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# Co-simulation domain decomposition algorithm for hybrid EMT-Dynamic Phasor modeling

Hélène Shourick · Damien Tromeur-Dervout · Laurent Chédot

**Abstract** An iterative coupling algorithm based on a restricted additive Schwarz domain decomposition is investigated to co-simulate electrical circuits with hybrid electromagnetic (EMT) and transient stability (TS) modeled using dynamic phasors. This co-simulation algorithm does not introduce any delay between the data exchanged at the co-simulation step. The pure linear convergence property of the iterative method allows it to be accelerated towards the true solution by a non-intrusive Aitken's acceleration of the convergence post-processing, even if the domain decomposition interface conditions make the iterative method divergent. This provides a method less sensitive to the splitting. This algorithm can then be implemented in a distributed master-slaves architecture. An example on a linear RLC circuit combining EMT and TS modeling, and a partitioning with overlap is given.

## 1 Introduction

The introduction of renewable energies into the power grid leads to the use of more components based on power electronics. These components imply faster dynamics. Power system safety simulations, which cannot be handled by traditional Transient Simulations (TS) conducted with dynamic

phasors (DP), require Electro-Magnetic Transient (EMT) simulations. Nevertheless, the advantage of TS programs is their computational speed which makes them suitable for handling large-scale networks, however, their modeling is not sufficiently detailed and can only catch slow dynamics. On the other hand, EMT simulators can capture fast dynamics, but are limited in computational speed; therefore, they are used to simulate only small portions of the network. For large power grids, it can be expected that the need for high-level detail requiring Electro-Magnetic Transient (EMT) modeling will be localized near disturbances, and other parts of the network will use TS modeling.

Based on this assumption, the co-simulation approach is an attractive candidate to handle these hybrid power system simulations. Nevertheless, the EMT-TS co-simulation has to face several locks as already underlined in [6] among which we can mention:

- the data exchange between TS and EMT simulators, including choice of the interface variables (i.e. the partitioning of the network), the data conversion between waveform and dynamic phasors or phasors;
- the interaction protocol between TS and EMT simulators, including the time step size difference between EMT and TS, the coupling algorithm (i.e. iterative or non iterative, the signal rebuilding Zero order hold or high order hold), the convergence of the resulting coupling algorithm.

Advances in these areas have been proposed in recent years. Among these is the partitioning based on traveling wave patterns of the transmission line which introduce natural decoupling into the nodal equations of an EMT simulator due to transmission line latency. Note that this latency limits the flexibility of choosing DP time steps [9]. Le-Huy & al [7] developed a simple hybrid line model that accounts for wave propagation in both electromagnetic and TS simulations i.e both ends exchange historical current and the delay

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is considered as a phase shift for the TS part. Another co-simulation that splits the transmission line is the one of Rupasinghe & al [12] in which a Base-Frequency Phasor Adaptive Simulation Transient solver is developed, which allows to derive frequency-dependant equivalent of network components using base-frequency dynamic phasors at the discrete level. Then the offset frequency parameter can switch from EMT to DP at the fundamental frequency. Plumier & al [10] has proposed a co-simulation algorithm coupling EMT and phasor which dynamically updates by an iterative procedure the equivalent impedance of Norton and Thevenin equivalent models representing the boundary conditions between each subsystem. Some acceleration of the coupling algorithm is achieved by a prediction scheme which is similar to a Richardson extrapolation of temporal quantities.

Shu & al [14] proposed a two-level Schur complement through which the Thevenin equivalent obtained for each EMT subsystem could fully consider the coupling among different EMT subsystems and the TS central system. Abhyankar and Flueck [1] proposed an implicitly coupled TSEMT algorithm where the set of TS and EMT equations are solved by a Newton's method at each TS time step. An important issue in this approach is the computation of the instantaneous Thevenin equivalent voltage for EMT time steps that are not on the temporal boundary. Their experiment has shown that it is preferable to use the TS solution at the end TS time step rather than a linear interpolation between the two TS interval steps, probably due to the Newton's algorithm. Rimorov & al [11] focused on the problems of co-simulation stability and precision in the presence of delays and proposed a generalized interface framework related to the search for a power-conjugate interface that combines current and voltage through an "impedance" parameter resulting from non-physical related boundary conditions.

