HAL open science

Preparation in Bohmian Mechanics

Carlo Rovelli

To cite this version:

Carlo Rovelli. Preparation in Bohmian Mechanics. 2024. hal-03612535

HAL Id: hal-03612535

https://hal.science/hal-03612535v1

Preprint submitted on 9 Jan 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Preparation in Bohmian Mechanics

Carlo Rovelli
Aix-Marseille University, Université de Toulon, CPT-CNRS, F-13288 Marseille, France.
Perimeter Institute, 31 Caroline Street N, Waterloo ON, N2L2Y5, Canada and
The Rotman Institute of Philosophy, 1151 Richmond St. N London N6A5B7, Canada
(Dated: April 26, 2021)

According to Bohmian mechanics, we see the particle, not the pilot wave. But to make predictions we need to know the wave. How do we learn about the wave to make predictions, if we only see the particle? I show that the puzzle can be solved, but only thanks to decoherence.

arXiv:2104.11339v1 [quant-ph] 22 Apr 2021

The problem

In Bohmiam mechanics [1], the state of a system of N degrees of freedom (or "particles") is given by a couple $(\Psi(x_n), X_n)$, namely a wave function $\Psi(x_n)$, or "pilot wave", and the positions X_n of the particles. Here $n = 1, ..., N$. The dynamics is given by the two equations

$$i\hbar\, \partial_t \Psi = H\Psi, \qquad \partial_t X_n = \frac{\hbar}{m_n}\, \frac{\partial_{x_n}\Psi}{|\Psi|}\bigg|_{x_n=X_n}, \qquad (1)$$

where H is the Hamiltonian operator and m_n the mass of the nth particle. The first is the Schrödinger equation, the second is Bohm's guidance equation.

A theorem states that if at some time we do not know the position of the particles but we know their probability distribution and this satisfies $\rho(X_n) = |\Psi(X_n)|^2$, then the dynamics conserves this equation. The value of this equation "initially" is either assumed or said to be attained via a suitable equilibration process [2], depending on the versions of the Bohmiam theory considered. In either case, if we know the wave function, we can give probabilistic predictions about where the particles will go.

According to the theory, what we see, are the actual positions X_n of the variables ("particles"). Hence, in a sense, the particle positions are "manifest" variables and the wave is "hidden". In a different sense, the particle positions are called the "hidden variables", because they are variables added to conventional wave mechanics to make it (non local and) deterministic.

There is one aspect of this theory that I have never understood. In order to make predictions, we have to know Ψ. But how can we know Ψ, if what we see is only the particle positions X_n? The theory gives us a probability distribution for the particle position, given the wave; but it does not give us a recipe for guessing the wave, given the particle positions that we see. (On the problem, see [3, 4].)

In textbook quantum mechanics, the state is determined by a *preparation*, which is essentially a measurement. A measurement, in conventional quantum theory, *collapses* the wave function on the eigenstate of the observed eigenvalue. Hence we know the state because we made an observation, and we the theory instructs us to *assume* that the state is then the corresponding eigenstate. If we see spin up, we then say the state is $|+\rangle$.

But nothing of the sort is directly true in Bohmiam mechanics: if we see the particle in the position X, the wave can still be virtually anything. Hence we cannot infer much about the wave, from observing particles. Hence we cannot measure the wave. But then how can we make predictions, given that the wave is needed for predictions?

Here, I give an answer to this question. Maybe this answer is known, but I have not found it in the literature. The answer is more subtle than what I expected, requires thinking about decoherence and does not require any hypothesis about special states in the past.

An insufficient solution

In Bohmian literature, one reads that after a standard measurement the state of a System (with variable s) and Apparatus (with pointer variable a) has the form

$$\Psi(s, a) = \sum_n c_v\ \psi_n(s)\phi_n(a), \qquad (2)$$

where n labels the eigenvalues of measured quantity, ψ_n are the corresponding eigenstates, and ϕ_n are states of the pointer variable that are peaked on well disjoint regions R_n of its configuration space. If the Bohmian pointer variable $a = A$ is in one of these regions, then the *effective* state of the system $\psi(s) = \Psi(s, A)$ is in fact in the eigenstate of the measured quantity.

