price of rye in Samara at t and its price one month earlier). During those years, Slutsky engaged in correspondence with mathematicians, statisticians and economists in the West (Pearson, Bortkiewich, Marschak and later, Frisch).⁶

Apart from his position at the Kiev Commercial Institute—where he moved to the teaching of political economy which "[he] considered [his] main speciality" (Slutsky, 1939a, 227)—Slutsky held various short term appointments, teaching on various subjects (mathematics, probability, history of economics, and history of socialist ideas) at the Ukrainian Cooperative Institute (1917) and at the Kiev Institute of National Economy (1919) (Barnett, 2011, 55), also working as statistician at the Kiev Statistical Bureau. During his years at the Commercial Institute (1913–1926), Slutsky's interests in political economy led to a handful of articles on various subjects (apart from the 1915 article) that did not circulate in the Western world, except maybe through private offprints. They can be read also as pedagogical material in relation

⁵The gist of the heated debate within mathematical circles is the status of the Law of Large Numbers as a mathematical statement and its relationship with the notion of probability (Seneta, 1994).

⁶Jacob Marschak attended Slutsky's lectures at Kiev Commercial Institute in 1915–1916. There is as yet no evidence that Slutsky ever corresponded with Moore and that his analysis on regression would have had any direct influence on Moore's *Forecasting the Yield and Price of Cotton* (Moore, 1917). According to Barnett (2011, 36) the fact that Moore calculated the correlation between fluctuations in the prices of New York cotton futures and the prices of spot cotton on exchanges in the South must have been induced through Pearson.

with Slutsky's involvements. "The essence of Cooperation and its Forms" (1913b) is devoted to the analysis of the motivations driving cooperatives and their history in Russia (Barnett, 2004). This interest for cooperatives is in line with a huge interest in the organization of cooperatives and in the cooperative movement in the wake of the search for alternatives to capitalistic concentration in private organizations. It was also a time of development of cooperatives in Russia (especially purchasing cooperatives and credit cooperatives) (Sobolev et al., 2018). As such Slutsky's contribution cannot claim originality (Barnett, 2011). To Slutsky, cooperation implies a sense of co-decision and equality within the organization (*Solidarism* in France) and can be applied to any important function in the economy (credit, consumption, production, distribution). It owes more to Utopian socialism and other non-Marxist theories about the way to address the social question than to a Marxist-Leninist theory of economic development, it promotes equality of its members and the interests of other stakeholders, as well as not-for-profit activities.

In 1914, Slutsky published an article on Sir William Petty (Slutsky, 1914) which can be read as an introduction for students to the economic views of a pre-classical economists with whom Slutsky could identify as a polymath searching for a synthesis between various founding elements of value and raising statistical analysis to a standard of economic practice. We shall postpone a more thorough presentation of this article until the last section. Be it enough to mention that Slutsky's presentation shows a synthetic view of Petty's contributions and invites readers to take it as a fundamental reading to uplift their formation to economic thinking.

The outburst of the Bolshevik revolution (October 1917) put more and more the intellectual elites under spotlight, to reach its acme in the 1930s during the Stalinist era. It is quite certain that, after his deliberate excursion at the heart of the marginalist theory of value, Slutsky had to retreat to more ideologically immune research. To accompany the transition towards a planned economic system in the 1920s' Soviet Russia, Slutsky contributed through his mathematical skills, only to deal tangentially with economic issues: he felt that the foundations of probability were weak and that only an axiomatic approach was acceptable, in which statistical tools would play a central role. Most of his research then centered on refining the Law of Large Numbers to account for a non-frequentist approach to probabilities, a view in which probabilities are defined as time-dependent stochastic processes. This approach relied on various authors (David Hilbert, Emile Borel), who aimed at connecting probability theory with set theory and functions. Slutsky expressed his views in this direction in "On the Question of the Logical

foundations of the Calculus of Probabilities" (Slutsky, 1922).

During this period (1917–1926), Slutsky made four excursions in the field of economics. In 1923, he published two connected articles on monetary issues (Slutsky, 1923a,b), more precisely on State's currency emission. Then, in 1926, he published a very abstract reflection on the foundations of economics, a pioneer contribution to praxeology, "On the Formal Praxeological Foundations of Economics" (Slutsky, 1926a, 2004b) (hereafter "Praxeology"). Last, a fourth article went to completion in 1926, only to be published in 1927 in German (Slutsky, 1927c). It is devoted to a critical assessment of Böhm-Bawerk's theory of value, "A critique of Böhm-Bawerk's concept of value and his theory of the measurability of value" (Slutsky, 1927c, 2004a) (hereafter, "Critique", see also Barnett 2011, 82sq.) The two papers on currency emission are motivated by the historical context of Soviet financial situation and the need to stabilize the monetary system. Praxeology is a highly abstract representation of economic activities and potentialities based on various concepts dealing with structural relationships between agents and their own representations. Critique bears on Slutsky's earlier account of the marginalist theory of value and capital. We shall postpone the analysis of these papers until the last section.

Year 1926 opens up a new period in Slutsky's life (1926-1930), away from teaching activities and from Kiev. In 1925, as Slutsky's reputation as a statistical theorist with an interest for economic issues was well established, he was invited by Nikolai Kondratiev to join the Conjoncture Institute of the People's Commissariat of Finance in Moscow. This opportunity to leave Kiev came in conveniently to Slutsky, since Ukrainian authorities had summon him to deliver all his lectures in the Ukrainian language, which he did not master and did not want to master. Slutsky moved to Moscow to become one of the three main consultants at the Conjoncture Institute. The Moscow Conjoncture Institute had been established in 1920, at times of severe depression in Soviet Russia. The Institute was involved in supporting the NEP put forth by Lenin, providing forecasting on economic fluctuations and developing various production indices and price indices (both complementing and competing with Gosplan) (Klein, 1999). At the Moscow Conjoncture Institute, Slutsky was in charge with basic economic processes. It was in this environment of interest for fluctuations (Franco et al., 2022) that he prepared his 1927 paper on moving summations of random series as a cause of cycles. During his stay at MCI all other published research by Slutsky focused either on statistical topics (regression and correlation theory) or on the theory of probability (stochastic limit theorems and the law of large numbers). Slutsky's research was certainly above the standard of mathematical

technics used to produce reports and conjunctural analysis: he was following his own path of research.

In 1928, Slutsky attended an international congress of mathematicians in Bologna during which he presented a contribution "Sur les fonctions éventuelles compactes" (published as Slutsky (1929b)), which revolves again on the issue of understanding new phenomena that cannot be simply built on the basis of past phenomena. During this famous conference, Slutsky was at the center of a mathematical quarrel with mathematician Cantelli about the priority of the strong law of large numbers (in the case of independent repetitions of a Bernoulli trial) (Seneta, 1992; Bru, 2003).

By the end of the 1920s, political pressures on intellectuals increased. In 1928, Nikolai Kondratiev was dismissed and the MCI was transferred from the Ministry of Finance to the Central Statistical Office, and was then officially closed after Stalin's arrival to full power in 1930. Kondratiev and several other members of the Institute were arrested, and rational scientific debate vanished from most disciplines in the USSR. Slutsky was not bothered, not having been involved in any compromising debate and journal (Barnett, 2011, 99sq). However, he was wise enough in later occasions to avoid connecting his own research on random processes with economic topics, now working essentially on probability theory. Slutsky then moved to work at the Central Institute of Meteorology. There his research focused on the study of solar activity and other geophysics' topics. Notably, he was able to contribute to work on the periodicity of sunspot, a theme he had also discussed with Vainshtein while at the Conjoncture Institute. Apart from this aspect of his post-1930 work, Slutsky's contributions dealt with the conditions for applying statistical tools to the study of geophysical phenomena, since the conditions for assuming independence of events in a series were not met and geophysical processes were likely to be non-stationary.

In 1935, Slutsky turned back to pure mathematics and statistical theory at the Mathematical Institute of Moscow, and in 1939 he held a chair of mathematical statistics at Moscow university. Among other works, he studied correlations of related series for a limited number of trials. He obtained conditions for measurability of random functions in 1937 (Slutsky (1937a), also in Russian as Slutsky (1939b)). During this period, he was assigned to monitor a Commission on the Application in Industry of Mathematical Statistics. The goal of this commission was to increase the performance of Soviet industry through application of statistical techniques. During the war, Slutsky engaged in completing a statistical handbook, *Tables for the Calculation of the Incomplete Γ -Function and the χ^2 Probability Function*. Unfortunately, he died March 1948 of a lately diagnosed lung cancer before

completing the book, which was published posthumously (Slutsky, 1950).

3 Slutsky's 1915 contribution to the theory of the consumer

Slutsky's name is foremost known to any student in economics for the Slutsky equation, which introduced to economists the decomposition of the total effect of a price change upon demand into an income effect and a substitution effect. The "Fundamental Equation of Value Theory" (Hicks, 1939, 309), as Hicks would later call it, was put forth in "Sulla teoria del bilancio del consumatore", an article published in 1915 in the *Giornale degli economisti* (Slutsky, 1915). To grasp the importance and originality of this contribution, three points need to be addressed. First, I shall present the content and structure of the article and discuss Slutsky's motivation to submit it to this journal. In the next section I discuss how Slutsky's contribution came to be incorporated into the modern presentation of the theory of value as developed independently from Slutsky by Hicks and Allen (Hicks and Allen, 1934a,b) and in Hicks' *Value and Capital* (Hicks, 1939). I also account for the spectrum of influences of Slutsky's 1915 article in the second half of the 20th century, once its widespread dissemination was made possible through publication of an English translation in 1952 (republished as Slutsky (2012)).

Slutsky's interest for utility and demand theory is a centerpiece of his master thesis *Theory of marginal utility* which contains already important developments on utility theory and Pareto's ordinalism. In this work, Slutsky shows a great familiarity with the thoughts of Menger, Jevons, Pareto, Marshall, Seligman, Edgeworth, Auspitz and Lieben, Cournot, Böhm-Bawerk, Wieser (with a specific critical focus of the Austrian school). The first chapters are dedicated to a discussion of the views of psychologists—Wundt, Ehrenfels, Brentano—on will, emotions, pleasure and displeasure and the meaning of decision. The master thesis also contains theoretical investigations on various ways to model utility, with a focus on additive utility functions. Hence, the 1915 article can be read as a direct continuation of the subject where it had been left in 1910, going beyond the Paretian analysis of the price-demand relationship and providing new results both in the case of a generalized utility function and in the case of an additive utility function.

