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# Phoresis in cellular flows: from enhanced dispersion to blockage

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In this article, we study numerically the dispersion of colloids in a 2-dimensional cellular flow in the presence of an imposed mean salt gradient. Due to the additional scalar, the colloids do not follow exactly the Eulerian flow-field, but have a (small) extra-velocity proportional to the salt gradient,  $\mathbf{v}_{dp} = \alpha \nabla S$ , where  $\alpha$  is the phoretic constant and  $S$  the salt concentration. We study the demixing of an homogenous distribution of colloids and how their long term mean velocity  $\mathbf{V}_m$  and effective diffusivity  $D_{eff}$  are influenced by the phoretic drift. We observe two regimes of colloids dynamics depending on a blockage criterion  $R = \alpha GL / \sqrt{4D_c D_s}$ , where  $G$  is the mean salt gradient amplitude,  $L$  the length-scale of the flow and  $D_c$ ,  $D_s$  the molecular diffusivities of colloids and salt. When  $R < 1$ , the mean velocity is strongly enhanced with  $V_m \propto \alpha G \sqrt{Pe_s}$ ,  $Pe_s$  being the salt Péclet number. When  $R > 1$ , the compressibility effect due to the phoretic drift is so strong that a depletion of colloids occurs along the separatrices inhibiting cell-to-cell transport.

## 1. Introduction

Advection-diffusion of a passive contaminant or a magnetic field by cellular flows with closed streamlines have been the focus of many studies in the past decades, see for instance Solomon & Gollub (1988); Shraiman (1987); Soward (1987); Young *et al.* (1989). In such configurations, it has been shown that the transport of brownian particles is greatly enhanced by advection: their dynamics at long time becomes diffusive, with the ratio of their effective diffusivity to their molecular diffusivity growing as the square root of the Péclet number,  $Pe = UL/D$ , where  $U$  is the velocity scale,  $L$  its the length-scale, and  $D$  the molecular diffusivity of the particles. Since these pioneering studies, modified versions of this problem have been addressed in the context of non tracer particles *i.e.*, particles which do not strictly follow the fluid motions because of their response time to flow modifications (Maxey 1987a). The question of long time dynamics of very heavy, weakly inertial, particles settling under the action of gravity was addressed by Pavliotis & Stuart (2005) and Afonso (2008) who derived expressions for the mean particle settling velocity, which was found to be larger than the settling velocity in a quiescent fluid. More recently, such study was extended to the case of arbitrary density ratio by Renaud & Vanneste (2020); in particular, the authors pointed out the role of compressibility effects due to particle inertia, which can lead to reduced transport in the case of light

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particles that tend to be trapped in vortical flow regions (Maxey 1987b; Wang & Maxey 1993). Reduction of effective diffusivity was also observed numerically by Li *et al.* (2021) who studied the case of passive Brownian particles submitted to the action of a mean transverse force in an array of vortex similar to the one of Solomon & Gollub (1988), and in the case of active particles moving in a laminar vortical flow as transport barriers develop (Berman *et al.* 2021).

Phoretic particles like colloids are another class of particles which do not follow exactly the fluid motion; indeed they are submitted to a drift velocity that originates from heterogeneities in the background of chemical –diffusiophoresis– or thermal –thermophoresis– origin. Anderson (1989) showed that the drift velocity for diffusiophoresis is of the type  $\mathbf{v}_{\text{dp}} = D_{\text{dp}} \nabla \log S$ , where  $D_{\text{dp}}$  is the diffusiophoretic mobility and  $S$  is the concentration in background chemical (*e.g.* salt), while Gupta *et al.* (2020) determined experimentally that the dependence can be rather more complex (in particular,  $D_{\text{dp}}$  may depend on the salt concentration). These observations are in accordance with a recent more complex expression for the phoretic velocity proposed in Menolascina *et al.* (2017); Salek *et al.* (2019). Thus depending on the species considered and its concentration, a drift velocity of the type  $\mathbf{v}_{\text{dp}} = \alpha \nabla S$  can also be observed in practice (corresponding to  $D_{\text{dp}} \propto S$ ), as discussed in Chu *et al.* (2022); this is the expression we consider in this article. Note finally that in a view of simplicity, in the following we mainly refer to diffusiophoresis. However, other mechanisms like chemotaxis (movement of a motile organism in a direction corresponding to a gradient concentration of a nutriment for instance) lead to a similar expression for the drift velocity, but with a much larger drift coefficient  $\alpha$ , as also discussed in Chu *et al.* (2022).

Mixing of phoretic particles in the presence of salt heterogeneities was first studied experimentally in a  $\Psi$ -shaped channel (Abécassis *et al.* 2009), and later in chaotic advection at the micro or macro-scale (Deseigne *et al.* 2014; Mauger *et al.* 2016). In particular, the authors showed that mixing is delayed when the colloids are introduced together with the salt, and enhanced when introduced in salted water. In this context, it has been shown that although the drift velocity is usually very weak as compared to the flow velocity, mixing is strongly modified because, due to diffusiophoresis, the particle velocity field is compressible (Volk *et al.* 2014; Raynal *et al.* 2018; Raynal & Volk 2019; Chu *et al.* 2020, 2021). One may thus wonder how the long time transport of such phoretic particles by a cellular flow is modified when a mean scalar gradient such as salt or temperature is imposed to the system. It can be anticipated that in the large time regime, a stationary state is reached, and we are interested here in the behavior of the effective velocity and diffusivity in this asymptotic regime: for instance, we wonder whether the average colloids velocity will be enhanced or reduced as compared to the case without advection. Answering this question is not trivial because the presence of strongly localized gradients may lead to enhanced cell-to-cell transport as well as arrested transport as previously observed in a simpler geometry (Chu *et al.* 2020).

In this article, we study the dispersion of colloids in a 2-dimensional cellular flow in the presence of an additional scalar (which we call salt) forced with an imposed mean salt gradient. The flow satisfies

$$\mathbf{u}(x, y) = u_0(\sin kx \cos ky, -\cos kx \sin ky), \quad (1.1)$$

with  $k = \pi/L$ ; it has closed streamlines in square cells of side  $L$  and is  $2L$ -periodic in space. Due to the presence of salt gradients, the colloids do not follow exactly the Eulerian flow-field  $\mathbf{u}$ , but have a (small) extra-velocity of the type

$$\mathbf{v}_{\text{dp}} = \alpha \nabla S, \quad (1.2)$$

where  $\alpha$  is the (small) phoretic coefficient and  $S$  is the salt concentration. Salt and colloids concentration fields ( $S$  and  $C$ ) satisfy the advection diffusion equations

$$\partial_t S + \nabla \cdot S \mathbf{u} = D_s \nabla^2 S, \quad (1.3)$$

$$\partial_t C + \nabla \cdot C(\mathbf{u} + \mathbf{v}_{dp}) = D_c \nabla^2 C, \quad (1.4)$$

where  $D_s$  and  $D_c$  are the molecular diffusion coefficients of salt and colloids respectively. The salt concentration is forced by an imposed mean salt gradient in the  $x$ -direction

$$\mathbf{G} = \langle \nabla S \rangle = G \mathbf{e}_x. \quad (1.5)$$

The problem is mainly governed by the salt and colloids Péclet numbers, respectively  $Pe_s = u_0 L / D_s$  and  $Pe_c = u_0 L / D_c$ , and by a new additional non dimensional number, a criterion of blockage, which will be shown to be the ratio  $R = \alpha G L / \sqrt{D_c D_s}$ . By means of high resolution numerical simulations performed both in the Eulerian and Lagrangian frameworks, we will study the demixing of an homogenous distribution of colloids and how their mean velocity  $\mathbf{V}_m$  and effective diffusivity  $D_{\text{eff}}$  (obtained in the long time regime) are influenced by the phoretic drift for a wide range of parameters  $\alpha G$ ,  $Pe_s = u_0 L / D_s$ ,  $Pe_c = u_0 L / D_c$ , with  $\alpha > 0$  and  $G > 0$ .

