Distributed hybrid observer with prescribed convergence rate for a linear plant subject to disturbances using multi-hop decomposition

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***Abstract*— With a continuous-time formulation of the multi-hop decomposition, we propose a distributed hybrid observer for a sensor network where the plant and local observers run in continuous time and the information exchange among the sensing nodes is sampled-data. Process disturbances, measurement noise and communication noise are considered, and we prove that under some necessary detectability assumptions the observer gains can be tuned to guarantee exponential ISS with a prescribeda convergence rate. Simulations illustrate the performance of the proposed observer.**

***Index Terms*— Distributed estimation; Hybrid observer; Linear systems; Input-to-State Stability; Sensor network**

I. Introduction

The last two decades have witnessed the development of the so-called smart sensors, that is, different types of hardware with the ability of interacting with the environment by measuring physical variables, processing information, and communicating. When a number of these devices are deployed around a physical, in general complex, large-scale system to perform a collaborative monitoring task, this setup is referred as a Wireless Sensor Network (WSN).

One of the most important tasks for a WSN is state estimation, which finds important applications in different areas such as robotics [1], power systems [2], smart grids [3], or traffic management [4], among others. In distributed state estimation, no central processing unit is assumed to gather and process the information from the WSN, but each smart sensor (referred usually as *agent*) is supposed to estimate the state of the system with some limited locally available information. This information typically consist of some local measurements of the plant and of data exchanged with a subset of the other agents, called neighbors. This configuration leads to important advantages such as flexibility, scalability or reconfigurability, but also poses important challenges in terms of estimation analysis, distributed design, convergence and performance.

Perhaps the most popular approach for this setting is the Distributed Kalman Filter (DKF) [5]. DKF algorithms involve several steps that typically include: i) prediction of prior states using the system model, ii) correction based on local measurements with a Kalman gain, and iii) information fusion from neighboring agents based on consensus [6], [7], [8], diffusion [9], [10], or gossip-based algorithms [11]. An excellent review of these techniques can be found in [12].

Unlike DKF, the so-called distributed Luenberger observers do not rely on the knowledge of the statistical description of the noises or disturbances [13] and resort to deterministic approaches. To solve the problem in a distributed way, different state-based partitions have been proposed. In [14], a new estimation structure based on the observability canonical form is given, which depends on locally measured outputs and an agent indexing or ordering. An alternative subspace decomposition method, called multi-hop decomposition, is proposed in [15], where each agent constructs its own observability staircase form. These works provide a fully distributed design for the estimator in discrete time [13], [14], [15] or continuous time [15]. However, situations in which local measurements are available in continuous time but the intercommunication between agents happens in discrete time cannot be analyzed with these approaches.

This kind of continuous time observers in the presence of sampled-data measurements has been widely studied over the last two decades for the centralized case. Several observer design strategies have been proposed in the literature see e.g. [16], [17], [18] and references therein.

This paper develops a new hybrid distributed observer built upon the multi-hop decomposition [15] for linear plants equipped with a sensor network. Each network node runs locally a hybrid observer in continuous time for the locally observable modes, exploiting the continuous availability of locally measured outputs through continuous Luenberger-like dynamics. To reconstruct the whole plant state, neighboring estimates are injected at sampled-data communication instants and are used through the design of suitable consensus matrices. Under a necessary assumption, the proposed design guarantees a prescribed convergence rate and ISS with respect to process disturbances, continuous-time measurement noises and discrete-time communication noises.

Notation: We use a_{ij} to denote the element in the i-th row and j-th column of matrix A. We use $X = \underset{i\in V}{\text{col}}\,(Y_i)$, where $V \subset \mathbb{N}$ and Y_i is a collection of matrices with the same number of columns, to denote that X is a block column vector obtained by vertically stacking Y_i, for decreasing values of i from top to bottom; we use $X = \underset{i\in V}{\text{diag}}\,(Y_i)$, where $V \subset \mathbb{N}$ and Y_i is a collection of matrices, to denote that X is

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a block diagonal matrix whose diagonal blocks are Y_i, with decreasing values of i from top-left to bottom-right. We use (u, v) to denote the vector $\left[u^\top, v^\top\right]^\top$. $|x| := \sqrt{x^\top x}$ denotes the Euclidean norm of vector x. $\|x\|_\infty$ denotes the $\mathcal{L}$-infinity norm of the signal x. Given two sets $\mathcal{V} \subset \mathbb{N}$ and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is the (directed) graph with nodes $\mathcal{V}$ and edges $\mathcal{E}$. Finally, $\mathcal{N}_i$ denotes the set of nodes j such that (j, i) is an edge of $\mathcal{G}(\mathcal{V}, \mathcal{E})$, called in-neighborhood of i.

II. System description and problem formulation

A. System data

Consider a linear plant subject to an external disturbance driven by the following equation

$$\begin{aligned} \dot{x} &= Ax + d_0, \\ y_i &= C_i x + d_i, \quad \forall i \in \mathcal{V} := \{1, \dots, p\} \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the plant state, d_0 is some external disturbance, matrices A, C_i, for all $i \in \mathcal{V}$ are assumed to be constant and known. Each sensor measures an output y_i affected by measurement noise d_i.

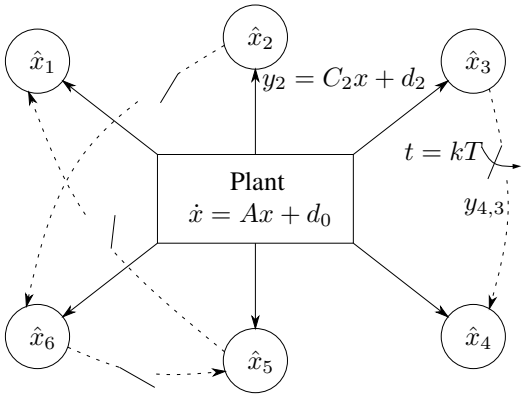

Fig. 1. Sketch of the system architecture for a set of $p = 6$ sensors. The continuous arrows represent the continuous-time measurement y_i available at a sensor's location, while the dashed lines depicts the exchange of information among the agents through a communication graph, that is performed only at periodic and synchronous sampling instants $t_k = kT$.

As shown in Figure 1, each output measurement $y_i \in \mathbb{R}^{m_i}$, with $m_i \leq n$, is available only locally at the sensor's location within our sensors network. Each pair (C_i, A) is not necessarily observable or detectable.