Budget is divided into 13 sections. Section 1 takes as a starting point the opposition between the hedonist school and the "modern" (positivist) school. Members of the hedonist school take as a grounding principle the idea that each good, as a provider of pleasure, is esteemed by individual

consumers through the law of diminishing marginal utility. Therefore the hedonist school takes psychological laws as its basis and remains dependent upon unsettled (endless) disputes about measurability of psychological values. On the contrary, the "modern" school relies on another conception of utility, an index function of utility first introduced by Pareto, which is "completely strict and abstract" (Slutsky, 2012, 174), formal and independent "of all psychological and philosophical hypotheses" (Slutsky, 2012, 174). This positivist view on utility—a word not used by Slutsky—assumes that utility can be recovered "by empirica data" (ibid.). Slutsky notes that, as shown by Pareto, such a function cannot be determined uniquely, thus leading, as Slutsky interprets it—to a separation between psychological analysis and economics. Going beyond Pareto's stance, who did not recognize in full the no-bridge between utility—Pareto's ophelimity—and an index utility function, section 1 ends on a teaser, that "we shall see later how it is possible to arrive at another, better defined concept" (ibid.)

Section 2 presents the mathematics of the index utility function and repeats the main tenets of the Paretian view on the empirics of utility. Utility as an index of utility expresses mathematically through a real valued function, the fact that the more a combination of goods is desired, the greater the *value* attributed to this combination. It is fundamentally a relativistic value, whose meaning is dependent upon the value attributed to a combination *as compared* to another. Assuming that agents are utility maximizers—by definition, searching for the most desired combination—a budget is in a state of stable equilibrium over a time span if no alternative budget affordable has strictly greater utility. It is the task of the mathematical economist to establish "stability" conditions. The fundamental goal of the economist, then, is to determine practically the utility function from the empirical data, "to face and solve the problem of the determination of the utility function by means that are practicably attainable, such as the variations of demand as a function of income and of prices" (ibid., 175). Slutsky announces that the solution to the problem is connected "with that of the possibility of an agreement between the formal and the psychological aspects of the problem of utility." (ibid., 175). Commenting on the main assumptions made on the utility function, he notes that the problem of non-derivability or variability through time can be addressed by considering groups instead of individuals, "applying statistical methods in the investigation" (ibid.).

Let U the index of utility being represented as a real-valued function Φ of the set of quantities of goods $\Phi(x_1, \dots, x_n)$. Each marginal utility is assumed to be positive ($u_i > 0$), and the law of decreasing marginal utility (Gossen's law) is replaced with a distinction between satiating goods and non-satiating

goods ($u_{ii} < 0$ and $u_{ii} > 0$)

Section 3 to 9 present the stability conditions for the budget of the consumer (i.e. for the way the consumer uniquely allocates his/her income in between the set of goods) and then the expression of the variation in demand as a consequence of the variation in income and price. Building upon Pareto's mathematical treatment in the *Manuel d'économie politique* (Pareto, 1909), Slutsky's aim is to obtain a more operational result. This is based on deriving first the variations of the individual's demand as a function of income (section 6), introducing the distinction between "relatively indispensable" ($\delta x_i/\delta r > 0$) and "relatively dispensable" ($\delta x_i/\delta r < 0$) goods (where r stands for income)⁷ (Slutsky, 2012, 183). In section 7, Slutsky establishes the variation in demand as a function of the variations in price and connects it with the previous result, thus obtaining the general formula

$$\frac{\delta x_i}{\delta p_j} = u' \frac{M_{ij}}{M} - x_j \frac{\delta x_i}{\delta r}$$

where x_i is the quantity demanded of good i , p_j is the price of good j , and u' is the marginal utility of income, while M is the determinant of the matrix of second-order partial derivatives u_{ij} of the utility function bordered by the prices, and M_{ij} is the minor of M with respect to u_{ij} . This is the fundamental equation of value, as it decomposes the reaction of a consumer to a price variation as the sum of a substitution term and an income term (Slutsky, 2012, eq.46 and 47,185). The demand for a relatively dispensable good can be abnormal in certain cases.

Slutsky also derives a property (the "law of reversibility") of the consumer behavior (section 9), namely of symmetry of "residual variation":

$$k_{ij} = \frac{\delta x_j}{\delta p_i} + x_i \frac{\delta x_j}{\delta r} = \frac{\delta x_i}{\delta p_j} + x_j \frac{\delta x_i}{\delta r} = k_{ji}$$

The residual variability k_{ij} corresponds to the change dx_j in the quantity demanded of good j resulting from a price change dp_i accompanied by a compensating adjustment in money income such as to make possible "the purchase of the same quantities of all the goods that had formerly been bought" (Slutsky, 2012, 186). In other words, the residual variability is a compensated variation of demand associated to a compensated variation of price. To Slutsky, the symmetry property $k_{ij} = k_{ji}$ (Slutsky, 2012, eq. 55, 188) is of utmost importance, since it is a quantitatively defined relation between observable quantities. It can be (and should be) confronted with data on individual budgets:

⁷In Slutsky's article, income is represented by s .

"Empirical confirmation is highly desirable, inasmuch as it would demonstrate the correspondence to the truth, or at least the plausibility, of the hypothesis that the increments of utility do not depend upon the mode of variation [$u_{ij} = u_{ji}$]." (ibid., 188)

From this later quotation, it seems that Slutsky endorses Pareto's interpretation of the symmetry relation as an expression of some assumed psychological law about the effect of the order of incremental variations of consumption upon utility (Chipman and Lenfant, 2002). However, he is not quite explicit about this, and it would certainly need to be confronted with his former remark about the statistical nature (possibly over several individuals) of the mathematical relationships.

Section 10 and 11 derive specific relationships when utility is assumed to be additively separable. The rationale for discussing the case is linked to its historical importance in the development of the marginalist theory of value. Slutsky starts from discussing the assumptions on u_{ii} . Gossen's law of satiation would imply that all u_{ii} are negative. If one discards this assumption, then the right way to address the problem is to consider the case of a system of goods, some of them exhibiting non satiation. In the case of a system of goods for which the individual's preferences can be represented with one non-satiating good ($u_{jj} > 0, u_{ii} < 0$ for all $i \neq j$), Slutsky discusses a result obtained by Ricci (1904), that the demand for the non satiating good could be increasing. He shows that the conditions obtained by Ricci for increasing demand are incompatible with the "stability conditions" for the consumer (i.e. second order sufficient conditions for a constrained maximum), hence "cannot occur in reality" (ibid., 191). Actually, for a system such that one good is non satiating and all other goods are satiating, either all goods will be normal, or there is a possibility only for a satiating good to be inferior and to exhibit $\partial x_i / \partial p_i > 0$.⁸

To some extent, Slutsky's attitude towards the theory of utility is ambiguous. The point is that although Slutsky regards the hedonist view as intrinsically wrong (as will be seen in his criticism of Böhm-Bawerk), he takes as a scientific issue the question of whether some assumptions about the utility functions (additivity, signs of the partial second derivatives) would mirror some behavioral properties in the price-quantity space. Beyond pushing forward Pareto's analytical results, Slutsky's main motivation is driven by the search for empirically meaningful definitions of the relationships between goods and consumers. A running idea in the article is to provide criteria

⁸Slutsky's contribution to the additive utility case is discussed at length in Weber (1999).

(by means of equalities or inequalities) expressed as relationships between *observable relationships* or *empirically measurable data*. Accordingly, the Slutsky equation is based on a notion of compensation that is observable under specific conditions. This empirical meaningfulness of the theory of the consumer is at stake in the two final sections of the article (section 11 and 12) where Slutsky aims at discussing precisely the relationships between utility and individual behavior, answering the heated question at the core of the marginalist theory of value of whether there are any empirical counterparts to the signs of the second derivatives of the utility function u_{ii} and u_{ij} . As can be expected, since those properties are assumed independently from any system of signs in the system of goods, the answer is negative.⁹ Formally, Slutsky expresses u_{ii} and u_{ij} as two functions of (i) the marginal utility of income, (ii) various determinants implying only observables ($\partial x_i / \partial r$, k_{ij}), and (iii) a term θ which it is *impossible* to express "as a function of empirical data" (Slutsky, 2012, 194-196).

This outcome of the empirical theory of (index) utility applied to consumer's behavior opens to an "irreconcilable conflict" (ibid.) with the traditional hedonistic-psychological theory. Values (and signs) attributed to an individual's utility function can be arbitrary, without any consequence for his observable conduct. To Slutsky—and undoubtedly so to any reflexive reader—such a consequence is but perplexing, "because, even though attaching great importance to the absolute logical independence of the methods of economic science from those of psychology, we could not ignore the existence of a very complicated interdependence between the *facts* studied by the two sciences." (Slutsky, 2012, 197). In retrospect, the only way out of this dead end would consist in establishing new psychological facts, involving at least some cognitive elements. Slutsky's attempt at finding out a bridge from the data of consciousness to an empirical utility function is soon abandoned. This way of solving the problem is much too fragile to stand as solution.¹⁰

⁹Fundamentally, only assumptions pertaining to the whole system of goods are likely to exhibit meaningful empirical properties independent of any arbitrary increasing transformations of the utility function. The additive utility assumption plus one increasing marginal utility is precisely an example of this, as is Chipman's case of Auspitz-Lieben-Edgeworth-Pareto complementarity between all pairs of goods (Chipman, 1977) (see also Weber (2000)).

¹⁰In a nutshell Slutsky (2012, 197-198) starting from the idea that any consumer would be able to identify for himself all the situations when any two goods α and β are independent, taking as data the list of those pairs of independent goods, Slutsky conceives of recovering from that list a complete set of equalities for θ . Fundamentally, if ever the different empirical values obtained for θ were to be the same, one would at best obtain a provisional utility function, likely to be discarded by any new empirical result contradict-

Section 13 draws the lessons from the full development of the theory of index utility functions. The whole apparatus of categories and assumptions based on u_{ij} (including the Paretian distinction between complementary and competitive goods) is deemed useless "if one remains loyal to the formal definition of utility, for it is impossible to deduce from the facts of conduct the character (that is, the sign) of the second derivatives of utility" (Slutsky, 2012, 198), which then must remain disconnected from any belief in some sort of "*internal evidence*" about the consciousness of motives by which we are guided. Here, Slutsky, though he embraces the formalist view on utility, is unable to discard the grounding assumption of some psychological parallelism between empirical behavior and some inner value of things to the consumer. He would make one step forward in his critical analysis of Böhma-Bawerk. In any event, Slutsky does not consider that precise and complete empirical data could conceivably be obtained to solve this question, except perhaps by way of an experiment. Thus, the future of the theory of utility depends on the development of experimental investigation.

4 Reception and importance of the 1915 article

It is not exaggerated to claim that Slutsky's 1915 article is a landmark in microeconomics. On the one hand, it completes the Paretian revolution, almost exhausting its meaning to a point that does not seem to have been fully acknowledged. On the other hand, it provides several tools that can be implemented in demand analysis, in accordance with the positivist and empirical orientation that would become prominent in the 1930s. More than that, Slutsky's decomposition has been adapted in various theoretical settings throughout 20th century, an indication of its methodological and epistemological power for modern economics. As Stigler (1950, 382) put it: "The beauty and power of the essay are unique."

