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A direct parameter-error co-design approach of discrete-time saturated LPV systems

C. de Souza, V. J. S. Leite *Member, IEEE*, S. Tarbouriech *Member, IEEE*, E. B. Castelan *Member, IEEE*, and L. F. P. Silva

Abstract—Considering a discrete-time linear parameter-varying (LPV) systems with input constraints, the paper deals with the co-design of a parameter-dependent state-feedback controller and a new parameter-dependent event-triggering mechanism. Two independent event-triggering policies are introduced at the sensor node to economize the limited network resources. They indicate whether the current state or the current scheduling parameters should be transmitted to the controller or not. In this sense, the controller scheduling parameters can differ from those of the system, yielding a certain degree of robustness concerning parameter deviations. Sufficient conditions, given in terms of linear matrix inequalities (LMIs), ensure the regional asymptotic stability of the closed loop. To formulated them, we use the Lyapunov theory along with the generalized sector condition. Moreover, we formulate a convex optimization procedure indirectly reducing the transmission activity over the network. The proposed methodology effectiveness is attested through numerical examples and comparisons with related works from the literature.

Index Terms—Co-design approach. Linear parameter-varying systems. Saturation. Event-triggered control. Parameter-deviations.

I. INTRODUCTION

Due to their advantages comprising lower costs and simple maintenance, networked control systems (NCSs) have attracted significant attention in the last two decades. However, in comparison with a point-to-point connection, the networked connection is unreliable due, for example, to the bounded communication bandwidth [16]. Assuming that the plant signals are periodically sampled, when the sampling period is small, the number of the data transmitted through the network will increase considerably, leading to network overload. If only some packets need to be transmitted from the sensor to the controller to ensure satisfactory system performance, then the inefficient packets can be avoided to save network resources. Hence, selecting an effective communication mechanism to

reduce the waste of communication resources is an essential issue for NCSs. The event-triggering approach has emerged as an attractive tool for dealing with limited network resources. The main idea of the event-triggering strategy is to perform the control tasks only when a particular event (generated by a specified event-triggering scheme) occurs [25]. Many significant results concerning event-triggered control (ETC) are reported in the literature [10], [15], see also [28] for ETC application to a multi-agent system. However, most of them are developed for linear time-invariant (LTI) and non-linear systems. Also, the ETC in the framework of discrete-time systems has its inherent benefits as the minimum inter-event time could always be guaranteed (see [17], for instance).

A large amount of works on linear parameter-varying (LPV) control can be found in literature in both continuous and discrete-time framework (see, for instance, [2], [4]). Noting that identification methods usually yield discrete-time LPV models [27], our approach focuses on such a framework. In such a context, the event-triggered control strategy has been little explored yet. [19] proposes the event-triggered H_∞ control for discrete-time polytopic LPV systems by jointly designing a mixed event-triggering mechanism and state feedback controllers. For the same class of systems, [22] presents a synthesis condition for event-triggered dynamic output feedback control with bounds on the ℓ_2 -performance. [17] addresses the event-triggered and self-triggered H_∞ output tracking control for discrete-time LPV systems with network-induced delays. However, in these works, it is assumed that scheduling parameters information is available for the controller all the time, which does not occur in practice. In contrast, [24] proposes the co-design of an event generator and a state-feedback controller for discrete-time LPV systems, where the parameters are not precisely known. However, their estimated values satisfy a known uncertainty level. The problem of discretization and event-based digital output control design for continuous-time LPV systems subject to a time-varying networked-induced delay is investigated in [1], where the event detection mechanism is based on a significant change of the scheduling parameters. An event-triggered state-feedback controller design method for discrete-time LPV systems is proposed in [12], where three event-triggering mechanisms are established for transmitting the states, the scheduling parameters, and controller output through the communication network. Other more involving approaches may use sampled-data control handling the process changes between consecutive aperiodic samplings [3], [21], but in these cases, without

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of data transmissions while preserving the desired system performance, as shown in Figure 1. Periodically, they make the decision, based on event-triggering rules, whether the current state and the current scheduling parameter should be sent to the controller via network or not.

i) State-based ETM: We assume that the updating of the current state is performed whenever $\|\hat{x}_{k-1} - x_k\|_{Q_{\Delta}(\hat{\vartheta}_k)}^2 \leq \|x_k\|_{Q_x(\hat{\vartheta}_k)}^2$ is violated, i.e.,

$$\hat{x}_k := \begin{cases} x_k, & \text{if } \|\hat{x}_{k-1} - x_k\|_{Q_{\Delta}(\hat{\vartheta}_k)}^2 > \|x_k\|_{Q_x(\hat{\vartheta}_k)}^2 \\ \hat{x}_{k-1}, & \text{otherwise.} \end{cases} \quad (7)$$

The positive definite parameter-dependent matrices $Q_{\Delta}(\hat{\vartheta}_k) = \sum_{i=1}^N \hat{\vartheta}_{k(i)} Q_{\Delta i}$, $Q_{\Delta i} \in \mathbb{R}^{n \times n}$ and $Q_x(\hat{\vartheta}_k) = \sum_{i=1}^N \hat{\vartheta}_{k(i)} Q_{xi}$, $Q_{xi} \in \mathbb{R}^{n \times n}$ are variables to be designed, leading to an additional degree of freedom in the co-design. As these matrices acts as weights on the terms of the event-triggering condition, their computation has a direct impact on the event-triggering policy and, consequently, on the way of reducing the data transmissions. Note that such an event-triggering rule is more involving than those considered, for instance, in [7], [12], [22], due to the parameter dependence of the weighting matrices.

