Bayesian Rationality Revisited: Integrating Order Effects

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Bayesian rationality revisited: integrating order effects

Abstract

Bayes' inference cannot reliably account for uncertainty in mental processes. The reason is that Bayes' inference is based on the assumption that the order in which the relevant features are evaluated is indifferent, which is not the case in most of mental processes. Instead of Bayes' rule, a more general, probabilistic rule of inference capable of accounting for these order effects is established. This new rule of inference can be used to improve the current Bayesian models of cognition. Moreover, it should play an essential role in the search for artificial emotional intelligence.

Keywords: Bayesian inference; Order effects; Decision-Making; Emotions; Emotional Intelligence

Introduction

It has already been shown that the paradigm of Bayesian rationality in cognitive science is faced with some general challenges, like the problem of connecting its probabilistic models to psychological mechanisms or that of justifying the priors used in Bayesian inference (Oaksford & Chater 2007) (Griffiths *et al.* 2008). However, beyond these challenges, Bayesian rationalism suffers from an even more fundamental problem: *Bayes' inference cannot account for mental processes because the latter give rise to order effects.* Order effects in mental processes denote the fact that the order in which the relevant mental features are evaluated is not indifferent. Being understood as contextual or as subjective effects, these order effects cannot be accounted by the classical, set theoretical, Kolmogorov probability calculus from which Bayes' rule is derived. These non-classical effects are actually quite similar to those encountered in the quantum realm when measuring non-compatible observables, which means that they can be accounted for within the mathematical framework of quantum theory. In quantum theory, the measurable properties of any system, its "observables", are represented by Hermitian operators of a non-commutative C* algebra acting on the vector space of the possible states of this system. The only probability measure that can be assigned to their possible values is a measure over the subspaces of this vector space (Gleason 1957) and not as a measure over subsets of the set of possible states –like in classical probability theory. On the basis of the quantum probability calculus so defined, a new paradigm of cognition, called "quantum cognition", has been developed for a few decades by several authors, including Aerts, Sozzo, Busemeyer, Bruza, Wang, Atmanspacher, Römer and Pothos (Aerts *et al.* 2011) (Aerts and Sozzo 2013) (Busemeyer and Bruza 2012) (Wang and Busemeyer (2013) (Atmanspacher and Römer 2012) (Pothos and Busemeyer 2019) (Busemeyer and Wang 2017). Quantum cognition can deal with the non-classical effects that are inherent to cognition and decision making, namely, order effects and interference effects, and then with all manifestations of these effects, like the "and fallacy", the "or fallacy" or Ellsberg paradox that regards decision making in uncertain situation (Bruza and Busemeyer 2012). For our purpose, the essential point is that the study of cognitive processes requires accounting for the non-commutativity of mental observables, which thus requires working within the paradigm of quantum cognition where *Bayes' rule is no more valid.*

However, insofar as Bayes' rule plays an essential role in the current models of cognition and decision making, it would be rather problematic to drop this mathematical tool definitely instead of trying to generalize it in order to make it also applicable to non-commutative observables. This article proposes such a solution: the general idea of Bayesian rationality is kept, namely the idea of modeling cognitive processes in probabilistic terms, by updating the prior distribution when a new event occurs, while *Bayes' classical rule is replaced by a more general probabilistic rule capable of dealing with these non-classical effects*. This change is absolutely required as soon as subjective experience is involved in the mental processes considered, that is, quite in all of them as soon as their reduction to pure mechanical reasoning is inappropriate. This change is all the more necessary insofar as Bayes' rule plays a fundamental role in the current research in artificial intelligence, for categorization tasks, and in the simulation of emotional intelligence.

We will first recall, in section 1, what is Bayesian rationality and emphasize the fact that Bayes' rule on which it presently relies does not hold in the case the considered mental processes give rise to order effects. Two paradigmatic successful applications of Bayes' inference will also be presented in order to analyze the reason of this success. In section 2, we will present the order effects inherent to decision making and to all mental processes involving emotional experience. In section 3 we will evaluate some currently proposed Bayesian models of cognition: are these models really successful, as too easily claimed by the supporters of the "all-Bayesian" rationality? We will explain that this apparent success has to be questioned when the commutativity of the considered observables is not satisfied. Section 4 will present the theoretical framework of quantum cognition, an alternative approach to cognition and decision making capable of accounting for these non-classical effects, which are indeed inherent to all mental processes. Within this generalized framework, we will propose an alternative to Bayes' rule where the conditional probabilities P(Ai/Ej) are not linearly related to the "inverse" conditional probabilities P(Ej/Ai), like in Bayes' rule. This new probabilistic rule of inference, whose classical limit, when there is no order effects, is

Bayes' rule, requires to compute the commutator of the couples of observables respectively defined from the series of events {Ai} and {Ej} in order to take into account their degree of non-commutativity. Section 5 will illustrate the fertility of this alternative, probabilistic rule of inference for modelling cognitive processes. It will discuss the way of improving the current models of visual perception and those of emotion recognition from the subject's face expression. Section 6 will explain why such a generalized, probabilistic rule of inference should play an essential role in the field of artificial intelligence, in particular for simulating the emotional aspect of intelligence.

1. Bayesian rationality and its limitations

1.1. Bayesian rationality

Bayesian rationality is an approach to cognition based on the idea that cognitive processes, and in particular reasoning, can be rationally modeled by appealing to probabilities, used as a tool to make predictions, rather than by logical rules (Griffiths, Kemp and Tenenbaum 2008) (Oaksford and Chater 2007). Many works have been developed in order to model high-level cognitive processes in probabilistic terms, in the range of perception (Knill and Pouget 2004) (Mamassian *et al.* 2002), categorization (Ashby and Alfonso-Reese 1995), (Griffiths et al. 2008) (Perfors and Tenenbaum 2009) or language processing (Xu and Tenenbaum 2007), among others. *It is moreover assumed that the good probabilistic calculus to use is the classical one, from which Bayes' rule is derived.* Bayes' rule extends the too narrow framework of binary classical logic by taking into account the uncertainty in the knowledge of premises and the acquisition of information, which is evaluated in terms of probabilities. Even when probabilities are interpreted subjectively, as degrees of belief (de Finetti 1970), Bayesian reasoning still satisfies the rules of the classical probability calculus, as shown by Cox-Jaynes theorem (Cox 1946), which strengthens the idea that Bayesian inference is appropriate for modeling mental (or even brain) processes.

In the literature about cognition, Bayes' rule is thus regarded as the key ingredient used for integrating uncertainty in cognition and decision-making (Griffiths *et al.* 2008) (Oaksford & Chater 2007) (Evans *et al.* 2015) (Cruz *et al.* 2015) (Mamassian *et al.* 2002). It is used to model many areas of human activities, like finance (for modelling risk), medicine (for diagnostic and decision making) or meteorology (for weather forecasting), and the success of Bayesian models of cognition seems certain, as well in reasoning, learning or making decision. Accordingly, Bayesian networks, based on Bayes' rule, are used in machine learning whose applications have been developed in image processing (Simonyan and Zisserman 2015), neuroscience[1] (Poggio 2016) (Mamassian *et al.* 2002) and medical diagnostics (Kubota 2017), among other domains. Bayes' inference is supposed to correctly represent the way we reason, we learn and make decision in uncertain situation, and therefore to be a key-ingredient for developing artificial intelligence. Let us now examine accurately the presumed success of Bayes' inference and show that its application to mental processes suffers from important bias.

1.2. Bayes' inference and its domain of validity

Bayes' inference allows to calculate how a priori probabilities are updated when new information is gathered (Hossein 2014): the posteriori probability P (A/E) of the event A given the evidence E, which can denote in particular the observation of some feature or some

[1] Reducing mental processes to brain's activity leads to the idea that Bayesian inference "takes part of the automatic and unconscious, elementary operations of our brain" (Dehaene 2012). According to a growing trend in theoretical neuroscience, the human perceptual system could thus be modeled as a Bayesian machine. The brain is supposed to represent sensory information probabilistically, in the form of probability distributions. This hypothesis, which presently lacks of experimental confirmation (Knill and Pouget 2004), is only an over-interpretation of Bayesian rationalism within a reductionist materialism. It will not be examined further in this article, which focuses on the rationality of mental processes.

property of the situation under consideration, is calculated from the prior probability P (A), estimated before the occurrence (or the knowledge of the occurrence) of E, and the "likelihood" P (E/A), which is the conditional probability of observing E when A is realized:

$$P\ (A/E) = [P\ (A).\ P\ (E/A)]\ /\ P\ (E), \tag{1}$$

where P(E), which appears in the denominator of this expression, is the probability of occurrence of the event E, which can for example denote the observation of a property and is a priori computed independently of A.

