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An Acoustic Resonance Study of Complex Three-Dimensional Cavities by a Particular Integral Method

Alexandre Leblanc, Antoine Lavie

Université d'Artois, LAMTI, Technoparc Futura, 62400 Béthune, France. antlavie@fsa.univ-artois.fr

Christian Vanhille

ESCET. Universidad Rey Juan Carlos, C/. Tulipán, s/n. 28933 Móstoles, Madrid, Spain

Summary

In this paper, a three-dimensional boundary element method developed to evaluate the acoustic resonance of complex cavity is presented. The formulation is based on a particular integral method. Neumann or mixed boundary conditions are considered. Some numerical tests are carried out and results compared to classic models. General agreement is observed. A practical eigenfrequency analysis is realized by applying the model to a complex car cabin. The Arnoldi method is used to achieve the eigenfrequencies. A good numerical behaviour is observed and its implementation and use are quite easy.

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1. Introduction

Since vehicle noise problems are now a serious concern in the automotive industry, there is a strong demand to implement a method which can predict the noise level in the early design stage of a vehicle [1], and particularly, the noise caused by structural vibration. This increasing demand for acoustic eigenvalues analysis calls for an efficient and reliable method of computation of resonance frequencies of acoustical cavities. Acoustic fields in this steady-state interior problem are governed by the Helmholtz equation and boundary conditions. Over the last two decades, both the finite element (FEM) and boundary element method (BEM) [2] were proposed to obtain numerical solutions of this problem [3]. For BEM, the discretization is confined to the fluid-structure boundary. In the case of the FEM, the mesh is extended to the fluid domain. Compared to FEM, BEM leads to significant decrease of the preprocessing time. However, it must be noted that the FE matrices are symmetric and band for which efficient solvers are available. Application of BEM to acoustic eigenvalue analysis leads to a half-dozen efficient formulations [4] like the determinant search method (DSM) [5], the dual reciprocity method (DRM) [6], the multiple reciprocity method (MRM) [7, 8] and the series expansion method (SEM) [9]. In this paper, the particular integral method (PIM) is chosen because it is, with the DRM, one of the most versatile eigenanalysis methods:

frequency parameter is taken out from the matrices which are computed just once; sound fields are easily obtained; better accuracy is possible with additional internal collocation points; absorption can be taken into account at the boundary.

In Section II, a brief review of boundary element methods used in the acoustic resonance problem, excepted PIM, is proposed. Afterwards, the problem to solve is stated and the PIM is described.

In Section III, simple problems with known analytic and experimental results are presented in order to show the validity of the proposed technique. A detailed presentation of the solution procedure for eigenvalues extraction follows to demonstrate the efficiency of the Arnoldi eigensolver and the usefulness of the proposed new technique to solve more practical problems.

2. Acoustic eigenvalue problem and numerical method

In this section, the acoustic resonance problem is given and the numerical method developed to solve it is described.

2.1. Brief review of boundary element methods

Until the year 1982, the DSM [5] was the only method available to extract eigenvalues using the boundary element method. This method suffers from two major drawbacks: it requires the system matrix to be formed repeatedly for different values of frequencies and it is extremely inefficient in the case of closely spaced frequencies. The

DRM [6], based on an approximated function related to the acoustic pressure, offers a clear advantage over the DSM: matrices are free from the frequency parameter and as a result, they need to be calculated only once. Nevertheless, matrices are fully populated and unsymmetric, and their conditioning can be damaged when additional field collocation points or zoned boundary techniques are used. The MRM [8] eigenvalue formulation uses Helmholtz fundamental solution with expansion of matrices in terms of the wave numbers [10] and can eliminate the need for internal collocation points. The MRM method has two important drawbacks: it is difficult to use and inadequate for a general problem. Although this technique increases the size of matrices, the MRM allows to use generalized eigensolvers readily available in standard eigensolver packages. Moreover, this increasing matrix procedure introduces spurious eigenmodes into the solution, which, if not automatically filtered out, damages the reliability of the eigensolution. The SEM [9] is equivalent to the MRM and suffers from the same drawbacks as outlined by the matrix augmentation procedure of Kamiya *et al.* [10].

