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► **To cite this version:**

Maria-Isabel Restrepo, Louis-Martin Rousseau, Jonathan Vallée. Home Healthcare Integrated Staffing and Scheduling. Omega, 2020. hal-03609450

HAL Id: hal-03609450

<https://hal.science/hal-03609450>

Submitted on 15 Mar 2022

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Home Healthcare Integrated Staffing and Scheduling

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Abstract

Workforce planning for home healthcare represents an important and challenging task involving complex factors associated with labor regulations, caregivers' preferences, and demand uncertainties. This task is done manually by most home care agencies, resulting in long planning times and suboptimal decisions that usually fail to meet the health needs of the population, to minimize operating costs, and to retain current caregivers. Motivated by these challenges, we present a two-stage stochastic programming model for employee staffing and scheduling in home healthcare. In this model, first-stage decisions correspond to the staffing and scheduling of caregivers in geographic districts. Second-stage decisions are related to the temporary reallocation of caregivers to neighboring districts, to contact caregivers to work on a day-off, and to allow under- and over-covering of demand. The proposed model is tested on real-world instances, where we evaluate the impact on costs, caregiver utilization, and service level by using different recourse actions. Results show that when compared with a deterministic model, the two-stage stochastic model leads to significant cost savings as staff dimensioning and scheduling decisions are more robust to accommodate changes in demand. Moreover, these results suggest that flexibility in terms of use of recourse actions is highly valuable as it helps to further improve costs, service level, and caregiver utilization.

Keywords: Staffing and scheduling, Home healthcare, Two-stage stochastic programming, Context-free grammars

1. Introduction

Home healthcare refers to any type of care given to a patient at his own home rather than in a healthcare facility like a hospital or a clinic. Caregivers (e.g., personal support workers,

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4 nurses, and therapists) meet the patients' needs by bringing all necessary equipments at their
5 homes and therein provide care. This activity increases the quality of life for the patients,
6 as they are allowed to remain at home where they are most comfortable. Moreover, it yields
7 relevant cost savings for the entire healthcare system as hospitalization costs are avoided
8 (Lanzarone & Matta, 2014).

9 Home healthcare planning includes different decision levels that are usually classified in
10 three main categories: *strategic planning*, *tactical planning*, and *operational planning* (Hulshof
11 et al., 2012). Strategic planning relates to problems addressing structural decision making
12 to design and to dimension the healthcare delivery process. This planning level often in-
13 volves long planning horizons in which decisions are based on aggregate information and
14 forecasts. Some applications include *districting* problems in which the geographic territory
15 where home care agencies operate is partitioned in districts (i.e. smaller geographic zones).
16 Tactical planning is related to medium-term decision making dealing with the implementation
17 of strategic decisions. Examples of problems in this decision level include *personnel scheduling*
18 problems, where work patterns are designed and allocated to caregivers to meet a forecasted
19 and often uncertain demand for services. Operational planning includes short-term decision
20 making related to the execution of the healthcare delivery process. Applications include *visit*
21 *rescheduling* where visit schedules are updated a few days in advance or during the execution
22 day, to respond to events such as caregiver absenteeism, incoming urgent care requests, and
23 changes in visit requirements.

24 The spatial distribution of patients and the uncertainty in demands represent some im-
25 portant features found in home healthcare workforce planning. The incorporation of these
26 aspects increases the complexity of the problems under study. However, including them in
27 the modeling and solution process could have a positive impact on an efficient service delivery
28 in terms of costs and quality. First, the integration of decisions in several districts usually
29 generates flexible staffing and scheduling solutions that respond in a better way to fluctuating
30 demand, since caregivers are allowed to work in a different district than the one they are ded-
31 icated to (Lahrichi et al., 2006). In a similar way, the incorporation of demand uncertainty
32 provides solutions that will be more robust to accommodate changes in demand associated
33 with the arrival of new patients and with changes in patients' conditions.

34 In this paper, we focus on the integration of two medium-term workforce planning prob-
35 lems: the *staff dimensioning problem* and the *caregiver scheduling problem*. This integration
36 deals with the definition of the number of caregivers to recruit per district, as well as with the
37 allocation of schedules to caregivers while considering demand uncertainty. Caregiver sched-
38 ules are defined by sequences of *work stretches* and *rest stretches*. Work stretches contain a
39 consecutive number of work days, where each work day contains exactly one *shift* (e.g., morn-
40 ing shift, night shift) executed in one district. Similarly, rest stretches represent a consecutive
41 number of *days-off*. The composition of feasible schedules is subject to work regulations en-

42 suring, among others, that there is a minimum rest time between consecutive shifts, that each
43 work stretch includes a sequence of shifts between a minimum and a maximum value, and
44 that each rest stretch contains a sequence of days-off between a minimum and a maximum
45 value.

46 Our work is motivated by the challenges experienced in AlayaCare, a start-up company
47 based in Canada developing software solutions for home healthcare agencies. Most of these
48 agencies currently lack the tools to forecast future demands, to manage their labour resources,
49 and to optimize work assignments. Hence, the staffing and scheduling planning is mostly done
50 manually by experienced coordinators. Since this planning method often fails to include most
51 of the rules for the composition of schedules, as well as accurate demand forecasts, it results
52 in the inability to hire an adequate number of caregivers, to retain current caregivers, and to
53 meet the needs of patients.

54 This paper has the following contributions. First, to the best of our knowledge, our work is
55 the first to propose an optimization approach that integrates staffing and scheduling decisions
56 in the context of home healthcare. To do so, we present a two-stage stochastic programming
57 model where *first-stage decisions* correspond to the staffing and scheduling of caregivers at
58 each geographic district, and *second-stage decisions* are related to the temporary reallocation
59 of caregivers to neighboring districts, to contact caregivers to work on a day-off, and to allow
60 under-covering and over-covering of demand. Second, although other authors have already
61 benefit from the expressiveness of *context-free grammars* to build short-term schedules with a
62 planning horizon of one day (see Restrepo et al. (2017); Côté et al. (2013)), we believe that our
63 work is the first that uses context-free grammars to build schedules over long time horizons
64 (i.e., one month or more) guaranteeing *horizontal work regulations* such as the minimum rest
65 time between consecutive shifts and the allocation of a minimum and a maximum number of
66 shifts to each work sequence. Context-free grammars allow to easily incorporate horizontal
67 regulations as a set of recursive rewriting rules (or productions) to generate patterns of strings
68 (Hopcroft et al., 2001), in our case, to generate caregiver schedules. Third, we discuss how
69 to forecast the demand of home care services and how to integrate these forecasts in a two-
70 stage stochastic programming model. Fourth, we perform an extensive computational study
71 on real-based data to evaluate the impact in costs, caregiver utilization and service level, by
72 using several recourse actions, various scheduling policies and different planning horizons.

73 The paper is organized as follows. In Section 2, we review related works on caregiver
74 staffing and scheduling for healthcare. In Section 3, we present the methodology to solve the
75 integrated caregiver staffing and scheduling for home healthcare. Computational experiments
76 are presented and discussed in Section 4. Concluding remarks and future work follow in
77 Section 5.

78 **2. Related Work**

79 Healthcare planning problems for hospitals have been extensively studied over the past
80 years. In particular, nurse staffing and scheduling problems have attracted most of the at-
81 tention from the operations research community since the generation of high-quality nurse
82 schedules can lead to improvements in hospital resource efficiency, in patient safety and sat-
83 isfaction, and in administrative workload (Burke et al., 2004). Recent approaches to this
84 problem include the works presented in Maenhout & Vanhoucke (2013) and Kim & Mehro-
85 tra (2015). Maenhout & Vanhoucke (2013) present a branch-and-price procedure to solve
86 an integrated nurse staffing and scheduling problem, where the number of nurses has to be
87 determined for each profession in order to balance, over several months, the workforce costs
88 and the coverage of patients in multiple hospital departments. Results indicate that staffing
89 multiple departments simultaneously and including nurse skills into the staffing decisions lead
90 to significant improvements in schedule quality in terms of cost, employees’ job satisfaction,
91 and effectiveness in providing high-quality care. Kim & Mehrotra (2015) present a two-
92 stage stochastic integer program with mixed-integer recourse to integrated nurse staffing and
93 scheduling. In the problem, first-stage decisions define initial staffing levels and schedules,
94 while second-stage decisions adjust these schedules at a time epoch closer to the actual date
95 of demand realization. Results show that, when compared with a deterministic model, the
96 two-stage stochastic model leads to significant cost savings. The work of Kim & Mehrotra is
97 similar to ours as the authors use a two-stage stochastic integer programming program with
98 recourse to solve integrated staffing and scheduling problems in healthcare. The objective of
99 both works is to find initial staffing levels and schedules to minimize overall labor costs by
100 right-sizing the staff and by balancing understaffing and overstaffing costs. However, their
101 work differs in some important aspects from ours. First, as opposed to our work, the work of
102 Kim & Mehrotra does not consider the spatial dimension in the planning, since the staffing
103 and scheduling is done for nurses in a hospital and not for caregivers that need to visit patients
104 in different geographic zones. Second, the authors assume that work patterns repeat from
105 week to week during the planning horizon and that all possible weekly patterns are generated
106 in advance. Instead, in our approach, caregiver schedules are allowed to be different from
107 week to week, and weekly schedules are not generated in advance, as one of the objectives of
108 our model is to build (with context-free grammars) caregiver schedules that guarantee several
109 work regulations. Third, regarding the use of recourse actions, both works allow for calling
110 in additional staff when needed. However, our work uses an additional recourse action corre-
111 sponding to the reallocation of caregivers to neighbor areas and, contrary to Kim & Mehrotra,
112 we do not allow to cancel shifts from the scheduled staff.

113 Problems related to the routing and scheduling of human resources involve the most im-
114 portant volume of existing investigations in home healthcare planning. These problems define

115 the assignment of caregivers to patients, as well as the design of caregivers routes to reduce
116 traveling distances, to decrease overtime costs, and to improve the *continuity of care*. Conti-
117 nuity of care guarantees that a patient is most of the time visited by the same caregiver in
118 the whole duration of the care plan. Home healthcare routing and scheduling problems often
119 require the incorporation of several constraints related to the management of caregivers' work
120 regulations, to the matching of caregivers' skills and patients' requirements, and to the satis-
121 faction of patients' and caregivers' preferences. Since the addition of these constraints often
122 makes the modelling and solution of this problem intractable, different authors have proposed
123 heuristic methods such as tabu search algorithms (Hertz & Lahrichi, 2009) and rolling horizon
124 approaches (Bennett & Erera, 2011; Nickel et al., 2012) to efficiently solve practical instances
125 of this problem. Exact approaches have also been developed in Bachouch et al. (2011) and
126 Cappanera & Scutellà (2014) to deal (in an integrated way) with assignment, scheduling, and
127 routing decisions.