Bayes' rule is derived from the definition of conditional probability in the classical probability calculus:

$$P\ (A/E) = P\ (A \text{ and } E)\ /\ P\ (E),$$

where "A and E" has no temporal connotation, meaning that A and E can occur in any temporal order or be simultaneous. This order-independent definition gives rise to the "rule of multiplication" of the classical probability calculus:

$$P\ (A \text{ and } E) = P\ (A/E)\ .\ P\ (E) = P\ (E/A).\ P\ (A), \tag{2}$$

and Bayes'rule (1) is then straightforwardly obtained by dividing the two terms of the second equality of (2) by P(E).

However, it is essential to notice that (2) is valid *on the condition that the value of the joint probability P (A and E) is independent from the order of occurrence of A and E*. If P (A and E) depends on the order of occurrence of A and E, we must clearly distinguish the calculation of

$$P\ (A \textit{ and then } E) = P\ (E/A).\ P\ (A),$$

from that of

$$P\ (E \textit{ and then } A) = P\ (A/E).\ P\ (E),$$

and in this case Bayes' rule (1) is not verified since (2) is no more valid. This means that *Bayes' inference is valid only if the condition of commutativity of events is satisfied.*

1.3. Two paradigmatic applications of Bayes' inference

In order to check the latter condition of commutativity in the use of Baye's probabilistic inference, let us mention two paradigmatic applications of it. The first one, presented in any text-book as an illustration of Bayes' rule, describes the drawing of balls in two urns. The second application of Bayes' rule, which is widely used in the medical field, regards diagnostic testing.

1.3.1. Drawing of balls in two urns

We have two urns I and II that respectively contain 2 yellow balls and 6 blue balls (for urn I) and 3 yellow balls and 9 blue balls (for urn II). A blue ball is drawn from one of these two urns but we do not know which one it is[2]. The problem is to evaluate the possibility that this ball was drawn from urn I or from urn II. Bayes' rule allows us to calculate these probabilities. The probability that this ball was drawn from urn I is:

$$P\ (I/Blue) = P\ (I).[P(Blue/I)\ /\ P(Blue)],$$

and, interpreting here probabilities as proportions (for example, P(I) = proportion of balls in urn I = 8/20 = 0.4), leads to the result P(I/Blue) = 0.4.

This result can be checked *by merely counting the proportion of blue balls in urn I*, which are in number of 6 for a total of 15 blue balls in urns I and II. This proportion is exactly 0,4 –which confirms the previous result. A quite similar reasoning can be done for evaluating P(II/Blue). No dispute can be made to this reasoning based on Bayes' rule (1) since the variables that are measured, called "observables" in the following of this article, are purely objective data, contextually independent and completely independent from any subjective interpretation and any judgment. *These observables are intrinsically defined as proportions of*

[2] It is supposed that the drawings are equiprobable.

yellow and blue balls in urn I and II, that is, as objective properties of the physical world. As a consequence, the order in which they are evaluated does not matter and Bayes' rule can be successfully applied.

1.3.2. Contamination screening

The second example of successful application of Bayes' rule is the classic case of diagnostic testing (Broemeling 2011) (Klement and Bandyopadhyay 2020). We want to know if someone is infected by a virus or not by performing a test whose response is positive or negative. The events "to be infected" and "not to be infected" are respectively noted as (V+) and (V-). The positive and negative results of the test are respectively noted as (T+) and (T-). Suppose that we know the sensitivity of the test P(T+/V+), which is the probability that the test is positive for an infected person, its specificity P(T-/V-), which measures the reliability of the negative tests, and the contamination prevalence P(V+), which estimates the proportion of persons infected in the population. If a person has obtained a positive test, what is the probability P(V+/T+) that this person is contaminated?

Bayes' rule allows us to calculate this probability:

$$P\,(V+/T+) = P(V+).[\,P(T+/V+)\,/\,P(T+)],$$

where P(T+) can be computed as:

$$P(T+) = P(T+/V+).P(V+) + P(T+/V-).P(V-),$$

with P(V-) = 1 - P(V+) and P(T+/V-) = 1 - P(T-/V-).

For example, if the reliability of the diagnostic test is 0.8 for the sensibility P(T+/V+) and 0,7 for the specificity P(T-/V-), while the prior probability of contaminated people (defined as a frequency) is known to be 0.01, Bayes' rule computes that P(V+/T+) = 0,026.

This result (which shows that the test is not very significant) can be considered as valid because:

(i) the two observables that respectively measure the existence or the absence of the contamination (V = V+ or V-) and the positivity or the negativity of the test (T = T+ or T-) *are supposed to measure purely objective data or states of the world, like in the previous example* (for the number of yellow and blue balls) and that, consequently,

(ii) the act of observing the state of contamination of a subject (for example, by referring to clinical signs, medical imaging or biological anomaly) has no influence on the result of the test performed on her, and, reciprocally, it is granted that performing this test and stating its result does not change the state of contamination of the patient, which is regarded as an intrinsic attribute. *Hence the commutativity of the observables that respectively measure the state of health of a patient (responding to the question: is this patient infected or not by the virus?) and its contamination rate (is this patient positive or negative to the test?).*

However, as will be emphasized in the next section it is not the case for cognitive processes: the condition of commutativity of cognitive observables is generally not fulfilled – which seriously questions the reliability of the current Bayesian models of cognition.

2. Order effects

As explained above, the commutativity of observables that are involved in the previous examples (section 1.3) relies on the assumption that the measured properties are *intrinsically possessed by the entities considered* (objects, animals, patients, …). However, as will be explained below, it is generally not the case of mental observables whose values are highly contextual and subject-dependent. Let us investigate this essential characteristic of mental activity on two paradigmatic examples.

2.1. Non-commutativity in decision-making

In a survey realized in 1997 (September 6-7) and involving 1002 respondents, half of the participants were asked the two questions ‘is Clinton honest and trustworthy?’, noted as A hereafter, *and then* ‘is Gore honest and trustworthy?‘, noted as B hereafter, while the other half were asked the same pair of questions in the opposite order. As reported by Moore (2002), the list of answers for the two groups shows that Clinton received 50% agreement when asked first (which defines the "non-comparative" context) but 57% when asked second (which defines the "comparative" context because this answer can be influenced by the first one). It also shows that Gore received 68% when asked first and 60% when asked second. This difference in the frequencies of the respondents’ answers shows that the order in which the questions are asked is significant since the frequency of the positive answers to the same question depends on whether this question is asked first or second. Focusing for example on positive answers for both questions A and B, respectively noted as Ay and By, this question order effect can be expressed by the following difference:

$$P\ (Ay\ By) \neq P\ (By\ Ay),$$

where P (Ay By) is the probability of responding “yes” to question A followed by “yes” to question B, and P (By Ay) is the probability of obtaining the same answer to these questions asked in the inverse order. Moore calls this type of question order effect “consistency effect” to denote the fact that the difference between the probabilities of positive answers for questions A and B decreases from the non-comparative context to the comparative context, which is here the case since in the non-comparative context $p(By) – p(Ay) = 18\%$ while in the comparative context $p(Ay/n\ By) – p(By/n\ Ay) = 3\%$. Note that other types of order effects in decision making have been observed in other similar survey experiments, for example a “contrast” order effect showing that, unlike the previous consistency effect, the difference between these probabilities is amplified in the comparative context.

