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## Highlights

### **Influence of turbulent natural convection on heat transfer in shallow caves**

B. Qaddah, L. Soucasse, F. Doumenc, S. Mergui, Ph. Rivière, A. Soufiani

- Large-eddy simulations of natural convection in a confined shallow cave are performed using wall temperature distributions representative of different times of the year.
- We propose a general approach to predict heat transfer in a shallow confined cave, and analyse the role of convection and radiation.
- Two flow regimes are found depending on the direction of the mean vertical temperature gradient.
- Heat transfer coefficients are calculated from the simulations and the use of the Newton's law to predict the heat flux at cave walls is discussed.

# Influence of turbulent natural convection on heat transfer in shallow caves

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## Abstract

We aim at analyzing in detail the different heat transfer mechanisms involved in a confined shallow cave embedded in a rock massif submitted to seasonal variations of the ground temperature. Heat conduction in the rock massif, radiative heat transfer between cave walls, and turbulent natural convection inside the cave are considered. The natural convection problem is solved by large-eddy simulations (LES) using a Chebyshev pseudo-spectral method associated with a spectral vanishing viscosity (SVV) model. The thermal boundary conditions applied to the cave walls are obtained from a large-scale model that takes into account heat conduction in the rock massif and radiative fluxes between cave walls. This approach allows us to characterize the relative strength of convective and radiative fluxes and to identify the regions of the cavity and times of the year of intense heat transfer. We identified two different flow regimes: (i) a one-cell flow regime associated with strong convection, high turbulence level and unstable mean vertical temperature gradient, (ii) a multiple-cell flow regime associated with weak convection, low turbulence level and stable mean vertical temperature gradient. The use of the Newton's law to describe convection heat fluxes at the cavity walls is discussed.

*Keywords:* Shallow caves, Turbulent natural convection, Heat transfer, Large-eddy simulation, Spectral vanishing viscosity

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## 1. Introduction

Karsts are landscapes formed from the dissolution of soluble rocks, for instance limestone or gypsum [1]. The chemical erosion due to rainwater results in the formation of an extensive network of caves. Heat transfer in karstic massifs is at the core of many issues, as diverse as paleoclimate reconstruction from speleothem analysis [2], consequences of tunneling on the environment [3], evolution of subterranean flora and fauna [4], or conservation of parietal prehistoric paintings [5, 6]. However, assessing temperature fields and heat fluxes in karstic massifs is complicated by the coupling between several heat transfer mechanisms, as heat conduction in the rock and convection due to air and water circulation in caves [7], possibly with latent heat exchanges due to condensation/evaporation or even ice formation [8].

In this work, we focus on caves located at a shallow depth, typically on the order of 10 meters. This configuration corresponds to that of many painted caves in France (Lascaux [9], Marsoulas and Pech Merle [10]) and throughout the world (Altamira in Spain [5], Takamatsuzuka Tumulus in Japan [11]). The exceptional state of conservation of parietal paintings (some of which are more than 10,000 years old) is mainly due to the high stability of cave microclimate. Painting damages can occur when this microclimate is disturbed. Human visits may result in significant climate perturbations in a number of different ways, directly (increase in temperature, humidity and CO<sub>2</sub> concentration, resulting in enhanced condensation and corrosion on cave walls [12]) or indirectly (need for an artificial ventilation, modification of cave entrance to allow visits as in Lascaux [9] or Marsoulas [10]). In some cases, the cessation or the limitation of visits is not enough to restore favourable conditions for conservation, and remediation may be necessary (e.g., thermal insulation of the Takamatsuzuka Tumulus [11]). A deep understanding of the physical mechanisms driving heat transfer inside a cave and between a cave and its external environment is therefore necessary to improve the conservation of painting cave heritage.

The damping of the external temperature fluctuations by the rock massif surrounding a confined cave is the main reason for its high thermal stability (by confined cave, we mean a cave for which mass transfer with the external environment can be neglected). Quindos and coworkers [13, 5] measured the amplitude and phase shift of the annual temperature variations at the roofs of Altamira Cave, at different locations of varying depth (from 3.5 m to 17.5 m), and found a good agreement with the prediction of the periodic



38 1D conduction model. In the range of depth from a few meters to approxi-  
39 mately 10 m, daily temperature fluctuations can be considered as completely  
40 damped, while annual temperature fluctuations are still perceptible.

41 Due to complex cave geometry and depth variations all along the cave,  
42 temperature levels inside the cave (i.e., wall and air temperatures) vary not  
43 only in time, but also in space. Quindos *et al* [5] found that the amplitude and  
44 phase shift at the floors of Altamira Cave were close to the values measured at  
45 the roofs, which they attributed to radiative heat transfer between cave walls.  
46 Guerrier *et al* [14] confirmed by numerical simulations the significant role of  
47 thermal radiation in the homogenization of the temperature field inside a  
48 confined cave. However, spatial temperature variations are not strictly zero.  
49 For instance, temperature differences on the order of 0.1 K were commonly  
50 measured between the walls of the Hall of Bulls in Lascaux Cave [15]. This is  
51 enough to trigger significant natural convection flow. Indeed, assuming a cave  
52 height of 5 m, the Rayleigh number comparing buoyancy and diffusion is on  
53 the order of  $10^9$ , denoting possible turbulent flow at least in some parts of the  
54 cave [16, 17]. In addition, relative humidity is often close to 100 % in confined  
55 caves [5, 10], due to the presence of thin liquid films of percolating water on  
56 the walls and weak ventilation. Therefore, small temperature variations can  
57 induce condensation/evaporation at the walls, so that latent heat exchange  
58 must be considered. Due to seasonal variations of water intake [18], air  
59 humidity [5] and wall temperatures, condensation/evaporation mass fluxes  
60 are expected to vary in time. In conclusion, a minimal model for a shallow  
61 confined cave must consider heat conduction in the rock massif, radiative heat  
62 transfer between cave walls, and turbulent natural convection inside the cave.  
63 In addition, the significance of latent heat exchanges due to condensation and  
64 evaporation must be assessed.

65 The numerical investigation of cave climate is clearly restricted by the  
66 computational effort required to simulate 3D turbulent natural convection.  
67 If the gas is assumed to be transparent, radiative transfer between opaque  
68 walls can be efficiently computed using view factors. A simple approach  
69 to account for convection without solving the Navier–Stokes equations is to  
70 estimate the wall convective heat fluxes from the Newton’s law [11, 14]. A  
71 considerable drawback of this method is the need for empirical correlations to  
72 estimate heat transfer coefficients, whereas available correlations refer to cav-  
73 ities of simple geometry with uniform thermal boundary conditions on each  
74 wall [19]. These conditions are far from being fulfilled in natural cavities,  
75 making the estimation of heat transfer coefficients inaccurate. In contrast,

76 some authors rely on Computational Fluid Dynamics (CFD) to get better  
77 insights on convection in caves [6, 20]. Assuming laminar flow, Lacanette *et*  
78 *al.* [20] developed specific numerical methods to solve for air velocity, temper-  
79 ature and moisture fields in Lascaux cave. Two sets of prescribed boundary  
80 conditions, representative of climate conditions in 1980 and 1999, were con-  
81 sidered.

82 In this article, we aim at analyzing in detail the different mechanisms  
83 involved in heat transfer in a confined shallow cave embedded in a rock  
84 massif. Ideally, we should consider a problem where heat conduction in  
85 the rock, radiative transfer between the cave walls, and turbulent natural  
86 convection inside the cave are fully coupled, and solve a 1-year periodic  
87 regime. However, a one-year CFD simulation is not practicable with current  
88 computational resources. We thus proceed as follows. We first define a  
89 large-scale model, including heat conduction in the rock massif and radiative  
90 heat transfer between the cave walls, but neglecting convection inside the  
91 cave. Solving the periodic regime provides temperature fields in the rock  
92 massif (including the cave walls) all along the year. Then we select six wall  
93 temperature fields (spaced two months apart) representative of the different  
94 thermal states encountered in the cave over the year. These temperature  
95 fields are used as boundary conditions to solve the natural convection problem  
96 inside the cave by a detailed flow simulation.

