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► **To cite this version:**

Darith-Anthony Hun, Julien Yvonnet, Johann Guilleminot, Abdelali Dadda, Anh-Minh Tang, et al..  
Desiccation cracking of heterogeneous clayey soil: experiments, modelling and simulations. *Engineering Fracture Mechanics*, 2021, 258, pp.108065. 10.1016/j.engfracmech.2021.108065 . hal-03621299

**HAL Id: hal-03621299**

**<https://hal.science/hal-03621299v1>**

Submitted on 28 Mar 2022

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# Desiccation cracking of heterogeneous clayey soil: experiments, modelling and simulations

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## Abstract

Experimental results for the cracking of heterogeneous clay samples during desiccation are reported, and an associated numerical model is developed for comparison. The clay samples contain embedded rigid inclusions to induce heterogeneous strain fields during drying. The crack paths and local strain fields are monitored during the desiccation process using digital image correlation. A numerical phase field model for crack initiation and propagation is introduced and compared with the experimental results. A qualitative agreement is found for the obtained crack paths, whereas discrepancies remain for the local strain fields. A discussion regarding the comparison between the experimental results and model is provided.

*Keywords:* Clay, shrinkage, desiccation, crack propagation, Digital Image Correlation, Finite Element Analysis, phase field method, Inclusions

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## 1. Introduction

Desiccation crack patterns can be observed in several natural systems, including complex crack networks in dried blood, cracks in old paintings, T/Y-

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shaped cracks in dried mud, and polygonal terrain cracks [1]. In soil science,  
5 soil cracks typically serve as preferential paths for water movement or solute  
transport [2] and contribute to the progressive deterioration of earth structures  
[3].

To understand the mechanism governing the desiccation cracking of clayey  
soils, other than a few observations at the field scale, most previous experimental  
10 studies have focused on crack production in laboratory settings using processed  
clays, often from slurry states [4]. These studies have shown that desiccation  
cracking is a complex coupled suction-contraction process [5, 6, 7, 8, 9] that is  
sensitive to the boundary conditions (air relative humidity, ambient tempera-  
ture, basal friction, clay layer thickness, etc.) and intrinsic soil properties (clay  
15 content).

Modeling the shrinkage crack process involves many challenges, including:  
(a) taking into account the multiphysics phenomena, (b) the possibly stochastic  
character of these phenomena, (c) the complex microstructure of the mate-  
rial, and (d) the inherent numerical difficulties related to the crack initiation,  
20 branching, and coalescence of many interacting cracks within the samples.

Many models and numerical techniques have been developed to reproduce  
the crack patterns during clay desiccation, including phenomenological spring  
models [10, 11, 12, 13, 14], discrete element method (DEM) models [15, 16,  
17], cohesive zone models [18, 19, 20], mesh fragmentation techniques [21], and  
25 eXtended Finite Element Method (XFEM) models [22]. Recently, Hu et al. [23]  
developed a phase field model to simulate crack patterns during the shrinkage of  
thin films on substrates and proposed a stochastic model of a random field for  
the fracture toughness and fracture energy to model the random distributions  
of cracks. One advantage of the phase field method for simulating fractures  
30 [24, 25, 26] is that it relies on a continuum mechanics model and thus allows the  
introduction of different behaviors and coupling in a more versatile manner.

In all the aforementioned numerical studies, only homogeneous samples were  
considered. The sole exception was [23], in which the existence of a heteroge-  
neous composition of the samples at a smaller scale was indirectly taken into

35 account through the stochastic description of the properties at the sample scale. Surprisingly, we are not aware of studies involving cracks in drying clay samples involving macro-scale heterogeneities. The objective of the present study is to conduct both experiments and modeling for this configuration. In the exper-  
40 iments, rigid inclusions are embedded into clay samples subjected to drying to analyze their influence on crack initiation and the final crack patterns. As in [27], where homogeneous samples were considered, digital image correlation is used to evaluate local strain fields during the entire process. A phase field crack propagation model is developed to simulate the process, and its predictions are compared with the experiments. A discussion of the agreement between the  
45 experiments and the model is provided.

The remainder of this paper is organized as follows. In Section 2, we describe the experimental setup for the desiccation of heterogeneous clay samples and provide the crack pattern and strain field results for different configura-  
50 tions. In Section 3, the numerical phase field fracture model is presented. In Section 4, the parameters of the model are identified based on the experimental results. Finally, a comparison between the experimental results and numerical predictions is provided in Section 5.

## 2. Experiments

### 2.1. *Experimental setup*

55 The novelty of the present experimental setup lies in the introduction of rigid inclusions within clay samples during the drying process to determine the influence of heterogeneities on the cracking process.

#### 2.1.1. *Material*

The studied material is Romainville clay, which can be found in the East  
60 Paris Basin. Its ability to shrink, swell and crack causes significant damage to buildings, especially in periods of severe drought, and several study campaigns have been conducted [28, 29] to understand and analyze these phenomena. This

clay is composed of different sediments: illite, smectite, carbonate, quartz, and feldspar . The physical properties of this clay are listed in Table 1.

Soil properties	Values
Density of solid phase [ $\text{Mg.m}^{-3}$ ]	2.79
Liquid limit <sup>a</sup> [%]	77
Plastic limit <sup>b</sup> [%]	40
Plasticity index <sup>c</sup> [%]	37
USUC classification	CH
Clay ( $< 2\mu\text{m}$ ) [%]	79
Clay composition	Illite and smectite
Specific surface area (methylene blue absorption) [ $\text{m}^2.\text{g}^{-1}$ ]	340

Table 1: Physical properties of Romainville clay [30].

<sup>a</sup>Liquid Limit is the water content (mass proportion of water in the mixture) at which soil changes from a plastic to a liquid state

<sup>b</sup>Plastic Limit is the water content at the change from a plastic to a semisolid state.

<sup>c</sup>The plasticity index of a soil is the difference between its liquid and plastic limits.

### 2.1.2. Experimental method

For the experiments, the clay paste was conditioned as described in [30, 31].

The samples were prepared as follows.



Figure 1: Experimental samples:  $S_0$ ,  $S_{1c}$ ,  $S_1$ ,  $S_3$  and  $S_6$ . Underscript  $c$  indicates that the inclusion is centered.

- The clay was taken directly from the site (Romainville, Paris-Est, France) as a block.
- The material was cut into small 2 cm pieces and immersed in distilled water for a period of 24 h to liquefy it.

- Then, the whole set was passed through a 2 mm sieve to filter out the larger particles.
- 75 • The mixed sludge had a water content of approximately 170% at this stage, and it was poured into  $D = 116$  mm diameter cups with a height  $h_0 = 8$  mm. Several ( $n = 1, 2, 3$  or 6) rigid inclusions (cylinders with circular cross sections made of polymethyl methacrylate (PMMA) and radius  $r_{inc} = 8$  mm) were randomly placed, as illustrated in Fig. 1. The radius of the inclusions should be small enough (compared to that of the soil sample) to minimize the boundary effects on the interaction between the soil and the inclusions. At the same time, the inclusions should be sufficiently large to ensure good observation of the soil/inclusion interface using the technique employed in this study. Note that each cup into which the clay
- 80
- 85 was poured was covered with a Teflon film on both the lateral walls and the bottom to limit the effects of friction on the surface, as shown in the right panel of Fig. 2 and the picture in Fig. 3.

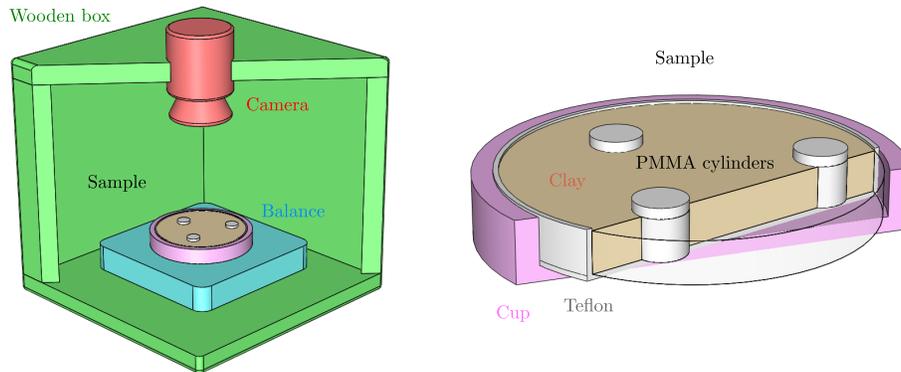


Figure 2: Experimental setup.

- The last step was the removal of air bubbles from the clay slurry. To this end, the samples were placed under vacuum for a period of 2 h and left
- 90 covered for sedimentation for 72 h.
- Before starting the analysis, the supernatant water on the surface of the

clay mud sample was removed. The water content at  $t = 0$  s of the test was  $w_c \sim 115\%$ , and the clay sample thickness was  $H = 8$  mm.

95 Then, the clay samples were placed in a wooden box with dimensions of  $50 \times 50 \times 50$  cm to protect them from rapid thermal or moisture variations or from the surrounding ambient air. However, the box was not tight, and its internal hydrothermal conditions evolved with the current laboratory conditions. Several measuring tools were placed in the box, namely a scale, a thermometer,  
100 and an ambient moisture probe, which provided access to the water loss in the sample, temperature, and moisture throughout the drying test.

After 72 h, the sample was sufficiently stiffened. Black speckles were painted on its upper surface, providing an appropriate contrast to follow the shrinkage kinematics and measure the local deformation using the digital image correlation  
105 (DIC) technique. Images were recorded with a digital camera (Canon EOS TTL camera, equipped with an 18–55 mm zoom lens, providing 8-bit color images with  $5184 \times 3456$  pixels). Image acquisition was performed at time intervals of  $\Delta t = 10$  min. Various samples (with different inclusion numbers and configurations) were considered, as shown in Fig. 2 (left panel) and Fig. 3.

- 110
- $S_0$ : a clay sample without inclusion.
  - $S_{1c}$ : a clay sample with a single centered inclusion.
  - $S_1, S_3, S_6$ : clay samples containing  $n = 1, 3, 6$  randomly-distributed inclusions.

On an orthonormal frame  $(0; \mathbf{x}, \mathbf{y})$  whose origin is at the center of the sample,  
115 the inclusions are located at the coordinates  $(-1.90, -24.7)$  mm for  $S_1$ ;  $(19.87, 3.18)$ ,  $(-20.84, 19.35)$ , and  $(-24.30, 20.17)$  mm for  $S_3$ ; and  $(-22.9, -30.7)$ ,  $(24.26, -8.63)$ ,  $(-2.46, -2.72)$ ,  $(-29.82, 19.01)$ ,  $(2.20, 27.36)$ , and  $(32.6, 31.40)$  for  $S_6$ .

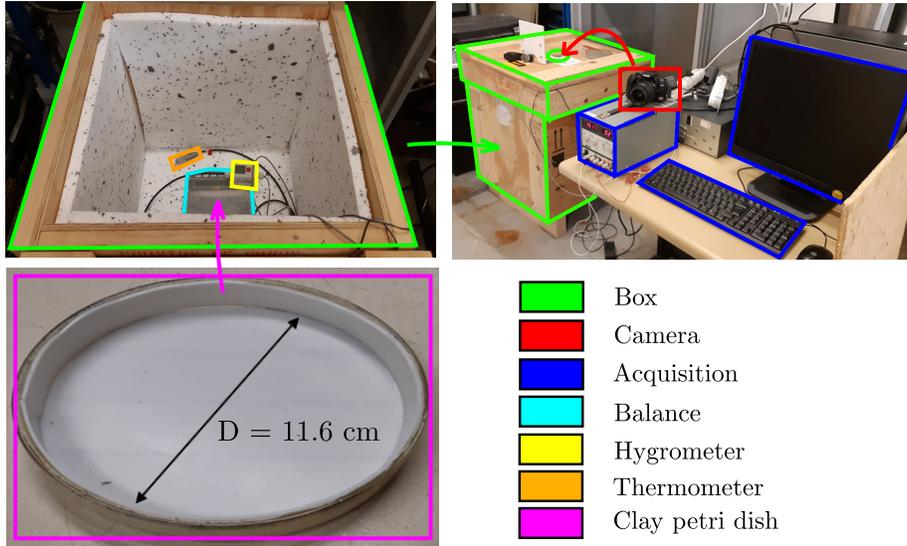


Figure 3: Experimental setup photos.

