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# On Blind Frame Synchronization of LDPC Codes

Rodrigue Imad and Sebastien Houcke

**Abstract**—We consider in this letter the problem of blind frame synchronization of Low Density Parity-Check (LDPC) codes. We present and compare new methods of blind frame synchronization based on the calculation of Likelihood functions of the syndrome. Simulation results show that the four proposed synchronization techniques are effective once applied to LDPC codes. The method that involves the maximization of the Likelihood Difference (LD) function of the parity-checks, presents the best results in terms of probability of false synchronization.

**Index Terms**—Blind frame synchronization, LDPC codes, parity-check matrix, Likelihood functions.

## I. INTRODUCTION

LOW Density Parity-Check (LDPC) codes have been widely used in the encoding and decoding of data as they can achieve very good performance even at low Signal-to-Noise Ratios (SNR) [1], [2]. However, in order for the decoder to decode the received codewords, frame synchronization should be performed. Data-Aided frame synchronization techniques involve the insertion of a predetermined synchronization word in a frame [3], [4]. Although these methods present good synchronization performance, they reduce the spectral efficiency of the system. Furthermore, the performance of data-aided frame synchronization is degraded at low SNRs. Blind frame synchronization techniques were proposed in [5], [6] wherein no additional bits are added to the coded ones. In [6], the authors showed that their method is based on a Maximum A Posteriori (MAP) probability approach. Even at low SNRs, the method presented in [6] presents very good synchronization capabilities and it outperforms the method of [5]. In [7], [8], frame synchronization was jointly performed with the decoding. In [9], the authors proposed to solve jointly the problem of blind frame synchronization and encoder identification for LDPC codes.

The frame synchronization technique presented in [6] involves the calculation of an estimate of the Log-Likelihood Ratio (LLR) of the syndrome, based on the parity-check matrix of the error correcting code. In this letter we propose new algorithms of blind frame synchronization, in which we propose to replace the LLR of the syndrome used in [6] by other alternatives, namely the Sum-Product (SP), the Cosine Conformity (CC), the Taylor Polynomial (TP) and the Likelihood Difference (LD) expressions. The functions of the CC, TP and LD were used in [10], [11], [12] to solve the problem of blind recognition of LDPC codes. In this letter, we will use these functions to implement new blind frame synchronization techniques and apply them to LDPC codes.

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This letter is organized as follows. In section II we present the context of our study and the existing blind frame synchronization technique that was first introduced in [6]. The proposed algorithms of blind frame synchronization are presented in Section III. Simulation results are shown in Section IV. Section V concludes the work.

## II. BLIND FRAME SYNCHRONIZATION ALGORITHM

We consider in this letter that an LDPC encoder is used at the transmitter. Each code has a rate  $R = \frac{k}{n}$ , where  $k$  is the length of information bits and  $n$  is the length of a coded block. We also assume in our study that each encoded bit is modulated by a Binary Phase Shift Keying (BPSK) modulation and transmitted over an additive white Gaussian channel. Note that the BPSK scenario was assumed in this paper to have easy access to the Likelihood functions of the bits. However, the blind synchronization techniques proposed in this paper are not restrictive to this type of modulation. In fact, from the received modulated symbols, it is enough to estimate the LLR of the coded bits [13], and then apply the blind synchronization methods proposed in this paper, using the estimates of the LLR of the bits.

At the reception, a received sample can be written as:

$$r(i) = x(i - t_0) + b(i), \quad (1)$$

where  $x(i)$  is the  $i^{th}$  encoded and modulated bit,  $b(i)$  is a noise sample and  $t_0$  is the delay of a codeword that we assume to be smaller than  $n$ . From a sequence of received samples, the target of frame synchronization is to find the position of a codeword so that the decoder can work properly. In other words, our target is to estimate  $t_0$  from a sequence of received blocks.