All this is fine, of course, but steps are missing. The key one is that system and apparatus do *not* end up in a state of the form (2) if they are in a generic state before the measurement interaction. This follows immediately from the fact the evolution during the interaction is unitary, hence a generic state ends up in a generic state, while the states (2) are not generic. To end up in the form (2), the combined system (system+apparatus) must be in a tensor state, namely uncorrelated, before the interaction. But how can we know that it is so, if not by measurement? So, to prepare a state, we already need to know a state. This appears to open a regress to infinity.

Furthermore, even if we get to know the effective state, how do we know that other components of the universal wave function don't come-in, and affect the evolution governed by effective states? Here I try to fill these gaps.

The key preliminary corollary

Consider a single particle. The wave is $\Psi(x)$ and the particle position is X. Imagine that at some time the wave has two components disjoint in configuration space. That is $\Psi(x) = \Psi_1(x) + \Psi_2(x)$ where the support of Ψ_1 is a compact region R_1 and the support of Ψ_2 is a compact region R_2 that does not overlap with R_1. Then by the probability assumption the particle must be either in R_1 or in R_2. Say it is in R_1. Then, as long as Ψ_1 and Ψ_2 do not interfere, we can replace Ψ with Ψ_1 in the guidance equation for X.

This is immediate, because the guidance equation depends only on the value of Ψ in the neighbourhood of $x = X$ and Ψ_2 vanishes there. Since the Schrödinger equation is linear, we can evolve Ψ_1 with this equation and disregard Ψ_2 entirely.

All this stops to hold, of course, if there is interference, namely if at some time R_1 and R_2 get to overlap.

Next consider the same single particle, but now interacting with an environment of many degrees of freedom e_n. The wave is $\Psi(x, e_n)$ and the positions are (X, E_n). Imagine, as before, that at some time Ψ has two components Ψ_1 and Ψ_2 such that the supports of $\Psi_1(x, E_n)$ and $\Psi_2(x, E_n)$ are disjoint in the space of the variable x. Then, as before, we can disregard Ψ_2 for the dynamics of the particle, as long as there is no interference. But let's furthermore, imagine that the interaction between the particle and the environment is non negligible (in a precise sense specified below). Then, interference is suppressed. This is because the probability distribution for the particle position is given by

$$\begin{aligned}\rho(x) &= |\Psi(x, E_n)|^2 = |\Psi_1(x, E_n) + \Psi_2(x, E_n)|^2 \quad (3)\\ &= |\Psi_1(x, E_n)|^2 + |\Psi_2(x, E_n)|^2\\ &\quad + Re[\overline{\Psi_1(x, E_n)}\Psi_2(x, E_n)].\end{aligned}$$

Interference is given by the last term, but if the coupling with the environment is sufficient, the two states Ψ_1 and Ψ_2 affect the environment differently and therefore $\Psi_1(x, e_n)$ and $\Psi_2(x, e_n)$ are going to have different support in the e_n variables. But then there is no value of E_n that can prevent this term from vanishing. This is essentially the Bohmian version of decoherence.

The consequence is that, once more, Ψ_2 can be discarded, as far the motion of X is concerned, but now without limitations: there is no risk of interferences bringing back Ψ_2 into the dynamics of X.

The effective loss of quantum entanglement

Consider now two systems, with position variables S and A, and state $\psi(s, a)$, in an entangled state. That is

$$\Psi(s, a) \neq \Psi(s)\Phi(a). \quad (4)$$

For simplicity, consider an entangled state of the form

$$\Psi(s, a) = \Psi_1(s, a) + \Psi_2(s, a) = \Psi_1(s)\Phi_1(a) + \Psi_2(s)\Phi_2(a). \quad (5)$$

Immagine that the system A interacts sufficiently strongly with an environment. Say $\Phi_1(a)$ and $\Phi_2(a)$ have disjoint support, as before, and the variable A is in the support of $\Phi_1(a)$. Then, for the argument above, the state Ψ_2 can be discarded, for the future evolution of the variable A and S.