### 2.1.3. DIC image analysis

The obtained images were analyzed using DIC [32] to evaluate the 2D displacement and deformation field on the upper surface of the sample. An in-house image correlation software (CMV) was used to process the images converted into gray levels. The software is based on a local DIC formulation using the zero-centered normalized cross-correlation criterion, an affine shape function of the displacement field within correlation windows, and biquintinc interpolation of the gray level to achieve subpixel accuracy for evaluation of the displacement fields. In-plane transformation gradient components were obtained from discrete evaluations of the displacement at the centers of adjacent non-overlapping correlation windows using relations detailed by Allais et al. [33]. Two quantities of interest are analyzed in the following: the spherical (or hydrostatic) strain,  $\epsilon_S$ , and the deviatoric strain,  $\epsilon_D$ :

$$\epsilon_S = \frac{1}{2} \text{tr}(\epsilon) , \quad \epsilon_D = \frac{2}{3} (\bar{\epsilon}_2 - \bar{\epsilon}_1) , \quad (1)$$

where  $\bar{\epsilon}_1$  and  $\bar{\epsilon}_2$  ( $\bar{\epsilon}_2 \geq \bar{\epsilon}_1$ ) are the eigenvalues of the 2D Green-Lagrange strain tensor, and  $\text{tr}(\epsilon) = \epsilon_1 + \epsilon_2$  measures the variation of the surface. It is empha-

sized that  $\epsilon_s$  is different from the volume variation, which cannot be measured based on purely surface investigations. These quantities were evaluated for a gauge length limited by the typical length scale of the speckle painting. With approximately 2700 pixels along the diameter of the cup and speckle sizes that  
125 allowed the use of 30-pixel-wide correlation windows, there were approximately 100 independent local evaluations of the 2D displacement field along such a distance. The accuracy of the measurements was limited by the image quality, noise, and other artifacts that are not detailed here; however, the accuracy was significantly better than 1%, and is thus sufficient to quantify the heterogeneity  
130 of the investigated strain fields.

## 2.2. Results

### 2.2.1. Evolution of experimental conditions

The first results, shown in Fig. 4, indicate the conditions of the experiment, with an ambient relative moisture of  $RH = 48\% \pm 6$ , a temperature of  $T =$   
135  $20 \pm 2^\circ\text{C}$ , and water content (ratio of the weights of water and clay) of  $wc = 115 - 10\%$ . The relatively small variations in these parameters from one sample to another show that the experiments were carried out under stable, repeatable conditions. Even if the conditions differed slightly between samples, it turns out that the drying rate was slow (the tests lasted approximately one week)  
140 with respect to the typical time of the moisture transfer within the clay matrix. At this stage, we consider that the water content in the clay material can be considered uniform and is directly given by the overall mass loss quantified by the scale. The latter was almost linear with time, as shown in Fig. 4 (c).

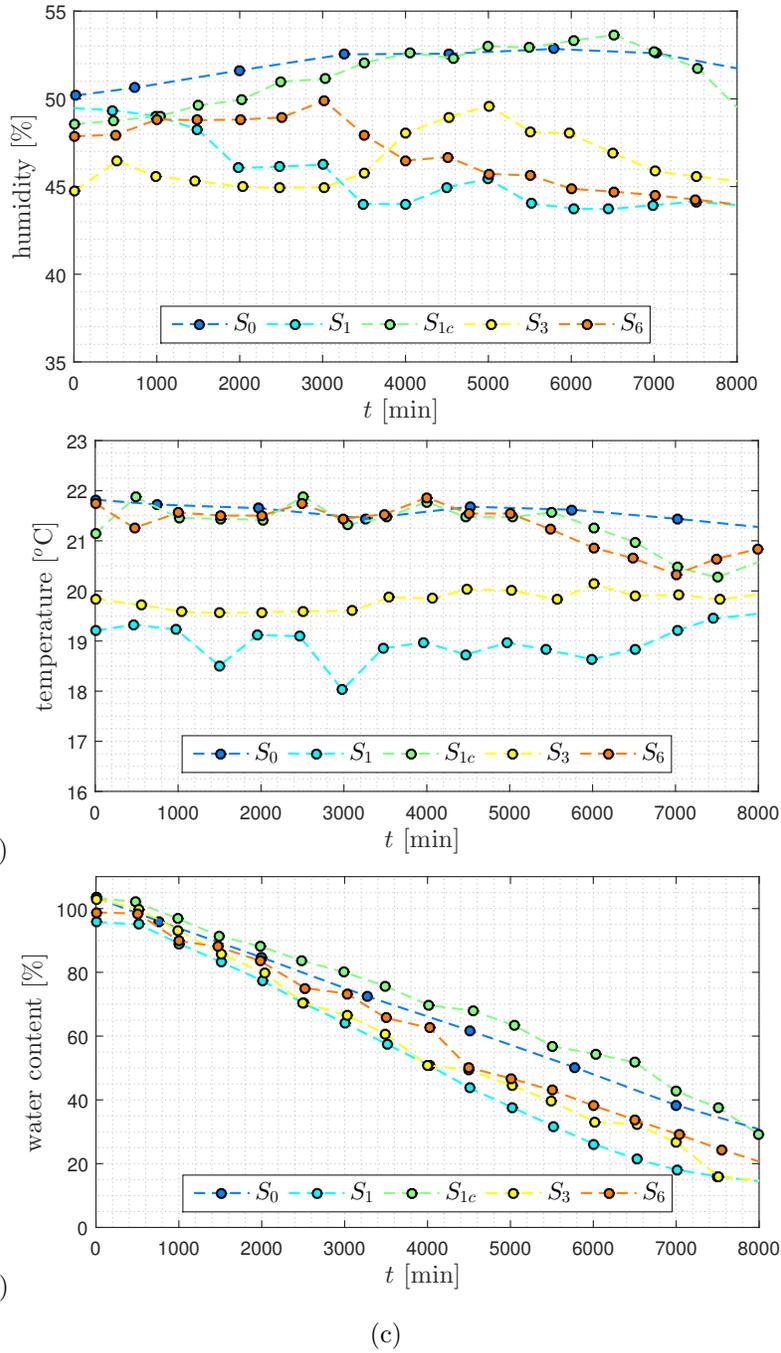


Figure 4: (a) Moisture, (b) temperature and (c) water content during drying of the sample.

### 2.2.2. Crack process

145 In the experiments, three characteristic times  $t_1 = 4000$  min,  $t_2 = 6000$  min, and  $t_3 = 8000$  min were chosen for the observations of all samples. These times correspond, respectively, to a time when cracks have not yet initiated, a time when cracks start to initiate, and a time when cracks have propagated through the sample. The drying process lasted 72 h.

150 Figs. 5,6,7,8, and 9 show the crack patterns in the samples at times  $t_1$ ,  $t_2$ , and  $t_3$ . The crack paths obtained for each specimen show some similar propagation pattern features. The cracks initiate near the rigid inclusions and propagate toward the outer edge of the specimen. Once all of the cracks have propagated, the material shrinks over its entire range, and the material is broken down into  
155 small independent pieces. It should be noted that the crack paths over the entire test set consist of three or four branches with angles between these branches,  $\theta_c$ , such that  $\pi/2 \leq \theta_c \leq \pi/3$ .

### 2.2.3. Strain analysis

In this section, the deformation maps obtained using the DIC technique are  
160 presented for the three characteristic times  $t_1$ ,  $t_2$ , and  $t_3$ . These maps were defined for the set of samples ( $S_0, S_{1c}, S_1, S_3, S_6$ ) and show the spherical and deviatoric strains in the samples. The spherical and deviatoric deformations are defined as Eq. (1). The intensities of these strains are indicated by colors that are associated with the kinematics of the drying cracking process in qualitative  
165 and quantitative ways.

For later purpose, we define three characteristic regions within the sample as illustrated in Fig. 10.

- $R_B$  is a ring surface next to the external and inclusion surfaces,
- $R_+$  is another ring surface interior to the external portion of  $R_B$ ,
- 170 •  $R_i$  is the rest of the interior surface.

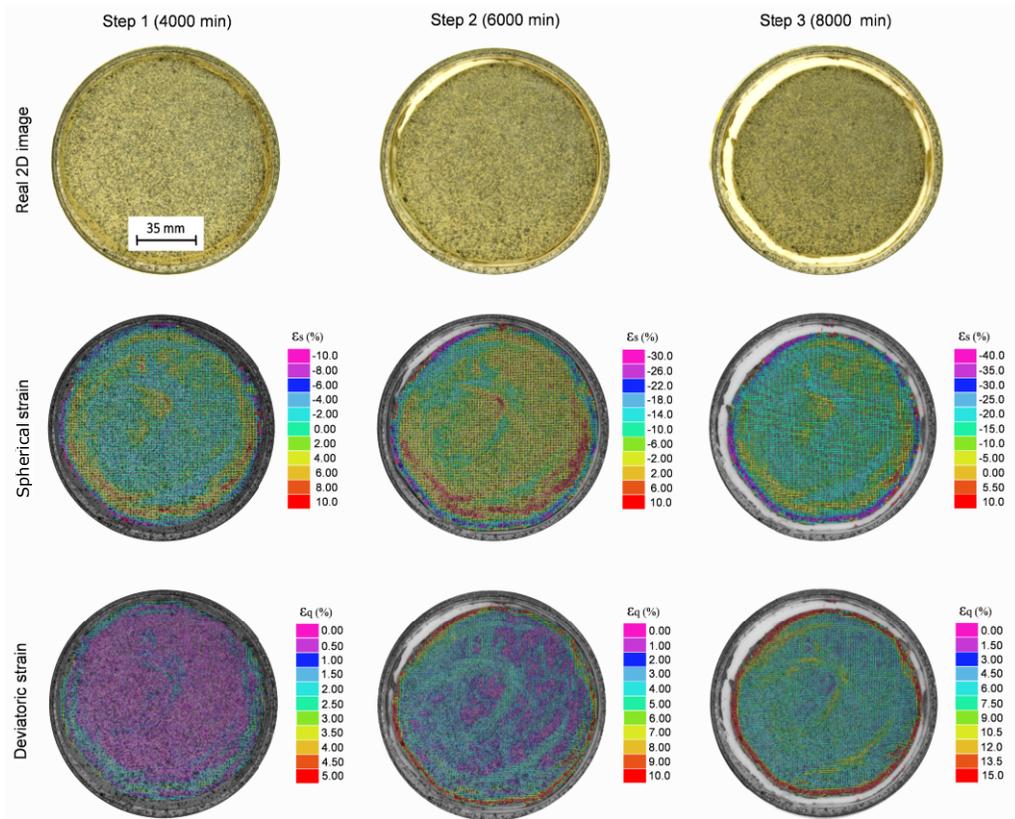


Figure 5:  $S_0$  test with shrinkage, spherical and deviatoric strains for  $t = 4000, 6000,$  and  $8000$  min.

