Estimating Drill String Friction with Model-Based and Data-Driven Methods

Jean Auriol\textsuperscript{1}, Roman Shor\textsuperscript{2}, Silviu-Iulian Niculescu\textsuperscript{1} and Nasser Kazemi\textsuperscript{2}

Abstract—Estimation of the behavior of long dynamic systems with limited sensing remains an open question. In this paper, we consider the rotational motion of a deep drilling system and compare three algorithms to estimate the friction factors along the drillstring and thus provide an estimate of bottomhole rotational velocity. These friction terms characterize the interaction between the drill pipe and the wellbore walls (Coulomb source terms) within the curving wellbore. This information is essential to design the next generation of stick-slip mitigation controllers, to develop real-time wellbore monitoring tools, and to enable effective toolface control for directional drilling. We propose two model-based algorithms (an adaptive observer and a recursive dynamics framework) and a machine learning-based algorithm to estimate friction parameters, all of them presenting advantages and drawbacks. The performances of the two model-based estimators are finally compared with the data-driven neural network.

I. INTRODUCTION

Extraction of resources at depths greater than a few hundred meters in the earth’s subsurface - oil, gas, minerals, and thermal energy - necessitates the drilling of long slender boreholes from the surface to the subsurface target. The diameters of these wells range from 10 to 50 cm, and lengths can frequently exceed 10,000 m, leading to mechanical systems with extreme aspect ratios. The drilling system is comprised of a drill string comprised of multiple sections of steel drill pipe and stiffer drill collars and is actuated by a geared AC motor at surface controlled by a variable frequency drive (VFD). Transfer of energy (rotational and axial) often leads to complex dynamic behaviors, which manifest themselves as drill string vibrations that have been extensively studied; see [6], [17].

A particular vibration pattern that is prevalent is called stick-slip, where self-excited torsional oscillations manifest themselves in the drill string and are caused by friction along the wellbore between the drill pipe and the wellbore wall and by the bit-rock interaction. These stick-slip oscillations are generally characterized by a series of stopping – “sticking” – and releasing – “slipping” – events of the bit.

This phenomenon has been studied extensively and has been explained through a non-linear, or velocity-weakening - effect of bit-rock interaction [11], through distributed friction between the drill pipe and the wellbore wall [1] or through a combination of both [8], [17], [23]. This is of particular importance in modern wellbores, which are rarely straight and must follow pre-planned well plans, ranging from simpler horizontal or deviated wells to complex three-dimensional paths, thus increasing the effect of torque and drag through a variation in normal forces between the drillpipe and borehole wall.

II. MODEL UNDER CONSIDERATION

During directional drilling operations, limited and frequently latent downhole measurements are available. Typical tools contain a combination of accelerometers, gyroscopes and one to three-axis magnetometers. These sensors are sampled at frequencies between 1-100 Hertz at the downhole tool but only averaged or windowed values are transmitted to surface [13]. Due to severe limitations in bandwidth, these measurements are transmitted to the surface infrequently (once per minute to once per hour) and with a significant latency (on the order of seconds). Although this is not a problem for human-in-the-loop operations, it becomes insufficient for many automated solutions. Such delays and sampled measurements imply significant performance degradation when using feedback controllers that aim to compensate stick-slip oscillations [3]. Real-time estimation of friction parameters [14] enable feedforward [4] control as well as a new generation of controllers based on the backstepping methodology [15] and thereby allow better performance.

In the present work, we propose three strategies (two of them being completely original) to estimate these Coulomb friction factors. The two first algorithms are model-based (adaptive observer and recursive dynamics framework) while the last one is based on machine-learning. The model we use for the two first approaches is adjusted from [1]: the bit is off-bottom (we do not consider bit-rock interaction), and Coulomb friction is included as a source term implemented as an inclusion. This paper is structured as follows: it begins with a brief derivation of the drill string model in section II, followed by the description of three methods to estimate friction: (i) Section III describes the adaptive observer; (ii) Section IV presents the recursive dynamics framework, and (iii) Section V describes the machine learning approach. Finally, in Section VI the three approaches are compared with data generated by the model described in Section II. Some concluding remarks end the paper.