Note that if x_k is updated at instant k , then from (7), we have that $e_k = \hat{x}_k - x_k = x_k - x_k = \mathbf{0}$, and if x_k is not updated at instant k , then from (7), we have that $e_k = \hat{x}_k - x_k = \hat{x}_{k-1} - x_k$. Thus, the following inequality is always satisfied.

$$\|e_k\|_{Q_{\Delta}(\hat{\vartheta}_k)}^2 \leq \|x_k\|_{Q_x(\hat{\vartheta}_k)}^2 \quad (8)$$

ii) Scheduling-parameter based ETM: Additionally, we assume that whenever the difference between the last transmitted scheduling parameter $\hat{\vartheta}_k$ and the current one, ϑ_k , multiplied by a given scalar $0 \leq g \leq 1$ reaches the lower bound 0 or the upper bound $1 - g$, then the current sample of ϑ_k is transmitted through the network. Therefore, the policy of the scheduling parameter transmission is based on the violation of the condition

$$0 \leq \hat{\vartheta}_{k(i)} - g\vartheta_{k(i)} \leq 1 - g, \quad (9)$$

for any $i \in \mathcal{I}[1, N]$ and a given $0 \leq g \leq 1$. Following [5, Lemma 2], we note that the last transmitted scheduling parameter, $\hat{\vartheta}_k$, can be written as

$$\hat{\vartheta}_{k(i)} = g\vartheta_{k(i)} + (1 - g)\varphi_{k(i)}, \quad (10)$$

for any vector $\varphi_k \in \mathbb{R}^N$ verifying $\sum_{i=1}^N \varphi_{k(i)} = 1$ and $\varphi_{k(i)} \geq 0$. Therefore, once both sides of inequality (9) are satisfied, equation (10) is ensured. Such a fact can be noted by summing (10) up for $i \in \mathcal{I}[1, N]$ and by considering $\vartheta_k, \hat{\vartheta}_k \in \Phi$, to obtain

$$\sum_{i=1}^N \varphi_{k(i)} = \frac{\sum_{i=1}^N \hat{\vartheta}_{k(i)} - g \sum_{i=1}^N \vartheta_{k(i)}}{1 - g} = 1. \quad (11)$$

Moreover, taking into account the positivity of $\hat{\vartheta}_{k(i)} - g\vartheta_{k(i)}$, one has $\varphi_{k(i)} \geq 0$.

Observe that the value of g has a direct influence on the size of the error allowed between the plant and controller parameters, and consequently, on the transmission rate. For

g close to 0, the greater the admissible parameter error, the lower the transmission rate. On the other hand, for g close to 1, the lower the admissible parameter error, the greater the transmission rate. Concerning the transmission rate, the worst case is reached with $g = 1$, which leads to a 100% transmission rate. Therefore, $g = 1$ implies that the controller shares the same scheduling parameter of the plant, then recovering the assumption of parameter sharing in [7], [8], [19], [22].

Note that the proposed ETM policy (9) differs from those usually found in the literature, such as those in [11], [12], [18], [24]. In our approach, the transmission policy depends directly on the scheduling parameters. In the previous works, the decision policy depends on the norm of the dynamic matrices variations, an indirect parameter-based decision, or on some prescribed (small) deviation of the parameter.

By using (9) and doing some manipulations, the closed-loop system (6) can be rewritten as:

$$\begin{aligned} x_{k+1} = & \sum_{i=1}^N \sum_{j=1}^N \nu_{ij} \vartheta_i \vartheta_j 0.5 ((A_i + A_j)x_k - (B_i + B_j)\Psi(u_k) \\ & + g(B_i K_j + B_j K_i)(x_k + e_k)) + \sum_{q=1}^N \sum_{i=1}^N \sum_{j=i}^N \nu_{ij} \vartheta_i \vartheta_j \varphi_q 0.5 \\ & \times (1 - g)(B_i K_q + B_j K_q)(x_k + e_k) \end{aligned} \quad (12)$$

with $\nu_{ij} = 1$ when $i = j$ and $\nu_{ij} = 2$ otherwise. Note that we omitted the dependence of ϑ , $\hat{\vartheta}$ and φ on k to simplify the expression.

Assuming the two ETM schemes (7) and (9), we intend to solve the following problem:

Problem 1: Consider the LPV system under saturating actuators (1). Given a scalar $0 \leq g \leq 1$, co-design the parameter-dependent state-feedback controller (4) and the two independent ETMs (7) and (9) that ensure the regional asymptotic stability of the closed-loop system while reducing data transmissions of states and scheduling variables on the network.

Let us emphasize that the presence of input saturation needs to characterize the region of attraction of the origin for the closed loop [26]. Due to the fact that we do not assume any stability hypothesis for the open loop, we are interested by the regional asymptotic stability of the closed-loop system and therefore the region of attraction will not be the whole state space [20], [13], [7]. As the exact characterization of such a region is extremely challenging, a workaround consists in characterizing an estimate of the domain of attraction of the origin for the closed-loop system (6). This is definitively an implicit objective in solving Problem 1, which we will address by building level sets from the Lyapunov function considered.