B. Cross-sensors information exchange

In our solution each sensor or agent is equipped with a local observer whose state estimate is denoted as $\hat{x}_i$, $i \in \mathcal{V}$. Since each measurement y_i is not necessarily sufficient for the local observer to reconstruct the whole state, some cross-sensors communication is needed. In this paper, we assume that the sensors $\mathcal{V}$ are connected through a communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, wherein continuous communication is impossible and the exchange of information among the network is only performed synchronously and periodically with a period $T > 0$. We also assume that the information exchange along each edge of the graph $(i, j) \in \mathcal{E}$ can be affected by transmission noise that can represent, for example, small delays or quantization errors. More formally, this means that each sensor $i \in \mathcal{V}$ has access to two types of measurements, namely

$$y_i(t) \qquad \forall t \in \mathbb{R} \tag{2}$$

$$y_{i,j}(kT) := \hat{x}_j(kT) + w_{i,j}(k) \quad \forall k \in \mathbb{N}, \forall j \in \mathcal{N}_i \tag{3}$$

where the second set of measurements is only available at periodic instants of time (sampled-data) $t_k = kT$.

In the previous equation $w_{i,j}$ is a (discrete-time) communication disturbance affecting the information transmitted from sensor j to sensor i. In the sequel, we will denote with $w_i := \operatorname{col}_{j \in \mathcal{N}_i}(w_{i,j})$ the collection of the communication disturbances transmitted to sensor i, and we will collect together all d_i and w_i as follows

$$d := \operatorname*{col}_{i \in \{0, \dots, p\}}(d_i), \qquad w := \operatorname*{col}_{i \in \mathcal{V}}(w_i). \tag{4}$$

C. Problem formulation

The goal of this paper is the design of a distributed pool of observers, located at each sensor, which only uses the available information $y_i(t), y_{i,j}(kT)$. Due to the presence of disturbances acting on the plant and on the measurements, it is likely that an exact estimation of the state is not achievable. Nevertheless, an ISS bound from the disturbances to the estimation errors can be derived. This corresponds to the following problem.

Problem 1: Design a *distributed hybrid or sampled-data observer* for system (1) such that, for a given exponential rate $\alpha > 0$, the following ISS property holds

$$\begin{aligned} \left| \operatorname*{col}_{i \in \mathcal{V}} (\hat{x}_i(t) - x(t)) \right| \leq \kappa \mathrm{e}^{-\alpha t} \left| \operatorname*{col}_{i \in \mathcal{V}} (\hat{x}_i(0) - x(0)) \right| \\ + \gamma_C \|d\|_\infty + \gamma_D \|w\|_\infty, \ \forall t \in \mathbb{R}_{\geq 0}. \end{aligned} \tag{5}$$

where $\hat{x}_i$ is the state estimate at the i-th node and $\kappa, \gamma_C, \gamma_D$ are some strictly positive, constant gains.

Note that considering (5) without noise we obtain a standard UGES property for the estimation error.

III. Continuous-discrete observer design

A. Multi-hop decomposition and necessary assumptions

The proposed distributed observer architecture relies on the multi-hop decomposition presented in [15] for a discrete-time setting and whose main definitions are easily extended hereafter to the continuous-time case.

For the linear system (1), the ϱ-hop output matrix, $C_{i,\varrho}$, of agent $i \in \mathcal{V}$ is recursively defined as $C_{i,0} := C_i$ and

$$C_{i,\varrho} := \begin{bmatrix} C_{i,\varrho-1} \\ \operatorname*{col}_{j \in \mathcal{N}_i} (C_{j,\varrho-1}) \end{bmatrix}.$$

In other words, $C_{i,\varrho}$ represents the outputs measured by agent i and by all the agents j such that in $\mathcal{G}$ there exists a (directed) path from j to i of length at most ϱ.

Consider now, for any $i \in \mathcal{V}$ and any "hop" $\varrho \in \{0, \dots, \ell_i\}$ of our multi-hop decomposition, the pair $(C_{i,\varrho}, A)$, where $\ell_i \in \mathbb{N}$ is specified in Assumption 1 below.

Based on the classical continuous-time observability concept, we can define the observable and unobservable sub-spaces of $\mathbb{R}^n$ related to $(C_{i,\varrho}, A)$, denoting them by $\mathcal{O}_{i,\varrho}$ and $\overline{\mathcal{O}}_{i,\varrho}$, respectively. Then, we can define the "innovation" matrix for agent i at hop $\varrho \in \{0, \dots, \ell_i\}$ as the matrix $W_{i,\varrho}$ whose columns generate the subspace of $\mathbb{R}^n$ that is observable at hop ϱ and unobservable at hop $\varrho - 1$. Namely, defining $\overline{\mathcal{O}}_{i,-1} := \mathbb{R}^n$, $W_{i,\varrho}$ is recursively defined as an orthogonal matrix such that

$$\begin{aligned} &\mathrm{Im}\,(W_{i,\varrho}) = \overline{\mathcal{O}}_{i,\varrho-1} \cap \mathcal{O}_{i,\varrho}, \qquad \varrho \in \{0, \dots, \ell_i\} \\ &\mathrm{Im}\,(W_{i,\ell_i+1}) = \overline{\mathcal{O}}_{\ell_i}. \end{aligned} \tag{6}$$

It is immediate to verify that horizontally stacking matrices $W_{i,\varrho}$ in (6), $\varrho \in \{0, \dots, \ell_i + 1\}$, provides an orthogonal matrix W_i, whose image is $\mathbb{R}^n$.

Remark 1: For linear systems, the observable sub-spaces related to a pair $(C_{i,\varrho}, A)$ are identical in the continuous-time and discrete-time cases. Thus, the multi-hop decomposition results presented in [15, Lemma 3-10] can and will be used for the next developments.

Note that matrix W_{i,ℓ_i+1} in (6) might be non-empty, regardless of the value ℓ_i. For this reason, some assumption on the unobservable modes is necessary for designing a distributed observer that solves Problem 1. In particular, given a desired exponential convergence rate α in (5), the least conservative assumption is based on the definition of collective α-detectability presented in [15].

Definition 1: [15] Given $\alpha > 0$, system (1) is collectively α-detectable with respect to $\mathcal{G}$ if, for each $i \in \mathcal{V}$, there exists a number of hops $\ell_i \in \mathbb{N}$ such that pair (C_{i,ℓ_i}, A) is $\alpha-$detectable.

Assumption 1: Given $\alpha > 0$, system (1) is collectively α-detectable. Moreover, ℓ_i in (6) is selected according to Definition 1.

Assumption 1 does not require any connectedness or spanning tree assumption on graph $\mathcal{G}$, which are typical assumptions in distributed estimation. Such assumptions may be conservative (see the example in [15, Figure 1]), while it is immediate to see that Assumption 1 is necessary for solving Problem 1.

