

Power definitions for three-phase systems in terms of instantaneous symmetrical components

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Abstract

This paper considers power definitions for three-phase power systems with non-sinusoidal and unbalanced voltages and currents. By applying the Lyon transformation to three-phase voltages and currents, instantaneous symmetrical components are obtained and used to define powers. Definitions of instantaneous apparent and reactive powers are derived, as well as definitions of well-known quantities based on rms values, such as apparent and reactive powers. The introduced set of power definitions based on the same mathematical framework could be used for time-instantaneous and time-average compensation. To provide correct calculation of powers for unbalanced three-phase four-wire systems, all wires and related quantities of the three-phase four-wire system are treated equally. The instantaneous apparent and reactive powers are also expressed in terms of instantaneous symmetrical components derived from the Clarke transformation. The proposed definitions are compared with other power definitions. In the case of a three-phase three-wire system, the proposed definitions give the same values of powers as the most known power definitions. In the case of a three-phase four-wire system, the proposed definitions give the same value of the apparent power as the DIN 40110 Standard and the same value of reactive power as the Currents' Physical Component theory.

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1. Introduction

Distortion and unbalance of voltages and currents are very common in modern power systems leading to lower power quality and transmission efficiency. Demand for power conditioning devices in low voltage distribution systems has been constantly growing over the last years [1]. The development of the new power conditioning devices and methods is closely related to the research on power theories. Two different concepts of power definitions have been developed: one based on the instantaneous power [2-11], and the other based on the average power [3,7,9-21]. Definitions based on the instantaneous power concept aim at keeping instantaneous power unchanged while minimizing transmission line losses. On the other hand, the average power concept aims at keeping active (average) power unchanged and results in greater reduction of transmission line losses. The instantaneous power theory proves to be useful in designing active power-line conditioners known as active filters [9,24]. The average power theories brought about the application of time-average compensation that enables the smallest rms values for the compensated

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currents keeping the same active (average) power of the original load currents [9-11]. Although there are many published power theories, there is not much research on the unification of the two concepts.

Fortescue's method of symmetrical components [25] is commonly used for the steady-state analysis of asymmetrical three-phase power systems. The method's application to time-dependent quantities is defined by Lyon transformation [26]. The Lyon approach also makes it possible to study power systems under transient and non-sinusoidal conditions [27], [28]. This is an important property due to many causes of distortion and unbalance of voltages and currents in power systems. Since it transforms instantaneous voltages and currents, the Lyon transformation is applicable in an unmodified form in the presence of voltage and current harmonics which makes it a promising mathematical framework for power definitions. Instantaneous symmetrical components obtained by Lyon transformation are used to define instantaneous power and its time-independent part in [29]. By replacing the complex operator used in Lyon transformation with a phase-shift operator in the time domain, the instantaneous power could be defined and divided on average and oscillating power components, as shown for arbitrary voltage and current waveforms in [30]. The instantaneous symmetrical components are used for dynamic compensation, balancing and control of power systems by means of static compensators [31]. Although the instantaneous power has been defined using Lyon transformation and divided into components in previous papers, none of them have considered the definitions of instantaneous reactive and apparent power. The set of power definitions in terms of instantaneous symmetrical components has not been proposed in literature yet.

The main contribution of this paper is the proposal and validation of the set of power definitions in terms of instantaneous symmetrical components intended to be used in three-phase three-wire and three-phase four-wire systems. The proposed definitions are valid for non-sinusoidal and unbalanced voltages and currents, and they can be used for time-instantaneous and time-average compensation. The proposed definitions also avoid mistakes caused by erroneous treatment of zero-sequence voltages when dealing with three-phase four-wire systems [9]. The results of the simulation tests have revealed the similarities of the proposed definitions with the various well-known definitions.

2. Instantaneous powers

2.1. Three-phase three-wire system

Let us assume that electric energy is delivered to a load by its three-phase three-wire supply lines, denoted by a, b, and c, as shown in Fig. 1.

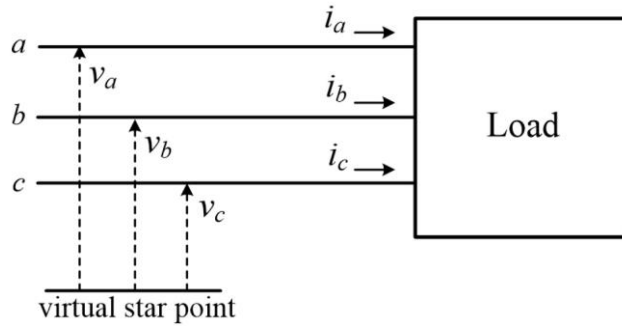


Fig. 1. Load in a three-phase three-wire system.

The instantaneous voltages v_a , v_b and v_c , and line currents i_a , i_b and i_c are assumed to be periodic quantities of the same period T :

$$v_j = \sum_{k=1}^n \sqrt{2}V_{jk} \cos(k\omega t + \theta_{jk}), \quad j = a, b, c \quad (1)$$

$$i_j = \sum_{k=1}^n \sqrt{2} I_{jk} \cos(k\omega t + \psi_{jk}), \quad j = a, b, c \quad (2)$$

where V_{jk} is the rms value of k -th voltage harmonic of phase angle θ_{jk} , I_{jk} is the rms value of k -th current harmonic of phase angle ψ_{jk} , n is the highest order of the voltage and current harmonics, and ω is angular velocity (equal to $2\pi f$, f being the basic frequency, i.e., $f=1/T$).

In the case of three-phase three-wire systems, a virtual star point is assumed for voltage reference, as suggested in some of the most known power theories [7,14,15,18]. Such virtual star point ensures the sum of the voltages equal to zero, i.e.: $v_a + v_b + v_c = 0$. These voltages can be calculated from measurable phase-to-phase voltages:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_{ab} \\ v_{bc} \\ v_{ca} \end{bmatrix} \quad (3)$$

Instantaneous voltages and line currents can be arranged in three-dimensional vectors \mathbf{v} and \mathbf{i} :

$$\mathbf{v} = [v_a \quad v_b \quad v_c]^T \quad (4)$$

$$\mathbf{i} = [i_a \quad i_b \quad i_c]^T \quad (5)$$

where T indicates vector or matrix transposition.

Application of the Lyon transformation to instantaneous voltage and current in a three-phase three-wire system can be presented by the following equations:

$$\underline{\mathbf{v}}_s = \underline{\mathbf{A}}^{-1} \mathbf{v} \quad (6)$$

$$\underline{\mathbf{i}}_s = \underline{\mathbf{A}}^{-1} \mathbf{i} \quad (7)$$

where $\underline{\mathbf{v}}_s$ and $\underline{\mathbf{i}}_s$ are three-dimensional vectors which contain voltage and current symmetrical components in the time domain:

$$\underline{\mathbf{v}}_s = [\underline{v}_{0'} \quad \underline{v}_{d'} \quad \underline{v}_{i'}]^T \quad (8)$$

$$\underline{\mathbf{i}}_s = [\underline{i}_{0'} \quad \underline{i}_{d'} \quad \underline{i}_{i'}]^T \quad (9)$$

In the previous expressions, $\underline{v}_{0'}$ and $\underline{i}_{0'}$ are instantaneous zero-sequence components, $\underline{v}_{d'}$ and $\underline{i}_{d'}$ are instantaneous positive-sequence components and $\underline{v}_{i'}$ and $\underline{i}_{i'}$ are instantaneous negative-sequence components.

Matrix $\underline{\mathbf{A}}$ and its inverse, used in the applications of Lyon transformation (6) and (7), are:

$$\underline{\mathbf{A}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{\alpha}^2 & \underline{\alpha} \\ 1 & \underline{\alpha} & \underline{\alpha}^2 \end{bmatrix} \quad (10)$$

$$\underline{A}^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{\alpha} & \underline{\alpha}^2 \\ 1 & \underline{\alpha}^2 & \underline{\alpha} \end{bmatrix} \quad (11)$$

where $\underline{\alpha} = e^{j2\pi/3}$.

Matrix \underline{A} has the following property [21,27]:

$$\underline{A}^T \underline{A}^* = \underline{A} \underline{A}^* = 3 \underline{A} \underline{A}^{-1} = 3[\mathbf{1}]_{3 \times 3} \quad (12)$$

where \underline{A}^* indicates the complex conjugate of matrix \underline{A} and $[\mathbf{1}]_{3 \times 3}$ is a three-dimensional identity matrix. T indicates nonconjugate (non-Hermitian) vector or matrix transpose operator.