1 Jean Auriol, and Silviu-Iulian Niculescu are with Université Paris-Saclay, CNRS, CentraleSupelec, Inria, Laboratoire des Signaux et Systèmes, 91190, Gif-sur-Yvette, France. 2 Roman Shor and Nasser Kazemi are with the University of Calgary, Calgary, Alberta, Canada. Corresponding author: jean.auriol@centralesupelec.fr
A. Torsional dynamics of the drill string

The distributed model we present here is adjusted from [1]. This kind of representation is popular in the literature [6], [17]. We give a schematic representation of the drill string in Fig. 1. Let us denote the angular velocity and torque as \( \omega(x,t) \) and \( \tau(x,t) \), respectively, with \( (t,x) \in [0,\infty) \times [0,L] \) \((L \text{ being the length of the drill string)\). We have

\[
\frac{\partial \tau(t,x)}{\partial t} + J_G \frac{\partial \omega(t,x)}{\partial x} = 0 \tag{1}
\]

where, \( J \) is the polar moment of inertia, \( G \) the shear modulus and \( \rho \) the drill string density. The source term \( S(t,x) \) is given as

\[
S(t,x) = -k_1 \rho J \omega(t,x) - F(t,x), \tag{3}
\]

where the damping constant \( k_1 \) is the viscous shear stress and \( F \) is a differential inclusion that corresponds to the Coulomb friction between the drill string and the borehole, also known as the side force. We use the following inclusion to implement the side force

\[
\begin{cases}
F(t,x) = r_o(x) \mu_s F_N(x), & \omega(t,x) > \omega_c, \\
F(t,x) \in \pm r_o(x) \mu_s F_N(x), & |\omega(t,x)| < \omega_c, \\
F(t,x) = -r_o(x) \mu_k F_N(x), & \omega(t,x) < -\omega_c,
\end{cases} \tag{4}
\]

where \( \mu_s \) is the static friction coefficient (i.e. the friction between two or more solid objects that are not moving relative to each other) and \( \mu_k \) is the kinetic friction coefficient (also known as dynamic friction or sliding friction, which occurs when two objects are moving relative to each other and rub together). \( \omega_c \) is the threshold on the torsional velocity where the Coulomb friction transits from static to dynamic. The function \( F_N \) is the normal force acting between the drill string and the borehole wall. The function \( F(\omega) \in \pm r_o(x) \mu_s F_N(x) \) denotes the inclusion where

\[
F(t,x) = -\frac{\partial \tau(t,x)}{\partial x} - k_1 \rho J \omega(t,x)
\in [-r_o(x) \mu_s F_N(x), r_o(x) \mu_s F_N(x)],
\tag{5}
\]

and take the boundary values \( \pm \mu_s F_N(x) \) if this relation does not hold. The shape of the friction source term is illustrated in Fig. 1. The method to calculate normal force, to account for discontinuities of a multiple sectioned drill string and the Riemann invariants may be found in [1].

B. Boundary condition

We consider that the drilling device is actuated at the top-drive by a motor torque: \( \tau_m \). To reach a given reference velocity set-point \( \omega_{SP} \), we consider a standard PI control law [7]:

\[
\tau_m = k_p (\omega_{SP} - \omega) + k_i \int_0^t (\omega_{SP}(\xi) - \omega(\xi)) d\xi, \tag{6}
\]

where \( k_p \) is a proportional gain and \( k_i \) an integral gain. We denote \( J_{TD} \) the top drive inertia and \( \omega_o \) verifies the following equation

\[
\dot{\omega}_0 = J_{TD}^{-1} (\tau_m - \tau(t,0)). \tag{7}
\]

Finally, since the bit is off-bottom, we have \( \tau(t,L) = 0 \).