97 The thermal conductive fluxes in the air at the cave walls (i.e., the ther-  
98 mal “convective” fluxes) obtained from the detailed flow simulation are then  
99 compared with the radiative fluxes predicted by the large-scale model. Two  
100 cases may arise:

- 101 • the thermal conductive fluxes at the cave walls are much smaller than  
102 the radiative fluxes, and disregarding convection in the large-scale  
103 model was a valid assumption. The temperature and velocity fields  
104 inside the cave are known from the detailed flow simulation. If hu-  
105 mid air was considered, the vapor concentration field in the cave and  
106 evaporation/condensation mass fluxes at the cave walls would also be  
107 known.
  
- 108 • the thermal conductive fluxes at the cave walls are larger than the ra-  
109 diative fluxes. In this case, natural convection significantly contributes  
110 to the uniformization of the wall temperature fields. Since this effect  
111 was not taken into account in the large-scale model, the intensity of  
112 the natural convection flow is likely overestimated. Conduction fluxes

113 at the cave walls can thus be regarded as higher bounds of the actual  
 114 ones, which is still a useful information. This approach also provides  
 115 higher bounds of evaporation/condensation mass fluxes when humid  
 116 air is considered.

117 For the sake of simplicity, we consider a parallelepiped cavity, but more  
 118 complex geometries could be treated with the same global approach. As a  
 119 first step, we only consider in this article the limiting case of dry air (latent  
 120 heat exchanges are thus disregarded). The large-scale model is solved using  
 121 the finite element method. The turbulent convection flow inside the cave  
 122 is obtained from large-eddy simulations (LES) performed with a Chebyshev  
 123 pseudo-spectral method associated with a spectral vanishing viscosity (SVV)  
 124 model [21].

125 The paper is organised as follows. We first describe the large-scale model  
 126 used to obtain temperature fields at cave walls (Sec. 2). Then we present  
 127 the LES model used for the simulation of the natural convection flow inside  
 128 the cave, and its numerical validation (Sec. 3). We discuss in section 4 the  
 129 different flow regimes observed depending on the season, as well as turbulent  
 130 statistics. We analyse in section 5 the heat flux distributions at the walls.  
 131 Conducto-convective fluxes are compared with radiative fluxes. Concluding  
 132 remarks are presented in section 6.

## 133 2. Large-scale model

### 134 2.1. Governing equations

135 The large-scale model is a 3D extension of the 2D model defined by  
 136 Guerrier *et al* [14]. We consider the confined parallelepiped cavity embedded  
 137 in the rock massif displayed in Fig. 1a. The ground surface is inclined at  $10^\circ$   
 138 from the horizontal direction. The left upper edge of the cavity is located at  
 139 a depth of 7.3 m. The cave dimensions are the height  $L_X = 5.3$  m, the width  
 140  $L_Y = 7$  m and the length  $L_Z = 17$  m (see Fig. 2a), which roughly reflect  
 141 the size of the Hall of Bulls in Lascaux Cave. The gravity acceleration field  
 142 corresponds to  $\mathbf{g} = -g\mathbf{X}$ .

143 Conductive heat transfer is assumed in the rock massif:

$$\frac{\partial T_r}{\partial t} = \alpha_r \nabla^2 T_r, \quad (1)$$

144 where  $T_r$  is the rock temperature,  $t$  the time and  $\alpha_r = 8 \times 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$  the  
 145 limestone diffusivity [14]. A time-periodic Dirichlet condition is imposed at

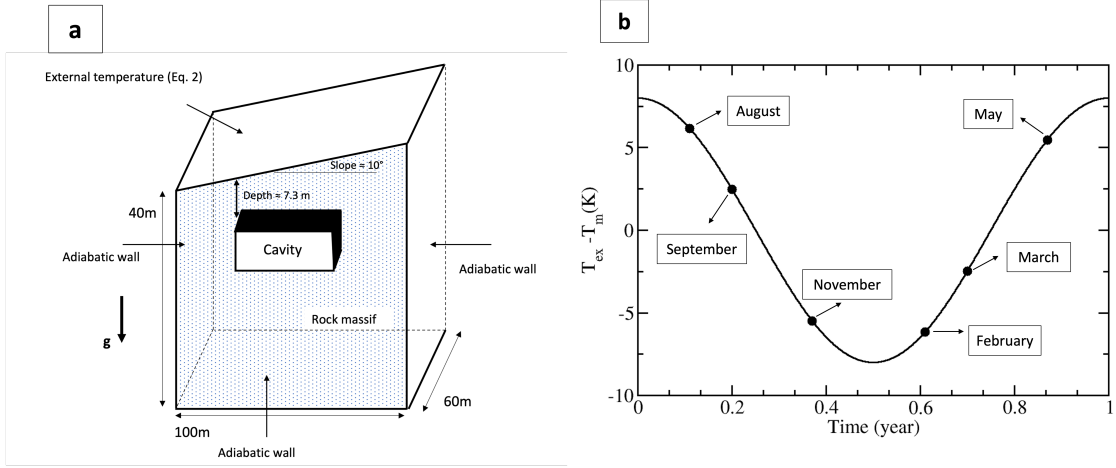


Figure 1: a) Geometry setup for the large-scale model (not at scale). b) Time evolution over a one-year period of the external temperature fluctuation  $T_{ex}(t) - T_m$  applied at the upper surface of the massif. The six filled circles correspond to the six months that we investigate (see the corresponding times in Tab. 1).

146 the upper surface of the rock massif (see Fig. 1a):

$$T_{ex}(t) = T_m + A \cos\left(2\pi \frac{t}{\tau}\right), \quad (2)$$

147 where  $T_{ex}$  is the external ambient temperature,  $\tau = 1$  year is the period,  
 148  $T_m = 12^\circ\text{C}$  is the annual average external temperature and  $A = 8^\circ\text{C}$  is the  
 149 amplitude of the temperature variations (these values of  $T_m$  and  $A$  are typ-  
 150 ical of the climate conditions in south-west of France). The time evolution  
 151 of  $T_{ex}$  is displayed in Fig. 1b, where the six months that will be investi-  
 152 gated using the LES model are highlighted. As we only consider the periodic  
 153 regime, we arbitrarily assume that the initial time corresponds to the hottest  
 154 temperature of the year that takes place in July.

155 The lateral and bottom sides of the massif are adiabatic. With the  
 156 approximation of black walls (the emissivity of limestone is 0.96 [19], i.e.,  
 157 close to 1), and disregarding convection as explained in the introduction, the  
 158 boundary condition at cave walls reads

$$-\lambda_r \nabla T_r \cdot \mathbf{n} = \sigma T^4 - \int_{\Omega \cdot \mathbf{n} < 0} I(\Omega) |\Omega \cdot \mathbf{n}| d\Omega, \quad (3)$$

159 where  $\lambda_r = 1.656 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  is the rock thermal conductivity [14],  $\mathbf{n}$  is the  
 160 normal vector pointing to the cavity,  $\sigma$  is the Stefan-Boltzmann constant

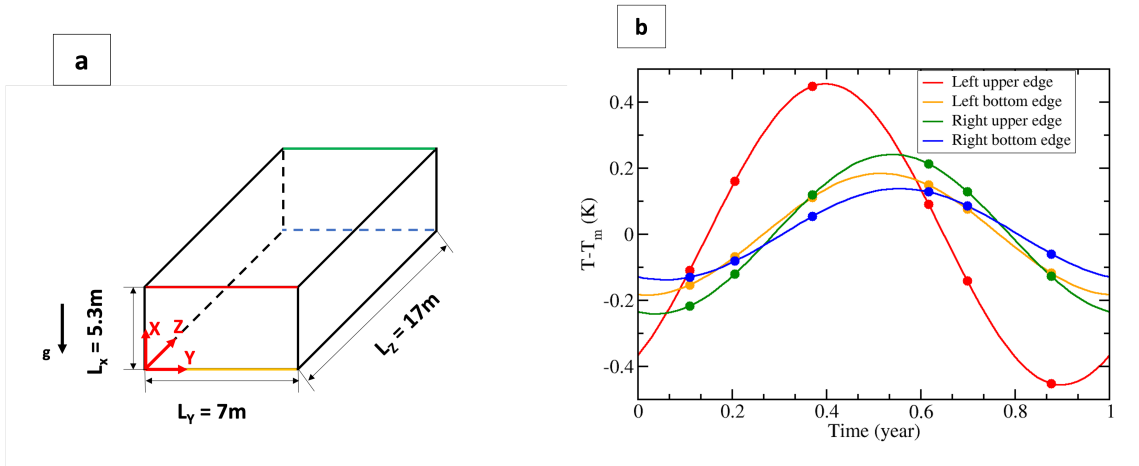


Figure 2: a) Geometry setup of the cavity. b) Time evolution of the temperature fluctuations  $T(t) - T_m$  averaged on four edges of the cave. Red, yellow, green and blue lines correspond to the left upper ( $Z = 0$  and  $X = L_X$ ), left bottom ( $Z = 0$  and  $X = 0$ ), right upper ( $Z = L_Z$  and  $X = L_X$ ) and right bottom ( $Z = L_Z$  and  $X = 0$ ) edges, respectively (the colors of the edges in Fig. 2a, and of the curves in Fig. 2b correspond to each other). The six months that we investigate are marked with filled circles.