128 Real applications of routing and scheduling of human resources in home healthcare often
129 require the optimization of multiple objectives, as well as the incorporation of uncertainty in
130 demands to obtain robust solutions that react better to changes in demand. In that order
131 of ideas, Duque et al. (2015) and Braekers et al. (2016) propose bi-objective optimization
132 approaches to maximize the quality of service and to minimize the distance travelled by the
133 caregivers. Lanzarone et al. (2012) formulate different scenario-based stochastic programming
134 models to solve the robust nurse-to-patient assignment problem that preserves the continuity
135 of care and balances the operators' workloads. Lanzarone & Matta (2012) use analytical poli-
136 cies to address the nurse-to-patient assignment problem, in which both continuity of care and
137 demand uncertainty are considered. Nguyen et al. (2015) present a variant of a home care
138 problem in which the availability of nurses is uncertain (e.g., nurses might call sick on short
139 notice). To address this problem, the authors propose to use a matheuristic optimization
140 approach for robust nurse-to-patient assignment and nurse scheduling and routing. Carello
141 & Lanzarone (2014) and Lanzarone & Matta (2014) present robust approaches for the nurse-
142 to-patient assignment under continuity of care. In the former work, the authors apply the
143 robust cardinality-constrained approach proposed in Bertsimas & Sim (2004) to incorporate
144 the uncertainty in patients' demands. In the latter work, the authors propose an analyt-
145 ical policy that takes into account the stochasticity of new patient's demand and nurses'
146 workloads. Hewitt et al. (2016) solve the nurse-to-patient assignment problem and develop a
147 solution method to incorporate uncertainty in demand, as future patient requests are often
148 unknown at the time of planning. Cappanera et al. (2018) extend the cardinality-constrained
149 robust approach presented in Cappanera & Scutellà (2014) to include uncertainty in patients'
150 demands in a home care problem integrating assignment, scheduling and routing decisions.
151 The interested reader is referred to Fikar & Hirsch (2017) for a recent survey of current works
152 in home healthcare routing and scheduling.

153 Contrarily to the routing and scheduling of caregivers, integrated staffing and scheduling
154 problems for home healthcare have been rarely studied in the literature. This problem is
155 highly relevant, as human resources need to be properly managed in order to avoid inefficient
156 visit schedules, treatment delays, and low quality of service (Matta et al., 2014). Two medium-
157 term home healthcare nurse scheduling problems are addressed in Trautsamwieser & Hirsch
158 (2014) and in Wirnitzer et al. (2016). In these works, a given set of nurses is allocated to
159 schedules which are built by including work regulations associated with the allocation of days-
160 off between work stretches, the allocation of rest times between consecutive working days, and
161 the allocation of a maximum working time per day and per week. Trautsamwieser & Hirsch
162 (2014) use a branch-and-price-and-cut solution approach to solve the problem over a one-week
163 planning horizon. Experiments on real-world based instances show that the proposed method
164 helps to significantly reduce the schedule planning time when compared to a manual planning
165 process. Wirnitzer et al. (2016) present a mixed integer programming (MIP) model to address
166 the nurse scheduling problem for longer planning horizons (e.g., one month). Experiments
167 on real-world instances suggest that using the MIP model not only helps to reduce the time
168 to generate the schedules, but also improves the solution quality from the patients and from
169 the nurses point of view. A home healthcare nurse staffing problem with uncertain demands
170 is studied in Rodriguez et al. (2015). The authors propose to use a two-stage stochastic
171 programming approach where first-stage decisions correspond to a global staff dimensioning,
172 while second-stage decisions are related to the allocation of schedules (that do not include
173 work regulations or continuity of care) to nurses with different skills. Results indicate that
174 the proposed approach helps decision-makers with staffing and scheduling decisions before
175 opening a home healthcare service or before hiring a new nurse.

176 Forecasting patients' demands represents an important step in robust approaches for plan-
177 ning and managing resources in health care. These forecasts can create alerts for the man-
178 agement of patient overflows, they can enhance preventive health care, and when used as
179 an input for planning human resources, they can significantly reduce the associated costs in
180 overstaffing and understaffing (Soyiri & Reidpath, 2013). Several methods have been pro-
181 posed in the literature to forecast demands and to support healthcare providers in human
182 resource planning before the care execution. These forecasting methods include, among oth-
183 ers, Markovian decision models (Lanzarone et al., 2010; Garg et al., 2010), Bayesian models
184 (Argiento et al., 2016), and autoregressive moving average models (Jalalpour et al., 2015). In
185 this paper, we use a *decomposable time series model* (Harvey & Peters, 1990) to forecast the
186 demand since this type of model is relatively easy to implement and to explain to the end
187 user.

188 The literature review in home healthcare planning reveals that no method has been pro-
189 posed to integrate caregiver staffing and scheduling when demand is stochastic and when
190 the composition of schedules includes complex work regulations, in particular, existing works

191 show that when rules for the composition of schedules are included in the problem, staffing
 192 decisions are not considered since it is assumed that these decisions have been already taken
 193 in a previous step of the decision process (Defraeye & Van Nieuwenhuysse, 2016). In a similar
 194 way, when staffing decisions are included in the problem, the composition of caregivers' sched-
 195 ules does not consider important work rules such as the allocation of a minimum rest time
 196 between consecutive shifts. This paper addresses these gaps in the literature by proposing
 197 a model that integrates staff dimensioning with staff scheduling decisions for a medium-term
 198 home healthcare problem. Furthermore, the proposed model includes uncertainty in demands
 199 and the incorporation of several work rules for the generation of caregivers' schedules, pro-
 200 viding solutions that are expected to react in a robust way to variations in demand and that
 201 comply with workplace agreements. We remark that although other works have already used
 202 context-free grammars to solve personnel scheduling problems under stochastic demand (see
 203 Restrepo et al. (2017)), our work is the first one that uses grammars to build schedules over
 204 time horizons longer than one day (i.e., a month or longer). Additionally, our work differs
 205 from the work in Restrepo et al. (2017) by three other aspects. First, this paper consid-
 206 ers staffing decisions, while the work presented in Restrepo et al. (2017) assumes that the
 207 number of employees is already given. Second, in this paper, employees can work in different
 208 geographic areas, while in Restrepo et al. (2017) all employees are assumed to work in a single
 209 place. Third, while this paper uses the reallocation of caregivers to neighboring areas and
 210 the possibility of calling caregivers to work during one of their days-off as the set of recourse
 211 actions, the work in Restrepo et al. (2017) uses the allocation of activities and breaks to daily
 212 shifts to protect against demand uncertainty.
 213 Next section presents the definition and formulation of the problem studied in this paper.

214 3. Problem Definition and Formulation

215 The integrated caregiver staffing and scheduling problem for home healthcare considers a
 216 territory divided into $|C|$ *geographic areas* or *districts*, each one covering several patients. We
 217 assume that each patient is assigned to only one district. The *planning horizon* includes $|D|$
 218 days, where each day $d \in D$ is covered by a set of *working shifts* S characterized by a set of
 219 attributes, namely: a start time b_s , a day of the week d_s (e.g. Monday, Tuesday,...), a length
 220 l_s , and a cost c_s that depends on the shifts's length l_s and the day of the week d_s . Each
 221 district $c \in C$ defines a different type of caregiver $e \in E$ ($E = C$) working in at most one shift
 222 $s \in S$ per day. To guarantee the continuity of care for patients, caregiver $e \in E$ should work
 223 most of the time in his district. However, caregivers might be temporarily reallocated (at
 224 the expense of an additional cost) to a *compatible* district $c \in C$ during shift $s \in S$ to meet
 225 unexpected demands. Campbell (2011) showed that schedule flexibility resulting from the
 226 reallocation of employees can be more valuable than the perfect information about demand,

227 especially when demand uncertainty is high.

228 We assume that demands (expressed as the number of visits during day $d \in D$ in district
229 $c \in C$ and shift $s \in S$) are uncertain. Hence, when solving the integrated caregiver staffing
230 and scheduling problem for home healthcare we consider two types of decisions. The first type
231 includes the first-stage decisions, which define the staffing levels (i.e. the number of caregivers
232 to hire), as well as the allocation of individual schedules to each caregiver. The second type
233 incorporates the second-stage decisions, which define the adjustment of caregivers' schedules
234 few days before their execution. These adjustments include the caregivers reallocation to
235 compatible districts, contacting caregivers to work during their day-off, and allowing demand
236 over-covering and under-covering. Because schedules must be available to caregivers at least
237 one month in advance to allow for choices, we assume that the planning horizon is larger
238 than or equal to 4 weeks. At the beginning of this planning horizon staffing and scheduling
239 decisions (first-stage decisions) are made to minimize the sum of the total staffing costs, the
240 expected recourse costs, and the expected over-covering and under-covering costs. Since the
241 actual demand is often revealed one week in advance, the planned schedules are adjusted at
242 the beginning of each week for the following week. These adjustment decisions (second-stage
243 decisions) are applied for each type of shift at each day of the week.

244 The methodology to solve the problem studied in this paper is divided in three steps.
245 The first step is related to the demand forecasting and scenario generation. The second step
246 involves the definition of caregivers' schedules by means of grammars. The third step uses
247 a two-stage stochastic programming optimization model for caregiver staffing and schedule
248 allocation. The description of these steps is presented next.

249 *3.1. Demand Forecasting and Scenario Generation*

250 The ability of accurately forecast the demand for visits is a fundamental requirement for
251 developing robust decision support tools in home healthcare resource planning. In fact, sev-
252 eral strategic and tactical decisions in home healthcare are based on forecasts of demand for
253 resources. For instance, recruitment decisions are mainly driven by forecasts on the amount
254 of visits required by the patients in a given planning horizon. If this demand is accurately
255 predicted, several operational problems such as under-utilization and over-utilization of care-
256 givers can be avoided. On the contrary, inaccurate forecasts threatens the quality of the plans
257 obtained leading to more expensive solutions that could be infeasible for some demand scenar-
258 ios. In this section, we present a methodology for demand forecasting and scenario generation
259 in home healthcare. We remark that the methods used to forecast and to generate scenarios
260 for the demand are possible approaches, developing and evaluating different methods for these
261 tasks is out of the scope of this work.

262 *3.1.1. Demand forecasting*

263 To estimate the number of patients b_{dcs} to visit during day $d \in D$ in district $c \in C$, and
 264 shift $s \in S$, we use a decomposable time series model with three main model components:
 265 *growth*, *seasonality*, and *holidays*. These components (included in equation (1)) represent
 266 the *growth* function (g_{dsc}) which models *non-periodic changes* in the value of the time series,
 267 the *periodic changes* function (s_{dsc}) modelling weekly or yearly seasonality, and the *effects*
 268 *of holidays* function (h_{dsc}) including effects from days such as christmas and new year’s day.
 269 The error term ϵ_{dsc} represents irregular changes in demand, which are not accommodated by
 270 the time series model.

$$b_{dsc} = g_{dsc} + s_{dsc} + h_{dsc} + \epsilon_{dsc}, \text{ for each } s \in S, c \in C \quad (1)$$

271 Equation (1) is estimated with Facebook Prophet which is an open source library to
 272 create quick, accurate and completely automated time series forecasts. This tool uses an
 273 *additive regression model* with four components: i) a piecewise linear or logistic growth curve
 274 to detect changes in trends by selecting change points from the historical data; ii) a yearly
 275 seasonal component modeled using Fourier series; iii) a weekly seasonal component using
 276 dummy variables; iv) a user-provided list of relevant holidays. Unlike with ARIMA models,
 277 the time series measurements do not need to have a regular period. Hence, there is no need
 278 to interpolate missing values to fit. The reader is referred to (Taylor & Letham, 2018) for
 279 more information on how Facebook Prophet works.