2.2. Non-commutativity of emotional observables

Order effects do not only occur in decision making but in all mental activity where subjective experience is involved. In particular, significant order effects can be observed and quantified in the domain of emotions, which should then be taken into account in the cutting-edge research in artificial intelligence (see section 6). As can be observed in daily life, felt emotions cannot be regarded as intrinsic features of a person since they are continuously changing according to our life experience. Their nature and their intensity is highly contextual since they strongly depends on our personal past and present experience of life, on our social environment and even on what we felt just a moment before (Stolorow 2005). For example, asking a subject about her degree of happiness and asking the same question after reminding her of a sad event in her life generally provides different results. As was the case for the previous example of surveys with two successive questions, the order effects relative to emotions can be evaluated from data on successive measurements of the intensities of emotions felt by subjects. These intensities can be collected by asking them to report discrete values on a graduate scale (Bachorowski and Braaten 1994) or to report them continuously, using a continuous response digital interface on which the subject moves a stylus or finger (Geringer *et al*. 2004).

For illustration, consider the following table, drawn from an article by Prkachin and team (1999). This table reports the average intensity of five emotions experienced by subjects conditioned in target emotional states[3] by Lang's method (Lang 1979).

[3] The numbers between brackets are the standard deviations.

Table I.—Average intensity of each of five emotions experienced on six trials

	Emotional rating				
Target	Happiness	Anger	Fear	Sadness	Disgust
Neu	0.24[a] (0.66)	0.03 (0.17)	0.09 (0.38)	0.15 (0.57)	0.03 (0.17)
Hap	**5.30** (1.36)	0.03 (0.17)	0.24 (0.71)	0.21[a] (0.49)	0.15 (0.87)
Ang	0.00 (0.00)	**5.15** (1.15)	0.39 (1.12)	0.81[a] (1.36)	1.21[a] (1.62)
Fea	0.21 (0.74)	0.73[a] (0.31)	**4.61** (1.50)	1.09[a] (1.59)	0.36 (1.08)
Sad	0.06 (0.24)	1.27[a] (1.81)	0.79[a] (1.50)	**5.15** (1.62)	0.36 (1.06)
Dis	0.06 (0.24)	1.58[a] (1.94)	0.58[a] (0.97)	0.85[a] (1.46)	**4.97** (1.40)

(from Prkachin *et al.* 1999)

Let us for example focus on the couple of emotions Anger and Fear. From this table the probabilities of the sequence of evaluation "Fear and then Anger" and of the reverse sequence can be evaluated as follows: 1) from the first line (neutral target), the prior probabilities P(A) and P(F) of respectively experiencing Anger and Fear, can be computed as the ratio of the average intensity of each of these emotions (respectively 0.03 and 0.09) to the sum of all the five average intensities (0.54); 2) from the third line (Anger conditioning), the conditional probability $P(F/W_A)$ of experiencing Fear for a subject conditioned in Anger state W_A can be computed. *In first approximation,* which can here be done in order to show the existence of order effects, W_A will be identified to the target emotional state[4] A. The conditional probability $P(F/W_A) \approx P(F/A)$ can then be evaluated as the ratio of the average intensity of Fear (0.39) to the sum of the five average intensities on this line (7.56); 3) similarly, from the fourth line, the conditional probability $P(A/W_F)$ of experiencing Anger for a subject conditioned in Fear state can be evaluated as the ratio of the average intensity of Anger (0.73) on the sum of the five numbers of this line (7). The resulting probabilities are

[4] Indeed, the target emotional state is not totally reached by the subject. The distinction between the latter state and her real emotional state is considered in the more precise calculations of section 5 and Appendix 1.

P (A F) = P(A) x P(F/A) = (0.03/0.54) x (0.39/7.56) = 2.87 10^{-3}

P (F A) = P(F) x P(A/F) = (0.09/0.54) x (0.73/7) = 1.74 10^{-2}

This clearly shows a net difference (of a factor of about 6) between the sequential probabilities P (A F) and P (F A).

It can also be shown that emotional observables do not commute neither with their physiological correlates nor with their behavioral correlates. This point can be established from data about the joint measurement of emotional observables and their physiological or behavioral correlates reported in the literature (Barrett *et al.* 2019) (Kassam and Mendes 2013) (Kreibig *et al.* 2007) (Prkachin *et al.* 1999) (Sinha *et al.* 1992). For example, as shown by Kassam and Mendes (2013) on experimental basis, the very act of reporting one's own emotional state generally changes one's physiological and behavioral "responses", this effect being particularly significant for subjects conditioned in angry state. In this experiment, the subjects are conditioned in such an angry state by delivering them a negative feedback to a difficult task they have done -for example by telling them that they are incompetent. The observed physiological responses are here evaluated by the values of cardiovascular observables, like heart rate and pre-ejection period[5], which can be measured almost continuously. The behavioral responses are evaluated by external experimenters through videos showing the participants performing the required tasks, by noting for example their facial expression and their body movements. Kassam's and Mendes' study clearly shows that for these subjects, conditioned in angry state, the changes in the values of the cardiovascular and behavioral observables are significantly different depending on whether or not they report their emotional state (of anger, in this example). This tells us that the successive measurement of emotional and physiological or behavioral observables gives rise to order effects since if

[5] The pre-ejection period is the time elapsed between the depolarization of the left ventricle and the beginning of ventricular ejection. Its value is strongly dependent on that of the volume of blood ejected by the left ventricle at each cardiac cycle.

these observables were commuting their values would not be inter-dependent, as is actually the case.

A more precise characterization of these order effects in emotional life requires a mathematical representation of the non-commutativity of the emotional observables with each other and with their physiological and behavioral correlates. This representation will be proposed in section 5 and Appendices 1 and 2.

3. Are current Bayesian models of cognition reliable?

As shown in section 2, mental processes can give rise to order effects. However, as emphasized in section 1, Bayes' rule is valid only under the assumption of commutativity of the observables involved in this probabilistic inference, which means that order effects cannot be taken into account. So, we can be suspicious about the validity of the current Bayesian models of cognition. Are these models reliable? Let us examine some of them.

3.1. Bayesian models of categorization

The ability to classify objects (or concepts) into categories is a fundamental cognitive task realized by humans in order to organize their mental representation of the world they live in and thus to cope with it. "Categories" form the basic cognitive mental representations in which and by which humans can organize their knowledge and make inferences about the world (Murphy and Medin 1985) (Rosch 1978). They group concepts which share similar properties, like their form, their color or their behavior. Categorization is the act of classifying a new object according to these categories. A Bayesian model of categorization computes the probability of classifying it into each of the categories, given the properties of this object, and determines the most probable category label it can be assigned to.

Category learning models aim to explain how category labels can be learned, for example by children. They are based on the same Bayesian method of classification of objects or concepts into appropriate categories and aim to explain that the acquired knowledge is used to make decisions about how to categorize new stimuli. Several rational analyses of category learning have been proposed (Perfors et al. 2011) (Ashby and Alfonso-Reese, 1995) (Nosofsky 1986). All these models compute via Bayes' rule the probability distributions associated with different category labels and select the most probable result.

However, like for any Bayesian model, these categorization models require defining the set of "primitive" properties for which the prior probability distribution and the likehood function are provided. *This is where the crucial question arises: can these properties be intrinsically defined, and therefore commute with each other, or not?* In the current Bayesian models of categorization or category learning this condition is satisfied only if these "primitive" properties can be regarded as purely objective ones on which anyone agrees, being of physical, biological or behavioral order. This is actually the case in most of the current Bayesian models of categorization. For example, in the model of category learning presented by Perfors *et al.* (2011), classifying a new item in the categories "dog", "fish", "bird", "tree", flower", fruit" involves the observation of some of its physical properties on which anyone agrees, like its size, its color, its hairiness or skin appearance (a crucial property to classify animals), or the fact it has leaves (a property of trees).

However, this kind of categorization based on intrinsically defined (and then commuting) properties covers only a small part of the real human's experience of categorization. As can be observed in everyday life, and as emphasized in section 2, mental activity generally involves contextual and subject-dependent observables, which do not commute with each other. Think about our very selective ability to perceive the world around us, this selective perception of the world being a form of subjective bias in the interpretation of information in order to fit with our values and our beliefs (Pronin 2007). This subjective tonality of cognitive processes has been observed for a long time by psychologists for "normal" as well as for pathological subjects (Cattell 1930) (Roiser and Sahakian 2013). Psychoanalysts go even so far as to talk about "resistance" to accessing memories, which is a form of cognition, when the latter are associated to traumatic feelings (Freud 1909). It seems then difficult to ignore the role of emotions in cognitive processes. *As a consequence, the observables involved in categorization tasks do not generally commute*.