This paper is organized as follows: in Section 2 we will address the well-documented case of mixing of diffusing salt in a cellular flow in the presence of an imposed mean salt gradient  $\mathbf{G}$ , which converges towards a stationary state. In Section 3 we will describe the demixing of diffusing colloids in this stationary salt concentration field; here again the colloids concentration field reaches a stationary state. We will consider Eulerian simulations, and explain the typical concentration fields observed. Then we will consider Lagrangian simulations for higher colloids Péclet number  $Pe_c$ , and study the net mean velocity  $V_m$  reached by the colloids, together with their effective diffusion; in particular, we will show that, depending on the parameters considered,  $V_m$  can be much higher than what could be expected, but also much less, up to vanishing, a situation referred to as ‘‘blockage’’. Finally in Section 4 we will go more into details concerning the mean colloids velocity analysis. We will deduce the order of magnitude of  $V_m$  when no blockage occurs, obtain the condition of blockage, and compare with numerical simulations in a wide range of parameters.

## 2. Case of salt

The transport of a scalar or a vector in a cellular flow has been the focus of many studies in the past decades (Shraiman 1987; Soward 1987; Young et al. 1989; Afonso 2008; Renaud & Vanneste 2020). We propose here to briefly outline the subject in the context of an imposed mean gradient which corresponds to the original configuration of Shraiman (1987).

The salt concentration  $S(x, y, t)$  evolves independently of the colloids and is forced by the imposed gradient  $\mathbf{G} = (G_x, G_y)$  following equations (1.3) and (1.5). In practice, we solve numerically the equation for  $S' = S - \mathbf{G} \cdot \mathbf{x}$ , which satisfies the equation

$$\partial_t S' + \nabla \cdot S' \mathbf{u} = D_s \nabla^2 S' - \mathbf{G} \cdot \mathbf{u}. \quad (2.1)$$

Because the velocity field is periodic in space with zero average

$$\langle \mathbf{u} \rangle = \frac{1}{4L^2} \iint_{[0, 2L]^2} \mathbf{u}(x, y) dx dy = 0, \quad (2.2)$$

the equation (2.1) for  $S'$  can be solved with periodic boundary conditions with  $\langle S' \rangle = 0$

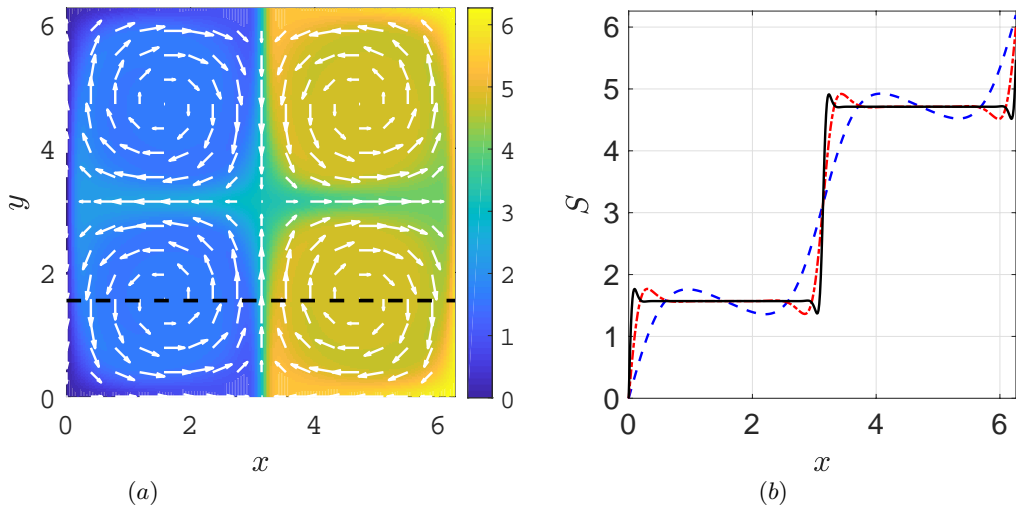


FIGURE 1. Salt case, with a positive mean imposed gradient  $\mathbf{G} = \pi/L \mathbf{e}_x$  ( $L = \pi$ ). (a): 2D concentration field for  $Pe_s = 314$ ; the concentration is almost constant in a given cell. (b): 1D-cut along the dotted line in figure 1a ( $y = \pi/2$ ) for three different salt Péclet numbers: ---:  $Pe_s = 31.4$ ; ----:  $Pe_s = 314$ ; —:  $Pe_s = 3141$ ; for  $Pe_s \geq 314$ , the profile is flat except for a very small layer around the vertical separatrix where the concentration jumps abruptly.

(Holzer & Siggia 1994). This ensures that the total salt concentration field  $S = S' + \mathbf{G} \cdot \mathbf{x}$  satisfies boundary conditions compatible with the imposed mean gradient:

$$S(x + 2L, y) = S(x, y) + 2G_x L \quad (2.3)$$

$$S(x, y + 2L) = S(x, y) + 2G_y L. \quad (2.4)$$

In the following, without any loss of generality, we set  $L = \pi$  for the numerical simulations. The equation for  $S'$  is solved in spectral space in a square domain  $(x, y) \in [0, 2\pi]^2$  using the method described in Volk et al. (2014) with a mean gradient in the  $x$  direction ( $\mathbf{G} = (G, 0)$ ). Because equation (2.1) is linear,  $S'$  is proportional to  $G$  so that we set  $G = 1$  without loss of generality. We set  $u_0 = 1$  and adjust the spatial resolution to  $256^2$  for Péclet number  $Pe_s = u_0 L / D_s \in [30, 300]$  and  $512^2$  for  $Pe_s = u_0 L / D_s \in [300, 3000]$ . Such very high resolutions are chosen to ensure high precision when interpolating the salt gradient in real space which is needed for the Lagrangian dynamics of colloids.

Integrating in time from an initial condition  $S' = 0$ , the salt concentration develops strongly localized gradients and reaches a stationary state ( $\partial_t S = 0$ ) thanks to the stationarity of the forcing term  $-G u_x$  (equation 2.1). Such localization can be understood in the limit of infinite Péclet numbers. Indeed, in the case of a non-diffusing scalar ( $D_s = 0$ ), equation 1.3 leads to  $\mathbf{u} \cdot \nabla S = 0$ , which admits a solution for which  $\nabla S$  is zero almost everywhere. Given the forcing, it implies that  $S$  is piecewise constant and increases by jumps by a value  $GL$  in the direction  $x$  of the forcing; the jump occurs at the vertical boundary of a cell, where the velocity is vertical and perpendicular to the salt-gradient so that  $\mathbf{u} \cdot \nabla S = 0$ .

This is indeed the picture that we obtain at large Péclet number, as shown in figure 1, which displays the Eulerian salt concentration field obtained for  $Pe_s = 314$  (Fig 1a) together with a 1D horizontal profile at mid-height of a cell for different Péclet numbers (Fig 1b): the concentration is almost constant in a given cell, and jumps abruptly (but on a finite slope) by a value  $\Delta S = GL$  from one cell to the next one in the  $x$ -direction.