In co-simulation algorithms such as non-iterative Jacobi, zero-order hold iterative co-simulation and non-iterative algorithm improving variables smoothing, the delay of one co-simulation step (i.e. TS time step delay) between the given inputs and the retrieved outputs of the TS and EMT systems can lead to instabilities. Some iterative techniques such as the fixed point method [4] or the Newton-like method [10, 1] can, even with a high order smoothing constraints, solve the so-called "constraint function" corresponding to the interface of the systems [5].

In this paper, we consider a fixed point coupling algorithm based on the Schwarz domain decomposition technique which can be related to the dynamic iteration method (DI) (i.e waveform relaxation of [8]) in which we used a restricted additive Schwarz (RAS) splitting [13]. These DI methods can be convergent or divergent depending on the domain partitioning and boundary conditions. Nevertheless, we used the good property of purely linear convergence or divergence (i.e. the error operator of the method does not

depend on the iteration number) to accelerate the iterative method towards the true solution with the Aitken's acceleration technique even with a divergent method [15]. The advantages of our approach are:

- the fundamental concept of the method is to post-process the sequence of interfaces solutions generated by the domain decomposition solver. It can use different boundary conditions for the acceleration as long as they are linear in the variables (i.e Dirichlet, Neumann, Robin, those of [11],...);
- as it is a post-process the method can be used non-intrusively in the local solver;
- it is not necessary for domain partitioning to be cut on the transmission lines;
- it can also support an overlap of TS and EMT parts. We then have some components with the two representations.

The outline of the paper is as follows: Section 2 describes the EMT and TS (modeled with dynamic phasors), as well as the translation techniques needed to exchange data from one model to the other. The co-simulation domain decomposition algorithm and its Aitken convergence acceleration technique are given in Sect. 3. Section 4 gives the concept that we are developing in the co-simulation platform based on co-simulation algorithm. Section 5 presents results obtained on a linear RLC circuit before concluding in Sect. 6.

## 2 EMT and Dynamic Phasor modelling

**EMT modeling** uses the Modified Augmented Nodal Analysis [16] to build the system of differential algebraic equations (DAE) (1) relating the current and voltage unknowns of the electrical network.

$$F(t, x(t), \dot{x}(t), y(t)) = 0, \text{ with Initial Conditions.} \quad (1)$$

where  $x$  (respectively  $y$ ) are the differential (respectively algebraic) unknowns.

The linearized backward differential formula (BDF) time discretization of (1) (here Backward Euler) leads to solving the linear system (2) in order to integrate the state space representation of the DAE from time step  $t^n$  to time step  $t^{n+1} = t^n + \Delta t_{emt}$  (the operator  $\mathbb{I}$  represents the difference between two potentials or the identity for intensity variables,  $G$  represents the voltage/current sources) :

$$\underbrace{\begin{pmatrix} \mathbb{I} - \Delta t_{emt} A & B \\ C & D \end{pmatrix}}_{\mathbf{H}_{\Delta t_{emt}}} \begin{pmatrix} x^{n+1} \\ y^{n+1} \end{pmatrix} = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x^n \\ y^n \end{pmatrix} + G^{n+1}. \quad (2)$$

We use Dynamic Phasors for **TS modeling**. Consider a  $T_0$ -periodic signal with its pulse  $\omega_0 = \frac{2\pi}{T_0}$ , by applying the

Fourier transform, the signal can be written on the interval  $\tau \in [t - T_0, t]$  as  $x(\tau) = \sum_{h=-\infty}^{+\infty} X_h e^{jh\omega_0\tau}$ , where the Fourier coefficients are:  $X_h = \frac{1}{T_0} \int_{t-T_0}^t x(\tau) e^{-jh\omega_0\tau} d\tau$ .