280 *3.1.2. Scenario generation*

281 In generating the different scenarios for our problem we only consider uncertainty in the
 282 number of visits per day, per shift, and per district. Therefore, we assume that the duration
 283 of patients’ visits and travel times are deterministic parameters which are included in the
 284 caregiver capacities (i.e. the number of patients visited per shift). We allow these capacities
 285 to vary with the day of the week, with the type of shift, and with the district where the
 286 caregiver is working. For instance, the capacity of night shifts is generally lower than the
 287 capacity of morning shifts, as patients visited at night need care for longer periods than
 288 patients visited in the morning. We assume that the number of visits per day, per shift, and
 289 per district is a random variable with finite support. In addition, we define Ω_d as a set of
 290 scenarios for the demand at each day $d \in D$, and $p_d^{(w)} > 0$ as the probability of occurrence of
 291 scenario $w \in \Omega_d$. Note that $\sum_{w \in \Omega_d} p_d^{(w)} = 1, \forall d \in D$.

292 The scenarios for the demand are generated with Monte Carlo simulation. We assume
 293 that given the estimated values for the mean of demand (\hat{b}_{dsc}) and the estimated values for
 294 the upper bound (\hat{b}_{dsc}^u) of a $(1 - \alpha)$ confidence interval returned by Facebook Prophet after

295 fitting model (1) to the historical data, the standard deviation $\hat{\sigma}_{dsc}$ can be computed with
 296 equation (2).

$$\hat{\sigma}_{dsc} = (\hat{b}_{dsc}^u - \hat{b}_{dsc}) \times \frac{\sqrt{n}}{Z_{1-\frac{\alpha}{2}}} \quad (2)$$

297 Where $Z_{1-\frac{\alpha}{2}}$ is the value for a standard normal variable with a $1 - \frac{\alpha}{2}$ probability to the
 298 right, and n denotes the size of the training set used to estimate time series model (1). Once
 299 the values for $\hat{\sigma}_{dsc}$ are obtained, we can compute the demand for the number of visits in
 300 district $c \in C$ and shift $s \in S$ during day $d \in D$ under scenario $w \in \Omega_d$ as:

$$b_{dsc}^{(w)} = \max \left\{ 0, \left[\hat{b}_{dsc} + R * \frac{\hat{\sigma}_{dsc}}{\sqrt{n}} \right] \right\} \quad (3)$$

301 Where R represents the value of a random variable that follows a standard normal distri-
 302 bution and $[\]$ denotes the nearest integer function.

303 An example on the scenario generation for a given day $d \in D$ is shown in Tables 1 and
 304 2. Table 1 presents for each combination of districts and shifts (denoted as $d_0, d_1, d_2,$ and d_3
 305 for the districts, and $a_4, m_4, m_8,$ and n_{10} for the shifts) the values for the forecasted mean
 306 demand (\hat{b}), the values for the lower bound and upper bound (\hat{b}^l, \hat{b}^u) of a 90% confidence
 307 interval for the forecasted demand, the values for the actual value of the demand (b), and
 308 the values for the possible values for the demand (list) with their corresponding frequency
 309 (count), after running 500 simulations. Table 2 shows a sample of 10 scenarios from the 500
 310 scenarios generated. Each column from this table presents the demand values (number of
 311 visits) during day d for each combination of districts and shifts.

district_shift	b	\hat{b}^l	\hat{b}^u	\hat{b}	list	count
$d_0_m_4$	1	1	2	1	[1]	[500]
$d_0_m_8$	2	1	3	1	[1, 2, 0, 3]	[314, 157, 25, 4]
$d_0_n_{10}$	2	1	2	1	[1, 2, 0]	[466, 30, 4]
$d_1_a_4$	7	5	9	7	[7, 6, 5, 8, 9, 4, 10, 3]	[151, 134, 87, 77, 23, 22, 5, 1]
$d_1_m_4$	5	2	8	4	[4, 5, 6, 3, 2, 7, 1, 8, 0, 9]	[122, 110, 84, 78, 49, 31, 14, 9, 2, 1]
$d_1_m_8$	14	12	17	14	[13, 14, 15, 12, 16, 11, 17, 18, 10]	[126, 122, 106, 56, 47, 22, 13, 5, 3]
$d_1_n_{10}$	9	6	10	7	[8, 9, 7, 6, 10, 11, 5, 12]	[182, 128, 116, 33, 32, 5, 3, 1]
$d_2_m_4$	2	1	3	1	[1, 2, 0, 3]	[304, 154, 40, 2]
$d_2_m_8$	2	1	2	1	[1, 2, 0]	[460, 34, 6]
$d_2_n_{10}$	2	1	2	1	[1, 2, 0]	[420, 78, 2]
$d_3_a_4$	3	1	5	2	[3, 2, 4, 1, 5, 0]	[179, 176, 67, 59, 10, 9]
$d_3_m_4$	4	3	6	4	[4, 3, 5, 2, 6, 7, 1]	[204, 129, 116, 26, 23, 1, 1]
$d_3_m_8$	4	2	6	4	[3, 4, 2, 5, 1, 6, 7]	[192, 147, 88, 55, 9, 8, 1]
$d_3_n_{10}$	4	3	6	4	[4, 3, 5, 2, 6, 1]	[212, 151, 93, 30, 11, 3]

Table 1: Results for the demand forecasting and Monte Carlo simulation.

district_shift	Scenario									
	1	2	3	4	5	6	7	8	9	10
$d_0_m_4$	1	1	1	1	1	1	1	1	1	1
$d_0_m_8$	1	1	1	2	2	3	1	2	2	1
$d_0_n_{10}$	1	1	1	1	2	1	1	1	1	1
$d_1_a_4$	7	5	5	6	8	8	5	7	6	7
$d_1_m_4$	3	3	5	5	5	4	6	6	6	2
$d_1_m_8$	11	15	13	13	14	13	13	11	16	11
$d_1_n_{10}$	7	11	9	9	5	7	6	8	8	6
$d_2_m_4$	2	2	1	2	1	1	1	1	1	2
$d_2_m_8$	1	1	2	1	1	1	1	1	1	2
$d_2_n_{10}$	1	1	1	1	1	2	1	2	1	1
$d_3_a_4$	3	3	3	2	2	2	3	4	3	1
$d_3_m_4$	4	5	4	4	3	3	5	5	3	3
$d_3_m_8$	3	3	5	3	3	5	3	2	3	4
$d_3_n_{10}$	3	5	4	5	3	4	4	3	6	3

Table 2: Example of 10 scenarios for a given day in the planning horizon.

3.2. Grammars

A *context-free grammar* is a set of recursive rewriting rules (or productions) used to generate patterns of strings, or (in the case of personnel scheduling) to generate schedules or daily shifts. Context-free grammars have been successfully used in the context of personnel scheduling. Applications include the solution of multi-activity and multi-task shift scheduling problems (Côté et al., 2013; Boyer et al., 2012) and multi-activity tour scheduling problems (Restrepo et al., 2017, 2016).

A context-free grammar consists of a four-tuple $G = \langle \Sigma, N, \mathcal{S}, P \rangle$, where Σ is an alphabet of characters called the *terminal symbols*, N is a set of *non-terminal symbols*, $\mathcal{S} \in N$ is the starting symbol, and P is a set of *productions* represented as $A \rightarrow \alpha$, where $A \in N$ is a non-terminal symbol and α is a sequence of terminal and non-terminal symbols. The productions of a grammar are used to generate new symbol sequences until all non-terminal symbols have been replaced by terminal symbols. A *context-free language* is the set of sequences accepted by a context-free grammar.

A *parse tree* is a tree where each inner-node is labeled with a non-terminal symbol and each leaf is labeled with a terminal symbol. A grammar recognizes a sequence if and only if there exists a parse tree where the leaves, when listed from left to right, reproduce the sequence. A *DAG* Γ is a *directed acyclic graph* that embeds all parse trees associated with words of a given length n recognized by a grammar. The DAG Γ has an and/or structure where the and-nodes represent productions from P and or-nodes represent non-terminals from N and letters from Σ . An and-node is true if all of its children are true. An or-node is true if one of its children is true. The root node is true if the grammar accepts the sequence encoded by the leaves. The DAG Γ is built with a procedure proposed in Quimper & Walsh (2007) using bottom-up parsing and dynamic programming.

In employee scheduling, the use of grammars allows one to include work rules regarding

337 the definition of *work stretches* and *rest stretches* in an easy way. Thus, feasible schedules can
 338 be represented as words in a context-free language. Specifically, for the problem addressed in
 339 this paper we use grammars to:

- 340 • Generate work stretches representing sequences of work spanning a minimum and a
 341 maximum number of days.
- 342 • Generate rest stretches denoting sequences of days-off spanning a minimum and a max-
 343 imum number of days.
- 344 • Define a minimum and a maximum consecutive number of morning, afternoon, and
 345 night shifts within a work stretch. For instance, a given work stretch cannot have more
 346 than 3 night shifts in a row.
- 347 • Forbid infeasible transitions between shifts by associating costs to productions. For
 348 instance, a night shift cannot be followed by a morning shift.
- 349 • Allocate a rest stretch between two work stretches.

350 **Example 1**

351
 352 *Consider the following grammar for an employee scheduling problem where the planning hori-
 353 zon consists of five days, work stretches have a length of three consecutive days, and days-off
 354 can be allocated in consecutive or nonconsecutive days:*

$$355 G = (\Sigma = (w, r), N = (\mathcal{S}, F, Q, W, R), P, \mathcal{S}),$$

357
 358 *Where productions P are: $\mathcal{S} \rightarrow RF|FR|QR$, $F_{[3,3]} \rightarrow WW$, $W \rightarrow WW|w$, $Q \rightarrow RF$,
 359 $R \rightarrow RR|r$ and symbol $|$ specifies the choice of production. Letter w represents the allocation
 360 of a working shift and letter r represents the allocation of a day-off. $P_{[min, max]}$ restricts the
 361 subsequences generated by production P to a length between a minimum and maximum num-
 362 ber of days.*

363
 364 *In this grammar, production $F_{[3,3]} \rightarrow WW$ generates two non-terminal symbols W , meaning
 365 that the schedule will include a work stretch of exactly three days. Production $Q \rightarrow RF$ gen-
 366 erates two non-terminal symbols R and F , meaning that the schedule will start with a rest
 367 stretch and then it will include a work stretch of exactly three days. Production $R \rightarrow RR$
 368 generates two non-terminal symbols R , meaning that the schedule will include a rest stretch.
 369 Productions $W \rightarrow w$ and $R \rightarrow r$ generate terminal symbols associated with the allocation of
 370 a shift and with the allocation of a day-off to the schedule of an employee, respectively. The*

371 last three productions are $\mathcal{S} \rightarrow RF$, $\mathcal{S} \rightarrow FR$, and $\mathcal{S} \rightarrow QR$. The first production generates
 372 a schedule starting with two days-off followed by a work stretch. The second production gen-
 373 erates a schedule starting with a work stretch followed by two days-off. The last production
 374 generates a schedule starting with one day-off, followed by a work stretch, to finish with one
 375 day-off. The three words recognized as valid schedules by the grammar in this example are
 376 $rrwww$, $wwrr$, and $rwrr$.