The typical width  $\ell_s$  on which the concentration increases by a value  $\Delta S$  (the salt

Batchelor scale (Batchelor 1971)) results from a competition between contraction and diffusion through the equation of salt gradient  $\tilde{\gamma}^s = \nabla S$ :

$$\frac{D\gamma_i^s}{Dt} = -\partial_i u_j \gamma_j^s + D_s \nabla^2 \gamma_i^s, \quad (2.5)$$

where  $D/Dt = \partial_t + u_k \partial_k$  is the material derivative. This horizontal gradient is built by bringing the scalar to the hyperbolic point  $(\pi, 0)$  by contraction by the velocity field along the horizontal separatrix ( $i = j = x$ ); because the concentration field is stationary its width is set when the two terms on the right hand side of equation (2.5) are of the same order of magnitude (Raynal & Gence 1997), that is  $u_0/L \sim D_s/\ell_s^2$ , therefore

$$\ell_s \sim \frac{L}{\sqrt{Pe_s}}. \quad (2.6)$$

Indeed, as the horizontal velocity reverses near the point  $(\pi, 0)$ , the advection term  $\mathbf{u} \cdot \nabla \gamma_x$  vanishes when averaged over the width  $\ell_s$  while the contraction term remains. This concentration gradient is then advected upwards by the velocity field along the vertical separatrix  $x = L = \pi$ . It reaches the top of the cell after a time  $\tau_{adv}^s \sim L/u_0$ , time after which it is dissipated by diffusion since the characteristic dissipation time is  $\tau_{diff}^s \sim \ell_s^2/D_s$ , and  $\tau_{adv}^s = \tau_{diff}^s$  because of equation (2.6). Thus this concentration discontinuity is only visible around the vertical separatrix, with an associated gradient

$$\tilde{\gamma}^s \sim \frac{\Delta S}{\ell_s} \mathbf{e}_x \sim G \sqrt{Pe_s} \mathbf{e}_x. \quad (2.7)$$

This scenario supposes that the salt can be transported on a large scale before diffusing, *i.e.*  $Pe_s \gg 1$ , in agreement with figure 1 *b*.

### 3. Case of colloids

In the following we suppose that the salt has reached its stationary state, and focus on the dynamics of the colloids which are advected by the velocity field  $\mathbf{v} = \mathbf{u} + \mathbf{v}_{dp}$  (with  $\mathbf{v}_{dp} = \alpha \nabla S$ ) and diffuse with a diffusion coefficient  $D_c$ . Starting from a *uniform distribution*, colloids will demix because  $\nabla \cdot \mathbf{v}_{dp} \neq 0$  (Volk *et al.* 2014) and reach a non uniform stationary state that we will characterize thereafter.

This problem can be addressed either in the Eulerian or in the Lagrangian frameworks. On the one hand one may use the Lagrangian approach by solving the stochastic differential equation governing the position  $\mathbf{X}_c$  of the colloids:

$$d\mathbf{X}_c = (\mathbf{u}(\mathbf{X}_c) + \alpha \nabla S(\mathbf{X}_c))dt + \sqrt{2D_c} d\mathbf{W}, \quad (3.1)$$

where  $\mathbf{W}$  is a 2D Wiener process (Van Kampen 2007). Such framework will be used in the following in cases when the colloid Péclet number,  $Pe_c = u_0 L/D_c$ , is very large, in order to compute the mean velocity of the colloids and their effective diffusivity at long times. In practice we compute a 3 cubic periodic interpolant of the over-resolved salt gradient for each value of  $Pe_s$  which is then used for all cases when varying  $\alpha \in [0, 10^{-2}]$  and  $D_c \in [10^{-6}, 10^{-1}]$ ; starting from at least  $\mathcal{O}(10^4)$  uniformly distributed colloidal particles, we solve equation (3.1) numerically for each trajectory by splitting the deterministic and stochastic parts at each time step. We use a 4th order Runge Kutta method for the deterministic part with  $dt = 10^{-2}$  and a standard Euler scheme Euler scheme for the stochastic part (Higham 2001). The simulation runs for long times as compared to the diffusion time,  $\tau_c = L^2/D_c$ , so that the long time dynamics of the colloids is well established.

On the other hand the problem may also be addressed in an Eulerian framework by investigating the colloids concentration field  $C$ , as  $C(\mathbf{x}, t)/\langle C \rangle$  is the probability of finding a colloid at location  $\mathbf{x}$  at time  $t$ . The concentration  $C$  is coupled with that of the salt through the diffusiophoretic velocity  $\mathbf{v}_{\text{dp}} = \alpha \nabla S$ . We use the Eulerian approach and solve the partial differential equation (1.4) in the case of moderate Péclet numbers ( $Pe_c \leq 3 \cdot 10^4$ ,  $D_c \in [10^{-4}, 10^{-1}]$ ) using the same code as for the salt; without loss of generality we set the initial uniform condition  $C(x, y, t) = C_0 = 1$  as the equation is linear in  $C$ . We run the simulation until  $C$  has reached the stationary regime ( $\partial_t C = 0$ ), which leads to an alternative way of measuring the mean colloid velocity through the relation  $\langle C(\mathbf{u} + \alpha \nabla S) \rangle$ . This value proved to compare successfully to its Lagrangian counterpart as a numerical test.

When dealing with diffusiophoresis, it is usual to consider separately the *salt-attracting* case where the colloids move toward the salt gradient ( $\alpha > 0$ ), and reversely the *salt-repelling* case ( $\alpha < 0$ ) (Deseigne *et al.* 2014; Volk *et al.* 2014). Indeed, changing the sign of the diffusiophoretic parameter may fully change the physics and the scaling involved (Raynal & Volk 2019; Chu *et al.* 2020). This is not the case here: the set of coupled equations (1.3–1.4), with flow-field (1.1) and  $\mathbf{v}_{\text{dp}} = \alpha \nabla S$  is actually invariant under transformation  $\alpha \rightarrow -\alpha$ ,  $x \rightarrow -x$  and  $y \rightarrow -y$ . This implies that the physics of the problem is unchanged when considering salt-attracted or salt-repelled colloids. Thus without loss of generality we set

$$\alpha \geq 0. \quad (3.2)$$

Finally, as the drift velocity ( $\mathbf{v}_{\text{dp}} = \alpha \nabla S$ ) is linear in  $\alpha$ , equations are unchanged if the imposed salt gradient  $\mathbf{G}$  is reversed and  $\alpha$  changed into  $-\alpha$ , so that we shall set  $G = 1$  in the following and only vary  $\alpha$ .

Because colloids are expected to have a small diffusion coefficient as compared to the salt, we shall consider only cases for which

$$D_c \ll D_s \quad (3.3)$$

(equivalently  $Pe_c \gg Pe_s$ ). In the case  $\alpha > 0$ , the colloids start to demix due to diffusiophoresis, and gradients of colloid concentration appear. In the stationary state, the width  $\ell_c$  of these gradients is governed by the competition between the dominant mechanism of gradient creation (contraction either by the velocity field or diffusiophoresis) and destruction by diffusion. In the case when contraction by the velocity-field is dominant, we obtain as for the salt an heterogeneity of width  $\ell_c = L/\sqrt{Pe_c}$  along the vertical separatrix, with  $\ell_c \ll \ell_s$ . When diffusiophoresis is dominant, it acts as an additional mechanism of creation of gradient, which leads to an even smaller Batchelor scale  $\ell_c$ . Finally we have whatever  $\alpha$

$$\ell_c \ll \ell_s. \quad (3.4)$$

### 3.1. Eulerian results: colloidal concentration field

The salt field is uniform, except in the vicinity of the vertical separatrices where there is a very strong salt gradient. It is therefore understood that far from the separatrices, diffusiophoresis does not operate and the colloids move only along the current lines; as a result the colloids concentration remains constant in the central vortex. Because non-trivial behavior takes place around the vertical separatrices, we consider the equation of

evolution of the gradient in colloids  $\vec{\gamma}^c = \nabla C$  (Raynal & Volk 2019) †:

$$\frac{D\gamma_i^c}{Dt} = \underbrace{-\partial_i u_j \gamma_j^c}_{(a)} + \underbrace{D_c \nabla^2 \gamma_i^c}_{(b)} - \underbrace{\alpha \partial_j \gamma_i^c \partial_j S}_{(c)} - \underbrace{\alpha \gamma_j^c \partial_i \partial_j S}_{(d)} - \underbrace{\alpha \gamma_i^c \partial_j^2 S}_{(e)} - \underbrace{\alpha C \partial_i \partial_j^2 S}_{(f)}. \quad (3.5)$$