Now consider a waveform that is not strictly periodic. (in an almost periodic state) so that the Fourier coefficients would vary over time:

$$X_h(t) = \frac{1}{T_0} \int_{t-T_0}^t x(\tau) e^{-jh\omega_0\tau} d\tau = \langle x \rangle_h. \quad (3)$$

This time-varying coefficient is conventionally called the h-dynamic phasor. Only relevant harmonics (a subset  $I = \{\dots, -1, 0, 1, \dots\}$ ) are computed and the signal is given by

$$x(t) = \sum_{h \in I} X_h(t) e^{jh\omega_0 t}. \quad (4)$$

The introduction of (4) in (1) leads, by using the orthogonality of the functions  $e^{jh\omega_0 t}$ , to a DAE system with smoother dynamics allowing the use of a bigger time step  $\Delta t_{TS}$ . The number of TS variables is then multiplied by the chosen number of harmonics. The structure of the matrix  $H_{TS}$  of the BDF scheme, by choosing two harmonics  $h = a$  and  $h = c$  and by solving the imaginary and real part separately and with  $\mathbf{S}$  the matrix holding account of the differential property of dynamic phasor modeling is written (with  $H_{\Delta t_{TS}}$  defined by (2)):

$$\mathbf{H}_{TS} = \begin{pmatrix} \left( \begin{array}{c|c} \mathbf{H}_{\Delta t_{TS}} & -a \omega_0 \mathbf{S} \\ \hline a \omega_0 \mathbf{S} & \mathbf{H}_{\Delta t_{TS}} \end{array} \right) & 0 \\ \hline 0 & \left( \begin{array}{c|c} \mathbf{H}_{\Delta t_{TS}} & -c \omega_0 \mathbf{S} \\ \hline c \omega_0 \mathbf{S} & \mathbf{H}_{\Delta t_{TS}} \end{array} \right) \end{pmatrix}.$$

Suppose  $\Delta t_{TS} = m \Delta t_{emt}$  with ( $m \in \mathbb{N}^*$ ). The TS and EMT parts of the network have to exchange interface solutions at each TS time step. These values must be transformed to be understandable from one part to another.

**Translation from EMT to Dynamic Phasor:** to translate the EMT values to dynamic phasor ones, a Fast Fourier Transform is applied on a history of the duration of  $T_0$  containing the values taken by the part computed by the EMT at the intermediate m EMT time steps. The history must be the size of a period  $T_0$ . We can therefore take a  $\Delta t_{TS}$  the size of  $T_0$  and fill the History with the m intermediate time steps, but it is better to be able to take a smaller TS time step. Let's take a  $\Delta t_{TS}$  shorter than a period, to perform the FFT a history of the size of  $T_0$  is still necessary, a "sliding" history will be performed by deleting the m oldest values from the beginning of the history and adding the m new values to the end of the history. This operation is repeated at each moment of passage of information, i.e. after each time step TS (and therefore after m EMT time steps). The observation window will no longer be  $[t - T_0, t]$  but  $[t - T_0 + \alpha \Delta t_{TS}, t + \alpha \Delta t_{TS}]$

with  $\alpha \in [0, T_0/\Delta t_{TS}]$ , by applying the suitable change of variable in (2), an offset of  $e^{jh\omega_0 \alpha \Delta t_{TS}}$  appears, it will then be compensated by applying a shift of  $e^{-jh\omega_0 \alpha \Delta t_{TS}}$  on the FFT. This translation from EMT to DP is defined at the discrete level by the linear operator  $E_{TS}^{emt}$ .

**Translation from Dynamic Phasor to EMT:** to rebuild an EMT signal from dynamic phasor coefficients, it suffices to apply (4) with the chosen harmonics at each EMT time step. This translation from DP to EMT is defined at the discrete level by the linear operator  $E_{emt}^{TS}$ .

