



**HAL**  
open science

# Complete measurement of the phase-space topology of fiber four-wave mixing using iterated initial conditions

Anastasiia Sheveleva, Ugo Andral, Bertrand Kibler, Pierre Colman, J.M. Dudley, Christophe Finot

## ► To cite this version:

Anastasiia Sheveleva, Ugo Andral, Bertrand Kibler, Pierre Colman, J.M. Dudley, et al.. Complete measurement of the phase-space topology of fiber four-wave mixing using iterated initial conditions. Conference on Lasers & Electro-Optics US, May 2022, San Jose, United States. hal-03610037

**HAL Id: hal-03610037**

**<https://hal.science/hal-03610037v1>**

Submitted on 24 Aug 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Complete measurement of the phase-space topology of fiber four-wave mixing using iterated initial conditions

A. Sheveleva<sup>1</sup>, U. Andral<sup>1</sup>, B. Kibler<sup>1</sup>, P. Colman<sup>1</sup>, J. M. Dudley<sup>2</sup>, C. Finot<sup>1</sup>

<sup>1</sup> Laboratoire Interdisciplinaire Carnot de Bourgogne, UMR 6303 CNRS-Université de Bourgogne-Franche-Comté, 21000 Dijon, France

<sup>2</sup> Institut FEMTO-ST, Université Bourgogne Franche-Comté CNRS UMR 6174, Besançon, 25000, France

[Anastasiia.Sheveleva@u-bourgogne.fr](mailto:Anastasiia.Sheveleva@u-bourgogne.fr)

**Abstract:** We experimentally demonstrate an optical fiber testbed enabling the quantitative study of ideal four-wave mixing. We directly measure the complex phase-space topology including features such as the separatrix, Fermi Pasta Ulam recurrence, and stationary waves.

## 1. Introduction

Nonlinearity is a key physical process that enables multiple wave mixing, paving the way for many important applications such as parametric amplification, frequency comb generation, and ultrashort pulse synthesis. Optical fibers exhibiting Kerr nonlinearity are well-known to exhibit a wide range of nonlinear dynamics, such as modulation instability, supercontinuum generation, and the emergence of coherent breathers and solitons [1]. Indeed, even very simple initial conditions such as a periodically-modulated continuous wave can lead to very complex longitudinal dynamics, potentially involving a very large number of spectral components. However, the essential physics of the underlying wave mixing processes can be captured using a truncated three-component model (pump and two sidebands) and linked to a one dimensional Hamiltonian formalism [2]. Similar to a nonlinear oscillator, the periodic nature of the interaction can lead to highly-complex dynamical trajectories when plotted in phase-space. On the other hand, in typical experimental studies, discrepancies between measurements and the results of the truncated model become quickly significant. Consequently, modelling previous experimental results has required the use of the full nonlinear Schrödinger equation to accurately reproduce the observed features [3, 4]. This is a clear limitation to exploring the fundamental properties of canonical nonlinear wave mixing processes.

In this paper, we present a solution to this problem with the development of a new experimental fiber-based platform that is able to rigorously excite truncated ideal four wave nonlinear dynamics. Based on iterated propagation of phase and intensity shaped discrete spectral components, we are able to excite a range of evolution trajectories and we retrieve a highly accurate map showing the existence of the dynamical separatrix, of stationary solutions and of Fermi-Pasta-Ulam recurrence.

## 2. Principle and experimental setup

The underlying concept of our approach is based on restricting the number of frequency components propagating in the fibre while also limiting all the effects that potentially lead to deviations from a conservative system. In order to do so, we spectrally imprint the initial spectral amplitude and phase on a frequency comb generated from the 40-GHz phase modulation of a continuous wave (see experimental setup Fig. 1). This signal then undergoes nonlinear evolution in a 500-m long single mode fiber with anomalous dispersion. Over such a short distance of propagation, development of additional components through multi-wave mixing are at a very low amplitude and Brillouin and Raman effects and loss are negligible.

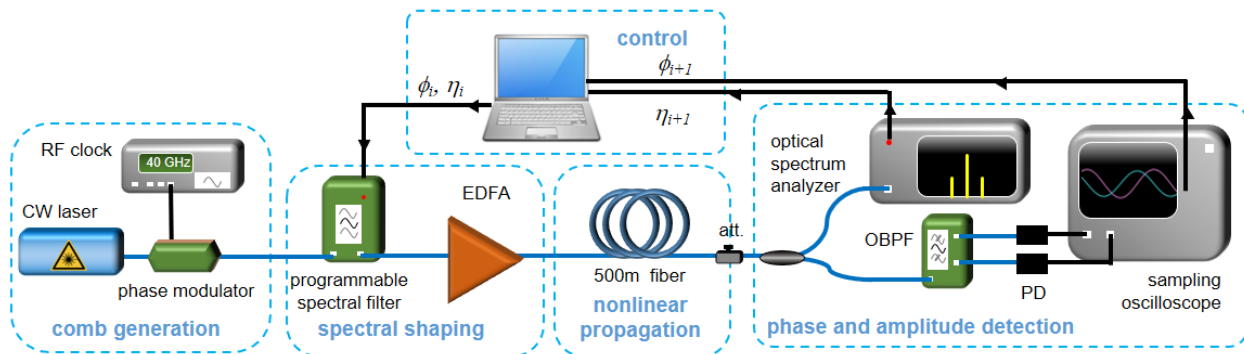


Fig. 1. Experimental setup. CW : Continuous Wave ; PD : Photodiode ; OBPF : Optical BandPass Filter

After propagation, the spectral properties of the central component and sidebands are characterized both in terms of amplitude and phase in order to derive canonical conjugate variables  $\eta$  and  $\phi$  which have physical interpretations as the fraction of the total power in the central frequency component and the sideband-pump frequency component phase difference respectively. The experimentally measured values can then be imprinted as new input values and the process can be iterated at will without any accumulation of higher-order sidebands or deleterious amplified spontaneous noise. Potentially unlimited propagation can therefore be emulated.