Let us evaluate each of those terms on the separatrix:

$$(a) \sim u_0 \gamma^c / L \quad (3.6)$$

$$(b) \sim D_c \gamma^c / \ell_c^2 \quad (3.7)$$

$$(c) \sim \alpha \gamma^c \Delta S / (\ell_c \ell_s) \quad (3.8)$$

$$(d) \sim \alpha \gamma^c \Delta S / \ell_s^2 \ll (c) \quad (3.9)$$

$$(e) \sim \alpha \gamma^c \Delta S / \ell_s^2 \ll (c) \quad (3.10)$$

$$(f) \sim \alpha \gamma^c \Delta S \ell_c / \ell_s^3 \ll (c), \quad (3.11)$$

where the three last terms are negligible compared to term (c) because of relation (3.4). Since the velocity field  $\mathbf{u}$  is perpendicular to the salt or colloids gradients along the vertical separatrix, like in Section 2 the left hand-side term of equation (3.5) is zero in this stationary regime. We consider the case when the mechanism of creation of gradients by diffusiophoresis is stronger than by advection, which gives (c)  $\gtrsim$  (a). The width of the heterogeneity in colloids is then given by the competition between the creation of gradients by diffusiophoresis, and its destruction by diffusion ((c)  $\sim$  (b)). Therefore we obtain

$$\ell_c \sim \frac{D_c \ell_s}{\alpha \Delta S} \sim \frac{D_c}{\alpha G \sqrt{Pe_s}}, \quad (3.12)$$

given equation (2.7). Combining condition (c)  $\gtrsim$  (a) with scaling (3.12) gives the following condition for the parameter  $\alpha$  to ensure diffusiophoresis has a stronger effect than advection:

$$\alpha \gtrsim \frac{\sqrt{D_c D_s}}{GL}. \quad (3.13)$$

In the case of salt, the gradient is advected along the vertical separatrix and then dissipated (see Section 2); what about colloids? The gradient of colloids is created in the vicinity of the vertical separatrix and then advected in the cell over a distance  $d$  for a time  $\tau_{adv}^c \sim d/u_0$ , until it is dissipated by diffusion, which happens roughly at time  $\tau_{diff}^c \sim \ell_c^2/D_c$ . We obtain the travelled distance  $d$  before dissipation by equaling the two times, *i.e.*

$$d \sim \frac{D_c D_s}{\alpha^2 G^2 L}, \quad (3.14)$$

where we have used equation (3.12) for  $\ell_c$ . Due to condition (3.13), this distance verifies

$$d \lesssim L. \quad (3.15)$$

In practice, since the gradient is present along the entire vertical, it is advected along the upper edge of the cell, then rapidly diffuses. If the condition (3.13) is not met, and if the diffusiophoretic effect is weak compared to contraction by the flow-field, then the phenomenology for colloids goes back to that of salt where the equilibrium state corresponds to a balance between advection and diffusion with (a)  $\sim$  (b), and the inhomogeneity is located only on the vertical separatrix. Therefore the colloidal heterogeneity would essentially be dissipated somewhere on the upper edge or rapidly

† Compared to our previous work (Raynal & Volk 2019), the diffusiophoretic velocity is here proportional to the salt gradient.



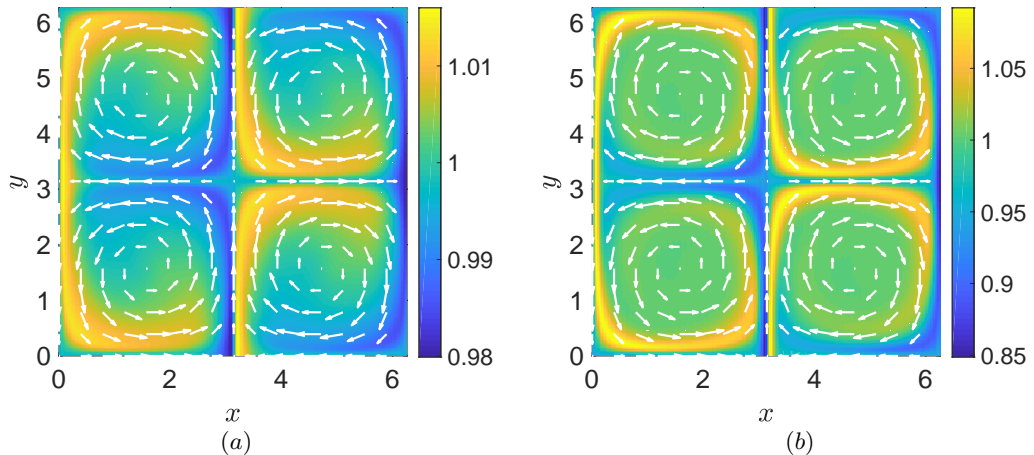


FIGURE 2. Eulerian colloid field  $c(y, y)$  in four adjacent cells for  $\alpha = 10^{-3}$  and  $Pe_c = 3141$ . (a):  $Pe_s = 31.4$ , advection and diffusiophoresis have comparable strength in the creation of gradients, the two terms in relation (3.13) are of the same order of magnitude; (b):  $Pe_s = 314$ : diffusiophoresis is more efficient and relation (3.13) is met.

after, depending on how the left and right hand-side of condition (3.13) compare. This is indeed what we observe in the Eulerian simulations when varying the different parameters  $\alpha$ ,  $Pe_s$  and  $Pe_c$ . As an illustration, Figure 2 shows two typical colloidal concentration fields for  $\alpha = 10^{-3}$  and  $Pe_c = 3141$  and two different salt Péclet numbers  $Pe_s = 31.4$  (a) and  $Pe_s = 314$  (b): in figure 2a, the two terms in equation (3.13) are of the same order of magnitude, while condition (3.13) is satisfied in figure 2b. Note the different scales in the colorbars: except in the cores of cells which seem more homogeneous, the heterogeneities are more pronounced when the diffusiophoretic effects are stronger, with more asymmetric positive and negative deviations from the mean  $C_0 = 1$ .

### 3.2. Lagrangian results: velocity and effective diffusion

The Eulerian simulations showed how the colloids are demixed due to diffusiophoresis in the stationary regime. As a result, it is expected that their averaged velocity is modified. In order to explore this phenomenon when  $Pe_c$  is large, we now perform Lagrangian simulations and investigate the statistics of colloids displacement at long times, denoted  $\Delta \mathbf{X}_c = \mathbf{X}_c(t) - \mathbf{X}_c(0)$ . We denote an ensemble average over trajectories by  $\langle \cdot \rangle_{\mathcal{L}}$ ; this allows to define the mean Lagrangian velocity  $\langle \mathbf{V} \rangle_{\mathcal{L}}$  through the relation  $\langle \Delta \mathbf{X}_c \rangle_{\mathcal{L}} \sim \langle \mathbf{V} \rangle_{\mathcal{L}} \times t$  at large times. We also measure an effective diffusion  $D_{\text{eff}}$ : we have  $\langle (\Delta X_c)^2 \rangle_{\mathcal{L}} - \langle \Delta X_c \rangle_{\mathcal{L}}^2 \sim 2D_{\text{eff}}^x \times t$  and  $\langle (\Delta Y_c)^2 \rangle_{\mathcal{L}} - \langle \Delta Y_c \rangle_{\mathcal{L}}^2 \sim 2D_{\text{eff}}^y \times t$  at large times, where we have treated separately the  $x$  and  $y$  directions in order to take into account a possible anisotropy.