The method proposed in [6] is based on the use of a sliding synchronization window of size  $Kn$ , where  $K$  is the number of blocks of length  $n$  contained in the window. By applying the sliding window on the received blocks and at each position  $t$  of this window, the LLR of the syndrome is calculated for each of the  $K$  blocks contained in the window. The LLR of the syndrome is a vector of  $n - k$  elements, and each of these elements is calculated using a parity equation defined by a row of the parity-check matrix  $H$  of the code.

At the synchronization position, the probability of having a verified parity equation is greater than at a non-synchronization position. This is due to the fact that at the synchronization position, the tested block corresponds to a valid codeword which is corrupted only by the noise of the channel, whereas at a position of non-synchronization and even in the absence of noise, the block in the synchronization window is not a codeword.

Based on the Min-Sum algorithm and the approximation introduced in [14] to compute the LLR of the syndrome, the algorithm proposed in [6] calculates at each position  $t$  of the sliding window, the LLR of an element of the syndrome by:

$$L_{MS}(S_t(i)) = (-1)^{u_i+1} \left( \prod_{j=1}^{u_i} \text{sign}(r(t+i_j)) \right) \cdot \min_{j=1, \dots, u_i} |r(t+i_j)|, \quad (2)$$

where  $u_i$  denotes the number of non zero elements in the  $i^{\text{th}}$  row of the parity-check matrix  $H$ , and  $i_j$  is the position of the  $j^{\text{th}}$  non zero element in this  $i^{\text{th}}$  row.

Having  $K$  blocks in the sliding window, the algorithm in [6] computes then the sum of the LLR of the syndrome by:

$$\phi_{MS}(t) = \sum_{i=1}^{K(n-k)} L_{MS}(S_t(i)), \quad (3)$$

and estimates the frame synchronization position by:

$$\hat{t}_{MS} = \underset{t=0, \dots, n-1}{\text{argmin}} \{ \phi_{MS}(t) \}. \quad (4)$$

This algorithm is able to estimate blindly the position of frame synchronization. It presents very good synchronization performance, even at low SNRs.

### III. PROPOSED BLIND FRAME SYNCHRONIZATION ALGORITHMS

We propose in this section new algorithms of blind frame synchronization that are based on the same concept as in Section II, however, in each algorithm, the LLR of the syndrome is replaced by another function of the syndrome.

#### A. The sum of the Log-Likelihood Ratio based on the Sum-Product algorithm

Instead of using the approximation of the LLR of the syndrome that is based on the Min-Sum algorithm, we propose in this section to compute this LLR as was defined in the Sum-Product algorithm in [15]. Hence, at each position  $t$  of the sliding window, our blind frame synchronization algorithm computes:

$$\phi_{SP}(t) = \sum_{i=1}^{K(n-k)} L_{SP}(S_t(i)). \quad (5)$$

with

$$L_{SP}(S_t(i)) = \text{atanh} \left( \prod_{j=1}^{u_i} \tanh \left( \frac{-r(t+i_j)}{\sigma^2} \right) \right), \quad (6)$$

The SP based blind frame synchronizer estimates the frame synchronization position by:

$$\hat{t}_{SP} = \underset{t=0, \dots, n-1}{\text{argmax}} \{ \phi_{SP}(t) \}. \quad (7)$$

#### B. The sum of the Likelihood Difference

We propose in this section a new blind frame synchronization technique that is based on the calculation of the Likelihood Difference (LD) of the parity-checks of the code. In [16], the authors define the Likelihood Difference (LD) of the parity-checks of the code and propose to use it for blind recognition of channel codes. They showed that the LD based codes identification algorithm achieves very good performance and has a reduced computational complexity.

Inspired by [16], we propose in this section a new blind frame synchronization technique that is based on the same concept as in [6], however, the LLR of the syndrome is replaced by its LD.