To chart the fate of Slutsky's 1915 contribution, one needs to figure out the state of demand analysis after Pareto's breakthrough. The mathematical theory of utility and demand was then a subject of interest only to a small community of searchers scattered throughout the Western world (United States, France, Switzerland, Germany, Austria, Italy, and England) and with only few connections with each others. It is only during the 1920s that stronger connections would develop through correspondence and travels, up to the creation of the Econometric Society in 1931. By that year, the political landscape had changed in Russia, preventing Slutsky to be one of its founding

ing the equalities.

members (Bjerkholt, 2017). One characteristic of this theoretical agenda is to let aside the fundamental issue of the foundations of utility, assuming that a utility function (in the ordinal meaning of the term) does exist and that individuals' behavior on markets derive from its maximization. In that way, reflection on the nature of rational behavior and its links with utility and observable behavior was ignored. It only became an issue in the 1930s, once researchers engaged in the search for an axiomatic representation of rational behavior and discussed the famous integrability conditions—i.e. conditions on a set of observable price-quantity behaviors to deduce an economically meaningful generating utility function.¹¹

The fact that the potential of Slutsky's article went unnoticed in published work until 1933 and that tribute would be paid to its contribution only after similar results had been obtained by Hicks and Allen (1934a; 1934b) has triggered the interest of historians of economic thought (Chipman and Lenfant (2002); Bjerkholt (2014)). The rationale for Slutsky's endeavor is still a matter of speculation. According to Barnett (2011, 40 sq) Slutsky's contribution could be linked with the interests of several economists in Russia who dealt with the analysis of peasants budget at a microeconomic level, in search for methodologically sounder foundations for such studies, within a war context when natural experimental conditions of price and income variations were operating. Though nothing in the article justifies this circumstantial connection, nonetheless Slutsky puts emphasis on the formulation of empirical relationships that could be confronted with actual observable behaviors, and it is reasonable to think that he conceived of the compensated demand and the income-demand relations as relations to be implemented in empirical studies. However, Slutsky's interest for the marginalist theory of value and the relationships between individual psychology and consumer behavior is the main topic of his Master thesis (Slutsky, 2010), which contains already a discussion of Jevons, Menger, Pareto and the contributions of German-Austrian schools of psychology (Wundt, Ehrenfels, Ebbinghaus, Brentano).

Many causes have been put forward to account for the fact that Slutsky's article did not catch attention for at least fifteen years. "Sulla teoria del bilancio del consumatore" was published in Italian in *Giornale degli Economisti* during war times.¹² It was at the vanguard in terms of mathematical formal-

¹¹Notable contributions to both issues in the 1930s are due to Frisch, Allen, Alt, Georgescu-Roegen and Samuelson (Chipman and Lenfant, 2002)

¹²The fact that Slutsky chose *Giornale degli economisti* as an outlet is quite understandable: he was completing Pareto's theory on utility published in the same journal and he commented on previous work by Ricci in the same journal. Also, *Giornale degli economisti*

ism, and quite at odds with the Marshallian style of thought then dominant (Johnson, 1913; Edgeworth, 1915): Indeed, Slutsky does not make any simplifying assumption (such as constancy of marginal utility of income); he does not use any diagram or figure to illustrate his thoughts in a two-good case; he does not consider the concept of indifference curves.

On the basis of published information, Valentino Dominedò (1933) was the first to grasp the importance of Slutsky (1915) for the theory of value and demand. Then, it was discovered (through Dominedò) by Henry Schultz and by Hicks and Allen. Actually, Dominedò may have known about Slutsky's article through Ricci (1932a), who acknowledged Slutsky's criticism of his account of the additive utility case (Ricci, 1904). Clearly, Ricci did not capture the importance of Slutsky for the theory of substitutes and complements since he did not mention him in his article on the subject in *Econometrica* (Ricci 1933). The same could be said about Rosenstein-Rodan (1933), who mentioned Slutsky in relation with the consciousness of economic conduct.

The first author that grasped the importance of Budget is Henry Schultz. The discovery of the relevance of Slutsky's paper takes place within a context of growing interplay between pure economics and statistical analysis, which developed during the 1920s in the US. During this span of time, Robert A. Lehfeldt, Holbrook and Elmer J. Working, Henry Moore, Henry Schultz, Ragnar Frisch, Elisabeth W. Gilboy, and others would confront methods to implement statistical methods (Pearson) into the theory of supply and demand functions (Morgan, 1990, chap 5 and 6). By the beginning of the 1930s, statistical studies of demand came into endless debates about time trends and the best method to obtain the "true" demand curve. Under constant marginal utility of money, Schultz had obtained symmetry conditions on cross demands for which statistical tests were disappointing (Schultz, 1933). He was thus looking for more severe theoretical properties in demand theory that could be used as a guiding principle in statistical studies of demand. We know that Schultz and Friedman (then Schultz's assistant) came across Slutsky's contribution around 1933-1934, and took benefit of his analysis in "Interrelations of Demand, Price, and Income" (Schultz, 1935) (see Chipman and Lenfant, (2002, 563sq)).¹³ Slutsky's article was providing the much wanted additional constraint at the individual level, offering as a second bird a new definition of complementarity (Schultz, 1935, 481), which

was in those years the journal most open to publishing mathematical economics.

¹³Bjerkholt (2014) has documented in detail Henry Schultz trip to Europe in 1933-1934, his contacts with Dominedò, Hicks, Allen, Robbins, Rosenstein-Rodan, and the fact that an ever-recurring subject of discussion was the treatment of complementary and substitute goods in demand theory.

he favored over Hicks and Allen's definition (Schultz, 1935, 447). Schultz praises Slutsky's operational spirit. The Slutsky equation contains a concept of compensated demand that can be observed empirically under some specific circumstances. Assume that an individual is subjected to changes in prices and income in such a way as to be able to observe his behavior when his new income allows him to buy exactly the same basket as before the price-income change, this would permit to identify a compensated price change under Slutsky's definition of it (but not a Hicksian compensated demand, which is based on the constancy of individual utility). However, Schultz identified that Slutsky symmetry condition is theoretically valid only at the individual level and not preserved at the aggregate level, while statistical data deal with aggregates. Consequently, statistical investigation on Slutsky symmetry did not improve on Hotelling's simpler conditions.

In February 1934, the first part of Hicks and Allen's article appeared in *Economica* (Hicks and Allen, 1934a). It contained a decomposition of the elasticity of demand for a good into a term involving the income elasticity of demand (i.e. an income effect) and an elasticity of complementarity (i.e. a substitution effect in modern terminology). Shortly after, Allen (1936) would acknowledge the pioneering work of Slutsky and Hicks (1937, 13). Their attention was driven to Budget through "various references" (Allen, 1950, 210) to it after they had come independently to similar results.

As a complement to published testimonies and recollections, we learn from Bjerkholt (2014) that Slutsky was not able to disseminate his article as he would have wished to do, only receiving the offprints of the 1915 paper in 1926. Besides, his attempt to inform Frisch (September 1926) about the content of the article was not met with success. After the publication of *Summation*, Slutsky's fame increased abroad, and this too may have contributed to making his previous works more visible.¹⁴

Precisely, the interpretation of Slutsky symmetry condition has been a subject of heated debates in the 1935-1950 period, as it was recast within the issue of integrability of demand. Some authors like Allen rejected the symmetry condition, while others took it as an established theoretical truth of the theory of rational behavior. Empirical rejection could indicate an irrational behavior. The subject was closed (except for particular cases) by Samuelson,

¹⁴Bjerkholt (2014) also documents that references to Slutsky appeared here and there. Thus, a brief summary was published in *Rivista Internazionale di Scienze Sociali e Discipline Ausiliarie* in 1915 and it was listed in the December 1915 issue of the *Economic Journal* (also in Murray (1915)). In our view, this is confirming the fact that Slutsky's paper could not attract attention as long as its potential for statistical demand studies based on utility foundations was not grasped and put to the fore by Schultz.

once the axiomatic setting of the theory of revealed preferences was put to an end with the Strong Axiom of Revealed Preferences (Samuelson, 1950).

Slutsky's equation and Slutsky symmetry conditions have become a cornerstone in microeconomics and have been applied to various fields of study beyond demand theory. Instances are the generalisation of Slutsky equation when prices enter the utility function (Kalman, 1968), when choice involves risky assets (Sandmo, 1969) or contingent commodities (with no effect on expected return) (Fischer, 1972) work-leisure trade-off (Gilbert and Pfouts, 1958), optimal taxation (Diamond and Mirrlees, 1971). Application of Slutsky decomposition for intertemporal choice or demand for durables turns out to be more difficult without specific assumption on the utility function.

Slutsky's symmetry conditions and Slutsky equation have been central also in the development of aggregate demand systems in econometrics. Notably, the use of Slutsky symmetry conditions at the aggregate level assumes a representative agent or specific aggregation conditions.

An interesting test for the power of Slutsky's theory of demand is whether it would still be an inspiration after the 1970s and 1980s, once the hopes to build demand systems on simple empirical features and rationality assumptions had vanished (Deaton and Muellbauer, 1980). The principle of analysing separately income effects and substitution effects at the individual or aggregate levels has proved fruitful, even in a framework disconnected from utility maximization arguments. For instance, Werner Hildenbrand's 2014 attempt at providing a socio-economic rationale for the market law of demand bears on the principle of identifying conditions on the aggregate income effect that are empirically supported. In technical terms, given the negative semi-definiteness of the average Slutsky matrix (matrix representing the compensated substitution terms) it is enough to assume that the dispersion of aggregate demand increases with income to obtain the Law of Demand. In other words, even though Hildenbrand operates a major shift as regards the principle of individual utility maximization, he nonetheless builds his whole argument on a decomposition between an income and a price effect that originates in Slutsky's Budget (see Lewbel (1994) for a discussion).