This means that the interaction of a system A with the environment has the effect to cancel any interference effect due to any entanglement between A and another system S. Once again, this can be recognised as a typical effect of decoherence. Hence if A interacts sufficiently with an environment, we can always effectively assume that its state is not entangled with another system S.

Armed with this, we can tackle the preparation of a state in Bohmian mechanics.

Preparation

Consider a variable S (System) and a variable A (apparatus, or pointer variable) interacting with an environment. Say the interaction hamiltonian between S and A depends on a function O on the phase space of the coupled system, and is turned on at some time t. Assume that before this time, A has sufficiently interacted with the environment. Then, because of the argument given above, we can assume that at time t the state is a tensor state. More precisely, as the variable A has sufficiently interacted with the environment, the system and the apparatus can be effectively considered uncorrelated. The empty components of the universal wave are there, but do not disturb. This is the missing ingredient in previous analysis of Bohmian measurement that I found.

Let $\psi(s, a) = \Psi(s, a, E_n)$ be the effective state of System+Apparatus. Its evolution is effectively governed by a Schrödinger equation. The standard Schrödinger evolution of an initial uncorrelated state $\psi(s, a) = \psi_A(a)\psi_S(s)$ by an interaction hamiltonian that depends on an observable O drives the state to the well known form

$$\psi(s, a) = \sum_n c_v \ \psi_n(s)\phi_n(a), \quad (6)$$

as above, where ϕ_n are states of the pointer variable that are peaked on well disjoint regions R_n of the configuration space of a. In Bohm theory, the pointer A is going to be in one of these, say the nth one. By the results of the previous section, because of the fact that A interacts with the environment, any interference between the terms of the last equation is suppressed and therefore the future evolution of the variable S is going to be guided by the single eigenstate ψ_n that correspond to the non empty branch of the apparatus wave. This is preparation of a state in Bohmian mechanics.

Conclusion

Contrary to the expectation I had when I started looking into this, it works. The process corresponding to decoherence plays a major role. The trick is that it is true that knowledge of the X_n gives us very little information about the Ψ, but this can be circumvented by having decoherence rendering most components of Ψ irrelevant and arranging the dynamics so that the relevant part of the effective state ψ is a finite sum of terms we know, so that the pointer variable being in one region restricts the effective wave function of the system to a known form. And this is of course exactly what experimenters do in a real laboratory experiment.

To summarize: first, the environment-apparatus interaction renders the interference between states with different values of the pointer variable irrelevant. Second, we can write a *generic* state of the combined (system+apparatus) as a sum of tensor terms peaked on different values of the pointer, and because of the previous point, we can forget the empty branches. Hence we can safely assume that before the start of the interaction system and apparatus are uncorrelated. The measurement interaction correlates them. Finally, the value of the pointer value tells us precisely that the only relevant branch of the effective state of the system is the corresponding eigenstate.

I embarked in this exploration thinking that Bohiam mechanics does not really work, but, as far as I can see, it works. There is no need for this to make any special ("past low entropy") assumption (as I at some point suspected), but decoherence is necessary to make predictions.

***

Many thanks to Wayne Myrvold, Davide Romano, and David Albert for comments, enlightening discussions, and corrections to the first draft of this paper.

[1] Sheldon Goldstein, "Bohmian Mechanics," in *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta (2017).

[2] Antony Valentini, "Inflationary Cosmology as a Probe of Primordial Quantum Mechanics," Phys. Rev. D **82**, 063513 (2010).

[3] D. Bohm and B.J. Hiley, "Measurement Understood Through the Quantum Potential Approach," Foundations of Physics **14**, 255 (1984).

[4] D. Bohm, B.J. Hiley, and P.N. Kaloyerou, "An Ontological basis for the Quantum Theory," Physics reports **144**, 321–375 (1987).