Because the mean Eulerian velocity field  $\langle \mathbf{u} \rangle$  is zero, one could reasonably think that the mean Lagrangian velocity  $\langle \mathbf{V} \rangle_{\mathcal{L}}$  is of the same order of magnitude as the mean salt gradient  $\alpha \mathbf{G}$ . However, this is not at all the case when looking at figure 3a, that shows the ratio of mean Lagrangian  $x$  and  $y$ -velocity components of  $\langle \mathbf{V} \rangle_{\mathcal{L}}$  to  $\alpha G$  as a function of the Péclet number  $Pe_c$ , at fixed  $Pe_s = 314$  and  $\alpha = 10^{-3}$ : while the  $y$ -component remains zero as expected, the  $x$ -component is much larger than 1 for most of the points. Moreover the ratio has a non trivial behaviour: at low to moderate Péclet numbers  $Pe_c$ , the velocity increases, but eventually decreases to zero at larger Péclet numbers.

Figure 3b shows the effective diffusion coefficient in the  $x$  and  $y$ -directions. The line is the prediction by Shraiman (1987) for a diffusing scalar *without diffusiophoresis* in a

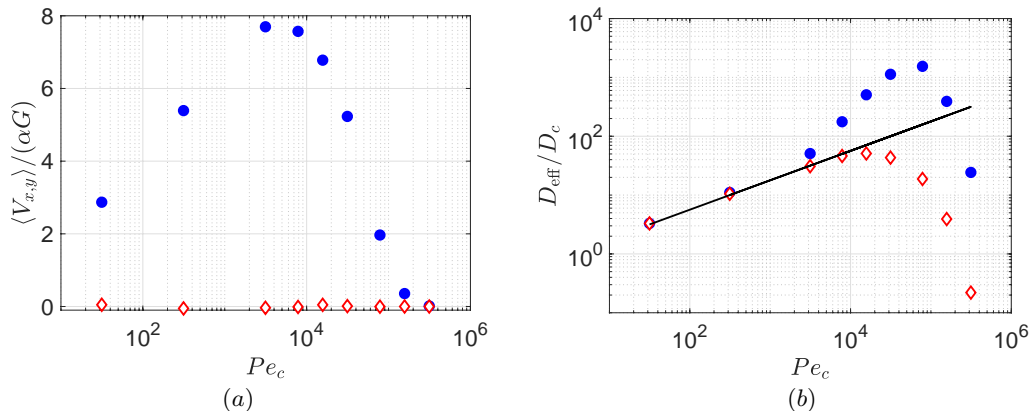


FIGURE 3. (a): non dimensional mean Lagrangian velocity  $\langle \mathbf{V} \rangle$  and (b): non dimensional effective diffusion  $D_{\text{eff}}$ , for  $Pe_s = 314$  and  $\alpha = 10^{-3}$  as a function of the Péclet number  $Pe_c$ .  $\bullet$ :  $x$ -direction;  $\diamond$ :  $y$ -direction. The velocity is made non dimensional using the diffusiophoretic velocity  $\alpha G$  based on the mean gradient; the effective diffusion is compared to the diffusion coefficient of the colloids  $D_c$ . The solid black line is the analytical solution proposed by Shraiman (1987):  $D_{\text{eff}} / D_c = Pe_c^{1/2} / \sqrt{2\pi}$ .

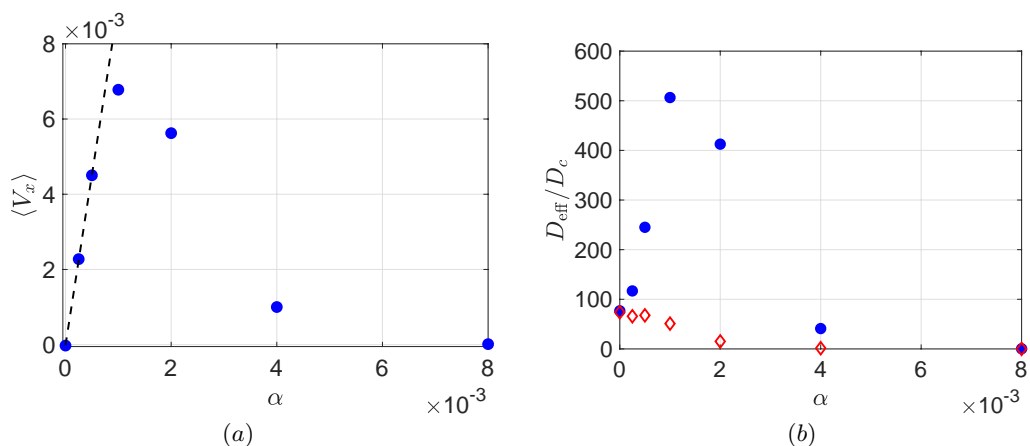


FIGURE 4. (a):  $x$ -component of the Lagrangian velocity  $\langle V_x \rangle$  and (b): non dimensional effective diffusion  $D_{\text{eff}} / D_c$ , for  $Pe_s = 314$ ,  $Pe_c = 15\,708$  as a function of  $\alpha$ .  $\bullet$ :  $x$ -direction;  $\diamond$ :  $y$ -direction.

cellular flow: he showed that the effective diffusion grows as the square root of the Péclet number, and becomes orders of magnitude higher than the diffusivity of the scalar. Very interestingly, we find that the effective diffusion at small to moderate Péclet number ( $Pe_c \leq 3000$ ) is isotropic, and follows the prediction by Shraiman, although the effects of diffusiophoresis are macroscopically visible on the velocity at those Péclet numbers. For  $Pe_c > 3000$ , while the mean Lagrangian velocity begins to decay (figure 3 a), the effective diffusivity becomes highly anisotropic, with an increase in the  $x$ -direction compared to Shraiman's prediction, and a decrease in the  $y$ -direction. At even higher Péclet numbers, the effective diffusivity in both directions finally decays to zero, although for larger Péclet numbers than for the velocity.

Figure 4 shows the mean Lagrangian velocity and effective diffusion as a function of the diffusiophoretic coefficient  $\alpha$  for the same salt Péclet number and  $Pe_c = 15\,708$ , a Péclet number for which the effective diffusion is anisotropic (figure 3 b), more realistic of the large Péclet numbers encountered for colloids. The mean velocity in the  $y$ -direction

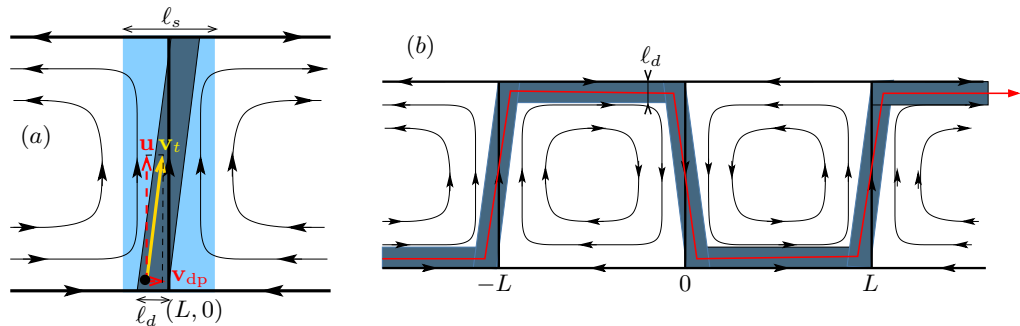


FIGURE 5. (a): schematic displacement of a particle in the salt gradient (light blue region) around the vertical ascending separatrix  $x = L$ . The Eulerian velocity field  $\mathbf{u}$  is vertical ascending, and  $\mathbf{v}_{dp}$  is horizontal; the total displacement is therefore  $\mathbf{v}_t = \mathbf{u} + \mathbf{v}_{dp}$ , with  $\mathbf{u} \approx u_0 \mathbf{e}_y$ . The particles that change or will change cell are located on the grey strip of width  $\ell_d$ . (b): in gray, schematic location of particles that change cell, for cells located between  $y = 0$  and  $y = L$ ; in red, example of trajectory. Outside this gray stripe, the particles remain trapped in a cell.

