Coupling of evolution strategy and lagrangian augmented algorithm for optimal design of mechanical system

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Abstract : This article presents a new optimization algorithm for nonlinear optimization problems with mixed variables. This algorithm is based on a coupling of deterministic and evolutionnary methods of optimization. We first point out the general context of optimal design problems and the characteristics of the these nonlinear problems with mixed variables. We briefly recall the general principles of the evolutionnary and deterministic methods. Then the principle of coupling between these two approaches is presented. Finally we discuss the results obtained on a test problem of the literature and a problem of dimensioning of a gear pair. We show in both cases the profits compared to evolutionnary methods in terms of reliability and speed of convergence.

Résumé : Cet article présente un nouvel algorithme d'optimisation destiné aux problèmes d'optimisation non linéaires en variables mixtes. Cet algorithme repose sur un couplage d'une méthode déterministe et évolutionnaire d'optimisation. Nous rappelons tout d'abord le contexte général des problèmes de conception optimale et les particularités de ces problèmes non linéaires en variables mixtes. Les principes généraux des méthodes évolutionnaires et déterministes sont ensuite exposés. Puis le principe du couplage entre ces deux approches est présenté. Enfin nous exposons les résultats obtenus sur un problème test de la littérature et ensuite sur un problème de dimensionnement d'un train engrenage. Nous montrons dans les deux cas les gains obtenus par rapport à une méthode évolutionnaire seule en termes de précision et de vitesse de convergence.

1. Introduction : In today's Computer Aided Design software, a mechanical system is essentially represented by the volumes of its parts. Even if concepts of feature and complex assembly structure have been added to this representation, it is not well suited for an easy design of the mechanical system. Dimensioning mechanical component is quite difficult with this kind of software. Our aim is to propose design-making tools, which can be introduced in Computer Aided Design software. Thank to the analysis of the design process of a known mechanical system, we can formulate the mechanical design problem as an optimization problem, called problem of optimal design. The latter contains non-linear equations, inequality constraints and mixed variables that are continuous and discrete. There are also interdependent discrete parameters whose values may be taken in normalized tables and which directly depend on the choice of one of the discrete variables.

These problems of optimal design have been solved with genetic algorithm [3] and evolution strategy [5]. The obtained results have shown the efficiency of these algorithms, but many evaluations of function

are necessary to reach a good optimal solution. In this paper, a hybrid evolution strategy is presented to solve this optimal design problems with a low number of evaluations.

2. The optimal design problem : The general mixed discrete continuous problem is defined as:
Minimize

$$f(x)$$

Subject to :

$$g_j(x) \le 0, j = 1..m$$

$$g_j(x) = 0, j = m+1..m+l$$

$$u \le x \le v, \quad u,v,x \in R^n = R^{n_C} \cup R^{n_D}$$

where f is the objective function and g_j are nonlinear inequality constraint functions. The components of the mixed variable vector $x = (x_C, x_D)^T$ are divided into n_C continous variables, expressed as $x_C \in R^{n_C}$ and n_D continous variables, expressed as $x_D \in E^{n_D} \subset R^{n_D}$. Here $E^{n_D} \subset R^{n_D}$ is the space of discrete values, acceptable by the problem and it can be randomly distributed. In this class of optimal design problem, we consider that the first order derivatives of the objective function and constraints with respect the continuous variables exist, but the first derivatives of the functions with respect to the discrete variables are supposed unknown. Moreover, this type of problems are strongly nonlinear and typically contain about ten variables.

3. Evolutionary strategies : Evolutionary algorithms have started to receive significant attention during the last decade [1,2,6,7]. Part of their success is due to the large domain of application of those methods, their robustness and their gain of flexibility. Those methods are specially well suited for solving difficult optimization problems [9,14,15,16]. However, their computational cost is generally very high. A large number of evaluations must generally be performed for a satisfying result to be found. Evolution strategies are based on the principle of evolution, i.e. survival of the fittest. Unlike classical methods, they do not use a single search point but a population of points called individuals. Each individual represents a potential solution to the problem. At each iteration, called a generation, a new population of χ offspring individuals is created by means of recombination and mutation, starting from the old population of μ parents ($\chi > \mu$). This new population is then evaluated with an evaluation function. This evaluation function is equal to the function to be minimized when the problem does not contain any constraint. It may be a quadratic penalty function for handling constraints of the optimization problem [10]. The μ best individuals out of the population of parents and offspring are selected to reproduce and replace the old population of parents. By means of these randomized processes, the population evolves toward better and better regions of the search space.