### 3 Co-simulation domain decomposition algorithm

The BDF scheme leads at each time step to solve a linear system  $Hx = b$ . We use the Restrictive Additive Schwarz domain decomposition as follows:

#### 3.1 Restrictive Additive Schwarz (RAS)

By adapting the notations of [3], we consider a non-singular matrix  $H \in \mathbb{R}^{n \times n}$  having a non-zero pattern and the associated directed graph  $G = (W, F)$ , where the set of vertices  $\Omega = \{1, \dots, n\}$  represents the  $n$  unknowns and the set of edges  $F = \{(i, j) | a_{i,j} \neq 0\}$  represents the pairs of vertices that are coupled by a non-zero element in  $H$ . Next, we assume that a graph partitioning was applied and resulted in  $N$  non-overlapping subsets  $\Omega_i^0$  whose union is  $\Omega$ . Let  $\Omega_i^p$  be the  $p$ -overlap partition of  $\Omega$ , obtained by including all the vertices immediately neighboring the vertices of  $\Omega_i^{p-1}$ . Let  $R_i^p \in \mathbb{R}^{n_i \times n}$  be the operator which restricts  $x \in \mathbb{R}^n$  to the components of  $x$  belonging to  $\Omega_i^p$ . Let  $\tilde{R}_i^0 \in \mathbb{R}^{n_i \times n}$  be the operator which restricts  $x \in \mathbb{R}^n$  to the components of  $x$  belonging to  $\Omega_i^0$  and 0 otherwise. Let  $\Omega_{i,e}^p = \Omega_i^{p+1} \setminus \Omega_i^p$  and  $R_{i,e}^p \in \mathbb{R}^{n_{i,e} \times n}$  the restriction operator which restricts  $x \in \mathbb{R}^n$  to the components of  $x$  belonging to  $\Omega_{i,e}^p$ . By defining  $H_i = R_i^p H R_i^{pT}$ ,  $F_i = R_i^p H (R_{i,e}^p)^T$ ,  $x_i = R_i^p x$  and  $b_i = R_i^p b$ ,  $x_{i,e} = R_{i,e}^p x$ , then the Restrictive Additive Schwarz (RAS) iteration  $k + 1$  to solve  $Hx^\infty = b \in \mathbb{R}^n$  is written locally for the  $\Omega_i^p$  partition:

$$x_i^{k+1} = H_i^{-1} (b_i - F_i x_{i,e}^k). \quad (5)$$

By defining  $M_{RAS}^{-1} \stackrel{def}{=} \sum_{i=0}^{N-1} \tilde{R}_i^{0T} H_i^{-1} R_i^p$  and adding the contribution of each partition  $\Omega_i^p$ , RAS can be viewed as a Richardson's process (using  $R_i^p A = R_i^p A (R_i^{pT} R_i^p + R_{i,e}^{pT} R_{i,e}^p)$ ):

$$\sum_{i=0}^{N-1} \tilde{R}_i^{0T} R_i^p x^{k+1} = \sum_{i=0}^{N-1} \tilde{R}_i^{0T} H_i^{-1} R_i^p b - \sum_{i=0}^{N-1} \tilde{R}_i^{0T} H_i^{-1} R_i^p H R_{i,e}^{pT} x_{i,e}^k \quad (6)$$

$$x^{k+1} = x^k + M_{RAS}^{-1} (b - Hx^k). \quad (7)$$

It can be reduced to a problem with the unknowns on the interface  $\Gamma = \{\Omega_{0,e}^p, \dots, \Omega_{N-1,e}^p\}$  of size  $n_\Gamma = \sum_{i=0}^{N-1} n_{i,e}$ , by

defining  $R_\Gamma = (R_{0,e}^p, \dots, R_{N-1,e}^p)^T \in \mathbb{R}^{n_\Gamma \times n}$  and by using the property  $R_{i,e}^{pT} R_{i,e}^p R_\Gamma^T R_\Gamma = R_{i,e}^{pT} R_{i,e}^p$ :

$$\underbrace{R_\Gamma x^{k+1}}_{z^{k+1}} = \underbrace{R_\Gamma (I - M_{RAS}^{-1} A)}_P \underbrace{R_\Gamma^T R_\Gamma x^k}_{z^k} + \underbrace{R_\Gamma M_{RAS}^{-1} b}_c. \quad (8)$$