Since the Lyon transformation is directly applied to real quantities, the obtained instantaneous zero-sequence components \underline{v}_0 and \underline{i}_0 are real quantities, i.e., $\underline{v}_0 = v_0$ and $\underline{i}_0 = i_0$. The obtained instantaneous positive-sequence components \underline{v}_d and \underline{i}_d and corresponding instantaneous negative-sequence components \underline{v}_i and \underline{i}_i are complex conjugate quantities, i.e., $\underline{v}_d = \underline{v}_i^*$ and $\underline{i}_d = \underline{i}_i^*$.

The instantaneous power transferred to the load shown in Fig.1 is:

$$p = \mathbf{v}^T \mathbf{i} = (\underline{A} \underline{v}_s)^T (\underline{A} \underline{i}_s)^* = \underline{v}_s^T \underline{A}^T \underline{A}^* \underline{i}_s^* = 3 \underline{v}_s^T \underline{i}_s^* \quad (13)$$

which means

$$p = 3(v_{0'} \underline{i}_{0'}^* + v_{d'} \underline{i}_{d'}^* + v_{i'} \underline{i}_{i'}^*) \quad (14)$$

The instantaneous apparent power s is defined as follows [4-6,8]:

$$s = \sqrt{(v_a^2 + v_b^2 + v_c^2)(i_a^2 + i_b^2 + i_c^2)} = \sqrt{(\mathbf{v}^T \mathbf{v})(\mathbf{i}^T \mathbf{i})} \quad (15)$$

The instantaneous apparent power s can be decomposed into two orthogonal components, as follows:

$$s^2 = p^2 + q^2 \quad (16)$$

where p is the instantaneous power and q is usually assigned to instantaneous reactive power [2,4-6].

The instantaneous power p determines the so-called power currents (arranged in vector \mathbf{i}_p) which instantaneously transfer energy to the load [4-10]:

$$\mathbf{i}_p = \frac{p}{\mathbf{v}^T \mathbf{v}} \mathbf{v} = \frac{\mathbf{v}^T \mathbf{i}}{\mathbf{v}^T \mathbf{v}} \mathbf{v} \quad (17)$$

Powerless currents (arranged in vector \mathbf{i}_q) result from the difference between actual currents and power currents [4-10]:

$$\mathbf{i}_q = \mathbf{i} - \mathbf{i}_p \quad (18)$$

Theoretically, powerless currents can be compensated immediately without energy storage capability of the compensator [2,5,9-11]. Such compensators are commonly known as "active filters" and are realised as PWM (Pulse Width Modulation) inverter-based switching compensators.

The following holds true for voltages and currents:

$$\mathbf{v}^T \mathbf{v} = v_a^2 + v_b^2 + v_c^2 = 3(v_{0'}^2 + v_{d'}^2 + v_{i'}^2) = 3\mathbf{v}_{s'}^T \mathbf{v}_{s'}^* \quad (19)$$

$$\mathbf{i}^T \mathbf{i} = i_a^2 + i_b^2 + i_c^2 = 3(i_{0'}^2 + i_{d'}^2 + i_{i'}^2) = 3\mathbf{i}_{s'}^T \mathbf{i}_{s'}^* \quad (20)$$

where $v_{0'}$, $v_{d'}$, and $v_{i'}$ are the absolute values of voltage symmetrical components, and $i_{0'}$, $i_{d'}$, and $i_{i'}$ are the absolute values of current symmetrical components.

By using (13) and (19), power currents can be defined in terms of instantaneous symmetrical components, as follows:

$$\mathbf{i}_p = \frac{p}{3\mathbf{v}_{s'}^T \mathbf{v}_{s'}^*} \mathbf{A} \mathbf{v}_{s'} = \frac{\mathbf{v}_{s'}^T \mathbf{i}_{s'}^*}{\mathbf{v}_{s'}^T \mathbf{v}_{s'}^*} \mathbf{A} \mathbf{v}_{s'} \quad (21)$$

The expression (21) gives the same currents as (17) which are power currents defined by instantaneous reactive power theories [4-10].

By using (19) and (20), the instantaneous apparent power s can also be defined in terms of instantaneous symmetrical components according to the following:

$$s = 3\sqrt{(v_{0'}^2 + v_{d'}^2 + v_{i'}^2)(i_{0'}^2 + i_{d'}^2 + i_{i'}^2)} \quad (22)$$

The zero sequence currents $i_{0'}$ are equal to zero, since the currents i_a , i_b and i_c have to fulfil Kirchhoff's law. Zero sequence voltages $v_{0'}$ are also equal to zero since the assumed virtual star for voltage reference ensures the sum of the voltages equal to zero. Therefore, zero sequence quantities can be omitted from expressions (14) and (22). The instantaneous power p , instantaneous apparent power s , and instantaneous reactive power q in terms of the instantaneous symmetrical components are:

$$p = 3(\underline{v}_{d'} \underline{i}_{d'}^* + \underline{v}_{i'} \underline{i}_{i'}^*) \quad (23)$$

$$s = 3\sqrt{(v_{d'}^2 + v_{i'}^2)(i_{d'}^2 + i_{i'}^2)} \quad (24)$$

$$q = 3j \cdot (\underline{v}_{i'} \underline{i}_{d'} - \underline{v}_{d'} \underline{i}_{i'}) \quad (25)$$

Since $Re\{\underline{v}_{i'} \underline{i}_{d'}\} = Re\{\underline{v}_{d'} \underline{i}_{i'}\}$, the instantaneous reactive power q defined by (25) is a real quantity. The previous expressions of powers (23), (24) and (25) avoid mistakes when the voltage reference differs from the virtual star point which ensures the sum of voltages equal to zero. Namely, the choice of voltage reference affect only the zero-sequence voltages. By omitting the zero-sequence voltages, the proposed definitions (23), (24) and (25) do not use the common content of voltages referred to an arbitrary reference [9]. In such a way, the same results are obtained for an arbitrary voltage reference as in the case when the virtual star point is chosen for voltage reference.

The instantaneous power p , instantaneous apparent power s , and instantaneous reactive power q for three-phase three-wire systems can also be expressed in terms of instantaneous symmetrical components derived from the Clarke transformation [2-4]:

$$p = v_{\alpha'} i_{\alpha'} + v_{\beta'} i_{\beta'} \quad (26)$$

$$s = \sqrt{(v_{\alpha'}^2 + v_{\beta'}^2)(i_{\alpha'}^2 + i_{\beta'}^2)} \quad (27)$$

$$q = v_{\beta'} i_{\alpha'} - v_{\alpha'} i_{\beta'} \quad (28)$$

where the instantaneous symmetrical components $v_{\alpha'}$, $v_{\beta'}$, $i_{\alpha'}$, and $i_{\beta'}$ are obtained from the following equations [9]:

$$\begin{bmatrix} v_{\alpha'} \\ v_{\beta'} \\ v_{0'} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad (29)$$

$$\begin{bmatrix} i_{\alpha'} \\ i_{\beta'} \\ i_{0'} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (30)$$

2.2. Three-phase four-wire system

In the case of three-phase four-wire systems, some of the most known power theories choose the neutral line for voltage reference and use three phase-to-neutral voltages for calculation of powers [15,19]. However, such approach does not result in a correct calculation of powers when voltages are unbalanced, i.e., in the presence of zero sequence voltages [8,9]. Also, the neutral line current contributing to line losses appears in the presence of voltage asymmetry, which should be considered.

The well-known concept for three-phase three-wire systems which used zero-sum voltages has to be extended to four-wire systems. Although the neutral line plays a special role in energy distribution systems, all conductors of the four-wire system should preferably be treated equally, and power definitions should use zero-sum voltages calculated for four conductors.

Fig.2 shows the load in a three-phase four-wire power system with a virtual star point assumed for voltage reference which ensures the sum of voltages equal to zero, i.e.: $v_a + v_b + v_c + v_n = 0$.

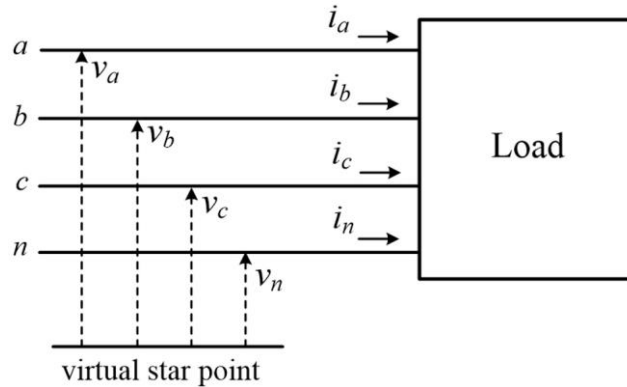


Fig. 2. Load in a three-phase four-wire system.