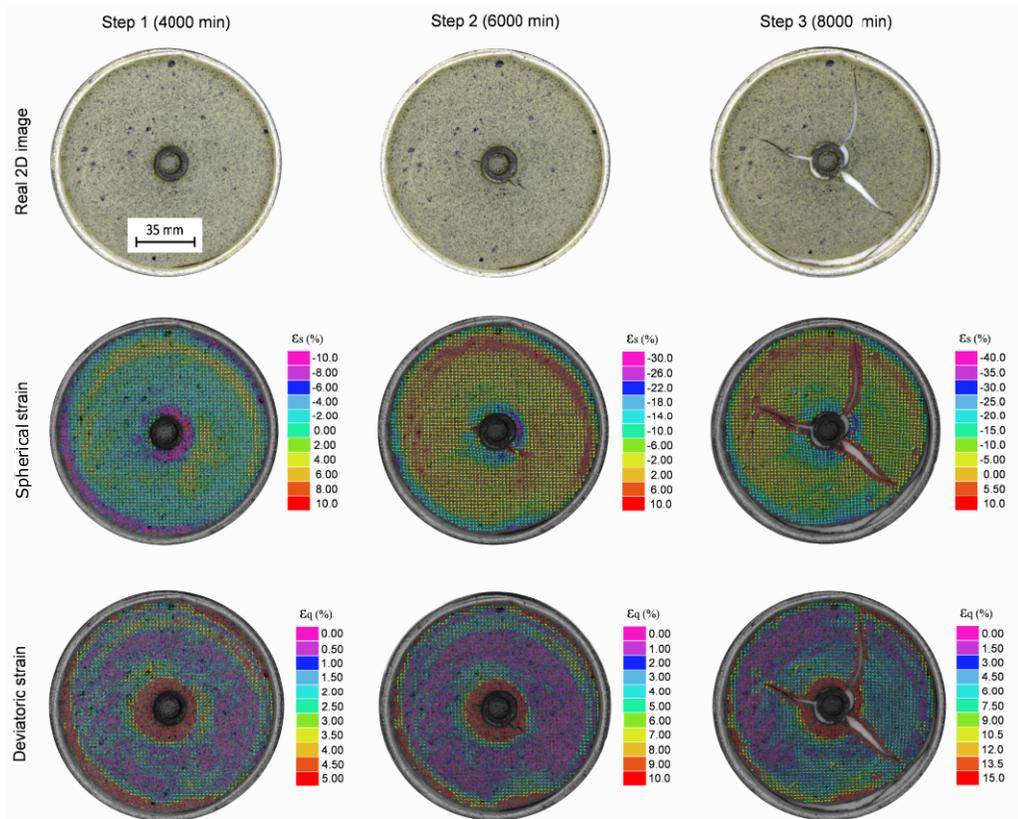


Figure 6:  $S_{1c}$  test with shrinkage, spherical and deviatoric strains for  $t = 4000, 6000,$  and  $8000$  min.

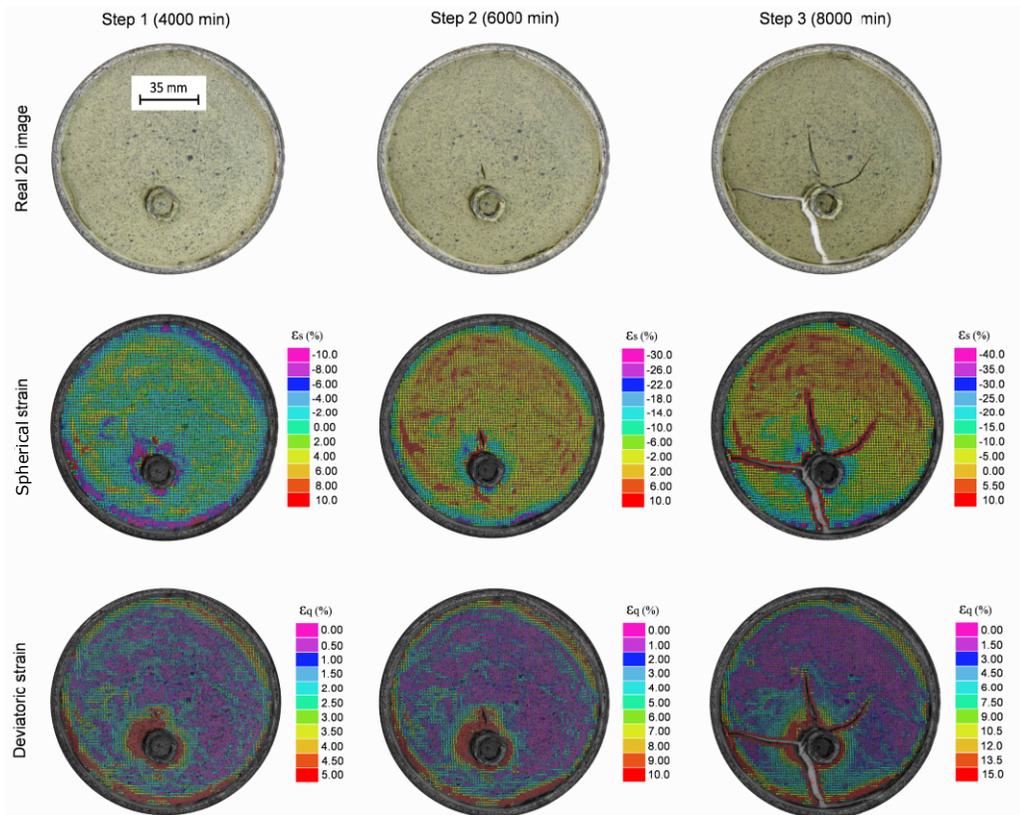


Figure 7:  $S_1$  test with shrinkage, spherical and deviatoric strains for  $t = 4000, 6000,$  and  $8000$  min.

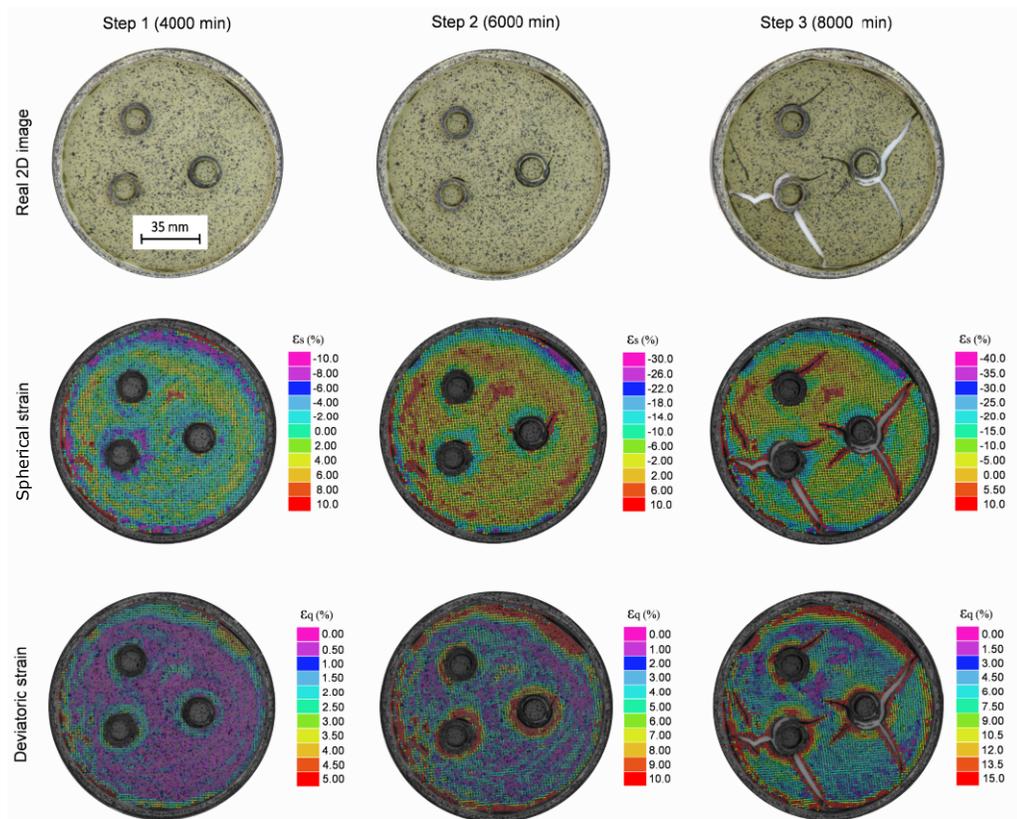


Figure 8:  $S_3$  test with shrinkage, spherical and deviatoric strains for  $t = 4000, 6000,$  and  $8000$  min.

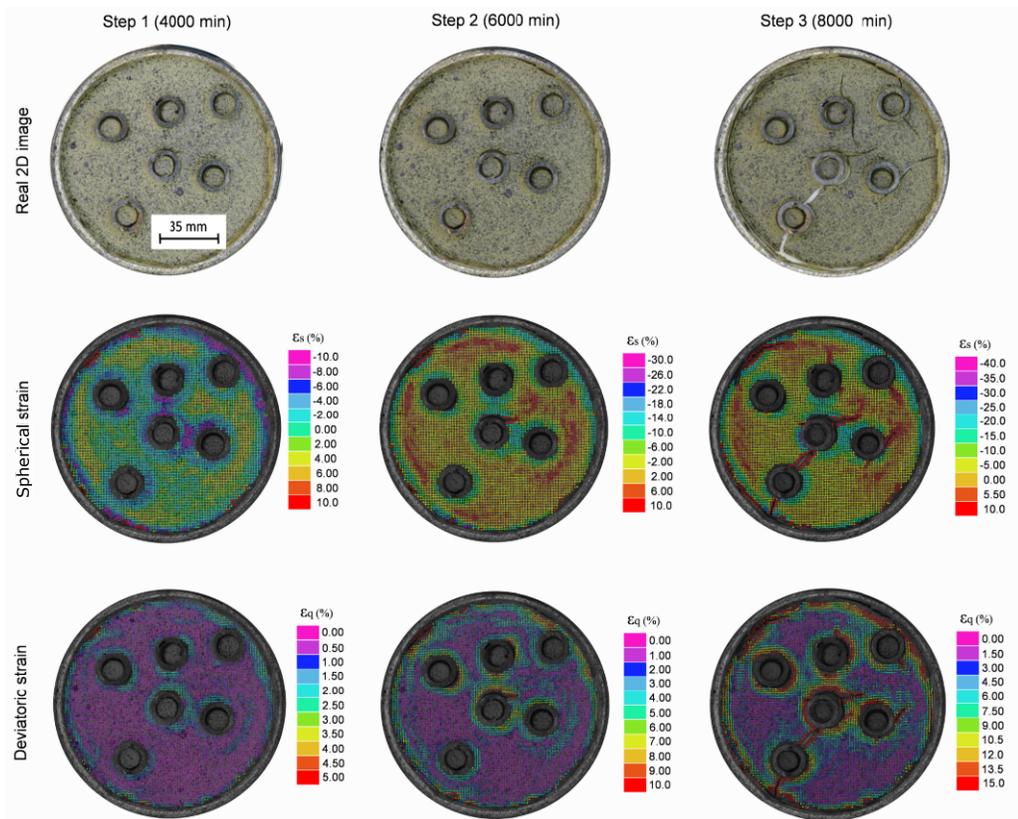


Figure 9:  $S_6$  test with shrinkage, spherical and deviatoric strains for  $t = 4000, 6000,$  and  $8000$  min.

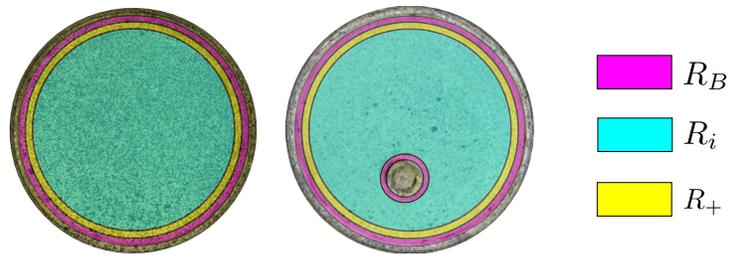


Figure 10: Definition of specific zones of interest in the clay samples.

$S_0$  test

In the first test, the sample does not contain rigid inclusions (see Fig. 1). From Fig. 5, we can observe that the spherical and deviatoric deformation fields remain roughly constant in the interior of the sample.