Given the received sequence of bits, the *a posteriori* LD of a parity-check  $S$  is defined in [16] by:

$$D(S) = p_0 - p_1 \quad (8)$$

with  $S$  a parity-check of a code  $C$ ,  $p_0 = P(S \text{ satisfied}/r)$  and  $p_1 = P(S \text{ unsatisfied}/r)$ .

The LD of a parity-check  $S$  is calculated by:

$$D(S) = \prod_{j=1}^{u_i} \tanh \left( \frac{-r(t+i_j)}{\sigma^2} \right) \quad (9)$$

In [10], the authors propose to maximize the average of the LD value in their codes recognition problem.

In this letter, we propose to maximize the sum of the LD value in order to find the correct frame synchronization position. Hence, our new blind frame synchronization technique calculates at each position  $t$  of the sliding window the LD function by:

$$L_{LD}(S_t(i)) = \prod_{j=1}^{u_i} \tanh \left( \frac{-r(t+i_j)}{\sigma^2} \right) \quad (10)$$

Then the algorithm computes the sum of the LD of the code by:

$$\phi_{LD}(t) = \sum_{i=1}^{K(n-k)} L_{LD}(S_t(i)). \quad (11)$$

The proposed LD based blind frame synchronizer estimates the frame synchronization position by:

$$\hat{t}_{LD} = \underset{t=0, \dots, n-1}{\text{argmax}} \{ \phi_{LD}(t) \}. \quad (12)$$

#### C. The sum of the Cosine Conformity

In [11], the authors present a new algorithm for LDPC codes recognition, based on the maximum average Cosine Conformity (CC) of parity-checks. Compared with the LLR algorithm, simulation results showed in [11] that the CC based codes recognition algorithm presents improved performance, with a reasonable computational complexity.

In this section, we propose to apply the formula of the Cosine Conformity in a new blind frame synchronization algorithm. Hence, the proposed CC based frame synchronizer computes

at each position of the sliding window, the CC of parity-checks by:

$$L_{CC}(S_t(i)) = \prod_{j=1}^{u_i} \left( \frac{\pi}{1 + e^{-\frac{2r(t+i_j)}{\sigma^2}}} \right). \quad (13)$$

Then, the algorithm computes the sum of the CC functions by:

$$\phi_{CC}(t) = \sum_{i=1}^{K(n-k)} L_{CC}(S_t(i)). \quad (14)$$

The proposed CC based blind frame synchronizer estimates the frame synchronization position by:

$$\hat{t}_{CC} = \underset{t=0, \dots, n-1}{\operatorname{argmax}} \{ \phi_{CC}(t) \}. \quad (15)$$

#### D. The sum of the Taylor Polynomial

In [12], the authors proposed to use the average Taylor Polynomial (TP) of the syndrome to perform codes recognition. When compared with other blind codes identification methods, the success rate of the TP based identification method is improved for the encoders with high rates [12]. In this paragraph, we propose to apply the Taylor Polynomial (TP) method to our frame synchronization problem. The blind frame synchronizer proposed in this section computes:

$$L_{TP}(S_t(i)) = \prod_{j=1}^{u_i} \sum_{n=0}^3 \frac{(-1)^n \pi^{2n}}{(2n)! \left( 1 + e^{-\frac{2r(t+i_j)}{\sigma^2}} \right)^{2n}} \quad (16)$$

and

$$\phi_{TP}(t) = \sum_{i=1}^{K(n-k)} L_{TP}(S_t(i)). \quad (17)$$

The proposed TP based blind frame synchronizer estimates the frame synchronization position by:

$$\hat{t}_{TP} = \underset{t=0, \dots, n-1}{\operatorname{argmax}} \{ \phi_{TP}(t) \}. \quad (18)$$

## IV. SIMULATION RESULTS

In this section, we test the performance of the four blind frame synchronization algorithms proposed in this letter and compare them with the existing method that was proposed in [6]. Monte Carlo simulations were performed on different LDPC codes and for each realization, information bits, noise and correct synchronization position were randomly chosen.