To calculate these zero-sum voltages, the following transformation of phase-to-neutral voltages v_{an} , v_{bn} and v_{cn} is used:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_n \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \\ -1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} v_{an} \\ v_{bn} \\ v_{cn} \end{bmatrix} \quad (31)$$

where v_a , v_b , v_c and v_n are the voltages against the virtual star point, i.e., the zero-sum voltages.

By using the zero-sum voltages, the instantaneous power p , the instantaneous apparent power s , and the modulus of the instantaneous reactive power q are defined as:

$$p = v_a i_a + v_b i_b + v_c i_c + v_n i_n \quad (32)$$

$$s = \sqrt{(v_a^2 + v_b^2 + v_c^2 + v_n^2)(i_a^2 + i_b^2 + i_c^2 + i_n^2)} \quad (33)$$

$$|q| = \sqrt{s^2 - p^2} \quad (34)$$

In the case of three-phase four-wire systems, symmetrical components are usually used to express asymmetrical phase-to-neutral voltages and phase line currents:

$$\begin{bmatrix} v_{an} \\ v_{bn} \\ v_{cn} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{\alpha}^2 & \underline{\alpha} \\ 1 & \underline{\alpha} & \underline{\alpha}^2 \end{bmatrix} \cdot \begin{bmatrix} \underline{v}_{0'} \\ \underline{v}_{d'} \\ \underline{v}_{i'} \end{bmatrix} \quad (35)$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{\alpha}^2 & \underline{\alpha} \\ 1 & \underline{\alpha} & \underline{\alpha}^2 \end{bmatrix} \cdot \begin{bmatrix} \underline{i}_{0'} \\ \underline{i}_{d'} \\ \underline{i}_{i'} \end{bmatrix} \quad (36)$$

By using (31) and (35), zero-sum voltages can be expressed in terms of the symmetrical components $\underline{v}_{0'}$, $\underline{v}_{d'}$, and $\underline{v}_{i'}$:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_n \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 1 & 1 \\ \frac{1}{4} & \underline{\alpha}^2 & \underline{\alpha} \\ \frac{1}{4} & \underline{\alpha} & \underline{\alpha}^2 \\ -\frac{3}{4} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \underline{v}_{0'} \\ \underline{v}_{d'} \\ \underline{v}_{i'} \end{bmatrix} \quad (37)$$

The neutral line current can also be expressed in terms of symmetrical components:

$$i_n = -i_a - i_b - i_c = -3\underline{i}_{0'} \quad (38)$$

When phase-to-neutral voltages are unbalanced, their zero sequence voltage components v_0 are not equal to zero. Also, in the presence of voltage or load asymmetry, zero sequence currents i_0 are also different from zero since the neutral wire provides a path for zero-sequence currents.

For three-phase four-wire systems, the instantaneous power (32) and instantaneous apparent power (33) can be expressed in terms of the instantaneous symmetrical components by using (37) and (38):

$$p = 3(\underline{v}_0 \cdot \underline{i}_0^* + \underline{v}_d \cdot \underline{i}_d^* + \underline{v}_i \cdot \underline{i}_i^*) \quad (39)$$

$$s = 3 \sqrt{\left(\frac{1}{4} v_0^2 + v_d^2 + v_i^2 \right) (4i_0^2 + i_d^2 + i_i^2)} \quad (40)$$

The previous expressions give the same results as (32) and (33), respectively.

2.3. Three-phase four-wire system considered as four-phase system

There is a more elegant approach to define powers for three-phase four-wire systems. The idea is to look at a three-phase system with a neutral wire as a four-phase system, since all the wires of a four-wire system should be treated equally. Therefore, symmetrical components for four-phase systems will be introduced.

The zero-sum voltages and line currents for three-phase four-wire systems can be expressed as instantaneous four-dimensional vectors \mathbf{v} and \mathbf{i} :

$$\mathbf{v} = [v_a \quad v_b \quad v_c \quad v_n]^T \quad (41)$$

$$\mathbf{i} = [i_a \quad i_b \quad i_c \quad i_n]^T \quad (42)$$

The application of the Lyon transformation to instantaneous voltage and current in a four-wire system can be represented by the following equations:

$$\underline{\mathbf{v}}_{s''} = \underline{\mathbf{B}}^{-1} \mathbf{v} \quad (43)$$

$$\underline{\mathbf{i}}_{s''} = \underline{\mathbf{B}}^{-1} \mathbf{i} \quad (44)$$

where $\underline{\mathbf{v}}_{s''}$ and $\underline{\mathbf{i}}_{s''}$ are four-dimensional vectors which contain voltage and current symmetrical components:

$$\underline{\mathbf{v}}_{s''} = [\underline{v}_0'' \quad \underline{v}_d'' \quad \underline{v}_{00}'' \quad \underline{v}_i'']^T \quad (45)$$

$$\underline{\mathbf{i}}_{s''} = [\underline{i}_0'' \quad \underline{i}_d'' \quad \underline{i}_{00}'' \quad \underline{i}_i'']^T \quad (46)$$

In the previous expressions, \underline{v}_0'' and \underline{i}_0'' are instantaneous zero-sequence components, \underline{v}_d'' and \underline{i}_d'' are instantaneous positive-sequence components, \underline{v}_{00}'' and \underline{i}_{00}'' are instantaneous pseudozero-sequence components, and \underline{v}_i'' and \underline{i}_i'' are instantaneous negative-sequence components [28]. The absolute values of instantaneous voltage symmetrical components are denoted as v_0'' , v_d'' , v_{00}'' , and v_i'' , while the absolute values of current symmetrical components are denoted as i_0'' , i_d'' , i_{00}'' , and i_i'' .

Matrix $\underline{\mathbf{B}}$ and its inverse, used in the applications of Lyon transformation (43) and (44), are:

$$\underline{\mathbf{B}} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \underline{\beta}^3 & \underline{\beta}^2 & \underline{\beta} \\ 1 & \underline{\beta}^2 & 1 & \underline{\beta}^2 \\ 1 & \underline{\beta} & \underline{\beta}^2 & \underline{\beta}^3 \end{bmatrix} \quad (47)$$

$$\underline{\mathbf{B}}^{-1} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \underline{\beta} & \underline{\beta}^2 & \underline{\beta}^3 \\ 1 & \underline{\beta}^2 & 1 & \underline{\beta}^2 \\ 1 & \underline{\beta}^3 & \underline{\beta}^2 & \underline{\beta} \end{bmatrix} \quad (48)$$

where $\underline{\beta} = e^{j2\pi/4} = j$.

Since $\underline{\mathbf{B}}^T \underline{\mathbf{B}}^* = \underline{\mathbf{B}} \underline{\mathbf{B}}^* = 4[\mathbf{1}]_{4 \times 4}$, where $[\mathbf{1}]_{4 \times 4}$ is a four-dimensional identity matrix, the following holds true:

$$\mathbf{v}^T \mathbf{i} = 4 \mathbf{v}_{\underline{s}''}^T \mathbf{i}_{\underline{s}''}^* \quad (49)$$

$$\mathbf{v}^T \mathbf{v} = 4 \mathbf{v}_{\underline{s}''}^T \mathbf{v}_{\underline{s}''}^* \quad (50)$$

$$\mathbf{i}^T \mathbf{i} = 4 \mathbf{i}_{\underline{s}''}^T \mathbf{i}_{\underline{s}''}^* \quad (51)$$

Zero sequence currents $\underline{i}_{0''}$ are equal to zero, since currents i_a, i_b, i_c and i_n have to fulfil Kirchhoff's law. Zero-sequence voltages $\underline{v}_{0''}$ are also equal to zero, because zero-sum voltages are used. Instantaneous positive-sequence components $\underline{v}_{d''}$ and $\underline{i}_{d''}$ and the corresponding instantaneous negative-sequence components $\underline{v}_{i''}$ and $\underline{i}_{i''}$ are complex conjugate quantities, i.e., $\underline{v}_{d''} = \underline{v}_{i''}^*$ and $\underline{i}_{d''} = \underline{i}_{i''}^*$. The remaining instantaneous pseudozero-sequence components $\underline{v}_{00''}$ and $\underline{i}_{00''}$ are real quantities, i.e., $\underline{v}_{00''} = v_{00''}$ and $\underline{i}_{00''} = i_{00''}$.

