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Experimental and Numerical Study for High Energy Impact Absorption with a Composite Material in Aeronautics

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Abstract. Aeronautics search high performance materials for structural weight reduction and impact energy absorption. Composite offers perspectives related to mass and stiffness ratio. However, the dispersion of their mechanical properties due to environmental conditions or impact behaviour hinders their development. Currently, honeycomb solutions and complex energy absorption mechanisms are used. The sizing of structures is controlled for conventional loads, but not for severe cases such avian impact. In the context of the development of an innovative reinforced aeronautical structure, the objective of this study is to identify and characterize a new concept of absorbent composite material at the coupon scale. In order to improve reliability and optimize its absorption characteristics during high-energy impact, works are realized to develop in parallel an experimental study methodology and a finite element model. The purpose of the latter is to have a predictive tool validated by correlation with experimental to ensure virtual testing. Lack of knowledge of the nonlinear behaviour of the composite material at high deformation, for different speeds, as well as the mechanisms of damage and fracture are locks to its development. Absence of a dedicated experiment matrix presents a difficulty for its characterization and sizing. The approach begins with the analysis and prioritization of the existing to determine the definition criteria of the new concept. After development of an experimental characterization study, the qualification and justification of the new concept is validated by correlation with a dedicated numerical simulation methodology. The results of the study highlight the analysis and development of an interesting concept.

Keywords: Aeronautics · Composite · Material · Finite element
Experimental · Transient dynamic · Nonlinear analysis · Reinforced structure
Avian impact

1 Introduction

The aeronautical industry is constantly looking for materials that offer the ability to limit inspection thresholds, lighten the structure or absorb a high level of energy during an impact such as bird strikes. In this context, composite materials offer interesting prospects, particularly for mass reduction objectives. However, the variability of their mechanical properties, due to environmental constraints, or their tolerance behavior to impact damage is a brake on their development. Aircraft and bird collision statistics are extensively studied [1–3]; they show the dangerousness, recurrence and cost of bird strikes to aviation. Various simulation methods have been developed for the representation of a bird [4–10]. Avian impacts, or impacts due to hail [11], tire burst, etc., on aeronautical structures [12–14], such as a bulkhead or a gearbox, require the development of technological solutions to absorb the energy resulting from these impacts or from extreme loads such as an emergency landing. These absorption systems guarantee a structural integrity threshold. Currently composite solutions consisting of a metallic alveolar reinforcement called honey-comb [15, 16], as well as sophisticated energy absorption mechanisms are used.

The objectives of the present study are related to this problematic. The work carried out is then a part of the development of a new generation of bulk-head structures with a new absorbent material. The aim is to develop a methodology for the experimental characterization of innovative concepts, combined with the development of a digital simulation model. The methodology developed must allow the implementation of a virtual qualification or “virtual testing”. Thus in this paper we will identify and characterize a reliable high-energy impact energy absorption system that constitutes the bulkhead.

Consequently, the performances to be achieved are listed as follow. First, master the representation of the various parameters governing coupled physical phenomena occurring during a high energy impact. Secondly, control the transient dynamic behavior of an absorbent material. Third, master the performance of the new absorbent concept which must be superior or equal to current systems. And finally, develop a predictive numerical simulation of the new concept by correlation with experimental studies.

In this kind of study, there is usually uncertainties, lack of knowledge and technical lock. In the context of the presented work, the search for a new innovative material allowing to absorb mechanical energy comes up against the lack of knowledge of its strongly non-linear behavior, due to impact deformation, geometrical characteristics of the system and contact management.

Furthermore, if the design of structures is controlled for load cases induced by the classic life cycle of an aircraft, it is otherwise for extreme load cases such as impact during aircraft navigation and in particular for bird strikes. To predict the behavior of the material, it is isolated independently of the mechanical structure to focus on its behavior under conventional mechanical loadings. Under these conditions the material is reduced to a specimen which in the industrial language is called a coupon. To design a new coupon-scale absorption system, the difficulty is to define a numerical macro-model representative of damage and fracture due to the absence of a dedicated experimental matrix. It is the removal of these various uncertainties that is the subject of our study.

The present paper is organized as follow: in Sect. 2 we present the procedure developed at the coupon scale; particularly, the state of the art and also the experimental study. Thus we conclude this section by presenting the experimental test plan. Section 3 is devoted to the numerical simulation methodology by finite element analysis. In Sect. 4 we present virtual test derived from numerical simulation. Finally, a conclusion and some perspectives are given in the last section.