We considered in Fig. 1 an LDPC code of length  $n = 1944$  bits, rate  $R = 1/2$ , and having 7 non zero elements in each row of its parity-check matrix. Five synchronization algorithms were applied on this code, including the Min-Sum based algorithm that was proposed in [6]. The sliding synchronization window that was applied in each of these algorithms contained  $K = 1$  block. The probability of false synchronization curves were plotted in terms of the Signal-To-Noise Ratio. As we can see in Fig. 1, all the four algorithms proposed in this letter were able to achieve good synchronization performance. Moreover, a slight difference between the synchronization capabilities of these algorithms can be seen. The LD and SP based algorithms present almost the same performance and

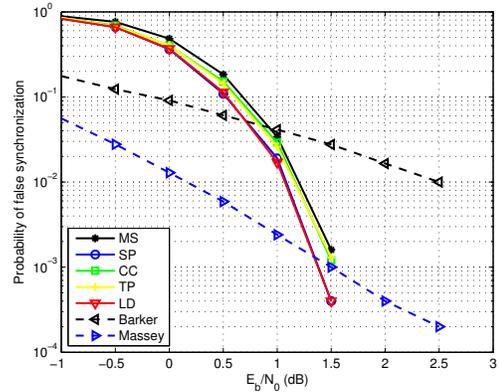


Fig. 1. Comparison between the proposed synchronization methods applied to an LDPC code of length  $n = 1944$ , rate  $R = 1/2$  and  $u_i = 7$ .

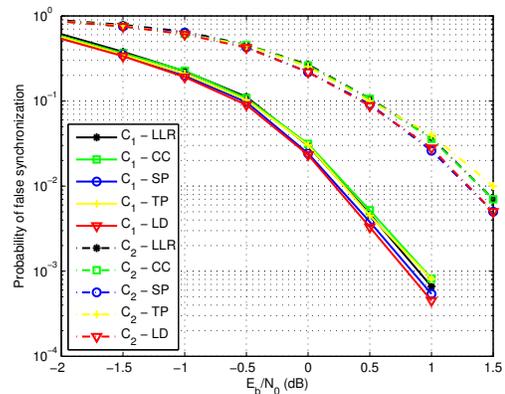


Fig. 2. Comparison between the proposed synchronization methods applied to an LDPC code of length  $n = 648$ , rate  $R = 1/2$  Code  $C_1$ :  $u_i = 4$ , Code  $C_2$ :  $u_i = 5$ .

they outperform the MS, CC and TP based synchronization algorithms.

Note that the computational complexity of the LD is less than the SP, and this is due to the fact that the inverse hyperbolic tangent function involved in the SP algorithm is not present in the LD based method. This yields to a reduction in the computational complexity of the proposed LD based synchronization algorithm.

We also plotted in Fig. 1 the Probability of False Synchronization curves of two reference data-aided synchronization algorithms that were presented in [3] (Barker) and [4] (Massey). A synchronization word of 40 bits was considered for each algorithm. It can be clearly seen from Fig. 1 that the proposed synchronization algorithms converge quicker than the ones presented by Barker and Massey. Furthermore, our methods present better performance in the zone where the LDPC codes are usually used.

In Fig. 2, the proposed synchronization algorithms were applied to two LDPC codes that have same length ( $n = 648$  bits), same rate ( $R = 1/2$ ), but with different parity-check matrices: Code  $C_1$  has 4 non zero elements in each row of its parity-check matrix while code  $C_2$  has 5. All synchronizers presented

