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Review of “Stress partitioning and effective behavior of N-phase laminates in anisotropic elasticity from a fast explicit method”

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Review of version 1

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Reviewer 1 (Anonymous)

Reviewer The manuscript proposes an algorithm for computing the effective thermoelastic properties of N-phase laminated solids with a large number of phases. The phases can exhibit an arbitrary elastic anisotropy as well as permanent plastic deformations. The idea is to group the N phases in pairs, apply the exact formula for two-phase laminates for each pair to obtain a new laminate with half the number of phases, and repeat the process iteratively until one phase with the effective properties is obtained. The algorithm also provides the full stress field within the laminate. The usefulness of the algorithm is illustrated by applying it to near-beta titanium alloys with elongated grains.

The topic may be of interest to the readership of the JTCAM; I therefore recommend publication.

Author First of all, I would like to thank the reviewer for the careful examination of the paper, the good appreciation of the work and the fruitful comments to improve the manuscript. Please see below detailed answers to the different issues raised. The amendments made appear in blue in the revised version of the manuscript.

Reviewer The author may find it interesting to mention the following two points:

The discussion of Section 5 states that, unlike the laminated model, self-consistent models cannot account for the direct interaction between two grains. True. But the laminated model is sensitive to the stacking order of the laminae, and hence requires much more information. How available is that information in applications? How sensitive to stacking order are the results of Section 5 and the conclusions drawn from them? May be the author could add a comment on this in the discussion.

Author As written in Section 3, the laminate under consideration is actually not sensitive to the stacking order of the phases. In the procedure consisting in grouping the phases in pair iteratively until one phase with the effective properties remains, the initial relative positions of the different phases have obviously no influence on the results obtained at the final step, i.e. they have no influence on the effective properties of the laminate. This was already noted by Glüge and Kalisch in 2014, 10.1016/j.compstruct.2014.01.027. Then, Equation (26) shows that the stresses in one

phase can be expressed only with respect to the effective properties of the laminate and the properties of the phase. Hence, the stresses in the phases are also not sensitive to the stacking order of the phases.

This issue is now much better highlighted by adding two sentences at the end of Section 3.1 (Effective behavior), rephrasing the comments just below Equation (26) in Section 3.2 (Stress partitioning) and adding a new sentence in the Conclusions.

Reviewer It may be possible to extend the algorithm to nonlinear viscoplastic behavior making use of compact expressions for two-phase viscoplastic laminates already available in the literature (e.g., Idiart, J. Mech. Phys. Solids 56 (2008) 10.1016/j.jmps.2008.03.004). Maybe the author could comment on this possibility as well.

Author Thank you for this very interesting remark. I agree. Working on effective strain energy potentials, it should indeed be possible to extend the present algorithm to obtain the effective strain energy potential of a N-phase viscoplastic laminate, i.e. grouping the N phases in pairs and applying the existing compact expressions for the effective strain energy potential of a two-phase laminate for each pair iteratively until one phase with the effective strain energy potential is obtained. The strains in each phase could then be deduced from a top-down algorithm.

A comment regarding this issue has been added at the end of the Conclusions section.

Reviewer 2 (Reinaldo RODRÍGUEZ-RAMOS)

Reviewer In the present work, an explicit method is proposed in order to compute the effective behavior and the distribution of stresses in a general N-phase laminate made of parallel, planar and perfectly bonded interfaces. The solutions are exact for a homogeneous far-field loading and work for an arbitrary number of phases, a general linear anisotropic elasticity, as well as different uniform thermal and plastic strains in the phases. A simple direct analytical formula is also derived to compute the stress in a given phase once the effective behavior of the laminate is known. Moreover, the correctness of the proposed method is checked by comparisons with finite element simulation results on a same boundary value problem, showing excellent agreements. Finally, an application of the method is performed for a near- $\beta\beta$ titanium alloy with elongated grains, by comparing the level of internal stresses for different elastic loadings within a N-phase laminate made of 100 000 orientations and a 2-phase laminate of equal volume fraction with maximal elastic contrast.

The approach makes a contribution solving micromechanical problems in practical engineering problems. This work is important in the field of numerical and experimental mechanics.

The present contribution has novelty aspects. Therefore, this work requires minor revision before to be published in Journal of Theoretical, Computational and Applied Mechanics (JTCAM).

Author First of all, I would like to thank the reviewer for the careful examination of the paper, the good appreciation of the work and the fruitful comments to improve the manuscript. Please see below detailed answers to the different issues raised. The amendments made appear in blue in the revised version of the manuscript.