161 and  $I(\boldsymbol{\Omega})$  is the radiative intensity (integrated over the infrared spectrum) in  
 162 direction  $\boldsymbol{\Omega}$ . The air is supposed to be transparent. Therefore,  $I(\boldsymbol{\Omega})$  depends  
 163 on wall temperatures, and does not depend on the temperature field of the  
 164 gas phase.

165 The model defined by Eqs.(1-3) is solved using the commercial software  
 166 Comsol Multiphysics (Galerkin method, time discretization based on implicit  
 167 backward differentiation formulas). The computational domain is discretized  
 168 with a total of approximately 530,000 quadratic Lagrangian tetrahedron el-  
 169 ements. The view factors related to surface elements on cave walls are cal-  
 170 culated using the hemicube method [22].

## 171 2.2. Results from the large-scale model

172 We discuss the six wall temperature fields resulting from the simulation  
 173 of the large-scale model, at months of the year indicated in Fig. 1b. The wall  
 174 temperature fields in February, March and May are displayed in Fig. 3. The  
 175 wall temperature fields for the months of August, September and November  
 176 (not shown) can be deduced by symmetry from those of February, March  
 177 and May respectively, thanks to the yearly periodicity. Despite the simple

178 geometry of the cave, the wall temperature fields are rather complex. Be-  
 179 cause of conductive damping in the rock and temperature uniformization by  
 180 radiative transfer inside the cavity, temperature gradients are larger along  
 181 the vault than along the floor, the latter being quasi-isothermal. The max-  
 182 imum wall-temperature difference  $\Delta T = T_{max} - T_{min}$  is reported in Tab. 1  
 183 for each month: it is minimal in February/August (0.124 K) and maximal in  
 184 May/November (0.492 K).

185 We can gain more insight by considering in Fig. 2b the time evolution  
 186 of the temperature averaged on the four edges highlighted in Fig. 2a. The  
 187 average temperature of the upper left edge, which is the closest to the ground  
 188 surface, evolves with larger amplitude and different phase shift compared to  
 189 that of the other edges, whose temperatures differ little from each other.  
 190 More specifically, the phase shifts of the left upper edge (depth  $d = 7.3$  m)  
 191 and of the right upper edge (depth  $d = 10.3$  m) are respectively 0.40 and  
 192 0.53 year. This is close to the values 0.41 and 0.58 year predicted by a 1D  
 193 semi-infinite model for which the phase shift is  $d\sqrt{\tau/(4\pi\alpha_r)}$ . The complexity  
 194 of the wall temperature fields thus results from the small  $10^\circ$  slope between  
 195 the ground and the horizontal plane (see Fig. 1a). Consequences on the flow  
 196 structure will be analysed in Sec. 4 by using these wall temperature fields as  
 197 thermal boundary conditions in the LES.

### 198 3. Large-eddy simulation model

#### 199 3.1. Governing equations

200 The air filling the cavity is assumed to be dry, transparent and at at-  
 201 mospheric pressure. Following the Boussinesq approximation, the physical  
 202 properties of the fluid are assumed to remain constant, except in the buoy-  
 203 ancy term of the momentum equation where the density is assumed to vary  
 204 linearly with temperature. The natural convection flow induced in the cavity  
 205 is therefore governed by the following dimensionless equations:

$$\nabla^* \cdot \mathbf{u}^* = 0, \quad (4)$$

$$\frac{\partial \mathbf{u}^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla^* \mathbf{u}^* = -\nabla^* p^* + Pr T^* \mathbf{x}^* + \frac{Pr}{Ra^{0.5}} \nabla^{*2} \mathbf{u}^* \quad (5)$$

$$\frac{\partial T^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla^* T^* = \frac{1}{Ra^{0.5}} \nabla^{*2} T^*, \quad (6)$$

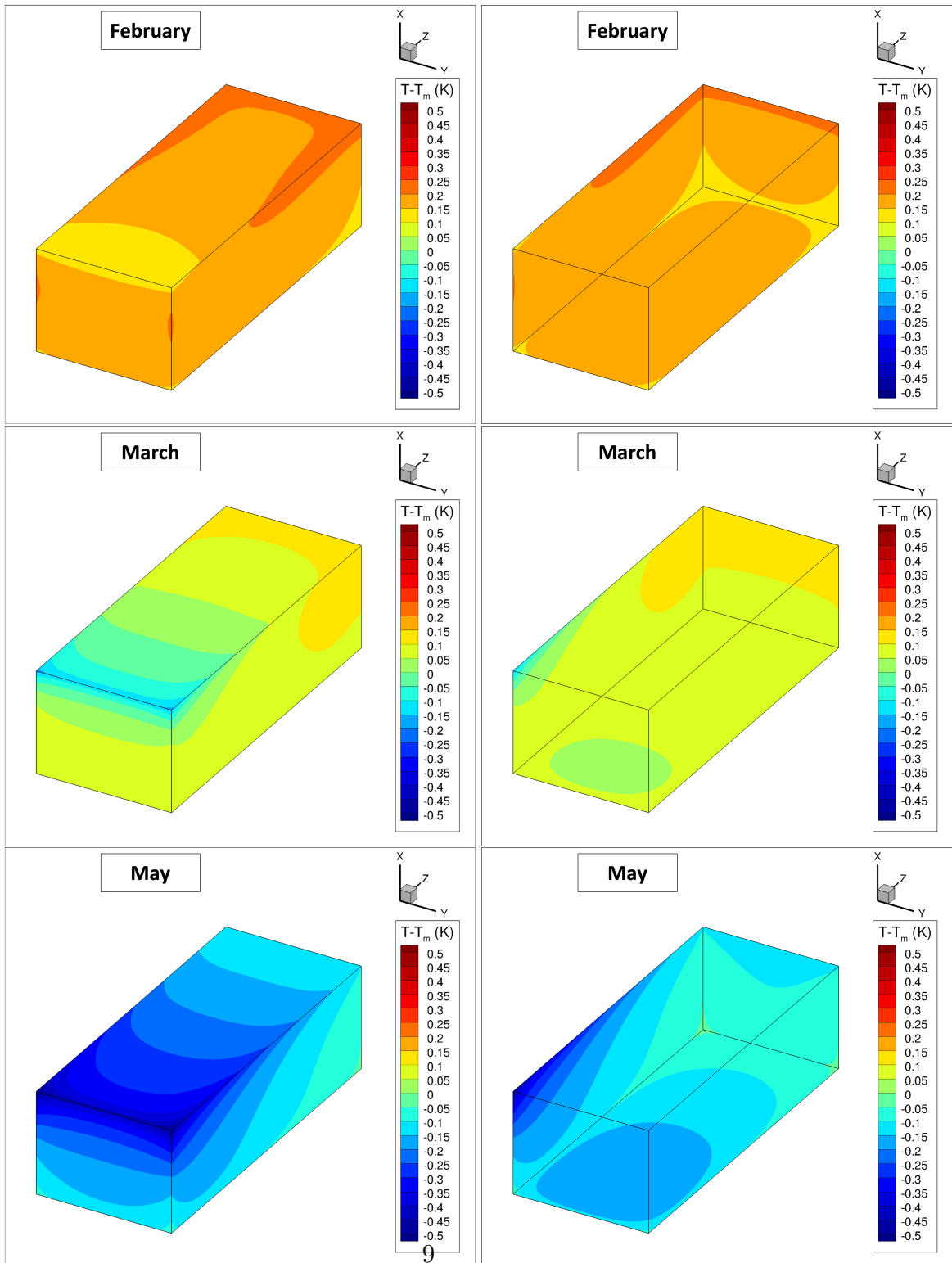


Figure 3: Wall temperature fields  $T - T_m$  computed from the large-scale model for three months: February, March and May, from top to bottom. The left panels correspond to the upper ( $X = L_X$ ), left ( $Z = 0$ ) and front ( $Y = L_Y$ ) cave walls. The right panels correspond to the bottom ( $X = 0$ ), right ( $Z = L_Z$ ) and back ( $Y = 0$ ) cave walls.