Finally, one may wonder what will remain of the Slutsky equation demand properties within the behavioral paradigm. This is open to question. It has been upheld (Gabaix, 2014) that measures of deviations from the Slutsky symmetry and negative semi-definiteness conditions in a model of bounded rationality might serve to measure "behavioral inattention". This view certainly demands very much in terms of individual rationality, and ignores the lot of experimental results that oppose such an instrumental use

of Slutsky (1952). It remains that Slutsky's exact stance as regards the proper role and nature of empirical or experimental data in demand theory and his contribution to the empiricist turn in economics needs a careful interpretation (Hands, 2010)

5 Slutsky's 1927/1937 contribution to the theory of random fluctuations

The other influential article by Slutsky appeared in 1927 in Russian in an outlet of the Moscow Conjoncture Institute (*Voprosy kon'yunktury*) and was titled "The Summation of Random Causes as the Source of Cyclic Processes" (Slutsky, 1927a). The goal of the article is to highlight that — and examine how — random series, under the effect of some lagged and weighted composition rule, can generate correlated series of values that exhibit non erratic wave movements. This time, Slutsky's article did not escape notice. The translation of Slutsky's article into English was soon undertaken and its publishing announced in the first issue of *Econometrica.*, to appear only in 1937 (Slutsky, 1937b).¹⁵ Between 1927 and 1937, the detailed summary that accompanied the 1927 article circulated among mathematical economists. "Summation" incorporated "a number of important results obtained after 1927" (ibid, editor's note, 105fn) while some tables were not reproduced. In this section, I present the content of the article. The goal of Summation is to challenge a common view on economic cycles that bears, one way or another, on superpositions of regularities in economic or natural events to account for economic cycles. According to this approach of cycles, exemplified by Moore (1923), random components are being assumed only to explain slight deviations from pure sinusoidal fluctuations (or harmonic fluctuations). Turning its back from this approach, Slutsky proposes to inquire whether the two main features of cycles—"the undulatory character of the processes and the approximate regularity of the waves" (Slutsky, 1937b, 107)—could be generated from a different starting point, assuming solely random causes and their combinations. This is the theory of chance waves. The novelty of the method is vindicated by Slutsky:

"The method of the work is a combination of induction and deduction. It was possible to investigate by the deductive method

¹⁵The spelling of Slutsky's name in the *Econometrica* article is "Slutzky". We have used the usual spelling.

only a few aspects of the problem. Generally speaking, the theory of chance waves is almost entirely a matter of the future. For the sake of this future theory one cannot be too lavish with experiments: it is experiment that shows us totally unexpected facts, thus pointing out problems which otherwise would hardly fall within the field of the investigator." (Slutsky, 1937b, 107)

The fundamental statistical idea of the article is this. Consider an "incoherent" series of values representing a phenomena, that is, a series whose values are disconnected from each other, each value being uncorrelated to its previous and upcoming value in the series. If, on the basis of such a series, one considers the construction of other series through various processes of summation (and so on possibly with resulting series), then one may isolate in the derived series patterns of behavior typical of cycles. To develop an analysis of this mathematical-statistical phenomenon, Slutsky argues in five steps. The first step consists in defining what makes a coherent series as against an incoherent (random) one. The second step consists in building various coherent series from incoherent series, that shall serve as test material for further analysis, similarities and difference between those derived series being taken as the data of inductive reasoning. In this respect, Slutsky has been identified as a pioneer of simulation in economics (Orcutt, 1960). The third step consists in establishing the undulatory character of the derived series, pointing out their properties of graduality and fluency. The fourth step deals with the regularity of the waves, an issue that "offers considerably greater difficulties" (Slutsky, 1937b, 118), involving Fourier's harmonic analysis and leading to identify a succession of "regimes" of cycles. The last step is a discussion of the pseudo-periodic character of the cycles. A mathematical Appendix on random waves completes the whole. In the following, I come back on each step.

To Slutsky, a fundamental starting point—which shall be related with his views on the foundations of probability—is a distinction between two kinds of chance series. They can be either coherent or incoherent. A coherent series is characterized by the fact that, within the finite number of terms that make this series, one can identify correlations. The most important of these correlations is the one linked with the distance between terms. The coefficient of correlation, for such coherent series, can be expressed as a function of the distance between the terms correlated (the *correlational function* of the series.) Even though such coherent series might exhibit other patterns, Slutsky limits his analysis to the case when the coefficient of correlation is determined solely by the distance between terms in the series. (r_t) figuring

the coefficient of correlation of any term with the t th term following, we have $(r_0) = 1$ and $(r_t) = (r_{-t})$.¹⁶

As a starting point for manipulations and inductive reasoning, Slutsky takes two experimental series obtained from the drawing of numbers for a national lottery, from which he extracts three incoherent basic series.¹⁷ As for coherent series, Slutsky views them as the offspring of incoherent ones, due to "an especially prominent role ... played in nature by the process of *moving summation* with weights of one kind or another." (Slutsky, 1937b, 108). Of course, coherent series once generated can in turn generate other coherent series through the same process of moving summation.

Consider the value of an event variable y at moment t as the "consequence" of several values taken through time by another causal variable x such that each consequence y is obtained through a linear combination of a finite incoherent series of n past values of x , with weights $A_0, A_1, A_2, \dots, A_{n-1}$. We get : $y_i = A_0x_i + A_1x_{i-1} + \dots + A_{n-1}x_{i-n+1}$. The same rule applied at various moment in time provides us with a series $(y_i, y_{i-1}, y_{i-2}, \dots, y_{i-n})$. Two adjacent values in the y series share a set of $(n - 1)$ common causes and the system of weights appears in a lagged way. This in itself is sufficient to make the terms of the derived series (or "consequences") correlated to a certain extent with one another, "even though the series of causes are incoherent" (Slutsky, 1937b, 109).

Using basic series, Slutsky then builds various derived series. One is obtained through ten-item simple moving summation. Another one is obtained by iterating the same process with the derived series. Still another one through iterating a two-item moving summation twelve times and a last one using specific weights based on the values of a Gaussian curve at given intervals. All these transformations are intended to provide a rich material—a set of Models—upon which inductive reasoning can be implemented. "We could not be satisfied by a smaller number of models because it was necessary to observe their various properties and to have illustrations for the elucidation of the different aspects of the problem." (Slutsky, 1937b, 111-113). It is by

¹⁶Note that Slutsky's notions of incoherent and coherent series are defined for finite series, and are probably conceived of as two relevant notions for empirical work : "I venture to propose this name [incoherent series] because it seems to me that it truly expresses what is intended, namely, the existence of some connection between the elements of parts of a thing (for example, of a series), but not a connection between this thing as a whole and another." (Slutsky, 1937b, 108, fn7)

¹⁷The first basic series is obtained through selecting only the last digit of the original series and the second one through replacing even numbers with 0 and odd numbers with 1. The second method was applied to another original series to obtain the third basic series.

selecting various such ways of composing incoherent series that one gets "an inductive proof ...*that the summation of random causes may be the source of cyclic, or undulatory processes.*" (ibid., 114)

Then comes the task of assigning properties of graduality and fluency to those waves. The principle of graduality states that within a series of correlated items variations from minimal to maximal values is gradual in the sense that small difference between values of neighboring items are more probable than between more distant ones, thus do not look like irregular zigzags. Fluency concerns the property of the series of first difference of adjacent values of a series. In the case of a simple moving summation, the series made of adjacent differences $y_i - y_{i-1}$ contains uncorrelated elements (except for Δy_i and Δy_{i+n}), thus in that case we have graduality without fluency.

The more derived series are produced through iterative processes of moving summations of random causes, the more they exhibit fluency of the undulatory process. Later in the article, Slutsky establishes also a tendency toward a sinusoidal form of series derived from random causes (i.e. tendency for second differences of the series $\Delta^2 y_i = (y_{i+2} - y_{i+1} - (y_{i+1} - y_i$ to be strongly negatively correlated with y_{i+1} .

That first result being established, the next step is to demonstrate "*the approximate regularity of the waves*" (Slutsky, 1937b, 118). This is, as Slutsky himself admits, a difficult task that he confronts first in an inductive way. Starting from the various experimental models built, Slutsky explains that a statistical treatment of the data in order to discard ripples from waves leads to identify relevant local maxima and minima, thus describing a number of half-waves of various length.¹⁸ It is then possible to calculate the frequency distribution of these waves. Waves identified with Model II exhibit about the same coefficient of variation as empirical waves observed for 12 countries¹⁹. Coefficients of variations and average lengths of waves are smaller for the other models. Here, Slutsky argues against Mitchell's attempt a denying regular waves. The apparent absence of periodicity is the result of a "tendency to stick to a purely descriptive point of view" (Slutsky, 1937b, 119) and to ignore "imperfection of a visual impression" (Slutsky, 1937b, 121). Crude evaluations of empirical results must be checked against alternative constructions of data: "Those investigators of economic life are right who believe in their acumen and instinct and subscribe to at least

¹⁸This idea is supported by Husserl in *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy* titled "Deskriptive und exacte Wissenschaften".

¹⁹Empirical data on business cycles are taken from Mitchell (1926, 32-33)

an approximate correctness in the concept of periodicity of business cycles" (Slutsky, 1937b, 119). Thus by confronting data from series of Model II and III with sums of the first five harmonics of Fourier series, Slutsky comes to the conclusion that a small number of harmonics is enough to obtain a significant correspondence in terms of periodicity of waves (at least for a subset of values corresponding to six periods) : "This hardly can be considered to be a chance occurrence; the explanation of such an effect must be found in the mechanism of the connection of the random values." (Slutsky, 1937b, 122). Beyond periodicity, Slutsky identifies regimes of cycles, that is, sequences of periodic waves with specific parameters. Transitions between regimes occur "sometimes rather gradually, sometimes more or less abruptly, around certain critical points)." (Slutsky, 1937b, 123) That way, Slutsky points out that non stationarity is a component of the analysis of cycles generated by random event.

In the last section of the paper, Slutsky inquires about the pseudo-periodic character of series. He does this through a physical metaphor echoing Wicksell's "rocking horse". Consider a system with damped oscillations constantly under random shocks that feed the system with energy. What would the system look like through time? Slutsky's answer is that if one were to consider a very long period of observation of the terms of series, the movement of the system would be reduced ultimately to a chance function, i.e. the result of "a particular instance of the summation of random causes" (Slutsky, 1937b, 132). Within a given finite time span—whose length depends on the value of random shock—the process is well described as a periodic sinusoidal movement.

Slutsky's contribution is remarkable in several respects. In a nutshell, the running thesis is that the moving weighted average of identical random series generate series of consequences that are correlated with each others in such a way as generating cycles. This thesis is scrutinized through a sophisticated method of inductive reasoning implying simulation and deductive reasoning based on statistical and probability theory. As Klein notes (1999, 153 and 158) Slutsky's analysis is anchored in a formalist thinking characteristic of Soviet mathematicians; it is also disconnected from materialistic causes of cycles and from any theoretical attempt at linking cycles with crises of capitalist economies. Notably, the notion of a regime change is not associated with a specific situation of crisis. As in Yule (1926) (who is mentioned by Slutsky at the beginning of Summation) random phenomena are important as data generating processes. Slutsky's contribution leads to a kind of small scale explanations, in which random causes are enough to exhibit approximate regularities over some period of time, without for all that pushing the

statistician into upholding a strict forever regularity of cycles, quite opposite to what Moore (1923) had done in his *Generating economic cycles*, where he advanced a strict eight-year periodic business cycle. This way, Slutsky is not committed to providing a unique causal explanation, putting instead the principle that various regimes of cycles are succeeding one another.