C. Estimation problem

As detailed in the introduction, the design of the next generation of stick-slip mitigation controllers requires reliable estimations of the friction parameters. In the rest of the paper, we present three methods to estimate all (or part) of the friction parameters. We emphasize their advantages and their drawbacks. Data measurements correspond to the top-drive torque and velocity.

III. FIRST APPROACH: ADAPTIVE OBSERVER

The first approach we propose is adjusted from the adaptive state-observer proposed in [2] (which can be adjusted to handle the case of a multi-sectional drill string [22]). This observer combines measurements from physical sensors (in particular top-drive angular velocity \( \omega_0 \)) with the proposed model of the system dynamics. It provides reliable estimates of the torque and RPM states and of the side forces friction parameters when the bit is off bottom [2]. We recall here the main ideas of this observer. Let us denote with \( \hat{\cdot} \) superscript the estimated states and \( e \) the measured estimation error of the top-drive angular velocity. The observer equations are given in [2] in terms of Riemann invariant \((\alpha_i, \beta_i)\) (the index \( i \) corresponds to the considered section of the drill-string). The Riemann invariants are the states corresponding to a transformation of the system which has a diagonalized transport matrix. We define them as \( \alpha_i = \omega_i + \frac{1}{\sqrt{\rho_G \tau_i}} \hat{r}_i \) and \( \beta_i = \omega_i - \frac{1}{\sqrt{\rho_G \tau_i}} \hat{r}_i \).

\[
\dot{\hat{\omega}}_0 = a_0 (\hat{\beta}_p(t,0) - \hat{\omega}_0) + \frac{1}{J_{TD}} \tau_m - p_0 e, \tag{8}
\]

\[
\frac{\partial \hat{\alpha}^i}{\partial t}(t,x) + (c_i) \frac{\partial \hat{\alpha}^i}{\partial x}(t,x) = \hat{\mathbf{S}}^i(t,x) - p_{\alpha}(x) e, \tag{9}
\]

\[
\frac{\partial \hat{\beta}^i}{\partial t}(t,x) - (c_i) \frac{\partial \hat{\beta}^i}{\partial x}(t,x) = \hat{\mathbf{S}}^i(t,x) - p_{\beta}(x) e, \tag{10}
\]

The source term in each section is computed from the estimated states and friction factors:

\[
\hat{\mathbf{S}}^i(t,x) = k_i (\hat{\alpha}^i(t,x) + \hat{\beta}^i(t,x)) + \frac{1}{J_i \rho^i} \hat{F}(t,x), \tag{11}
\]

The observers equations read as follows

\[
\dot{\hat{\alpha}}(t,0) = 2\hat{\omega}_0(t) - \hat{\beta}(t,0) - P_0 e,
\]
\[ \dot{\beta}(t, x_{i+1}) = \frac{\dot{\alpha}(t, x_{i+1})(1 - Z^i) + 2Z\dot{\beta}(t, x_{i+1})}{1 + Z^i} - P_l e, \]
\[ \dot{\alpha}^{i+1}(t, x_{i+1}) = \frac{2Z\dot{\alpha}(t, x_{i+1}) - \dot{\beta}^{i+1}(t, x_{i+1})(1 - Z^i)}{1 + Z^i}, \]
\[ \dot{\beta}(t, L) = \dot{\alpha}(t, L), \]
and the estimates of the friction factor are updated according to
\[ \dot{\hat{\mu}}_s(t) = \begin{cases} -l_s e, & \text{if } \min_x \hat{\omega}_x(t, x) \leq \hat{\omega}_c, \\ 0, & \text{if } \min_x \hat{\omega}_x(t, x) > \hat{\omega}_c, \end{cases} \]
\[ \dot{\hat{\mu}}_k(t) = \begin{cases} 0, & \text{if } \min_x \hat{\omega}_x(t, x) \leq \hat{\omega}_c, \\ l_k e, & \text{if } \min_x \hat{\omega}_x(t, x) > \hat{\omega}_c, \end{cases} \]