III. MAIN RESULTS

In this section, we present a solution to Problem 1. The theoretical conditions appearing in Theorem below are based on the Lyapunov theory and, more specifically, on the use of the following candidate Lyapunov function $V(x_k, \hat{\vartheta}_k) = x_k^T P(\hat{\vartheta}_k) x_k$, where $P(\hat{\vartheta}_k) = \sum_{i=1}^N \hat{\vartheta}_{k(i)} P_i$, $\hat{\vartheta}_k \in \Phi$ with

$\mathbf{0} < P_i = P_i^\top \in \mathbb{R}^{n \times n}$. Their feasibility ensures that the level set \mathcal{L}_V associated with $V(x_k, \hat{\vartheta}_k)$, i.e. $\mathcal{L}_V = \{x_k \in \mathbb{R}^n, \hat{\vartheta}_k \in \Phi : V(x_k, \hat{\vartheta}_k) \leq 1\}$, is a contractive set with respect to the trajectories of the closed-loop system (6), and, therefore, constitutes an estimate of the domain of attraction for the system. Furthermore, we use the well-known generalized sector condition approach [26, Lemma 1.6, p.43] to handle the non-linearity $\Psi(\cdot)$. A direct adaptation to the current setting leads to the following lemma.

Lemma 1: Consider u_k given by (5), $\bar{u} \in \mathbb{R}^m$, and a matrix $G(\hat{\vartheta}_k) = \sum_{i=1}^N \hat{\vartheta}_{k(i)} G_i$ with $G_i \in \mathbb{R}^{m \times n}$, $i \in \mathcal{I}[1, N]$ and $\hat{\vartheta}_k \in \Phi$, and define the following set

$$\mathcal{S}(\bar{u}) \triangleq \{x_k \in \mathbb{R}^n : |G(\hat{\vartheta}_k)x_k| \leq \bar{u}\}. \quad (13)$$

If $x_k \in \mathcal{S}(\bar{u})$, then for any diagonal positive definite matrix $T(\hat{\vartheta}_k) = \sum_{i=1}^N \hat{\vartheta}_{k(i)} T_i$ with $T_i \in \mathbb{R}^{m \times m}$, $i \in \mathcal{I}[1, N]$ and $\hat{\vartheta}_k \in \Phi$, the following inequality is verified.

$$\Psi(u_k)^\top T(\hat{\vartheta}_k) (\Psi(u_k) - (K(\hat{\vartheta}_k) - G(\hat{\vartheta}_k))x_k - K(\hat{\vartheta}_k)e_{x,k}) \leq 0. \quad (14)$$

Theorem 1: Given a scalar $0 \leq g \leq 1$, suppose there exist symmetric positive definite matrices $W_i \in \mathbb{R}^{n \times n}$, $\bar{Q}_{\Delta i} \in \mathbb{R}^{n \times n}$, and $\bar{Q}_{xi} \in \mathbb{R}^{n \times n}$, matrices $Z_i \in \mathbb{R}^{m \times n}$, $Y_i \in \mathbb{R}^{m \times n}$, and $U \in \mathbb{R}^{n \times n}$, and positive definite diagonal matrices $S_i \in \mathbb{R}^{m \times m}$, with $i \in \mathcal{I}[1, N]$, such that the LMIs in (15) (given at the top of the next page) and the following ones

$$\left[\begin{array}{c|c} gW_i + (1-g)W_q - U - U^\top & \star \\ \hline gZ_{i(\ell)} + (1-g)Z_{q(\ell)} & -\bar{u}_{(\ell)}^2 \end{array} \right] < \mathbf{0}, \quad (16)$$

$$q, i \in \mathcal{I}[1, N], \ell \in \mathcal{I}[1, m].$$

are feasible. Then, the saturated LPV system (1) in closed loop with the state-feedback controller (5) computed as

$$K_i = Y_i U^{-1} \quad (17)$$

subject to the ETMs (7) and (9) with matrices $Q_{\Delta i} = U^{-\top} \bar{Q}_{\Delta i} U^{-1}$ and $Q_{xi} = U^{-\top} \bar{Q}_{xi} U^{-1}$, is asymptotically stable and has a reduced number of data transmissions. Moreover, the level set \mathcal{L}_V , computed as¹

$$\mathcal{L}_V = \bigcap_{\forall \hat{\vartheta}_k \in \Phi} \mathcal{E}(P(\hat{\vartheta}_k)) = \bigcap_{\forall i \in \mathcal{I}[1, N]} \mathcal{E}(P_i), \quad (18)$$

with $\mathcal{E}(P, 1) = \{x_k \in \mathbb{R}^n : x_k^\top P_i x_k \leq 1\}$ for $i \in \mathcal{I}[1, N]$, is a region of asymptotic stability for the closed-loop system.

Proof: We split the proof into two steps. In the first one, we show that any trajectory initialized in \mathcal{L}_V belongs to the polyhedral set $\mathcal{S}(\bar{u})$, which guarantees the validity of the generalized sector condition (14), i.e., Lemma 1 applies. In the second, we prove the regional asymptotic stability of the closed-loop system. In other words, the asymptotic stability of the closed-loop system is guaranteed for any initial condition belonging to \mathcal{L}_V .