- 175 • First, at  $t_1$ , the spherical deformation is negative (as expected) in  $R_i$ , with  $\epsilon_S \sim -2\%$ , and is roughly homogeneous except in  $R_B \cup R_+$ . A positive deformation is observed in  $R_+$ , while another negative strain zone appears in  $R_B$  with a value different from that in the rest of the sample. The deviatoric strain is almost zero in the central area of  $R_i$ , indicating a  
180 purely radial surface deformation, as expected in a homogeneous shrinkage process. However, the deviatoric strain is non-zero with  $\epsilon_D \sim 2\%$  in  $R_B \cup R_+$ . These heterogeneities may be explained by the adhesion of the sample material to the external boundary of the dish.
  
- Second, at  $t_2$ , debonding occurs at the top left external boundary of the  
185 sample. The spherical and deviatoric strain maps are then perturbed. In  $R_i$ , the spherical strain remains negative, with  $\epsilon_S \sim -2\%$ , while the strain is no longer homogeneous and compression occurs ( $\epsilon_S \sim -14\%$ ) in  $R_+$ . Regarding the deviatoric strain, the main observations made at  $t_1$  remain valid, whereas more localized regions and heterogeneities appear in the  
190 central region.
  
- In the last step,  $t_3$ , the sample is fully detached from the dish boundaries. The strain field remains globally radial, and more pronounced heterogeneities appear.

### $S_{1c}$ test

195 The results for the  $S_{1c}$  test involving a centered inclusion are presented in Fig. 6.

- At step  $t = t_1$ , we observe an additional characteristic zone near the inclusion boundary ( $R_B$ ). In this zone, important compressive strains occur, with  $\epsilon_S = -10\%$ , which is equal to the deformation on the outer edge. As in the homogeneous case ( $S_0$  test), a zone  $R_i$  of weakly negative strain  $\epsilon_S \sim -2\%$  is observed. Then, in ( $R_+$ ), we observe an extension with ( $\epsilon_D \sim 4\%$ ). The deviatoric strain is almost zero except near the boundaries in  $R_B$ , with  $\epsilon_D \sim 5\%$ . We can see that the perturbations induced by the inclusion are localized in a radial area with a diameter of roughly three times the diameter of the inclusion.  
200
- At  $t = t_2$  debonding occurs at the bottom of the sample. The strain maps are modified in the same manner as in the case without an inclusion. In the subregion  $R_B$  around the inclusion, the shrinkage is high ( $\epsilon_S \sim -14\%$ ). In  $R_i$ , a small shrinkage remains ( $\epsilon_S \sim -2\%$ ). Then, in  $R_+$ , the situation is similar to that without inclusions. The deviatoric strain increases up to  $\epsilon_D \sim 10\%$  in  $R_B$  and remains quite low ( $\epsilon_D < 4\%$ ) in  $R_+ \cup R_i$ . The debonding at the lower part also induces a local increase in the deviatoric strain.  
210
- Finally, at  $t = t_3$ , the clay separates from the external boundary along 40% of the external surface. In contrast to the previous case ( $S_0$  test), the upper half of the ring  $R_+$  remains in extension ( $\epsilon_S \sim 5\%$ ). In the other part, as in the previous observation, there is a strong zone of shrinkage ( $\epsilon_S \sim -20\%$ ) on the border  $R_B$  and a less intense zone ( $\epsilon_S \sim -5\%$ ) in the remaining area  $R_i$ . For the deviatoric strain, high strains are again observed in  $R_B$  (outer part and near the inclusion). The deviatoric strain is almost null in areas far from the borders and the inclusion, especially in the non-debonded half of the sample.  
215  
220

It should also be noted that the very high values of the deviatoric strains at the borders of the cracks are post-processing artifacts and should not be interpreted as local deformation of the clay. Indeed, the displacement jump over the crack is integrated in the local evaluation of the strain, which is based on some finite differences (see [33] for the details about the calculation of the strain).

### $S_1$ test

The main difference between this test and the previous test is the lack of symmetry induced by the position of the inclusion.

- At  $t = t_1$ , as in the previous cases, the localized strain areas are located on the edges  $R_B$  of the sample and exhibit strong compression ( $\epsilon_S \sim -10\%$ ). It can also be observed that the field surrounding the inclusion is also more heterogeneous than in the  $S_{1c}$  sample. In this test, the ring  $R_+$  does not appear clearly, and the remaining surface  $R_i$  is a mixture of low-intensity positive and negative surface stresses  $|\epsilon_S| < 5\%$ . For the deviatoric strain, the intensity is relatively low, as in the previous samples, high  $\epsilon_D \sim 5\%$  at the outer areas  $R_B$ , and then low again in the inner area  $R_+ \cup R_i$ , but with some noticeable heterogeneities.
- At  $t = t_2$ ,  $\epsilon_S$  reaches  $-14\%$  in  $R_B$  near the external boundary. The  $R_+$  zone does not appear to develop in this case. In  $R_i$ ,  $\epsilon_S \sim 2\%$ ; nevertheless, some scattered zones have more localized values, where  $\epsilon_S \sim 10\%$ . Regarding the deviatoric strain, the intensity continues to increase on the edges  $R_B$  and remains small on the interior surface  $R_+ \cup R_i$ .
- At the last step  $t = t_3$ , the specimen again exhibits strong shrinkage  $\epsilon_S \sim 20\%$  on the outer edges  $R_B$ . The inner zone  $R_i$  has extensional strains  $\epsilon_S \sim [5\%, 10\%]$ , which are larger than those in the previous  $S_0$  and  $S_{1c}$  tests but remain relatively low. For the deviatoric strain, the trend seems to be similar to those in the previous cases.

In this sample with an off-center inclusion, debonding at the outer edge is limited. Adhesion induces very strong compression strains in areas very close to the boundaries, together with extensive strains of lower intensity in larger zones and far from the edges. Some fluctuations are observed in these areas in extension, but are not associated with strong deviatoric strains. The inclusion  
 255 located in the lower part first generates localized deviatoric strains and then cracks. This allows the surrounding clay to shrink, first in the neighborhood near the inclusion and then in the larger areas around the cracks.

### *S<sub>3</sub> and S<sub>6</sub> tests*

260 The final tests were carried out to investigate the effects of interactions between several inclusions during the drying process (see Fig. 8 and Fig. 9).

- At  $t = t_1$ , for both samples, a strain localization is observed on the surface near the inclusions and near the external boundary  $R_B$  with  $\epsilon_S \sim -10\%$  and  $\epsilon_D \sim 2\%$ . These values gradually decrease and finally vanish far from  
 265 the sample and inclusion edges. However, the symmetry of the revolution of this expansive strain field around the inclusions is broken in sample  $S_6$ , while it is essentially preserved in  $S_3$ , where the distance between the inclusions is larger. Within the  $R_+$  zone,  $\epsilon_S \sim 4\%$ .
- In the subsequent state at  $t = t_2$ , the heterogeneities in the strain field  
 270 are increased. It can be observed that there is no significant additional debonding at the outer edge of the sample, unlike in samples  $S_0$  and  $S_{1c}$ .
- At the final stage  $t = t_3$ , the extension zones  $R_+$  have been replaced by contraction zones or cracking. A few expansive zones remain, which are the locations of the secondary cracks.

275 A general observation is that the overall features of the strain field around the inclusions in  $S_3$  are similar to those observed in  $S_1$  and  $S_{1c}$ . This is also true for the crack geometries: cracks initiated from inclusions do not interact with the others. However, this is not true for the  $S_6$  sample, in which positive  $\epsilon_S$

strains are not observed between the closer inclusions. In addition, the cracks  
280 have different features: some are generated in areas with positive  $\epsilon_S$ , and some  
clearly connect two inclusions.

### 3. Numerical model

#### 3.1. Phase field mode of crack shrinkage

In this study, crack initiation and propagation are modeled using the phase  
285 field method [24, 25, 26, 34, 35, 36]. In this context, crack discontinuities are  
represented by a regularized approximation through a field  $d(\mathbf{x})$  associated with  
damage. A value of  $d = 0$  corresponds to undamaged material, whereas  $d = 1$  is  
associated with fully damaged material. The main advantage of this method is  
that the initiation, propagation, and merging of complex crack patterns can be  
290 achieved without any special numerical treatment using classical finite elements.  
In addition, this formulation naturally regularizes the strain localization and  
does not suffer from mesh-dependency issues. The main concept of this approach  
is to formulate the damage problem in a variational manner. Starting from  
the energy, minimization with respect to the displacement field on one side  
295 and maximization of the dissipation on the other side under the constraint of  
irreversible damage evolution yields two sets of coupled equations whose solution  
provides the displacement field and the damage field  $d(\mathbf{x})$  at each load increment.  
Here, we formulate this problem in the context of finite strains as shrinkage  
can induce large deformations. In addition, the constitutive law is modified to  
300 consider compressive hydric strains related to shrinkage.

Considering a domain  $\Omega \subset \mathbb{R}^d$ ,  $d = 2, 3$  containing the sample material in  
its initial configuration and its boundary  $\partial\Omega$  with an external normal unitary

vector  $\mathbf{N}$ , the equations are summarized as follows:

$$\left\{ \begin{array}{l} \nabla_X \cdot \mathbf{P} = 0 \text{ on } \Omega \\ \mathbf{S}(\mathbf{E}(\mathbf{u}), \mathbf{E}_h, d) = \frac{\mathcal{D}(d)}{h^2} \mathbb{C}(\mathbf{E} - \mathbf{E}_h) \\ \mathbf{E}_h = \alpha \mathbf{I} \\ \mathbf{u} = \bar{\mathbf{u}} \text{ on } \partial\Omega_u \\ \mathbf{P} \cdot \mathbf{N} = \bar{\mathbf{t}} \text{ on } \partial\Omega_t, \end{array} \right. \quad (2)$$

and

$$\left\{ \begin{array}{l} 2(1-d)\mathcal{H} - \frac{g_c}{\ell} \{d - \Delta_X d\} = 0 \text{ on } \Omega \\ \nabla_X d(\mathbf{x}) \cdot \mathbf{N} = 0 \text{ on } \partial\Omega. \end{array} \right. \quad (3)$$

Above,  $\mathbf{P}$  and  $\mathbf{S}$  are the first and second Piola-Kirchhoff stress tensors, respectively, which are related by  $\mathbf{P} = \mathbf{F}\mathbf{S}$ , where  $\mathbf{F} = \mathbf{I} + \nabla_X \mathbf{u}$  is the gradient of the transformation tensor,  $\nabla_X(\cdot)$  is the gradient with respect to the initial configuration, and  $\nabla_X \cdot (\cdot)$  is the divergence operator with respect to the initial configuration. The boundaries  $\partial\Omega_u$  and  $\partial\Omega_t$  denote the portions of  $\partial\Omega$  where the displacements  $\bar{\mathbf{u}}$  and tractions  $\bar{\mathbf{t}}$  are prescribed, respectively. In Eq. (2),  $\mathcal{D}(d)$  is a degradation function used to penalize the elastic tensor  $\mathbb{C}$ , and it is defined here as  $\mathcal{D}(d) = (1-d)^2$ . Furthermore,  $\mathbf{E}$  is the Green-Lagrange strain tensor, and  $\mathbf{E}_h$  is the hydric strain, with  $\mathbf{I}$  as the identity second-order tensor, and  $\alpha$  as an evolution parameter defined later in Section 4.3. Next, in Eq. (3),  $g_c$  is the fracture toughness,  $\ell$  is a regularization length parameter controlling the width of the crack, and  $\Delta_X(\cdot)$  is the Laplacian operator in the reference configuration. The function  $\mathcal{H}$  is a history function, which is introduced to satisfy the irreversibility of damage. It is expressed as (see [37]):

$$\mathcal{H}(\mathbf{x}, t) = \max_{\tau \in [0, t]} \{\Psi(\mathbf{E}(\mathbf{x}, \tau))\}, \quad (4)$$

with

$$\Psi(\mathbf{E}) = \frac{\lambda}{2} (\text{Tr}(\mathbf{E}))^2 + \mu \text{Tr}\{\mathbf{E}\}, \quad (5)$$

where  $\lambda$  and  $\mu$  are the elastic Lamé's parameters of the clay material in its solidified, uncracked state. The corresponding weak forms, which are applied

320 to solve the problem using finite elements, are provided as follows:

$$\int_{\Omega} \mathbf{S}(\mathbf{E}(\mathbf{u}), \mathbf{E}_h, d) : \delta \mathbf{E}(\mathbf{u}, \delta \mathbf{u}) d\Omega = \int_{\partial\Omega_N} \bar{\mathbf{t}} \cdot \delta \mathbf{u} d\Gamma, \quad (6)$$

and

$$\int_{\Omega} \left[ \left( \frac{g_c}{\ell} + 2\mathcal{H} \right) d \delta d + g_c \ell \nabla d \cdot \nabla (\delta d) \right] d\Omega = \int_{\Omega} 2\mathcal{H} \delta d d\Omega. \quad (7)$$

These equations are solved alternately in a so-called "staggered" scheme (see [26]). More details regarding the phase field method and its numerical solution procedures using FEM can be found in, e.g., [26, 36].