206 where \* denotes dimensionless variables and  $\mathbf{u}$ ,  $p$ ,  $T$  are respectively the  
 207 velocity vector, the motion pressure and the temperature. Equations are  
 208 made dimensionless using the reference height  $L_X$ , the reference time  $t_{ref} =$   
 209  $L_X^2/(\alpha Ra^{0.5})$ , and the reference temperature scale  $\Delta T$ . The reduced tem-  
 210 perature thus reads  $T^* = (T - T_0)/\Delta T$ , where  $T_0 = (T_{max} + T_{min})/2$  is  
 211 the reference temperature.  $Pr = \nu/\alpha = 0.712$ , and  $Ra = g\beta\Delta TL_X^3/(\alpha\nu)$   
 212 are respectively the Prandtl number, and the Rayleigh number, where  $\alpha =$   
 213  $2.05 \times 10^{-5} \text{ m}^2.\text{s}^{-1}$  is the thermal diffusivity,  $\nu = 1.46 \times 10^{-5} \text{ m}^2.\text{s}^{-1}$  is the  
 214 kinematic viscosity and  $\beta = T_0^{-1}$  is the thermal expansion coefficient.

215 Hydrodynamic and thermal boundary conditions at cave walls are zero  
 216 velocity (no-slip) and one of the temperature fields provided by the large-  
 217 scale model. Wall temperatures are assumed to be time-independent. This  
 218 is justified by the large time scale associated with the time evolution of the  
 219 wall temperature compared to the small characteristic time scale associated  
 220 with the convection inside the cave. Indeed, the characteristic time scale of  
 221 conduction in the rock for a depth equal to 7.3 m is about 2 years, which is of  
 222 the same order as the period of 1 year characterizing the change of external  
 223 boundary conditions, while the circulation time (or convection time), based  
 224 on the computed velocities is of the order of a few minutes. The time required  
 225 to reach statistically steady flows in the numerical simulations is of the order  
 226 of 1 hour, which is also much smaller than one year.

227 One of the main outcome of the simulation is the distribution of the  
 228 conductive flux at the walls, to be compared with the radiative flux from the  
 229 large-scale model. The conductive flux through the walls in the gas phase  
 230 reads

$$q_{con} = -\lambda \nabla T \cdot \mathbf{n}, \quad (7)$$

231 where  $\lambda = 2.51 \times 10^{-2} \text{ W.m}^{-1}.\text{K}^{-1}$  is the air conductivity. The dimensionless  
 232 counterpart of Eq. (7) is

$$Nu = -\nabla^* T^* \cdot \mathbf{n}, \quad (8)$$

233 where  $Nu = q_{con} L_X / (\lambda \Delta T)$  is the Nusselt number at the walls.

234 The values of  $\Delta T$ ,  $T_0$  and  $Ra$  for the six cases considered in the LES are  
 235 listed in Tab. 1. Notice that similar values of  $Ra$  may correspond to different  
 236 wall temperature fields resulting in different flow structures, as will be seen  
 237 in section 4.



Time	Month	$\Delta T$ (K)	$T_0$ (K)	$Ra \times 10^{-9}$
0.11 $\tau$	August	0.124	284.985	2.13
0.20 $\tau$	September	0.312	285.165	5.35
0.37 $\tau$	November	0.492	285.419	8.41
0.61 $\tau$	February	0.124	285.315	2.13
0.70 $\tau$	March	0.312	285.135	5.35
0.87 $\tau$	May	0.492	284.881	8.43

Table 1: Maximum temperature difference  $\Delta T$ , reference temperature  $T_0$  and Rayleigh number  $Ra$  for the six wall temperature fields used as boundary conditions in the LES ( $\tau = 1$  year).

238 *3.2. Numerical methods*

239 The large-eddy simulation (LES) approach is used in this study to save  
240 computational time. Simulations are performed using a Chebyshev pseudo-  
241 spectral method (detailed in Sec. 3.2.1), associated with a spectral vanishing  
242 viscosity (SVV) method (detailed in Sec. 3.2.2) to model the effects of the  
243 subgrid scales. In Sec. 3.3, we analyse the sensitivity of LES results to SVV  
244 parameters and we compare LES results with Direct Numerical Simulation  
245 (DNS) results in February, which corresponds to the smallest Rayleigh num-  
246 ber investigated.

247 *3.2.1. Chebyshev pseudo-spectral method*

248 The flow governing equations are implemented in a spectral code close  
249 to the one developed by Xin and Le Quéré [23], using a Chebyshev collocation  
250 method for the three spatial dimensions. This type of spectral method  
251 reaches a high spatial accuracy at a reasonable numerical cost. It assumes  
252 that the required solution is represented on a finite basis of orthogonal func-  
253 tions. The basis functions considered for the spatial discretization are the  
254 Chebyshev polynomials, suitable for the development of non-periodic func-  
255 tions. A projection method is used to ensure the pressure-flow coupling:  
256 first, the momentum and heat equations are solved using the pressure field  
257 from the previous time step; second, a pressure correction term is calculated  
258 from a Poisson equation and the predicted velocity is corrected to force the  
259 velocity divergence free condition. Time integration is performed using a  
260 second-order semi-implicit temporal scheme, coupling a backward differenti-  
261 ation (BDF2) scheme for the linear diffusion terms and an Adams Bashforth

262 extrapolation for the convective terms. Moreover, the computational domain  
 263 is decomposed along the largest spatial direction (here the  $Z$ -horizontal di-  
 264 rection) in order to perform parallel computations [24].

### 265 3.2.2. Spectral vanishing viscosity model

266 LES involves filtering the Navier-Stokes equations and solving only the  
 267 large scales of a turbulent flow. The small scales of the flow are not solved but  
 268 must be modeled. According to the filtering operation applied to the equa-  
 269 tions (4)-(6), additional non-linear terms are generated with supplementary  
 270 unknowns to be modeled by expressing them in terms of the filtered variables.  
 271 For the momentum and heat transfer equations, the dimensionless supple-  
 272 mentary terms are the subgrid scale stress tensor  $\nabla^* \cdot \boldsymbol{\tau}_{LES}^* = \nabla^* \cdot (\overline{\mathbf{u}^* \mathbf{u}^*} -$   
 273  $\overline{\mathbf{u}^*} \overline{\mathbf{u}^*})$  and the subgrid scale heat flux  $\nabla^* \cdot \mathbf{q}_{LES}^* = \nabla^* \cdot (\overline{\mathbf{u}^* T^*} - \overline{\mathbf{u}^*} \overline{T^*})$  where  
 274  $\bar{a}$  denotes the spatial filtering operator of the variable  $a$ . These terms are  
 275 usually modelled as diffusion terms [25].

276 Conventional LES models based on a subgrid viscosity are not suited  
 277 for spectral methods, and we rely in this work on the Spectral Vanishing  
 278 Viscosity (SVV) method [26, 27], which has been specially developed for  
 279 them. It consists in the introduction of an artificial dissipation term to ensure  
 280 the spectral convergence, i.e. dissipate the high modes of the Chebyshev  
 281 polynomial development. The main feature of the SVV method is to maintain  
 282 the spectral accuracy i.e. the exponential rate of convergence of the numerical  
 283 solution [21]. This SVV method has been used for several applications such  
 284 as turbulent channel flows [28], turbulent flows within rotating cavities [29]  
 285 and turbulent wakes [30].