B. Hybrid observer structure

Based on the multi-hop decomposition, each observer located at node $i \in \mathcal{V}$ is defined by the sampled-data system

$$\dot{\hat{x}}_i = A\hat{x}_i + W_{i,0}L_i(y_i - C\hat{x}_i), \quad t \notin \{kT, k \in \mathbb{N}\}, \tag{7a}$$

$$\hat{x}_i^+ = \hat{x}_i + \sum_{\varrho=1}^{\ell_i} \sum_{j \in \mathcal{N}_i} W_{i,\varrho} N_{i,j,\varrho} W_{j,\varrho-1}^\top (y_{i,j} - \hat{x}_i), \quad t \in \{kT, k \in \mathbb{N}\}, \tag{7b}$$

where $W_{i,\varrho}, i \in \mathcal{V}, \varrho \in \{0, \dots, \ell_i\}$ are defined in (6).

The architecture of observer (7) comprises three main parts:

1) *a copy of the observed plant*, which evolves in continuous time.
2) *a local output injection*, which can also be performed in continuous time, since each sensor has access to its own measurement in continuous time.
3) *a consensus correction term*, based on the information coming from the neighbors of agent i. Since the estimates provided by the other sensors/observers are only received periodically, this output injection corresponds to an impulsive modification of the state $\hat{x}_i$ of the i-th observer in the subspace generated by matrices $W_{i,\varrho}$, $\varrho \in \{1, \dots, \ell_i\}$.

The novelty in this model is given by the fact that the information coming from the neighbors is not available at all times, which makes the sampled-data architecture relevant in practical implementation.

The distributed observer tuning requires selecting matrices L_i and $N_{i,j,\varrho}$, of appropriate dimensions, for all (i, j, ϱ) in $\mathcal{V} \times \mathcal{N}_i \times \{0, 1, \dots, \ell_i\}$. The design of these gains will be addressed in the sequel.

C. Stability guarantees

We now present a tuning method for the observer gains to solve Problem 1.

The following property presents sufficient conditions for the observer in (7) to solve Problem 1. We will show in Section III-D that, under the necessary Assumption 1, there always exist gains satisfying this property.

Property 1: For each agent i, the local observer gains L_i are chosen so that

$$\overline{A}_{i,0} := \left(W_{i,0}^\top A - L_i C_i\right) W_{i,0} \tag{8}$$

is Hurwitz, with spectral abscissa $\overline{\alpha} \leq -\alpha$. Moreover, for every $\varrho \in \{1, \dots, \ell_i\}$, the consensus gains $N_{i,j,\varrho}$ are chosen so that

$$\overline{A}_{i,\varrho} := \mathrm{e}^{(W_{i,\varrho}^\top A W_{i,\varrho})T} \Big(I - \Big(\sum_{j \in \mathcal{N}_i} N_{i,j,\varrho} W_{j,\varrho-1}^\top \Big) W_{i,\varrho} \Big) \tag{9}$$

is Schur, with spectral radius $\overline{\beta} \in [0, \mathrm{e}^{-\alpha T}]$.

We now state the main result of this work, which will be proven in Section V-C.

Theorem 1: Given $\alpha > 0$, if the gains of observer (7) satisfy Property 1, then this observer solves Problem 1.

D. Feasibility guarantees and design guidelines

Inspired by the proof of [15, Th.14], we show next that Assumption 1 is not only necessary, but also sufficient for solving Problem 1.

Theorem 2: Under Assumption 1, it is always possible to select matrices L_i and $N_{i,j,\varrho}$ satisfying Property 1.

Proof: The existence of matrices L_i is trivial, as (8) considers only the locally observable modes.

Concerning the matrices $N_{i,j,\varrho}$, we can rewrite each matrix $\overline{A}_{i,\varrho}$ in (9) as

$$\overline{A}_{i,\varrho} = E_{i,\varrho} - \overline{N}_{i,\varrho} \overline{C}_{i,\varrho},$$

where

$$E_{i,\varrho} := \mathrm{e}^{W_{i,\varrho}^\top A W_{i,\varrho} T},$$
$$\overline{N}_{i,\varrho} := E_{i,\varrho} \operatorname*{col}_{j\in\mathcal{N}_i} \left(N_{i,j,\varrho}^\top\right)^\top,$$
$$\overline{C}_{i,\varrho} := \operatorname*{col}_{j\in\mathcal{N}_i} \left(W_{j,\varrho-1}^\top\right) W_{i,\varrho}.$$

Using this structure, we know from linear systems theory that, if the pair $\left(\overline{C}_{i,\varrho}, E_{i,\varrho}\right)$ is observable, then it is possible to find $\overline{N}_{i,\varrho}$ that places the eigenvalues of $\overline{A}_{i,\varrho}$ anywhere in the unit circle. Note also that any selection of $\overline{N}_{i,\varrho}$ corresponds to a unique selection of the gains $N_{i,j,\varrho}$ for all $j \in \mathcal{N}_i$, because $E_{i,\varrho}$ is invertible for any $T > 0$. We then complete the proof by showing observability of $\left(\overline{C}_{i,\varrho}, E_{i,\varrho}\right)$.

According to the PBH test for observability (see [19, Theorem 15.9]), the pair $\left(\overline{C}_{i,\varrho}, E_{i,\varrho}\right)$ is observable if and only

$$\operatorname{rank}\left(\begin{bmatrix} E_{i,\varrho} - \lambda I \\ \overline{C}_{i,\varrho} \end{bmatrix}\right) = n_{i,\varrho}, \quad \forall \lambda \in \mathbb{C}, \tag{10}$$

where the size $n_{i,\varrho}$ of $E_{i,\varrho}$ corresponds to the number of columns of $W_{i,\varrho}$. In turn, (10) is true if $\overline{C}_{i,\varrho}$ is full column rank. From item (iii) of [15, Lemma 3] we know that the image of $W_{i,\varrho}$ is a subset of the sum of the images of $W_{j,\varrho-1}, j \in \mathcal{N}_i$, or equivalently

$$W_{i,\varrho} = \operatorname*{col}_{j\in\mathcal{N}_i} \left(W_{j,\varrho-1}^\top\right)^\top Q, \tag{11}$$

where Q is some selection matrix. Since $W_{i,\varrho}$ is full column rank by definition, we may use (11) to obtain $Q^\top \overline{C}_{i,\varrho} = Q^\top \operatorname*{col}_{j\in\mathcal{N}_i} \left(W_{j,\varrho-1}^\top\right) W_{i,\varrho} = W_{i,\varrho}^\top W_{i,\varrho}$, which implies $\operatorname{rank}\left(\overline{C}_{i,\varrho}\right) \geq \operatorname{rank}\left(Q^\top \overline{C}_{i,\varrho}\right) = n_{i,\varrho}$, and since $\overline{C}_{i,\varrho}$ has exactly $n_{i,\varrho}$ columns, the equality is immediately obtained, thus concluding the proof. ∎

Remark 2: While Theorem 1 ensures that, under Assumption 1, any choice of the observer gains satisfying the bounds in Proposition 1 solves Problem 1, a good practical rule is to tune the observer gains so that $\overline{\alpha} \ll -\alpha$ while $\overline{\beta}$ is closer to $\mathrm{e}^{-\alpha T}$. This way, observers make local errors converge to zero faster than consensus errors, which improves the transient behaviour.