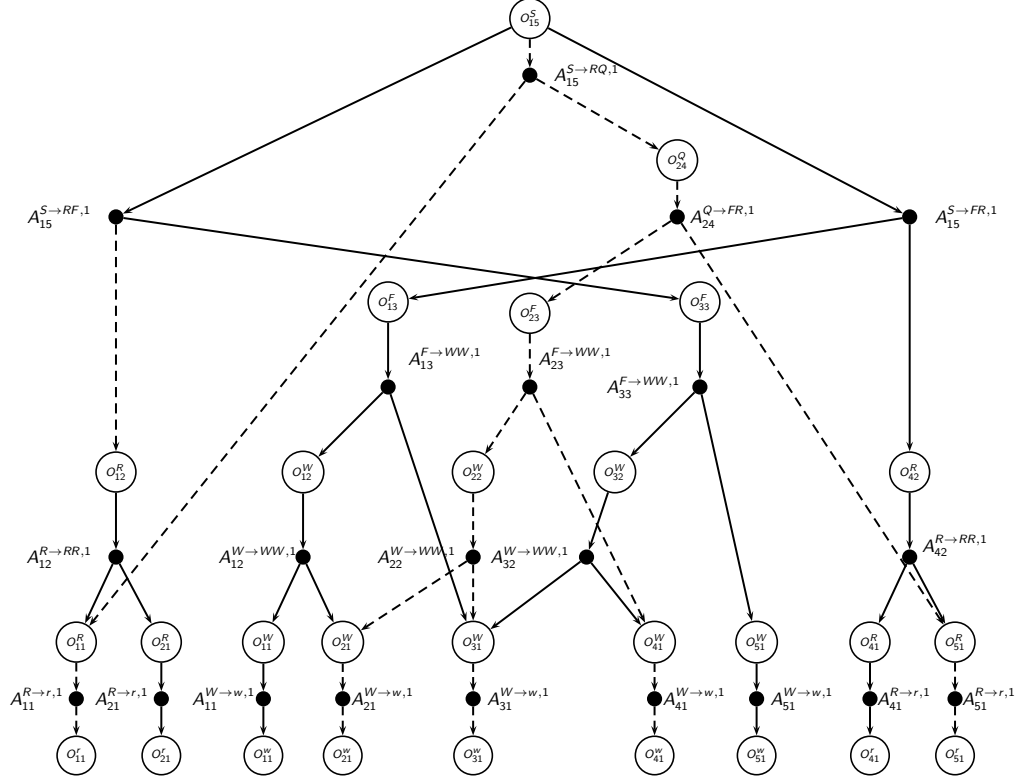
377

378 Let O_{dl}^π be the or-nodes associated with $\pi \in N \cup \Sigma$ (i.e. with non-terminals from N or
 379 letters from Σ) that generate a subsequence from day d of length l . Note that if $\pi \in \Sigma$,
 380 the node is a leaf and l is equal to one. On the contrary, if $\pi \in N$ the node represents a
 381 non-terminal symbol and $l \geq 1$. $A_{dl}^{\Pi,k}$ is the k^{th} and-node representing production $\Pi \in P$
 382 generating a subsequence from day d of length l . There are as many $A_{dl}^{\Pi,k}$ nodes as there
 383 are ways of using Π to generate a sequence of length l from day d . As previously mentioned,
 384 undesired productions (i.e. transitions between a night shift and a morning shift) are penalized
 385 by a cost denoted as $c_{dl}^{\Pi,k}$. The sets of or-nodes, and-nodes, and leaves of DAG Γ are denoted
 386 by O , A , and L , respectively. The root node is described by O_{1n}^S and its children by $A_{1n}^{\Pi,k}$.
 387 The children of or-node O_{dl}^π are represented by $ch(O_{dl}^\pi)$ and its parents by $par(O_{dl}^\pi)$. Similarly,
 388 the children of and-node $A_{dl}^{\Pi,k}$ are represented by $ch(A_{dl}^{\Pi,k})$ and its parents by $par(A_{dl}^{\Pi,k})$. For
 389 more details on the use of grammars in employee scheduling we refer the reader to [Côté et al.](#)
 390 [\(2011\)](#).

391 Figure [1](#) shows the DAG Γ associated with the grammar from Example 1. Observe that
 392 this figure includes three parse trees, each one representing one word (schedule) recognized by
 393 the grammar. As an example we present in dashed lines the parse tree generating schedule
 394 $rwrr$.

395 The works of [Restrepo et al. \(2017\)](#) and [Côté et al. \(2011\)](#) on anonymous tour scheduling
 396 problems with multiple activities are examples of the use of context-free grammars to represent
 397 the work rules involved in the composition of shifts. In both works, the authors present
 398 implicit grammar-based integer programming models where the word length n corresponds to
 399 the number of periods in the planning horizon, the set of work activities corresponds to letters
 400 in the alphabet Σ , and each employee is allowed to work in any work activity. In the model,
 401 the logical clauses associated with Γ are translated into linear constraints on integer variables.
 402 Each and-node A and each leaf L in Γ are represented by an integer variable denoting the
 403 number of employees assigned to a specific subsequence of work. Since this grammar-based
 404 model efficiently encapsulates the constraints for the generation of the schedules, it is used as
 405 a component in the formulation of the two-stage stochastic problem presented next.

Figure 1: DAG Γ on schedules of length five.



406 3.3. Two-Stage Stochastic Optimization Model

407 The formulation of the two-stage stochastic programming model requires a previous defini-
 408 tion of the grammars and DAGs Γ containing specific work regulations for the composition
 409 of valid caregiver schedules. Since work regulations could vary depending on the type of care-
 410 giver, we define a different grammar and a different DAG Γ^e for each $e \in E$. The notation
 411 used for the formulation of the problem is as follows:

412 *Parameters:*

- 413 – κ_{dsc}^e : number of visits a caregiver of type $e \in E$ working on shift $s \in S$ can perform in
 414 district $c \in C$ during day $d \in D$;
- 415 – c_{ds}^e : non-negative cost associated with one caregiver of type $e \in E$ working on shift $s \in S$
 416 during day $d \in D$;
- 417 – $c_{dl}^{\Pi, k, e}$: non-negative cost associated with the k^{th} and-node representing production Π from
 418 Γ^e , producing a sequence from day $d \in D$ of length l for caregiver $e \in E$;

- 419 – \hat{b}_{dsc} : mean demand for the number of visits in district $c \in C$ and shift $s \in S$ during day
420 $d \in D$;
- 421 – $b_{dsc}^{(w)}$: demand for the number of visits in district $c \in C$ and shift $s \in S$ during day $d \in D$
422 under scenario $w \in \Omega_d$;
- 423 – c_{dsc}^+, c_{dcs}^- : non-negative demand over-covering and under-covering costs for district $c \in C$
424 and shift $s \in S$ during day $d \in D$, respectively;
- 425 – t_{dsc}^e : non-negative transition cost associated with the reallocation one caregiver of type
426 $e \in E$ to district $c \in C$ during day d and shift $s \in S$;
- 427 – r_{ds}^e : non-negative cost associated with assigning shift $s \in S$ to a caregiver of type $e \in E$
428 during its rest day $d \in D$;
- 429 – δ_{sc}^e : binary parameter that takes value 1 if caregiver $e \in E$ admits a reallocation to district
430 $c \in C$ during shift $s \in S$, and it assumes value 0 otherwise.

431 *Decision variables:*

- 432 – u^e : variable that denotes the number of caregivers of type $e \in E$ to hire;
- 433 – $v_{dl}^{\Pi,k,e}$: variable that denotes the number of caregivers of type $e \in E$ assigned to the k^{th}
434 and-node representing production Π from Γ^e producing a sequence from day $d \in D$ of
435 length l ;
- 436 – y_{ds}^e : variable that denotes the number of caregivers of type $e \in E$ working on shift $s \in S$
437 during day $d \in D$ (equivalent to the number of caregivers of type $e \in E$ assigned to leaf
438 $O_{d1}^{s,e}$);
- 439 – y_{dr}^e : variable that denotes the number of caregivers of type $e \in E$ having rest during day
440 $d \in D$ (equivalent to the number of caregivers of type $e \in E$ assigned to leaf $O_{d1}^{r,e}$);
- 441 – $x_{dsc}^{e(w)}$: variable that denotes the number of caregivers of type $e \in E$ assigned to work in
442 district $c \in C$ and shift $s \in S$ during day $d \in D$ under scenario $w \in \Omega_d$;
- 443 – $z_{ds}^{e(w)}$: variable that denotes the number of caregivers of type $e \in E$ assigned to work during
444 a day-off on shift $s \in S$ during day d and scenario $w \in \Omega_d$;
- 445 – $s_{dsc}^{+(w)}$ and $s_{dsc}^{-(w)}$: slack variables denoting demand over-covering and under-covering in
446 district $c \in C$ and shift $s \in S$ during day $d \in D$ under scenario $w \in \Omega_d$, respectively.