### 3.2. Numerical Finite Element model

325 The numerical model is schematically described in Fig. 11. For a given test (i.e., for one sample and a given number and distribution of inclusions), a digital twin is defined using the experimental data described in Section 2. Based on the measurements, the initial radius of the cylindrical cup is set to  $r_0 = 58$  mm, the radius of the inclusions is  $r_{inc} = 8$  mm, and the height is  $h_0 = 8$  mm. The clay material is contained within the domain  $\Omega$  (see Fig. 11 (a) for an example with three inclusions).  
330

To account for the damage mechanisms observed experimentally, the boundary conditions are defined as follows.

- Zero-displacement Dirichlet boundary conditions are initially applied on  $\partial\Omega$ , while stress-free conditions are considered at the boundaries of the inclusions, denoted collectively as  $\partial\Omega'$ . Shrinkage is then induced by increasing the eigenstrain  $\mathbf{E}_h = \alpha_h \mathbf{I}$  (see Eq. (2)<sub>3</sub>), where the parameter  $\alpha_h$  is defined in Section 4.3. The problem (2) is then solved at each time step.  
335
- When the condition  $d = 1$  is met for a subset  $\partial\Omega_{deb}$  of  $\partial\Omega$  (note that  $\partial\Omega_{deb}$  is not connected in general), the boundary conditions on  $\partial\Omega_{deb}$  are switched to free traction with the aim of modeling the sudden debonding observed during the experiments.  
340

- Concerning the bottom of the sample, denoted by  $\partial\Omega_{z=0}$ , the boundary conditions are vertically fixed to 0, but are free within the  $\mathbf{e}_x$ - $\mathbf{e}_y$  plane.

345

From a computational standpoint, both 2D plane strain and 3D simulations are carried out. The geometry of the sample is discretized using

350

- T3 elements in the 2D case, resulting in a mesh with  $N_{dof}^d = 15 \times 10^3$  degrees of freedom for the damage problem and  $N_{dof}^e = 2N_{dof}^d$  degrees of freedom for the elastic problem; and
- T4 elements for the 3D case. The mesh contains  $N_e = 400 \times 10^3$  elements, corresponding to  $N_{dof}^d = 80 \times 10^3$  degrees of freedom for the damage

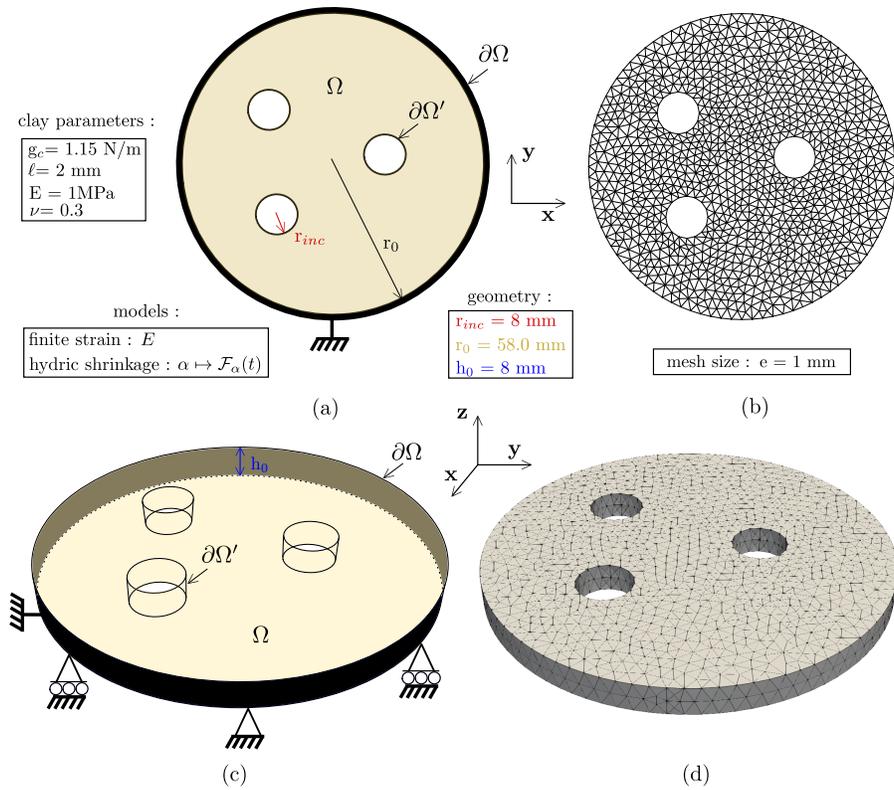


Figure 11: Numerical FEM model for a specific configuration: (a) geometry and boundary conditions for the 2D model; (b) 2D mesh; (c) geometry and boundary conditions for the 3D model; (d) 3D mesh.

problem and  $N_{dof}^e = 3 \times N_{dof}^d$  for the elastic problem.

An in-house implementation combining C++ and MATLAB codes was used.

355 The computational times on a computer equipped with an Intel(R) Xeon (R) platinum 8168 CPU (with 1 TB of RAM and  $2 \times 24$  cores rated at 2.7 GHz) ranged from 0.7 to 3 s per time step for the 2D case and from 55 to 150 s for the 3D case. The total computational times based on 1000 time steps were 2.5 h and 42 h for each 2D or 3D case, respectively.

#### 360 4. Model identification

In this section, we provide the values of the model parameters and identification procedures when required.

##### *4.1. Mechanical parameters*

The mechanical parameters used in the model described in Section 3 were  
365 extracted from the literature. The Young's modulus for the clay material was taken as  $E = 1$  MPa according to the study in [20], in which a simulation of crack propagation in the 2D plane was conducted with a cohesive zone model on clayey materials. This modulus was defined experimentally in [38], where a variable Young's modulus was defined as a function of the compaction of  
370 the material. In our clayey material, we assumed a similar situation when the material solidified according to the decrease in the water content. In [20], the modulus was averaged over the compaction range. This hypothesis was retained for the numerical study. Other authors [39] have proposed a variation of this modulus according to the water content in the experimental investigations of  
375 [40] on Barcelona soil and [41] on Bioley clayey silt. The Poisson's ratio was chosen as  $\nu = 0.3$  [20].

##### *4.2. Fracture parameters*

The damage parameters used in the following simulations require estimation of the toughness  $g_c$  and the regularization length  $\ell$ , as this parameter is interpreted here as a material parameter (see the discussion in [42]). Many authors  
380

have studied the impact of water content on crack resistance. In [43], a comprehensive review of the environmental factors in the cracking process was provided through multiple experiments. More specifically, numerous studies have allowed the identification of the parameters for linear mechanical fracture models through experiments (see [44, 45, 41]). The parameters used are often the stress intensity factor  $K_I$  or the tensile limit stress  $\sigma_t$ , which can then be theoretically related to the pair of parameters  $(g_c, \ell)$  in the phase field method (see [42]). Following these studies, a numerical investigation of clay desiccation using the phase field method was reported in [39]. The parameters used were the toughness depending on the water content ( $g_c = 1.12$  ( $w_r = 0\%$ ) –  $1.71$  ( $w_r = 100\%$ ) N/m) and a constant regularization length  $\ell = 2$  mm. In our study, we consider the toughness as a constant, with  $g_c = 1.15$  N/m, and the regularization length is set as  $\ell = 2$  mm.

### 4.3. Hydric model

In this section, we describe the procedure for identifying the function  $\alpha_h(t)$  of the coupled hydric shrinkage model (see Eq. (2)) based on the experiments. Experimental test  $S_0$  described in Section 2.2.3 was used, where a homogeneous circular sample was considered. In this test, the sample shrank without cracking and remained circular during the drying process. Denoting the experimental radius as  $R^{\text{exp}}(t)$ , it is possible to record the evolution of this radius as a function of time. This evolution is illustrated in Fig. 5 and the radius is reported in Fig. 12 (left).

The experimental evolution of the radius (red dots) was fitted using the  $R_t$  function, defined as follows:

$$R_t(t) = A(1 + \tanh(B(t - \phi))) + R_\infty, \quad (8)$$

Here,  $A$  is the amplitude, which is defined as

$$A = \frac{1}{2}(R_\infty - R_0), \quad (9)$$

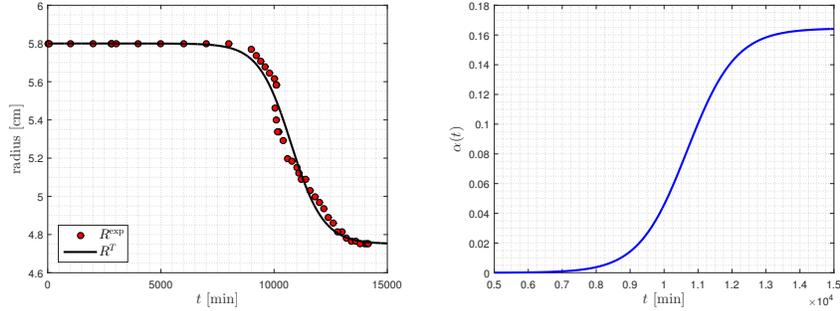


Figure 12: (Left) Identified  $R_t$  function, (right) the identified shrinkage parameter  $\alpha_h$ .

where  $R_0$  and  $R_\infty$  denote the initial and final radii, respectively.  $B$  is the transition time between radii  $R_0$  and  $R_\infty$  according to

$$B = \frac{1}{\Delta T}, \quad (10)$$

where  $\Delta T$  is the transition time, and  $\phi$  represents the shift in the hyperbolic tangent function. The other parameters are obtained for  $\phi = 10750$  and  $B = 7.0 \cdot 10^{-4}$ , corresponding to  $\Delta T = 1.4 \cdot 10^3$ . Fig. 12 shows both the experimental results and the fitted radius function (black line).

Then, the parameter  $\alpha_h$  was identified using the numerical model presented in the previous section without any inclusions (sample  $S_0$ , see Fig. 1); it is directly related to the radius shrinkage measurements. Thus, we obtained the time function for  $\alpha_h$ , as illustrated in Fig. 12 (right). In the following simulations, the identified function  $\alpha_h(t)$  is used, even for the simulations involving cracks, bearing in mind that this is a strong simplification of the model. The other parameters are summarized in Table 2.

E	$\nu$	$g_c$	$\ell$	e
1 MPa	0.3	1.15 N/m	2 mm	1 mm

Table 2: Identified parameters for the shrinkage model.

## 5. Comparison between experiments and simulations

In this section, the numerical model presented in Section 3 is compared with the experimental results presented in Section 2. In all cases, comparisons are made at time  $t_3$  (see Section 2.2.2).