2 Program of the Study

This section presents the procedure developed at the coupon scale in this study. It adopts the following steps:

- A state of the art;
- The experimental study;
- The finite element numerical simulation study;
- The correlation or “Virtual Testing”.

2.1 State of the Art

An aircraft nose is a framework on which a skin is fixed. This framework is stiffened by an assembly of longitudinal and circumference stiffeners. The nose of the aircraft is closed by a bulkhead, CF. Fig. 1. In the event of an avian impact, a system of energy absorption called a shield is placed on the bulkhead. For the shield, a classic honeycomb type solution is mainly used. The Fig. 2 presents for the reference structure the coupling of the phenomena of damage mechanics and fracture mechanics describing the avian impact.

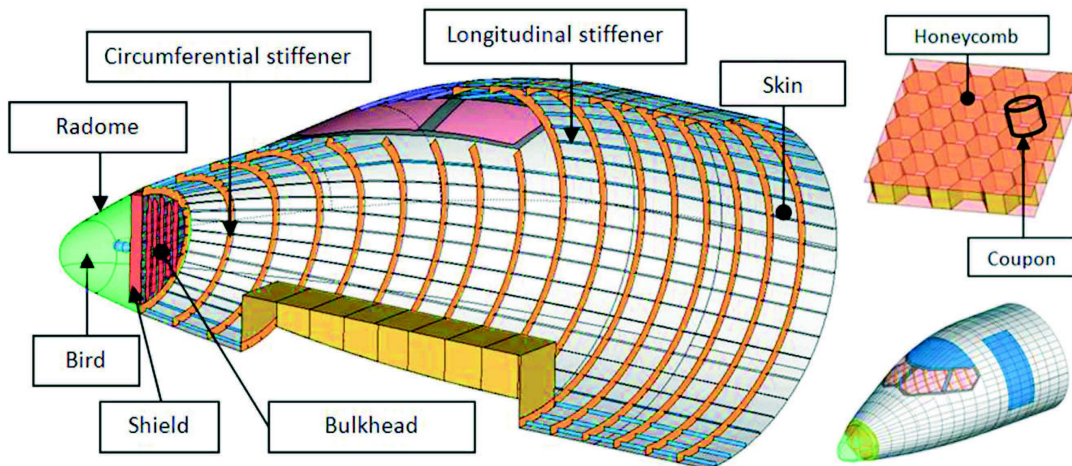


Fig. 1. Reference structure of an aircraft front nose concept.

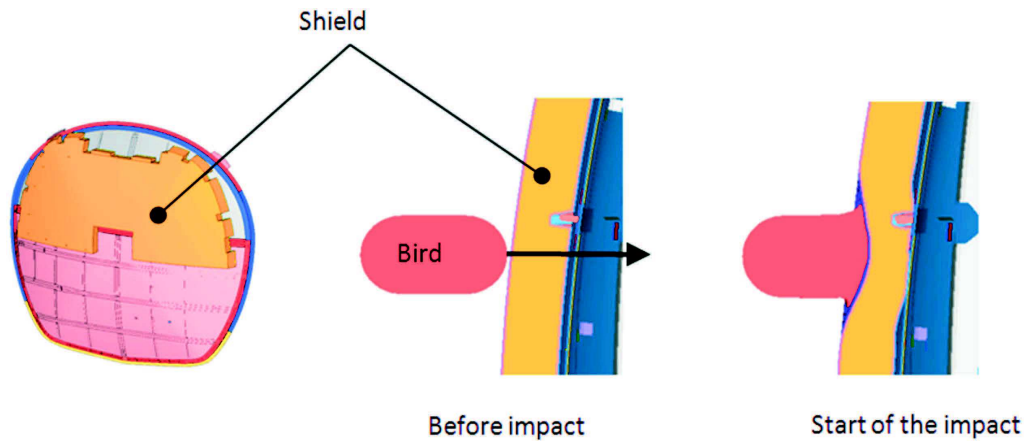


Fig. 2. Numerical finite element model of the reference structure.

To define the performance to be delivered by the material, the absorbed energy is evaluated; the study carries out an energy balance of the numerical model of the reference solution presented in Table 1.

Table 1. Distribution of impact energy by main mechanical elements.

Absorbed energy	Percentage of the total energy
Honeycomb absorber	46.6%
Bulkhead	33.3%
Bird	20.1%

The energy to be absorbed by the shield is associated with a maximum force on the structure, as well as maximum absorption distance, in order to preserve the bulkhead. These parameters are related to compression stiffness and material thickness. The absorbent material in front of the bulkhead absorbs part of the bird's kinetic energy and reduces the forces introduced on the structure by the bird.