286 The SVV method is implemented in the form of a modified Laplacian  
 287 operator combining the viscous and the SVV terms [21]. Hence, the modified  
 288 Laplacian operator  $\nabla_{SVV}^2$  is given by

$$\nabla_{SVV}^2 = \nabla \cdot (1 + \nu^{-1} Q) \nabla, \quad (9)$$

289 where  $Q$  is the viscosity kernel and  $\nu$  is the actual viscosity (in dimensionless  
 290 form, equal to  $PrRa^{-0.5}$  for momentum and  $Ra^{-0.5}$  for energy equation).  
 291 The viscosity kernel acts on each spatial direction independently. In spectral  
 292 space, it is given for the  $i^{\text{th}}$  direction by

$$\hat{Q}_i(k) = \epsilon_i e^{\frac{-(k-N_i)^2}{(k-M_i)^2}}, \quad \text{if } k > M_i \quad (10)$$

$$\hat{Q}_i(k) = 0, \quad \text{if } k \leq M_i \quad (11)$$

294 where  $k$  is the Chebyshev polynomial order,  $\epsilon_i$  is the viscosity amplitude,  $N_i$   
 295 is the number of collocation point in the  $i^{\text{th}}$  direction and  $M_i \leq N_i$  is the cut-  
 296 off spectral mode.  $\epsilon_i$  and  $M_i$  are the control parameters of the SVV method.  
 297 The numerical modeling reduces by either increasing  $M_i$  or decreasing  $\epsilon_i$ . It  
 298 is worth noting that the same SVV parameters are used in the momentum  
 299 equation and in the energy equation. This would be similar to considering a  
 300 subgrid Prandtl number equal to one in a conventional LES model based on  
 301 subgrid viscosity and diffusivity.

### 302 3.3. Numerical validation

303 In order to study the accuracy of the SVV approach, we carry out a sen-  
 304 sitivity analysis on the SVV parameters  $M_i$  and  $\epsilon_i$  and on the mesh size. To  
 305 this aim, we focus on the simulation of February (see Tab. 1), which corre-  
 306 sponds to the smallest Rayleigh number investigated here (the less turbulent  
 307 case) and for which DNS is practicable with current computational resources.

308 In a first step, we set the mesh size to  $N_X \times N_Y \times (N_Z \times N_p) = 160 \times 160 \times$   
 309  $(20 \times 32)$  (LES<sub>160</sub> mesh), where  $N_{X/Y/Z}$  is the number of collocation points  
 310 in directions  $X/Y/Z$  in each subdomain and  $N_p$  is the number of processors,  
 311 and we vary  $M_i$  and  $\epsilon_i$  as described in Tab. 2. The results are presented  
 312 in terms of time-averaged variables over a dimensionless time period of 100.  
 313 The time-averaged fluxes through each of the six walls of the cavity are  
 314 computed at each time step. The flow field is assumed to be statistically  
 315 steady when the sum of these six fluxes is less than 1% of the maximum one.  
 316 In Tab. 2, we calculate for each set of SVV parameters the volume-averaged  
 317 temperature ( $\langle T^* \rangle$ ), the volume-averaged kinetic energy of the mean flow  
 318 ( $k_{kin}^* = \langle u_i^* \rangle \langle u_i^* \rangle / 2$ ), the volume-averaged turbulent kinetic energy ( $k_{tur}^* =$   
 319  $\langle u_i^{*'} u_i^{*'} \rangle / 2$ ), and the Nusselt number ( $Nu = -\nabla \langle T^* \rangle \cdot \mathbf{n}_{int}$ ) averaged over  
 320 the upper ( $X^*=1$ ), left ( $Z^*=0$ ), and side ( $Y^*=0$ ) walls separately ( $\langle a \rangle$  and  
 321  $a'$  denote the time average and the fluctuation of  $a$  respectively). There is  
 322 no influence of the SVV parameters on the average wall heat flux and the  
 323 volume-averaged temperature in the system (the change is less than 1%). In  
 324 addition, the SVV parameters have little effects on the kinetic energy of the  
 325 mean flow (less than 3%) but significant effects (up to 10%) on the turbulent  
 326 kinetic energy. Therefore, varying the SVV parameters has little influence on  
 327 the time averaged fields as previously found in the application of turbulent  
 328 wakes by Pasquetti [21].

329 In a second step, we fix the SVV parameters to  $M_i = 3N_i/4$  and  $\epsilon_i =$   
 330  $1/(4N_i)$  and consider two LES meshes:  $160 \times 160 \times (20 \times 32)$  (LES<sub>160</sub>) and

SVV parameters	$\langle T^* \rangle \times 10^2$	$k_{kin}^* \times 10^4$	$k_{tur}^* \times 10^4$	$Nu_{up, X^*=1}$	$Nu_{left, Z^*=0}$	$Nu_{sides, Y^*=0}$
$M = N/2, \epsilon = 1/2N$	5.370	2.865	2.368	-16.385	6.046	7.192
$M = 2N/3, \epsilon = 1/3N$	5.367	2.889	2.274	-16.351	6.049	7.196
$M = 3N/4, \epsilon = 1/4N$	5.350	2.803	2.547	-16.385	5.979	7.219
$M = 4N/5, \epsilon = 1/5N$	5.341	2.880	2.504	-16.431	5.978	7.219

Table 2: Sensitivity to the SVV parameters of the volume-averaged temperature, the volume-averaged kinetic energy of mean flow, the volume-averaged turbulent kinetic energy and the wall-averaged Nusselt number for February.

331  $240 \times 240 \times (20 \times 32)$  (LES<sub>240</sub>). Results are compared with those of a DNS  
332 (no model or  $M_i = N_i$  and  $\epsilon_i = 0$ ) with a mesh of  $320 \times 410 \times (32 \times 32)$  points.  
333 The time required to perform the simulation for the dimensionless time  $\Delta t^* =$   
334 1000 with the DNS approach using the available resources is approximately  
335 128000 hours. With the LES approach using LES<sub>160</sub> and LES<sub>240</sub> meshes, the  
336 time decreases to 24576 hours and 55360 hours, respectively. In Tab. 3, the  
337 same averaged variables described in Tab. 2 are recalculated for each case.  
338 The results obtained with the LES<sub>240</sub> mesh are in good agreement with the  
339 DNS (differences below 10%), while results obtained with the LES<sub>160</sub> mesh  
340 show significant discrepancies with the DNS (up to 20%).

Case	$\langle T^* \rangle \times 10^2$	$k_{kin}^* \times 10^4$	$k_{tur}^* \times 10^4$	$Nu_{up, X^*=1}$	$Nu_{left, Z^*=0}$	$Nu_{sides, Y^*=0}$
DNS	5.760	2.540	2.710	-14.755	4.968	6.825
LES <sub>240</sub>	5.610	2.638	2.998	-15.605	5.381	6.961
LES <sub>160</sub>	5.350	2.803	2.547	-16.385	5.979	7.219

Table 3: Comparison between LES and DNS methods in terms of the volume-averaged temperature, the volume-averaged kinetic energy of mean flow, the volume-averaged turbulent kinetic energy and the average wall Nusselt number for February.