An important example of such non-classicality is provided by the task of assigning emotional states (which play the role of categories) to human subjects from the observation of their facial expression. Computing the probability that a subject be labeled as "sad", "happy" or "angry" given the observation of some features of her face expression *must take into account the non-commutativity of the relevant (emotional and behavioral) observables,* which gives rise to some unavoidable uncertainty in the correspondence between a subject's facial expression and the nature (and/or the intensity) of the experienced emotion –uncertainty which has been noticed by several authors (Barrett et al. 2017) (Duran and Fernandez-Dols 2019). This paradigmatic example will be developed in section 4 with the appropriate mathematical tools.

3.2. Bayesian models of visual perception

The quasi-impossibility to separate subjective experience from the idealized "cognition" of intrinsic properties of the world has also been observed in visual perception, even if this subjective component relies on less personal features. In the current Bayesian models of visual perception (Knill and Pouget 2004) (Kersten and Yuille, A., 2003) (Mamassian *et al.* 2002), the task to accomplish is to predict the most probable perceived image given the prior probability distribution of *physical properties*, like reflectances, shape and orientation of surfaces, object size or wavelength, and the likelihood function that a given set of values of these physical properties generate a particular image. The “primary” properties are then defined as *intrinsic features of the physical world*, as can be checked for example in the 3-D model of visual perception proposed by Mamassian and team (2002).

However, as noted by Owe, Lotto and Purves (2006), visual perception (like any perception, indeed) cannot be exclusively based on intrinsic features of the physical world. Visual illusions show that what we see is not the “objective” reality, exclusively describable in terms of physical properties of the world. For example, in White’s optical illusion the same target luminance can elicit different perceptions of color and brightness in different contexts (see figure 1). This figure shows that black and white horizontal bars alternate and that shorter grey bars cover the white bars at left (A) and the black bars on right (B). Though the shorter grey bars have the same color and the same opacity in (A) and (B) and therefore reflect the same amount of light, the grey bars in (B), surrounded by white stripes *appear* lighter and brighter than the grey bars in (A), which is surrounded by black stripes. This means that how a subject *perceives* the shorter grey bars is not exclusively determined by their intrinsic, physical properties but it strongly depends on the subject taking into account the background on which they are perceived. Visual perception, like any perception, is highly contextual and subject-dependent.

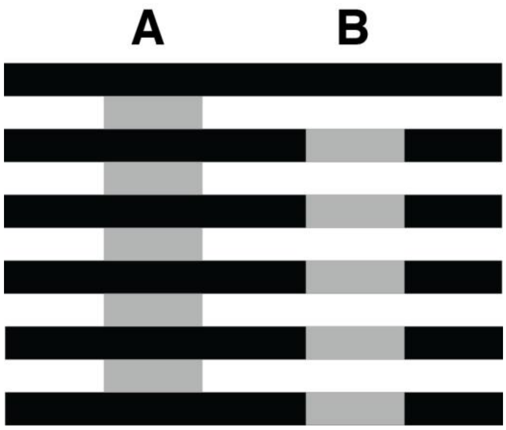

Figure 1. White’s illusion

A few solutions for tackling this problem has been proposed. Mamassian *et al.* (2002) suggest to allow the components of Bayes' inference, in particular the likelihood function and the prior distribution, to vary from trial to trial, that is, to be stimulus-dependent, in contrast with the strict Bayesian method where the latter are supposed to be intrinsically defined (see section 1). How *et al.* (2006) have suggested to explicitly take into account the context of perception, which can be illustrated by the mention of the white surrounding or the black surrounding in the previous example. Taking into account the context could be implemented by trying to compute the posterior conditional probability distribution for the target *and* the surround, which, as emphasized by How (How *et al.* 2006, section 7), seems rather complicated to implement for complex contexts. These authors have proposed to take into account the contextual effect within their "empirical ranking approach", by computing the probability distribution associated to the values of the considered properties (the target luminance, for example) that have *co-occurred* with each of the possible contexts.

However, these interesting attempts to improve Bayesian models of visual perception ultimately result in *models that are not any more strictly "Bayesian"* since the properties on which the Bayesian inference bears cannot be defined intrinsically, as properties of the physical, objective world -like in the paradigmatic uses of Bayes' rule presented in section 1.3. The resulting models actually deal with non-commutative observables, this non-commutativity being nothing but the very expression of the fact that the relevant properties to consider in visual perception (being understood as contextual, stimulus-dependent or subject-dependent) cannot be intrinsically defined.

As shown by the previous developments, the Bayesian models of cognition, which rely on categorization tasks, must be revisited in order to account for the order effects inherent to any mental process. This change can be done elegantly by deriving an appropriate probabilistic rule that generalizes Bayes' rule. This new rule will be derived in the next section within the mathematical framework of quantum cognition, which will be first presented.

4. Integrating order effects in Bayesian models of cognition

4.1. Theoretical framework

Integrating order effects in Bayesian models of cognition requires leaving the classical probability calculus since the latter cannot account for them. It requires appealing to a generalized probability calculus where the probability of a sequence of events P(A and then B) can be distinguished from that of the reverse sequence P(B and then A). Such a generalized probability calculus already exists, it has been built by physicists in order to deal with quantum phenomena, which generally give rise to order effects, and it has already been applied in the field of cognition and decision-making. "Quantum cognition" thus deals with cognitive processes within the same mathematical framework than that of quantum theory. It has been developed for a few decades by several authors, including Aerts, Sozzo, Busemeyer, Bruza, Wang, Atmanspacher, Filk, Pothos and Wang (Aerts *et al.* 2011) (Aerts and Sozzo 2013) (Busemeyer, Bruza 2012) (Wang and Busemeyer (2013) (Atmanspacher and Filk 2013), (Pothos and Busemeyer 2019) (Busemeyer and Wang 2017). Its basic idea is to represent geometrically the observation of a mental feature of a subject by the action of a projector on the vector-state representing her mental state (or her belief state). The latter is defined as an element of the Hilbert vector space H of all possible mental states (or belief states) and, like in quantum theory, the observables are represented by Hermitian operators forming a *non-commutative* algebra and whose (real) eigenvalues are the possible results of their measurement. Within this geometrical representation, measuring the observable $\mathbb{A}$ and obtaining the outcome A_i is represented by the projection $P_{Ai}|\psi\rangle$, where, in Dirac notation, $|\psi>$ is the state of the subject, P_{Ai} the projector on the eigenspace associated to Ai, which, for sake of simplicity, has been assumed to be one-dimensional, spanned by the vector $|Ai>$ of H. This projection can be illustrated as follows:

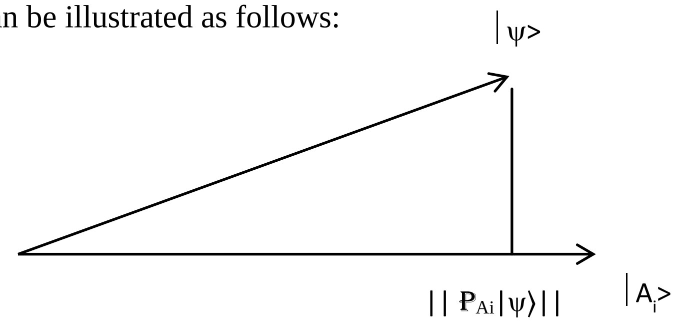

Fig 1. A simplified representation of the measurement of the observable $\mathbb{A}$ with result A_i

By Born rule, which defines the only probability measure in such a state space according to Gleason's theorem (Gleason 1957), the probability $P(A_i)$ of occurrence of A_i can be computed from the projection $P_{Ai}|\psi\rangle$ of $|\psi>$, as $P(Ai) = || P_{Ai}|\psi\rangle ||^2 \equiv <\psi|P_{Ai}|\psi>$.