IV. Error dynamics

Define the estimation error of each agent as

$$e_i := x - \hat{x}_i, \quad i \in \mathcal{V}, \tag{12}$$

and apply the multi-hop decomposition to the error dynamics, which yields, for each $i \in \mathcal{V}$ and each $\varrho \in \{0, \dots, \ell_i+1\}$

$$\varepsilon_{i,\varrho} := W_{i,\varrho}^\top e_i \quad \longrightarrow \quad e_i := \sum_{r=0}^{\ell_i+1} W_{i,r} \varepsilon_{i,r}. \tag{13}$$

Define also the index sets

$$\mathcal{V}_\varrho := \{i \in \mathcal{V} : \ell_i + 1 \geq \varrho\}, \quad \varrho \in \{1, \dots, \overline{\ell}\}, \tag{14}$$

where $\overline{\ell} := \max_i\{\ell_i + 1\}$. Then, for each $\varrho \in \{1, \dots \overline{\ell}\}$, we can denote with ε_ϱ the vector obtained by stacking all the transformed error coordinates at hop ϱ, and with ε the vector obtained stacking all the ε_ϱ, namely

$$\varepsilon_\varrho := \operatorname*{col}_{i\in\mathcal{V}_\varrho} (\varepsilon_{i,\varrho}), \qquad \varepsilon := \operatorname*{col}_{\varrho\in\{0,\dots,\overline{\ell}\}} (\varepsilon_\varrho) \in \mathbb{R}^{n_\varepsilon}. \tag{15}$$

Given these definitions, the error dynamics can be described by means of a hybrid dynamical system. To this end, we augment the error state ε with a timer element τ, which will be used to trigger periodic jumps, representing the sampled-data exchange of information. In particular, we prove in the sequel that the error dynamics results in

$$\begin{aligned} \begin{bmatrix} \dot{\varepsilon} \\ \dot{\tau} \end{bmatrix} &= \begin{bmatrix} A_\varepsilon \varepsilon + Rd \\ 1 \end{bmatrix}, \quad (\varepsilon, \tau) \in \mathcal{C}, \\ \begin{bmatrix} \varepsilon^+ \\ \tau^+ \end{bmatrix} &= \begin{bmatrix} J_\varepsilon \varepsilon + Sw \\ 0 \end{bmatrix}, \quad (\varepsilon, \tau) \in \mathcal{D}, \end{aligned} \tag{16}$$

where the state evolves in $X := \mathbb{R}^{n_\varepsilon} \times [0, T]$ and the flow and jump sets are defined as

$$\mathcal{C} := X, \qquad \mathcal{D} := \{(\varepsilon, \tau) \in X : \tau = T\}. \tag{17}$$

The matrices in (16) are defined as

$$A_\varepsilon := \begin{bmatrix} A_{\overline{\ell}} & & & \\ & A_{\overline{\ell}-1} & \star & \\ & & \ddots & \\ & 0 & & A_1 \\ & & & & A_0 \end{bmatrix}, \quad J_\varepsilon := \begin{bmatrix} \Delta_{\overline{\ell}} & & & \\ & \Delta_{\overline{\ell}-1} & \star & \\ & & \ddots & \\ & 0 & & \Delta_1 \\ & & & & \Delta_0 \end{bmatrix}, \tag{18}$$

$$R := \operatorname*{col}_{\varrho\in\{0,\dots,\overline{\ell}\}} (R_\varrho), \quad S := \operatorname*{col}_{\varrho\in\{0,\dots,\overline{\ell}\}} (S_\varrho), \tag{19}$$

where $\star$ represents some possibly non-zero terms. Using the definition of $\mathcal{V}_\varrho$ in (14) and adopting the convention that the unobservable modes of node i correspond to $N_{i,j,\ell_i+1} := 0$ for all $j \in \mathcal{N}_i$, the entries of (18)-(19) are given by

$$A_\varrho := \begin{cases} \operatorname*{diag}_{i\in\mathcal{V}} \left(W_{i,0}^\top A W_{i,0} - L_i C_i W_{i,0}\right), & \text{if } \varrho = 0, \\ \operatorname*{diag}_{i\in\mathcal{V}_\varrho} \left(W_{i,\varrho}^\top A W_{i,\varrho}\right), & \text{if } \varrho \in \{1, \dots, \overline{\ell}\}, \end{cases} \tag{20a}$$

$$R_\varrho := \begin{cases} \left[\operatorname*{diag}_{i\in\mathcal{V}} (-L_i) \;\middle|\; \operatorname*{col}_{i\in\mathcal{V}} \left(W_{i,0}^\top\right)\right], & \text{if } \varrho = 0, \\ \left[\mathbf{0} \;\middle|\; \operatorname*{col}_{i\in\mathcal{V}_\varrho} \left(W_{i,\varrho}^\top\right)\right], & \text{if } \varrho \in \{1, \dots, \overline{\ell}\}, \end{cases} \tag{20b}$$

$$\Delta_\varrho := \begin{cases} I, & \text{if } \varrho = 0, \\ \operatorname*{diag}_{i\in\mathcal{V}_\varrho} \left(I - \left(\sum_{j\in\mathcal{N}_i} N_{i,j,\varrho} W_{j,\varrho-1}^\top\right) W_{i,\varrho}\right), & \\ & \text{if } \varrho \in \{1, \dots, \overline{\ell}\}. \end{cases} \tag{20c}$$

$$S_\varrho := \begin{cases} \mathbf{0}, & \text{if } \varrho = 0, \\ \operatorname*{col}_{i\in\mathcal{V}_\varrho} (S_{i,\varrho}), & \text{if } \varrho \in \{1, \dots, \overline{\ell}\}, \end{cases} \tag{20d}$$

$$S_{i,\varrho} = -\left[\mathbf{0} \quad \operatorname*{col}_{j\in\mathcal{N}_i} \left(W_{j,\varrho-1} N_{i,j,\varrho}^\top\right)^\top \quad \mathbf{0}\right], \tag{20e}$$

where the only non-zero element of the block vector $S_{i,\varrho}$ is in position i. We can finally state the following lemma, whose proof is reported in the appendix.