### 420 5.1. Final morphology of crack patterns

Comparisons between the final crack patterns in the experiments and those obtained with the simulation model are presented in Figs. 13-16. A summary of the observations and qualitative agreement is provided in Table 3. Based on these results, we can note that a qualitative agreement regarding the final  
425 crack patterns is obtained. In the different cases, the simulation model correctly captures most of the phenomena, including the initiation of cracks from the inclusion boundaries and radial development of cracks. Moreover, when many inclusions are involved, the cracks merge between the inclusions and then propagate through the external boundary of the sample. In the 3D simulations,  
430 there is disagreement in the crack morphology in the case of a single inclusion: while three cracks were observed in the experiment, two branching cracks propagate in the model results. One possible reason is the implementation of a zero-displacement boundary condition along the external boundary in the 3D model while the interior of the sample is allowed to shrink along the  $z$  vertical  
435 direction, which creates a more complex strain state. In spite of this, most of the qualitative results are found to be in fairly good agreement with the experimental observations. However, it is worth noting that the experimental and simulation times at which the images are shown are not always the same. This is mainly because the shrinkage model (12) was identified experimentally on  
440 a sample without any inclusions. Thus, the present shrinkage model does not properly consider the interactions with other inclusions. Therefore, the simulations that exhibited the best agreement with the experiments were taken for times that did not always match those of the experiments. We are aware that the present shrinkage model is overly empirical and that it should be improved

445 by using more micro-mechanical foundations to consider the hydro-mechanical coupling and allow its use in more complex configurations.

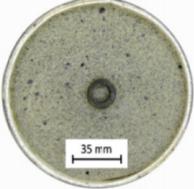
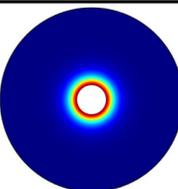
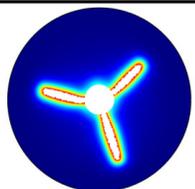
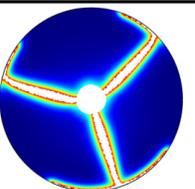
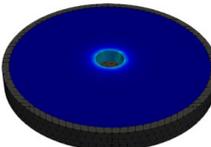
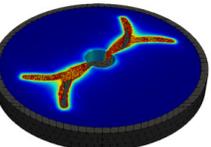
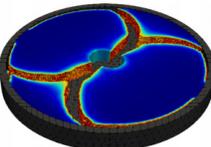
$S_{1c}$	$t_1$	$t_2$	$t_3$
Experimental	 4000 min	 6000 min	 8000 min
Simulation 2D	 6000 min	 7000 min	 8000 min
Simulation 3D	 5700 min	 6150 min	 8000 min

Figure 13: Crack pattern in  $S_{1c}$ .

### 5.2. Spherical strains

In Figs. 17-20, the local spherical strains measured experimentally as described in Section 2.2 are compared with the numerical simulations.

450 A global negative shrinkage was obtained in the experiments and is well reproduced in the simulations (see Fig. 17). This agreement is less obvious in the case of multiple inclusions (see Figs. 19 and 20) where the local fields are strongly perturbed by the open cracks. Furthermore, we note that even in the  $S_{1c}$  case, regions of compressive strain localization are observed in the  
 455 experiments near the external boundary, but these are not reproduced in the

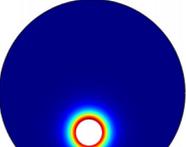
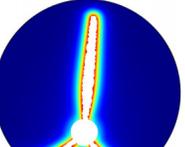
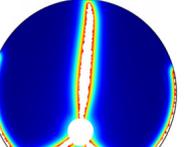
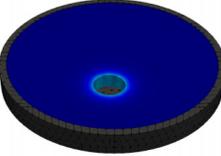
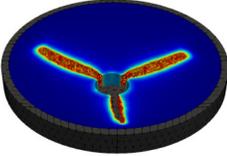
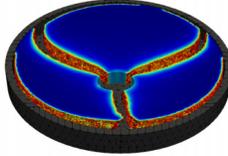
$S_1$	$t_1$	$t_2$	$t_3$
Experimental	 4000 min	 6000 min	 8000 min
Simulation 2D	 6000 min	 7000 min	 8000 min
Simulation 3D	 5900 min	 6200 min	 8000 min

Figure 14: Crack pattern in  $S_1$ .

simulations, even when cracks are not initiated (see Fig. 20). The compression ring around the inclusion is reproduced well by the simulation model.

We note that the strain localizes in the vicinity of the cracks in both the experiments and simulations. The graphic representations were adapted by  
460 thresholding the damage to  $d = 0.8$ .

### 5.3. Deviatoric strains

In Figs. 21-24, local deviatoric strains measured experimentally as described in Section 2.1.3 are compared with the numerical simulation results.

The conclusions are similar to those for spherical strains. Even though the  
465 global distribution is satisfactorily reproduced by the numerical model at the beginning of the simulation (without cracks), regions of deviatoric strain local-

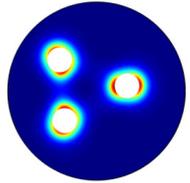
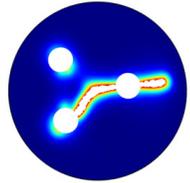
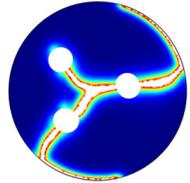
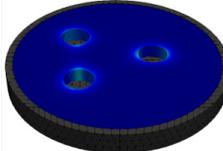
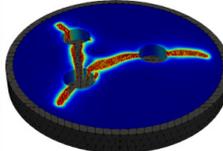
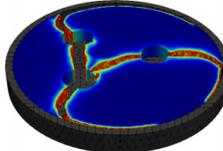
$S_3$	$t_1$	$t_2$	$t_3$
Experimental	 4000 min	 6000 min	 8000 min
Simulation 2D	 6000 min	 7000 min	 8000 min
Simulation 3D	 5820 min	 6000 min	 7000 min

Figure 15: Crack pattern in  $S_3$ .

ization near the external boundary that are observed experimentally are not reproduced by the present model. The origin of this phenomenon has not yet been clearly identified. However, once the cracks propagate, the simulation  
470 generates a dilatation deformation of high intensity along its path, as in the experiment.

#### 5.4. Remarks about experimental variability

As expected, the experiments show a major influence of the presence and positions of the inclusions on the final crack patterns. Another important  
475 observation is that in all of the experiments presented in this study, the cracks initiate from the boundary of the inclusions and then propagate into the matrix to the external boundary of the sample. This is also shown in the strain fields,

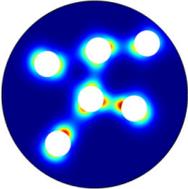
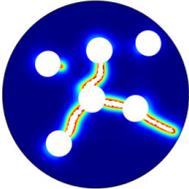
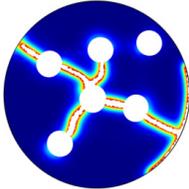
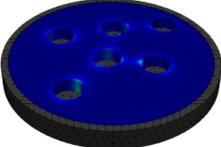
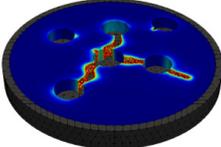
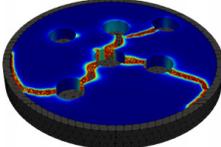
$S_6$	$t_1$	$t_2$	$t_3$
Experimental	 4000 min	 6000 min	 8000 min
Simulation 2D	 6000 min	 7000 min	 8000 min
Simulation 3D	 5550 min	 5700 min	 6000 min

Figure 16: Crack pattern in  $S_6$ .

which exhibit strain concentrations near the boundaries of the inclusions at the early stages of the experiments, while this is not the case away from the inclusions. However, the deformation and cracking patterns are also characterized by a strong stochastic nature, as illustrated in Fig. 25. Here, three experimental realizations of the  $S_3$  test were performed. It can be seen that the final crack geometry and local strain fields can differ significantly. We can assume that several phenomena are at the origin of the variability in the final crack distribution, listed as follows.

1. The friction on the bottom of the sample was experimentally limited by the application of a Teflon layer. However, this surface interaction remains to be quantified. Numerical studies on an experimental basis [46, 47] have

Exp.	2D simulation agreement	3D simulation agreement
$S_{1c}$		
3 cracks initiated from the inclusion and developing radially	Good agreement	Partial agreement: 2 cracks initiated from the inclusion but branching
$S_1$		
4 cracks initiated from the inclusion and developing radially	Partial agreement: 3 cracks initiated from the inclusion and developing radially	Partial agreement: 3 cracks initiated from the inclusion and developing radially
$S_3$		
Several cracks initiated from the inclusions	Good agreement	Good agreement
Inclusions cracks merge and propagate to the external boundary	Good agreement	Good agreement
$S_5$		
Several cracks initiated from the inclusions	Good agreement	Good agreement
Inclusions cracks merge and propagate to the external boundary	Good agreement	Good agreement

Table 3: Qualitative comparisons between experimental and simulated crack patterns during shrinkage of clay samples.

investigated the influence of this interaction on the cracking process.

- 490 2. The material parameters within the sample are time-dependent and heterogeneous at the micro-scale. The drying process may increase these heterogeneities. For example, the local fluctuations observed in the cen-

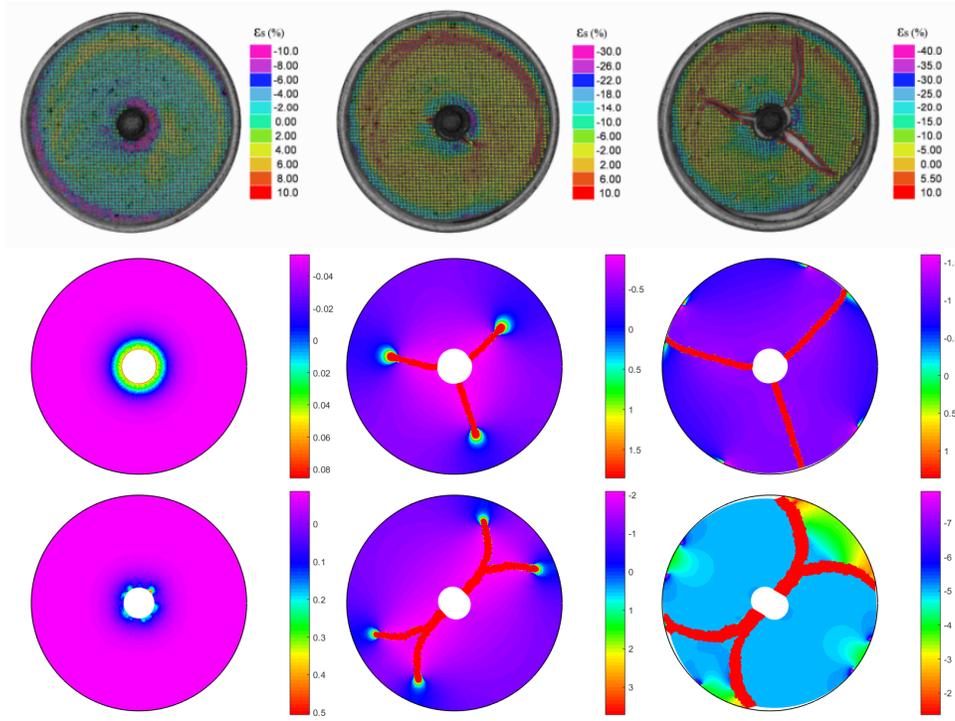


Figure 17: Spherical strain map in  $S_{1C}$ : experimental results (top), numerical results in 2D (middle), and numerical results in 3D (bottom); the times are the same than in Fig. 13.

tral part of the  $S_1$  sample (see Fig. 18) may be associated with such constitutive heterogeneities.

- 495 3. The debonding at the external surface of the sample occurs randomly during the experiments.

To investigate the effects of decohesion on the boundary, we conducted additional simulations, as shown in Fig. 26, where debonding was artificially induced at several locations on the external boundary. From Fig. 26, we can  
 500 note that the location of decohesion on the boundary has a strong impact on the distributions of cracks within the sample and seems to be consistent with the experimental observations in Fig. 25. It is worth noting that in all cases, the cracks still started from the inclusion boundaries and propagated to the external boundary, which was also observed experimentally.