6 Reception and importance of the 1927/1937 article

The fate of Slutsky's 1927 article is quite different from that of the 1915 article. As a contribution to economic cycles, it has been quickly associated to Yule's name in the "Slutsky-Yule effect". First of all, the publication contained a substantial summary that was enough to draw attention, and the publications of the Moscow Conjoncture Institute were focused enough to have a watchful readership. Besides, it is known that Slutsky sent an offprint to Ragnar Frisch and that it would be influential on his own theory of cycles (Bjerkholt, 2007; Holta, 2014, 456-57). After 1937, as the Cowles structural econometrics program and Keynesian macro-econometric modeling went under growing pressure, Summation became a subject of renewed interest (Dimand and Veloce, 2007; Dimand, 2020), especially within the New Classical Macroeconomics movement of the 1970s-1980s. Throughout, Summation has been interpreted in various ways, with only rare accounts for its methodological scope. However, its richness and absence of involvement within a specific way of theorizing about cycles or thinking about exogenous vs endogenous variables made it a prominent reference during the 20th century (Legrand and Hagemann, 2019)

6.1 Summation between 1927 and 1937: "Spurious correlations" vs "Inverted inference"

The early reception and discussion of Slutsky's analysis of cycles is well documented, though archival work may still bring relevant information (Barnett, 2011; Bjerkholt, 2014; Barnett, 2006; Bjerkholt, 2017). On the one hand, Ragnar Frisch (1933) would exploit Slutsky's random shocks only to provide exogenous impulses to his own dynamic device, warning economists that Slutsky's analysis by itself is unsound as a foundation for economic theorizing. On the other hand, Simon Kuznets would highlight the methodological importance of Slutsky's contribution as a potential cause for cyclic movements. The central question that is implicit in these two approaches is: If

random causes can generate cycles that look almost exactly like cycles observed in economic activity, should we consider that cycles are of necessity the result of random causes and abandon any causal economic interpretation beyond random causes or should we confine its use to a minimum set of exogenous shocks providing the proper amount of randomness to describe the facts of business fluctuations? Summation does not offer any definitive statement in favor of one interpretation over the other—though it contains enough to discard extreme interpretations—and this explains its openness to various uses and interpretations over the last century.

The first mention of Slutsky (1927a) in a published outlet seems to have been Mitchell (1927, 478) ("a most interesting analysis") at the very end of an addenda chapter to a book ready for publication *Business Cycles : The Problem and Its Setting*, where he notes Slutsky's disagreement with him as regards the hypothesis of periodicity of cycles. Then, Simon Kuznets (1929) noted that Slutsky's analysis of random cycles provided as a self-contained source of cyclic movements (Kuznets, 1929, 258). To Kuznets (1929, 258), "These results were in no way accidental. It is not only 'possible' that a summation of a random series will yield cycles, but also quite certain that this will be the case." Kuznets' interpretation makes random causes an actual source for genuine cycles, something that must be accounted for one way or another. In economic life, events may exert an influence for a while, and only for a while, on other relevant economic facts, thus giving them a shape well represented by moving summations.²⁰ Having studied the effect of various kinds of moving averages, Kuznets concludes by wondering whether such robust results obtained by Slutsky may be taken as the rationale for a realistic-genetic interpretation of cycles. This is the "inverted inference" thesis:

"can one ...say that ... cyclical oscillations may be conceived primarily as results of summation of random causes, and that

²⁰"In the moving average two considerations are of importance: (1) the period taken; (2) the weights assigned to the single values. The period taken determines the time limit within which a certain event is to be counted as an effective cause. It is clear that the longer the period of the average, the longer this extension of an influence of either an exceptional item or an exceptional cluster of items. As a result the cyclical swings will tend to be of longer duration. On the other hand, the exceptional occurrences are combined with a large number of other less exceptional items, and therefore the amplitude is likely to be mild. By the same reasoning a moving average of shorter duration is likely to give a cycle that is shorter on the average and more violent in the amplitude of its fluctuations." (Kuznets, 1929, 269) Kuznets's article presents Slutsky's results in a pedagogic way, stressing the most important features for an economic theory of cycles. He probably contributed to making Slutsky's ideas known to a wide readership.

the characteristics of some of these cyclical oscillations can best be grasped as a result of some peculiarities of the underlying random events or of the process of cumulation?" (Kuznets, 1929, 273-74)

Kuznets does not uphold such a radical stance, however he intends to account for its consequences on economic theorizing:

"Indeed, if one can explain how in certain processes of economic life, the response to stimuli is cumulative, then the whole discussion of the cause of business cycles becomes supererogation. If the business economy runs at a certain high or low plateau level, and the conditions of business behavior are in such shape as to cumulate favorable or unfavorable random events, then we are bound to have a cyclical up or down swing sooner or later. It is to be seen that the so-called institutional explanations of business cycles deal mainly with the economic forces that make for cumulation, with forces that explain why a given random event is not immediately cancelled by an opposite reaction but allowed to exert its influence for some time to come, an economic counterpart of the statistical mechanism of a moving average." (Kuznets, 1929, 274-75)

Also, Kuznets notes that differences between the features of two fluctuating series (e.g. index of production vs shares at NYSE) may be ascribed to differences in the way past events are likely to operate through time as relevant and highly weighted causes. In other words, Kuznets, without fully endorsing a exclusively random-based rationalizing of cycles, points out that Slutsky's results lead to reallocate the use of causality in the economic theory of cycles. Cycles are now seen as the outcome of the various ways by which economic agents and the institutional framework of an economy account for past events, be there purely random or derived from random causes. Kuznets also highlights the role of frequency distribution of random events (skewness). A critical point, not addressed by Kuznets, is to settle to what extent those behavioral features should be linked with individuals' maximizing behavior and should be compatible with a deterministic (dynamic) system representing an economy.

Kuznets interpretation is often set against Frisch's idea that moving average summation of random distributions leads to spurious correlations.

²¹ Two contributions by Frisch witness for his complex relationship with Slutsky's results. Frisch (1931, 78) focused on the "Slutsky effect" as essentially the effect of applying a linear operator upon a random variable as a source of cyclical fluctuations. In other words, what Frisch did is to confine the scope of Summation to a methodological paper that any econometrician or mathematical economist should take into account for understanding the undesirable effect of random causes that may blur the understanding and mere elucidation of true empirical cycles. Thus, random cycles are seen as a source of "spurious cycles" and should be eliminated as noisy fluctuations in order to focus on the interpretation of the deterministic component of cycles.²²

Frisch attempts is then to devise various linear operators to be applied on time series in order to reveal the main relevant components of the series, that is, the various genuine cycles lying behind and a trend component as well. Actually, it is a matter of theory how many components will be searched for, and it will be deduced from the modeling of the main relationship at work to describe the working of a capitalist economy (a complete description being out of reach and incompatible with quantification of economic phenomena). Thus, through manipulations of time series data, it is expected both to smooth the effect of erratic shocks and to reveal genuine cyclic components of the time series (amplifying some and discarding some insignificant ones). Now, the problem that arises is simple: applying linear operators onto the series will import into it spurious fluctuations. To remove them, Frisch claims to have studied the laws of those "spurious cycles" (in terms of period and relative amplitude with genuine ones). Frisch's interpretation of Slutsky is linked with his methodology and the way he articulates the study of data and theory. Given a time series, the first task is to identify different components in the series, then to inquire about the theoretical explanation of components, meaning by that to provide a rational explanation of the phenomenon identified in time series by way of a modeling strategy borrowed from physical systems (Frisch, 1931, 74).

The relationship between theory and manipulation of data would be somewhat different in Frisch's next venture into the theory of cycles. In his groundbreaking "Propagation Problems and Impulse Problems in Dynamic

²¹In private discussion with Slutsky, Frisch reacted to the 1927 paper. In 1931, Schultz also asked Slutsky a copy of the article in order to initiate a translation (Bjerkholt, 2014).

²²Actually, Kuznets' contention already arose a reaction by Souter (1930, 72), who held the view that the economic nature of the operations leading to the cumulative reactions to random shocks needs to be addressed. Thus, the static general equilibrium model would remain an adequate paradigm to the understanding of economic dynamics.

Economics", (Frisch, 1933) launched classic distinctions in modern economic thought regarding dynamic and statics, exogenous and endogenous variables, and provided the first macro-dynamic model engendering fluctuations characteristic of cycles, through causal macro-economic relationships involving few variables (net capital stock, consumption), time to build capital goods and *encaisses désirées* as a factor influencing consumption. As compared with the 1931 article, Frisch is accomplishing a step forward in providing a broad "mechanistic" theory of a dynamic capitalist economy. This time, the treatment of time series data is left aside and what comes first is the understanding of how the dynamic system is likely to exhibit permanent fluctuations. Indeed, the dynamic linear propagation model could not account for persisting movements of swings of regular amplitude, and something more was needed to prevent the dampening of oscillations. To solve the "impulse problem" Frisch assumed that random shocks (technological shocks) could change the value of parameters of the simultaneous equation system and trigger regular perturbations fostering the fluctuations in the system. From this purely theoretical standpoint, Slutsky's effect is ignored and random shocks are assumed to be just sufficient in order to provide the necessary impulses whose influences over the system are entirely captured by the propagation structure. The seminal idea for this random causal mechanism is attributed to Yule and Slutsky. The study of the "*mechanism* by which such irregular fluctuations may be transformed into cycles" was undertaken "independently of each other by Eugen Slutsky and G. Udny Yule" (Frisch, 1933, 198):

"Slutsky studied experimentally the series obtained by performing iterated differences and summations on random drawings... Yule only used second order differences, but tried to interpret the random impulses concretely as shocks hitting an oscillating pendulum. By the experimental numerical work done by these authors, particularly by Slutsky, it was definitely established that some sort of swings will be produced by the accumulation of erratic influences, but the exact and general law telling us what *sort* of cycles that a given kind of accumulation will create was not discovered....But still the main problem remained, both with regard to the mechanism by which the *time shapes* of the resulting curves are determined and with regard to the concrete economic interpretation." (Frisch, 1933, 199)

In so doing, certainly Frisch reinterprets Summation (Bjerkholt, 2007) more than he elucidates it (Barnett, 2006). He is however forced to admit this time that as a causal structure for swings, random shocks need some further

analysis (this is sketched in a discussion about Schumpeterian innovations). To Frisch, the explanatory mechanism for cycles needs to be complemented to account for endogenous explanations, connected with some long term tendencies of capitalist systems to influence the path of dynamical systems (such as Schumpeterian innovations). Thus, Frisch lays the foundations for a development of structural econometric program in which random causes offer useful but still under-theorized complement to the cyclical behavior of capitalist economies (Legrand and Hagemann, 2019). He would soon abandon his research in this direction due to flaws inherent in the linear dynamic model (Chen, 2010; Louçã, 2004).

6.2 From 1937 to Lucas

Frisch's propagation-impulse modeling became a standard way of doing research on business cycles, and has been interpreted as superior to other attempts in terms of realism (obtaining irregular cycles) and subjecting random causes and their cumulative effects to an explanatory scheme. Opposing views by Mitchell (1927) or Burns and Mitchell (1946) were famously criticized by Koopmans (1947) as "Measurement Without Theory". What counted first was the articulation between theoretical structure and data handling as developed at the Cowles commission. As far as business cycles are part of the research program developed at Cowles, they are defined as models with dynamic properties (Dimand, 2020, 565). Frisch and Slutsky as read through Frisch's glasses appear as two basis reference in Haavelmo (1938). In a structural dynamic system, each structural relation, or relations obtained from their combination (confluent relationships) are not exact laws and "*must be taken as laws in the statistical sense, i.e. as average laws*" (ibid. 204). Klein (1999, 161) notes that "Slutsky's moving average of random disturbances became the key building block in the articulation and modeling of stationary stochastic processes. Isolating a stationary process is like constructing an inertial frame of reference."