4. Deterministic methods : On the other hand, deterministic methods, which exploit local information like gradient information are rapid methods when the gradients of the functions can be calculated. But their domain of application is reduced to the class of differentiable problems. Deterministic methods assume that the objective and constraint functions are differentiable. They do not use probabilistic rules and they generally converge quickly. Those methods have a single search point. At each iteration *k*, a new point x_C^{k+1} is determined by the point x_C^k and a step in a calculated direction d^k such as : $x_C^{k+1} = x_C^k + \alpha^k . d^k$. The calculation of the step α^k and the direction d^k depend on the chosen deterministic method. They are generally based on the gradients of the functions.

5. Coupling evolutionary and deterministic methods : In order to combine the respective advantages of the deterministic

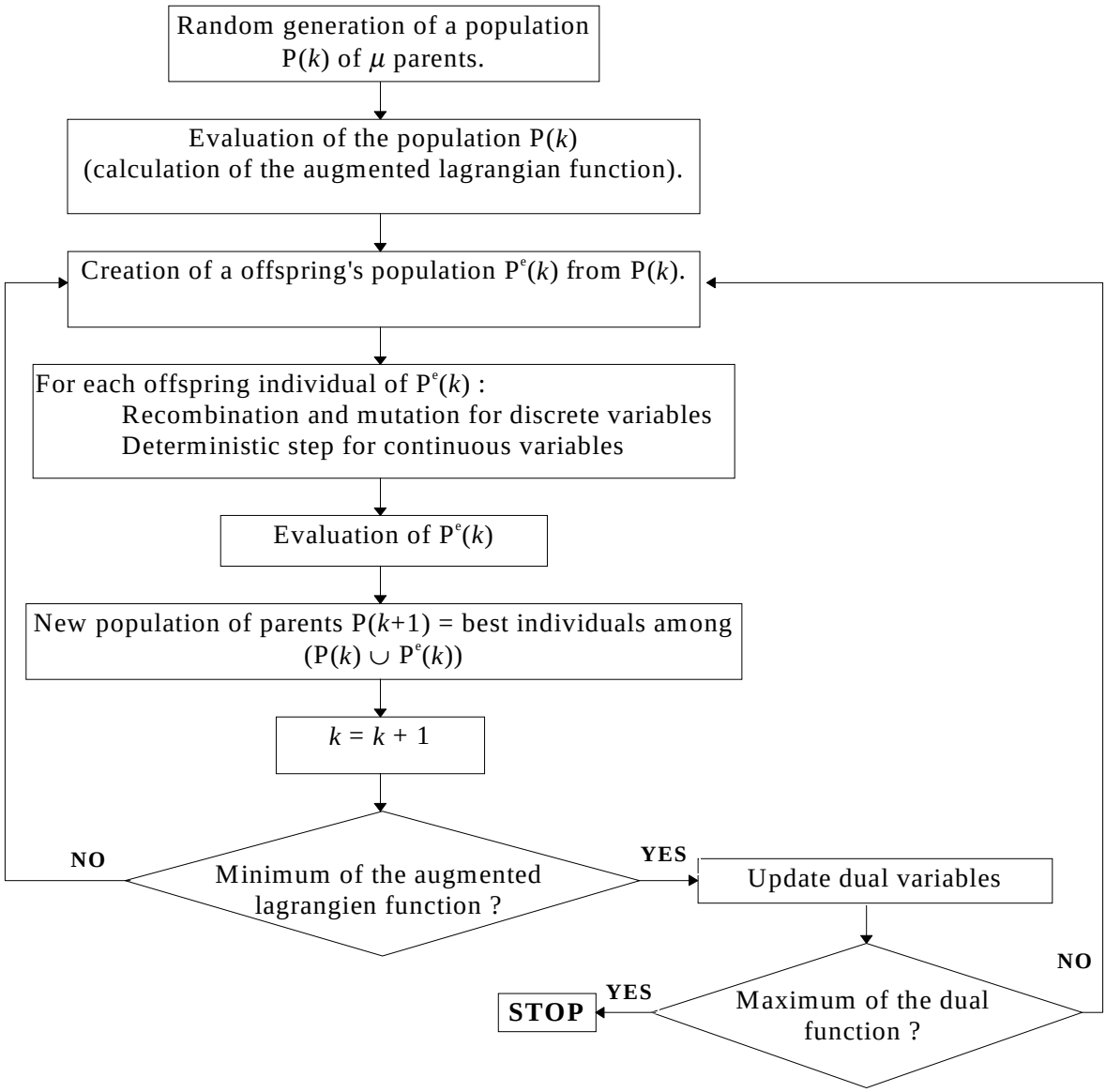

Figure 1 : Hybrid algorithm

method and evolution strategy we have proposed to couple the two methods. We have chosen to use a dual formulation of the problem with the augmented lagrangian function. This function is used to evaluate each individual in the evolution strategy. The algorithm is based on a maximization of the dual augmented function with respect to the dual variables. This dual augmented function is obtained by a minimization of the augmented lagrangian function with respect the primal variables, i.e the mixed-continuous variables of the optimization problem. In this paper we are handling a population of μ parents, where each individual is represented by a 3 vectors $x = (x_C, x_D, p)^T$. x_C, x_D are the continuous and discrete variables associated to the individual. p are strategy parameters which control the application of mutation.

The basic iteration of the algorithm consists in updating the discrete variable x_D by creating χ offspring individuals by means of recombination and mutation of the parent individuals. The continuous variables x_C of each individual are updated by a minimization along a descent direction d of the augmented lagrangian function. These updated continuous variables are the new continuous variables of each offspring.