2.2. Equations of the problem

An acoustical cavity (domain Ω) filled with fluid, e.g. air, is considered (see Figure 1). The variables exhibit an implicit $e^{+j\omega t}$ time dependence. The harmonic wave propagation problem in terms of acoustic pressure amplitude is governed by the Helmholtz equation

$$\Delta P + k^2 P = 0, \quad (x, y, z) \in \Omega \subset \mathbb{R}^3, \quad (1)$$

where Δ is the Laplacian operator, P is the amplitude of the acoustic pressure, k is the wave number ω/c , ω is the circular frequency and c is the sound velocity in the fluid medium. To solve the acoustic eigenvalue problem, three types of boundary conditions are possible:

$$P = 0 \quad \text{on } \Gamma_1 \quad (\text{Dirichlet}) \quad (2a)$$

$$\partial P / \partial n = 0 \quad \text{on } \Gamma_2 \quad (\text{Neumann}) \quad (2b)$$

$$\partial P / \partial n = \gamma P \quad \text{on } \Gamma_3 \quad (\text{Robin}) \quad (2c)$$

where γ is a function of the location of the point that depends on the frequency, and $\partial/\partial n$ is the outward normal derivative on the boundary. This last condition is used when sound absorption at boundaries is taken into account. In the following, homogeneous Neumann and/or Dirichlet boundary conditions are considered. The acoustic problem consists in computing the eigenvalues - wave number k - in the acoustic fluid within the enclosure.

2.3. Particular Integral Method

In order to use an eigenanalysis solver, the frequency must be removed from the final set of equations. To this end, Ahmad *et al* [11] proposed the PIM. In this method, the total pressure and its gradient are decomposed into two functions, a complementary function and a particular solution:

$$P = P^c + P^i \quad \text{and} \quad \frac{\partial P}{\partial n} = \frac{\partial P^c}{\partial n} + \frac{\partial P^i}{\partial n}. \quad (3)$$

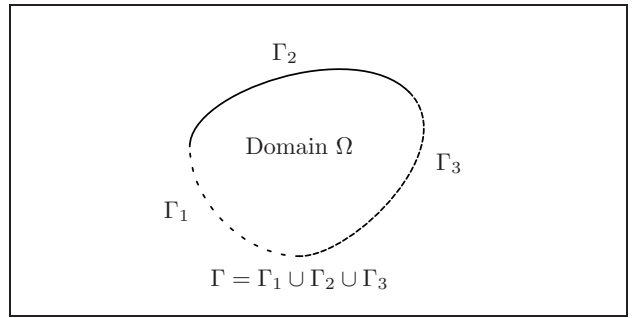


Figure 1. An acoustic cavity.

The complementary function satisfies the homogeneous part of the Helmholtz equation (1) without the inertial term

$$\Delta P^c = 0. \quad (4)$$

The particular solutions is solved by using

$$\Delta P^i + k^2 P = 0. \quad (5)$$

The boundary integral statement of equation (4) can be written as [2]

$$\alpha P^c(\vec{r}) + \int_{\Gamma} \frac{\partial G(\vec{r}, \vec{r}_s)}{\partial n} P^c(\vec{r}_s) d\Gamma(\vec{r}_s) - \int_{\Gamma} G(\vec{r}, \vec{r}_s) \frac{\partial P^c(\vec{r}_s)}{\partial n} d\Gamma(\vec{r}_s) = 0, \quad (6)$$

where α is the so-called solid angle [12] at the source point \vec{r} , n is the outward normal at the field point \vec{r}_s and Γ is the boundary of the domain Ω . $G(\vec{r}, \vec{r}_s)$ is the free-space Green's function for the Laplace's equation:

$$G(\vec{r}, \vec{r}_s) = \frac{1}{4\pi|\vec{r} - \vec{r}_s|} \quad (3-D). \quad (7)$$

This Green's function does not depend on the frequency parameter k . After discretization of the boundary and performing the integrations, equation (6) is re-written in a discrete form

$$[A]\{P^c\} = [B]\left\{\frac{\partial P^c}{\partial n}\right\}. \quad (8)$$

The complementary function and its derivative are eliminated from this equation by means of equation (3) to obtain

$$[A]\{P\} - [B]\left\{\frac{\partial P}{\partial n}\right\} = [A]\{P^i\} - [B]\left\{\frac{\partial P^i}{\partial n}\right\}. \quad (9)$$