In this geometrical representation, the order effects relative to the successive measurement of the observables $\mathbb{A}$ and $\mathbb{B}$ with respective results A_i and B_j are thus captured by the non-commutativity of the projectors P_{Ai} and P_{Bj} :

$$P_{Ai}\, P_{Bj} \neq P_{Bj}\, P_{Ai}.$$

Many "fallacies" in cognition and decision-making can be explained within this quantum-like approach (Busemeyer and Bruza 2012) (Aerts *et al.* 2011). For example, the "conjunction fallacy" is the fact that, in contrast with the classical probability calculus, the probability of occurrence of the conjunction of two events reported by human subjects is often greater than the probability of the occurrence of each of them. This phenomenon finds an explanation within this quantum-like framework. The probability of a sequence of events "A and then B", noted as P (A B), can be computed as $|| P_B\, P_A\, |\psi\rangle ||^2$, where P_B and P_A are, respectively, the projector on the subspace of the Hilbert space spanned by the eigenstates of

B and A, and $|\psi\rangle$ is the mental state of the subject. However, as shown by the following diagram, where, for sake of simplicity of presentation, the eigenspaces associated with the measurements of A and B are again supposed to be one-dimensional subspaces, the probability P (A B) = $|| P_B P_A |\psi\rangle||^2$ can be bigger than the probability P(B) of occurrence of B, which is $|| P_B|\psi\rangle||^2$:

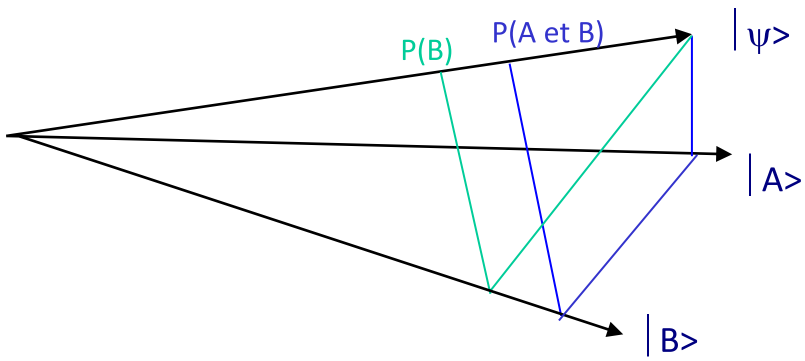

Fig 2. P (B) can be smaller than P (A and B) in contrast with the classical probability calculus

This generalized probabilistic framework has also been applied to deal with decision making in uncertain situation. For example, the famous Elsberg's paradox (Ellsberg 1961) bearing on the behavior of economic agents in a situation of uncertain knowledge can also be solved within this mathematical framework. Elsberg's paradox puts into question the law of total probabilities of the classical probability calculus. According to the quantum-like approach to cognition and decision making presented here, the paradoxal difference between the probability of the agent's choice provided by the classical probability calculus and the experimental result comes from the *interference term* between the possible belief states of a participant (Aerts and Sozzo 2013) (Busemeyer and Bruza 2012) (Uzan 2014).

Also note that Wang and Busemeyer (2013) have represented the question order effects presented in section 2.1 above within this quantum-like framework. If the sequence of questions A and then B asked in the Clinton/Gore survey obtains the positive answers Ay and By, the corresponding construction in the vector space of all possible mental sates consists in first applying to the subject's mental state $|\psi\rangle$ the projector P_{Ay} onto the eigenspace associated to the answer Ay and then by applying to the result $P_{Ay}|\psi\rangle$ of this first operation the projector P_{By} onto the eigenspace associated with the value By. The probability of the sequence of answer Ay By is then calculated (by Born rule) as:

$$P\,(Ay\ By) = \|P_{By}P_{Ay}|\psi\rangle\|^2 = <\psi|P_{Ay}P_{By}P_{Ay}\,|\psi>,$$

while the probability of the inverse sequence (By and then Ay) is calculated as:

$$P\,(By\ Ay) = \|P_{Ay}P_{By}|\psi\rangle\|^2 = <\psi|P_{By}P_{Ay}P_{By}\,|\psi>.$$

If the projectors P_{Ay} and P_{By} are non-commuting, which means that the commutator $[P_{Ay}, P_{By}] = P_{Ay}P_{By} - P_{By}P_{Ay}$ is different from the null operator, P (Ay By) and P (By Ay) are different[6]. Order effects are then nicely and successfully represented in this geometrical construction.[7]

4.2. A new probabilistic rule of inference

Within this quantum-like framework, Bayes' rule can be generalized in order to take into account the non-commutativity of mental observables which, as seen above, is involved in most of the mental processes. As explained in Introduction, the derivation of this new rule is justified by the will to continue working in the paradigm of Bayesian rationality, while making it capable to deal with order effects. This generalization of Bayes' rule will thus greatly improve the current Bayesian models of cognition where the role of subjective experience cannot be ignored and thus paves the way to a new, more realistic, implementation of emotional intelligence.

[6] The precise calculation of the relevant commutators and probabilities for this survey will not be presented here since we essentially focus on the order effects involved in cognitive processes (and not in decision making).
[7] Also note that another characterization of this order effect has been proposed more recently by Busemeyer and Wang (2017), by referring to a so-called "ABA experiment", where a measurement of B is inserted between two measurements of A. It shows that the second measurement of A can be different from the initial one and the degree of incompatibility of A and B can thus be evaluated from this difference. This order effect has been checked on the sequence ABA with a population of 325 participants on a wide range of 12 different set of issues.

To compute the conditional probability P (E / F) for the mental events ot the mental properties E and F, let us first compute the difference of the sequential probabilities P(E F) - P (F E) within this quantum-like framework:

(3) $$P(E\,F) - P(F\,E) = \langle\psi|P_E\, P_F\, P_E\,|\psi\rangle - \langle\psi|P_F\, P_E\, P_F\,|\psi\rangle$$

$$= \langle\psi|P_E\, P_F\, P_E - P_F\, P_E\, P_F\,|\psi\rangle$$

$$= \langle\psi|[P_E, P_F]\,(P_E + P_F - I)\,|\psi\rangle,$$

where the last equality has been obtained by factorizing the expression between the bra $\langle\psi|$and the ket$|\psi\rangle$, and by using the definition of the commutator $[P_E, P_F] = P_E\, P_F - P_F\, P_E$. Defining the operator Q as:

$$Q =_{df} [P_E, P_F]\,(P_E + P_F - I),$$

the difference of sequential probabilities P (E F) - P (F E) can be written as the expectation value of Q in the mental state $|\psi\rangle$:

(4) $$P(E\,F) - P(F\,E) = \langle\psi|\, Q\, |\psi\rangle \equiv \langle Q\rangle_\psi.$$

Consequently, by the definition of the conditional probability of the occurrence of the event F given that of the event E (or given the knowledge of E), which is here supposed to occur *before* F:

$$P(F/E) = P(E\,F)\,/\,P(E),$$

by the symmetrical definition of the conditional probability of the occurrence of the event E given that of F, which is here supposed to occur *before* E:

$$P(E/F) = P(F\,E)\,/\,P(F)$$

and by using equation (4) above we obtain the following new rule of probabilistic inference:

(5) $$P(F/E) = [P(E/F) \times P(F) + \langle Q\rangle_\psi]\,/\,P(E).$$

This new generalized probabilistic rule of inference computes the conditional probability of occurrence of the event F given the occurrence (or the knowledge of the occurrence) of the event E for a subject in the mental state $|\psi\rangle$. Its *classical limit*, when the projectors associated to the event E and F are commuting, and then when $<\mathbf{Q}>_\psi = 0$, is nothing but Bayes' rule (1): P (F/E) = [P (E/F) x P (F)] / P (E).

5. Bayesian models of cognition revisited

As noted above, the current Bayesian models of cognitive processes are based on categorization tasks, which are modelled on the basis of Bayes' inference. In order to take into account the order effects inherent to most of mental processes, these models should then be revisited. Still keeping the Bayesian approach to cognition (see section 1.1), this change only regards the inference rule that must be used to update the probabilities of realization of some assumption when new information is gathered. These models must use the general probabilistic rule (5) instead of Bayes' rule (1) insofar as the latter cannot account for order effects. Such a change thus requires to first compute the commutators of the couples of observables involved in these models.