This descent direction d is a quasi-Newton direction based on the approximate Hessian

matrix of the augmented lagrangian function. The step α along this direction is calculated by using the monotonicity analysis [13] when the constraints and objective function of the optimal design problem are monotonic. Here $\nabla f(x^k)$ and $\nabla g_j(x^k)$ are the gradients of the objective and constraint functions with respect to the continuous variables, the discrete variables being kept fixed. The method of monotonicity analysis is based on the identification of active or inactive constraints.

$$\left.\frac{df}{d\alpha}\right|_{\alpha=0} = \nabla f(x^k)^T d \qquad (1)$$

and $$\left.\frac{dg_j}{d\alpha}\right|_{\alpha=0} = \nabla g_j(x^k)^T d \text{ , j=1,.., m} \qquad (2)$$

For motion in the descent direction *d*, we identify constraints for which the derivatives in equations (1) and (2) have opposite signs. Only constraints showing an opposite derivative sign to d*f*/dα are able to be active.
We use a linear approximation of the constraints to compute the motion α_j corresponding to each of these candidate active constraints.

$$\alpha_j = -\frac{g_j(x^k)}{\nabla g_j^T(x^k).d}, \qquad (3)$$

The constraint showing the smallest value of α_j is considered dominant and defines the step size along the direction *d*.
When the objective and constraints function are not monotonic, the step α along the descent direction *d* is calculated with a golden section search coupled with a polynomial interpolation.
This basic iteration is repeat until the minimum of the augmented lagrangian function is reached. The dual variables, i.e the lagrange multipliers are then updated and the overall procedure is repeated until the maximum of the dual function is reached.

6. Application and results :

We have applied this hybrid algorithm to several problems of optimal design. We present here the detail of the results obtained on two of these problems.

6.1 Pressure vessel : The first one is the design of pressure vessel [17]. The objective is to minimize the total manufacturing cost for the pressure vessel. The design variables are the dimensions T_s (the shell thickness), T_h (the spherical head thickness), R (the radius of the cylindrical shell), and L (the length of the shell).