For three-phase four-wire systems, the instantaneous power p and instantaneous apparent power s in terms of the introduced instantaneous symmetrical components for four-phase systems are:

$$p = 4(\underline{v}_{d''} \underline{i}_{d''}^* + \underline{v}_{00''} \underline{i}_{00''}^* + \underline{v}_{i''} \underline{i}_{i''}^*) \quad (52)$$

$$s = 4\sqrt{(\underline{v}_{d''}^2 + \underline{v}_{00''}^2 + \underline{v}_{i''}^2)(\underline{i}_{d''}^2 + \underline{i}_{00''}^2 + \underline{i}_{i''}^2)} \quad (53)$$

The previous expressions give the same results as (39) and (40), respectively.

For three-phase four-wire systems, instantaneous reactive power behaves as instantaneous space-vector $\underline{q} = 4 \cdot \underline{v}_{s''} \times \underline{i}_{s''}$ with components $q_{d''}$, $q_{00''}$ and $q_{i''}$:

$$q_{d''} = 4j \cdot (\underline{v}_{00''} \underline{i}_{i''} - \underline{v}_{i''} \underline{i}_{00''}) \quad (54)$$

$$q_{00''} = 4j \cdot (\underline{v}_{i''} \underline{i}_{d''} - \underline{v}_{d''} \underline{i}_{i''}) \quad (55)$$

$$q_{i''} = 4j \cdot (\underline{v}_{d''} \underline{i}_{00''} - \underline{v}_{00''} \underline{i}_{d''}) \quad (56)$$

Since $Re\{\underline{v}_{00''}\dot{\underline{i}}_{\alpha''}-\underline{v}_{\beta''}\dot{\underline{i}}_{00''}\}=0$, $Re\{\underline{v}_{\beta''}\dot{\underline{i}}_{\alpha''}-\underline{v}_{\alpha''}\dot{\underline{i}}_{\beta''}\}=0$, and $Re\{\underline{v}_{\alpha''}\dot{\underline{i}}_{00''}-\underline{v}_{00''}\dot{\underline{i}}_{\alpha''}\}=0$, the instantaneous reactive power components $q_{\alpha''}$, $q_{00''}$ and $q_{\beta''}$ are real quantities.

The modulus of the instantaneous reactive power for three-phase four-wire systems in terms of its components is:

$$|q| = \sqrt{q_{\alpha''}^2 + q_{00''}^2 + q_{\beta''}^2} \quad (57)$$

The previous expression gives the same result as (34). The sign of the instantaneous reactive power for three-phase four-wire systems is disputable since it behaves as a space-vector [5].

The power currents (arranged in vector \underline{i}_p) are:

$$\underline{i}_p = \frac{p}{\underline{v}^T \underline{v}} \underline{v} = \frac{p}{4 \cdot \underline{v}_{s''}^T \underline{v}_{s''}^*} \underline{B} \underline{v}_{s''} = \frac{\underline{v}_{s''}^T \underline{i}_{s''}^*}{\underline{v}_{s''}^T \underline{v}_{s''}^*} \underline{B} \underline{v}_{s''} \quad (58)$$

The previous expression (58) gives the same power currents defined in [7,9,10]. The powerless currents \underline{i}_q are defined in the same way as for three-phase three-wire systems, according to (18).

The instantaneous symmetrical components derived from the Clarke transformation for four-phase systems can also be used [28]:

$$\begin{bmatrix} v_{\alpha''} \\ v_{\beta''} \\ v_{0''} \\ v_{00''} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_n \end{bmatrix} \quad (59)$$

$$\begin{bmatrix} i_{\alpha''} \\ i_{\beta''} \\ i_{0''} \\ i_{00''} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_n \end{bmatrix} \quad (60)$$

where pseudozero-sequence components ($v_{00''}$ and $i_{00''}$) also appear besides α -sequence ($v_{\alpha''}$ and $i_{\alpha''}$), β -sequence ($v_{\beta''}$ and $i_{\beta''}$), and zero-sequence components ($v_{0''}$ and $i_{0''}$).

The instantaneous power p , the instantaneous apparent power s , and the modulus of the instantaneous reactive power q for three-phase four-wire systems in terms of the instantaneous symmetrical components derived from the Clarke transformation for four-phase systems are:

$$p = v_{\alpha''} \dot{i}_{\alpha''}^* + v_{\beta''} \dot{i}_{\beta''}^* + v_{00''} \dot{i}_{00''}^* \quad (61)$$

$$s = \sqrt{(v_{\alpha''}^2 + v_{\beta''}^2 + v_{00''}^2)(i_{\alpha''}^2 + i_{\beta''}^2 + i_{00''}^2)} \quad (62)$$

$$|q| = \sqrt{q_{\alpha''}^2 + q_{\beta''}^2 + q_{00''}^2} \quad (63)$$

where the instantaneous reactive power components are:

$$q_{\alpha''} = v_{\beta''} i_{00''} - v_{00''} i_{\beta''} \quad (64)$$

$$q_{\beta''} = v_{00''} i_{\alpha''} - v_{\alpha''} i_{00''} \quad (65)$$

$$q_{00''} = v_{\alpha''} i_{\beta''} - i_{\beta''} i_{\alpha''} \quad (66)$$

Expressions (61), (62) and (63) render the same results as (52), (53) and (57).

3. Apparent, active and reactive powers

3.1. Three-phase three-wire system

The apparent power for three-phase three-wire systems can be calculated according to the following:

$$S = \sqrt{\frac{1}{T} \int_0^T (v_a^2 + v_b^2 + v_c^2) dt} \cdot \sqrt{\frac{1}{T} \int_0^T (i_a^2 + i_b^2 + i_c^2) dt} \quad (67)$$

which is the well-known Buchholz's definition of apparent power based on the product of collective rms values of voltages and currents [12]:

$$S = \sqrt{V_a^2 + V_b^2 + V_c^2} \cdot \sqrt{I_a^2 + I_b^2 + I_c^2} \quad (68)$$

where V_a , V_b , and V_c are rms voltage values, and I_a , I_b and I_c are rms current values.

Apparent power S can also be defined in terms of instantaneous symmetrical components for three-phase systems as follows:

$$S = 3 \cdot \sqrt{\frac{1}{T} \int_0^T (v_{d'}^2 + v_{i'}^2) dt} \cdot \sqrt{\frac{1}{T} \int_0^T (i_{d'}^2 + i_{i'}^2) dt} \quad (69)$$

Active power in terms of instantaneous symmetrical components for three-phase systems is defined as:

$$P = \frac{1}{T} \int_0^T p dt = 3 \cdot \frac{1}{T} \int_0^T (v_{d'} i_{d'}^* + v_{i'} i_{i'}^*) dt \quad (70)$$

For three-phase systems with non-sinusoidal voltages and currents, two reactive power definitions stand out: the Currents' Physical Component (CPC) theory [14] and Conservative Power Theory (CPT) [15]. By employing instantaneous symmetrical components for three-phase systems, both definitions can be obtained.

The expression for reactive power for three-phase three-wire systems in line with the CPC theory is derived using the instantaneous voltage and reactive current symmetrical components:

$$Q = 3 \cdot \sqrt{\frac{1}{T} \int_0^T (v_{d'}^2 + v_{i'}^2) dt} \cdot \sqrt{\frac{1}{T} \int_0^T (i_{rd'}^2 + i_{ri'}^2) dt} \quad (71)$$

The instantaneous reactive current symmetrical components $i_{rd'}$ and $i_{ri'}$ are obtained by summing the symmetrical components of reactive current harmonics:

$$i_{rd'} = \sum_{k=1}^n \frac{\frac{1}{T} \int_0^T (\dot{v}_{d'k} \dot{i}_{d'k}^* + \dot{v}_{i'k} \dot{i}_{i'k}^*) dt}{\frac{1}{T} \int_0^T (\dot{v}_{d'k}^2 + \dot{v}_{i'k}^2) dt} \cdot \dot{v}_{d'k} \quad (72)$$

$$i_{ri'} = \sum_{k=1}^n \frac{\frac{1}{T} \int_0^T (\dot{v}_{d'k} \dot{i}_{d'k}^* + \dot{v}_{i'k} \dot{i}_{i'k}^*) dt}{\frac{1}{T} \int_0^T (\dot{v}_{d'k}^2 + \dot{v}_{i'k}^2) dt} \cdot \dot{v}_{i'k} \quad (73)$$

where $\underline{v}_{d'k}$ and $\underline{i}_{d'k}$ are instantaneous positive-sequence components for k -th voltage and current harmonics, $\underline{v}_{i'k}$ and $\underline{i}_{i'k}$ are instantaneous negative-sequence components for k -th voltage and current harmonics, $\dot{v}_{d'k} = \frac{dv_{d'k}}{dt}$ and $\dot{v}_{i'k} = \frac{dv_{i'k}}{dt}$.