The Bayesian models of perception involve two types of observables that do not commute: those characterizing the subject's perception and those characterizing the physical situation. A typical example of observable of the first type is the observable "Luminosity", noted as $\mathbf{L}$ in the following, which measures the *perceived* luminosity of an object by the subject. The observables of the second type measure the relevant *physical* features of the object or those of the context in which this object is perceived, noted as Φ_i. The commutator of the couples of observables ($\mathbf{L}$, Φ_i) can be computed from data relating the change of the perceived luminosity of an object to the change of its physical features, which allows us to compute the likehood function. However, such data, which could be easily gathered by asking subjects to report on a scale the perceived luminosity corresponding to different values of the physical observables Φ_i *seem to be missing for now*. Consequently, we will not deepen here these calculations for models of perception but this method will be applied below to other categorization models for which more data are available. Moreover, we would like to focus on the Bayesian models of cognition that explicitly involve *experienced emotions* for their central role in the search of emotional intelligence. The widely spread (mis)use of algorithms supposed to detect the experienced emotions of subjects from their behavior, in particular from their facial expression, deserves some priority.

Assigning an emotional state to a subject requires finding the greatest conditional probability that the subject could be classified in this emotional state given the knowledge of some of her behavioral or physiological features. In more technical language, this task requires computing the conditional probabilities P (E_k / {Φ_i } and {B_j}) that the emotional observable takes the value E_k given the knowledge of the values of a set of physiological observables {Φ_i} and those of a set of behavioral observables {B_j}. However, as shown above (sections 2.2), emotional observables do not commute with each other and do not commute neither with their physiological nor with their behavioral observables. Classifying emotions from the knowledge of behavioral and/or physiological features thus requires to take into account these order effects, which means that the generalized probabilistic rule (5) must be used instead of Bayes' classical inference rule. This can be done by first computing the relevant commutators. Within the quantum-like model presented in section 4, an emotional observable, which is formally defined as a projector acting on the vector-space of all possible mental states, measures the intensity of a specified emotion experienced by the subject. As mentioned in section 2.2, this measurement can be realized by asking the subject to report discrete values on a rating scale or to report them continuously, for example by using a continuous response digital interface.

The commutators of couples of emotional observables measure their degree of incompatibility. A method of computation of these commutators from the data collected by

Pkachin *et al.* (1999) has been proposed by Uzan in reference (Uzan 2016). In the case analysed in (Uzan 2016) the experimenter tells the subjects stories that are directly related to these specific incidents, but other methods based on the projection of films have also been shown reliable (Kassam and Mendes 2013). The five considered emotional contents are here “happiness” (noted as H), “sadness” (S), “anger” (A), “fear” (F) and “disgust” (D). The commutator of a couple of emotional observables ($\mathbb{A}$, $\mathbb{B}$), which are projectors, can be computed if the conditional probabilities that an individual be “observed” (through a questionnaire or other means) in the emotional state A if she/he has been conditioned or “prepared” in some specified emotional state B. The way these conditional probabilities can be computed for the table of data drawn from Prkachin’s and team’s article have been explained in section 2.2: the conditional probability P (A/W_B) that a subject conditioned in the emotional state W_B experiences the emotion A can be assimilated with the rate of the reported average intensity of A with respect to the sum of all the reported average intensities of emotions for subjects prepared in the same target emotional state B. This computation has been done from Prkachin’s data that report these average intensity for each emotion. Appendix 1 computes the commutator of the emotional observables Anger ($\mathbb{A}$) and Disgust ($\mathbb{D}$), which are interpreted in this model as projectors acting on the same 2-dimensional complex Hilbert space.

The degree of non-commutativity of emotional observables and their physiological correlates can be evaluated from data reporting the outcomes of joint measurements of intensity of emotions and physiological variables. This computation can be done from the data provided by Pkachin *et al.* (1999), Kassam and Mendes (2013), Pauls and Stemmler (2003) and Sinha *et al.* (1992). These articles report the changes in the values of several cardiovascular variables of subjects experiencing emotional states induced by stimuli designed according to reliable procedures, like for example the one proposed by Lang (1979).

In the case analysed in (Uzan 2016) the experimenter tells the subjects stories that are directly related to these specific incidents, but other methods based on the projection of films have also been shown reliable (Kassam and Mendes 2013). From the data provided by the latter article it is possible to first compute the components of a subject's cardiovascular state (specified by her heart rate, for example, and noted as $|HR>$) in the 2-dimensional basis of the eigenstates $|A_n>$, for n = 1 or 2, of an observable $\mathbf{A}$ which notifies whether or not the subject has reported her emotion of Anger. These components can be computed by the respective variations ΔHR in heart rate when this subject respectively reports her emotional state (of Anger) and does not report it –these data are provided on page 3 of the previously mentioned article by Kassam and Mendes (2013), in the section "Cardiovascular reactivity". The commutator of the two projectors respectively associated with the measure of HR and that of Anger can then be easily computed. This commutator, which is different from the null operator, is computed in Appendix 2.

Finally, an estimation of the degree of non-commutativity between emotional and behavioral observables can be done from the very comprehensive data presented in the articles by Duran and Fernandez-Dols (2010) and by Barrett *et al.* (2019). These data question the common view according to which there would exist a well-established and universal correspondence between a subject's experienced emotion and her facial expression. Roughly speaking, these articles show that if subjects smile when they are happy and scowl when they are angry more often than would be expected by chance, significant variations of these correspondences exist across subjects, contexts and cultures (Barrett *et al.* 2019). The common-view correspondences between the nature of experienced emotions and the expressive facial configurations of the Facial Action Coding System[8] (FACS) are tested by referring to meta-analysis of experimental data and they are shown to actually be rather weak.

For our purpose, which is to evaluate the degree of non-commutativity between the observables involved in these correspondences, we will focus on the conditional probabilities to detect the nature (and/or the intensity) of a subject's experienced emotion from her facial expression. Figure 8B of the article by Barrett *et al.* (2019), which refers to the article by Duran, Reisenzein and Fernandez-Dols (2017), reports the proportion of successful correspondences between emotions (induced by presenting objects or events to the subjects) and the common view facial expression of these emotions. For example, the tested subjects assign the emotional experience of Anger to the common view facial expression of Anger (which is characterized by brows furrowed, eyes wide, lips tightened and pressed together – see figure 2 A of Barrett *et al.* 2019) with a (weak) proportion of .28 with regard to the six considered main emotions of the experiment. This means that the conditional probability of detecting Anger given its common view facial expression, noted as F_A, can be evaluated as $P(A/F_A) = .28$. The commutator of the observables $\mathbb{A}$ and $\mathbb{F}_A$, which respectively evaluate the intensity of Anger and the degree of resemblance of the subject's facial expression with the common view facial expression of Anger, can then be computed within the theoretical framework presented in section 4. This computation is explicitly done in Appendix 3.

Generally speaking, the task of assigning emotions to facial expressions have to take into account the degree of non-commutativity between the relevant emotional observables and the observables that measure the subject's facial expressions.

6. Implementing emotional intelligence

[8] The Facial Action Coding System has been developed by the psychologists Paul Ekman and Wallace Friesen in 1978. It is now the standard tool used in psycho-physiological studies of facial expression.

As is well known, human intelligence is not reduced only to the intellectual performance (or the IQ) but it has an important *emotional* part. Emotions play a crucial role in any aspect of life (Wang and Ross 2007) (Elfenbein and Ambady 2002) and emotional intelligence is the way we deal with them. Salovey and Mayer defined emotional intelligence as (Salovey and Mayer 1990):

"…the ability to perceive and express emotions, to integrate them to facilitate thought, to understand and to reason with emotions, as well as to regulate emotions in oneself and in others".

For doing that, the human uses verbal communication, that is, speaking and writing, but also, consciously or not, non-verbal communication, like reading persons' face expression, observing body movements and postures, and also physiological manifestations. By giving us access to our own emotions and to those of other persons, emotional intelligence allows us to develop empathy and to find the appropriate behavior in real time. Emotional intelligence thus has a fundamental social aspect since it helps us to connect our internal world to social reality by making decision based on the knowledge of the emotional tone of our social environment.