If one focuses on the material characteristics, the dynamic behavioral curve of the compression material (see Fig. 3) during impact is represented below by characteristic parameters. The dynamic compression effort F_c is associated with the displacement under compression L_c .

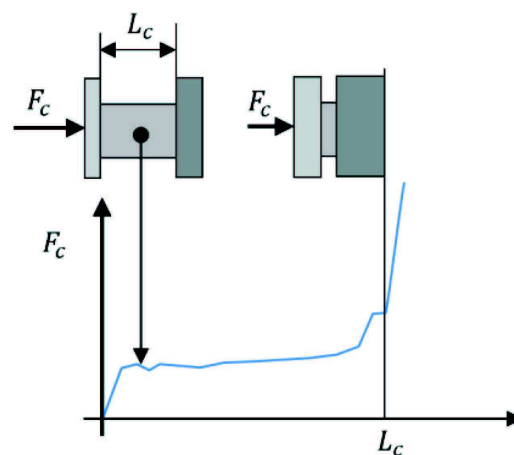


Fig. 3. Compressive curve force versus displacement of the honeycomb material.

After several analysis step, previous forces and displacements are converted (see Fig. 4) into a dynamic stress noted σ_d , as well as a dynamic deformation ε_d . The σ_{pl} plateau stress is identified and corresponds to the compressive strength of the material.

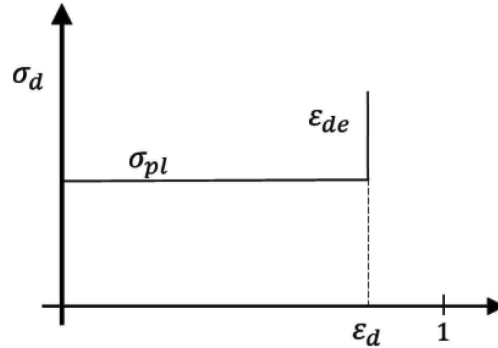


Fig. 4. Behavioural plateau curve stress versus strain up to densification.

During impact, the concept volume is reduced by eliminating porosity induced by the cells, ε_{de} corresponds to the densification deformation value.

It is necessary to identify the energy absorbed by the material up to densification noted \sum_{ad} . In purpose, the compression behavioral curve of the material can be simplified by a plateau curve (see Fig. 4).

2.2 Experimental Study

This part develops the study of parameters of physical phenomena. The dimensional and mechanical characteristics of the absorbent material define its performance. The material must absorb a high percentage of the bird's kinetic energy.

It is essential that the material deform during impact without reaching the area of densification in order to preserve the structure.

Indeed, when the material reaches the deformation value at densification, the force introduced on the structure becomes high. It is therefore necessary to choose the thickness of the material and to know the percentage of elongation at densification.

The energy absorbed by the material is deduced from the dynamic compressive force of the material and the absorption distance. This force depends on the compressive stiffness of the material associated with the contact surface with the bird during impact (Fig. 5, Table 2).

At this step of the study a new absorbent material is defined. The brainstorming of concepts and materials led to the definition of a thick composite with orthotropic mechanical characteristics.

The works join in the optimization of data such as nonlinear deformation behavior, strain velocity sensitivity, damage and fracture mechanisms.

The criticality of the avian shock imposes different deformation scenarios. During impact, the material is loaded in the three spatial directions in a coupled way in tension, compression, and shear.

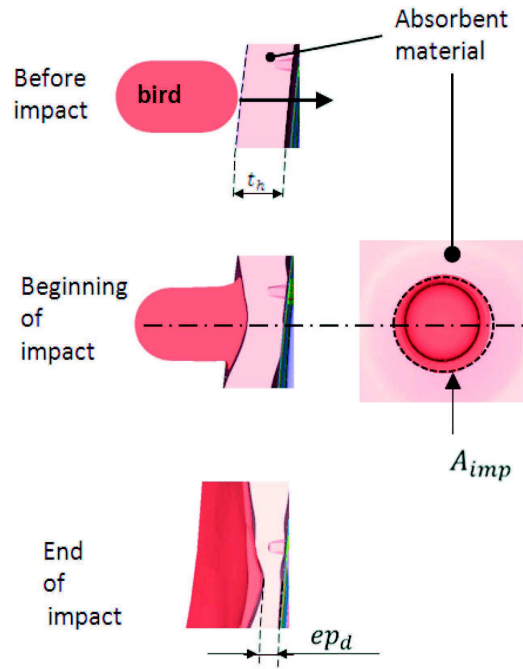


Fig. 5. Geometrical parameters.