The expression for reactive power for three-phase three-wire systems in line with CPT is also derived in terms of instantaneous voltage and current symmetrical components:

$$Q_C = 3 \cdot \frac{\sqrt{\frac{1}{T} \int_0^T (v_{d'}^2 + v_{i'}^2) dt}}{\sqrt{\frac{1}{T} \int_0^T (\tilde{v}_{d'}^2 + \tilde{v}_{i'}^2) dt}} \cdot \frac{1}{T} \int_0^T (\tilde{v}_{d'} \dot{i}_{d'}^* + \tilde{v}_{i'} \dot{i}_{i'}^*) dt \quad (74)$$

where $\tilde{v}_{d'} = \int \underline{v}_{d'} dt$ and $\tilde{v}_{i'} = \int \underline{v}_{i'} dt$ are unbiased integrals of voltages $\underline{v}_{d'}$ and $\underline{v}_{i'}$. Since $Im\{\tilde{v}_{d'} \dot{i}_{d'}^* + \tilde{v}_{i'} \dot{i}_{i'}^*\} = 0$, expression (74) gives a real quantity.

The reactive power definition according to CPT represents the extension of inductive reactive power definition according to Kusters and Moore to three-phase three-wire systems [17,20]. The reactive power definitions according to Kusters and Moore are based on optimum shunt reactive elements (capacitors or inductors) used for the minimization of the rms value of the current in the line supplying the load [13]. However, in electrical circuits under non-sinusoidal conditions, optimal compensation using simple reactive elements does not eliminate all the non-active power, nor reactive power. Therefore, the reactive power defined by the CPT does not represent the reactive power of the load but the reactive power of the optimal compensator.

Non-active power is defined in [22,23] as:

$$N = \sqrt{S^2 - P^2} \quad (75)$$

Non-active power can be considered as useless power which causes losses in feeding conductors without transferring energy to the load.

Active currents (arranged in vector \mathbf{i}_a) are defined as line currents of minimal rms values (i.e., the minimal line currents) which provide the same active power P as actual currents:

$$\mathbf{i}_a = \frac{P}{\frac{1}{T} \int_0^T \mathbf{v}^T \mathbf{v} dt} \mathbf{v} = \frac{P}{3 \cdot \frac{1}{T} \int_0^T \mathbf{v}_s^T \mathbf{v}_s^* dt} \underline{\mathbf{A}} \mathbf{v}_s \quad (76)$$

Non-active currents (arranged in vector \mathbf{i}_n) result from the difference between actual currents and active currents:

$$\mathbf{i}_n = \mathbf{i} - \mathbf{i}_a \quad (77)$$

The power factor can be defined by using active and apparent powers or by using the rms values of active and actual currents:

$$PF = \frac{P}{S} = \frac{\sqrt{\frac{1}{T} \int_0^T \mathbf{i}_a^T \mathbf{i}_a dt}}{\sqrt{\frac{1}{T} \int_0^T \mathbf{i}^T \mathbf{i} dt}} = \frac{\sqrt{I_{aa}^2 + I_{ba}^2 + I_{ca}^2}}{\sqrt{I_a^2 + I_b^2 + I_c^2}} \quad (78)$$

where I_{aa} , I_{ba} , and I_{ca} are the rms values of active currents (76) and I_a , I_b , and I_c are the rms values of actual line currents.

3.2. Three-phase four-wire system

By using zero-sum voltages, the apparent power for three-phase four-wire systems can be calculated as follows:

$$S = \sqrt{\frac{1}{T} \int_0^T (v_a^2 + v_b^2 + v_c^2 + v_n^2) dt} \cdot \sqrt{\frac{1}{T} \int_0^T (i_a^2 + i_b^2 + i_c^2 + i_n^2) dt} \quad (79)$$

The apparent power S for three-phase four-wire systems can be expressed in terms of instantaneous symmetrical components for three-phase systems as

$$S = 3 \cdot \sqrt{\frac{1}{T} \int_0^T \left(\frac{1}{4} v_0^2 + v_d^2 + v_i^2 \right) dt} \cdot \sqrt{\frac{1}{T} \int_0^T (4i_0^2 + i_d^2 + i_i^2) dt} \quad (80)$$

or in terms of instantaneous symmetrical components for four-phase systems as

$$S = 4 \cdot \sqrt{\frac{1}{T} \int_0^T (v_d^2 + v_{00}^2 + v_i^2) dt} \cdot \sqrt{\frac{1}{T} \int_0^T (i_d^2 + i_{00}^2 + i_i^2) dt} \quad (81)$$

The active power for three-phase four-wire systems can be expressed in terms of instantaneous symmetrical components for three-phase systems as

$$P = \frac{1}{T} \int_0^T p \, dt = 3 \cdot \frac{1}{T} \int_0^T (\underline{v}_{0'} \underline{i}_{0'}^* + \underline{v}_{d'} \underline{i}_{d'}^* + \underline{v}_{i'} \underline{i}_{i'}^*) dt \quad (82)$$

or in terms of instantaneous symmetrical components for four-phase systems as

$$P = \frac{1}{T} \int_0^T p \, dt = 4 \cdot \frac{1}{T} \int_0^T (\underline{v}_{d''} \underline{i}_{d''}^* + \underline{v}_{00''} \underline{i}_{00''}^* + \underline{v}_{i''} \underline{i}_{i''}^*) dt \quad (83)$$

The expression for reactive power for three-phase four-wire systems can be derived using the instantaneous voltage and reactive current symmetrical components for three-phase systems:

$$Q = 3 \cdot \sqrt{\frac{1}{T} \int_0^T \left(\frac{1}{4} v_{0'}^2 + v_{d'}^2 + v_{i'}^2 \right) dt} \cdot \sqrt{\frac{1}{T} \int_0^T (4i_{r0'}^2 + i_{rd'}^2 + i_{ri'}^2) dt} \quad (84)$$

The instantaneous reactive current symmetrical components $i_{r0'}$, $i_{rd'}$ and $i_{ri'}$ are obtained by summing the symmetrical components of reactive current harmonics:

$$\underline{i}_{r0'} = \sum_{k=1}^n \frac{\frac{1}{T} \int_0^T (\dot{\underline{v}}_{0k} \underline{i}_{0k}^* + \dot{\underline{v}}_{dk} \underline{i}_{dk}^* + \dot{\underline{v}}_{ik} \underline{i}_{ik}^*) dt}{\frac{1}{T} \int_0^T \left(\frac{1}{4} \dot{v}_{0k}^2 + \dot{v}_{dk}^2 + \dot{v}_{ik}^2 \right)} \cdot \frac{1}{4} \dot{v}_{0k} \quad (85)$$

$$\underline{i}_{rd'} = \sum_{k=1}^n \frac{\frac{1}{T} \int_0^T (\dot{\underline{v}}_{0k} \underline{i}_{0k}^* + \dot{\underline{v}}_{dk} \underline{i}_{dk}^* + \dot{\underline{v}}_{ik} \underline{i}_{ik}^*) dt}{\frac{1}{T} \int_0^T \left(\frac{1}{4} \dot{v}_{0k}^2 + \dot{v}_{dk}^2 + \dot{v}_{ik}^2 \right)} \cdot \dot{\underline{v}}_{dk} \quad (86)$$

$$\underline{i}_{ri'} = \sum_{k=1}^n \frac{\frac{1}{T} \int_0^T (\dot{\underline{v}}_{0k} \underline{i}_{0k}^* + \dot{\underline{v}}_{dk} \underline{i}_{dk}^* + \dot{\underline{v}}_{ik} \underline{i}_{ik}^*) dt}{\frac{1}{T} \int_0^T \left(\frac{1}{4} \dot{v}_{0k}^2 + \dot{v}_{dk}^2 + \dot{v}_{ik}^2 \right)} \cdot \dot{\underline{v}}_{ik} \quad (87)$$