The pure linear convergence of the RAS at the interface given by  $z^k - z^\infty = P(z^{k-1} - z^\infty)$  (the error operator  $P$  does not depend on the iteration  $k$ ) allows to apply the Aitken's convergence acceleration technique, in order to obtain the true solution  $z^\infty$  on the interface  $\Gamma$ :  $z^\infty = (I - P)^{-1}(z^k - Pz^{k-1})$ , and so after another local solve, the true solution  $x^\infty$ . Let us note that one can accelerate the convergence towards the solution for a convergent or a divergent iterative method. The only requirement is that 1 is not one of the eigenvalues of  $P$ . Considering  $e^k = z^k - z^{k-1}$ ,  $k = 1, \dots$ , the operator  $P \in \mathbb{R}^{n_\Gamma \times n_\Gamma}$  can be computed algebraically after  $n_\Gamma + 1$  iterations as  $P = [e^{n_\Gamma+1}, \dots, e^2][e^{n_\Gamma}, \dots, e^1]^{-1}$ .

### 3.2 RAS for EMT-TS

Let us now adapt the RAS method to the hybrid TS-EMT modeling of a linear electrical network. The  $x_{i,e}^k$  term of Eq. (5) for TS (respectively EMT) part comes from the EMT (respectively TS) part. Let  $x_{TS}^{N+1}$  and  $y_{TS}^{N+1}$  (respectively  $x_{emt}^{n+1}$  and  $y_{emt}^{n+1}$ ) be the differential and algebraic unknowns of TS (respectively EMT). The time discretisation of the DAE for TS to integrate from  $T^N$  to  $T^{N+1}$ , assuming that  $\Delta t_{TS} = m\Delta t_{emt}$  can be written as:

$$\underbrace{\begin{pmatrix} \mathbb{I} - \Delta t_{TS} A_{TS} & B_{TS} \\ C_{TS} & D_{TS} \end{pmatrix}}_{H_{TS}} \underbrace{\begin{pmatrix} x_{TS}^{N+1} \\ y_{TS}^{N+1} \end{pmatrix}}_{w_{TS}^{N+1}} = \underbrace{\begin{pmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{pmatrix}}_{\Theta_{TS}} \underbrace{\begin{pmatrix} x_{TS}^N \\ y_{TS}^N \end{pmatrix}}_{w_{TS}^N} + \underbrace{\begin{pmatrix} E_{TS}^A & E_{TS}^B \\ E_{TS}^C & E_{TS}^D \end{pmatrix}}_{E_{TS}^{emt}} \underbrace{\begin{pmatrix} x_{emt}^m \\ y_{emt}^m \end{pmatrix}}_{w_{emt}^m} + G_{TS}^{N+1}. \quad (9)$$

Similarly one time step for the EMT side to integrate from  $t^n$  to  $t^{n+1} = t^n + \Delta t_{emt}$  is written:

$$\underbrace{\begin{pmatrix} \mathbb{I} - \Delta t_{emt} A_{emt} & B_{emt} \\ C_{emt} & D_{emt} \end{pmatrix}}_{H_{emt}} \underbrace{\begin{pmatrix} x_{emt}^{n+1} \\ y_{emt}^{n+1} \end{pmatrix}}_{w^{n+1}} = \underbrace{\begin{pmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{pmatrix}}_{\Theta_{emt}} \underbrace{\begin{pmatrix} x_{emt}^n \\ y_{emt}^n \end{pmatrix}}_{w^n} + \underbrace{\begin{pmatrix} E_{emt}^A & E_{emt}^B \\ E_{emt}^C & E_{emt}^D \end{pmatrix}}_{E_{emt}^{TS}} \underbrace{\begin{pmatrix} x_{TS}^{N+1}(t^{n+1}) \\ y_{TS}^{N+1}(t^{n+1}) \end{pmatrix}}_{W^{N+1}(t^{n+1})} + G_{emt}^{n+1}. \quad (10)$$