Simulating emotional intelligence algorithmically is now an important subject of research in the field of artificial intelligence, namely for improving human-machine interaction. It first requires *emotion recognition*, which can be assimilated to a classification task from multimodal sensory, behavioral or physiological data (Poria *et al.* 2017). Human emotions are "recognized" by using several types of sensors that detect speech signal, voice tone, facial expressions and body language, and by appealing to data on previously observed correlations (called above "the common-view correspondence") between the nature and intensities of emotions, on the one hand, and the nature of values of physiological and behavioral observables, on the other hand. Actual results based on facial expression or bodily movement recognition utilize statistical techniques, namely the use of supervised machine learning algorithms. Several deep learning algorithms compute the most probable emotional state which can be assigned to a subject by analyzing her behavior, like her facial expression (Li and Deng 2018) or her body language and posture, which seems the most informative observations even realized from afar and from any angle of view (Santhoshkumar and Geetha 2019).

However, in the learning phase of these algorithms, the primitive properties that are observed on a huge number of images (like a collection of facial expressions, of postures or body gestures) are presently regarded as objective properties that could be observed independently of the possible subject's emotional state. Similarly, the likehood function, which relates them, through conditional probabilities, to the emotional categories to recognize (generally, five or seven "primary" emotions) are regarded as objective relationships, always carried out, independently of any other relevant parameter. However, as explained above (sections 2.2 and 5.2), this classical approach to cognitive processes involving emotional experience cannot be held anymore because there are significant order effects in the successive measurement of the intensity of emotions and their behavioral or physiological correlates.

Moreover, the emotion recognition task is only the first step to achieve in order to simulate emotional intelligence. Similar to the case of a physician whose diagnosis is not only based on medical imaging and blood analysis but also on thinking about the history of the patient, the best interpretation of her symptoms and their possible link, *emotional intelligence requires much more capabilities than only detecting emotions*. In addition to understanding our own emotions and those of our social environment, emotional intelligence requires the ability to make probabilistic inferences from these data. In particular, in order to respond appropriately to a situation, emotional intelligence requires to make *predictions* about the emotional state of a subject and her behavior given her present emotional state and new data, like information about the change of her face expression and her voice tone. This requires again the use of a probabilistic rule capable of accounting for the non-commutativity of emotional observables with each other and with their behavioral and physiological correlates.

Consequently, we need to use the generalized probabilistic rule (5) instead of Bayes' classical inference rule (1) in order to compute the required conditional probabilities $P(Ek/\{\Phi_i\})$, $P(E_i/\{Bj\})$, or even $P(E_k /\{\Phi_i\}$ and $\{Bj\})$ that the presented item can be classified as being in the emotional state E_k given the values, respectively indexed by i and j, of the physiological and the behavioral observables. The deep learning algorithms that realize the task of classifying face expressions in various emotional categories and predicting emotion from emotional, sensory, behavioral or physiological data, which involve measuring non-commutative observables (Singh, Majumber and Behera 2014) (Cohen et al. 2003) can thus be improved by using the generalized probabilistic rule (5) for probability computations instead of Bayes' inference (1). This can be done the same way as in the developments presented in section 5.2, by first computing the relevant commutators.

We have to notice that the previous conclusion on the simulation of emotional intelligence indeed applies for the simulation of *all* aspects of mental activity, not only because this non-commutativity also holds for various other aspects of cognitive activity but also because emotions are involved in them to varying degrees. As shown by Wang and Ross (2007) and as reported by well-known studies in the field of psychoanalysis (Van Der Linden et d'Argembeau 1999), the emotions felt during life situations connect its different aspects in our mind and thus play an essential role in their memorization. In daily life, "negative" emotions, like anger or sadness, can disturb our concentration and make it difficult remembering a telephone number or performing a simple mental calculus, while “positive” emotions, like a feeling of happiness, can improve our ability to perform these tasks. More generally, we can say that emotions play an essential role in all human activities, being of psychological, social or cultural order (Hwang and Matsumoto 2021).

Thus, it seems that the generalized probabilistic inference rule (5), which, in contrast with Bayes’ rule, accounts for order effects, must be used for realistically modeling all aspects of human activity and for improving the algorithms that simulate these activities. Future research in the field of artificial intelligence and robotic should integrate this new probabilistic rule.

Conclusion

This article has questioned the “all-Bayesian” dogma defended by most researchers in cognitive science and artificial intelligence. Its central rule, namely Bayes’ rule, is derived from the classical probability calculus, which, as shown in this article, cannot reliably deal with mental processes. The reason is that mental processes give rise to order effects which cannot be accounted by the classical probability calculus –and then by Bayes’ rule. Keeping the general idea of Bayesianism, according to which cognitive processes can be rationally modeled by using probabilities rather than rigid rules of inference, a new, probabilistic rule generalizing Bayes’ rule and capable of accounting for order effects has been proposed in section 4. The application of this new rule requires to compute the commutators of the observables involved in the studied situation, which can be done within the generalized probability theory developed by physicists to deal with quantum phenomena and briefly presented in this same section. As explained in sections 5 and 6, the use of this generalized probabilistic rule instead of the classical Bayes’ rule will make the current models of cognition and the algorithms of artificial intelligence more reliable.

References

Aerts, D. and Sozzo, S. (2013) Quantum Structure in Economics: The Ellsberg Paradox. *ArXiv:1301.0751 v1 [physics.soc-ph] 4 Janv 2013*.

Aerts, D., Sozzo S., Gabora, L. and Veloz, T. (2011). Quantum structure in cognition: Fundamentals and applications. In ICQNM 2011: *The fifth international conference on quantum, nano and micro technologies*.

Ashby, F. and Alfonso-Reese, L. (1995). Categorization as probability density estimation. Journal of Mathematical Psychology, 39, 216–233.

Atmanspacher, H., & Römer, H. (2012) Order effects in sequential measurements of non-commuting psychological observables. *Journal of Mathematical Psychology, 56*, 274-280.

Barrett, L. F., Adolphs, R., Marsella, S., Martinez, A. M. and Pollak, S. D. (2019) Emotional expressions reconsidered: challenges to inferring emotion from human facial movements. *Psychological Science in the Public Interest*, 20(1), pp. 1-68.

Broemeling L. D. (2011) Bayesian Methods for Medical Test Accuracy. *Diagnostics (Basel, Switzerland), 1*(1), 1–35.

Busemeyer, J. R., & Bruza, P. (2012) *Quantum models of cognition and decision*. Cambridge: Cambridge University Press.

Busemeyer, J. and Wang, Z. (2017) Is there a problem with quantum models of psychological measurements? *Plos One* Nov. 8, 2017.

Cattell, R. B. (1930). The subjective character of cognition and the pre-sensational development of perception. *British Journal of Psychology*, *14,* 166.

Cohen, I., Sebe N., Gozman, F.G., Cirelo, M.C. and Huang, T.S. (2003) Learning Bayesian network classifiers for facial expression recognition both labeled and unlabeled data, *2003 IEEE Computer Society Conference on Computer Vision and Pattern Recognition, Proceedings 2003.*

Cox R. T. (1946) Probability, Frequency, and Reasonable Expectation, *Am. Jour. Phys*., 14: 1–13.

Cruz N., Baratgin J. Oaksford M. and Over D. (2015) Bayesian reasoning with ifs and ands and ors. *Frontiers in Psychology* 6:192.

De Finetti B. (1970) Logical foundations and measurement of subjective probability. *Acta Psychologica*, 34 : 129-145.

Dehaene S. (2012): course on the Bayesian Brain. https://www.college-de-france.fr/media/stanislas-dehaene/UPL5981541562447201298_Cours2012_CerveauStatisticien_6.pdf

Duran, J. I., and Fernandez-Dols J-M. (2018) Do emotions result in their predicted facial expressions? A meta-analysis of studies on the link between expression and emotion. *PsyArXiv* : https://psyarxiv.com/65qp7

Elfenbein, H. A., and Ambady, N. (2002). On the universality and cultural specificity of emotion recognition: A meta-analysis. *Psychological Bulletin, 128*(2), 205–235.