The expression for reactive power for three-phase four-wire systems is also derived using the instantaneous voltage and reactive current symmetrical components for four-phase systems:

$$Q = 4 \cdot \sqrt{\frac{1}{T} \int_0^T (v_{d''}^2 + v_{00''}^2 + v_{i''}^2) dt} \cdot \sqrt{\frac{1}{T} \int_0^T (i_{rd''}^2 + i_{r00''}^2 + i_{ri''}^2) dt} \quad (88)$$

The instantaneous reactive current symmetrical components $i_{rd''}$, $i_{r00''}$, and $i_{ri''}$ are obtained by summing the symmetrical components of reactive current harmonics:

$$\underline{i}_{\text{d}''} = \sum_{k=1}^n \frac{\frac{1}{T} \int_0^T (\dot{v}_{\text{d}''k} \underline{i}_{\text{d}''k}^* + v_{00''k} \underline{i}_{00''k}^* + \dot{v}_{\text{i}''k} \underline{i}_{\text{i}''k}^*) dt}{\frac{1}{T} \int_0^T (\dot{v}_{\text{d}''k}^2 + v_{00''k}^2 + \dot{v}_{\text{i}''k}^2) dt} \cdot \dot{v}_{\text{d}''k} \quad (89)$$

$$\underline{i}_{\text{r}00''} = \sum_{k=1}^n \frac{\frac{1}{T} \int_0^T (\dot{v}_{\text{d}''k} \underline{i}_{\text{d}''k}^* + v_{00''k} \underline{i}_{00''k}^* + \dot{v}_{\text{i}''k} \underline{i}_{\text{i}''k}^*) dt}{\frac{1}{T} \int_0^T (\dot{v}_{\text{d}''k}^2 + v_{00''k}^2 + \dot{v}_{\text{i}''k}^2) dt} \cdot \dot{v}_{00''k} \quad (90)$$

$$\underline{i}_{\text{i}''} = \sum_{k=1}^n \frac{\frac{1}{T} \int_0^T (\dot{v}_{\text{d}''k} \underline{i}_{\text{d}''k}^* + v_{00''k} \underline{i}_{00''k}^* + \dot{v}_{\text{i}''k} \underline{i}_{\text{i}''k}^*) dt}{\frac{1}{T} \int_0^T (\dot{v}_{\text{d}''k}^2 + v_{00''k}^2 + \dot{v}_{\text{i}''k}^2) dt} \cdot \dot{v}_{\text{i}''k} \quad (91)$$

where $\underline{v}_{\text{d}''k}$ and $\underline{i}_{\text{d}''k}$ are instantaneous positive-sequence components for k -th voltage and current harmonics, $\underline{v}_{00''k}$ and $\underline{i}_{00''k}$ are instantaneous pseudozero components for k -th voltage and current harmonics, $\underline{v}_{\text{i}''k}$ and $\underline{i}_{\text{i}''k}$ are instantaneous negative-sequence components for k -th voltage and current harmonics, $\dot{v}_{\text{d}''k} = \frac{dv_{\text{d}''k}}{dt}$, $\dot{v}_{00''k} = \frac{dv_{00''k}}{dt}$ and $\dot{v}_{\text{i}''k} = \frac{dv_{\text{i}''k}}{dt}$.

Non-active power for three-phase four-wire systems is defined in the same way as for three-wire systems, according to (75).

Active currents (arranged in vector \underline{i}_{a}) are defined as:

$$\underline{i}_{\text{a}} = \frac{P}{\frac{1}{T} \int_0^T \underline{v}^T \underline{v} dt} \underline{v} = \frac{P}{4 \cdot \frac{1}{T} \int_0^T \underline{v}_{\text{s}}^T \underline{v}_{\text{s}}^* dt} \underline{B} \underline{v}_{\text{s}} \quad (92)$$

Non-active currents are defined according to (77).

The power factor for three-phase four-wire systems is defined as:

$$PF = \frac{P}{S} = \frac{\sqrt{\frac{1}{T} \int_0^T \underline{i}_{\text{a}}^T \underline{i}_{\text{a}} dt}}{\sqrt{\frac{1}{T} \int_0^T \underline{i}^T \underline{i} dt}} = \frac{\sqrt{I_{\text{a}a}^2 + I_{\text{b}a}^2 + I_{\text{c}a}^2 + I_{\text{n}a}^2}}{\sqrt{I_{\text{a}}^2 + I_{\text{b}}^2 + I_{\text{c}}^2 + I_{\text{n}}^2}} \quad (93)$$

where $I_{\text{a}a}$, $I_{\text{b}a}$, $I_{\text{c}a}$ and $I_{\text{n}a}$ are the rms values of active currents (92) and I_{a} , I_{b} , I_{c} and I_{n} are the rms values of actual line currents.

4. Simulation results

Here, simulation examples show how the proposed definitions can be applied for determining power quantities. Fig.3 shows the example of a three-phase supply of a single-phase load [14]. The load and its supply are simplified to the structure with a loss-less transformer and parameters shown in Fig.3. The reactive power of the load at the fundamental frequency can be entirely compensated by turning on the switch S and connecting the capacitor, as shown in Fig.3. In that case, the remaining amount of reactive power results from the presence of fifth harmonic.

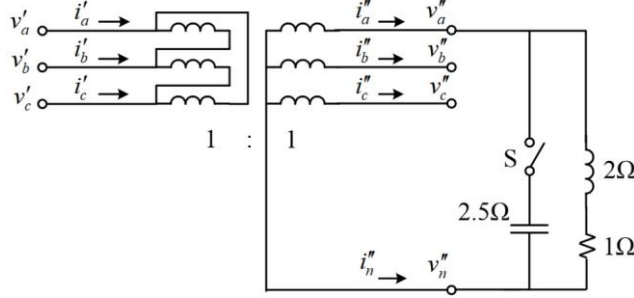


Fig. 3. Load in a three-phase four-wire system.

The following unbalanced source voltages for a three-phase three-wire system on the supply side of the transformer have been assumed:

$$v_{ar} = \sqrt{2} (220 \cos(\omega_1 t) + 10 \cos(5\omega_1 t)) \quad (94)$$

$$v_{br} = \sqrt{2} \left(220 \cos\left(\omega_1 t - \frac{2\pi}{3}\right) + 10 \cos\left(5\omega_1 t - \frac{10\pi}{3}\right) \right) \quad (95)$$

$$v_{cr} = \sqrt{2} \left(55 \cos\left(\omega_1 t + \frac{2\pi}{3}\right) + 2.5 \cos\left(5\omega_1 t + \frac{10\pi}{3}\right) \right) \quad (96)$$

The assumed source voltages are referred to an arbitrary reference r and they are unbalanced since the voltage drop of 75% in the third phase line has been assumed.

To calculate powers for a three-phase three-wire system on the supply side of the transformer, the voltages against the virtual star point, i.e., the zero-sum voltages v'_a , v'_b and v'_c shown in Fig.4(a) have been calculated according to:

$$\begin{bmatrix} v'_a \\ v'_b \\ v'_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} v_{ar} \\ v_{br} \\ v_{cr} \end{bmatrix} \quad (97)$$

First, the system shown in Fig.3 is analysed before connecting the capacitor. The line currents on the source side (three-phase three-wire system) and the corresponding power currents calculated according to (17) or (21) are also shown in Fig.4(a). The instantaneous power p , instantaneous apparent power s and instantaneous reactive power q on the source side, calculated according to (23), (24) and (25), are shown in Fig.4(b). The same quantities can also be obtained using Clarke transformation, according to (26), (27) and (28).

Due to the assumed Dyn transformer connection, the voltages on the load side are balanced ($v''_a + v''_b + v''_c = 0$, $v''_a = v'_a - v'_b$, $v''_b = v'_b - v'_c$, $v''_c = v'_c - v'_a$ and $v''_n = 0$). These voltages are shown in Fig.5(a). The line currents on the load side and

the corresponding power currents calculated according to (58) are also shown Fig.5(a). The instantaneous power p , instantaneous apparent power s and the modulus of instantaneous reactive power q on the load side, calculated according to (52), (53) and (57), are shown in Fig.5(b). The quantities shown in Fig.5(b) can also be obtained by using Clarke transformation for four-phase systems, according to (61), (62) and (63), or by using the hyper-space vector transformation [8,9].