Finally, we use a saturation to improve the robustness of the approach: \( \tilde{\mu}_s \) and \( \tilde{\mu}_k \). The different constants \( a_0, p_0, p_1, p_2, P_0, P_1, l_s, l_k, h_s, h_k, \) can be found in [2]. The estimation procedure is summarized by Algorithm 1. The observer equations (8)-(13) correspond to a copy of the original dynamics expressed in the Riemann coordinates to which are added output correction terms. The convergence of the estimated states towards the real states is proved in [2] in the absence of friction terms. Note that the observer can be adjusted to handle the cutting action [8].

**Algorithm 1** Adapative observer

**Require:** \( \tau_m, \hat{\omega}_0, \omega_c \)

1. Simulate the observer with the adaptive laws (12)-(13)

**Output**
\[ \mu_k \leftarrow \lim_{\infty} \hat{\mu}_k(t), \quad \hat{\mu}_s \leftarrow \lim_{\infty} \hat{\mu}_s(t). \]

IV. SECOND APPROACH: RECURSIVE DYNAMICS FRAMEWORK

The second approach we propose corresponds to a recursive dynamics interconnection framework. Such a framework is particularly suited for sensing and estimation since it allows structuring the model on simpler and more realistic interconnected dynamical subsystems (blocks) and better exploiting the interconnection structure. Let us mention that if all the points of the drill string reach the kinematic mode, then the function \( \mathcal{F}(t, x) \), defined by (4), does not (directly) depend on time anymore. Adjusting the method developed in [9] (computations are omitted due to space constraints), we have
\[ \omega^i(t, x_{i+1}) = f^i(\omega^i(t, x_i), \tau^i(t, x_i)) + h^i_\omega \]
\[ \tau^i(t, x_{i+1}) = g^i(\omega^i(t, x_i), \tau^i(t, x_i)) + h^i_\tau, \]
where \( f^i \) and \( g^i \) are two functions that only depend on the physical parameters of the drilling device (except \( \mu_k \)) and past and future values of \( \omega^i(\cdot, x_i) \) and \( \tau^i(\cdot, x_i) \), the largest time-shift being fixed by the parameters of the system. The functions \( h^i_\omega \) and \( h^i_\tau \) are constant functions that linearly depend on \( \mu_k \). Using equations (14) and (15), we can express \( \tau(t, L) \) as a function of top-drive angular velocity and torque measurements. Since the bit is off-bottom, we know that \( \tau(t, L) = 0 \). Thus, it becomes possible to use classical regression techniques (such as Recursive Least Squares) to estimate the kinematic coefficient \( \mu_k \). Indeed, if we have \( M \) measurement points, this coefficient is estimated by minimizing the cost function \( \hat{\mu}_k \in \arg\min_{\mu_k} \sum_{i=1}^{M}(\hat{\tau}(t, L))^2 \), where \( \hat{\tau}(t, L) \) is the estimated version of the torque obtained using equations (14) and (15). Obviously, the proposed estimation of the torque-on-bit only holds if we have \( \omega > \omega_c \) all over the drill string (otherwise, non-linear terms appear, which makes the estimation extremely difficult). Thus, we need to apply equations (14) and (15) to all points of the drill string (in practice, to a sufficiently large number of points) to verify that it is the case. Among all the estimations of the drill-bit source signature, only the ones for which this condition is fulfilled can be used to estimate the kinematic friction term.