is again zero and is not shown here. For small  $\alpha$ ,  $\langle V_x \rangle$  increases linearly, before vanishing also at higher values. In figure 4 *b*, we recover at  $\alpha = 0$  the value predicted by Shraiman (1987) for the effective diffusion in both directions. Then  $D_{\text{eff}}^x$  increases rapidly for small  $\alpha$ , before decreasing to zero at larger values; in the  $y$ -direction, the effective diffusivity decreases to zero as  $\alpha$  increases.

In the next section we deepen the analysis of the colloids velocity and study the physical mechanisms at stake which explain the observations we just reported based on the Lagrangian simulations.

## 4. From enhanced dispersion to blockage

### 4.1. Enhanced dispersion

Figures 3 *a* and 4 *a* showed that transport is strongly enhanced at small diffusiophoretic forcing, and blocked at higher forcing. In order to explain these observations, we shall return to the behaviour of colloids in the salt gradient along the vertical separatrices. Colloids that are in this region are shifted to the right under the effect of diffusiophoresis: they move with a velocity  $\mathbf{v} = \mathbf{u} + \mathbf{v}_{dp}$  with  $\mathbf{v}_{dp} = \alpha \nabla S$ . The velocity  $\mathbf{u}$  is roughly vertical, while the diffusiophoretic velocity is roughly horizontal, see figure 5 *a* around the separatrix  $x = L$  (upward velocity).

We denote by  $\ell_d$  the horizontal displacement travelled by a colloid under the effect of the salt gradient as it rises or falls along the vertical separatrix; in a first step we suppose  $\ell_d \leq \ell_s$ . Along the vertical separatrix we have  $u \sim u_0$ , so that

$$\frac{\ell_d}{v_{dp}} \sim \frac{L}{u_0}, \quad (4.1)$$

see figure 5 *a*. Because of equation (2.7) the diffusiophoretic velocity in the salt gradient writes

$$v_{dp} \sim \alpha G \sqrt{Pe_s} \quad (4.2)$$

and finally

$$\ell_d \sim \frac{\alpha GL}{u_0} \sqrt{Pe_s}. \quad (4.3)$$

Hence the particles which change vortex during their ascent (or descent) in the salt

gradient are located in a strip of width  $\ell_d$ , indicated in gray in figure 5 *a*. In figure 5 *b* we follow those particles on several vortices: in the horizontal strips, the salt gradient is negligible, so that the colloids follow the Eulerian flow-field, mostly horizontal here and such that  $\mathbf{u} \sim u_0 \mathbf{e}_x$ . Therefore the total displacement of particles is horizontal, as found in our Eulerian and Lagrangian simulations, see for instance figure 3 *a*. Note also that the strips colored in gray in figure 5 *b* are in good qualitative agreement with the regions of high concentration in the Eulerian colloidal fields in figure 2. The regions not coloured in figure 5 *b* are the locations of the particles that are trapped inside a cell: by construction, the Lagrangian velocity of these trapped particles is zero on average. We can thus evaluate the spatial average horizontal velocity  $V_m$  on a cell of size  $L^2$ :

$$V_m \sim \frac{1}{L^2} [(\ell_d \times L) v_{\text{dp}} + (\ell_d \times L) u_0] \quad (4.4)$$

$$\sim \frac{\ell_d}{L} (v_{\text{dp}} + u_0). \quad (4.5)$$

With equation (4.1), we obtain  $v_{\text{dp}}/u_0 \sim \ell_d/L$ , so that  $v_{\text{dp}}$  can be neglected, and, using equation (4.3):

$$V_m \sim \alpha G \sqrt{Pe_s}. \quad (4.6)$$

This simple result leads to some remarks:

- the mean colloids velocity is predicted to be much larger than that corresponding to the mean salt gradient ( $\alpha G$ ), in accordance with figure 3 *a*;
- we recover the proportionality in  $\alpha$  observed in figure 4 *a* for small  $\alpha$ ;
- relation (4.6) is expected to slightly overestimate the actual velocity. Indeed, we assumed  $u \sim u_0$  on the separatrices; however, the mean of  $u_0 \sin kx$  on the side of a cell is rather  $u_0/\pi$ ; changing  $u_0$  into  $u_0/\pi$  leads to

$$V_m \sim \alpha G \sqrt{\frac{Pe_s}{\pi}}. \quad (4.7)$$

Anyway, the velocity of a colloid is actually still lower when not laying exactly on the separatrix, so that even equation (4.7) slightly overestimates the velocity.

Let us test this prediction by performing Lagrangian numerical experiments for many different values of  $D_s$ ,  $D_c$  and  $\alpha$ . For all those values we measure the  $x$ -components of the velocity and display  $\langle V_x \rangle_{\mathcal{L}} / (\alpha G \sqrt{Pe_s})$  as function of  $Pe_c$  in figure 6. It can be observed that the formula predicts the correct order of magnitude for the maximal value of the velocity ( $1/\sqrt{\pi} \simeq 0.56$ ) at a given colloid Péclet number. However the expression (4.7) does not depend on  $Pe_c$  disagreeing with figure 3 *a*, and fails to predict the rapid decay of the mean velocity observed in figures 3 *a* and 4 *a* when increasing  $Pe_c$  and  $\alpha$ . There is thus a need to refine the analysis and figure out why the dispersion of the particles stops, both for the velocity and the effective diffusion.

#### 4.2. Blockage

In order to address the question of blockage, we consider again the displacement of colloids in the layer of salt gradient around the vertical separatrices (figure 5 *a*). If the diffusiophoretic effect is strong enough, an area completely empty of colloids of width  $\ell_d$  forms in a layer at the left of the separatrix in the salt gradient (on the low salt concentration side). To effectively observe this depleted zone, and eventually for it to become a real barrier to transport from one cell to another, this band without colloids must be advected all around the cell, without having the time to diffuse. Blockage therefore occurs if the diffusion time of this depleted region,  $\ell_d^2/D_c$ , is large compared to

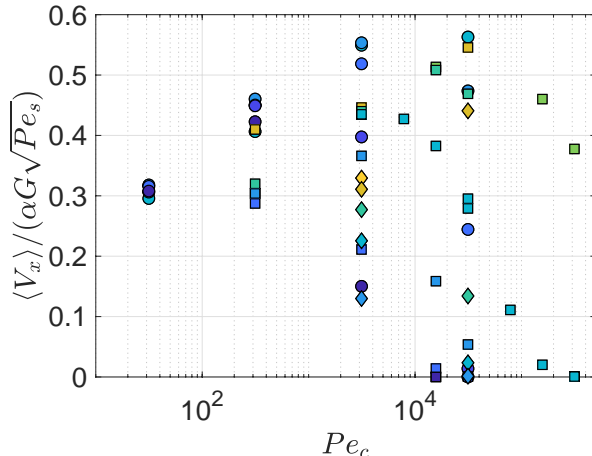


FIGURE 6. Normalized horizontal velocity  $\langle V_x \rangle_{\mathcal{L}} / (\alpha G \sqrt{Pe_s})$  as a function of  $Pe_c$  for the different numerical cases performed. Symbols ( $\circ$ )  $Pe_s = 31.4$ , ( $\square$ )  $Pe_s = 314$ , ( $\diamond$ )  $Pe_s = 3144$ . The color of symbol codes for the value of  $\alpha \in [5 \times 10^{-6}, 1.6 \times 10^{-3}]$ , the darker the larger the value.