P can be approximated by a global shape function:

$$P(\vec{r}) = \sum_{m=1}^{\infty} C(\vec{r}, \vec{r}_m) \phi(\vec{r}_m), \quad (10)$$

where ϕ is a fictitious density function at \vec{r}_m [13]. The shape function used is

$$C(\vec{r}, \vec{r}_m) = R - r(\vec{r}, \vec{r}_m), \quad (11)$$

where $r(\vec{r}, \vec{r}_m)$ is the distance between this two points and R is a suitable constant which is generally the largest distance between any two points on the body [13]. The sum (10) is truncated: the number of terms is equal to the number of nodes of the mesh. So, we obtain the following matrix form:

$$\{P\} = [C]\{\phi\}. \tag{12}$$

From equations (5), (10) and (11), we get

$$P^i(\vec{r}) = k^2 \sum_{m=1}^{\infty} D(\vec{r}, \vec{r}_m)\phi(\vec{r}_m), \tag{13}$$

with

$$D(\vec{r}, \vec{r}_m) = \frac{1}{6} \left[\frac{1}{2}r(\vec{r}, \vec{r}_m)^3 - Rr(\vec{r}, \vec{r}_m)^2 \right]. \tag{14}$$

Just as for $[C]$, we obtain:

$$\{P^i\} = k^2[D]\{\phi\}. \tag{15}$$

The normal derivative $\partial P^i / \partial n$ at \vec{r} can be computed from equation (13) and equation (14):

$$\frac{\partial P^i(\vec{r})}{\partial n} = k^2 \sum_{m=1}^{\infty} T(\vec{r}, \vec{r}_m)\phi(\vec{r}_m), \tag{16}$$

with

$$T(\vec{r}, \vec{r}_n) = \left[\frac{1}{4}r(\vec{r}, \vec{r}_n) - \frac{1}{3}R \right] (\vec{r} - \vec{r}_n) \cdot \vec{n}. \tag{17}$$

Its matrix form is

$$\left\{ \frac{\partial P^i}{\partial n} \right\} = k^2[T]\{\phi\}. \tag{18}$$

After combining equations (9), (15) and (18), the following equation is obtained:

$$[A]\{P\} - [B] \left\{ \frac{\partial P}{\partial n} \right\} = k^2([A][D] - [B][T])\{\phi\}. \tag{19}$$

By expressing the acoustic eigenvalue problem in terms of the fictitious function ϕ with equation (12), rather than the pressure amplitude P , the inversion of matrix is avoided. This yields [14]:

$$[A][C]\{\phi\} - [B] \left\{ \frac{\partial P}{\partial n} \right\} = k^2[M]\{\phi\}, \tag{20}$$

with

$$[M] = [A][D] - [B][T].$$

Including the appropriate boundary conditions in equation (20) leads to a generalized eigenvalue problem (GEP)

$$[H]\{x\} - k^2[G]\{x\} = 0. \tag{21}$$

For pure homogeneous Neumann boundary conditions, $\{x\} = \{\phi\}$, $[H] = [A][C]$ and $[G] = [M]$ [13]:

$$[A][C]\{\phi\} - k^2[M]\{\phi\} = 0. \tag{22}$$

With mixed homogeneous boundary conditions, i.e. $P_1 = 0$ on Γ_1 and $\partial P_2 / \partial n = 0$ on Γ_2 , equations (12) and (19) can be partitioned [15] as:

$$\begin{aligned} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} - \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{Bmatrix} \frac{\partial P_1}{\partial n} \\ \frac{\partial P_2}{\partial n} \end{Bmatrix} \\ = k^2 \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}, \end{aligned} \tag{23}$$

and

$$\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}.$$

Then the following system is obtained:

$$\begin{aligned} \begin{bmatrix} B_{11} & A_{12} \cdot (C_{21}C_{11}^{-1}C_{12} - C_{22}) \\ B_{21} & A_{22} \cdot (C_{21}C_{11}^{-1}C_{12} - C_{22}) \end{bmatrix} \begin{Bmatrix} \frac{\partial P_1}{\partial n} \\ \phi_2 \end{Bmatrix} \\ = k^2 \begin{bmatrix} 0 & (M_{11}C_{11}^{-1}C_{12} - M_{12}) \\ 0 & (M_{21}C_{11}^{-1}C_{12} - M_{22}) \end{bmatrix} \begin{Bmatrix} \frac{\partial P_1}{\partial n} \\ \phi_2 \end{Bmatrix}. \end{aligned} \tag{24}$$