To verify that the condition \( \omega > \omega_c \) is satisfied, we need the knowledge of \( \omega_c \). However, knowing a decent upper-bound could be sufficient. More precisely, let us denote \( \bar{\omega}_c \) such an upper-bound. Applying the previous procedure for \( \omega > \bar{\omega}_c \) all over the drill string, we can estimate \( \mu_k \), assuming that enough data points verify these conditions (to apply the least-squares algorithm). Note that we can modify the reference set-point in the PI control law to guarantee this last condition. Then, since \( \mu_k \) is now known, we can reiterate the estimation procedure while decreasing the bound \( \omega_c \). Once the quadratic error \( \sum_{i=1}^{M}(\hat{\tau}(t, L))^2 \) becomes larger than a chosen threshold \( \epsilon \), it means that we have reached \( \omega_c \), since the non-linear static friction terms are now modifying our estimation of the torque on the bit. This procedure is described by Algorithm 2. The values of \( \omega_c \) are updated using a dichotomy approach (tolerance \( \eta \)). The threshold \( \epsilon \) should be large enough to encompass measurement noise.

**Algorithm 2** Recursive dynamics framework.

**Require:** \( \tau(t, 0), \omega(0), \hat{\omega}_0, \omega_c, \epsilon, \eta \)

1. Compute \( \hat{\tau}(t, L) \) with equations (14) and (15) for all the data points that verify \( \hat{\omega}(t, x) > \hat{\omega}_c \) for all \( t \).

2. **Recursive Least Squares algorithm:** \( \{\hat{\mu}_k^{\text{in}}\} = \arg\min_{\mu_k} \sum_{i=1}^{M}(\hat{\tau}(t, L))^2 \).

3. Update \( \hat{\omega}_c \): \( \hat{\omega}_c \leftarrow \min \hat{\omega}_x \).

4. Compute \( S = \sum_{i=1}^{M}(\hat{\tau}(t, L))^2 \) for all the data points that verify \( \hat{\omega}(t, x) > \hat{\omega}_c \).

5. If \( S > \epsilon \), \( \hat{\omega}_c \leftarrow \hat{\omega}_c^k \). Else, \( \hat{\omega}_c \leftarrow \hat{\omega}_c^k \).

6. If \( \hat{\omega}_c - \omega_c > \eta \) go to Step 3.

**Output**
\[ \mu_k \leftarrow \hat{\mu}_k, \quad \hat{\omega}_c \leftarrow \hat{\omega}_c^k. \]

V. THIRD APPROACH: MACHINE LEARNING ESTIMATION

The third approach we present in this paper is based on Machine Learning ([18]) where we show that a model trained on a range of depths and drillstring dimensions, can predict friction parameters for a new well. Hence, it does not require any specific knowledge of the system or an accurate model. Note that adaptive neural networks have already been applied in [16] or [26] for control purposes. The main idea behind this approach is that the measured top-drive torque and angular velocity (or in closed-loop the output of the PI control law) depend on the geometry and the physical parameters of the drilling device (and in particular of the friction factors). Thus, the spectrum of these signals should contain sufficient information to estimate the
Advantages

- Procedure to estimate
- Near-real time algorithm.
- Estimation of
- Convergence guaranteed in

details of the algorithm are summarized in Algorithm 3.

algorithm is now ready to be tested on a new dataset. The
layers, weights) and train it using
our neural network (i.e., we choose the number of neurons,
gains (and their corresponding frequencies). We then design
features. While training our neural network, it appeared that
the correct values of the friction parameters are known. Our
training set gives us a set of data (known as training set) for which
the well (including its geometry), we can run thousands
of simulations, only modifying the parameters we want to
estimate (namely \( \mu_k \) and \( \mu_s \)) between each simulation. This
gives us a set of data (known as training set) for which
the correct values of the friction parameters are known. Our
algorithm will learn from this data set and find the suitable
correlations between the previously defined attributes and the
unknown parameters. Once adequately trained, the algorithm
can be applied to make accurate predictions for new data sets.

We choose to develop a neural network regressor using a
sigmoid activation function [18], [21]. We use an optimizer
in the family of quasi-Newton methods (Adam algorithm
with 150 layers). The learning rate is constant. The network is
trained by simulating thousands of test points for which we
know the correct values of the friction parameters. From these simulations, we generate the spectrum of the
corresponding output signals and compute several relevant
features. While training our neural network, it appeared that
the relevant attributes for learning are the two dominant
gains (and their corresponding frequencies). We then design
our neural network (i.e., we choose the number of neurons,
layers, weights) and train it using cross-validation. The
algorithm is now ready to be tested on a new dataset. The
details of the algorithm are summarized in Algorithm 3.