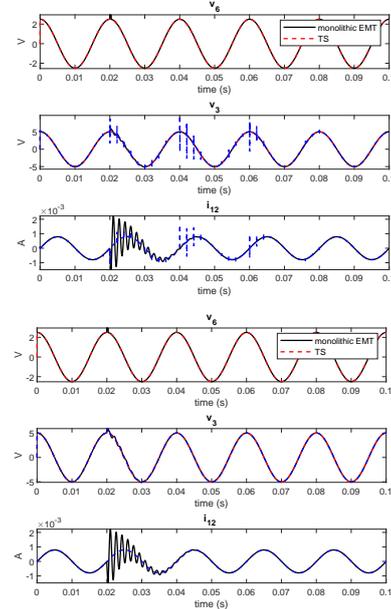
The  $m$  time steps can be gathered in one larger system considering  $t^n = T^N$ :

$$\underbrace{\begin{pmatrix} I \\ -\Theta_{emt} H_{emt} \\ \vdots \\ -\Theta_{emt} H_{emt} \end{pmatrix}}_{\mathbb{H}_{emt}} \underbrace{\begin{pmatrix} w_{emt}^n \\ w_{emt}^{n+1} \\ \vdots \\ w_{emt}^{n+m-1} \\ w_{emt}^{n+m} \end{pmatrix}}_{W_{emt}} = \underbrace{\begin{pmatrix} I \\ E_{emt}^{TS} \\ \vdots \\ E_{emt}^{TS} \end{pmatrix}}_{\mathbb{E}_{emt}^{TS}} \underbrace{\begin{pmatrix} (x^n, y^n)^T \\ W^{N+1}(t^{n+1}) \\ \vdots \\ W^{N+1}(t^{n+m-1}) \\ W^{N+1}(t^{n+m}) \end{pmatrix}}_{W_{TS}^{N+1}} + \underbrace{\begin{pmatrix} 0 \\ G_{emt}^{n+1} \\ \vdots \\ G_{emt}^{n+m-1} \\ G_{emt}^{n+m} \end{pmatrix}}_{G_{emt}^{N+1}}. \quad (11)$$

These systems need the values of the TS (respectively EMT) solution at the instant  $T^{N+1} = T^N + m\Delta t_{emt}$  to be connected to the EMT (respectively TS) part. We need the RAS iterative algorithm to obtain the exact values at time  $T^{N+1}$ . Iteration  $p + 1$  is then iterated by taking the connected values, at the iteration  $p$ , of the other subdomain. We can use the multiplicative form or the additive form as follows:

$$\begin{cases} H_{TS} w_{TS}^{N+1, k+1} = \Theta_{TS} w_{TS}^N + E_{TS}^{emt} w_{emt}^{m, k} + G_{TS}^{N+1}, \\ \mathbb{H}_{emt} w_{emt}^{N+1, k+1} = \mathbb{E}_{emt}^{TS} W_{TS}^{N+1, k} + G_{emt}^{N+1}. \end{cases} \quad (12)$$

Note that  $W_{TS}^{N+1}$  in Eq. (11) may present a jump between



**Fig. 1** TS-EMT hybrid simulation of the test of section 5 with a jump in the voltage source at  $t = 0.2s$  that last less than  $\Delta t_{TS}$ : without (top) and with (bottom) linear interpolation of  $W_{TS}^{N+1}$

$(x^n, y^n)^T$  and  $W^{N+1}(t^{n+1})$  which may produce some spurious oscillations in the solution (see Fig. 1(top)  $v_3$ ,  $i_{12}$ ). That is why we use a linear interpolation on  $q$  time points between

$(x^n, y^n)^t$  and  $W^{N+q}(t^{n+1})$  (contrary to the experiment of [1], see Fig. 1(bottom)  $v_3, i_{12}$ )).