Lemma 1: Consider the transformed error coordinates in (13), the vector ε defined in (15) and the noise vectors d and w in (4). Then, ε evolves according to (16)-(20).

V. Proof of Theorem 1

A. Nonsmooth Lyapunov function

Consider, for some $P = P^\top > 0$ to be specified later, the Lyapunov function candidate

$$V(\varepsilon,\tau) := \sqrt{\varepsilon^\top \overline{P}(\tau)\varepsilon} := \sqrt{\varepsilon^\top \mathrm{e}^{A_\varepsilon^\top (T-\tau)} P \mathrm{e}^{A_\varepsilon (T-\tau)}\varepsilon}, \tag{21}$$

with A_ε defined in (18). Introduce the attractor

$$\mathcal{A} := \{(\varepsilon,\tau) \in X : \varepsilon = 0\}, \tag{22}$$

and notice that V is smooth outside A_ε, so we can define its gradient $\nabla V(\varepsilon,\tau)$ for all $(\varepsilon,\tau) \in \mathcal{C} \setminus \mathcal{A}$.

B. Properties of V and some useful results

We will now present some properties of triangular and exponential matrices, needed to prove the main stability result. Their proofs are trivial applications of linear algebra concepts, thus they are not reported.

Lemma 2: Consider $A, B \in \mathbb{R}^{n\times n}$. The following holds

(i) if A, B are upper triangular, then AB is also upper triangular, and $(ab)_{ii} = a_{ii}b_{ii}$;
(ii) if A is upper triangular and $B = \mathrm{e}^{At}$, then B is also upper triangular, and $b_{ii} = \mathrm{e}^{a_{ii}t}$
(iii) if λ is an eigenvalue of A and $B = \mathrm{e}^{At}$, then $\mathrm{e}^{\lambda t}$ is an eigenvalue of B.

Next, we prove some properties of V which will be used to prove the main theorem in the next section.

Proposition 1: Consider the Lyapunov function in (21). There exist positive constants c_1, c_2, M, c_C, c_D and $\eta \le \mathrm{e}^{-\alpha T}$ such that

(i) $c_1|\varepsilon| \le V(\varepsilon,\tau) \le c_2|\varepsilon|$, for all $(\varepsilon,\tau) \in X_\varepsilon$,
(ii) $|\nabla_\varepsilon V(\varepsilon,\tau)| \le M$, for all $(\varepsilon,\tau) \in X \setminus \mathcal{A}$,
(iii) $\dot V \le c_C|d|$, for all $(\varepsilon,\tau) \in \mathcal{C} \setminus \mathcal{A}$,
(iv) $V^+ \le -\eta V + c_D|w|$, for all $(\varepsilon,\tau) \in \mathcal{D} \setminus \mathcal{A}$.

Proof: We prove the statements one by one.

Proof of (i)*:* Since P is positive definite, we have

$$\sqrt{\lambda_m}\left|\mathrm{e}^{A_\varepsilon(T-\tau)}\varepsilon\right| \le V(\varepsilon,\tau) \le \sqrt{\lambda_M}\left|\mathrm{e}^{A_\varepsilon(T-\tau)}\varepsilon\right|, \tag{23}$$

where λ_m and λ_M are the minimum and maximum eigenvalues of P, respectively. Recalling that, for any invertible matrix $T \in \mathbb{R}^{m\times m}$ and any vector $y \in \mathbb{R}^m$, we have

$$|Ty| = \frac{|T^{-1}||Ty|}{|T^{-1}|} \ge \frac{|T^{-1}Ty|}{|T^{-1}|} = \frac{1}{|T^{-1}|}|y|,$$

from (23) we can obtain

$$c_1|\varepsilon| := \sqrt{\lambda_m}\vartheta_1|\varepsilon| \le V(\varepsilon,\tau) \le \sqrt{\lambda_M}\vartheta_2|\varepsilon| =: c_2|\varepsilon|,$$

where $\vartheta_1 := \min_{\tau\in[0,T]} \frac{1}{\left|\mathrm{e}^{A_\varepsilon(\tau-T)}\right|}$ and $\vartheta_2 := \max_{\tau\in[0,T]} \left|\mathrm{e}^{A_\varepsilon(T-\tau)}\right|$, are both strictly positive, because of continuity and since the matrix exponential is always invertible.

Proof of (ii)*:* From the explicit calculation of the derivatives we get, for all $(\varepsilon,\tau) \in X \setminus \mathcal{A}$,

$$|\nabla_\varepsilon V(\varepsilon,\tau)| = \frac{2\left|\overline{P}\varepsilon\right|}{2\sqrt{\varepsilon^\top \overline{P}\varepsilon}} = \sqrt{\frac{\varepsilon^\top \overline{P}^2\varepsilon}{\varepsilon^\top \overline{P}\varepsilon}} \le \frac{\lambda_M\vartheta_2}{\sqrt{\lambda_m}\vartheta_1} =: M.$$

Proof of (iii)*:* Computing the evolution of V along flows of the solutions to (16)-(17) we get

$$\begin{aligned}\dot V &= \langle \nabla_\varepsilon V(\varepsilon,\tau), A_\varepsilon\varepsilon + Rd\rangle + \langle \nabla_\tau V(\varepsilon,\tau), 1\rangle \\ &= \frac{2\varepsilon^\top \overline{P}A_\varepsilon\varepsilon}{2\sqrt{\varepsilon^\top\overline{P}\varepsilon}} + \langle \nabla_\varepsilon(\varepsilon,\tau), Rd\rangle - \frac{2\varepsilon^\top\overline{P}A_\varepsilon\varepsilon}{2\sqrt{\varepsilon^\top\overline{P}\varepsilon}} \\ &\le M|R||d| =: c_C|d|, \qquad \forall (\varepsilon,\tau) \in \mathcal{C}\setminus\mathcal{A}.\end{aligned}$$

Proof of (iv)*:* For the evolution of V across jumps of the solutions to (16)-(17), denoting $g(\varepsilon,\tau) := (J_\varepsilon\varepsilon + Sw,\ 0)$, we have

$$V^+ := V(g(\varepsilon,w)) = V(g(\varepsilon,0)) + V(g(\varepsilon,w)) - V(g(\varepsilon,0)).$$