Algorithm 3 ML algorithm

Require: \( \tau_m(t) \), training dataset \( \tau_m^k(t), \mu_k^p, \mu_s^p \)

Training of the network

1: Compute the Fourier transform of \( \tau_m^k(t) \)
2: Compute the two dominant gains and the corresponding frequencies
3: Using cross-validation, design a neural network

Estimation

4: Compute the Fourier transform of \( \tau_m(t) \)
5: Compute the two dominant gains and the corresponding frequencies
6: Run the neural network with this new point

Output

\( \hat{\mu}_s \leftarrow \mu_s, \hat{\mu}_k \leftarrow \mu_k \)

The generalizability of the proposed neural network depends on the number of learnable free parameters and the depth of the neural network [10], [27]. In most applications, the comprehensive dataset is not available and the learned network cannot properly model the complexities of the system. To remedy this shortcoming, one needs to take advantage of active learning, model-based learning, and transfer learning to make the learned mapping function

general enough so that the learned operator gives proper parameter estimation when applied on a new dataset [12], [25], [19].

VI. SIMULATION RESULTS

We now test our three estimation algorithms against the
simulation model given in [6] using the wellbore survey from
[1], where the well is vertical for 300m, builds to 40 degrees
inclination and holds until to a measure depth of 1250m and
then builds and holds at 45 degrees for the remainder of the well.

A. Comparison of the three algorithms on two test cases

We aim to compare the performance of our three different algorithms on test case studies. We have given in Table I
the theoretical advantages and drawbacks of the different
procedures. The drilling device is simulated using system (1)-(7). This model has been tested against field data in [5] and
has been shown to qualitatively (with high accuracy) capture the
dynamics apparent in the field data. Unfortunately, we do
currently have any field data for which the friction factors
are available. We compare the three estimation algorithms on two test case studies. All the numerical values for the
different parameters can be found in [6]. We consider a reference set-point for the top drive angular velocity of 60
RPM. In the first case study, the side forces static friction term is chosen to be equal to 0.42, while the kinematic
friction term is equal to 0.29. In the second one, we have
\( \mu_s = 0.45 \) and \( \mu_k = 0.28 \). In both cases, we have \( \omega_c = 14 \)
RPM. These parameters are chosen so that there are stick-slip oscillations [5]. We assume that the measured signals
are subject to a white Gaussian noise that is characterized by its signal-to-noise ratio (SNR). Seventy-five simulations and estimations have been performed for each algorithm. The
SNR for all the signals is equal to 10. We give in Table II and
Table III the mean and standard deviation for the estimations
given by each algorithm.

Our three algorithms allow satisfying estimations of the friction parameters. The two first algorithms are model-based
and require a reliable model of the drill string dynamics (which is not the case of the machine learning algorithm).
The Recursive dynamics framework-based appears (theoretically and in simulations) to provide the most accurate
estimation of the kinematic friction factor. Moreover, it gives an estimation of the threshold \( \omega_c \). This is because

TABLE I

<table>
<thead>
<tr>
<th>Algo.</th>
<th>Advantages</th>
<th>Drawbacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algo. 1</td>
<td>-Correct estimations ( \mu_s, \mu_k ) when testing in simulations and against field data [2]. -Real-time estimation robust to uncertainties and delays.</td>
<td>-Requires the knowledge of ( \omega_c ). -No proof of convergence for the adaptive part. -Persistent excitation for the adaptive part.</td>
</tr>
<tr>
<td>Algo. 2</td>
<td>-Reliable estimation of ( \mu_s ). -Procedure to estimate ( \omega_c ). -Good robustness properties. [9]</td>
<td>-Near-real time algorithm. -( \mu_s ) is not estimated -Computationally expensive (( \mu^2 )).</td>
</tr>
<tr>
<td>Algo.3</td>
<td>-Reliable, and easy to implement. -Estimation of ( \mu_k ) and ( \mu_s ). -Fast algorithm (once properly trained).</td>
<td>-Requires thousands of training points. -Lacks generalizability. -Depends on the initial condition of the system.</td>
</tr>
</tbody>
</table>