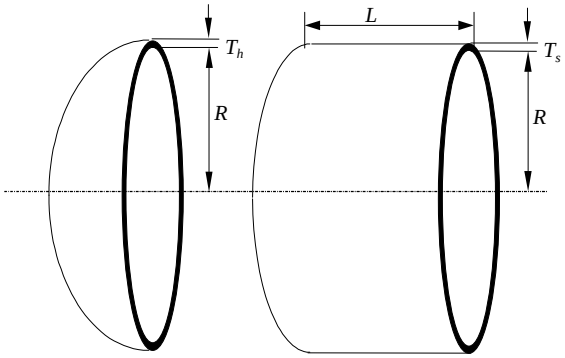

Figure 2: Pressure vessel

The optimization problem contains 2 discrete variables (T_s, T_h), 2 continuous variables (R, L) and 7 inequality constraints. Its equations are as follows:

Minimize the objective function :
$f(x) = 0{,}6224.T_s.R.L + 1{,}7781.T_h.R^2$
$+ 3{,}1611.T_s^2 L + 19{,}84.T_s^2.R$

Subject to the non linear inequality constraints :

$g_1(x) = 0{,}0193.R - T_s \le 0$
$g_2(x) = 0{,}00954.R - T_h \le 0$
$g_3(x) = 752 \text{ x } 1728 - \pi.R^2.L^2 - (4\pi.R^2)/3 \le 0$
$g_4(x) = 1{,}1 - T_s \le 0$
$g_5(x) = 0{,}6 - T_h \le 0$

Variables : $x = \{ T_s, T_h, R, L \}^T$

Bounds on variables :

$1{,}1 \le T_s \le 1{,}9375$

$0{,}5625 \le T_h \le 1{,}5$

$50 \le R \le 70$

$30 \le L \le 50$

This problem belong to a set test problems for which we know the theoretical optimal solution. Here , this solution is :

$$x^{theo} = \{\ 1{,}125;\ 0{,}625;\ 58{,}29;\ 43{,}69\ \}^{T}.$$
$$f^{theo} = 7197{,}7289$$

The results presented in table 1 were obtained at the end of 15 executions for a population of 7 parents and 50 offsprings. The error between the theoretical optimum and the best solution identified by the hybrid algorithm is indicated in the column "Error". The column "Average Error %" specifies the average error on 15 executions. The column "Eval 10%" indicates the average number of evaluations of functions necessary to reach the theoretical optimum with 10% of error. The column " Eval total" gives the total number of evaluations for 15 runs. This table allows to compare the hybrid algorithm with the evolution strategy alone.

	Error	Average Error	Eval 10%	Eval total
ES	0 %	1,69 %	267	37605
hybrid	0 %	0,84 %	174	17507

Table 1 : Results for the pressure vessel problem

Figure 3 shows the evolution of the objective function during generations. It makes it possible to highlight the best speed of convergence of the hybrid algorithm.

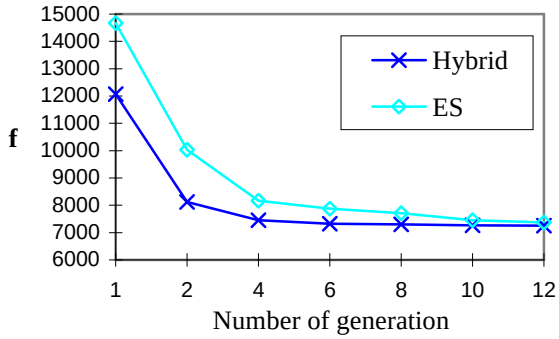

Figure 3 : Evolution of objective function along with the number of generation

6.2 : Cylindrical gear pair : The second problem of optimal design is a problem of optimization of transmission of power by a cylindrical gear pair with helical teeth.