Whenever the line segment $\Gamma := \{\gamma g(\varepsilon,w) + (1-\gamma)g(\varepsilon,0), \gamma \in [0,1]\}$ does not pass through the origin, we can use the mean value theorem to rewrite V^+ as

$$V^+ = V(g(\varepsilon,0)) + \langle \nabla V(\xi), g(\varepsilon,w) - g(\varepsilon,0)\rangle, \tag{24}$$

for some $\xi \in \Gamma$. If instead $0 \in \Gamma$, then we can use a similar argument based on perturbations and converging sequences. Using (24), we obtain

$$\begin{aligned}V^+ &= V\left(J_\varepsilon\varepsilon, 0\right) + \langle \nabla_\varepsilon V(\xi), Sw\rangle + \langle \nabla_\tau V(\xi), 0\rangle \\ &= \sqrt{\varepsilon^\top J_\varepsilon^\top \mathrm{e}^{A_\varepsilon^\top T} P e^{A_\varepsilon T} J_\varepsilon\varepsilon} + \langle \nabla_\varepsilon V(\xi), Sw\rangle. \end{aligned} \tag{25}$$

Let us consider the first term in the last line of (25). Since both A_ε and J_ε are block upper triangular, using items (i) and (ii) of Lemma 2 we conclude that $\mathrm{e}^{A_\varepsilon T}J_\varepsilon$ is block upper triangular as well. Moreover, we know that the blocks on the main diagonal of $\mathrm{e}^{A_\varepsilon T}J_\varepsilon$ are given by $\mathrm{e}^{A_\varrho T}\Delta_\varrho$, for all $\varrho \in \{0,\dots,\overline{\ell}\}$. Since these are in turn block diagonal matrices, we conclude that the eigenvalues of $\mathrm{e}^{A_\varepsilon T}J_\varepsilon$ are given by the union of the eigenvalues of the following matrices, issued from (20),

$$\begin{cases} \mathrm{e}^{\left(W_{i,0}^\top A W_{i,0} - L_i C_i W_{i,0}\right)T}, & \forall i \in \mathcal{V}, \\ \mathrm{e}^{\left(W_{i,\varrho}^\top A W_{i,\varrho}\right)T}\left(I - \left(\sum_{j\in\mathcal{N}_i} N_{i,j,\varrho} W_{j,\varrho-1}^\top\right) W_{i,\varrho}\right), & \\ & \forall i \in \mathcal{V}, \varrho \in \{1,\dots,\ell_i\}, \\ \mathrm{e}^{\left(W_{i,\ell_i+1}^\top A W_{i,\ell_i+1}\right)T}, & \forall i \in \mathcal{V}. \end{cases}$$

These correspond to the eigenvalues of

- $\mathrm{e}^{\overline{A}_{i,0}T}$ for all $i \in \mathcal{V}$,
- $\overline{A}_{i,\varrho}$ for all $i \in V, \varrho \in \{1,\dots,\ell_i\}$,
- $\mathrm{e}^{\left(W_{i,\ell_i+1}^\top A W_{i,\ell_i+1}\right)T}$ for all $i \in \mathcal{V}$.

In view of Property 1, item (iii) of Lemma 2 and Assumption 1, we can then conclude that $\mathrm{e}^{A_\varepsilon T}J_\varepsilon$ has spectral radius $\eta \le \max\{\mathrm{e}^{\overline{\alpha}T}, \overline{\beta}, \mathrm{e}^{-\alpha T}\} = \mathrm{e}^{-\alpha T} < 1$. From basic linear systems theory (see for example [19, Theorem 8.4]) we know that this fact is equivalent to the existence of a solution P to the discrete-time Lyapunov inequality

$$J_\varepsilon^\top \mathrm{e}^{A_\varepsilon^\top T} P \mathrm{e}^{A_\varepsilon T} J_\varepsilon < \eta^2 P.$$

Using this fact in (25), we obtain

$$\begin{aligned}V^+ &\le \eta\sqrt{\varepsilon^\top P\varepsilon} + M|S||w| = \eta V + M|S||w| \\ &=: \eta\, V + c_D|w|, \qquad \text{for all } (\varepsilon,\tau) \in \mathcal{D}_\varepsilon \setminus \mathcal{A}_\varepsilon.\end{aligned}$$

■

C. Proof of Theorem 1

We will use a proof technique similar to the one in [20, Theorem 1], which needs to be carefully revisited in some of its parts, as the conditions are slightly different. For each solution ξ_ε to (16)-(17), we can use items (iii), (iv) of Proposition 1 to integrate the value of $v(t,k) = V(\xi_\varepsilon(t,k))$ as follows

$$\begin{aligned} v(t,k) &\leq v(t_k,k) + c_C(t-t_k)\|d\|_\infty \\ &\leq (\eta v(t_k,k-1) + c_D|w|) + c_C T\|d\|_\infty \\ &\leq \eta\left(v(t_{k-1},k-1) + c_C T\|d\|_\infty\right) \\ &\qquad + c_D\|w\|_\infty + c_C T\|d\|_\infty \\ &\leq \eta\left(\eta v(t_{k-1},k-2) + c_D\|w\|_\infty\right) \\ &\qquad + c_D\|w\|_\infty + (1+\eta)\, c_C T\|d\|_\infty, \end{aligned}$$

where $\{t_k\}$ is the sequence of jump times of ξ_ε, and we used the fact that $0 \leq (t-t_k) \leq T$ for all $(t,k) \in \operatorname{dom} \xi_\varepsilon$. Iterating this process, we obtain

$$\begin{aligned} V(\xi_\varepsilon(t,k)) &\leq \eta^k V(\varepsilon(0,0)) \\ &\quad + \sum_{h=0}^{k-1} \eta^h c_D\|w\|_\infty + \sum_{h=0}^{k} \eta^h T c_C\|d\|_\infty. \end{aligned} \tag{26}$$

Recalling that $\eta \leq \mathrm{e}^{-\alpha T}$, we can write

$$\eta^k = \mathrm{e}^{\alpha T}\mathrm{e}^{-\alpha(k+1)T} \leq \mathrm{e}^{\alpha T}\mathrm{e}^{-\alpha t}. \tag{27}$$

Applying (27) and item (i) of Proposition 1 to (26), we get

$$|\varepsilon(t,k)| \leq \frac{c_2}{c_1}\mathrm{e}^{\alpha T}\mathrm{e}^{-\alpha t}|\varepsilon(0,0)| + \frac{T c_C\|d\|_\infty}{c_1(1-\eta)} + \frac{c_D\|w\|_\infty}{c_1(1-\eta)}.$$