Reviewer The reviewer would like to make the following remarks:

Brief information about the novelty of this work should be suitable.

Author At the difference of previous approaches where explicit formulas were restricted to a finite number of phases or to isotropic elasticity or to purely elastic behavior, this contribution provides explicit compact formulas for an arbitrary number of phases, a completely general anisotropic elasticity as well as different thermal and plastic strains in the phases. In particular, a simple direct analytical formula is derived in order to compute the stress in a given phase once the effective behavior of the laminate has been determined. Furthermore, the code to compute the effective behavior and the stress distribution in an arbitrary N-phase laminate is made freely available for download at <https://github.com/AniPlas/Laminate>. In addition, the developed method proves that both the effective properties of the laminate and the stresses in the phases are insensitive to the stacking order of the phases. Finally, it is shown that a convergence towards the laminate solution is obtained by increasing the aspect ratio of oblate spheroidal grains in self-consistent models.

Reviewer Validation of the present model with other theoretical approach is recommended. In particular, for polycrystals with elongated grains.

Author Following the reviewer’s recommendation, a new section is now dedicated to comparisons with elastic self-consistent models that consider oblate spheroidal grains of different aspect ratios (see Section 6 of the new manuscript). A convergence towards the laminate solution is observed by increasing the aspect ratio of the oblate spheroids (see new Figure 6). The new Figures 7 and 8 show also that very small relative difference ($< 10^{-5}$) of the norm of the effective stiffness tensor are obtained between the two models when oblate spheroids with $a = b = 100c$ are considered in the self-consistent models.

Reviewer The author explains advantageous of the present model in comparison with other models. It is convenient to show some numerical facts related to this point.

Author In the new Section 6, Figure 7 shows the relative difference of the norm of the effective stiffness tensor of 100 000 uniform orientations computed by the laminate model and an elastic self-consistent model considering oblate spheroids with $a = b = 100c$ as a function of the absolute tolerance set in the quadv function in Matlab to evaluate the Eshelby tensor. It is known that the more the grains are elongated, the more accuracy is needed on the numerical evaluation of the Eshelby tensor. For the present extreme case with $a = b = 100c$, Figure 7 shows that an absolute tolerance lower than 10^{-12} should be set in order to obtain a convergence towards an accurate result. At the same time, Figure 7 exhibits also the average ratio between the CPU time needed by the elastic self-consistent to achieve the calculation over the one needed by the laminate model. This ratio is about 7 showing, as expected, that the laminate is much faster. Moreover, this ratio increases exponentially for absolute tolerance smaller than 10^{-13} . This means that, when using a self-consistent model, a preliminary numerical study is needed to fix the value of the absolute tolerance (or the number of Gauss points if another numerical method is used) in order to both ensure a sufficient accuracy on the evaluation of the Eshelby tensor and also to maintain a reasonable CPU time.

Reviewer What is the disadvantageous of this method?

Author The disadvantage of this method lies in the relevance of using a laminate configuration to describe the material under study. This is illustrated in the new Figure 6 where it is observed that the maximal von Mises stresses are always overestimated compared to the values provided by self-consistent models with more realistic grain aspect ratios.

Reviewer The author stated: “...Another difference is that the methodology of Section 3 is as accurate for a very small or a very large number of phases whereas self-consistent models only work well for a very big number of phases...” The author could show numerical evidence regarding this point.

Author This sentence has been removed in the new manuscript as it was not really exact for self-consistent models. The new Figure 8 shows indeed that the relative difference of the norm of the effective stiffness tensor between the two models decreases with the number of phases considered. However, it must be underlined that the relative difference is already pretty small (i.e. $< 4 \times 10^{-5}$) when considering only two phases. As shown by the new Figure 6, a convergence towards the laminate solution is observed by increasing the aspect ratio of the oblate spheroids in the self-consistent models when considering both a very large number of phases (100 000) or only two phases.

Editor’s assessment (Laurence BRASSART)

The paper proposes an original explicit method to calculate the distribution of stresses and overall response of laminates with an arbitrary number of phases, building on standard solutions for two-phase laminates. General elastic anisotropy of the phases is considered, as well as the presence of permanent strains due to plasticity or temperature change. The paper makes a useful contribution to the field of micromechanics and fits well within the scope of JTCAM. The fact that the code is made available is particularly beneficial. Both reviewers requested only minor revisions. The authors comprehensively addressed the reviewers’ comments (including the

addition of a new section presenting a comparison of the proposed method to the predictions of the self-consistent model). Both reviewers recommended the second version of the paper to be accepted as is.

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