The $\{x\}$ vector of equation (21) contains the unknown boundary values of ϕ and $\partial P / \partial n$. The PIM is also able to handle sound absorption at the boundary [16]. Absorption in an acoustic cavity can be incorporated in the discretized boundary element equation (20) with the following expression of boundary pressure gradient at the sound absorbing boundary, Robin conditions (equation 2c):

$$\left\{ \frac{\partial P}{\partial n} \right\} = -jk\beta \{P\} \tag{25}$$

If pressure gradients at the absorbing and non-absorbing boundary nodes are respectively denoted by $\{\partial P_1 / \partial n\}$ and $\{\partial P_2 / \partial n\}$, then, substituting equation (25) into equation (20), with hard boundary condition for the non-absorbing nodes, yields:

$$\begin{aligned} jk\beta \begin{bmatrix} B_{11}C_{11} & B_{11}C_{12} \\ B_{21}C_{11} & B_{21}C_{12} \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} \\ + [A][C]\{\phi\} = k^2[M]\{\phi\}. \end{aligned} \tag{26}$$

Including the coefficient β into the matrix and denoting the absorption matrix as $[C_{abs}]$ and $[K] = [A][C]$, equation (26) is rewritten as

$$[K]\{\phi\} + \lambda[C_{abs}]\{\phi\} + \lambda^2[M]\{\phi\} = 0, \tag{27}$$

with the eigenvalues $\lambda = jk$. This is a quadratic eigenproblem.

2.4. Eigenvalues extraction

The eigenvalue problem formulated in the previous part is a generalized eigenproblem with real unsymmetric and fully populated matrices. The two-sided Lanczos method [17] is quite popular to solve such a problem, because the Lanczos subspace matrix is tridiagonal, and then saves memory space. However the major problem of the Lanczos method is its potential breakdown problem. The look-ahead Lanczos method [18], the block Lanczos

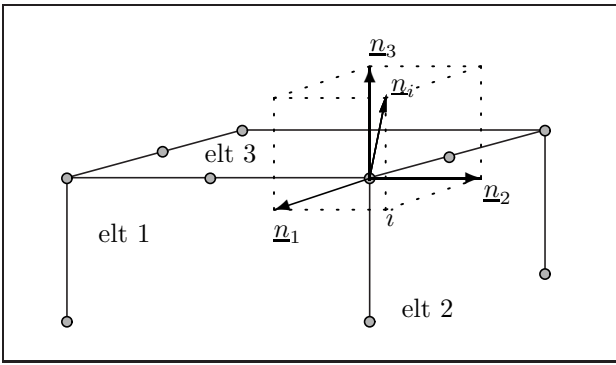


Figure 2. Normal vector determination.

method [19], the nested Lanczos method [20] and implicitly restarted Lanczos method [21] all offer solutions to this problem. Here, a variant of the Arnoldi procedure called the implicitly restarted Arnoldi method (IRAM) is chosen to solve the eigenfrequency problem. This method, provided by the ARPACK software [22], is designed to solve large scale GEPs with unsymmetric matrices. In Section III, it shows its efficiency by comparison to a direct algorithm of LAPACK [23], as well as the analytical values of the studied problems when available.

2.5. Geometry difficulties

The discretization of the problem is carried out with isoparametric quadratic elements [24]. This process duplicates the number of normal vectors at some nodes, belonging to two or more elements. To overcome this difficulty, the average normal is used, as shown in Figure 2.

3. Numerical experiments

These numerical developments written in Fortran 77 are included in the boundary element code EQI [25].

The above numerical method, called the generalized particular integral method (GPIM), is tested by comparison with conventional methods and its validation is given. Some improvements are also developed and the model is finally applied to a complex car cabin.

3.1. Validity of the method

The first simulation case involves determination of the eigenmodes of a rectangular parallelepiped with Neumann conditions (cf. equation 2b called hard boundary conditions). The dimension are $L_x = 0.6a$, $L_y = 0.8a$ and $L_z = a$ (Figure 3), where a is a typical length.