\[ \text{TABLE I: ADVANTAGES AND DRAWBACKS OF THE DIFFERENT ALGORITHMS} \]
TABLE II
MEAN AND STANDARD DEVIATION (IN PARENTHESES) OF THE FRICTION FACTORS ESTIMATION USING THE THREE PROPOSED ALGORITHMS FOR THE FIRST TEST CASE ($\mu_s = 0.34$ and $\mu_k = 0.187$).

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$\mu_s$</th>
<th>$\mu_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adaptive observer</td>
<td>0.338 (0.012)</td>
<td>0.192 (0.011)</td>
</tr>
<tr>
<td>Recursive dynamics framework</td>
<td>N.A.</td>
<td>0.1897 (0.035)</td>
</tr>
<tr>
<td>Machine learning algorithm</td>
<td>0.332 (0.025)</td>
<td>0.19 (0.041)</td>
</tr>
</tbody>
</table>

TABLE III
MEAN AND STANDARD DEVIATION (IN PARENTHESES) OF THE FRICTION FACTORS ESTIMATION USING THE THREE PROPOSED ALGORITHMS FOR THE SECOND TEST CASE ($\mu_s = 0.45$ and $\mu_k = 0.28$).

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$\mu_s$</th>
<th>$\mu_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adaptive observer</td>
<td>0.50 (0.015)</td>
<td>0.289 (0.004)</td>
</tr>
<tr>
<td>Recursive dynamics framework</td>
<td>N.A.</td>
<td>0.281 (0.003)</td>
</tr>
<tr>
<td>Machine learning algorithm</td>
<td>0.463 (0.03)</td>
<td>0.285 (0.02)</td>
</tr>
</tbody>
</table>

the disturbance terms can theoretically directly be expressed in terms of the available measurements (the mismatch in the simulations being the consequences of the presence of noise and uncertainties). However, it fails (in its current form) to provide an estimation of the static friction term (which explains the N.A. in Table II and Table III). In the case of a one-sectional drilling device, this could be overcome by considering a lumped approximation of the system [2]. Finally, even if the Machine Learning algorithm shows satisfactory performance, it requires sizeable training datasets, which can be time-consuming to both generate and then subsequently train. Note that since the proposed model has been tested against field data in [5], we believe that our machine learning algorithm can be efficiently trained using these simulated datasets.

B. Consequences in terms of control design

It is essential to mention that the PI controller that has been considered here (in equation (7)) may not be able to suppress stick-slip oscillations [4] (we chose it for its simplicity as the objective of the paper was to estimate friction parameters and not to compare different control strategies). More involved control strategies may require estimations of the distributed states, and at some point of the friction factors (which is one of the stakes of the present contribution) [4]. However, it is possible to envision a switching mode strategy to reduce the performance limitations related to a wrong estimation of the friction terms. We would impose a large enough control input to guarantee the BHA release from the stick phase in the first mode (using an improved PI controller as $Z$-torque [20]). Once we have left this mode (i.e., once we have broken the static torque and $\omega > \omega_s$ all over the drill string), we switch to the second mode, for which the disturbance term induced by the Coulomb side forces is now constant and only depends on the kinematic friction term (that is usually known more accurately).