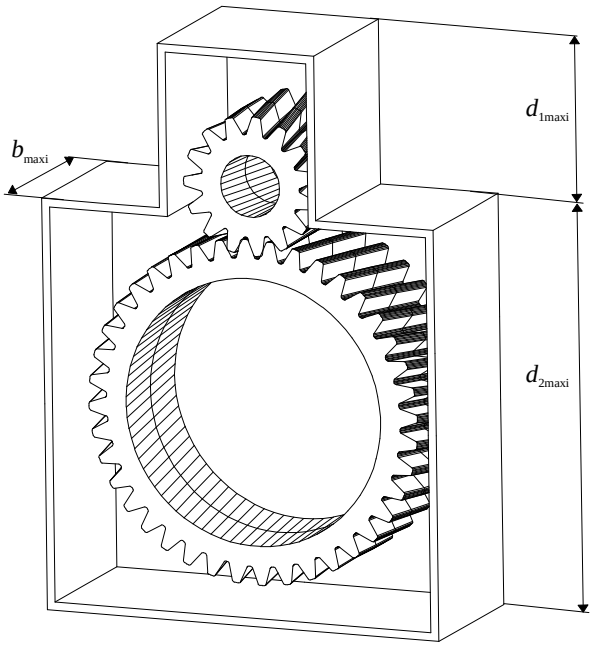

Figure 4 : Parallel helical gear pair

The determination of the teethes is a complex operation. There are however simplified and standardized methods usually used in the engineering and design departments. The formulation of the problem that we present is based on the simplified "C" method recommended by standard [18].

From kinematics imposed conditions (a given speed ratio reduction), the normal module m_{n0}, the helix angle β_0, the number of teeth of the pinion Z_1, the facewidth b and finally the addendum modification of the pinion x_1 and

the wheel x_2, are determined. The standard recommends to first determine the parameters m_{n0}, β_0, Z_1, b checking of the conditions of strength on surface pressure and of rupture on the level of teeth. The addendum modifications x_1 and x_2 are then calculated from conditions on the balancing of the specific slips and fatigue strength Although these parameters are all interdependent, the calculation complexity does not allow to tackle the problem differently.

The formulation of this problem like a problem of optimal design was already undertaken [3]. However because of the intrinsic limitations of the used optimization methods , the authors had to solve this problems in three steps. At the time of the first step, the values of b and d_{10} are determined. This allows to obtain an optimization problem with continuous variables. Then the values of m_{n0}, β_0, Z_1, are obtained by an optimization method based on the graphs of variations [8], and finally the addendum modification of teeth are determined, thanks to a particular algorithm.

We propose to formulate this problem of optimal design overall, in order to simultaneously determine the values of the variables of design: m_{n0}, β_0, Z_1, Z_2, x_1, x_2, b. The objective function is formulated to minimize the mass of the gears and the relative variation of the specific slips of the wheel and the pinion. The importance of each criterion is given by the values of α_1 and α_2. The functions $g_1(x)$ and $g_2(x)$ are conditions on the acceptable maximum powers with the surface pressure and the rupture of teeth.

The functions $g_3(x)$ and $g_4(x)$ are conditions on the transverse contact ratio ε_α and the overlap ratio ε_β. The function $g_5(x)$ expresses a condition on the linear velocity of teeth. The functions $g_6(x)$ and $g_7(x)$ limit the values of the addendum modification. The functions $g_8(x)$ and $g_9(x)$ express conditions on meshing interference. The functions $g_{10}(x)$ and $g_{11}(x)$ are conditions on the minimal and maximum values of the diameter of the pinion.

The function $g_{12}(x)$ imposes a maximum error on the speed reduction ratio u. Finally the functions $g_{13}(x)$ and $g_{14}(x)$ limit the minimal value of the facewidth compared to the diameters of the pinion and the wheel. The expression of this problem thus requires 14 nonlinear constraint functions, and 7 variables including 4 continuous and 3 discrete. It is thus a particularly complex problem of optimal design, the more so as the calculation of the factor C_{B4} requires to solve, for each evaluation of the function $g_2(x)$, the equation : inv(α) = 0 (inv(α) = α - tan(α)).