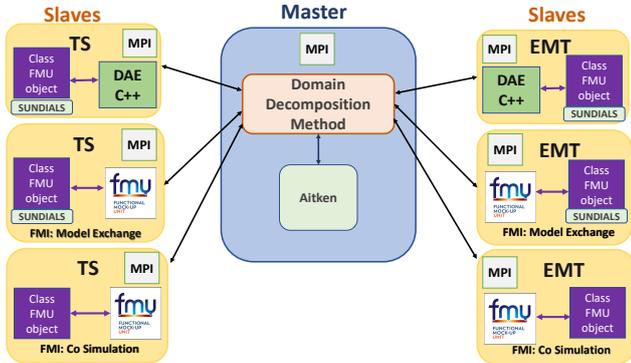
#### 4 Co-simulation platform

The platform under development at SuperGrid-Institute is based on the co-simulation algorithm. It is designed in a master-slaves approach (see Fig. 2) where the co-simulation algorithm is the master in charge of doing the orchestration to call the slaves EMT and TS models. Slaves and Master are launched as Message Passing Interface (MPI) processes.

The slaves can be either Functional Mock-up Units [2] (in order to be able to develop models with dedicated tools such as Modelica electrical libraries ) or C++ code implementing the DAE function.

To obtain more parallelism, the local resolutions are carried out on the slaves side via an instance of the FMUobject architecture. Each slave contains an instance of a FMUobject C++ class that embeds a local DAE solver such as IDA of Sundials and is also in charge of instantiating the FMU when it applies.

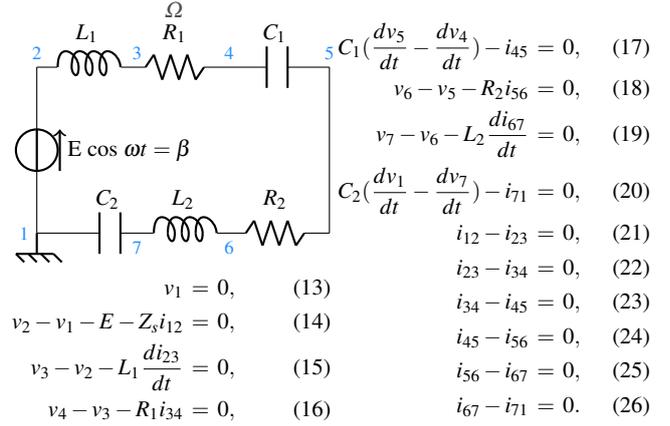
The master manages the co-simulation algorithm by sending commands, using the MPI point to point communication routines, to FMUobject which then manages the FMU with FMI commands. Then the master receives the interface values from the slave and applies the Aitken's acceleration before sending the true solution interface values back to the slaves.



**Fig. 2** Co-simulation platform architecture with MPI Master-Slaves communications and using local solver in the FMI standard or C++ DAE functions.

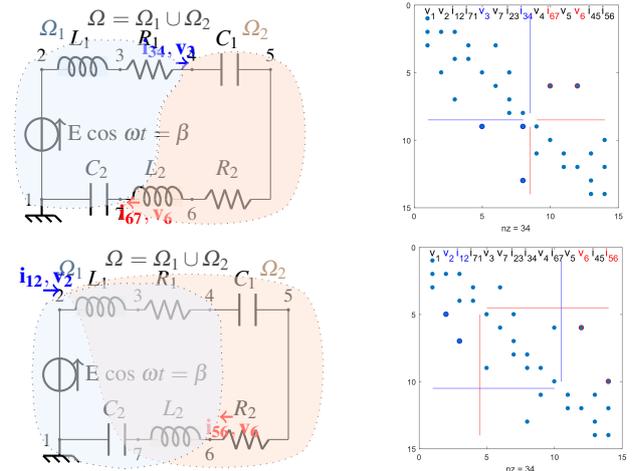
#### 5 Results

In order to illustrate our co-simulation algorithm, we consider the RLC circuit of Fig 3. The small linear system associated with the RLC circuit is partitioned into two subdomains using graph partitioning without overlap (Figure 4 top) and with an overlap of 1 (Figure 4 bottom). Each subdomain needs two values from the other to solve its equa-



**Fig. 3** Linear RLC circuit and its associated EMT modeling DAE system with  $L_1 = L_2 = 0.7$ ,  $C_1 = C_2 = 1.10^{-6}$ ,  $R_1 = R_2 = 77$ ,  $Z_s = 1.10^{-6}$ ,  $\omega = 2\pi 50$ ,  $E = 5$ .