Lastly, we note that

$$\begin{aligned} |\varepsilon|^2 = \varepsilon^\top\varepsilon &= \sum_{i\in\mathcal{V}}\sum_{\varrho=0}^{\ell_i+1} e_i^\top W_{i,\varrho} W_{i,\varrho}^\top e_i \\ &= \sum_{i\in\mathcal{V}} e_i^\top W_i \sum_{\varrho=0}^{\ell_i+1} \left(Q_{i,\varrho}Q_{i,\varrho}^\top\right) W_i^\top e_i \end{aligned}$$

where each $Q_{i,\varrho}$ selects unique columns of W_i, because of the orthogonality of the multi-hop decomposition, therefore it is immediate to verify that $\sum_{\varrho=0}^{\ell_i+1} Q_{i,\varrho}Q_{i,\varrho}^\top = I$. This implies

$$|\varepsilon|^2 = \sum_{i\in\mathcal{V}} e_i^\top W_i W_i^\top e_i = \left|\operatorname*{col}_{i\in\mathcal{V}}(e_i)\right|^2.$$

Therefore, we showed that (5) holds with

$$\begin{aligned} \kappa &:= \frac{c_2}{c_1}\mathrm{e}^{\alpha T} \\ \gamma_C &:= \frac{T c_C}{c_1(1-\eta)} = \frac{TM|R|}{\sqrt{\lambda_m}\vartheta_1(1-\eta)} = \frac{\lambda_M\vartheta_2}{\lambda_m\vartheta_1^2}\frac{T|R|}{1-\eta}, \\ \gamma_D &:= \frac{c_D}{c_1(1-\eta)} = \frac{\lambda_M\vartheta_2}{\lambda_m\vartheta_1^2}\frac{|S|}{1-\eta}, \end{aligned}$$

thus completing the proof.

VI. Numerical simulations

In order to illustrate the proposed observer, we simulate the continuous-time system (1), selecting

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{bmatrix}, \qquad C_i = e_i^\top, \quad i \in \{1,\dots,4\},$$

where e_i represents the i-th vector of the Euclidean basis, i.e. a vector with all zeros and a one in position i. The agents are interconnected with a directed ring graph, i.e. agent 1 sends information to agent 2, 2 to 3, and so on.

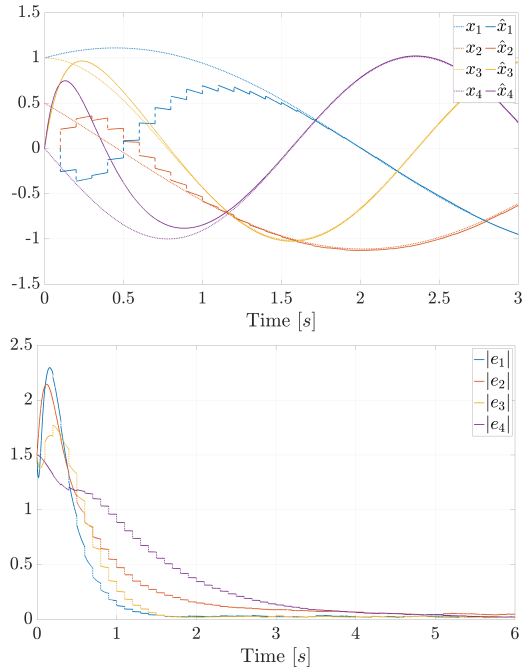

Fig. 2. Evolution of the estimates of agent 3, compared with the plant states (top); evolution of the norm of the estimation error for each of the agents (bottom).

Figure 2 shows the simulation results for the proposed observer. For this simulation, we selected $T = 0.1s$ and $\alpha = 1$. The observer gains were chosen so that Property 1 was satisfied with $\overline{\alpha} = -5\alpha$ and $\overline{\beta} = \mathrm{e}^{-\alpha T}$. The noises d and w were simulated as random signals, with $\|d\|_\infty = 0.04$ and $\|w\|_\infty = 0.02$.

In the first plot, we can see the comparison among the state estimates of agent 3 and the plant states. We can see that the estimates of the locally observable modes, namely $\hat{x}_3$ and $\hat{x}_4$, converge continuously to the corresponding plant states. On the other hand, convergence of the locally unavailable states $\hat{x}_1$ and $\hat{x}_2$ is achieved through the jumps emerging from the periodic information exchange with the neighbouring agents. The second plot shows the evolution of the error norm for each agent: all the error norms converge to a neighborhood of the origin, thus highlighting the ISS property proven for the proposed observer.

VII. Conclusions

In this work we exploited a continuous-time formulation of the multi-hop decomposition to construct a novel dis-

tributed continuous-discrete observer. The hybrid nature of the proposed observer allowed us to analyze a scenario where the plant and the local observers work in continuous time, while the information exchange happens only periodically. Disturbances acting on the plant, on the measurements and on the communication, were also considered. Under some necessary collective detectability assumptions, we showed that it is possible to tune the observer gains to ensure exponential ISS from the noises to the estimation error, with a prescribed exponential convergence rate. Simulations were included to show the effectiveness of the proposed estimation algorithm.

APPENDIX: PROOF OF LEMMA 1

We start by writing the flow and jump dynamics of the error coordinates. From (1) and (7), these are given by

$$\begin{aligned}\dot{e}_i &= \dot{x} - \dot{\hat{x}}_i = Ax + d_0 - A\hat{x}_i - W_{i,0}L_i\left(y_i - \hat{y}_i\right)\\ &= \left(A - W_{i,0}L_iC_i\right)e_i + d_0 - W_{i,0}L_id_i\\ e_i^+ &= x^+ - \hat{x}_i^+\\ &= x - \hat{x}_i - \sum_{\varrho=1}^{\ell_i}\sum_{j\in\mathcal{N}_i} W_{i,\varrho}N_{i,j,\varrho}W_{j,\varrho-1}^\top\left(y_{i,j} - \hat{x}_i\right).\end{aligned}$$

Then, we can use (13) to obtain

$$\begin{aligned}\dot{\varepsilon}_{i,\varrho} &= W_{i,\varrho}^\top A\sum_{r=0}^{\varrho} W_{i,r}\varepsilon_{i,r} - W_{i,\varrho}^\top W_{i,0}L_iC_i\sum_{r=0}^{\varrho} W_{i,r}\varepsilon_{i,r},\\ &\qquad + W_{i,\varrho}^\top\left(d_0 - W_{i,0}L_id_i\right)\\ \varepsilon_{i,\varrho}^+ &= \varepsilon_{i,\varrho} - W_{i,\varrho}^\top\sum_{r=1}^{\ell_i}\sum_{j\in\mathcal{N}_i} W_{i,r}N_{i,j,r}\\ &\qquad W_{j,r-1}^\top\left(\hat{x}_j - \hat{x}_i + w_{i,j}\right).\end{aligned}$$