447 The formulation for the stochastic caregiver staffing and scheduling problem, is as follows.

$$\min \sum_{d \in D} \sum_{s \in S} \sum_{e \in E} c_{ds}^e y_{ds}^e + \sum_{d \in D} \sum_{e \in E} \sum_{A_{dl}^{\Pi,k,e} \in A^e} c_{dl}^{\Pi,k,e} v_{dl}^{\Pi,k,e} + \mathcal{Q}(\mathbf{y}) \quad (4)$$

$$y_{ds}^e = \sum_{A_{d1}^{\Pi,1,e} \in \text{par}(O_{d1}^{s,e})} v_{d1}^{\Pi,1,e}, \forall d \in D, e \in E, s \in S, \quad (5)$$

$$y_{dr}^e = \sum_{A_{d1}^{\Pi,1,e} \in \text{par}(O_{d1}^{r,e})} v_{d1}^{\Pi,1,e}, \forall d \in D, e \in E, \quad (6)$$

$$u^e = \sum_{A_{1n}^{\Pi,k,e} \in \text{ch}(O_{1n}^{S,e})} v_{1n}^{\Pi,k,e}, \forall e \in E, \quad (7)$$

$$\sum_{A_{dl}^{\Pi,k,e} \in \text{ch}(O_{dl}^{\pi,e})} v_{dl}^{\Pi,k,e} = \sum_{A_{dl}^{\Pi,k,e} \in \text{par}(O_{dl}^{\pi,e})} v_{dl}^{\Pi,k,e}, \quad \forall e \in E, O_{dl}^{\pi,e} \in O^e \setminus \{O_{1n}^{S,e} \cup L^e\}, \quad (8)$$

$$u^e \geq 0 \text{ and integer}, \forall e \in E, \quad (9)$$

$$v_{dl}^{\Pi,k,e} \geq 0 \text{ and integer}, \forall d \in D, e \in E, A_{dl}^{\Pi,k,e} \in A^e, \quad (10)$$

$$y_{ds}^e \geq 0 \text{ and integer}, \forall d \in D, e \in E, s \in S, \quad (11)$$

$$y_{dr}^e \geq 0 \text{ and integer}, \forall d \in D, e \in E. \quad (12)$$

448 The objective of model (4)-(12) is to minimize the total staffing cost (i.e. allocation of
449 working shifts to caregivers), the penalization for certain transitions between shifts (i.e. transi-
450 tion from night shifts to morning shifts), and the *expected recourse function* $\mathcal{Q}(\mathbf{y})$. Constraints
451 (5)-(6) set the value of variables y_{ds}^e and y_{dr}^e as the summation of the value of the parents of
452 leaf nodes $O_{d1}^{s,e}$ and $O_{d1}^{r,e}$, respectively. Constraints (7) define the number of caregivers of type
453 $e \in E$ to hire. Constraints (8) guarantee, for every or-node in Γ^e , $e \in E$ excluding the root
454 node $O_{1n}^{S,e}$ and the leaves L^e , that the summation of the value of its children is the same as the
455 summation of the value of its parents. Constraints (8) can be seen as flow conservation equa-
456 tions where or-nodes $O_{dl}^{\pi,e}$ represent “transition nodes”. The constraints for those transition
457 nodes guarantee that if m caregivers of type e are allocated to the productions generating the
458 subsequence associated with node $O_{dl}^{\pi,e}$, those m caregivers have to be distributed along all the
459 possible ways to use π to generate a sequence of length l from position d ($\text{ch}(O_{dl}^{\pi,e})$). Consider
460 the following example using the DAG Γ from Figure 1. Assume that three caregivers are
461 assigned to and-node $A_{22}^{W \rightarrow WW,1}$ (represented by variable $v_{22}^{W \rightarrow WW,1}$) and that one employee
462 is assigned to and-node $A_{12}^{W \rightarrow WW,1}$ (represented by variable $v_{12}^{W \rightarrow WW,1}$). Since these two and-
463 nodes have one child in common (i.e. or-node O_{21}^W) the number of employees allocated to
464 O_{21}^W is four. Now, since or-node O_{21}^W has one child ($A_{21}^{W \rightarrow w,1}$) these four employees must be
465 allocated to a working shift during day 2.

466

467 The expected recourse function $\mathcal{Q}(\mathbf{y})$ is denoted by $\mathcal{Q}(\mathbf{y}) \equiv \mathbb{E}_\xi[\mathcal{Q}(\mathbf{y}, \xi)]$. The *recourse*
 468 *function* $\mathcal{Q}(\mathbf{y}, \xi_d(w))$ for a given realization w of ξ and fixed values for the allocation of
 469 caregivers to shifts and days-off ($\bar{y}_{ds}^e, \bar{y}_{d-1r}^e, \bar{y}_{dr}^e$, and \bar{y}_{d+1r}^e) is represented by:

$$\min \sum_{e \in E} \sum_{s \in S} \sum_{c \in C} t_{dsc}^e x_{dsc}^{e(w)} + \sum_{e \in E} \sum_{s \in S} r_{ds}^e z_{ds}^{e(w)} + \sum_{s \in S} \sum_{c \in C} (c_{dsc}^+ s_{dsc}^{+(w)} + c_{dsc}^- s_{dsc}^{-(w)}) \quad (13)$$

$$\sum_{c \in C} \delta_{sc}^e x_{dsc}^{e(w)} = (z_{ds}^{e(w)} + \bar{y}_{ds}^e), \forall s \in S, e \in E, \quad (14)$$

$$\sum_{s \in S} z_{ds}^{e(w)} \leq \bar{y}_{d-1r}^e, \forall e \in E, \quad (15)$$

$$\sum_{s \in S} z_{ds}^{e(w)} \leq \bar{y}_{dr}^e, \forall e \in E, \quad (16)$$

$$\sum_{s \in S} z_{ds}^{e(w)} \leq \bar{y}_{d+1r}^e, \forall e \in E, \quad (17)$$

$$\sum_{e \in E} \kappa_{dsc}^e x_{dsc}^{e(w)} - s_{dsc}^{+(w)} + s_{dsc}^{-(w)} = b_{dsc}^{(w)}, \forall s \in S, c \in C, \quad (18)$$

$$x_{dsc}^{e(w)} \geq 0 \text{ and integer}, \forall e \in E, s \in S, c \in C, \quad (19)$$

$$z_{ds}^{e(w)} \geq 0 \text{ and integer}, \forall e \in E, s \in S, \quad (20)$$

$$s_{dsc}^{+(w)}, s_{dsc}^{-(w)} \geq 0, \forall s \in S, c \in C. \quad (21)$$

470 The objective of model (13)-(21) is to minimize the reallocation costs, the costs of con-
 471 tacting caregivers to work on a day-off, and the penalization for demand over-covering and
 472 under-covering. Constraints (14) define the reallocation of caregivers of type $e \in E$ working
 473 on shift $s \in S$ to compatible districts. Constraints (15)-(17) set the valid conditions to contact
 474 caregivers to work on a day-off. That is, if an employee is having three days-off in a row only
 475 the day-off in the middle of the rest stretch can be assigned to a working shift. Constraints
 476 (18) ensure that the total number of caregivers working on day $d \in D$, shift $s \in S$, and district
 477 $c \in C$ is equal to the demand subject to some adjustments related to demand under-covering
 478 and over-covering. Constraints (19)-(21) set the non-negativity and integrality of variables
 479 $x_{dsc}^{e(w)}$ and $z_{ds}^{e(w)}$, and the non-negativity of variables $s_{dsc}^{+(w)}$ and $s_{dsc}^{-(w)}$.

480 Since we assumed that the number of visits per day, per shift, and per district is a random
 481 variable with finite support, where Ω_d is the set of scenarios for the demand at each day and
 482 $p_d^{(w)} > 0$ is the probability of occurrence of scenario $w \in \Omega_d$, the expected recourse function
 483 $\mathcal{Q}(\mathbf{y})$ can be expressed as:

$$\mathcal{Q}(\mathbf{y}) \equiv \sum_{d \in D} \mathbb{E}_\xi[\mathcal{Q}(\mathbf{y}, \xi_d)] \equiv \sum_{d \in D} \sum_{w \in \Omega_d} p_d^{(w)} \mathcal{Q}(\mathbf{y}, \xi_d(w)) \quad (22)$$

484 With this result, recourse functions (13)-(21) can be incorporated in (4)-(12) to obtain an
 485 *deterministic equivalent problem* given by:

$$\begin{aligned}
 f(\mathcal{Z}) = \min & \sum_{d \in D} \sum_{s \in S} \sum_{e \in E} c_{ds}^e y_{ds}^e + \sum_{d \in D} \sum_{e \in E} \sum_{A_{dl}^{\Pi,k,e} \in A^e} c_{dl}^{\Pi,k,e} v_{dl}^{\Pi,k,e} + \\
 & \sum_{d \in D} \sum_{w \in \Omega_d} p_d^{(w)} \left(\sum_{e \in E} \sum_{s \in S} \sum_{c \in C} t_{dsc}^e x_{dsc}^{e(w)} + \sum_{e \in E} \sum_{s \in S} r_{ds}^e z_{ds}^{e(w)} \right) \\
 & \sum_{d \in D} \sum_{w \in \Omega_d} p_d^{(w)} \left(\sum_{s \in S} \sum_{c \in C} c_{dsc}^+ s_{dsc}^{+(w)} + c_{dsc}^- s_{dsc}^{-(w)} \right) \\
 & \text{(5) - (12) and} \\
 & \text{(14) - (21), } \forall d \in D, w \in \Omega_d.
 \end{aligned}$$

486 Observe that model \mathcal{Z} could involve a large number of variables and constraints, espe-
 487 cially when the number of days in the planning horizon is large. However, since context-free
 488 grammars allow to handle multiple shift types and to represent complex work regulations in
 489 an implicit (compact) way, and since the size of the model does not depend on the number of
 490 caregivers to hire at each district, model \mathcal{Z} can be efficiently solved for large instances without
 491 the need of decomposition methods.

492 4. Computational Experiments

493 In this section, we test the proposed approach on real-world instances from a home health-
 494 care agency working with AlayaCare. First, we present information related to the agency's
 495 operations and to the rules for schedule generation. Second, we describe the procedure adopted
 496 for the generation of the instances and present the size of these instances. Third, we report
 497 and analyze the computational results and present a discussion on the practical aspects and
 498 managerial insights of the proposed approach.

499 The computational experiments were performed on a Linux operating system, 16 GB of
 500 RAM and 1 processor Intel Xeon X5675 running at 3.07GHz. The algorithm to solve the
 501 problem was implemented in C++. The deterministic equivalent problem \mathcal{Z} was solved with
 502 CPLEX version 12.7.0.0. The time limit to solve each instance is proportional to the length
 503 of the planning horizon. For example, if a given instance is defined over 4 weeks, the time
 504 limit is set to 2 hours. Similarly, if a given instance is defined over 12 weeks, the time limit
 505 is set to 6 hours. A relative gap tolerance of 0.01 was set as a stopping criterion for solving
 506 the MILPs with CPLEX.

507 4.1. Operations and Schedule Generation

508 • Operations: The test instances are generated based on 8-month historical data from
 509 operations of one private agency operating in Greater Toronto Area. This region is
 510 divided in four districts (i.e. $|C| = 4$). The agency operates in these districts 24
 511 hours per day from Monday to Sunday. We only consider the staffing and scheduling
 512 of personal support workers, as they represent the largest portion of employees in the
 513 agency (70% of the total number of caregivers). Based on the agency’s operations we
 514 defined four types of shifts: morning shifts of type 1 (denoted as m_8) starting at 7:00
 515 with an 8-hour length; morning shifts of type 2 (denoted as m_4) starting at 10:00 with
 516 a 4-hour length; afternoon shifts (denoted as a_4) starting at 14:00 with a 4-hour length;
 517 and night shifts (denoted as n_{10}) starting at 18:00 with a 10-hour length. We assume
 518 that the base cost of each working time interval is 1\$ and that the shift allocation
 519 cost depends on the shift length, as well as on the day covered (weekend shifts are more
 520 expensive than weekday shifts). Because one of the objectives of the agency is to increase
 521 the service level, demand under-covering costs are set to a large value equal to the cost of
 522 each visit (c_{ds}^e/κ_{ds}^e) multiplied by 10. Similarly, the costs for the demand over-covering
 523 are equivalent to the cost of each visit (c_{ds}^e/κ_{ds}^e) multiplied by 0.5. The values for these
 524 costs, for the capacities of shifts, as well as other parameters characterizing each type
 525 of shift are presented in Table 3. Observe that the costs presented in this table do
 526 not consider a 20% surcharge for weekend days. In addition, the cost of contacting a
 527 caregiver to work on a day-off is $r_{ds}^e = c_{ds}^e * 2$, the surcharge for allowing transitions
 528 between districts is $t_{dsc}^e = 10\%$, and the transition costs between forbidden shifts is
 529 equal to $c_{dl}^{\Pi,k,e} = 1000\$$.