Power currents are the aim of time-instantaneous compensation (compensation without energy storage). These currents have smaller rms values, but they are more distorted than actual currents. Power currents are time-dependent on voltages but also on actual currents, which makes them unsuitable as compensation objectives. Differently from power currents, active currents are proportional only to voltages. Besides that, active currents minimize losses in feeding conductors providing the same active power P as actual currents. By assuming that active currents are the aim of compensation, the load will be compensated to a unity power factor [10,11,16,23]. Active currents on the source side calculated according to (76) are shown in Fig.6(a). Active currents on the load side calculated according to (92) are shown in Fig.6(b). Active currents appear in all line conductors although a single-phase load is considered.

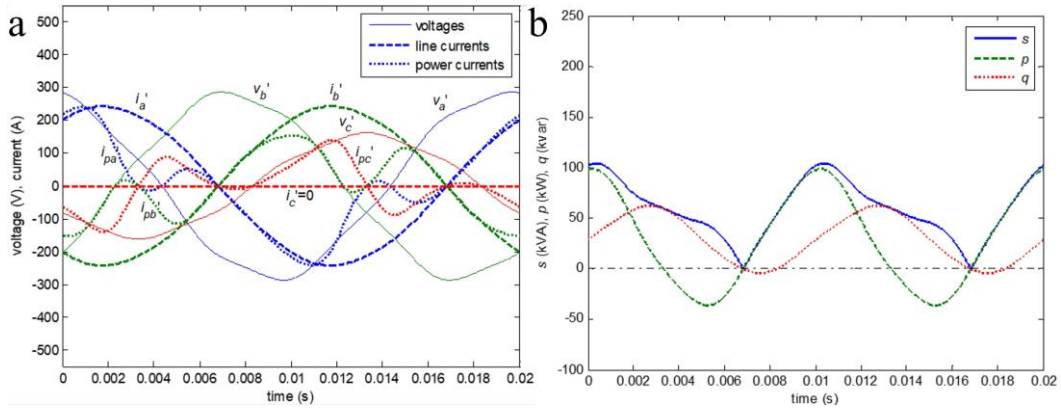


Fig. 4. (a) Voltages, line currents and power currents on the source side before connecting the capacitor; (b) Instantaneous power, instantaneous apparent power, and instantaneous reactive power on the source side before connecting the capacitor.

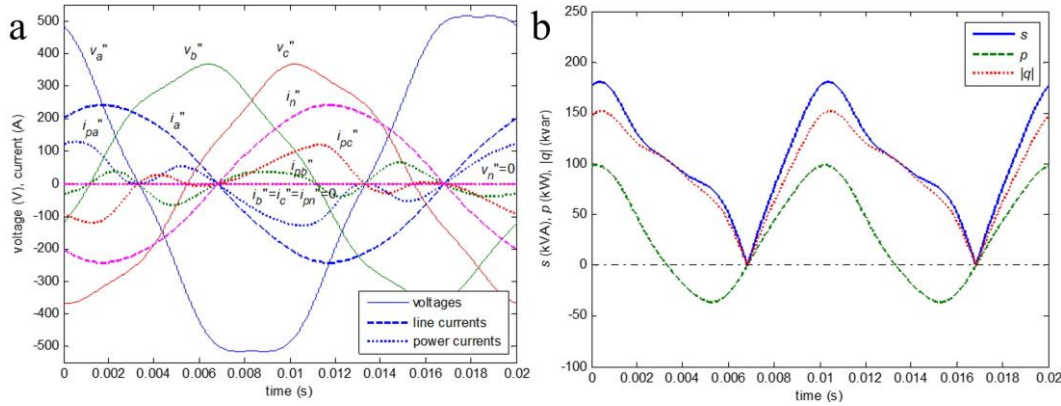


Fig. 5. (a) Voltages, line currents and power currents on the load side before connecting the capacitor; (b) Instantaneous power, instantaneous apparent power, and instantaneous reactive power on the load side before connecting the capacitor.

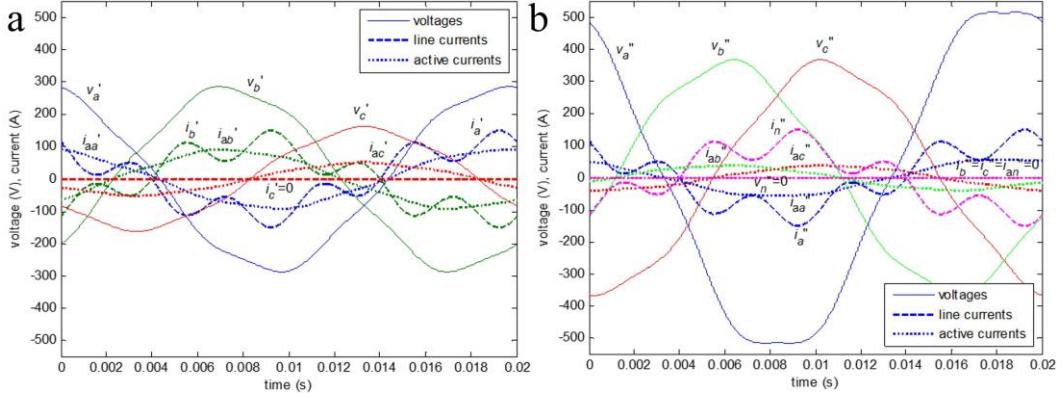


Fig. 6. (a) Voltages, active currents, and line currents on the source side (three-phase three-wire system) after connecting the capacitor; (b) Voltages, active currents and line currents on the load side (three-phase four-wire system) after connecting the capacitor.

Fig.6(a) and Fig.6(b) also show the line currents after connecting the capacitor and compensating the reactive power of the load at the fundamental frequency. The obtained line currents after the compensation of the load are reduced and oscillate around the active currents which are the optimal currents providing a unity power factor.

For the system shown in Fig.3, the values of the apparent and reactive powers on the source side (three-phase three-wire system), calculated according to various definitions before and after connecting the capacitor, are shown in Table 1. The same powers are also calculated for the load side (three-phase four-wire system) and they are shown in Table 2. All the considered theories give the same value of active power $P=29.05$ kW.

In the case of a three-phase three-wire system on the source side, the considered power definitions [14-23] use zero-sum voltages. Therefore, all the considered definitions give the same value of the apparent power as the proposed apparent power definition in terms of instantaneous symmetrical components (69). However, the considered theories [14,15] give various values of reactive power after connecting the capacitor that compensates reactive power of the load at the fundamental frequency. The CPT [15] gives a negative value of inductive reactive power for the load that is compensated to zero value of reactive power at the fundamental frequency, which means that the remaining reactive property of the load at the frequency of the fifth harmonic is not identified correctly. The proposed reactive power definition in terms of instantaneous symmetrical components (71) gives the same value of reactive power as the CPC theory [14].

Table 1. Apparent and reactive powers for the three-phase three-wire system.

	Without capacitor		With capacitor	
	S (kVA)	Q (kvar)	S (kVA)	Q (kvar)
IEEE Standard [22]	72.68		35.41	
DIN Standard [23]	72.68		35.41	
CPC theory [14]	72.68	58.15	35.41	12.56
CPT [15]	72.68	58.15	35.41	-0.11
Proposed definitions	72.68	58.15	35.41	12.56

Table 2. Apparent and reactive powers for the three-phase four-wire system with balanced voltages.

	Without capacitor		With capacitor	
	S (kVA)	Q (kvar)	S (kVA)	Q (kvar)
IEEE Standard [22]	125.89		61.33	
DIN Standard [23]	125.89		61.33	
CPC theory [14]	89.02	58.15	43.37	12.56
CPT [15]	89.02	58.15	43.37	-0.11
Proposed definitions	125.89	58.15	61.33	12.56

On the load side, the voltages of the considered three-phase four-wire system are balanced, due to the assumed Dyn transformer connection. The proposed apparent power definitions in terms of instantaneous symmetrical components

(80) and (81) give the same value of the apparent power obtained by IEEE and DIN standards. However, the considered theories [14,15] give the smaller values of apparent power compared to standards [22,23] because these theories omit the contribution of neutral line currents to the collective rms value of the currents. Again, the proposed reactive power definitions in terms of the instantaneous symmetrical components (84) and (88) give the same value of reactive power as the CPC theory [14].

In order to have unbalanced source voltages supplying the unbalanced load in a three-phase four-wire system, the load shown in Fig.3 is directly supplied by source voltages (94), (95) and (96) which are referred to the neutral line ($v_{ar}=v_{an}$, $v_{br}=v_{bn}$, $v_{cr}=v_{cn}$, $v_n=0$). In this case, the load is connected between the phase lines a and b and supplied by phase-to-phase voltage v_{ab} . The lines c and n are unloaded. Values of the apparent and reactive powers, calculated according to various definitions before and after connecting the capacitor, are shown in Table 3.