### 3. Experimental results

We first study the dynamics of the system at maximum gain (average power of 21.5 dBm). Experimental phase space portraits obtained for different initial values  $\eta_0$  and  $\phi_0$  are shown in Fig. 2a, yielding immediate insight into the phase space topology. The experimental orbits are seen to be in very good agreement with the theoretical predictions from numerically integrating the ideal system [2] and clearly confirm the importance of the separatrix dividing the phase plane into two well-defined regions. The different experimental trajectories are nearly closed orbits and do not intersect. We can also fully reconstruct the evolving intensity profiles in the temporal domain: results over a distance of 50 km (Fig. 2b) show the expected recurrence dynamics with a possible phase shift of half-temporal period. The influence of the average power for  $\eta_0 = 0.65$  is shown in Fig. 2c. For the lowest powers, the initial condition  $\phi_0 = 0$  leads to orbits that are on the left side of the separatrix and that surround the trajectory obtained for  $\phi_0 = \pi$ . When increasing the power, the separatrix is crossed. The orbits then get smaller and smaller up to the stage where they reach a fixed point for 22.7 dBm. When further increasing the average power, the orbit becomes increasingly open. These results show the success of our experimental approach, and open up new perspectives for the study of the fundamental properties of nonlinear wave mixing and frequency comb dynamics.

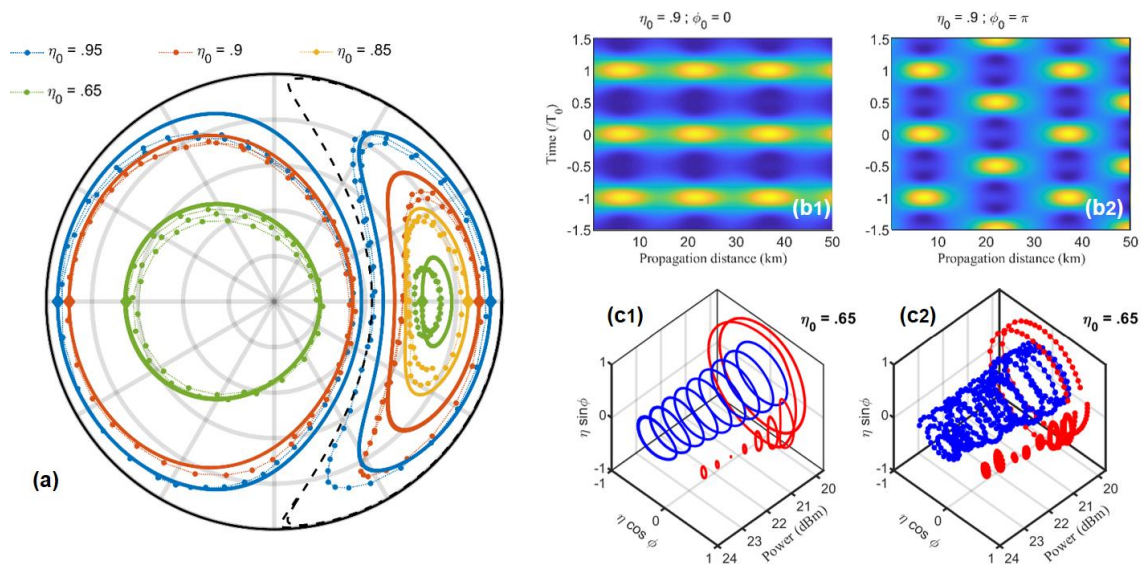


Fig. 2. (a) Phase space portraits for various initial values of  $\eta_0$ . Initial phase offset  $\phi_0$  of 0 or  $\pi$  appear respectively on the right and left side of the separatrix (dashed black line). The experimental results over 50 iterations (circles joined with dotted lines) are compared with the theoretical solution (solid thick line). (b) Longitudinal evolution of the temporal intensity profiles for initial values of  $\eta_0$  of 0.65. The theoretical predictions (panels 1) are compared with the experimental results (panels 2).

### References

- [1] G. P. Agrawal, *Nonlinear Fiber Optics, Fourth Edition* (Academic Press, San Francisco, CA, 2006).
- [2] S. Trillo and S. Wabnitz, "Dynamics of the nonlinear modulational instability in optical fibers," *Opt. Lett.* **16**, 986-988 (1991).
- [3] G. Xu, K. Hammani, A. Chabchoub, J. M. Dudley, B. Kibler, and C. Finot, "Phase evolution of Peregrine-like breathers in optics and hydrodynamics," *Phys. Rev. E* **99**, 012207 (2019).
- [4] A. Mussot, C. Naveau, M. Conforti, A. Kudlinski, F. Copie, P. Szriftgiser, and S. Trillo, "Fibre multi-wave mixing combs reveal the broken symmetry of Fermi–Pasta–Ulam recurrence," *Nat. Photon.* **12**, 303-308 (2018).