the advection time over the entire perimeter of cell  $4L/u_0$ . This competition is governed by the ratio  $R^2$  of the two time scales, such that

$$R = \sqrt{\frac{\ell_d^2/D_c}{4L/u_0}}. \quad (4.8)$$

Using relation (4.3) for  $\ell_d$ , one obtains the condition of blockage:

$$R = \frac{\alpha GL}{2\sqrt{D_c D_s}} \gg 1, \quad (4.9)$$

where we recognize  $\Delta S = GL$  the jump in concentration from one cell to the other (section 2). Note that we recover a condition similar to what we had found before (equation (3.13)). Interestingly, this condition is independent of the Eulerian velocity field  $\mathbf{u}$ , although the blockage would not occur without the velocity field.

For all this section, we have assumed so far that  $\ell_d \leq \ell_s$ . However, using equations (2.6) and (4.3), we obtain

$$\frac{\ell_d}{\ell_s} \sim \frac{\alpha G}{u_0} Pe_s = \frac{\alpha GL}{D_s}; \quad (4.10)$$

therefore  $\ell_d$ , as calculated using (4.3), may be larger than  $\ell_s$ . While  $\ell_d$  measures the deviation of colloids by diffusiophoresis, and since the diffusiophoretic velocity is zero outside the salt gradient, this implies that at the most

$$\ell_d \sim \ell_s \quad (4.11)$$

(see also figure 5a). We can therefore wonder whether the expressions obtained above for the velocity and the blockage condition remain valid in this case. Indeed, if equation (4.3) overestimates  $\ell_d$ , then condition (4.9) could falsely lead to a condition of blockage. We will show that in that case condition (4.9) remains a valid criterion for blockage.

As explained above a *salt* heterogeneity of width  $\ell_s$  diffuses after having travelled a distance  $L$  along the vertical separatrix; hence a *colloidal* heterogeneity of width  $\ell_s$  should travel much longer in space and time before diffusing, since  $D_c \ll D_s$ . This can be checked using the same criterion as before, the ratio  $R'^2$  between the diffusing time

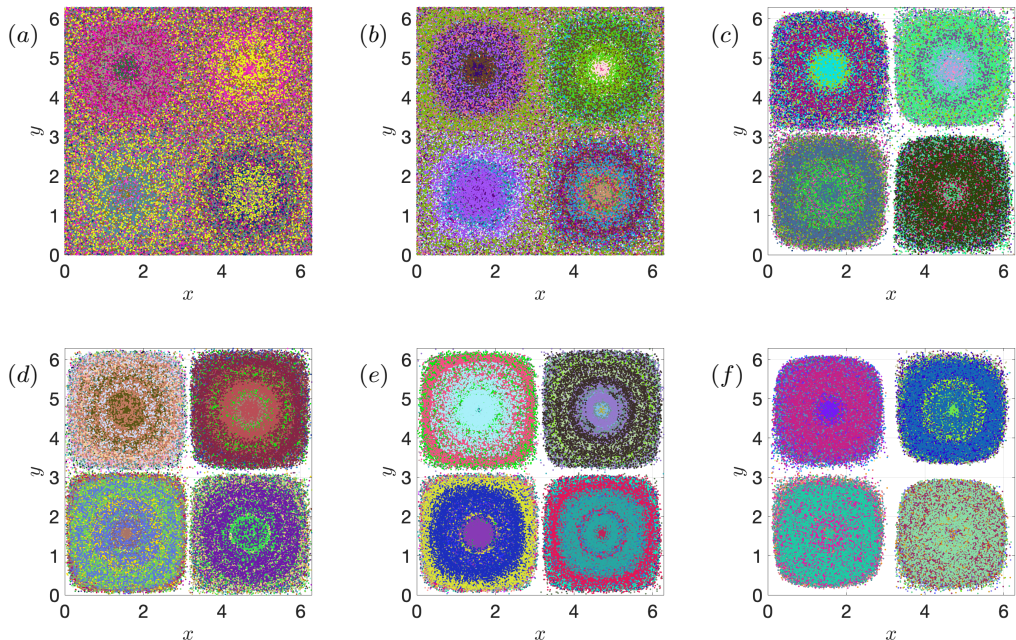


FIGURE 7. Successive positions of  $N = 100$  long trajectories of colloids, plotted modulo  $2L$ , in four adjacent cells. Particles were initially distributed uniformly and randomly inside the cells with positions dumped long after release, each color coding for one trajectory. Parameters are  $Pe_s = 314$  for each figure and different values of  $Pe_c$  and  $\alpha$ . The criterion  $R$  is increasing from (a) to (f). (a):  $\alpha = 10^{-3}$ ,  $Pe_c = 3141$ ,  $R = 0.497$ ; (b):  $\alpha = 10^{-3}$ ,  $Pe_c = 15700$ ,  $R = 1.11$ ; (c):  $\alpha = 10^{-3}$ ,  $Pe_c = 157000$ ,  $R = 3.51$ ; (d):  $\alpha = 4 \cdot 10^{-3}$ ,  $Pe_c = 15700$ ,  $R = 4.44$ ; (e):  $\alpha = 10^{-3}$ ,  $Pe_c = 314000$ ,  $R = 4.97$ ; (f):  $\alpha = 8 \times 10^{-3}$ ,  $Pe_c = 15708$ ,  $R = 8.88$ .

and the travelling time on the perimeter of a cell, *i.e.*, taking  $\ell_d = \ell_s$  in equation (4.8); we obtain

$$R' = \sqrt{\frac{\ell_s^2/D_c}{4L/u_0}} = \frac{1}{2} \sqrt{\frac{D_s}{D_c}} \gg 1, \quad (4.12)$$

where  $\ell_s$  was evaluated using equation (2.6). Therefore this case when  $\ell_d \sim \ell_s$  also corresponds to a situation of blockage, with no transport from one cell to the other. Let us show that in this situation of blockage we also have  $R \gg 1$ : we denote by  $\ell_d^{est}$  the overestimated width of the depleted band  $\ell_d$  given by relation (4.3). Then  $\ell_d^{est} \geq \ell_d \sim \ell_s$ , and according to equations (4.8) and (4.12),  $R \geq R'$ . Thus we have both  $R \geq R'$  and  $R' \gg 1$ , and finally  $R \gg 1$ : the criterion (4.9) on  $R$

- (i)  $R \ll 1$ : enhanced dispersion;
- (ii)  $R \gg 1$ : blockage

is valid for all situations.