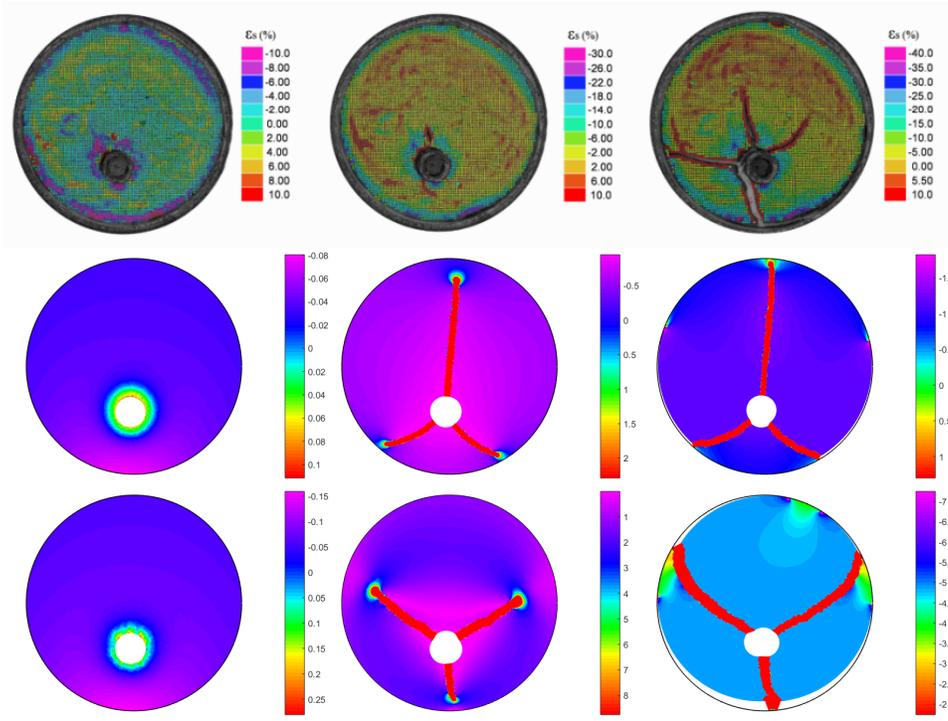


Figure 18: Spherical strain map in  $S_1$ : experimental results (top), numerical results in 2D (middle), and numerical results in 3D (bottom); the times are the same than in Fig. 14.

505 *5.5. Discussion*

In the above comparisons, we observed some partial, yet encouraging, qualitative agreement between the experiments and simulations. Globally, the initiation of cracks is well reproduced near the inclusions, and the propagation paths between inclusions and to the external boundary of the samples are well reproduced by the model. The global shrinkage of the sample is modeled satisfactorily through both spherical and deviatoric strains for simple configurations, but zones of localizations observed in the experiments are not captured by the present model. Possible reasons for the observed discrepancies and suggestions for future investigations are summarized as follows.

- 515 1. The zero-displacement boundary condition applied at the outer edge is a first approximation of the frictional forces that prevent the clay matrix

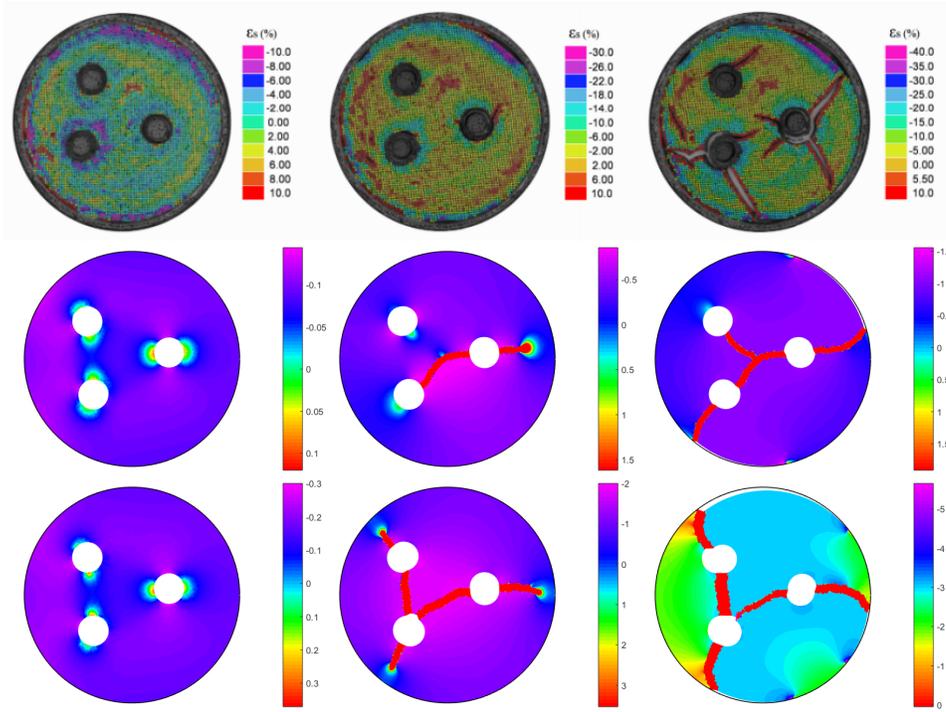


Figure 19: Spherical strain map  $S_3$ : experimental results (top), numerical results in 2D (middle), and numerical results in 3D (bottom); the times are the same than in Fig. 15.

from shrinking freely, hence creating the conditions for cracking. However, it was observed that the clay matrix slightly lifted off the bottom at the edges, and the amplitude of this phenomenon increased with time. To illustrate this point, results are shown in Fig. ?? where the sample is not restrained on its outer edges, but in a zone of small thickness close to the edges and in the clay. In this case, the edge effects observed experimentally are qualitatively reproduced.

520

2. A localized, compressive strain zone is experimentally observed close to the outer edge and at the interface between the inclusions and the matrix. This complex interface phenomenon requires further study and is presently not captured by our model.

525

3. The parameters that were used in the simulations were considered to be

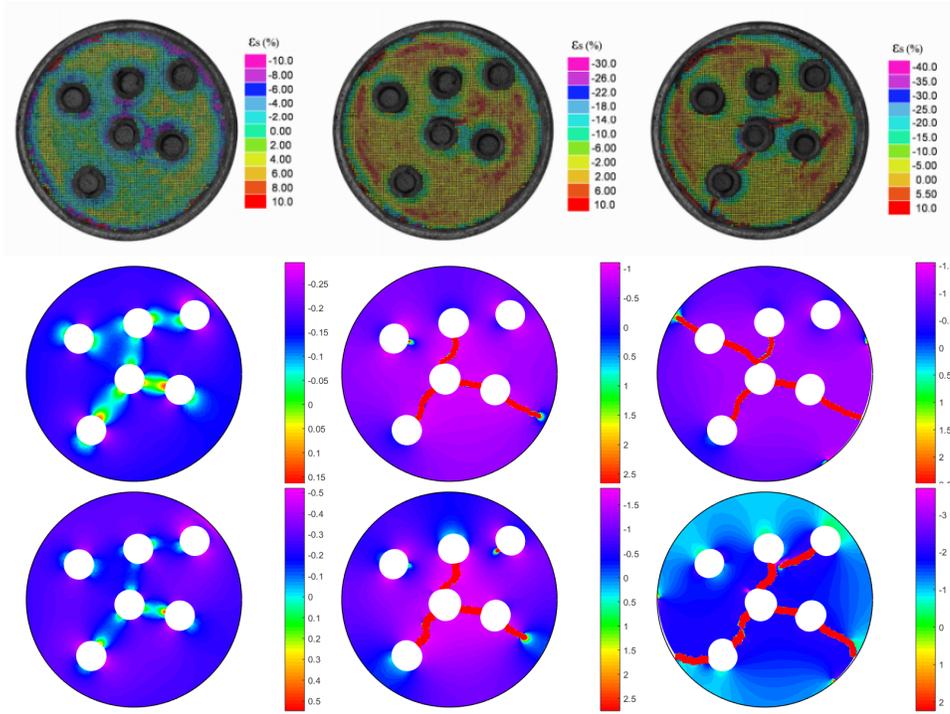


Figure 20: Spherical strain map in  $S_6$ : experimental results (top), numerical results in 2D (middle), and numerical results in 3D (bottom); the times are the same than in Fig. 16.

constants. However, some experimental results available in the literature  
 530 suggest that these parameters may vary spatially and depend on the water  
 content. In this context, a more advanced experimental campaign involv-  
 ing variable water contents could enhance the predictive capability of the  
 computational model.

4. A large discrepancy is also observed in terms of the crack propagation ve-  
 535 locity (quantified by the crack extension in a quasi-static regime). While  
 crack propagation is typically observed over long time periods in the ex-  
 periments (approximately 3000 to 4000 min), the crack propagation is  
 approximately 10 to 20 times faster in the corresponding simulations.

5. The frictional forces on the bottom of the specimen are not included in  
 540 the present model.

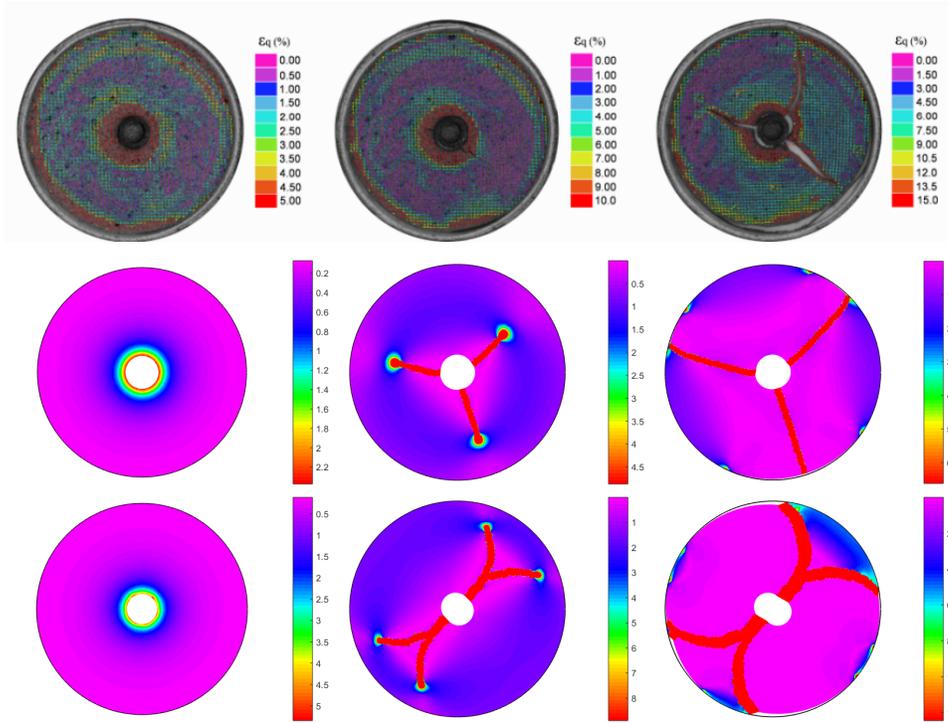


Figure 21: Deviatoric strain map in  $S_{1C}$ : experimental results (top), numerical results in 2D (middle), and numerical results in 3D (bottom); the times are the same than in Fig. 13.

6. As shown in Fig. 26, debonding of the sample at the external boundary has a direct impact on the local crack path. In our simulations, the cohesion at the external boundaries is assumed to be perfect until a crack reaches the external boundary.
- 545 7. The different sources of uncertainties mentioned in Section 5.4 are not included in the numerical model, which is another large source of discrepancies. Therefore, more quantitative comparisons should involve statistical data instead of direct deterministic comparisons.