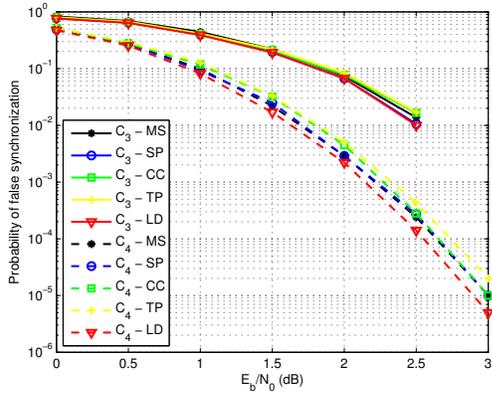


Fig. 3. Comparison between the proposed synchronization methods applied to two LDPC codes of length  $n = 648$  and  $u_i = 7$ . Code  $C_3$ :  $R = 1/2$ , Code  $C_4$ :  $R = 5/6$ .

the same behaviour: The probability of false synchronization increases with the density of the parity-check matrix of the LDPC codes.

In Fig. 3, we applied the five frame synchronization algorithms to two LDPC codes having a length  $n = 648$  bits and 7 non zero elements in each row of their parity-check matrix. These codes differ only by their rate: Code  $C_3$  has a rate  $R = 1/2$ , while the rate of code  $C_4$  is  $5/6$ . Although the five synchronization algorithms presented close performance, the LD based method showed to be the most efficient one. Note that in [12], the authors showed that the TP based codes recognition algorithm presented improved performance for codes with high rates. However, it can be clearly seen in Fig. 3 that in the case of frame synchronization, the TP based frame synchronizer did not bring any improvement in the probability of false synchronization of high rates LDPC codes.

Furthermore, and as shown in Fig. 3, the performance of the proposed synchronization algorithms is improved with the code rate. In fact, it was shown in [6] and [17] that the MS based synchronization method presents very good performance once applied to codes having a sparse parity check matrix, i.e., when the number of nonzero elements in the rows and columns of the parity check matrix is small. This same conclusion applies for all the synchronization algorithms presented in this paper. For a fixed number of ones in each row of the parity check matrix, the number of ones in each column decreases with the code rate. Therefore, an improvement in the synchronization performance of the proposed algorithms is expected for codes with higher rates.

In Fig. 4, a sliding window of size  $K = 2$  blocks was applied in the five synchronization methods. As expected, the performance of our algorithms is improved when the number of blocks in the sliding window increases. Even at very low SNRs, we are still able to obtain a very good synchronization performance. Once again, the LD based synchronizer presented the best performance.

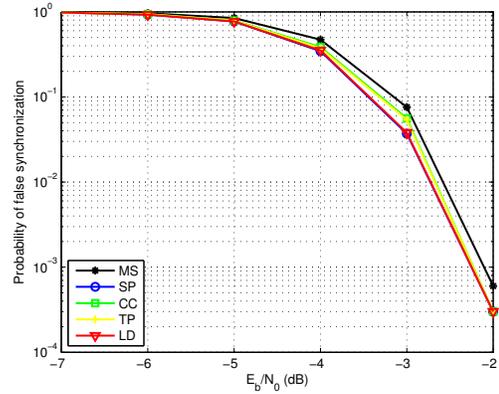


Fig. 4. Comparison between the proposed synchronization methods applied to an LDPC code of length  $n = 1944$ , rate  $R = 1/2$  and  $uk = 4$ , with a synchronization window of size 2 blocks.

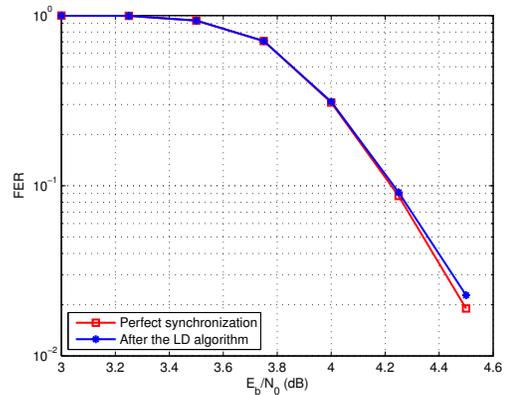


Fig. 5. FER curves of an LDPC code of length  $n = 480$  bits and rate  $R = 1/2$ .