In the second half of 20th century, Slutsky's contribution would still be regularly mentioned as a methodological source for thinking about cycles. However, focus on growth and structural macroeconomic modeling lead to a disinterest in random shocks as a rationale for cycles (Duarte and Hoover, 2012).

Along the Burns and Mitchell tradition, Adelman and Adelman (1959) offer a critical assessment of the Klein-Goldberger Model of the American economy (Klein and Goldberger, 1955). They provide a test aimed at distinguishing between economic series generated by the Klein-Goldberger model

and pure random series. The question is whether the KG model is supported to model the dynamics of a modern industrial country. This is done first by constructing the values of exogenous variables in the model through a linear trend. A simulation of the Model with those values leads to a smooth linear system. The next step assumes that cyclical movement originate from random shocks on this exact trend, in accordance with Slutsky and Frisch (Adelman and Adelman, 1959, 606, fn19). To the authors, the necessity of adding random shocks to obtain cyclical movements comes from the impossibility to model a purely micro-founded model whose behavioral equations would strictly proceed from a heterogeneous set of individuals. Ignoring heterogeneity leads to inexact behavioral assumptions at the macroeconomic level. Imposing some type of random shocks in the model leads to oscillations of periods and amplitudes similar to those observed empirically. The cyclical fluctuations obtained through stochastic impulses are similar to those described by the NBER as characterizing the United States economy. However, one should not conclude “that the type of perturbations actually responsible for the observed cyclical behavior are exogenous to economic theory in general” (Adelman and Adelman, 1959, 621).²³

Even though Slutsky’s contribution had been confined to a provider of random shocks, issues about serial correlation and the role of random causes in the analysis of propagation mechanism was still relevant in the Cowles program of structural economics in the 1970s (Heckman, 2000).²⁴

The revitalization of Summation as a methodological and even foundational reference took place in the 1980s, when it became a reference in the rational expectation revolution in macroeconomics, through the works of Robert Lucas and the rational expectations revolution on the one hand, and Real business cycle theorists on the other hand. Actually, among pioneers of cycle analysis, Slutsky is probably the most consensual reference for New Classical macroeconomists.

Interestingly, the somewhat instrumental use of Slutsky and Yule work

²³Adelman (1960) complements this article with a methodological contribution involving the comparison of the Klein-Goldberger model plus random shocks vs an artificial model made of accounting equations. To Adelman, it is expected that the approximation obtained with a structural model such as the Klein-Goldberger is much better than the one obtained with a model with minimal economic content.

²⁴Some aside comments on Slutsky’s discovery deserve mention. In a book titled *L’ère logique* (1969) Jacques Bureau, a man trained as agricultural engineer who is notably remembered as an enthusiastic promoter of jazz in France through the association *Hot Club de France*, interpreted Slutsky’s random cycles as an expression of a “microclimate of chance events”. In 1976, the book *Hasard et Prévision* (1976) by Daniel Zajdenweber contained a comprehensive comment on Slutsky’s discovery

would evolve in the context of the rational expectations revolution. In his seminal article, Muth (1961, section 5) confronts the usual empirical implications of a Cobweb-type model with that obtained through rational expectations plus random shocks. "From a purely theoretical standpoint, there are good reasons for assuming rationality. ... The only real test, however, is whether theories involving rationality explain observed phenomena any better than alternative theories." (Muth, 1961, 330). An empirically confirmed implication of the Cobweb model is the quasi-periodic fluctuations in prices of a number of commodities. However, the same pattern could be obtained through "a dynamic system forced by random shocks" (ibid, 333). As one can understand it, the fact of random shocks blurs the interpretation of dynamic systems, a view that leads us back to Frisch (1931): "Slutsky and Yule first showed that moving-average processes can lead to very regular cycles. A comparison of empirical cycle periods with the properties of the solution of a system of differential or difference equations can therefore be misleading whenever random shocks are present." (Muth, 1961, 133) ²⁵ However, the rational expectation assumption leads to associate closely unexpected events with their exclusive counterpart (the difference between expected prices and actual prices).

Lucas' (1980) epistemology of modeling stresses the critical importance of random shocks. Against structural modeling, Lucas advocates that an economic model must be erected as an artificial construct, based only on some behavioral and informational assumptions (rational behavior, rational expectations, structurally efficient information). Lucas' view echoes Adelman's idea of testing a model against another through its differential performance when subjected to random shocks: "not all well-articulated models will be equally useful. ... we need to test them as useful imitation of reality by subjecting them to shocks for which we are fairly certain how actual economies, or parts of economies, would react." (Lucas, 1980, 696-97). The contribution of Slutsky (1937b) (and that of Tinbergen) are interpreted as independent advances in statistical and economic theory that promoted "the idea that one might describe an economy as a system of stochastically disturbed difference equations, the parameters of which could be estimated from actual time series." (Lucas, 1980, 701).²⁶ In this respect, Lucas promotes a unified

²⁵Here, Muth follows Haavelmo (1940) who alerted economist about the risk of introducing into the theoretical structure things borrowed from empirical data, that are merely the effect of random causes.

²⁶A statement by Lucas echoes also Slutsky's ideas that within a long time span, the apparent regularity of cycles is blurred and becomes a series of random events "If the Depression continues, in some respects, to defy explanation by existing economic analysis

modeling of random shocks and propagation mechanism that does more justice to the ideas of Slutsky than did Frisch (1933), a view also set forth by (Lines, 1990) about Lucas' (1975) equilibrium business cycle model in which erratic monetary-fiscal shocks undergo lagged information processing.

The importance of Slutsky has been underlined also within the group of Real Business Cycle theorists. Hartley et al. (1997) show how RBC theory mixes layers of methodological underpinnings and strategies and complete recasting of the rules for assessing them. They also point out the weaknesses in their attempts at incorporating dead economists on their side. RBC theorists after Kydland and Prescott (1982) have promulgated several new principles tied together to erect a new view on cycles, now viewed essentially as optimal paths of reactions of agents to present shocks (interpreted as temporary or permanent) in the economy. The group of New classical macroeconomists promoting RBC puts to the fore new methodological principles and a new language (rational expectations, microeconomic foundations based on a representative optimizing agent credo, calibration, perfectly competitive economies with efficient, clearing markets, and random shocks—essentially technological— as the necessary complement to the intertemporal coordinated decision plan of Robinson Crusoe's economies). According to RBC, the usual business cycle models linked to the structural econometrics and Keynesian tradition are mistaken in their attempt at separating trends from cycles, at building business cycles as a necessary complement to growth theory.

Prescott (1986) and Kydland and Prescott (1990) depart radically from the standard view of the 1950s and 1960s that cycles are the outcome of endogenous phenomena around a trend, as exemplified in Goodwin (1967). Instead, they advocate real shocks are constantly operating on the economic system, triggering optimal reactions from individuals and permanent consequences on the economy with no tendency to come back to a previous trend, whose values are inferred from neoclassical growth theory. The whole visualization of cycles is discarded. This new interpretation of business fluctuations is often associated with a come-back to Frisch and Slutsky.

Another aspect of Slutsky's influence on RBC theorists is linked with the methods used to detrend data. Since Solow's growth model is used to define the steady state path of macroeconomic variables (consumption, investment, output) it is necessary to produce these data. However, nothing justifies a constant trend at all times over a period, hence the trend and the deviations

(as I believe it does), perhaps it is gradually succumbing to the Law of Large Numbers" (Lucas, 1980, 706).

from trend at each time should be estimated jointly. In practice, RBC theorists apply a technic of filtering data that relies on some a priori specification of the trend. These filtering technics (notably the Hodrick-Prescott filter) can induce artificial fluctuations or correlations between series that are not present in the original data, which are interpreted as spurious correlations. In this matter, RBC theorists fail to adopt the same rigorous treatment to data that Slutsky had done in his simulations (induction on the basis of a rich set of simulations). Hence the whole stage of detrending appears as a source of creation of artificial correlations and fluctuations around an artificially created trend, thus casting a doubt on the whole interpretation of facts (the steady state) against whose competitive theories might be erected (Hartley et al., 1997)

In “Understanding Real Business Cycles”, Plosser (1989) imagines Robinson Crusoe’s reactions to productivity shocks (temporary or persistent), the channels by which intertemporal substitution leads to reasonably expected correlations between time series of consumption, investment, labor time, output... Productivity shocks could as well be preference shocks. To Plosser it is common to think of the business cycle as separate from growth and to characterize the cycle as deviation from a smooth deterministic trend that proxies for growth, due to productivity shocks that follow a random walk. “While rarely explicitly recognized, tests of these business cycle theories are actually joint tests of the model for growth (the trend) and the model for the cycle” (5) What is retained from Slutsky, then. It is mainly an output of the summation of random causes, the fundamental fact that they generate fluctuations and not cycles: “My own preference is to use the term “fluctuations” since “business cycle” frequently carries the connotation that there is true periodicity present in economic activity. Virtually all of modern macroeconomics dismisses the view that there are actual periodic cycles in economic activity. Instead it follows the important work of Slutsky (1937) and interprets the ups and downs in economic activity as the accumulation of random events or a stochastic process” (Plosser, 1989, 54)

Real business cycle theorists do not engage in the understanding of the propagation process as did Lucas. Yet, Slutsky is providing the rationale for understanding the claim that random shocks can generate various kind of reactions on the part of economic agents, that eventually exhibit regularities in co-movements of some fundamental macroeconomic variables and "fluctuations" rather than "cycles" (Prescott, 1986). The identification of Slutsky as an autonomous reference (kept apart from Frisch) is confirmed in (Kydland et al., 1990):

“An entirely different way [from that of Frisch] of generating cycles is suggested by the statistical work of Eugen Slutsky (1937). Slutsky shows that cycles resembling business fluctuations can be generated as the sum of random causes—that is, by a stable, low-order, stochastic difference equation with large positive real roots.” (Kydland et al., 1990, 6)

The recognition of Slutsky’s Summation as a mould for thinking about random causes of economic fluctuations in the real business cycles tradition is now well established (Chatterjee et al., 2000; Ramey, 2016)

Further advances in the assessment of lasting influence of Summation need to address more directly Slutsky’s inference from his own simulations with random series. This is done in passing by Blanchard and Watson Blanchard and Watson (1986) who attribute to Frisch and Slutsky the use of the propagation-impulse framework. Both are identified with the idea that small random shocks transmitting energy to the propagation system, while other authors would focus on ceiling and floor assumptions (e.g. Hicks (1949), see also Eckstein and Sinai (2007)). The article discusses the characteristics of shocks that provide the best emulation of cycles and concludes that large infrequent shocks provide a better description of cycles than do frequent small shocks as exemplified by Slutsky and reinstated by Lucas (1977). They conclude that economy is best described through a mixture of infrequent large shocks and more frequent small shocks of various nature (monetary, supply side, demand side, fiscal). The influence of Slutsky’s methodology as a representation of the causes of regular fluctuations in the economy has thus been revived since the 1980s. Within the context of a general increase in strong shocks on economic agents (financial crises, extreme meteorological events, pandemics) the interest for deepening the understanding of various probability distributions of events, notably the consequences of skewness is well in tune with the spirit Slutsky’s 1927/1937 article.