## 6. Conclusion

550 In this study, we provided experimental results for a heterogeneous clay sample during desiccation and compared them with a numerical simulation model.

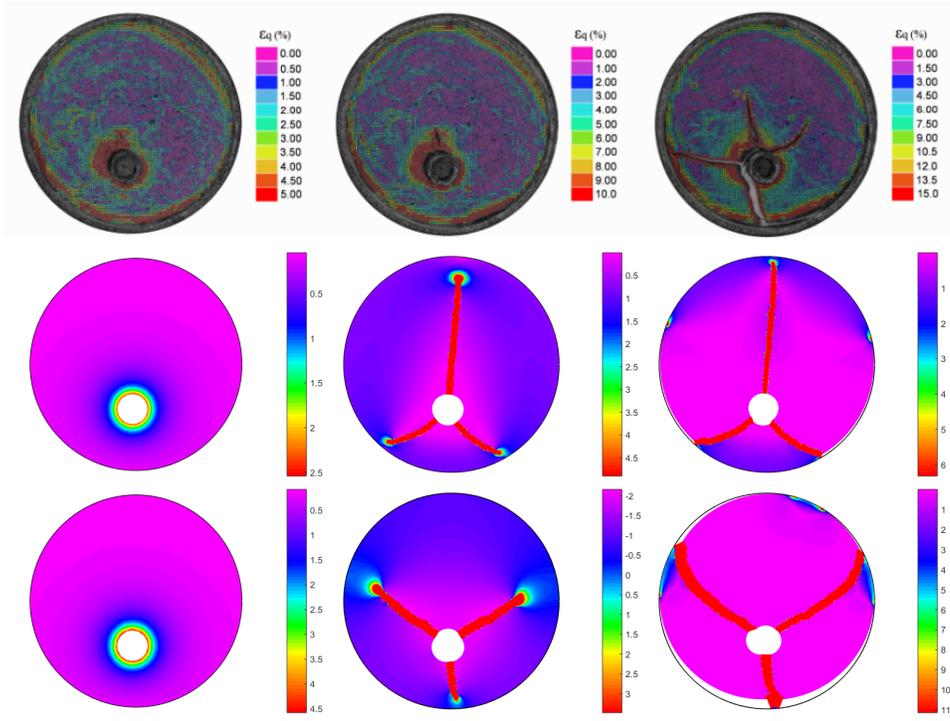


Figure 22: Deviatoric strain map in  $S_1$ : experimental results (top), numerical results in 2D (middle), and numerical results in 3D (bottom); the times are the same than in Fig. 14.

The main contributions and results of this study are highlighted as follows.

- We provided original experimental results for clay samples containing rigid inclusions, inducing more localized strain fields and a reduction in the uncertainties related to crack propagation during drying.
- For samples including one, three, and six inclusions, we provided experimental observations of final crack paths as well as local strain fields using DIC.
- We observed that in all cases, the cracks initiated from inclusion boundaries and then propagated to the external boundary of the sample.
- It was shown that debonding of the sample from the external boundary strongly affected the direction and number of cracks.

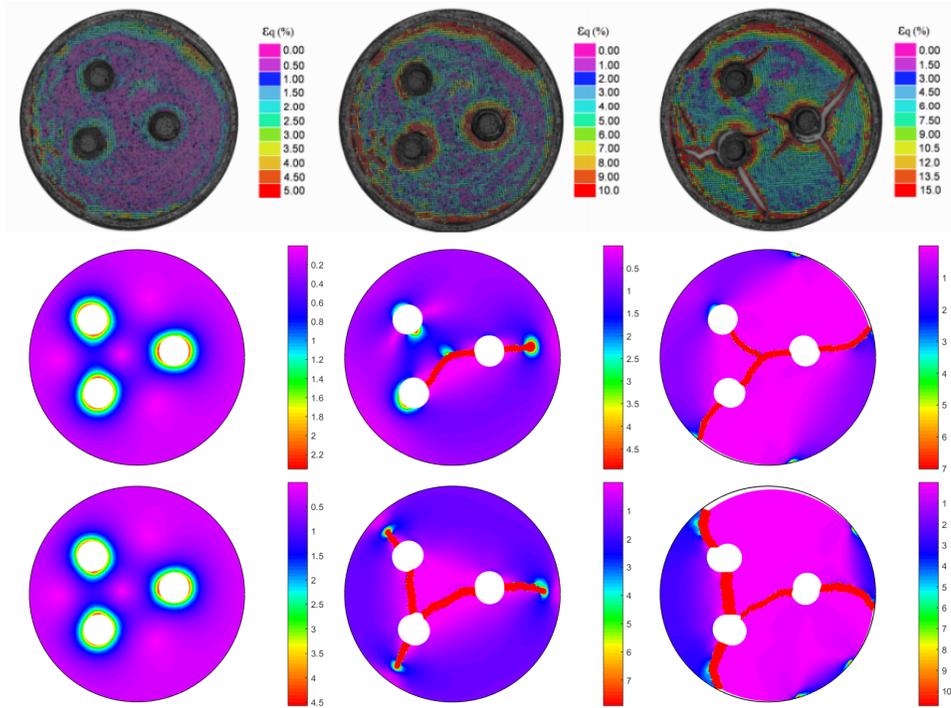


Figure 23: Deviatoric strain map in  $S_3$ : experimental results (top), numerical results in 2D (middle), and numerical results in 3D (bottom); the times are the same than in Fig. 15.

- A numerical continuous crack propagation model based on a phase field was used to simulate the heterogeneous shrinking process and for comparison with the experiments.
- While good qualitative agreement with the experimental results was generally observed, several discrepancies remain: (i) in the experiments, a region of compressive strain occurs near the external boundary, which is not reproduced in the simulations; (ii) the global compressive strain within the samples observed experimentally is well reproduced by the model, but local strains remain poorly captured; (iii) crack paths, including the number of cracks, directions, and branching is only qualitatively reproduced by the numerical model.
- We introduced a discussion of the possible sources of these discrepancies,

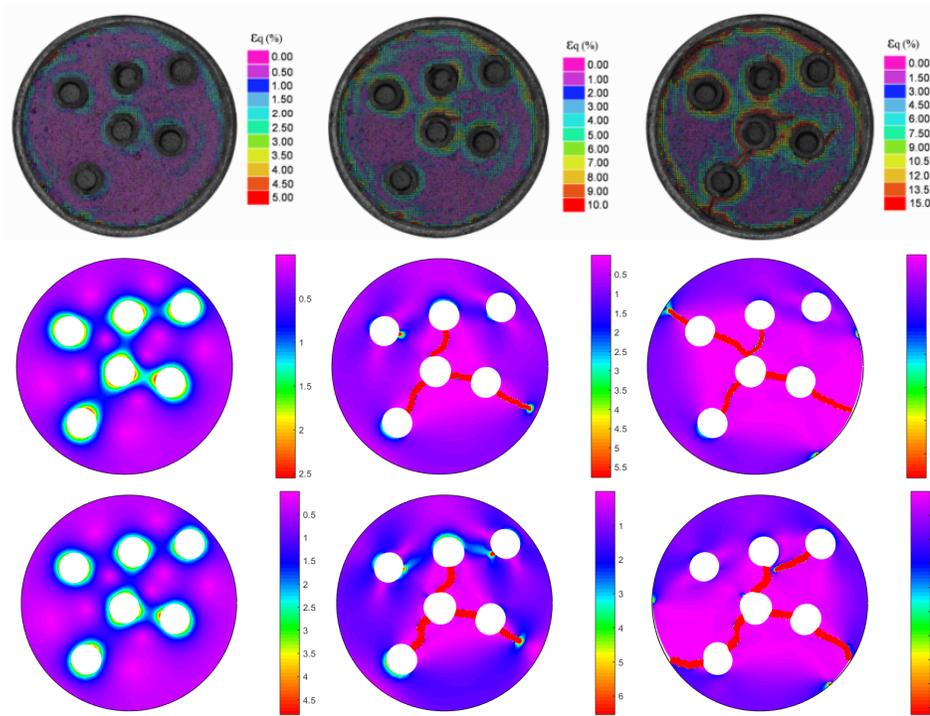


Figure 24: Deviatoric strain map in  $S_6$ : experimental results (top), numerical results in 2D (middle), and numerical results in 3D (bottom); the times are the same than in Fig. 16.



Figure 25: Different crack patterns observed for the same configuration with 3 inclusions.

575 including: (i) uncertainties related to external boundary decohesion and variations in local material parameters; (ii) the frictional effects of the lower boundary during drying; and (iii) the possibility of a more complex hydro-thermo-mechanical coupling, especially during the phase change of the clay matrix from liquid to solid, which is not included in the present

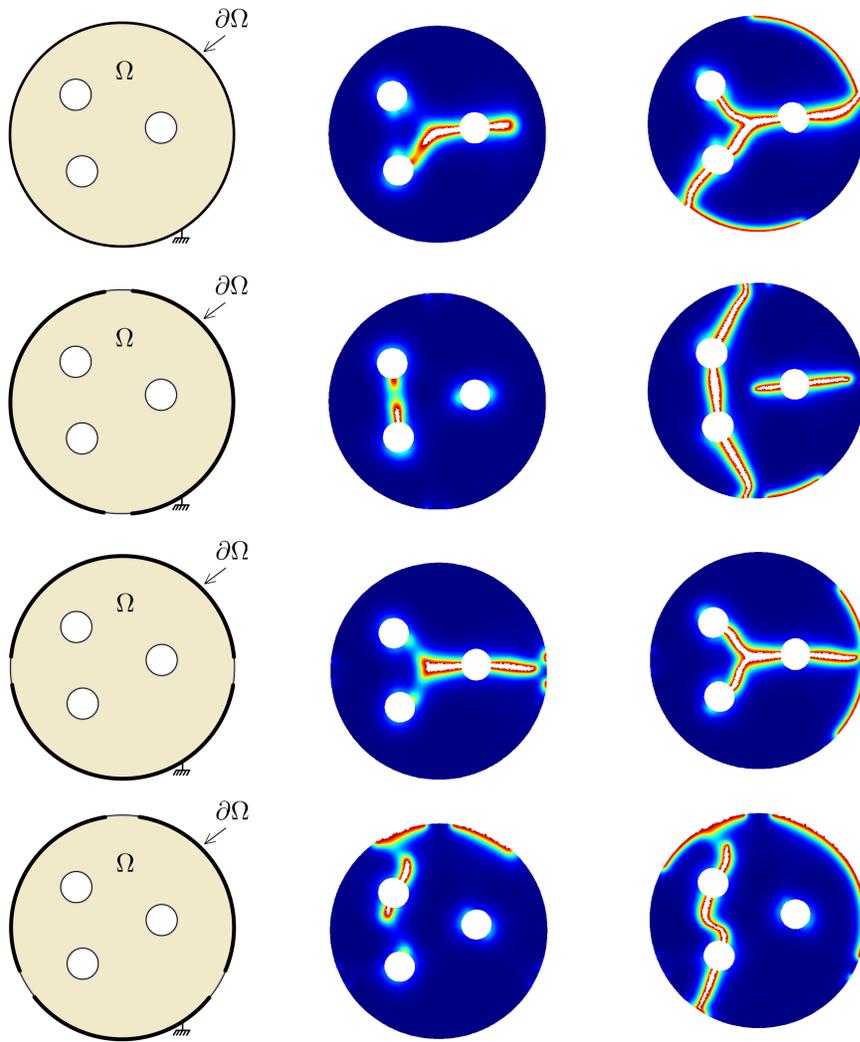


Figure 26: 2D crack evolution for different wall debonding locations: boundaries conditions, initiation and crack paths.

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model.

## 7. Acknowledgements

The authors gratefully acknowledge the financial support from the LABEX Multi-Scale Modeling & Experimentation of Materials for Sustainable Construction (MMCD) through ANR Investments for the Future program ANR-585 11-LABX-022-01.

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