Step 1: By supposing the feasibility of (16), multiply its left-hand side by $\vartheta_{k(i)}$ and $\varphi_{k(q)}$, sum it up to $i, q \in \mathcal{I}[1, N]$ and replace Z_i by $G_i U$. Then, according to (10), replace

$(1-g) \sum_{q=1}^N \varphi_{k(q)}$ by $\sum_{q=1}^N \hat{\vartheta}_{k(q)} - g \sum_{q=1}^N \vartheta_{k(q)}$. Note that, in this step, the matrices W and G become functions only of the most recent transmitted scheduling parameter $\hat{\vartheta}_k$. So, assume that $W(\hat{\vartheta}_k) = P^{-1}(\hat{\vartheta}_k)$ and use the fact that $[P^{-1}(\hat{\vartheta}_k) - U]^\top P(\hat{\vartheta}_k) [P^{-1}(\hat{\vartheta}_k) - U] \geq \mathbf{0}$ to replace $P^{-1}(\hat{\vartheta}_k) - U - U^\top$ by $-U^\top \sum_{i=1}^N \hat{\vartheta}_{k(i)} P_i U$. Next, with the regularity of U , pre- and post-multiply the resulting inequality by $\text{diag}\{U^{-\top}, 1\}$ and its transpose, respectively, to get

$$\left[\begin{array}{c|c} -P(\hat{\vartheta}_k) & \star \\ \hline G_{(\ell)}(\hat{\vartheta}_k) & -\bar{u}_{(\ell)}^2 \end{array} \right] < \mathbf{0}. \quad (19)$$

Finally, apply Schur complement, pre- and post-multiply the resulting inequality by x_k^\top and x_k , respectively, we have that

$$x_k^\top \frac{G(\hat{\vartheta}_k)^\top_{(\ell)} G(\hat{\vartheta}_k)_{(\ell)}}{\bar{u}_{(\ell)}^2} x_k < x_k^\top P(\hat{\vartheta}_k) x_k$$

, which yields for $x_k \in \mathcal{L}_V$:

$$x_k^\top G(\hat{\vartheta}_k)^\top_{(\ell)} G(\hat{\vartheta}_k)_{(\ell)} x_k < x_k^\top P(\hat{\vartheta}_k) x_k \bar{u}_{(\ell)}^2 \leq \bar{u}_{(\ell)}^2.$$

That allows us to conclude that $|G(\hat{\vartheta}_k)_{(\ell)} x_k|^2 \leq \bar{u}_{(\ell)}^2$ for $x_k \in \mathcal{L}_V$, ensuring the inclusion $\mathcal{L}_V \subseteq \mathcal{S}(\bar{u})$ and, therefore, Lemma 1 applies. Therefore, any trajectory of the closed-loop system starting inside \mathcal{L}_V remains in $\mathcal{S}(\bar{u})$.

Step 2: It remains to demonstrate the regional asymptotic stability of closed-loop system. By supposing the feasibility of (15), multiply its left-hand side by $\vartheta_{k+1(r)}$, $\vartheta_{k(i)}$, $\vartheta_{k(j)}$, $\varphi_{k+1(s)}$ and $\varphi_{k(q)}$, sum it up to $r, s, q, i \in \mathcal{I}[1, N]$ and $j \in \mathcal{I}[i, N]$, and replace K_i by $Y_i U^{-1}$, Z_i by $G_i U$, $\bar{Q}_{\Delta i}$ by $U^\top \bar{Q}_{\Delta i} U$, and \bar{Q}_{xi} by $U^\top \bar{Q}_{xi} U$. Then, make some algebraic manipulations to simplify the expressions, as in [5]. Next, according to (10), replace $(1-g) \sum_{q=1}^N \varphi_{k(q)}$ by $\sum_{q=1}^N \hat{\vartheta}_{k(q)} - g \sum_{q=1}^N \vartheta_{k(q)}$. Note that, in this step, the matrices W , Q_x , Q_Δ , G , S , and K become functions only of the most recent transmitted scheduling parameter $\hat{\vartheta}_k$. So, assume that $W(\hat{\vartheta}_k) = P^{-1}(\hat{\vartheta}_k)$ and use again the fact that $-U^\top P(\hat{\vartheta}_k) U \leq P^{-1}(\hat{\vartheta}_k) - U - U^\top$. Then, with the regularity of U , pre- and post-multiply the resulting inequality by $\text{diag}\{U^{-\top}, U^{-\top}, S^{-1}(\hat{\vartheta}_k), \mathbf{I}_n\}$ and its transpose, respectively. After that, apply Schur complement to obtain $\mathcal{M} < \mathbf{0}$ with \mathcal{M} given by (20) (depicted at the top of the next page). Then, pre- and post-multiply (20) by the augmented vector $\xi_k^\top = [x_k^\top \quad e_{x,k}^\top \quad \Psi(u_k)^\top]^\top$ and ξ_k , respectively. Next, according to (6), replace $(A(\vartheta_k) + B(\vartheta_k)K(\hat{\vartheta}_k))x_k + B(\vartheta_k)K(\hat{\vartheta}_k)e_{x,k} - B(\vartheta_k)\Psi(u_k)$ by x_{k+1} . Also, observe that $x_{k+1}^\top P(\hat{\vartheta}_{k+1})x_{k+1} - x_k^\top P(\hat{\vartheta}_k)x_k = V(x_{k+1}, \hat{\vartheta}_{k+1}) - V(x_k, \hat{\vartheta}_k) = \Delta V(x_k, \hat{\vartheta}_k)$. Finally, denote $S^{-1}(\hat{\vartheta}_k) = T(\hat{\vartheta}_k)$ and take into account (7) and (8), to conclude that $\xi_k^\top \mathcal{M} \xi_k = \Delta V(x_k, \hat{\vartheta}_k) - 2\Psi(u_k)^\top T(\hat{\vartheta}_k) (\Psi(u_k) - (K(\hat{\vartheta}_k) - G(\hat{\vartheta}_k))x_k - K(\hat{\vartheta}_k)e_{x,k}) - e_{x,k}^\top Q_\Delta(\hat{\vartheta}_k)e_{x,k} + x_k^\top Q_x(\hat{\vartheta}_k)x_k < 0$. Therefore, the feasibility of (15) ensures the positivity of $V(x_k, \hat{\vartheta}_k)$ and the negativity of $\Delta V(x_k, \hat{\vartheta}_k)$ for any $x_k \in \mathcal{L}_V$. Then, \mathcal{L}_V is an invariant and contractive set with respect to the closed-loop system, and therefore, constitutes an estimate of the domain of attraction for the system, concluding the proof. ■