341 In order to get further insights on the accuracy of the SVV-LES model,  
342 we compare LES<sub>240</sub> results and DNS results for the local distribution of key  
343 quantities such as the Nusselt number and the turbulent kinetic energy. In  
344 Fig. 4,a and b, we present the evolution of the Nusselt number along a hori-  
345 zontal line at the upper wall ( $X^* = 1, Y^* = 0.6604$ ) and along a vertical line  
346 at the left wall ( $Y^* = 0.6604, Z^* = 0$ ), respectively. The LES<sub>240</sub> mesh pro-  
347 vides satisfactory results compared to the DNS with a small overestimation at

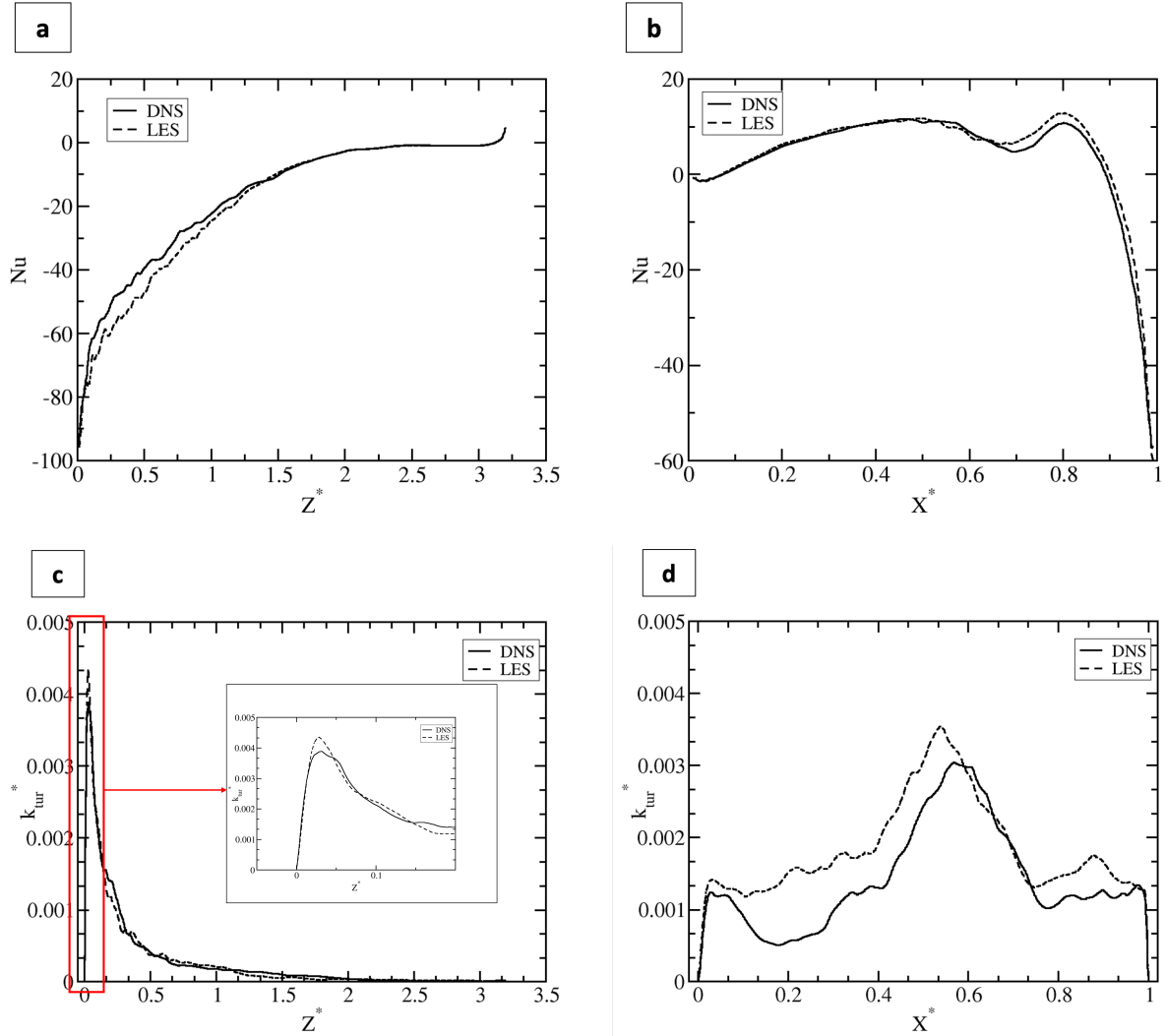


Figure 4: Nusselt number along the horizontal line  $X^* = 1, Y^* = 0.6604$  (a) and the vertical line  $Y^* = 0.6604, Z^* = 0$  (b) and turbulent kinetic energy along the horizontal line  $X^* = 0.6, Y^* = 0.6604$  (c) and the vertical line  $Y^* = 0.6604$  and  $Z^* = 0.05$  (d) for February. The straight and dashed lines correspond to the DNS and LES<sub>240</sub> results, respectively.

348 locations associated with significant turbulent fluctuations. In Fig. 4, c and  
 349 d, we plotted the turbulent kinetic energy along a horizontal line ( $X^* = 0.6$ ,  
 350  $Y^* = 0.6604$ ) and along a vertical line ( $Y^* = 0.6604$ ,  $Z^* = 0.05$ ), where  
 351 these turbulent fluctuations are significant. The agreement between LES<sub>240</sub>  
 352 and DNS is rather good within the vertical boundary layer (c) while discrep-  
 353 ancies are more important near the bottom wall (d) with overpredictions  
 354 of the tubulent kinetic energy up to a factor of 2. Consequently, the SVV  
 355 method with the LES<sub>240</sub> mesh preserves the heat flux at the walls but tends  
 356 to overpredict the turbulent kinetic energy.

357 Since the main objective of this paper is to determine the role of turbulent  
 358 convection on the heat fluxes at the cave walls and in order to save compu-  
 359 tational time, we use the LES<sub>240</sub> mesh (CPU time savings of approximately  
 360 44% compared to DNS) and SVV parameters  $M_i = 3N_i/4$  and  $\epsilon_i = 1/(4N_i)$   
 361 for the simulation of all cases of Tab. 1.

## 362 4. Flow field analysis

### 363 4.1. One-cell and multiple-cell convection patterns

364 Figure 5 shows the streamlines of the mean airflow colored by the kinetic  
 365 energy for each month studied, in the vertical  $Y$  mid-plane (the mean flow and  
 366 mean temperature fields are mostly bidimensional thanks to the symmetry  
 367 of the equations and boundary conditions with respect to the  $Y$  mid-plane).  
 368 For all cases ascending or descending flows develop along the left and right  
 369 vertical walls. They are connected through horizontal flows along the floor  
 370 and the ceiling, giving rise to a primary rotating circulation in the cavity.  
 371 However, we can define two distinct flow regimes depending on the period  
 372 of the year: a one-cell regime in March, May and August (left panels in  
 373 Fig. 5) characterized by a single large-scale circulation which extends over  
 374 the entire cavity, and a multiple-cell regime in September, November and  
 375 February (right panels in Fig. 5) for which the primary rotating flow near  
 376 the walls is associated with more complex flow patterns within the core.

377 The different patterns can be schematically classified according to (i)  
 378 the direction of rotation of the primary circulation along the walls, (ii) the  
 379 number of cells in the bulk of the cavity. We are going to show that this  
 380 classification results from the relative temperatures of the four edges men-  
 381 tioned in Sec. 2.1, as illustrated in Fig. 6 where rather cold edges are marked  
 382 in blue and rather hot in red. Indeed, it can be seen in Fig. 2b, that for each  
 383 month there is one edge that is significantly hotter or colder than the three