Table 2. Definition of the parameters.

Parameters	Definition
t_h	Material thickness
e_{pd}	Strain densification
x_d	Displacement to densification
\sum_{imp}	Material absorbed energy
F	Bird force
σ_d	Dynamic compressive stress
A_{imp}	Contact area between bird and material
d	Average diameter contact area between bird and material

Also, during impact, the deformation velocities of the materials are variable and range from almost static to high speeds corresponding to those of the aircraft in flight.

It is therefore necessary to define the behavior of the material subjected to a high level of deformation and its sensitivity at different speeds. The behavioral law has a configuration ranging from quasi-static to an impact speed of more than 150 m/s. The different means of testing eligible for research work are summarized in Table 3.

After analyzing the existing means of characterization, the selection is oriented towards two industrial tools:

- A gas gun for dynamic characterization in compression.
- A static test machine for the characterization under compression, tensile and shear loads.

Table 3. Main means of industrial testing

Test machine	Speed limitation
a) Gas gun	> 1200 m/s
b) Taylor impact test	> 200 m/s
c) Hopkinson bar test	70 to 220 s^{-1}
d) Dynamic traction jack	up to 30 m/s
e) Drop weight impact machine	up to 15 m/s
f) Quasi static test machine: compression, tension and shear	10^{-4} to $10^{-1}s^{-1}$

The experimental characterization strategy leads to the extraction of a network of stress versus displacement curves for different configurations of coupons, loads, and material directions. Each of the curves is analyzed and processed to have a stress versus strain representation for each load configuration. The characterization of the material is completed by the extraction of mechanical characteristics corresponding to the elastic limit, damage, fracture mechanisms and densification value. Particular attention is given to the analysis of the dispersion of results. An experimental plan has been defined for this purpose.

2.3 Experimental Test Plan

During this experiment, it is necessary to carry out a quasi-static and dynamic test campaign at different speeds to determine the mechanical behavior of the new material in its three spatial directions. These tests are carried out on coupons taken from the raw material. These coupons are geometrically controlled to measure, verify, and record the dimensions of each one. Ideally, five coupons per test type and direction should be tested to verify repeatability of the process.

All these measurements are summarized in a database leading to the extraction of the following characteristics:

- Young's modulus, compression and shearing modulus;
- Compression densification limit;
- Limit at fracture;
- Behavioral stress versus strain curve of the material, for almost static and dynamic speed with different values up to a maximal value issued of the air-worthiness requirement certification;
- Compressive plateau stress.

Figure 6 presents the quasi-static compression test curve for the direction 1 of the material and for coupon number 2.

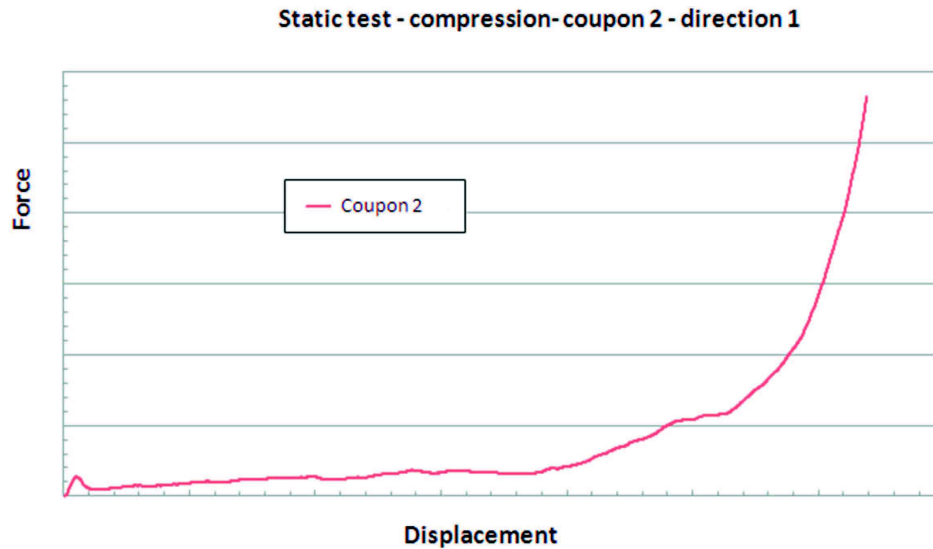


Fig. 6. Static test in compression for coupon number 2 with force versus displacement curve for material direction number 1.