¹Details on this computation can be found in Lemma 1 in [6].

$$\begin{bmatrix}
 0.5g(W_i + W_j + \bar{Q}_{xi} + \bar{Q}_{xj}) \\
 +(1-g)(W_q + \bar{Q}_{xq}) - U - U^\top & \star & \star & \star \\
 \mathbf{0} & -0.5g(\bar{Q}_{\Delta i} + \bar{Q}_{\Delta j}) \\
 & -(1-g)\bar{Q}_{\Delta q} & \star & \star \\
 0.5g(Y_i + Y_j - Z_i - Z_j) \\
 +(1-g)(Y_q - Z_q) & 0.5g(Y_i + Y_j) \\
 & +(1-g)Y_q & -g(S_i + S_j) \\
 & & -2(1-g)S_q & \star \\
 0.5(A_i + A_j)U + 0.5g(B_i Y_j + B_j Y_i) \\
 +0.5(1-g)(B_i + B_j)Y_q & 0.5g(B_i Y_j + B_j Y_i) \\
 & +0.5(1-g)(B_i + B_j)Y_q & -0.5g(B_i S_j + B_j S_i) \\
 & & -0.5(1-g)(B_i + B_j)S_q & -gW_r \\
 & & & -(1-g)W_s
 \end{bmatrix} < \mathbf{0}. \quad (15)$$

$r, s, q, i \in \mathcal{I}[1, N], j \in \mathcal{I}[i, N].$

$$\mathcal{M} = \begin{bmatrix}
 -P(\hat{\vartheta}_k) + Q_x(\hat{\vartheta}_k) & \star & \star \\
 \mathbf{0} & -Q_{\Delta}(\hat{\vartheta}_k) & \star \\
 S^{-1}(\hat{\vartheta}_k)(K(\hat{\vartheta}_k) - G(\hat{\vartheta}_k)) & S^{-1}(\hat{\vartheta}_k)K(\hat{\vartheta}_k) & -2S^{-1}(\hat{\vartheta}_k)
 \end{bmatrix} + \begin{bmatrix}
 (A(\vartheta_k) + B(\vartheta_k)K(\hat{\vartheta}_k))^\top \\
 (B(\vartheta_k)K(\hat{\vartheta}_k))^\top \\
 -B(\vartheta_k)^\top
 \end{bmatrix} P(\hat{\vartheta}_{k+1})$$

$$\times \begin{bmatrix}
 (A(\vartheta_k) + B(\vartheta_k)K(\hat{\vartheta}_k)) & B(\vartheta_k)K(\hat{\vartheta}_k) & -B(\vartheta_k)
 \end{bmatrix}. \quad (20)$$

Observe that g parameter may affect the feasibility of the LMIs condition in Theorem 1 and a search on this parameter over the interval $0 \leq g \leq 1$ can be used to improve the ETM performance. In general, the feasibility of conditions (15) and (16) is easier to verify for $g = 1$, since this case corresponds to consider a full transmission rate and no parameter error. Examples presented later illustrate such a possibility.

A. Optimization procedure

This section aims to propose an optimization procedure that indirectly reduces the number of state signal updates. By looking at the event-triggering condition (7), we can see that it is a relative measure of the deviation between the last sampled state and the current state with $Q_{\Delta i}$ and Q_{xi} acting as weights on this measure. Thus, we have that the ‘‘smaller’’ $Q_{\Delta i}$ and the ‘‘larger’’ Q_{xi} are, the more the current state is allowed to deviate from the last sampled one and the fewer transmissions events are expected. However, the matrices $Q_{\Delta i}$ and Q_{xi} do not appear explicitly as decision variables in the conditions of Theorem 1, due to the congruence transformation required to formulate them in terms of LMIs. To overcome such an issue, we propose to consider the following constraints with the aim to get lower bound on Q_{xi} and upper-bound on $Q_{\Delta i}$ to recover representative optimization scheme:

$$\begin{bmatrix}
 g\bar{Q}_{xi} + (1-g)\bar{Q}_{xq} & \star \\
 U & g\hat{Q}_{xi} + (1-g)\hat{Q}_{xq}
 \end{bmatrix} > \mathbf{0}, \quad (21)$$

$$\begin{bmatrix}
 g\hat{Q}_{\Delta i} + (1-g)\hat{Q}_{xq} & \star \\
 \mathbf{I}_n & U + U^\top - g\bar{Q}_{\Delta i} \\
 & -(1-g)\bar{Q}_{\Delta q}
 \end{bmatrix} > \mathbf{0}. \quad (22)$$