Parameter	Shift			
	m_8	m_4	a_4	n_{10}
Shift allocation cost c_{ds}^e (\$)	8	4	4	10
Under-covering cost c_{dsc}^- (\$)	40	40	20	100
Over-covering cost c_{dsc}^+ (\$)	2	2	1	5
Capacity κ_{ds}^e (number of visits)	2	1	2	1
Max_days	6	4	4	3

Table 3: Costs and capacity values for each type of shift.

District	District			
	d_0	d_1	d_2	d_3
d_0	1	0	0	0
d_1	0	1	1	0
d_2	0	1	1	1
d_3	0	0	1	1

Table 4: District compatibilities.

- 530 • Schedule composition: The work regulations for the schedule composition are the fol-
531 lowing
- 532 1. The minimum and maximum number of days in each work stretch are 4 and 6,
533 respectively.
 - 534 2. The minimum and maximum number of days in each rest stretch are 1 and 3,
535 respectively.
 - 536 3. A rest stretch is necessary between two work stretches.
 - 537 4. Each shift has a maximum number of consecutive times it can appear in a work
538 sequence. These values are presented in row Max_days of Table 3. For instance, a
539 work stretch cannot contain more than 3 night shifts in a row.
- 540 • Grammar: Let w_s be a terminal symbol that defines working on shift $s \in S$. Let
541 r be a terminal symbol that represents a rest period. Let F and R be non-terminal
542 symbols representing work and rest stretches, respectively. Let s_u be the maximum
543 number of consecutive times shift s can appear in a work sequence. In productions
544 $\Pi \in P$, $\Pi \xrightarrow{ctr}_{[\min, \max]}$ restricts the subsequences generated by a given production to
545 a length between a minimum and maximum number of days, and ctr denotes a cost
546 associated with the production. The grammar and the productions that define valid
547 schedules for caregiver of type $e \in E$ during a planning horizon of four weeks are as
548 follows:

$$\begin{aligned}
G^e &= (\Sigma = (w_s \ \forall s \in S, r), \\
N &= (\mathcal{S}, F, H, J_s, J'_s, J_s^2, J_s^3 \ \forall s \in S, R), P, \mathcal{S}), \\
\mathcal{S}_{[28,28]} &\rightarrow RHR|RH|HR, \\
H &\rightarrow FRFRFRF, \\
F_{[4,6]} &\xrightarrow{ctr} J_s J'_s, \ \forall s \in S; F_{[4,6]} \rightarrow J_s^2 J_s^3, \ \forall s \in S, \\
J'_s &\xrightarrow{ctr} J_s J'_s, \ \forall s \in S, \ \forall s' \in S \setminus \{s\}; J'_s \rightarrow J_s^2 J_s^3, \ \forall s \in S, \ \forall s' \in S \setminus \{s\}; \\
J_{s[0,s_u]} &\rightarrow J_s^2 J_s^3, \ \forall s \in S, \\
J_s^2 &\rightarrow J_s^2 J_s^3, \ \forall s \in S; J_s^3 \rightarrow w_s, \ \forall s \in S; \\
R_{[1,3]} &\rightarrow rR; R \rightarrow r.
\end{aligned}$$

549 *4.2. Instances Generation and Size of Problems*

550 Three instances spanning planning horizons from 4 to 12 weeks and including 500 demand
551 scenarios were generated to test our model. These instances were built with the procedures
552 presented in Sections [3.1.1](#) and [3.1.2](#). Table [5](#) presents for each instance (denoted as I1, I2, and
553 I3), the number of or-nodes, the number of and-nodes, and the number of leaves in each DAG
554 $\Gamma^e, \forall e \in E$. This table also presents the number of variables and the number of constraints
555 for the first-stage and second-stage components of model \mathcal{Z} . Note that the size of the model
556 is not proportional to the number of caregivers, as the employee dimension is included in the
557 model in an implicit way.

	Instance		
	I1 (4 weeks)	I2 (8 weeks)	I3 (12 weeks)
Or-nodes	1,551	3,102	6,204
And-nodes	3,614	7,228	14,456
Leaves	140	280	560
First-stage constraints	6,208	12,416	24,832
Second-stage constraints	305,000	610,000	1,120,000
First-stage integer variables	15,016	30,032	60,064
Second-stage integer variables	560,000	1,120,000	2,240,000
Second-stage continuous variables	224,000	448,000	896,000

Table 5: Instances size.

558 Since the size and complexity of problem \mathcal{Z} increase with the number of scenarios, we
559 decided to perform an analysis to evaluate how staffing and scheduling decisions (including
560 a fraction of the scenarios) accommodate the real demand, and how these decisions react
561 when they are evaluated on all generated scenarios (500). Specifically, for each instance
562 we first solve problem \mathcal{Z} with a fraction of the scenarios (e.g., 50 out of 500) to get the

563 optimal solution for variables y_{ds}^e and y_{dr}^e . These optimal values are fixed in second-stage
564 problems (13)-(21), which are solved with the actual demand information and with all 500
565 scenarios. Table 6 presents the results for this evaluation on problem \mathcal{Z} including reallocation
566 of caregivers (Realloc. = 1) and contacting caregivers to work on a day-off (RestToW = 1).
567 For each type of instance (Instance) and number of scenarios (Scen.), we present the status of
568 the solution (Status), the recourse cost when the schedule is evaluated with the real demand
569 (Real.C), and the recourse cost when the schedule is evaluated with 500 scenarios (Recour.C).
570 The percentage increase in these two costs (Real.C and Recour.C) by using a fraction of the
571 scenarios is presented in columns %I.Real.C and %I.Recour.C. This percentage is computed
572 as: $\%I = 100 \times \frac{Cost - base_cost}{base_cost}$, where $Cost$ represents the value for Real.C and Recour.C, and
573 $base_cost$ denotes the recourse cost obtained after solving problem \mathcal{Z} with the largest possible
574 number of scenarios (300 for I1 instances, 125 for I2 instances, and 25 for I3 instances).

Instance	Scen.	Realloc.	RestToW	Status	Real.C (\$)	Recour.C (\$)	%I.Real.C	%I.Recour.C
I1	5	1	1	Optimal	1,679.6	2,297.34	14.63%	28.45%
I1	25	1	1	Optimal	1,505.8	1,807.29	2.77%	1.05%
I1	100	1	1	Optimal	1,464.6	1,801.97	-0.04%	0.75%
I1	150	1	1	Optimal	1,508.6	1,800.15	2.96%	0.65%
I1	200	1	1	Optimal	1,638.8	1,827.56	11.85%	2.18%
I1	250	1	1	Optimal	1,506.8	1,824.26	2.84%	2.0%
I1	300	1	1	Optimal	1,428	1,794.67	-2.54%	0.34%
I1	350	1	1	Optimal	1,567	1,835.46	6.95%	2.62%
I1	400	1	1	Optimal	1,503.8	1,827.55	2.63%	2.18%
I1	450	1	1	Optimal	1,465.2	1,788.53	0.0%	0.0%
I2	5	1	1	Optimal	3,829	4,048.22	24.77%	11.92%
I2	25	1	1	Optimal	3,106	3,832.19	1.21%	5.95%
I2	50	1	1	Optimal	3,212	3,786.17	4.67%	4.67%
I2	100	1	1	Optimal	3,100.8	3,685.55	1.04%	1.89%
I2	150	1	1	Optimal	3,062.2	3,618.67	-0.22%	0.04%
I2	200	1	1	Optimal	3,068.8	3,617.14	0%	0.0%
I3	5	1	1	Optimal	6,187.4	6,315.87	6.68%	9.18%
I3	25	1	1	Optimal	5,904.4	5,827.59	1.8%	0.74%
I3	50	1	1	Optimal	5,800	5,784.86	0%	0%

Table 6: Costs on stochastic instances for different number of scenarios.

575 To choose the number of scenarios that will be used in each instance we observed the values
576 for the percentage differences in the recourse costs (%I.Recour.C). Since these differences are
577 smaller than 0.5% for 300 scenarios for instances I1 and for 150 scenarios for instances I2, we
578 decided to set $|\Omega_d| = 300$ for I1 and to set $|\Omega_d| = 150$ for I2. Regarding instances I3, we set
579 $|\Omega_d|$ to 50 as the expected recourse cost (Recour.C) was smaller than the value of Recour.C
580 for the other number of scenarios (5 and 25), and as the model was not able to solve instances
581 with a larger number of scenarios.

582 *4.3. Computational Results*

583 In this section, we present the computational results after testing our model on real-world
 584 instances. First, we present the performance of the proposed model for different planning
 585 horizons. Second, we introduce an example to illustrate a typical output of the problem.
 586 Third, we analyze the impact of the type of recourse actions used in the costs and number of
 587 caregivers staffed. An analysis of the impact of schedule flexibility in the costs and number
 588 of caregivers staffed is presented at the end of this section.

589 Table 7 presents for each instance and each combination of recourse actions allowing care-
 590 giver reallocation (Realloc.) and working on a day-off (RestToW), the CPU time in seconds
 591 to solve the problem (Time), the status of the solution (Status), the total cost (Total.C), and
 592 the total number of caregivers to hire.

Instance	Scen.	Realloc.	RestToW	Time (s)	Status	Total.C (\$)	Caregivers				
							d_0	d_1	d_2	d_3	Total
I1	300	0	0	8.04	Optimal	12,858.8	4	43	6	22	75
I1	300	1	0	418.45	Optimal	11,773.5	4	39	13	15	71
I1	300	0	1	166.78	Optimal	11,528.5	4	39	5	20	68
I1	300	1	1	1,121.53	Optimal	10,843	4	37	11	17	69
I2	150	0	0	53.54	Optimal	24,990.7	4	43	6	21	74
I2	150	1	0	1,876.03	Optimal	22,620.7	4	38	13	14	69
I2	150	0	1	675.45	Optimal	21,441.6	3	38	5	17	63
I2	150	1	1	5,212.85	Optimal	20,116.4	4	35	11	14	64
I3	50	0	0	1,139.62	Optimal	35,995.5	4	34	6	17	61
I3	50	1	0	10,522.7	Optimal	32,341.8	4	30	11	13	58
I3	50	0	1	2,799.27	Optimal	30,033.5	3	30	6	16	55
I3	50	1	1	11,983.2	Optimal	27,805.5	3	27	10	12	52

Table 7: Computational effort and results on stochastic instances.