7 Slutsky’s other contributions: recharting economic ontology

Slutsky’s two famous and long-lasting contributions to economics deserve recognition owing to the methodological questions they address to economics and economists. Budget opens to the question of the proper balance between observable behavior and rationality assumptions in consumer theory. Summation leads to address the issue of how economic systems transform

random events into structured economic behaviors through layers of intermediate phenomena involving human behaviors and institutional settings supporting them. It is probable that those two contributions will remain to some extent open systems of interpretation. Nevertheless, Slutsky's lesser-known contributions to economics may shed some light and help build a bigger picture of his views on economics.

Three articles deserve peculiar attention. The first one is a short presentation of William Petty's thoughts (Slutsky, 1914). The second one is a critical assessment of Böhm-Bawerk's theory of value (Slutsky, 1927c), the third one is a programmatic article on the praxeological foundations of economics (Slutsky, 1926a).

"Sir William Petty" appeared in 1914. The content of this short article was originally presented on November 1913 at the Society of Economists in Kiev, and published first in the student's bulletin of the Kiev Commercial Institute. It presents the ideas of a "wonderful economist" to readers together with a translation into Ukrainian of a selection of Petty's writings covering various subjects (Barnett, 2011, 130). Under the Marxist doctrine, Petty was seen as a bourgeois economist. Slutsky wanted to let know to readers that Petty's work could be read as a pre-classical theory of value, whose richness and originality might be a source of inspiration for thinking about economics. After an exalted presentation of Petty's life and brilliancy, Slutsky presents his political-economic views, which he deems of "greatest interest to us" (Slutsky, 1914, 132). In Petty, Slutsky views the thinker who rejects metaphysical thinking in favor of "numbers, weights and measures". Slutsky links Petty's appeal to natural price with that of just price and market price, "and the very concept of justice in market affairs is discussed as the result of mutual voluntary evaluation of both parties acting with normal good conscience." (ibid., 133). And Slutsky goes on:

"Thus we should represent this as being that the concept of a natural price is a free price, forming outside the actions of whatever power there might be, in only one mutual action between sellers and buyers freely competing with each other" (ibid, 133).

Slutsky criticizes Marx for presenting Petty simply as a predecessor of the labor theory of value. Writing that "labour is the father and active beginning of wealth, as the land is its mother" (from *Treatise on taxes and contributions*, 1662) cannot be taken as a formulation of the labour theory of value. To Petty, at best, Slutsky goes on, "labour is not the substance of value but merely a factor that defines the exchange proportion", that is

a relative price (extrinsic value in Slutsky's wording). The notion of natural price is altered by several factors affecting agent's behavior (habits and principles, monetary influences, providing a "reasonably defined empirical theory of value formation, which should not on any account be confused with Marx's point of view, which considers the actual substance of value to be in the quantity of expended (socially necessary) labour" (ibid., 135-136).

Slutsky is manifestly in proximity with Petty's endeavor to search for empirically meaningful measures or comparative measures (quantity of food produced by land without added work; equivalence between types of food). Though he rejects several regulations, "[Petty] is not blinded by the phantom of automatic harmony" (ibid., 140) At some points in the article, Slutsky makes interesting digressions, exposing his opinion on some important matter, notably as regards intertemporal valuation, introducing a distinction between immediate feeling and a rationally constructed view on value. If I exchange a durable good A for a disposable good B, once B has been consumed, I am forced to consider the future utility accruing from the abandoned good A as "lower than its future significance". Thus, Slutsky points out already an argument that will be developed in his critique of Böhm-Bawerk, an essential distinction between "utility" evaluation and a more general evaluation of "significance".

"We cannot escape our capability to look ahead into the future and be concerned about it, and this creates disharmony in our evaluation of economic goods: *the economic evaluation of the significance of a particular good does not exhaust our attitude to the good.* Were this not so, then there would be no place for the concept of exploitation, since in any and every exchange one would choose that which had the greater subjective value for him, rather than that which he gave away." (emphasis added, ibid., 141)

As a concluding remark to this presentation, Slutsky underlines that the economist cannot dispense from being engaged in the philosophical foundations of his own approach:

"To bring the study of political economy to this state of wholeness, to link all its sources with all the cultural, scientific and philosophical foundation in which it developed, is the study of the history of science's only worthy final aim." (ibid., 142)

The theme of value and the tension between immediate evaluation and rational evaluation as part of a whole comes back as the central issue in

"A Critique of Böhm-Bawerk's concept of value and his theory of the measurability of value" (Slutsky, 2004a). It is presented as the groundwork for a more ambitious project of a positive theory of value measurement that never appeared (Slutsky, 2004a, 357). This masterful criticism echoes some questions raised in Budget as regards individuals' capacity of valuing objects of choice. In this article, Slutsky provides a straightforward criticism of Böhm-Bawerk's theory of value in *Kapital und Kapitalzins* (Böhm-Bawerk, 1891) and indirectly a "final clarification of the received marginal utility theory...[which] has not ceased to exert a hampering effect...on the continual advancement of the science." (Slutsky, 2004a, 357)

Slutsky focuses first on the issue of utility measurement and the theory of action. Slutsky puts to the fore the internal inconsistencies in Böhm-Bawerk's statements about utility, desire, judgment of intensity and duration of pleasures and pain as psychic acts, summation of intensities of sensations, and our presumed ability to judge in advance the feelings associated to the consumption of a good. To Slutsky, in any act of choice involving several units of goods, the assumption that "several pleasures could constitute in a direct manner, an immediate discernible unit of pleasure" goes against psychological knowledge that "if desire is a quantity, then it is a quantity of intensity. For this reason, it has no parts that could be contained in the unit of a whole in any intuitive way." ((Slutsky, 2004a, 359). In other words, the imagined summation of pieces of intensities when considering a lot made of several goods or units of the same good is contradicting Böhm-Bawerk's own assumptions, a criticism that had been already addressed to Böhm-Bawerk by Frantz Čuhel (Čuhel, 1907). More broadly, Slutsky rejects Böhm-Bawerk's idea that value of a thing is essentially reduced to a measurement of *experienced* feelings that contradicts all his arguments in which desires and imagination about pleasures are used as equivalent. The second step in Slutsky's criticism deals with the idea of the mind as a calculating machine in practice. "The fact that people economize rationally, weighing benefits and detriments" (ibid., 360), is no proof that calculation is processed by comparing intensities of sensations. Besides, the fact of measurement, if accepted is no justification for the logical possibility of measurement; and the fact of choice is no proof that this kind of measurement has been operated, to the exclusion of all other motivational theories. Besides, it ignores the debates in the field of psychology over the possibility of measuring sensations, not to mention the meaning of measurement itself (ibid., 361). The third criticism addresses the incapacity of Böhm-Bawerk to take distance with a hedonist terminology and way of thinking and to confront with alternatives that involve other dimensions than pleasure and pain. Slutsky is there a

fierce opponent to the "alchemy of hedonism [that] supposes itself capable of transforming all of these things : honor, duty, passion and who knows what else into sheer sensations of pleasure and pain." (ibid. 362)²⁷ Because he does not consider the variety in the circumstances and objects of choices and the impossibility to avoid to use imagination, "Böhm-Bawerk ...cannot do justice to the actual phenomenon of value in the primary and genuine sense of the term." (ibid. 363). To this, Slutsky opposes the view that value proceeds from the recognition of primary goods having primary values, the economic value of economic goods being derived from their ability to provide these primary values in a given situation: "*desire, honor, duty, and so forth are customarily primary goods, and ... their values are primary values; ... the choice between primary goods is determined by the comparison between their values; ... economic value is derived from the primary, the realization of which depends upon one's disposal over the corresponding economic good in a given situation.*" (ibid. 363) Fundamentally, Slutsky's analysis leads to flush out the fundamental flaw of hedonism:

"it consists not solely in denying primary character to all goods other than one's own pleasure and pain, but *even more importantly, in conferring a motivational meaning on pleasure as such, whose value, however, as a phenomenon sui generis is misapprehended and left out of play*" (ibid. 364)

The analysis of intertemporal choice leads to highlight the intrinsic contradictions of Böhm-Bawerk and to put pleasure in its right place with other primary values. The fact of being able to identify a future good as offering a greater value than a present one is contradictory with the fact of attributing the choice of the present good to a lack of will, since BB builds his theory of choice on the assumed ability to choose the greatest value. The only way out of this contradiction, Slutsky notes, is to conceive pleasure as something to be valued for itself (and not as a value by itself). Consequently, the fact of choice of a present good in the case of a choice between a present and a future good "means nothing other than I *regard the value of the deferred realization as a smaller one in comparison with that value standing immediately before me.* ...The essence of value cannot consist in the sensation of

²⁷To Slutsky, inner calculations do not lead to unequivocal decisions: "Let us assume that someone must choose between his own pleasure and that of his wife or his brother. Let the second pleasure, *according to his imagination*, be, for example, greater than his own. Is it not then clear that the direction of his choice still is not by any means unequivocally prescribed thereby. A man can sacrifice a greater pleasure for a smaller one to his wife, yet not forgo his smallest pleasure for a larger one to his brother." (Slutsky, 2004a, 362).

pleasure. *Pleasure is not value, but rather merely a possible bearer of value* and if and if, as is usual, it is such a bearer, *it is then a good.*" (Slutsky, 2004a, 366)

Eventually, in his incapacity of giving recognition to the meaning of primary goods (including pleasure) as the motivational support for desire and action, Böhm-Bawerk theory "proves itself to be evidently, a nominalist or even (ontologically) nihilist theory, since it overlooks and negates precisely the very phenomenon whose elucidation *should be its true task*" (ibid, 369).

The Böhm-Bawerk article stands as an informative complement to Budget, offering new avenues to think at the kind of articulation between psychology and economics, in which goods are not seen as the direct bearer of utility and claiming for a refined theory of rational behavior. It also points to the implicit necessity to provide an ontology of economics.

What can be said about the influence of Böhm-Bawerk article in the West? On the basis of published articles and books, one is tempted to say that this article had little to no recognition in Western literature. To my knowledge, one of the first mention is in an article by Bagiotti commemorating the centenary of Gossen's book, where he advances that Slutsky's critical account of Böhm-Bawerk "is still ignored by its admirers"(Bagiotti, 1955, 249). The next step in getting the Böhm-Bawerk article out of oblivion is Emil Kauder's famous *A History of Marginal Utility* where Slutsky's article is briefly presented (Kauder, 1965, 129-30).