$i, q \in \mathcal{I}[1, N]; j \in \mathcal{I}[i, N].$

The first one enforces the lower bound $Q_{xi} > \hat{Q}_{xi}^{-1}$ through the direct application of the Schur’s complement. The second enforces the upper bound $Q_{\Delta i} < \hat{Q}_{\Delta i}$ through the direct application of Lemma 1 in [23]. In this case, the fact of minimizing the variables $(\hat{Q}_{\Delta i}, \hat{Q}_{xi})$ effectively minimizes the variables $(Q_{\Delta i}, Q_{xi})$. Therefore, we consider the following optimization procedure:

$$\mathcal{O} : \begin{cases} \min & \sum_{i=1}^N \text{tr}(\hat{Q}_{\Delta i} + \hat{Q}_{xi}), \\ \text{subject to} & (15), (16), (21), \text{ and } (22). \end{cases} \quad (23)$$

Let us recall that the scalar g plays a relevant role on the transmission activity of the scheduling parameter, making its choice very important. As discussed earlier, the closer g is to zero, the smaller the transmission activity. Such a fact will be highlighted in Section IV.

IV. SIMULATIONS RESULTS

This Section illustrates the effectiveness of our results through numerical examples borrowed from the literature [14], [18], [19].

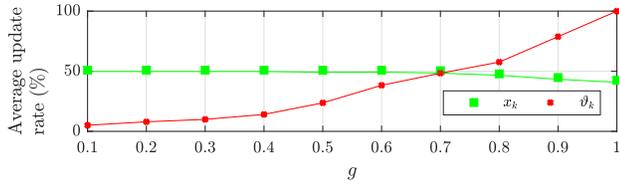


Fig. 2. Average update rate of the scheduling parameter and the state.

Example 1. Consider saturated LPV system (1) with matrices

$$A = \begin{bmatrix} 0.25 & 1 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.6 + \rho_k \end{bmatrix}, B = \begin{bmatrix} 1 - 0.8\rho_k \\ 0 \\ 1 - \rho_k \end{bmatrix}, \quad (24)$$

where the time-varying parameter $\rho_k \in \mathcal{I}[0, 0.5]$ and the symmetric saturation limit is $\bar{u} = 0.5$. In this case, we can take $\vartheta_{k(1)} = 1 - 2\rho_k$ and $\vartheta_{k(2)} = 2\rho_k$ with $A_1 = A(0)$, $B_1 = B(0)$, $A_2 = A(0.5)$ and $B_2 = B(0.5)$.

First, we are interested in investigating the influence of parameter g (see (9)) in the update rate of the scheduling parameter and the state. With this purpose, for $g = 0.1, 0.2, \dots, 1$, we use the optimization procedure \mathcal{O} given in (23) to design the control gains and the event-triggering parameters. For each case, we simulate the closed-loop response for 1000 initial conditions belonging to \mathcal{L}_V and ϑ_k chosen randomly. The average update rate (%) of the scheduling parameter, marked with \times , and the state, marked with \blacksquare , as a function of g are illustrated in Figure 2. We can see that as g approaches to 1, the update rate increases until it reaches 100% for $g = 1$, i.e. $\hat{\vartheta}_k = \vartheta_k$. In such a case, we also have the smallest update rate of the states, which is expected as the controller is better adjusted to the conditions of the plant.

For $g = 0.8$, we plot for a specific initial condition $x_0 = [-0.035 \ -0.025 \ -0.589]^\top$, the closed-loop temporal response and the inter-event interval of the event generators, as seen in figures 3 and 4, respectively. In the first one, we can observe that the states converge to origin despite the saturation of the actuator in the first two instants of simulation. In the second figure, we have (on the bottom) the inter-event interval of the scheduling parameter, marked with \times , and the state, marked with \blacksquare . Note that both mechanisms are independent, although one can affect the other indirectly, as discussed previously. Moreover, we have (on the top) the difference between $\hat{\vartheta}_k$ and ϑ_k , which is marked with \bullet (\times) when its value is inside (outside) the region of validity imposed by (9), traced with black lines. With g close to 1, the smaller the error allowed between $\hat{\vartheta}_k$ and ϑ_k , and therefore, the higher the update rate. The control gain and the event-triggering parameters used for this simulation were $K = [K_1 \ | \ K_2] = [-0.054 \ -0.084 \ -0.148 \ | \ -0.088 \ -0.222 \ -1.046]$ and

$$Q_\Delta = [Q_{\Delta 1} \ | \ Q_{\Delta 2}] = \begin{bmatrix} 0.033 & 0.053 & 0.106 & | & 0.013 & 0.032 & 0.149 \\ 0.053 & 0.089 & 0.212 & | & 0.032 & 0.080 & 0.377 \\ 0.106 & 0.212 & 0.727 & | & 0.149 & 0.374 & 1.752 \end{bmatrix},$$