GPIM results are compared to the corresponding analytical eigenvalues, computed by the following formula [26]:

$$k(n_x, n_y, n_z) = \pi \sqrt{\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2}. \quad (28)$$

An eigenmode is found by a particular combination of integers n_x , n_y and n_z . Each of these three integers is assumed non-negative. Four meshes composed by identical

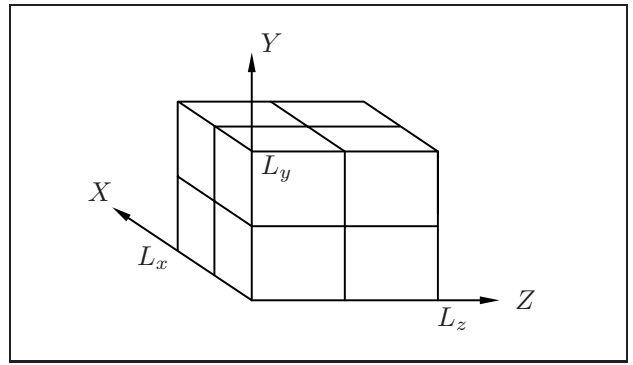


Figure 3. The 24-elements mesh of the rectangular parallelepiped acoustical cavity.

Table I. Comparison of natural frequencies (ka) of the 3-D rectangular parallelepiped with hard boundary.

Mode	GPIM 24 elts	GPIM 94 elts	GPIM 384 elts	GPIM 864 elts	Analyt. solution
1	3.602	3.224	3.213	3.188	3.142
2	4.602	4.126	4.028	3.999	3.927
3	5.991	5.326	5.184	5.128	5.029
4	6.248	5.717	5.389	5.371	5.236
ka max	3.142	7.854	12.566	18.849	

8-nodes quadrilateral elements are used to investigate the numerical behaviour of GPIM (respectively 24, 94, 384 and 864 elements). In Table I, the approximations of the first four Neumann eigenvalues by using the different discretizations are given. To ensure convergence, every mesh must meet the ‘ $\lambda/4$ criterion’ [27]: the length of elements must be smaller than a quarter of wavelength at the highest computed frequency. This condition defines a maximal value of ka for each mesh, which is also indicated in Table I, and is often used for radiation or scattering problem with isoparametric quadratic elements. We observe that this condition is not sufficient to guarantee accuracy of the solution.

The second example takes into account Dirichlet boundary conditions. We consider the same rectangular parallelepiped, but with Dirichlet condition on every nodes for $L_z = a$ and Neumann condition everywhere else. The results of this analysis are compared in Table II to the MRM solution [10] and the analytical solution for this problem given by [26]:

$$k(n_x, n_y, n_z) = \pi \sqrt{\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z + 1/2}{L_z}\right)^2} \quad (29)$$

3.2. Improvements

To solve the GEP with dense matrices of the GPIM, we use two different algorithms. The first can be found in LAPACK, driven by the procedure xGGEV, a direct solver

Table II. Comparison of natural frequency (ka) of the 3-D rectangular parallelepiped with mixed boundary.

Mode	24 elements		94 elements		Analyt. solution
	MRM	GPIM	MRM	GPIM	
1	1.640	1.534	1.590	1.537	1.517
2	4.440	4.999	4.280	4.449	4.229
3	4.760	5.446	4.710	4.834	4.712
4	5.600	6.316	5.580	5.942	5.467
ka max	3.142		7.854		

which uses the generalized Schur decomposition of the pair of matrices (H, G) of (21) to obtain all the generalized eigenvalues of these matrices, and optionally their left or right eigenvectors (or both). This procedure is easily implemented and offers accurate results but needs a lot of computation time. The second algorithm used can be found in the subroutines package ARPACK. This software is designed to compute few eigenvalues, in opposition to LAPACK. It is based on IRAM, which is able to solve large-scale Hermitian, non-Hermitian, standard or generalized eigenvalue problems. However, when applied to the GEP developed in this paper, its uses proves to have more difficulties than for LAPACK. This fact is mainly due to the structure of the matrices, which are not Hermitian positive semidefinite. Thus, a direct transformation of equation (21) to a standard eigenvalue problem $Cx = x\lambda$ leads to $C = G^{-1}H$. To avoid it, a direct factorization of G can be obtained, and replaced, when matrix vector product involving $G^{-1}H$ is called. However, experiments showed the limits of such a procedure. Thus, the choice of the number of Arnoldi vectors to be generated at each iteration of the method is not easy in this case. Depending on the size of the problem to solve rather than the number of eigenvalues to calculate, the method leads, for the same accuracy, to higher computing time than the direct method. A better use of ARPACK for GPIM is to use the shift-invert mode applied to the standard eigenvalue problem $Cx = x\lambda$ defined previously.