C. Machine Learning Results

To extend the usefulness of the data-driven model, we considered expanding the training dataset to include a range of bit depths, leading to a range in static and kinematic torques. Using two training data sets, we evaluate the performance of the neural network on a test data set generated for the considered well. Training data set 1 (DS1) contains 1000 data points with parameters ranges of $\mu_s \in [0.2, 0.83]$, $f_{rat} \in [0.2, 0.65]$ (where $f_{rat}$ is such that $\mu_k = f_{rat} \mu_s$) and for well depths ranging from 2,500 m to 2,770 m. Training data set 2 (DS2) contains 1000 data points with the same ranges for $\mu_s$ and $f_{rat}$, however, well depths range from 1,700 m to 2,870 m. The test data set (TS) contains 125 data points with $\mu_s \in [0.21, 0.76]$, $f_{rat} \in [0.23, 0.6]$ and measure depths ranging from 1,760 m to 2,752 m. Using the algorithm described in Algorithm 3, a neural network is trained on DS1 (NN1) and on DS2 (NN2) and then applied to TS. As the test data set contains depths outside of the range of DS1, the performance of NN1 is poor, as expected. Bit depth and the weighted average of the inclination of the drill pipe are used as additional input parameters. However, the neural network cannot generalize the predictions as no implicit knowledge of drill string dynamics is embedded in the neural network. Figure 2 shows the actual vs predicted friction values for the training data set, with static friction shown in blue and kinematic friction shown in red. NN2 performs well when applied to the data in TS as the TS2 spans the input parameter range, for $\mu_s$, $\mu_k$ and bit depth. It is noted that estimates of static friction are worse than the estimated of kinematic friction, as is expected. Figure 3 shows the actual vs predicted values of friction for the training data set. This shows that the machine learning model
is generalizable so long as the training data set spans a sufficient parameter space. Future work in embedding some of the basic physical properties of the drill string system into the neural network may improve performance outside of the training parameter space. Some preliminary work has been in this space for seismic data processing, and inversion shows promise for application to problems governed by the wave equation [24].

D. Towards a combined methodology

Due to the couplings between the static and kinematic friction terms, the machine learning algorithm cannot correctly always handle the simultaneous estimation of \( \mu_b \) and \( \mu_s \) (see the results of Section VI-C). A possible solution to overcome this problem is to combine the machine learning procedure with one of the other estimation techniques (e.g., the adaptive observer). For instance, the static friction parameter can only be estimated when the drill string comes to a complete stop, which explains why it is harder to estimate using the adaptive observer. This occurs at the beginning of the rotation and then only during severe stick-slip. Using the estimated value of \( \mu_b \) (provided by the adaptive algorithm) as an input for our machine learning algorithm, we could improve the global estimation. Moreover, by fixing one parameter in the machine learning algorithm and training for the other one, we could explore the coupling issue and sensitivity of the machine learning operator to our Fourier features. Finally, when generating synthetic data with an approximated input for our machine learning algorithm, we could improve the generalizability of the operator. Generalizability is the main factor when applying the machine-learned operator to field data, which will be the purpose of our future works.

VII. Concluding remarks

In this paper, we have designed three algorithms that provide real-time or near real-time estimations of the friction factors corresponding to the side-forces acting on a drilling device. Estimating these parameters is a well-known challenge in the drilling industry and, if understood, presents the opportunity to improve drilling operations through feedforward or model predictive control. It is a crucial step to develop stick-slip mitigation strategies. The three proposed procedures correspond to an adaptive observer (originally introduced in [2]), to an algorithm based on recursive dynamics framework, and to a machine learning algorithm. All of them only require surface measurements, making them realistically implementable on drilling devices. Their specific advantages and drawbacks have been emphasized. These algorithms have then been implemented on a previously validated torsional drill string model, with specific attention paid to the machine learning procedure. This algorithm is easy to implement and fast to run once properly trained but may not be robust to changes in the system parameters (as a new long training phase would be necessary). The three algorithms provide satisfying estimations of the friction parameters. Future works should include testing against field data and an analysis of the computational effort for each procedure. The expected outcome is a hybrid strategy to estimate friction parameters in drilling.