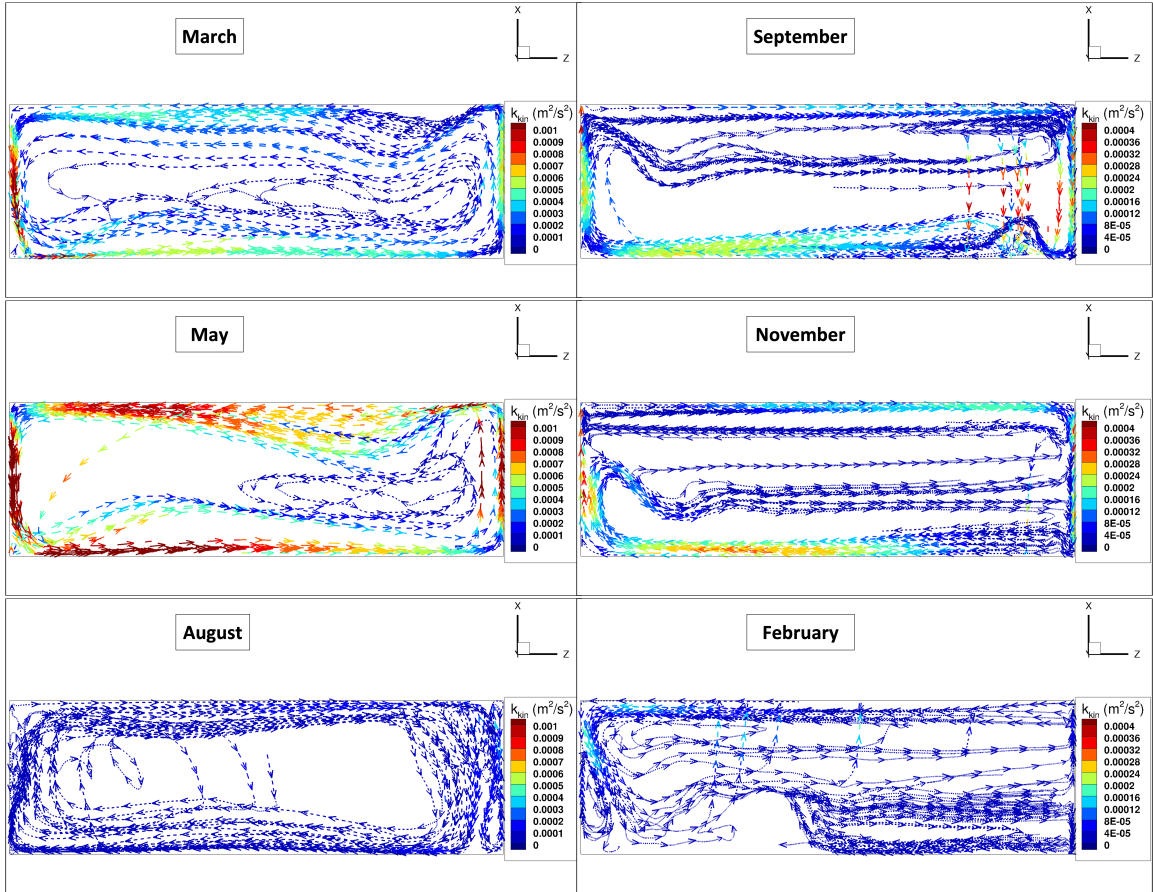


Figure 5: Flow streamlines colored by the kinetic energy of the mean flow. Streamlines are drawn from the  $Y$  mid-plane and then projected onto the  $Y$  mid-plane when they deviate from it.

384 others, which are almost at the same temperature. The direction of rotation  
 385 of the primary circulation along the walls in the  $X$ - $Z$  plane is determined by  
 386 the sign of the horizontal temperature gradient along the  $Z$  axis. In March,  
 387 May and February the right vertical wall is on average warmer than the left  
 388 wall (the horizontal gradient is positive) giving rise to a counterclockwise  
 389 rotation. Indeed, due to the buoyancy forces, the hot wall and the cold wall  
 390 drive the air flow upward and downward, respectively. Conversely, the hori-  
 391 zontal temperature gradient is negative in August, September and November  
 392 resulting in a clockwise rotation. On the other hand, when the floor is on  
 393 average warmer than the ceiling (March, May, August), the vertical temper-  
 394 ature gradient is negative resulting in an unstable thermal stratification in  
 395 the core and this corresponds to the one-cell regime. When the floor is on av-  
 396 erage colder than the ceiling (September, November, February), the vertical  
 397 temperature gradient is positive resulting in a stable thermal stratification  
 398 and this corresponds to the multiple-cell regime. For example in November,  
 399 three convection cells of weak intensity are observed. The air layer adjacent  
 400 to the left wall is heated then rises but the outer part of this layer is cooled by  
 401 the core then slows down and generates a horizontal current at mid-height.  
 402 This current travels through the core to the right side then splits with a part  
 403 incoming to the downward flow adjacent to the cold wall and another part,  
 404 slightly warmer, moving upward thus creating recirculation cells.

405 Based on this simplified analysis of thermal boundary conditions, we can  
 406 conclude that the global circulation along the vertical and horizontal walls  
 407 is governed by the temperature variations between the vertical walls, i.e., by  
 408 the horizontal temperature gradient, whereas the nature of the regime, one-  
 409 cell or multiple-cell, is determined by the temperature variations between the  
 410 horizontal walls, i.e., by the vertical temperature gradient.

411 In the following, we analyse the flow dynamics, the thermal features and  
 412 the turbulence level of the one-cell and multiple-cell regimes. The volume-  
 413 averaged kinetic energy of the mean flow,  $k_{kin}$ , the volume-averaged turbulent  
 414 kinetic energy,  $k_{tur}$ , the maximum velocity,  $\langle u \rangle_{max}$ , and the volume-averaged  
 415 standard deviation of the dimensionless mean temperature,  $\sigma(\langle T^* \rangle)$ , are given  
 416 in Table 4 for each month. The one-cell regime is characterized by a strong  
 417 mean flow compared to the multiple-cell regime, as indicated by relatively  
 418 high  $k_{kin}$  and  $\langle u \rangle_{max}$  values in May and March. In August, these quantities  
 419 are comparable to those of September and November although the temper-  
 420 ature difference  $\Delta T$  is approximately 3 or 4 times smaller (see Tab. 1). The  
 421 intense convection flow in the one-cell regime extends throughout the cavity



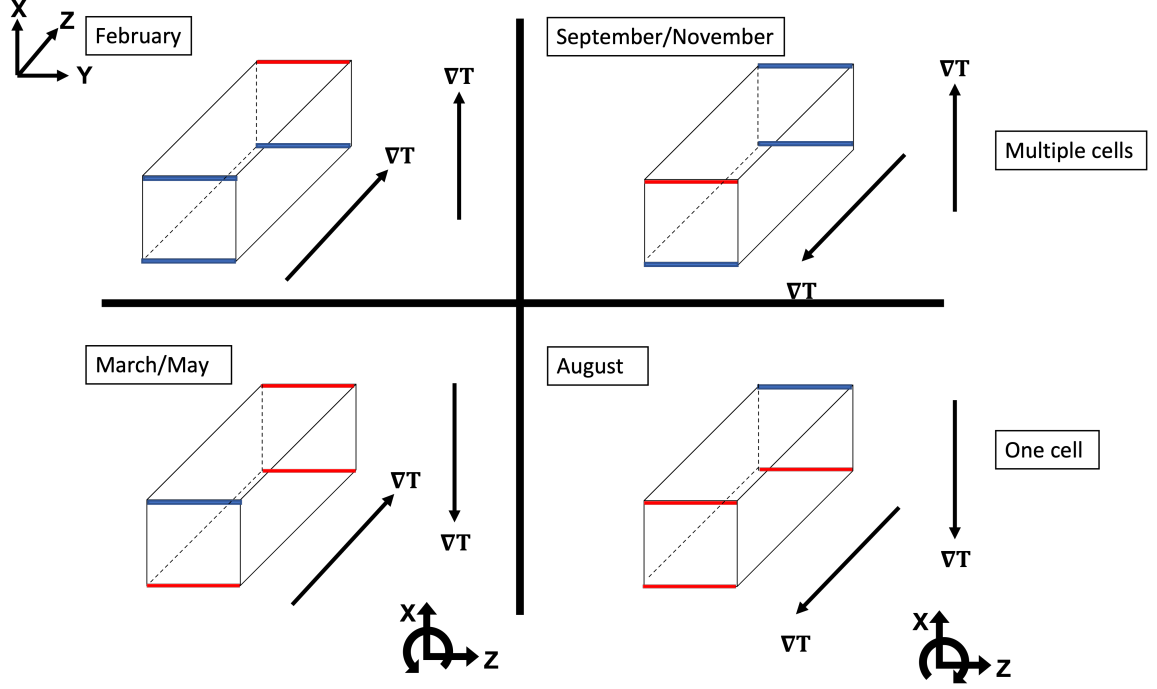


Figure 6: Simplified representation of thermal boundary conditions and associated flow regimes. Blue and red lines correspond to cold and hot edges, respectively.