The optimization problem formulation is :

Minimize the objective function :

$$f(x) = \alpha_1 \frac{b(m_{n0}Z_1)^2}{(\cos^2\beta_0 b_{\max i} d_{1\max i}^2)} + \alpha_2\left(\frac{Z_1Y_1U_1 - Z_2Y_2U_2}{Z_2Y_2U_2}\right)$$

Subject to the non linear inequality constraints :

$$g_1(x) = 1 - \frac{C_1 \cdot C_2 \cdot C_3 \cdot C_4 \cdot C_5 \cdot C_6}{K_{B_p} P_{\min i}} \le 0$$

$$g_2(x) = 1 - \frac{C_{B1} \cdot C_{B2} \cdot C_{B3} \cdot C_{B4} \cdot C_{B5} \cdot C_{B6}}{K_{B_R} P_{\min i}} \le 0$$

$$g_3(x) = 1.3 - \varepsilon_\alpha \le 0$$

$$g_4(x) = 0.8 - \varepsilon_\beta \le 0$$

$$g_5(x) = \frac{Z_1 V}{100}\sqrt{\frac{Z_2^2}{Z_2^2 + Z_1^2}} - 10 \le 0$$

$$g_6(x) = -x_1 - x_2 \le 0$$

$$g_7(x) = x_1 + x_2 - 1 \le 0$$

$$g_8(x) = \pi Y_1 U_1 - \frac{Z_1}{2}\tan\alpha_t' \le 0$$

$$g_9(x) = \pi Y_2 U_2 - \frac{Z_2}{2}\tan\alpha_t' \le 0$$

$$g_{10}(x) = 1 - \frac{m_{n0}Z_1}{d_{1\min i}\cos\beta_0} \le 0$$

$$g_{11}(x) = \frac{m_{n0}Z_1}{d_{1\max i}\cos\beta_0} - 1 \le 0$$

		$\alpha_1 = 1$ $\alpha_2 = 0$		$\alpha_1 = 0$ $\alpha_2 = 1$		$\alpha_1 = 1$ $\alpha_2 = 1$	
	Solution of [3]	ES	Hybrid	ES	Hybrid	ES	Hybrid
Z_1^*	21	22	22	30	30	26	22
Z_2^*	58	60	61	82	83	71	61
m_{no}^*	18	18	18	14	28	16	18
x_1^*	0.273	0.551	0.000	0.810	0.817	0.821	0.805
x_2^*	-0.273	0.449	1.000	0.163	0.171	0.170	0.101
β_0^*	19.127	19.575	19.975	15.168	15.098	17.660	19.906
b	182.0	152.95	143.360	320.216	472.273	147.55	159.525
d_1^*	400.092	420.291	421.347	435.160	870.032	436.574	412.163
f^*		$2.402\ 10^{-2}$	$2.310\ 10^{-2}$	$3.994\ 10^{-11}$	0.000	$2.500\ 10^{-2}$	$2.515\ 10^{-2}$
$f^{\text{Daidié}}$		$2.589\ 10^{-2}$		0.4664		0.492	
		1944740	1986958	1893340	1675088	19313990	1748959
			+42218		-218252		-165031

Table 2 : Results for the gear pair optimization

$$g_{12}(x) = \left| \bar{u} - \frac{Z_2}{Z_1} \right| - 0.01\bar{u} \le 0$$

$$g_{13}(x) = 1 - \frac{m_{n0} Z_1}{b \cos\beta_0} \le 0$$

$$g_{14}(x) = 0.1 \frac{m_{n0} Z_2}{b \cos\beta_0} - 1 \le 0$$

Bounds on variables :

$$b_{\text{mini}} \le b \le b_{\text{maxi}}$$
$$\beta_{0\text{mini}} \le \beta_0 \le \beta_{0\text{maxi}}$$
$$m_{n0\text{mini}} \le m_{n0} \le m_{n0\text{maxi}}$$
$$Z_{1\text{mini}} \le Z_1 \le Z_{1\text{maxi}}$$
$$Z_{2\text{mini}} \le Z_2 \le Z_{2\text{maxi}}$$
$$x_{1\text{mini}} \le x_1 \le x_{1\text{maxi}}$$
$$x_{2\text{mini}} \le x_2 \le x_{2\text{maxi}}$$