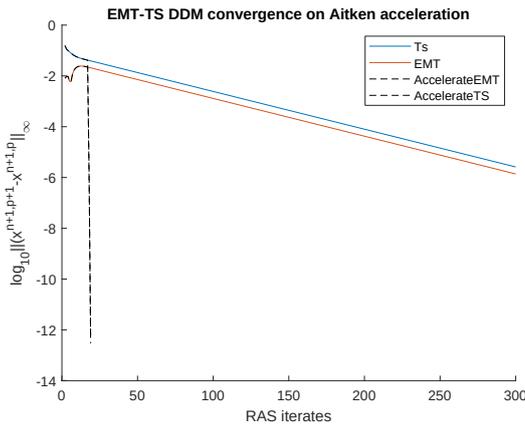
tions (i.e  $i_{34}$  and  $v_3$  for  $\Omega_2$ ,  $i_{67}$  and  $v_6$  for  $\Omega_1$  in the non-overlapping subdomains case).



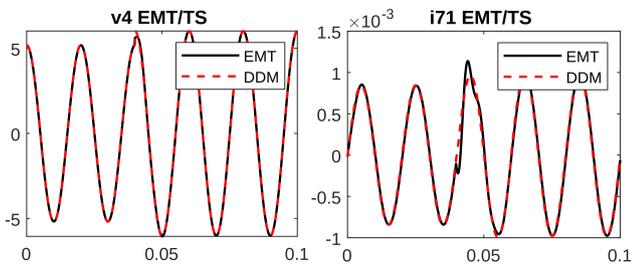
**Fig. 4** Graph partitioning of the RLC circuit in two subdomains and the associated matrix partitioning without overlap (top) and with overlap of 1 (bottom).

Figure 5 gives the  $\log_{10}$  of the error between two consecutive RAS iterations at  $t = 0.02$ . It shows a linear convergence behavior and can therefore be accelerated by the Aitken's convergence acceleration technique after the 9 iterations needed to numerically construct the error operator  $P$  of Eq. 8.

Figure 6 shows the solutions  $v_4$  EMT (left) and  $i_{71}$  TS (right) of heterogeneous DDM EMT ( $\Delta t = 2.10^{-4}$ )-TS ( $\Delta T = 2.10^{-2}$ ) with comparison with the DAE solution on monodomain. We proceed to an amplitude jump at  $t = 0.04$  for the voltage source.



**Fig. 5** Heterogeneous EMT ( $\Delta t = 2.10^{-4}$ )-TS( $\Delta T = 2.10^{-2}$ ) RAS convergence error for each subdomain at  $t = 0.02$  and its Aitken's acceleration with  $P$  computed numerically from 9 iterations ( $n_r = 8$ ).



**Fig. 6** Heterogeneous EMT ( $\Delta t = 2.10^{-4}$ )-TS( $\Delta T = 2.10^{-2}$ ) DDM results comparison with DAE monodomain.

## 6 Conclusion

An iterative co-simulation algorithm based on a restricted additive Schwarz heterogeneous DDM was used to co-simulate an RLC electrical circuit where part of the domain is modeled with EMT modeling and the other part with TS modeling. It integrates a non-intrusive aitken technique to accelerate convergence, possible thanks to the pure linear convergence or divergence property of the RAS of the heterogeneous DDM TS-EMT, with or without overlap. This co-simulation algorithm is the orchestrator of a Master-Slaves co-simulation platform under development.

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