The choice of the shift value parameter influences the efficiency of this method, i.e. zero value leads to spurious eigenvalues. The number of Arnoldi vectors is driven by an empirical rule which can be written as [22]: $ncv = 2 * nev$, where nev is the number of eigenvalues to be computed and ncv is the number of Arnoldi vectors. With this configuration, the average CPU gain is 6 compared to the direct solver implemented on LAPACK.

As an integral formulation, the GPIM must verify the ‘ $\lambda/4$ criterion’. Experiments show a good adjustment of this criterion for the GPIM, but attention must be paid to certain eigenvalues which do not meet this criterion fully. Table III shows the ability of the GPIM for a cube with hard boundary and four different discretizations (N is the number of nodes). Values that are given in bold types correspond to frequencies that do not fit the ‘ $\lambda/4$ criterion’.

Convergence is slower than expected. Accuracy can be improved using internal collocation points [15] and a dif-

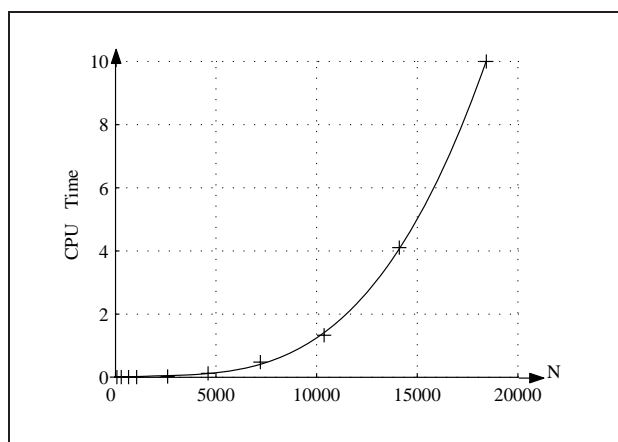


Figure 4. 3-D cube with hard boundary: computation cost versus number of nodes of the GEP for the first 12 eigenvalues.

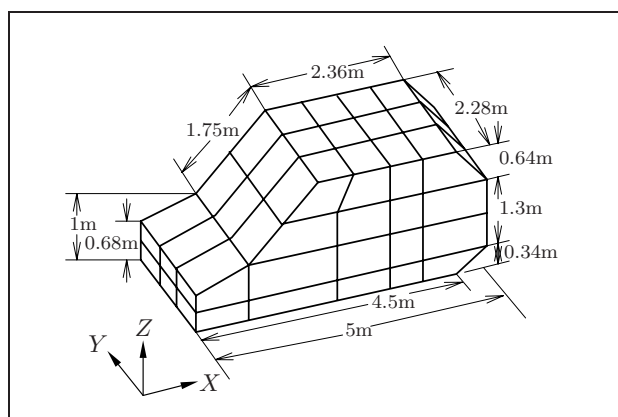


Figure 5. 90-elts meshed car interior.

ferent shape function from equation (11) [28]. Moreover, it has to be noted that spurious eigenvalues exist, for structures with one or more geometrical symmetric planes : multiple modes for undersized meshes are seen like different modes as we seen in Table III for the fourth and fifth modes.

Figure 4 shows the behaviour of central processor unit (CPU) time to solve GEP. With the same accuracy, three symmetric planes allow an average decrease factor equal to 10 on computation times.

3.3. Application

The usefulness of the BEM-based eigenanalysis technique is now tested in the case of noise control in automotive design [29]. A complex-shaped boundary (see Figure 5), for which no analytic solution is available, is considered. The model is compared to other numerical methods.

Table IV shows GPIM performances compared to the results data given by [30]. The mesh is made of 90 rectangular boundary elements, while the mesh of the FEM consists of 54 bricks elements. The GPIM is prone to compute higher frequencies because it is less affected by the increasing density of eigenmodes than the DSM [30].

For larger meshes (Table V), eigenvalues of the GPIM are closer to FEM and DSM results. The slower conver-

Table III. Natural frequency (ka) for the 10 first modes of the 3-D cube with hard boundary.