3 Numerical Simulation Methodology by Finite Element Analysis

3.1 Different Aspect of Avian Impact

Avian shock [1–4] is a dynamic phenomenon [17–19] in an extremely short time of the range of one millisecond. The forces on the structure and its response evolve over time; it is a transient phenomenon of extreme complexity.

The displacements, rotations and deformations of the structure are important; the hypothesis of small deformation is no longer valid. It is therefore necessary to consider geometric non-linearity.

Similarly, a linear relationship between strain and stress is no longer a valid assumption. Materials have a nonlinear behavior that covers many aspects such as plasticity, sensitivity to strain rate or temperature, viscoelasticity. But also, it is to involve very particular damage and fracture mechanisms, especially for composite.

Finally, contact [20–23] is a non-linearity intervening during impact, the central idea of which remains the condition of impenetrability.

The work carried out develops methodologies adapted to the numerical simulation by finite element of the physical phenomenon and the resolution of each type of coupled non-linearity.

3.2 Mathematical and Numerical Model

The strong formulation, or differential equation of local equilibrium, allows the resolution of problems of extreme simplicity. In the majority of cases, it is preferable to use a weak equivalent formulation called a variational formulation, known as the principle of virtual work.

$$\delta P^{int} - \delta P^{ext} + \delta P^{inert} = 0 \quad (1)$$

In Eq. (1) δP^{int} is the total internal power, δP^{ext} is the virtual external power and δP^{inert} is the virtual inertial power.

The finite element method is widely used [24, 25] from the weak formulation. The classic discretization process consists first in making a finite element spatial approximation by selecting appropriate basic functions. The elementary matrices of mass and stiffness are then obtained and finally the equation of motion in matrix form is written.

$$M \frac{d^2 u}{dt^2} + Ku = f^{ext} \quad (2)$$

In the Eq. (2) M is the mass matrix, u is the displacement vector, f^{ext} represents the external applied load vector, K is the stiffness matrix.

The impact is a transient dynamic phenomenon, the load on the structure and its response changes over time. The equation of motion is solved by adopting a scheme of evolution in time. There are different schemes for solving dynamic problems. Nevertheless, one distinguishes the implicit scheme and the explicit scheme [26].

The implicit scheme is used to solve quasi-static, static or low-frequency dynamic problems. Its unconditional stability is an advantage; the time step can be great compared to the explicit method. But the implicit scheme requires the resolution of algebraic systems of large dimensions at each time step.

The explicit scheme that is adapted to the phenomena of wave propagation and high speed impact is used for this study. This explicit scheme is conditionally stable. The stability limit is linked to the highest natural frequency of the system.

The time step for resolving the calculation algorithm must be less than a critical value. This time step is small, in the order of a microsecond. The use of a diagonal mass matrix for Eq. (2) simplifies the solution of the problem; this method requires little memory for solving and stocking matrices compared to the implicit method. It is an element by element resolution method that does not require assembly of the stiffness matrix.

Moreover the problem is parallelizable, which represents an interesting gain in time of resolution for problems with a large number of degrees of freedom.

For the reason of generality in the implementation, the Newmark method [27] is chosen. This method can be implicit or explicit in nature by the choice of two parameters that define the stability of the algorithm and its character.

4 Numerical Simulation Model and Virtual Testing

The numerical simulation model extracts the behavior of the material at the coupon scale, its ability to absorb energy for severe shocks such as bird impact on aircraft. A non-linear transient dynamic numerical study by finite elements using an explicit schema is carried out. Material, geometric and contact nonlinearities are represented.

The characterization of the behavioral law of the material at the coupon scale is carried out through the development of two studies led in parallel:

- A methodology for the treatment of experimental results and the creation of a behavioral law correlated to the experimental results of quasi-static and dynamic tests at different speeds for load configurations in tensile, compression and shear.
- The development of a finite element numerical simulation model representing the influential components of the test cell, the physical phenomena, the sensors from which are extracted the forces, displacements and means to visualize the deformations according to different time scales.

The behavioral law developed represents the damage and densification of the material; it represents the compaction of the composite material. This law represents large deformation and thus treats geometric non-linearity.

The Young modulus, tensile and shear modules are integrated into the behavioral law and simulate the discharge slope of the material.

Behavioral curves specific to each direction of the material reference axis are defined, in order to consider the orthotropic characteristics of the coupon.