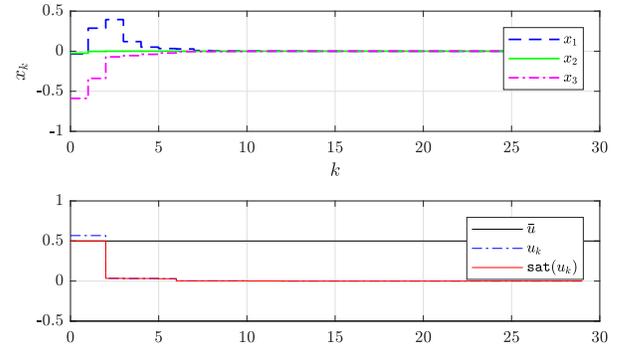


Fig. 3. The closed-loop temporal response for $x_0 = [-0.035 \ -0.025 \ -0.589]^\top$.

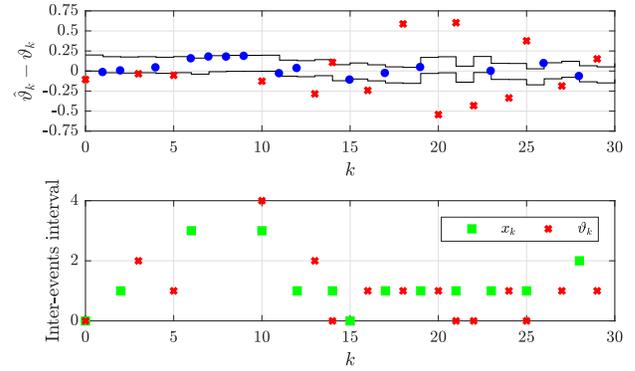


Fig. 4. Inter-event interval of the event generators.

$$Q_x = [Q_{x1} \ | \ Q_{x2}] = \begin{bmatrix} 0.847 & -0.034 & 0.040 & | & 0.836 & -0.057 & 0.086 \\ -0.034 & 2.113 & 0.036 & | & -0.057 & 2.048 & 0.105 \\ 0.040 & 0.036 & 1.269 & | & 0.086 & 0.105 & 0.670 \end{bmatrix}.$$

To complete the illustration of the results, let us make some consideration on the estimate of the region of attraction \mathcal{L}_V . In Figure 3, we plot the estimate of the region of attraction \mathcal{L}_V (light gray lines) with some state trajectories starting from the border of \mathcal{L}_V (colored lines). As we can see, the state trajectories converge to the origin. For initial conditions outside of \mathcal{L}_V , our method does not ensure convergence. The initial condition, $x_0 = [-0.1750 \ -0.1250 \ -2.9450]^\top$, for instance, results in a divergent trajectory.

Example 2. By assuming that the LPV system (24) is not subject to any restriction on the input signal, we compare our approach with [11], [12]. The authors in [11] consider the co-design of an event-triggering condition and a state-feedback controller for LPV systems. The proposed ETM sends information on states and scheduling variables simultaneously to the controller when needed. On the other hand, the authors in [12] propose an event-triggered reference tracking control design method for the LPV system, in which are designed a state-feedback controller and three separate ETMs for transmitting the state, the scheduling variables, and the controller output over the network.

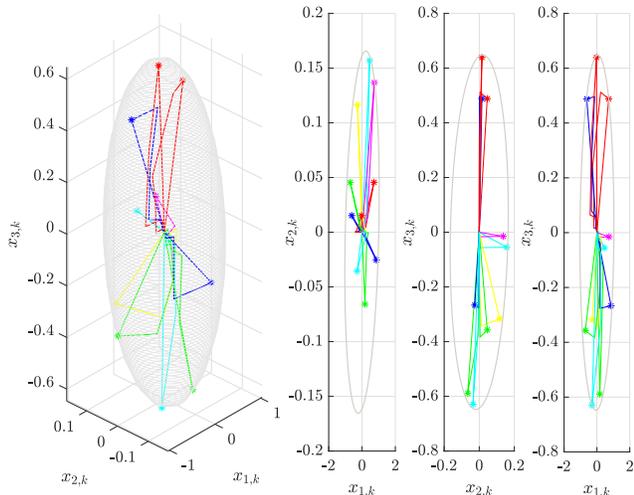


Fig. 5. The estimate of the region of attraction of the origin for Example 1.

By solving Theorem 1 in [11], we got the control gain $K = \begin{bmatrix} -0.095 & -0.373 & -0.368 & | & -0.179 & -0.728 & -1.205 \end{bmatrix}$ and the triggering parameter $\sigma = \sigma_x^{-1} = 0.048$. Also, by disregarding the reference tracking setup, we solve Theorem 2 in [12] with the scheduling parameter error threshold $\delta_0 = 0.05$ obtaining the control gain $K = \begin{bmatrix} -0.032 & -0.111 & -0.322 & | & -0.030 & -0.1388 & -1.071 \end{bmatrix}$ and the triggering parameters $\sigma_\Psi = 0.0226$ and $\sigma_u = 0.0250$. We use the data of both approaches to simulate the system's closed-loop response for $x_0 = [1 \ -1 \ -0.5]^\top$ and 1000 values of ϑ_k generated randomly.