Now, as highlighted in Remark 1, we can apply the results in [15, Lemma 3] and [15, Lemma 10], thus the flow dynamics in the previous equation becomes

$$\begin{aligned}\dot{\varepsilon}_{i,0} &= \left(W_{i,0}^\top A - L_iC_i\right)W_{i,0}\varepsilon_{i,0} + W_{i,0}^\top d_0 - L_id_i,\\ \dot{\varepsilon}_{i,\varrho} &= W_{i,\varrho}^\top A\sum_{r=0}^{\varrho} W_{i,r}\varepsilon_{i,r} + W_{i,\varrho}^\top d_0,\end{aligned}$$

while the jump dynamics gives

$$\begin{aligned}\varepsilon_{i,0}^+ &= \varepsilon_{i,0},\\ \varepsilon_{i,\varrho}^+ &= \varepsilon_{i,\varrho} - \sum_{j\in\mathcal{N}_i} N_{i,j,\varrho}W_{j,\varrho-1}^\top\left(\hat{x}_j - \hat{x}_i + w_{i,j}\right)\\ &= \varepsilon_{i,\varrho} - \sum_{j\in\mathcal{N}_i} N_{i,j,\varrho}W_{j,\varrho-1}^\top\Bigg(\sum_{r=0}^{\varrho} W_{i,r}\varepsilon_{i,r}\\ &\qquad - W_{j,\varrho-1}\varepsilon_{j,\varrho-1} + w_{i,j}\Bigg)\\ &= \varepsilon_{i,\varrho} - \sum_{r=0}^{\varrho}\sum_{j\in\mathcal{N}_i} N_{i,j,\varrho}W_{j,\varrho-1}^\top W_{i,r}\varepsilon_{i,r}\\ &\qquad + \sum_{j\in\mathcal{N}_i} N_{i,j,\varrho}\varepsilon_{j,\varrho-1} - \sum_{j\in\mathcal{N}_i} N_{i,j,\varrho}W_{j,\varrho-1}^\top w_{i,j},\end{aligned}$$

which correspond to (16) when expressed in matrix form.

REFERENCES

[1] Reza Khodayi-mehr, Yiannis Kantaros, and Michael M Zavlanos. Distributed state estimation using intermittently connected robot networks. *IEEE Transactions on Robotics*, 35(3):709–724, 2019.

[2] Le Xie, Dae-Hyun Choi, Soummya Kar, and H Vincent Poor. Fully distributed state estimation for wide-area monitoring systems. *IEEE Transactions on Smart Grid*, 3(3):1154–1169, 2012.

[3] Soummya Kar, Gabriela Hug, Javad Mohammadi, and Jose MF Moura. Distributed state estimation and energy management in smart grids: A consensus + innovations approach. *IEEE Journal of selected topics in signal processing*, 8(6):1022–1038, 2014.

[4] Antonella Ferrara, Simona Sacone, and Silvia Siri. State estimation in freeway traffic systems. In *Freeway Traffic Modelling and Control*, pages 169–190. Springer, 2018.

[5] Reza Olfati-Saber. Distributed Kalman filtering for sensor networks. In *2007 46th IEEE Conference on Decision and Control*, pages 5492–5498. IEEE, 2007.

[6] Ruggero Carli, Alessandro Chiuso, Luca Schenato, and Sandro Zampieri. Distributed Kalman filtering based on consensus strategies. *IEEE Journal on Selected Areas in Communications*, 26(4):622–633, 2009.

[7] Giorgio Battistelli and Luigi Chisci. Kullback–Leibler average, consensus on probability densities, and distributed state estimation with guaranteed stability. *Automatica*, 50(3):707–718, 2014.

[8] Xingkang He, Chen Hu, Yiguang Hong, Ling Shi, and Hai-Tao Fang. Distributed Kalman filters with state equality constraints: Time-based and event-triggered communications. *IEEE Transactions on Automatic Control*, 65(1):28–43, 2019.

[9] Federico S Cattivelli and Ali H Sayed. Diffusion strategies for distributed kalman filtering and smoothing. *IEEE Transactions on automatic control*, 55(9):2069–2084, 2010.

[10] Guoqing Wang, Ning Li, and Yonggang Zhang. Diffusion distributed Kalman filter over sensor networks without exchanging raw measurements. *Signal Processing*, 132:1–7, 2017.

[11] Soummya Kar and José MF Moura. Gossip and distributed Kalman filtering: Weak consensus under weak detectability. *IEEE Transactions on Signal Processing*, 59(4):1766–1784, 2010.

[12] Shaoming He, Hyo-Sang Shin, Shuoyuan Xu, and Antonios Tsourdos. Distributed estimation over a low-cost sensor network: A review of state-of-the-art. *Information Fusion*, 54:21–43, 2020.

[13] Shinkyu Park and Nuno C Martins. Design of distributed LTI observers for state omniscience. *IEEE Transactions on Automatic Control*, 62(2):561–576, 2016.

[14] Aritra Mitra and Shreyas Sundaram. Distributed observers for LTI systems. *IEEE Transactions on Automatic Control*, 63(11):3689–3704, 2018.

[15] Álvaro Rodríguez del Nozal, Pablo Millán, Luis Orihuela, Alexandre Seuret, and Luca Zaccarian. Distributed estimation based on multi-hop subspace decomposition. *Automatica*, 99:213–220, 2019.

[16] L. Etienne, L. Hetel, D. Efimov, and M. Petreczky. Observer synthesis under time-varying sampling for Lipschitz nonlinear systems. *Automatica*, 85:433–440, 2017.

[17] F. Ferrante, F. Gouaisbaut, R. G. Sanfelice, and S. Tarbouriech. State estimation of linear systems in the presence of sporadic measurements. *Automatica*, 73:101–109, 2016.

[18] A. Sferlazza, S. Tarbouriech, and L. Zaccarian. Time-varying sampled-data observer with asynchronous measurements. *IEEE Transactions on Automatic Control*, 64(2):869–876, 2018.

[19] João P. Hespanha. *Linear Systems Theory*. Princeton Press, Princeton, New Jersey, 2009.

[20] Dragan Nešić, Andrew R. Teel, Giorgio Valmorbida, and Luca Zaccarian. Finite-gain Lp stability for hybrid dynamical systems. *Automatica*, 49(8):2384–2396, 2013.