N	Modes											
	1	2	3	4	5	6	7	8	9	10		
74	1.797	2.824	3.293	3.334	3.770	4.146	4.603	4.827	5.863	5.923	6.188	6.692
290	1.647	2.371	2.804	3.234	3.401	3.711	3.847	4.074	4.879	5.115	5.410	5.556
650	1.621	2.310	2.763	3.234	3.303	3.623	3.729	3.972	4.708	4.944	5.226	5.278
1154	1.610	2.285	2.749	3.241	3.274	3.597	3.691	3.944	4.661	4.904	5.178	5.219
theoretical	1.571	2.221	2.721	3.142	3.512			3.848	4.443	4.712	4.967	5.210

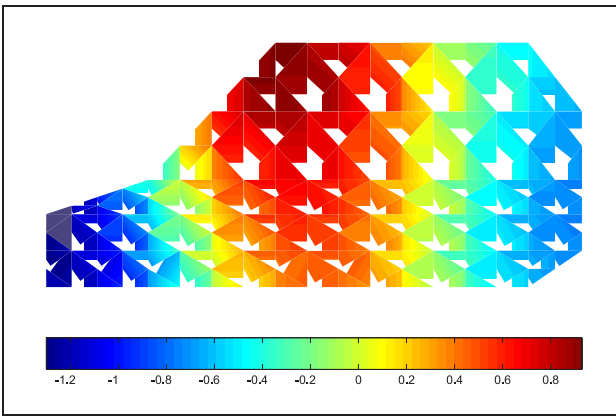


Figure 6. Car interior (272 nodes) with hard boundary: relative pressure amplitude at the second eigenmode in the symmetric plane.

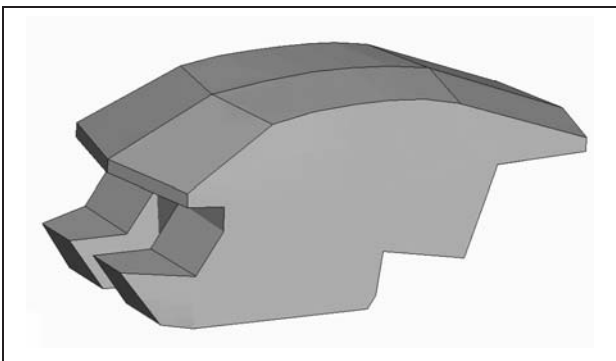


Figure 7. Geometry of the sedan compartment without seats.

gence of GPIM is compensated by its low computation time versus the DSM.

So, the GPIM eigenanalysis technique exhibits its effectiveness in extracting eigenmodes for an enclosure of complex geometry. Eigenmodes are easily obtained with equation (10) by using eigenvector of equation (21) as the fictitious function.

An example for the second mode shape is shown in Figure 6, an excellent agreement has been achieved with Bai's results [30].

A second application on a more complex Sedan compartment (cf. Figure 7) is carried out. This mesh contains 330 8-node boundary elements. In Table VI, comparisons with results given in Ref. [31] are shown and good agreement is observed.

Table IV. Eigenvalues of the sound field in the car cabin with hard boundary computed with GPIM versus others numerical methods.

Mode	FEM	BEM	
		DSM	GPIM
1	0.710	0.700	0.743
2	1.220	1.200	1.275
3	1.380	1.400	1.550
4	1.490	1.480	1.573
5	1.550	1.570	1.745

Table V. Eigenvalues of the sound field in the car cabin with hard boundary for three different meshes.

Mode	GPIM		
	90 elts	172 elts	688 elts
1	0.743	0.723	0.719
2	1.275	1.242	1.221
3	1.550	1.458	1.418
4	1.573	1.563	1.526
5	1.745	1.644	1.592
ka max	1.060	2.454	3.595

Table VI. Eigenvalues of the sound field in the second car cabin with hard boundary computed with the GPIM versus others numerical BEM results.

Mode	BEM	GPIM
	330 elts	330 elts
1	1.561	1.612
2	2.007	1.946
3	2.494	2.619
4	2.771	2.829
5	2.800	2.888

4. Conclusion

A boundary element eigensolution formulation (GPIM) is presented and tested for different theoretical and practical acoustical cavities. Results are compared to existing robust numerical methods. The accuracy and speed of the GPIM eigenfrequency calculations, combined with the greatly simplified meshing requirements for three-dimensional problems, gives great prospects for this technique as a design tool. An application of ARPACK solvers

is also outlined as an easy implementable and accurate eigenproblem solver for the GPIM. Moreover, these computations could be faster thanks to the possibility offered by the process of eigenvalues search employed in IRAM to work on parallel computers. Results show unexpected occurrences of spurious eigenvalues, whose behaviour in case of internal collocation points will have to be studied with care.

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