Table 3. Apparent and reactive powers for the three-phase four-wire system with unbalanced voltages.

	Without capacitor		With capacitor	
	S (kVA)	Q (kvar)	S (kVA)	Q (kvar)
IEEE Standard [22]	74.48		36.28	
DIN Standard [23]	73.59		35.85	
CPC theory [14]	76.23	58.15	37.14	12.56
CPT [15]	76.23	58.15	37.14	-0.11
Proposed definitions	73.59	58.15	35.85	12.56

As can be seen from Table 3, there is a slight difference between the values of the apparent power calculated according to DIN and IEEE standards. The proposed apparent power definitions in terms of instantaneous symmetrical components (80) and (81) give the same value of the apparent power obtained by DIN Standard. The considered theories [14,15] give different values of apparent power compared to standards because these theories use phase-to-neutral voltages which are unbalanced in this case. These theories give the unchanged values of reactive power in all considered cases, just as the value of active power remains unchanged. This is expected since the voltage applied to load terminals is the same and the load impedance and compensating capacitance are unchanged in all considered cases. Consequently, the CPT [15] does not give the correct value for the reactive power of the load that is compensated to zero value of reactive power at the fundamental frequency. As in previous cases, the proposed reactive power definitions in terms of the instantaneous symmetrical components (84) and (86) again give the same value of reactive power as the CPC theory [14].

5. Conclusion

The set of power definitions based on instantaneous symmetrical components has been proposed in this paper. The proposed definitions can be used for time-instantaneous and time-average compensation. The instantaneous and average powers are defined using the same mathematical framework. By applying the Lyon transformation to three-phase voltages and currents, instantaneous symmetrical components are obtained and used to define the powers for three-phase power systems with non-sinusoidal and unbalanced voltages and currents. To provide the correct calculation of powers for unbalanced three-phase four-wire systems, the instantaneous symmetrical component for the four-phase systems has been introduced and used for power definitions. The proposed definitions avoid common mistakes caused by erroneous treatment of zero-sequence voltages. In the case of a three-phase three-wire system, the proposed definitions give the same values of powers as the most known power definitions. In the case of a three-phase four-wire system, the proposed definitions give the same value of instantaneous reactive power as the methods based on Clarke transformation for four-phase systems and hyper-space vector transformation. Also, in the case of a three-phase four-wire system, the proposed definitions give the same value of the apparent power as the DIN Standard and the same value of reactive power as the Currents' Physical Components (CPC) theory.

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References

- [1] He J, Liang B, Li YW, C. Wang C. Simultaneous microgrid voltage and current harmonics compensation using coordinated control of dual-interfacing converters. *IEEE Trans. Power Electron.* 2017; **32**: 2647-2660.
- [2] Akagi H, Kanazawa Y, Nabae A. Instantaneous reactive power compensators comprising switching devices without energy storage components. *IEEE Trans Ind Appl IA* 1984; **20**: 625-630.
- [3] Ferrero A, Supeti-Furga G. A new approach to the definition of power components in three-phase systems under nonsinusoidal conditions. *IEEE Trans Instrum Meas* 1991; **40**:568-577.
- [4] Willems J. A new interpretation of the Akagi-Nabae power components for nonsinusoidal three-phase situations. *IEEE Trans Power Deliv* 1992; **41**: 523-527.
- [5] Peng FZ, Lai JS. Generalized instantaneous reactive power theory for three-phase systems. *IEEE Trans Instrum Meas.* 1996; **45**: 293-297.
- [6] Rosseto L, Tenti P. Evaluation of instantaneous power terms in multi-phase systems: techniques and application to power-conditioning equipment. *International Trans on Electrical Energy Systems* 1998; **4**: 469-474.
- [7] Depenbrock M. The FBD-method a generally applicable tool for analyzing power relations. *IEEE Trans Power Syst* 1993; **8**: 381-387.
- [8] Depenbrock M, Staudt V. Hyper space vectors: A new four-quantity-extension of space-vector theory. *International Trans on Electrical Energy Systems* 1998; **8**: 241-248.
- [9] Staudt V, Wrede H. Compensation strategies for four-conductor systems. *L' Energia Elettrica* 2004; **81**: 182-189.
- [10] Salmeron P, Montano L. Compensation in nonsinusoidal, unbalanced three-phase four-wire systems with active power conditioner. *IEEE Trans Power Deliv* 2004; **19**: 1968-1974.
- [11] Šekara T, Mikulović J. Optimal non-active power compensation under non-sinusoidal conditions. *Electrical Engineering* 2009; **88**: 423-429.
- [12] Emanuel AE. The Buchholz-Goodhue apparent power definition: the practical approach for nonsinusoidal and unbalanced systems. *IEEE Trans Power Deliv* 1998; **13**: 344-350.
- [13] Kusters NL, Moore WJM. On the definition of reactive power under nonsinusoidal conditions. *IEEE Trans Pow Appr Syst PAS* 1980; **99**: 1845-1854.
- [14] Czarnecki L. Currents' Physical Component (CPC) in circuits with nonsinusoidal voltages and currents, part 2: three phase three wire linear circuits. *Elect Power Qual Util J.* 2006; **12**: 1-13.
- [15] Tenti P, Mattavelli P, Paredes MHK. Conservative power theory, sequence components and accountability in smart grids. *Proc 10th Conf Int Sch Nonsinusoidal Curr Compens*, Poland, 2010.
- [16] Šekara T, Mikulović J, Djurišić Ž. Optimal reactive compensators in power systems under asymmetrical and nonsinusoidal conditions. *IEEE Trans Power Deliv* 2008; **23**: 974-984.
- [17] Mikulović J, Šekara T. A new reactive power definition based on the minimization of the load non-reactive currents. *Int Sch on Nonsinusoidal Curr and Compens*, Poland, 2015.
- [18] Czarnecki L, Bhattacharai P. Currents' Physical Components (CPC) in three-phase systems with asymmetrical voltage. *Przegląd Elektrotechniczny* 2015; **91**: 40-47.
- [19] Czarnecki LS, Haley PM. Power properties of four-wire systems at nonsinusoidal supply voltage. *IEEE Trans on Power Delivery* 2016; **31**: 513-521.
- [20] Czarnecki L. Critical comments on the Conservative Power Theory (CPT). *International School on Nonsinusoidal Currents and Compensation*, Lagow, Poland, 2015.
- [21] Mikulović J, Škrbić B, Đurišić Ž. Power definitions for polyphase systems based on Fortescue's symmetrical components. *International Journal of Electrical Power and Energy Systems* 2018; **98**: 455-462.
- [22] IEEE Standard 1459: 2010. Definitions for the measurement of electric power quantities under sinusoidal, non-sinusoidal, balanced or unbalanced conditions.
- [23] Späth H. A general purpose definition of active current and non-active power based on German standard DIN 40110. *Electrical Engineering* 2007; **89**: 167-175.
- [24] Akagi H, Watanabe EH, Aredes M. *Instantaneous power theory and applications to power conditioning*. 2nd ed. IEEE Press, Wiley, 2017.
- [25] Fortescue CL. Method of symmetrical co-ordinates applied to the solution of polyphase networks. *Proc Am Inst Electr Eng* 1918; **37**: 629-716.
- [26] Lyon WV. *Transient analysis of alternating current machinery*. New York: Technology Press of Massachusetts Institute of Technology, and Wiley, 1954.
- [27] Grandi G, Serra G, Tani A. General analysis of multi-phase systems based on space vector approach. *Int Power Electronics and Motion Control Conference EPE-PEMC*, Portorož, Slovenia, 2006.
- [28] Torre F, Leva S, Morando A. Symmetrical and Clarke-Park transformations for four-phase systems. *Compel Journal* 2008; **27**: 1370-1386.
- [29] Paap GC. Symmetrical components in the time domain and their application to power network calculations. *IEEE Trans Power Syst* 2000; **15**: 522-528.
- [30] Chen CC, Hsu YY. A novel approach to the design of a shunt active filter for an unbalanced three-phase four-wire system under nonsinusoidal conditions. *IEEE Trans Power Deliv* 2000; **15**: 1258-1264.
- [31] Irvani MR, Karimi-Ghartemani M. Online estimation of steady state and instantaneous symmetrical components. *IEE Proc.-Gener. Transm. Distrib.* 2003; **150**: 616 - 622.