The sensitivity of the material to the strain rate is considered. Experimental tests are performed at a limited number of reference impact velocities. The behavioral law also develops the other untested velocities by interpolating the behavioral laws of all velocities from the defined reference behavioral curves.

The numerical simulation model is developed after analysis of the test cell components and observed physical phenomena. It represents the characteristic elements of the problem. It extracts and compares physical quantities, such as forces and displacement, to compare them with the test values. It uses numerical simulation to qualify the material at the coupon scale. Three-dimensional numerical simulation by finite element modeling is used for the thick composite and other components of the test cell.

The macro-model finally represents the absorbent material and defines the test conditions, experimental tools, the characteristics of the coupons and the test results (virtual testing). The objective is to integrate this numerical macro-model into the global model of the bulkhead, in order to carry out dimensioning studies and validate the structure at the scale of the concept at a lower cost (Fig. 7).

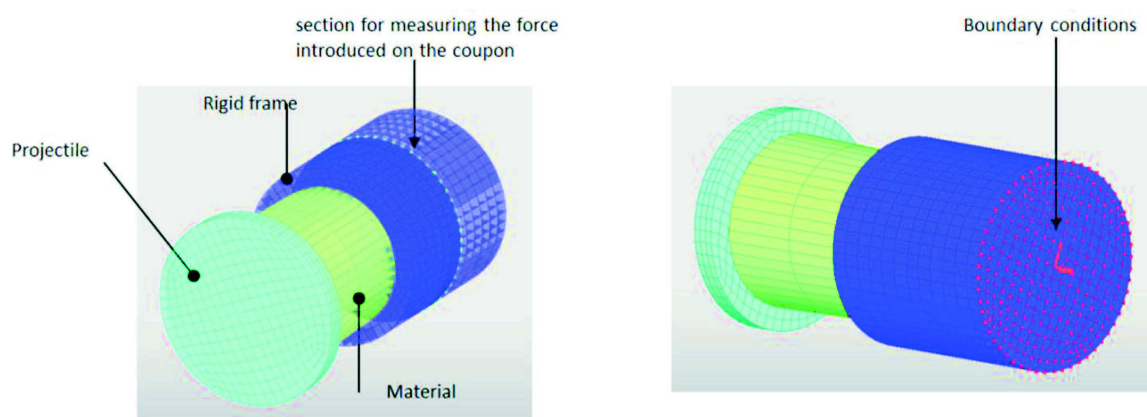


Fig. 7. Numerical finite element model (virtual testing).

The numerical simulation model is validated by correlating the results of tests with numerical simulation. To this end, an iteration phase on parameters of the model is carried out in order to improve its behaviour, i.e. the representativeness of the results.

The parameters tested are the fineness of spatial discretization, modelling of components, also some parameters of the material's behavioural curve. The method for resolving the nonlinearity of the contact has been defined. Test phases on mass realignment due to inertial effects are necessary.

Tests at different speeds are carried out. To verify the repeatability of the experiment, several tests with the same characteristics are also performed.

The suitability of the behavioural law with different tests has been tested. The objective is to obtain a calculation model representative of the test.

5 Conclusion and Perspectives

5.1 Conclusions

The originality of the approach lies in the methodology deployed and the technological solution designed based on a predictive numerical simulation model.

The interest of the development methodology and absorbent material goes beyond avian impact for an aircraft.

The numerical simulation model is validated by correlating test results with numerical finite element results. Integrated in a sensitivity study, an iteration phase on parameters of the model is carried out in order to improve its behaviour, i.e. the representativeness of the results.

The parameters tested were the spatial discretization, boundary conditions. But also parameters of the behavioural curve of the material. The method for resolving the nonlinearity of the contact has been defined. Test phases on mass realignment due to inertial effects were necessary.

Tests at different speeds are carried out. To verify the repeatability of the experiment, several tests with the same characteristics are also performed.

The suitability of the behavioural law with different tests was tested in order to obtain a calculation model representative of the test.

5.2 Perspectives

This approach is interesting and efficient. It is envisaged to continue the study on the scale of the bulkhead, through a phase of numerical simulation or Virtual Testing which will be followed by a real test carried out by a qualification and certification institution.

For the aeronautics sector, the new material is a test solution for "containment" problems such as the retention of a broken blade on engines, turbo machines.

With regard to the absorbent material, other sectors such as Defence in the case of shielding solutions or the Space sector for problems of acoustic wave resistance could be interested.

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