The third important contribution to economic thought to be commented in this section is "An Enquiry into the Formal Praxeological Foundations of Economics" (hereafter Praxeology). The article appeared in German in a Ukrainian journal. Praxeology is an attempt at building a formal-ontological building block to think of any kind of domain of knowledge involving the description of a system of elements linked together through parameters and dependence between past, present and future states. The most general presentation of the concepts involved in this system (state parameters, possibility sets, optimality, directive forces, time processes, components of a state) constitutes a *formal physics*. When adapted to a field of thought by way of mapping an axiomatic and conceptual structure onto it, one obtains subdivisions of this formal physics. If this axiomatic structure involves "actions performed by conscious beings" (373), one gets a *formal praxeology*. Still another narrowing of the conceptual apparatus—dealing with the administration of some objects of power called "assets"—leads to a *formal economics*" (373). Praxeology can be read as a first attempt at establishing a balance between empiricism and *a prioristic* elements in economics, between determinateness and randomness, between systemic constraints and

individuals' expectations about their power over objects in the present and the future. In this respect, it connects several topics that are central in all the other contributions to economics by Slutsky. Slutsky's goal in Praxeology is to provide us with a "formal-ontological" approach which, if successful, would be observable "as categorial forms in the corresponding substantive phenomena". (Slutsky, 2004b, 372) As he puts it, the substantive content or the essence of economic structures "cannot be construed as consisting entirely of logical building blocks; human activity certainly cannot be decomposed without residual into quantitative relationships among the specific components of a quasi-mechanical system. Yet, on the other hand, reality is not by its very essence a seamless entity that defies any analysis." (Slutsky, 2004b, 372)

An important point deals with the role of value or consciousness in the formal-ontological structure. To some extent, the description of the state of a system "without at all employing the concept of value or any categories of consciousness serves to demonstrate that certain empiricist trends of contemporary thought cannot be denied a relative validity" (ibid, 372). In this respect, the influence of Russel and Husserl on Slutsky's economic thought is fully acknowledged at several points in the essay, as it was already for Husserl in Critique and would be again in Summation. Once expectations about future possible states of a system enter stage, praxeology appears as the "foundation for a widely ramified deductive theory, ...the boundless richness of [which] ...represents a 'definite multiplicity' in the Husserlian sense." (Slutsky, 2004b, 380)²⁸

We can only give a partial account of the Praxeology article, which is extremely dense and contains a set of fundamental definitions of the categories of a formal praxeology and then of the basic concepts of a formal economics. A system is made of a set of partly interdependent variables defined by means of definite values (parameters). The whole set of conceivable parameters defines a state space. Any subset of values of the parameters in a state is a component. Some parameters—*determining circumstances*—determine the possibility set while others determine the state that will be selected: they represents the *directive force* of the system that determines the *optimal point* that results from the operations of the directive force (*selection*) through a series of transformations of the state of the system (*effect*). The presentation of the series of *effects* that can take place through time is the *perspective* of the system.

²⁸Chipman (2004) insists on this "surprising influence" of Husserl and points out the implicit references to Husserl in his editor's comments to Slutsky's articles

At the center of the conceptual apparatus pertaining to agent's behaviors is the concept of power. It is tempting and legitimate to interpret those elements within the narrow framework of a consumer budget share and a producer plan of production within a perfectly competitive environment. However, Slutsky warns the reader that various interpretative systems could be implemented on these praxeological categories. The objects on which agent's perform elementary operations could be economic goods or other agents, they could be rights, and even the kind of property on things is not strictly defined. The kind of decision taken may be parametric, but other kinds of interactions can be conceived of.

To Slutsky, a system can exhibit *passive* changes due to external circumstances, but only temporarily, that may induce some inertia into a system until the directive force becomes again the only driving force. As long as one considers the evolution of states from past to present, the optimal selection and temporal path depends on past history and initial values of the parameters. Once future values are taken into account as perspectives (involving views of human beings about the future), then the formal model is a *purposive process*, "a model that permits modification by various kinds of randomization effects" (Slutsky, 2004b, 373). The fact of introducing purposive processes, hence a consciousness about views on the future, establishes a watershed between the use of this formal apparatus for mechanics and for economics. The temporal evolution of a system is characterized by the definition of components that could possibly be part of the system at various point in time. If two situations differ only by one component and each situation is stable, then the component is said *irrelevant per se*. A past component may be relevant at some later point in time, and a future component too (retrospective or prospective relevance).

A formal physics becomes a formal praxeology once agents (conscious human beings) are considered in the system as the vectors of the purposiveness of the system, whose actions are effectuated upon various objects of power. Praxeological concepts deal with the definition of agents. Agents are the directive forces in a purposive system, and the changes of the system under their purposive action are *activities* (Slutsky, 2004b, 375). The elementary *existential operations* performed on objects by agents are *creation*, *maintenance* and *destruction* and transformation (*substitution*).

Slutsky has a complete terminology to describe operations of exchange:

"The capacity of performing existential operations on an object we call power. The agent in question shall be called a *power wielder*, the object on which he operates the *object of power*. By

capacity we mean here the objective possibility. If it is not absolute, the agent does not have an *absolute* but only a *relative power*, that is, a larger or smaller probability of being able to execute the operation in question. Any existential operation executed on power itself, considered an operation on this object of power, we call *power operation*. The operations of creation, maintenance or destruction of power are the elementary power operations; we shall call them *income earning*, *saving* and *spending*, and the corresponding objects are *incomes*, *savings*, and *expenditures*. The substitution of power over one object by power over another object is also a power operation, which we call *change*. If the power of one agent is replaced by the power of another agent, we talk of *power transfer*. If the change is effectuated by means of two interacting operations of power transfer, we shall speak of *exchange*." (Slutsky, 2004b, 375)

Agents are seen as performing operations on objects and on powers, that is disposing of things in order to gain incomes, to spend or to save or to exchange (economic activities). The whole set of objects that can come under power of agent's disposition in the present and future are assets, and economic activity is the *administration of assets*. The assets owned by an agent are his endowments. An agent and his assets is an economic unit. All the concepts presented here are orientating interpretation but are not necessarily synonymous with the usual concepts of economics, the point is that to Slutsky "no thorough understanding of economic phenomena can ever be attained without a closer analysis of the formal praxeological underpinnings of economics, and in particular without a closer analysis of the complex structures formed by power relationships as elements" (Slutsky, 2004b, 376). Objects of power can be primary or derived objects. Agents may have a disposition potential over a power in the future. A disposition potential over a future disposable object is part of the endowment of an agent. Expectations about the future disposability of an object (*mere expectations*) are also part of the endowment. In the final analysis, endowments are characterized in their most general form as composed of disposition potentials and mere expectations about disposition potentials.

Then Slutsky considers changes occurring to the praxeological structure when randomness is added into this determinate structure. The most difficult question arises as to whether randomness in a component of a system still leads to a unique selection from the purposive process. Slutsky introduces into his system considerations about the probabilities of carrying out

an existential operation of various disjunctive objects of power or probabilities over disjunctive outcomes. (The analysis would be even more complex if choices were treated as random.) The last section of the article deals with potential inconsistencies within an economic system, due to an ineffective effect of future expectations on present states. A specific praxeological device, taking the form of a complementary system of representation of agents' activities, allows to think of various situations and how the fulfilment or non-fulfilment of expectations will lead to *corrective actions*.

The directive force is premier in fostering changes in any state-system, but the system itself is liable to passive transformations. However, in economics, it is usual to go beyond the mechanistic approach of a deterministic system through implementing in it a random component and the idea that agents within the system hold perspectives and induce purposive processes, implemented through "actions of beings endowed with consciousness" (373)

Slutsky's contribution to praxeology can be put in perspective with his other contributions to economics. Consciousness and randomization are to recurrent issues and are witness for the importance he attributes both to a probabilistic representation of economic activities on the one hand, and for the necessity to integrate agent's own evolving representations in the understanding of economic choices. The importance of expectations and of views on the future thus appear in Praxeology and in the Böhm-Bawerk as a possible nexus to understand how individual agents' transform new data into new views about the future and update their plans. Unfortunately, on the basis of published archives, we cannot do more than guess in what directions Slutsky could have develop his thoughts.

In his praxeological framework, Slutsky elaborates on the notion of assets, viewed exclusively from the perspective of power on primary objects (individuals being endowed with some power on those primary objects) and of mere expectations about future possibilities to derive powers from the use of those primary objects in the present and in the future. A full formal-praxeological interpretation of economics needs to account for the gap between real components of a system (and of its future) and perceived components (presumed or expected in the future). Through time, agents would consider how their expectations are fulfilled and enter into corrective actions. Eventually, the whole presentation does not make reference to such concepts as value, capital, labor, and, prices, property (a concept necessarily linked with power). It seems that in this article, Slutsky aimed at clarifying his position as regards empiricism, that is, the idea that economic concepts should derive from common sense or introspection, an attitude he had not discarded completely in his 1915 article when imagining introspective experiments as regards utility

variations. As he declared to Frisch in a letter dated July 9, 1926, “Even though I always highly value the role of empirical experience and especially experiments in theoretical economics, I could not now subscribe to certain utterances in my earlier work, since, as I may believe, I have now arrived at a clearer insight into the relationship between the empirical and a priori elements of our knowledge.” (cited in (Chipman, 2004, 349)) This confession, however, should not be taken as a plea for *a priorism* in economics, since Slutsky does not engage with the issue of rational behavior and lets it open as to the various ways to use the praxeological system in economics. Though individuals’ actions are grounded in past experience, they are also to a high point motivated by representations about their potentialities and expectations. For that reason, the world and its possibilities for the future cannot be packed within a deterministic and naturalistic framework, and the empirical description and analysis of future events cannot rest upon a frequentist or empirical knowledge of probabilities. Slutsky’s praxeology was referred to by Ludwig von Mises (2002; 1933, 16) as a formal praxeological system “patterned after the science of logic” different from the experienced-based approach he favors (see Barnett (2011, 74)). Oskar Lange would also identify Slutsky as an independent founder of praxeology with Kotarbiński (Lange, 1971). Hence, Slutsky is probably the first author to explicitly connect praxeology and economics.²⁹

8 Conclusion

It is beyond the purpose of this essay to speculate more about what could have been a unified view of political economy according to Slutsky and the proper role of theory and empirical work would have played in it. Beyond doubt, through only two articles, Slutsky has been among the most inspiring economists in the 20th century. It is hoped that further research on Slutsky’s archives will add new insights on his thoughts.

²⁹On Slutsky and the origins of praxeology, see Chipman (2004) and Gasparski (1996). To Lange (1971, 1-2) “The Austrian economist Ludwig Mises also used the same term, although he erroneously identified praxeology with political economy and misconceived its foundations.”, and praxeology in economics is definitely associated with the principle of economic rationality which, when applied to magnitudes or quantities, takes the form of the principle of greatest efficiency or the principle of economy of means.

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