We performed the same procedure using our approach for some values of $0 \leq g \leq 1$. Table I shows the respective results. One can see that the average update rates of the states and the scheduling parameter obtained by [11] and [12] are higher than ours in almost all cases. Unlike this approach, [12] considers a third ETM between controller and actuator. However, such a mechanism had a high transmission rate, almost matching our case where all control signals are sent to the plant, reason why the last line in Table I is marked with “-” for Theorem 1 and [11]. For $g = 0.2$, for instance, our transmission activity are about 88% and 90% smaller than of [11] and [12] w.r.t. the scheduling parameter, respectively, and about 26% and 51% smaller than [11] and [12] with respect to the state, respectively.

TABLE I

PERCENTAGE OF TRANSMISSION RATE ACHIEVED BY THE PROPOSED METHOD AND THOSE FROM [11], [12].

	[12]	[11]	Theorem 1				
			g				
			0.2	0.4	0.6	0.8	1.0
$\vartheta_k(\%)$	78.33	66.66	8.02	14.42	38.22	57.56	100
$x_k(\%)$	100	66.66	49.05	49.05	55.41	47.50	42.68
$u_k(\%)$	98.4	-	-	-	-	-	-

Example 3. By assuming that the LPV system (24) is not subject to any restriction on the input signal, and

has a fixed matrix of control i.e. $B_1 = B_2 = B(0)$, we compare our approach with [24]. The authors in [24] propose the co-design of an ETM and a state-feedback controller for LPV systems where the parameters are not exactly known, but estimated parameters satisfying certain level are known. The robustness of the proposed event-triggered control system concerning the uncertainty of the parameter is indicated by the scalar δ_0 for the method in [24], according to the inequality $\|\Delta A(\vartheta_k, \hat{\vartheta}_k)x_k\|^2 \leq \delta_0^2 \|x_k\|^2$, with $\Delta A(\vartheta_k, \hat{\vartheta}_k) = A(\vartheta_k) - A(\hat{\vartheta}_k)$. Thus, we solve Theorem 1 in [24], with $\delta_0 = 0.1$ and the triggering parameter $\eta = 0.99$, obtaining the control gain $K = \begin{bmatrix} -0.095 & -0.354 & -0.414 & | & -0.091 & -0.335 & -0.768 \end{bmatrix}$. By using the ETM and the controller designed, we simulated the closed-loop response for $x_0 = [1 \ -1 \ -0.5]^\top$ and 1000 values of $\Delta A(\vartheta_k, \hat{\vartheta}_k)$, generated randomly within the allowed range, obtaining a percentage average update rate of x_k equal to 84.11%, as indicated in Table II. Note that, from the values of $\Delta A(\vartheta_k, \hat{\vartheta}_k)$ and $\hat{\vartheta}_k$, we can determine ϑ_k . We used these values of ϑ_k to establish a fair comparison with [24].

To compare our approach with that proposed by [24], we measured, for some values of $0 \leq g \leq 1$, the robustness of our event-triggered control system and computed the average update rate of both x_k and ϑ_k . The robustness of our method was determined by computing the following scalar δ : $\delta = \max_k \left\{ \sqrt{\|\Delta A(\vartheta_k, \hat{\vartheta}_k)x_k\|^2 / \|x_k\|^2} \right\}$ among the instants of simulation. The results are shown in Table II. As we can see, for values of g smaller than 0.8, our approach is more robust to the variation of parameters than [24], almost 4 times for $g = 0.2$. In addition, we found an average update rate of x_k smaller than [24] in all cases. For $g = 0.8$, for instance, we obtained a rate 36% smaller, approximately. As [24] does not consider an ETM to send the information about the scheduling parameter over the network, these data are not comparable.

TABLE II

RELATIVE ROBUSTNESS (δ/δ_0) ACHIEVED BY THE PROPOSED METHOD AND THAT FROM [24] AND THE PERCENTAGE OF TRANSMISSION RATE.

	[24]	Theorem 1				
		g				
		0.2	0.4	0.6	0.8	1.0
δ/δ_0	1	3.86	2.87	1.95	0.96	0
$\vartheta_k(\%)$	-	4.80	10.90	44.40	74.75	100
$x_k(\%)$	84.11	55.63	55.61	55.59	53.86	40.21

V. CONCLUSION

This paper examines the state-feedback event-triggered control design problem for a discrete-time LPV system subject to saturating actuators. We proposed two separate ETMs for transmitting the states and the scheduling variables through a communication network. In this sense, it is assumed that the scheduling parameters of the controller and the event-triggering mechanism may differ from those of the plant. The convex conditions based on linear matrix inequalities (LMIs) ensure the regional asymptotic stability of the closed-loop

system for every initial condition belonging to the estimated attraction region. An optimization procedure is derived, allowing the minimization of the data transmission over the network. Future work could be dedicated to expanding the results to polynomial scheduling parameters, which can lead to lower update rates for the scheduling parameters and states and larger estimates of the region of attraction. Furthermore, it could be interesting to address the case where the states are not fully available for building the ETM, adopting, for example, an observer-based strategy or the use of only local information.

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