In order to evaluate the effectiveness of the proposed LD based synchronization algorithm on LDPC codes, we plotted in Fig. 5 the Frame Error Rate (FER) curves obtained after applying the Belief Propagation (BP) decoding algorithm to an LDPC code of length  $n = 480$  bits and rate  $R = 1/2$ , chosen from the IEEE 802.22 standard [18]. Two cases were considered: In the first case we assumed perfect synchronization, while in the second case the synchronization position was estimated using the LD algorithm. As shown in Fig 5, the FER curves are almost the same. Therefore, the LD based synchronization algorithm is well suited for this type of codes.

The theoretical analysis of the LD based frame synchronization algorithm presented in Section III-B is also studied in this letter. We can easily show that the LD based synchronization criterion of (11) follows a Gaussian distribution:

$$\begin{aligned} \phi_{LD}(t) &\sim \mathcal{N}(0, K(n-k)\sigma^2) & t \neq t_0 \\ \text{and } \phi_{LD}(t_0) &\sim \mathcal{N}(K(n-k)m_{t_0}, K(n-k)\sigma_{t_0}^2). \end{aligned}$$

The analytical expressions of the mean  $m_{t_0}$  and variances  $\sigma^2$  and  $\sigma_{t_0}^2$  are computed in the same way as in [17]. In

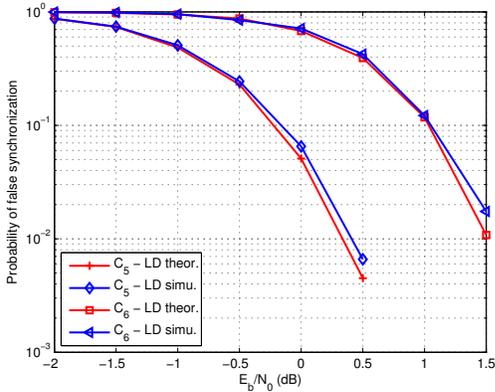


Fig. 6. Performance of the proposed LD based frame synchronization algorithm in simulation and theory.

Fig. 6, we plotted the theoretical and simulated performance of the proposed LD based frame synchronization algorithm. Two LDPC codes of same length ( $n = 1944$ ) and same rate ( $R = 1/2$ ), were considered. These codes differ only by the weight of their parity-check matrix. Codes  $C_5$  and  $C_6$  have  $u_i = 6$  and  $8$ , respectively. As shown in Fig. 6, the theoretical and simulation results of the proposed LD based frame synchronizer are almost the same.

Note that the blind synchronization algorithms introduced in this paper present better performance when the parity check matrix of the corresponding code is sparse, i.e., when the number of nonzero elements in the rows and columns of the parity check matrix is small. Hence, since codes  $C_5$  and  $C_6$  have 6 and 8 nonzero elements in each row of their parity check matrix, respectively, then the proposed synchronization algorithms are expected to have better performance on  $C_5$ , rather than  $C_6$ .

Furthermore, we assumed in the theoretical analysis that the elements of the syndrome are all independent. However, as shown in [17], this assumption depends on the number of nonzero elements in the parity check matrix of the code. The number of independent syndrome elements decreases with the number of ones in the parity check matrix, which explains why the theoretical and simulated curves in Fig. 6 are not exactly the same, but are nevertheless very close.

## V. CONCLUSION

We have considered in this letter the problem of blind frame synchronization of LDPC codes. We have presented four new blind frame synchronization methods based on the calculation of functions of the syndrome. Simulation results have shown that the four proposed methods have very good synchronization capabilities. Although the presented approaches have very close performance, the Likelihood Difference (LD) based frame synchronization algorithm proposed in this letter presents the best performance. Moreover, compared to the SP approach, the LD based algorithm has a lower computational complexity, which makes it a very good choice for other

synchronization problems, such as time synchronization or Carrier Frequency Offset (CFO) recovery.

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