Ellsberg D. (1961) Risk, Ambiguity and the Savage Axioms. *Quaterly Journal of Economics* 75 (4): 643-669.

Evans J., Thompson V. and Over D. (2015). Uncertain deduction and conditional reasoning. *Frontiers in Psychology* 6: 398.

Freud, S. (1909) Cinq leçons sur la psychanalyse, deuxième leçon : Résistance et refoulement, in *Leçons d'introduction à la psychanalyse*, Presses Universitaires de France, « Quadrige », 2013, p. 297-312

Geringer, J. M., Madsen, K. M., & Gregory, D. (2004) A fifteen-year history of the continuous response digital interface: Issues relating to validity and reliability. *Bulletin of the Council for Research in MusicEducation, 160,* 1–15.

Gleason AM (1957) Measures on the closed subspaces of a hilbert space. J Math Mech 6:885–893.

Griffiths, L., Kemp, T. and Tenenbaum, J.B. (2008). Bayesian models of cognition. The Cambridge Handbook of Computational Psychology. Cambridge University Press.

Griffiths, T. L., Sanborn, A., Canini, K., and Navarro, D. (2008). Categorization as nonparameteric Bayesian density estimation. In M. Oaksford & N. Chater (Eds.), The probabilistic mind: Prospects for Bayesian cognitive science. Oxford: Oxford University Press.

Hossein Pishro-Nik H. (2014) *Introduction to Probability, Statistics, and Random Processes.* https://www.probabilitycourse.com/

How C.Q., Lotto R.B. and Purves D. (2006) Comparison of Bayesian and empirical ranking approaches to visual perception. *Journal of Theoretical Biology* 241: 866-875.

Hwang, H. & Matsumoto, D. (2021). Functions of emotions. In R. Biswas-Diener & E. Diener (Eds), *Noba textbook series: Psychology.* Champaign, IL: DEF publishers

Kassam, K.S., and Mendes W.B. (2013) The effects of measuring emotion: physiological reactions to emotional situations depend on whether someone is asking. *Plos One*

Kersten, D., Yuille, A. (2003) Bayesian models of object perception. Curr. Opin. Neurobiol. 13 (2), 150–158.

Klement, R. J., & Bandyopadhyay, P. S. (2020) The Epistemology of a Positive SARS-CoV-2 Test. *Acta Biotheoretica*(Sept 4), 1-17.

Knill, D.C., Pouget, A. (2004) The Bayesian brain: the role of uncertainty in neural coding and computation. *Trends Neurosci.* 27: 712–719.

Kreibig, S.D. Wilhelm, F.H. Roth, W. and Gross, J.J. (2007) Cardiovascular, electrodermal, and respiratory response patterns to fear- and sadness-induced films. *Psychophysiology* 44:787–806.

Kubota T. (2017) Artificial intelligence used to identify skin cancer. http://news.stanford.edu/2017/01/25/artificial-intelligence-used-identify-skin-cancer/

Lang P.J. (1979) A bio-informational theory of emotional imagery. *Psychophysiology* 16:495-512.

Li S. and Deng W. (2018) Deep facial expression recognition: A Survey. Computer science, Psycxhology.

Mamassian P., Landy M. and Maloney L.T. (2002) Bayesian modelling of visual perception, in: Rao R.P.N. Probabilistic Models of the Brain: Perception and Neural Function. *MIT Press*, 2002: 13-36.

Moore DW (2002) Measuring new types of question order effects. *Public Opinion* Q*uarterly* 66 (1): 80–91.

Murphy, G. L. and Medin, D. L. (1985). The role of theories in conceptual coherence. *Psychological Review*, 92: 289–316.

Nosofsky, J. (1986). Attention, similarity, and the identification–categorization relationship. Journal of Experimental Psychology: General, 115(1): 39–57.

Oaksford, M. and Chater, N. (2007). Bayesian rationality: The probabilistic approach to human reasoning. Oxford University Press.

Pauls C.A. and Stemmler G. (2003) Repressive and Defensive Coping During Fear and Anger. *Emotion*, Vol. 3 (3): 284-302.

Perfors, A., Tenenbaum, J. B., Griffiths, T. L., & Xu, F. (2011). A tutorial introduction to Bayesian models of cognitive development. *Cognition*, *120*(3), 302-321.

Perfors, A., and Tenenbaum, J. B. (2009). Learning to learn categories. In N. Taatgen, H. van Rijn, L. Schomaker, & J. Nerbonne (Eds.), Proceedings of the 31st annual conference of the cognitive science society (pp. 136–141). Austin, TX: Cognitive Science Society.

Poggio T. (2016) Deep Learning: Mathematics and Neuroscience. *A Sponsored Supplement to Science*, Brain-Inspired intelligent robotics: The intersection of robotics and neuroscience: 9–12.

Poria S., Cambria E., Bajpai R. and Hussain A. (2017) A review of affective computing: From unimodal analysis to multimodal fusion. *Information Fusion* 37: 98–125.

Pothos, E. M. & Busemeyer, J. R. (2009) A quantum probability explanation for violations of "rational" decision theory. *Proceedings of the Royal Society B* 276:2171–78.

Prkachin KM, Williams-Avery RM, Zwaal C, Mills DE (1999) Cardiovascular changes during induced emotion: an application of Lang's theory of emotional imagery. *J Psychos Res* 47(3):255–267.

Pronin E. (2007) Perception and misperception of bias in human judgment *Trends in Cognitive Sciences,* Volume 11, Issue 1, pp. 37–43.

Roiser J.P. and Sahakian B.J. (2013) Hot and cold cognition in depression. CNS Spectr. 18(3):139-49.

Rosch E. (1978). Principles of categorization. In E. Rosch and B. B. Lloyd (Eds.), *Cognition and categorization* (pp. 27–48). Lawrence Erlbaum.

Salovey, P. and Mayer, J.D. (1990). Emotional intelligence. *Imagination, Cognition, and Personality*, 9, 185-211.

Santhoshkumar R. and Geetha M. (2019) Deep Learning Approach for Emotion Recognition from Human Body Movements with Feedforward Deep Convolution Neural Networks. *Procedia Computer Science*, Vol 152: 158-165.

Savage L.J. (1954). *The Foundations of Statistics*. New York, Wiley.

Simonyan K. and Zisserman A. (2015) Very deep convolutional networks for large-scale image recognition. *Computer Science*.

Singh, M., Majumder, A. and Behera, L. (2014) Facial expressions recognition system using Bayesian inference, *2014 International Joint Conference on Neural Networks (IJCNN)*, 2014, pp. 1502-1509, doi: 10.1109/IJCNN.2014.6889754

Sinha R, Lovallo WR, Parsons OA (1992) Cardiovascular differentiation of emotions. *Psychosom Med* 54:422–435.

Stolorow, R. D. (2005). The Contextuality of Emotional Experience. *Psychoanalytic Psychology, 22*(1), 101–106.

Xu, F. and Tenenbaum J.B. (2007) Word Learning as Bayesian Inference: Evidence from Preschoolers, *Psychol Rev* Apr;114(2):245-72.

Uzan P. (2014) Psychologie cognitive et calcul quantique. *Implications Philosophiques*, section Epistémologie, Avril 2014.

Uzan P. (2016) Complementarity in Psychophysics. *Lecture Notes in Computer Science*, Atmanspacher H., Filk T. and Pothos E. eds, Springer, Vol. 9535 : 168-178.

Van Der Linden, M., et d'Argembeau, A. (1999) L'émotion, ciment du souvenir. *Cerveau et Psycho* 28.

Wang, Z. and Busemeyer J.R. (2013) A Quantum Question Order Model Supported by Empirical Tests of an *A Priori* and Precise Prediction. *Topics in Cognitive Science* 5(4): 689-710.

Wang Z., Solloway T., Shiffrin R.M. and Busemeyer J.M. (2014) Context effects produced by question orders reveal quantum nature of human judgments. *PNAS* 111 (26): 9431–9436.

Wang, Q. and Ross, M. (2007) Culture and memory. In S. Kitayama & D. Cohen (Eds.), *Handbook of cultural psychology* (pp. 645–667). New York, NY: Guilford.