We first test the blockage criterion for different values of the parameters  $D_s$ ,  $D_c$  and  $\alpha$  performing simulations in the Lagrangian framework: as a proxy for the stationary distribution of colloids, we display in figure 7 successive positions of  $N = 100$  long trajectories of colloids, initially randomly and uniformly distributed inside four adjacent cells, for growing values of the criterion  $R$ . These trajectories, plotted modulo  $2L$  and dumped long after release so that their position distribution is stationary, allow to see if particles sample uniformly the cells or regroup in some localized regions. While the cells

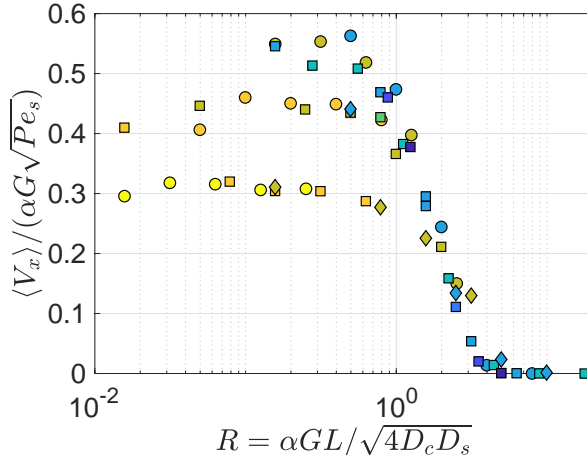


FIGURE 8. Normalized horizontal velocity  $\langle V_x \rangle_{\mathcal{L}} / (\alpha G \sqrt{Pe_s})$  versus the blockage criterion  $R = \alpha GL / 2\sqrt{D_c D_s}$  for the same numerical cases as in figure 6. Symbols are  $\circ$   $Pe_s = 31$ ,  $\square$   $Pe_s = 314$ ,  $\diamond$   $Pe_s = 3144$ . The color of symbol codes for the value of  $Pe_c \in [31; 3.14 \times 10^5]$ , the darker the larger the value.

are hardly distinguishable in figure 7a, they become more distinct as  $R$  increases, and finally become completely disjoint. In this last regime, unfolded trajectories are trapped in cells and their mean velocity drop to zero.

This proves that  $R$  is indeed a good indicator of trapping so that we choose to plot the rescaled mean velocity  $\langle V_x \rangle_{\mathcal{L}} / (\alpha G \sqrt{Pe_s})$  of figure 6 as a function of the blockage criterion  $R = \alpha GL / 2\sqrt{D_c D_s}$  instead of  $Pe_c$ . The result is displayed in figure 8: points are no longer scattered but regroup on a bundle of three curves which collapse provided  $R$  is large enough, when blockage dominates over other mechanisms of transport. Finally we note that the separation at small  $R$  coincides with the (weak) logarithmic dependence on the colloidal Péclet number, as pointed out from figure 3a, and which was not taken into account in the present model. Despite this observation, the order of magnitude obtained for  $\langle V_x \rangle_{\mathcal{L}}$  is rather good.

## 5. Summary and conclusion

We have studied the joint mixing/demixing of salt and colloids in a cellular flow with closed streamlines, and investigated how the long term dispersion of the colloids is modified by a linear phoretic drift  $\mathbf{v}_{dp} = \alpha \nabla S$ . In the chosen configuration of an imposed salt mean gradient along the  $x$ -direction  $\langle \nabla S \rangle = G \mathbf{e}_x$ , it is known that the salt concentration reaches a stationary state characterized by the presence of strong gradients localized along the vertical separatrices, so that the ratio of its effective diffusivity to the molecular diffusivity grows as the square root of the salt Péclet number (Shraiman 1987).

By means of high resolution numerical simulations performed both in the Eulerian and Lagrangian frameworks, we have shown that, starting from a uniform colloid concentration field (with no imposed colloid gradient), the colloids will demix. We have considered the long time dynamics of this demixing process, characterized by their mean velocity  $\mathbf{V}_m$  and their effective diffusivity  $D_{\text{eff}}$ ; in particular, we have studied how  $\mathbf{V}_m$  and  $D_{\text{eff}}$  are influenced by the phoretic drift for a wide range of parameters ( $\alpha G$ ,  $Pe_s = UL/D_s$ ,  $Pe_c = UL/D_c$ ) with  $\alpha$  and  $G > 0$ .

The main finding is that we observe substantially two regimes of colloids dynamics

depending on the blockage criterion  $R = \alpha GL / \sqrt{4D_c D_s}$ . When  $R < 1$ , the colloids are free to move and a strong demixing occurs at high  $Pe_c$ ; the concentration becomes homogeneous in the core of the flow cells while it is higher along the vertical separatrices, and in regions along the horizontal separatrices where the velocity  $\mathbf{u}$  of the flow is in the same direction as  $\mathbf{G}$  ( $\mathbf{G} \cdot \mathbf{u} > 0$ ). This is well explained as a combination of the non divergence free drift velocity, advection from the mean flow  $\mathbf{u}$ , and molecular diffusion. In this regime, the effective diffusivity is very close to  $D_c$  but the mean colloids velocity is strongly enhanced in the direction of the salt gradient. Using a model which takes into account the thickness of the high concentration region with an enhanced cell-to-cell transport due to the phoretic drift, we have shown that  $\langle V_x \rangle_{\mathcal{L}} \propto \alpha G \sqrt{Pe_s}$ . When  $R = \alpha GL / \sqrt{4D_c D_s} > 1$  the compressibility effect due to the phoretic drift is so strong that a depletion of colloids occurs along the separatrices so that colloids can no longer migrate from one cell to another and transport is suppressed. In this regime of blockage, all transport properties (mean velocity, effective diffusivity) go to zero when  $R$  is large enough. We also made an interesting observation in the transition regime  $R \sim 1$  (although we could not explain it), for which the effective diffusivity along the mean gradient,  $D_{\text{eff},x}$ , first strongly increases when increasing  $R$  while  $D_{\text{eff},y}$  and  $\langle V_x \rangle_{\mathcal{L}}$  are decreasing functions of  $R$ .

One may wonder if such behaviours as enhanced dispersion or blockage could be observed in practice. In the case of chemotaxis, because of the very large values of the drift coefficient  $\alpha$  involved (Chu *et al.* 2022), a situation of blockage would be likely to happen in such flow configurations, where the separatrices would become real transport barriers (Berman *et al.* 2021). However, when the nutrients would lack, and the gradient  $G$  would decrease sufficiently, the motile organisms would become free to leave their cell, and maybe find some food somewhere else. The case of diffusiphoresis could be studied in a similar configuration to the one of Solomon & Gollub (1988), for which a laminar flow with closed streamlines is created in an elongated volume ( $H = 0.75$  cm height,  $L_t = 15$  cm large,  $e = 1.5$  cm deep) by creating a laminar and stationary Rayleigh-Bénard flow with a small temperature different  $\Delta T \simeq 1^\circ$  C. Whether enhanced transport or blockage would be observed in such configuration by introducing salt and colloids at the same side (salt-in configuration), or at the opposite sides (salt-out configuration), remains an open question. Considering a drift velocity of the type  $\mathbf{v}_{\text{drift}} = D_{\text{dp}} \nabla \log S$  as in Mauger *et al.* (2016), with  $D_{\text{dp}} = 290 \mu\text{m}^2 \cdot \text{s}^{-1}$  for LiCl, one may obtain an estimate of the blockage criterion  $R_{\text{exp}}$  for the experiments by replacing  $\alpha$  with  $D_{\text{dp}}/S$  in  $R$  to get  $R_{\text{exp}} = D_{\text{dp}} GL / S \sqrt{4D_c D_c}$ . In the case of the aforementioned configuration one would have long after salt injection  $G = \Delta S / L_t$  and  $L = H$  so that an estimate of the criterion becomes

$$R_{\text{exp}} \simeq \frac{1}{40} \frac{D_{\text{dp}}}{\sqrt{D_c D_c}} \frac{\Delta S}{S}. \quad (5.1)$$

Taking roughly  $\Delta S/S \sim 1$ , one would then have  $R_{\text{exp}} = \mathcal{O}(0.1)$  which corresponds to enhanced transport velocity and a weak modification of the effective diffusivity. One may also argue that before developing a salt gradient over the whole width, it is developed over a much limited length at shorter times which increases the value of the criterion. One may then observe blockage at short time followed by enhanced transport at long time depending on the time delay between injection of salt and colloids. We let such experimental study for future work.



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