593 Results from Table 7 indicate that the computational effort increase with the length of the
 594 planning horizon, as well as with the flexibility related to the recourse actions. Observe that
 595 it was possible to find an optimal solution for all instances. We can conclude that the recourse
 596 action that contributes the most to an increase in the CPU time is allowing the reallocation
 597 of caregivers (Realloc. = 1). Specifically, for instances I1, I2, and I3 and when Realloc. =
 598 1 CPLEX was respectively 52, 35 and 10 times slower to solve the model when compared to
 599 solving the model with simple recourse, i.e. Realloc = 0 and RestToW = 0. When recourse
 600 RestToW is included in the model (contact caregivers to work on a day-off), these values
 601 increase to 140, 97, and 10 for instances I1, I2, and I3, respectively.

602 Results on staff dimensioning suggest that the number of caregivers to hire in districts
 603 d_0 , d_2 , and d_3 is very similar for instances spanning different planning horizons. However, for
 604 districts d_1 and d_3 we can observe some significative differences in the number of caregivers
 605 to hire (e.g., 27 caregivers for d_0 in instance I3 when Realloc = 1 and RestToW = 1 versus 37

606 caregivers for d_0 in instance I1 when $\text{Realloc} = 1$ and $\text{RestToW} = 1$). We remark that this
607 result might be due to forecasting errors and changes in the magnitude of demands from one
608 month to the other one.

609 *4.4. Example 2: Output Illustration*

610 Tables 8 and 9 present an example of the schedules and the use of recourse actions af-
611 ter solving the two-stage stochastic programming model on an instance including a 4-week
612 planning horizon and 10 scenarios. This example incorporates the use of recourse actions asso-
613 ciated with under-covering, with over-covering, with the reallocation of caregivers to neighbor
614 districts, and with contacting caregivers to work on a day-off. Table 8 shows four sched-
615 ules (one per district) including the shift and day-off allocation at each day in the planning
616 horizon. Recall that r represents the allocation of a day-off, and that m_4 , m_8 , a_4 , and n_{10}
617 denote different types of shifts. For instance, a caregiver hired to work in district 3 (d_3) will
618 be allocated in his last week to: afternoon shifts (a_4) in the first 2 days of the week; then he
619 will work in the next 2 days in night shifts (n_{10}); the caregiver will finish the week with 3
620 consecutive days-off (r).

	Date						
district	2017-07-03	2017-07-04	2017-07-05	2017-07-06	2017-07-07	2017-07-08	2017-07-09
d_0	m_8-d_0	m_8-d_0	m_8-d_0	m_4-d_0	m_4-d_0	r	m_8-d_0
d_1	m_4-d_1	m_4-d_1	m_4-d_1	m_8-d_1	m_8-d_1	r	r
d_2	r	r	r	m_4-d_2	m_4-d_2	$n_{10}-d_2$	$n_{10}-d_2$
d_3	m_4-d_3	m_4-d_3	a_4-d_3	a_4-d_3	r	r	r
	2017-07-10	2017-07-11	2017-07-12	2017-07-13	2017-07-14	2017-07-15	2017-07-16
d_0	m_8-d_0	$n_{10}-d_0$	$n_{10}-d_0$	r	r	r	m_8-d_0
d_1	r	m_4-d_1	m_4-d_1	m_8-d_1	m_8-d_1	r	r
d_2	r	r	r	m_8-d_2	m_8-d_2	$n_{10}-d_2$	$n_{10}-d_2$
d_3	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	r	r
	2017-07-17	2017-07-18	2017-07-19	2017-07-20	2017-07-21	2017-07-22	2017-07-23
d_0	m_8-d_0	$n_{10}-d_0$	$n_{10}-d_0$	r	r	r	m_8-d_0
d_1	r	m_4-d_1	m_4-d_1	m_8-d_1	m_8-d_1	r	r
d_2	$n_{10}-d_2$	r	r	r	m_4-d_2	m_4-d_2	$n_{10}-d_2$
d_3	m_8-d_3	m_8-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	r	r
	2017-07-24	2017-07-25	2017-07-26	2017-07-27	2017-07-28	2017-07-29	2017-07-30
d_0	m_8-d_0	m_8-d_0	$n_{10}-d_0$	$n_{10}-d_0$	r	r	r
d_1	r	m_8-d_1	m_8-d_1	m_8-d_1	a_4-d_1	a_4-d_1	r
d_2	$n_{10}-d_2$	r	m_4-d_2	m_4-d_2	m_4-d_2	m_8-d_2	m_8-d_2
d_3	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	r	r

Table 8: Example of the schedules obtained with the two-stage stochastic programming model.

621 The shift and day-off allocation of the schedule for d_3 is used as an example to show the
622 use of recourse actions related to the reallocation of caregivers to neighbor districts, and with
623 contacting caregivers to work on a day-off. Table 9 shows for each day of the week from
624 2017-07-24 to 2017-07-30 the changes in the schedules due to the recourse actions used for 10
625 demand scenarios. Values in bold indicate that a recourse action was used to protect against

626 uncertainty. For instance, during day 2017-07-24 and under scenario 10 the model decided to
627 include a district reallocation (a caregiver from district d_3 is reallocated to district d_1). In a
628 similar way, during day 2017-07-29 the model chose to use the recourse work on a day-off for
629 scenarios 2, 7, 9, and 10 (e.g., in scenario 2, a caregiver is called to work on his day-day in a
630 morning shift in district 3 (m_4-d_3)).

	Date						
	2017-07-24	2017-07-25	2017-07-26	2017-07-27	2017-07-28	2017-07-29	2017-07-30
Master schedule	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	r	r
Scen. 1	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	r	r
Scen. 2	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	m_4-d_3	r
Scen. 3	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	r	r
Scen. 4	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	r	r
Scen. 5	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	r	r
Scen. 6	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	r	r
Scen. 7	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	a_4-d_3	r
Scen. 8	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	r	r
Scen. 9	a_4-d_3	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	a_4-d_3	r
Scen. 10	a_4-d_1	a_4-d_3	$n_{10}-d_3$	$n_{10}-d_3$	r	$n_{10}-d_3$	r

Table 9: Illustration on the use of recourse actions in a schedule of a caregiver working in d_3 .

631 4.5. Assessing the Impact of Different Recourse Actions

632 In this section, we perform a comparison among the different types of recourse actions used
633 in the two-stage stochastic programming model. The impact of allowing caregiver reallocation
634 and working on a day-off is evaluated. Table 10 reports the percentage difference in the
635 total cost (%D.Total.C), the percentage difference in the scheduling cost (%D.Sched.C), the
636 percentage difference in the recourse cost (%D.Recour.C), and the percentage difference in the
637 total number of caregivers staffed (%D.Staff), when flexibility regarding the use of different
638 recourse actions is introduced in the model. These percentage differences are computed as
639 $\%D = 100 \times (Value - base_value) / base_value$. *Value* represents the final value for the total
640 cost, for the staffing cost, for the recourse cost, and for the total number of caregivers staffed,
641 and *base_value* denotes the value for the same attribute obtained after solving problem \mathcal{Z} (on
642 each instance I1, I2, and I3) with the base scenario. Since the base scenario corresponds to the
643 use of simple recourse in the second-stage model (i.e. only allowing demand under-covering
644 and over-covering) the differences in the recourse costs are mainly due to the reduction in
645 demand under-covering and over-covering costs.

Instance	Scen.	Realloc.	RestToW	%D.Total.C	%D.Sched.C	%D.Recour.C	%D.Staff
I1	300	1	0	-8.44%	-5.35%	-21.17%	-5.33%
I1	300	0	1	-10.35%	-12.7%	-0.65%	-9.33%
I1	300	1	1	-15.68%	-12.87%	-27.22%	-8%
I2	150	1	0	-9.48%	-6%	-23.03%	-6.76%
I2	150	0	1	-14.2%	-17.84%	-0.03%	-14.86%
I2	150	1	1	-19.5%	-17.36%	-27.85%	-13.51%
I3	50	1	0	-10.15%	-6.16%	-24.73%	-4.92%
I3	50	0	1	-16.56%	-18.91%	-7.99%	-9.84%
I3	50	1	1	-22.75%	-20.5%	-25.26%	-14.75%

Table 10: Impact of the type of recourse action used in the costs and number of caregivers staffed.

646 Results from Table [10](#) suggest that the introduction of flexibility in the use of recourse
647 actions significantly reduces the total costs, as well as the number of caregivers staffed. These
648 reductions appear to be larger for instances spanning planning horizons of 8 weeks or longer
649 than for instances spanning 4 weeks. The recourse action with larger impact is contact care-
650 givers to work on a day-off, and when this recourse action is integrated with the reallocation
651 of caregivers, the reductions in costs become even larger. Solving an integrated problem
652 including all districts instead of solving independent problems for each district generates a
653 supplementary cost reduction, as well as an improvement in caregivers' utilization. In partic-
654 ular, allowing reallocation of caregivers to neighbor districts gives planners the flexibility to
655 occasionally use resources from other districts to respond to changes in demands.

656 4.6. Assessing the Impact of Schedule Flexibility

657 Since the two-stage stochastic programming problem becomes harder to solve with the
658 length of the planning horizon, we perform an analysis on the impact of reducing schedule
659 flexibility. Specifically, for instances including more than 4 weeks (I2 and I3), we solve the
660 two-stage stochastic programming problem by imposing schedules starting at week 5 to be
661 exactly the same as schedules from the previous 4 weeks. For instance, in a problem with a
662 8-week planning horizon, the schedules in week 5 must be the same as the schedules for week
663 1, the schedules in week 6 must be the same as the schedules for week 2, and so on.

664 Table [11](#) reports an analysis on the impact of schedule flexibility in the computational
665 effort and results of model \mathcal{Z} . In particular, this table presents a comparison of the CPU
666 times in seconds (Time (s)), of the total costs (Total.C), and of the number of caregivers
667 staffed (Total.Staff) when schedules are completely flexible (Flex.) and when the scheduling
668 flexibility is reduced (No.Flex) as explained above.

Instance	Scen.	Realloc.	RestToW	Time (s)		Total.C (\$)		Total.Staff	
				Flex	No.Flex	Flex	No.Flex	Flex	No.Flex
I2	150	0	0	53.54	12.36	24,990.7	25,879.5	74	71
I2	150	1	0	1,876.03	1,397.98	22,620.7	23,583	69	67
I2	150	0	1	675.45	547.97	21,441.6	22,344.3	63	62
I2	150	1	1	5,212.85	3,442.57	20,116.4	20,858.4	64	58
I3	50	0	0	1,139.62	14.6	35,995.5	40,120.3	61	70
I3	50	1	0	10,522.7	704.88	32,341.8	36,155	58	69
I3	50	0	1	2,799.27	533.79	30,033.5	33,006.6	55	57
I3	50	1	1	11,983.2	2,668.15	27,805.5	30,678.8	52	54

Table 11: Impact of schedule flexibility in the costs and number of caregivers staffed.