Variables :

$$x = \{ m_{n0}, Z_1, Z_2, \beta_0, x_1, x_2, b \}^T$$

Data associated with bounds on variables :

b_{mini} =10 mm, b_{maxi} =500 mm, $\beta_{0\text{mini}}$ =10°, $\beta_{0\text{maxi}}$ =20°, $m_{n0\text{mini}}$ =0,5 mm, $m_{n0\text{maxi}}$ =50 mm, $Z_{1\text{mini}}$ =15, $Z_{1\text{maxi}}$ =45, $Z_{2\text{mini}}$ =40, $Z_{2\text{maxi}}$ =150, $x_{1\text{mini}} = x_{2\text{mini}}$ =-1 mm, $x_{1\text{maxi}} = x_{2\text{maxi}}$ =1 mm

$$d_{1\text{mini}} = Max\left\{ d_{1m}, \frac{d_{2m}}{\bar{u}} \right\};$$

$$d_{1\text{maxi}} = Max\left\{ d_{1M}, \frac{d_{2M}}{\bar{u}}, \frac{60 V_{\max}}{\pi N_1} \right\}$$

Data on the materials :

$\sigma_{H\lim}$ =1500 MPa, $\sigma_{F\lim}$ =460 MPa, R_m =3,5 µm, H_{Broue} =560 HB, $\nu_1 = \nu_2$ =0,3, $E_1 = E_2$ =206 GPa

Specific data problems :

N_1 =263 tr/mm, $\bar{u}$ =2,75, P_{mini} =2740 KW, $V_{\max}$ =25 m/s, d_{1m} =20 mm, d_{1M} =1500 mm, d_{2m} =20 mm, d_{2M} =15000 mm, K_A =1.5, K_R =1, Z_N =1, Y_{NT} =1.

Data for $K_{H\beta}$ coefficient [18] :

A_1 =1,11, B_1 =0,18, C_1 =1,5 10^{-4}, D_1 =1

Data for $Y_{\delta relT}$ coefficient [18] :

A_2 =0,04, B_2 =0,93

Data for Y_{RrelT} coefficient [18] :

A_3 =1,674, B_3 =0,329, C_3 =0,1

Data for Y_X coefficient [18] :

A_4 =1,03, B_4 =0,003

Data for Y_{FA} coefficient [18] :

h_{a0} =1.25, ς_{a0} =0.375

Detail expressions for coefficient C_j and C_{Bj} for j=1..5 can be found in [18].

Table 2 shows the computation results obtained at the end of 20 executions with an initial population of 7 parents and 50 offspring individuals, for several values combinations for α_1 and α_2.
The case $\alpha_1 = 1$ and $\alpha_2 = 0$ corresponds to a minimization of the mass of the gears. For $\alpha_1 = 0$ and $\alpha_2 = 1$, we only minimize the relative difference between the specific slips of the pinion and the wheel. Finally for $\alpha_1 = 1$ and $\alpha_2 = 1$, the two criteria are simultaneously optimized. In table 2 we present the optimal values of the design variables after convergence of the method, as well as the results obtained by Daidié [3].
These last results are calculated with the same data values. By comparing the lines "f^*" and "$f^{\text{Daidié}}$", we can note that the solutions identified by the evolution strategy (column "ES") and the hybrid algorithm (column "hybrid") are better than those obtained by [3]. This difference is explained by the fact that the sum of the addendum modification is null in the solution calculated by Daidié. These restrictions were made in order to simplify the calculation of the addendum modification. The difference between the number of evaluation of functions with ES and this with the hybrid method is much less significant than in the case of the problem of the pressure vessel. This can be explained by the fact that the monotonous analysis can not be used for this problem to determine the continuous variables of each offspring individual.

5. Conclusions : In this paper, we have proposed a hybrid method, based on the coupling of the evolution strategy with deterministic methods. Our aim was to use in parallel the advantages of evolutionary algorithms for the discrete variables and those of deterministic methods for the continuous variables. In the proposed hybrid method, the creation of a new individual is realized at each generation by a step in the discrete search space by mean of recombination and mutation, and a step in the continuous search space by mean of a move in a Lagrangian based quasi-Newton direction. The hybrid method, applied to two strongly constrained mechanical design problems with mixed variables, has given in association with the monotonicity analysis interesting results.

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