669 Results from Table 11 indicate that the method is in average 14 times faster when flexibility
670 in the allocation of schedules is limited. This speed-up is more substantial for instances with
671 type I3 as a longer time horizon is being considered. Observe that the total cost presents
672 an increase when there is less flexibility associated with the allocation of schedules, as the
673 two-stage model has less freedom to use recourse actions when needed. However, the number
674 of caregivers to hire shows a different behavior for instances I2 and I3. Specifically, in I2
675 instances the value of Total.Staff becomes smaller when the schedule flexibility is reduced. On
676 the contrary, when the schedule flexibility is reduced, the value of Total.Staff becomes larger
677 for instances I3. This might be explained by the fact that for short time horizons (8 weeks),
678 the model with less schedule flexibility (No.Flex) decides to hire less employees (even if this
679 means to have some extra under-covering) in order to reduce the employee underutilization
680 (visits over-covering). On the contrary, for longer time horizons (12 weeks) the No.Flex model
681 decides to hire more employees as this restriction in the schedule allocation might significantly
682 increase the visits under-covering and hence the total costs.

683 4.7. Value of the Stochastic Solution

684 The VSS is a standard measure that indicates the expected gain from solving a stochastic
685 model rather than its deterministic counterpart, the *expected value problem* (EV). The value of
686 the stochastic solution is defined as $VSS = EEV - RP$, where RP corresponds to the optimal
687 value of problem (4)-(12) and EEV corresponds to the expected value of using the EV solution.
688 EV is problem (4)-(12) evaluated using the mean scenario $\bar{\xi}_d = \hat{\mathbf{b}}_d$ for each day $d \in D$. Given
689 an EV solution $(\bar{\mathbf{y}}^*)$, EEV corresponds to: $EEV = \sum_{d \in D} \sum_{w \in \Omega_d} p_d^{(w)} \mathcal{Q}(\bar{\mathbf{y}}^*, \xi_d(\mathbf{w}))$. A large
690 VSS means that uncertainty is important for the quality of the resulting optimal solution. On
691 the contrary, a small VSS means that a deterministic approach based on the expected values
692 of the random variables might be sufficiently good to take a decision. The reader is referred
693 to Birge & Louveaux (2011) for an overview of stochastic programming.

694 Table 12 presents a comparison of the computational effort between the two-stage stochas-
695 tic programming model (denoted as Stochastic) and the mean value problem (denoted as De-

Instance	Scen.	Realloc.	RestToW	VSS_{Cost}	$VSS_{Scheduling}$	$VSS_{Recourse}$	VSS_{Staff}
I1	300	0	0	15.53%	-17.62%	60.89%	-13.64%
I1	300	1	0	14.12%	-11.34%	59.68%	-5.97%
I1	300	0	1	5.61%	-2.04%	25.75%	-1.49%
I1	300	1	1	5.12%	-1.74%	28.77%	-1.47%
I2	150	0	0	16.33%	-20.5%	61.8%	-17.46%
I2	150	1	0	14.62%	-13.8%	60.97%	-9.52%
I2	150	0	1	3.35%	0.86%	10.56%	0.0%
I2	150	1	1	4.27%	-0.2%	20.15%	-3.23%
I3	50	0	0	18.67%	-22.81%	63.58%	-19.61%
I3	50	1	0	17.25%	-15.61%	63.92%	-13.73%
I3	50	0	1	5.34%	0.26%	18.68%	-7.84%
I3	50	1	1	6.56%	0.64%	24.13%	-4%

707 schedules obtained after solving the deterministic problem are evaluated on the real demand
708 these schedules perform worse (as the Real.C is larger in most instances), when compared to
709 the performance of the schedules obtained with the stochastic problem. The differences in the
710 real cost between the deterministic model and the stochastic model are of great importance in
711 practice, since Real.C indicates how well the caregivers schedules react to the actual demand.
712 Since the majority of values for Real.C are lower when demand uncertainty is included in
713 the model, we can conclude that the schedules obtained with the stochastic model are more
714 robust than the schedules obtained with the deterministic model (EV problem).

715 We remark that since the schedules obtained with the stochastic model are usually more
716 robust than the schedules obtained with a deterministic model, Real.C is expected to be lower
717 when evaluated with the stochastic schedules than when evaluated with the deterministic
718 schedules. However, it may happen that in some cases this is not true. For example, in
719 instance I1 with $Realloc.=0$ and $RestToW=0$. In this case, what could have happened was that
720 the actual demand was very similar to the mean demand. Hence, when the dimensioning and
721 scheduling decisions obtained with the deterministic model are evaluated on a single instance
722 corresponding to the actual (observed) demand, Real.C is lower than the cost obtained with
723 the stochastic model.

724 Results from Table 13 suggest that the two-stage stochastic model can lead to significant
725 reductions in the total cost when compared to the mean value program, since all the VSSs
726 associated with the total cost are positive values ranging from 3.35% to 18.67%. This result
727 is mainly due to a reduction in the recourse costs associated with demand under-covering
728 and over-covering. Observe that some instances have negative VSS for the scheduling costs
729 ($VSS_{Sched.}$) and for the the staffing decisions (VSS_{Staff}). This means that the two-stage
730 stochastic model selects a larger workforce than the deterministic model, resulting in more
731 robust staffing and scheduling decisions that accommodate better to changes in demands.

732 4.8. Practical Aspects and Managerial Insights

733 The methodology developed in this paper represents an important and general decision
734 support tool for home care agencies interested in staff dimensioning and caregiver scheduling.
735 Specifically, our computational experiments indicate that:

- 736 • The design of robust staffing and scheduling decisions require the incorporation of un-
737 certainty in demands, as expected costs are smaller when uncertainty is included. This
738 is explained by the fact that opposite to deterministic models, the strength of stochastic
739 programming arises from the ability to represent solutions that protect against multiple
740 possible future outcomes (Birge, 1995). Hence, the aptitude to identify solutions that
741 handle or adapt best to the set of potential outcomes, relative to their probability of oc-
742 ccurring, is expected to generate costs that are smaller when compared to a deterministic
743 model when evaluated on several possible demand realizations.

- 744 • Including recourse actions such as allowing caregiver reallocation to neighbor districts
745 and working on a day-off significantly improves the costs associated with the dimen-
746 sioning decisions (staffing), as well as with demand under-covering and over-covering,
747 resulting in the improvement of caregiver utilization and quality of service.
- 748 • Solving an integrated problem including all districts instead of solving independent
749 problems for each district, generates supplementary cost reductions. In particular, al-
750 lowing reallocation of caregivers to neighbor districts gives planners the flexibility to
751 occasionally use resources from other districts to respond to changes in the demand or
752 in caregivers' availabilities.

753 Even though the case study was done for a specific agency from AlayaCare, this agency was
754 selected because it includes most of the key features of the consider problem (e.g., stochastic
755 demands, several geographic areas, different types of shifts, several work regulations for the
756 composition of schedules). Therefore, we believe that our study is general and that the
757 conclusions drawn from the computational experiments can be similar if the methodology is
758 tested in other practical cases.

759 The proposed model could be useful to evaluate the impact in costs and in the quality
760 of solutions by using different recourse actions. Specifically, recourse actions including the
761 allocation of overtime and the use of part-time caregivers could be tested to evaluate if an
762 increase in recourse flexibility helps to decrease the scheduling costs and demand under-
763 covering and over-covering costs. The model could also be used as a tool to detect the
764 lack/excess of caregivers due to changes in demand. For instance, given a fix number of
765 caregivers, the model will incur large under-covering costs if the size of the workforce is
766 inadequate to satisfy all patient visits when demand increases. On the contrary, the solutions
767 of the model will return large over-staffing costs if the size of the permanent workforce is
768 too large for the demand. Moreover, the two-stage stochastic programming model could
769 be extended to incorporate multiple types of caregivers with different skills, and to include
770 information about current employees with their preferences and availabilities.

771 Regarding the computational effort and limits of the two-stage stochastic programming
772 model, computational experiments indicate that the CPU time increases with the length
773 of the planning horizon, with the number of scenarios, and with the flexibility in recourse
774 actions. For each type of instance tested, we observed that the most important factor in this
775 computational time increase was the value of RestToW (i.e. contact caregivers to work on
776 their day-off) since problem \mathcal{Z} was in average 100 times slower when RestToW was set to 1.
777 We also observed that the computational time required to solve the problems can be reduced
778 by 5 times in average by limiting the schedule allocation flexibility. One idea to deal with
779 the computational limits of the method on larger planning horizons could be to use a rolling
780 horizon approach. In this way, the complexity of the problem will be reduced as this method

781 will gradually move along the planning horizon to incorporate stochastic information of the
782 demand.

783 The work presented in this paper has some limitations that could be addressed in future
784 work. These limitations are mainly related to the assumptions adopted to facilitate the
785 modeling and solution of the problem under study. For instance, assuming that the duration
786 of patients' visits and travel times are deterministic parameters could lead to suboptimal
787 solutions, especially if caregivers perform several short visits within one day and the variability
788 in these times is large. In the case of AlayaCare, this variability does not affect significantly
789 the solution of the problem, as most of the caregivers are personal support workers that
790 perform long visits during their shift. In addition, the practical use of the work presented
791 in this paper can be affected by assuming that caregivers will accept to work when called
792 during their day-off, since from time to time caregivers are free to reject this type of request
793 from their employer. Moreover, in a home care setting where caregiver absenteeism rates
794 are high, assuming that the workforce capacity is deterministic could lead to problems in
795 the implementation of the solutions obtained. The last limitation of this work is related to
796 the demand forecasting methods used, as other techniques could be explored to predict the
797 demand in a more accurate way.

798 **5. Concluding Remarks**

799 We presented a two-stage stochastic programming model for integrated staffing and schedul-
800 ing in home healthcare. In this model first-stage decisions correspond to staff dimensioning
801 and to the allocation caregivers to schedules. Second-stage decisions are related to the tem-
802 porary reallocation of caregivers to neighbor districts, to contact caregivers to work on their
803 day-off, and to allow under-covering and over-covering. Results on real-world instances show
804 that the use of the two-stage stochastic programming model helps to reduce demand under-
805 covering and over-covering costs when compared to a deterministic approach using the mean
806 demand. Moreover, computational results indicate that the use of flexible recourse actions
807 significantly reduces the total costs, improves caregiver utilization, and increases the level of
808 service.

809 An interesting avenue for future research is related to the development of specialized
810 solution methods to tackle larger instances commonly found in practice. Future research
811 could also include the use of different techniques for demand forecasting and for scenario
812 generation to assess the impact of demand estimation accuracy in the solutions obtained with
813 the two-stage stochastic programming problem.

814 **Acknowledgments**

815 The authors would like to thank MEDTEQ which supported this work a grant. The
816 authors would also like to thank to the data team at AlayaCare for their support in getting
817 access to anonymized data. We also thank the two anonymous referees and the Associate
818 Editor for their insightful comments and suggestions which allowed us to improve the paper.

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