Case	Regime	$k_{kin}$ ( $\text{m}^2/\text{s}^2$ )	$k_{tur}$ ( $\text{m}^2/\text{s}^2$ )	$\langle u \rangle_{max}$ (m/s)	$\sigma(\langle T^* \rangle)$
March	one-cell	$1.6 \times 10^{-4}$	$4.4 \times 10^{-5}$	0.076	0.018
May	one-cell	$2.8 \times 10^{-4}$	$9.3 \times 10^{-5}$	0.095	0.015
August	one-cell	$3.5 \times 10^{-5}$	$2.8 \times 10^{-5}$	0.045	0.011
September	multiple-cell	$3.6 \times 10^{-5}$	$8.8 \times 10^{-6}$	0.047	0.023
November	multiple-cell	$3.1 \times 10^{-5}$	$6.6 \times 10^{-6}$	0.050	0.044
February	multiple-cell	$8.4 \times 10^{-6}$	$1.0 \times 10^{-5}$	0.028	0.036

Table 4: Volume-averaged kinetic energy of the mean flow  $k_{kin}$ , volume-averaged turbulent kinetic energy  $k_{tur}$ , maximum velocity of the mean flow  $\langle u \rangle_{max}$  and volume-averaged standard deviation of the dimensionless mean temperature  $\sigma(\langle T^* \rangle)$  calculated for all months under study.

422 (see Fig. 5) leading to a strong mixing effect and thus the homogenisation  
 423 of the temperature field. This gives rise to small values of  $\sigma(\langle T^* \rangle)$  compared  
 424 to the values related to the multiple-cell convection of weak intensity. To  
 425 illustrate the analysis of the thermal feature, Fig. 7(top) displays the spatial  
 426 distribution of the mean temperature in the  $Y$  mid-plane of the cavity. We  
 427 focus on May (left panel) and November (right panel) as they correspond to  
 428 the months with the greatest temperature amplitude (see Tab. 1) in the one-  
 429 cell and multiple-cell regimes, respectively. In May, the temperature field is  
 430 nearly isothermal thanks to the mixing effect of the large-scale convection  
 431 cell. The temperature gradients are thus confined very close to the walls. By  
 432 contrast, the temperature field in November shows significant temperature  
 433 variations near the ceiling and a thermal stratification in the core related to  
 434 the presence of the recirculation cells.

435 From the data reported in Table 4, we can see that the turbulent fluctu-  
 436 ation level is higher in the one-cell regime than in the multiple-cell regime.  
 437 It should be noted that, although the kinetic energy of the mean flow is very  
 438 low in February, turbulent fluctuations are detected due to the presence of  
 439 counter-rotating cells near the left wall (see Fig. 5). The spatial distribution  
 440 of the turbulent kinetic energy is presented in Fig. 7 (bottom panels) for May  
 441 and November. In May, significant turbulent fluctuations are noticeable in  
 442 the lower left corner where the descending vertical boundary layer hits the  
 443 floor and to a less extent in the upper right side. The asymmetry between  
 444 the lower left and upper right corners is due to larger temperature gradients  
 445 in the left part of the cavity than in the right part. In November, the tur-  
 446 bulent kinetic energy is very small and turbulent fluctuations are detectable  
 447 only in the left part of the cavity near the flow division and in the top right  
 448 region where the downward flow in the cold boundary layer is sheared by the  
 449 ascending recirculation flow (see Figure 5).

450 In order to visualize and compare the 3D turbulent structures in May  
 451 and November, we make use of the  $Q$ -criterion  $Q = [\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij}]/2$   
 452 [31], where  $\Omega_{ij} = [\partial u_i/\partial x_j - \partial u_j/\partial x_i]/2$  is the vorticity tensor and  $S_{ij} =$   
 453  $[\partial u_i/\partial x_j + \partial u_j/\partial x_i]/2$  is the strain tensor. This criterion compares the rates  
 454 of rotation and deformation. Turbulent structures correspond to positive val-  
 455 ues of  $Q$  [32]. Figure 8 shows the  $Q$ -criterion colored by the kinetic energy  
 456 of the mean flow for these two months, for a given instantaneous flow field  
 457 in the asymptotic regime. Vortices almost spread everywhere in May, while  
 458 they are restricted to regions near the left and right planes in November. It  
 459 is interesting to note that the instantaneous flow is fully three-dimensional

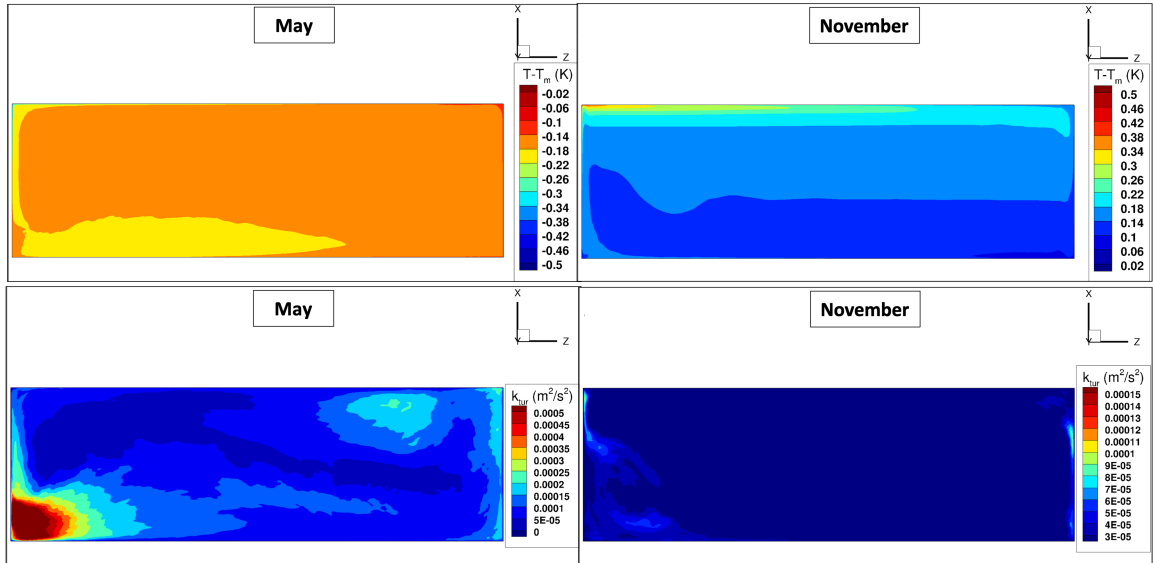


Figure 7: Mean temperature (top panels) and turbulent kinetic energy (bottom panels) fields in the  $Y$  mid-plane of the cavity for May and November.

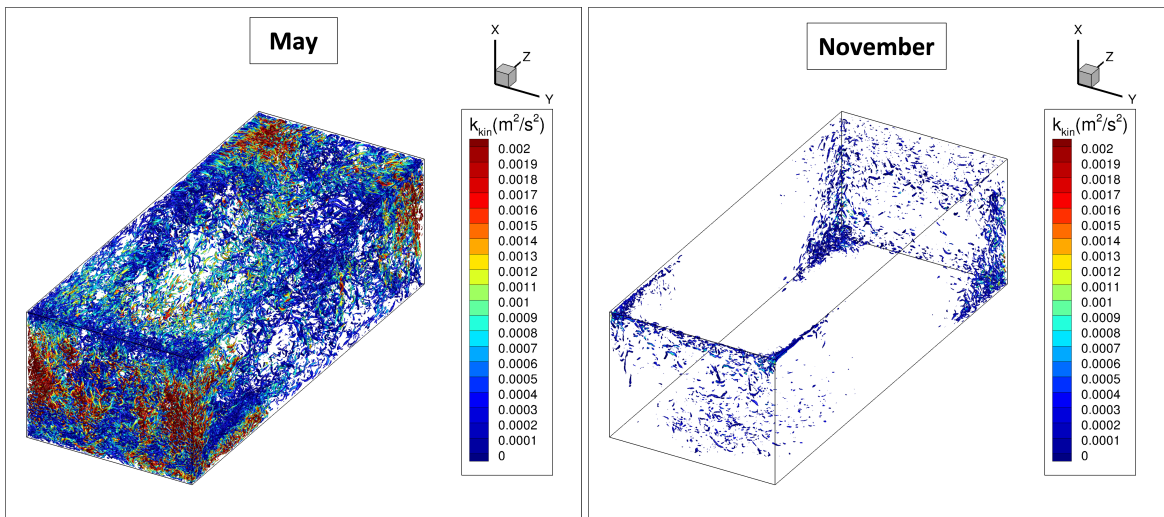


Figure 8: Isosurfaces of the  $Q$ -criterion colored by the kinetic energy of the mean flow for an instantaneous flow field for May and November.  $Q = 1 \text{ s}^{-2}$  for all cases.

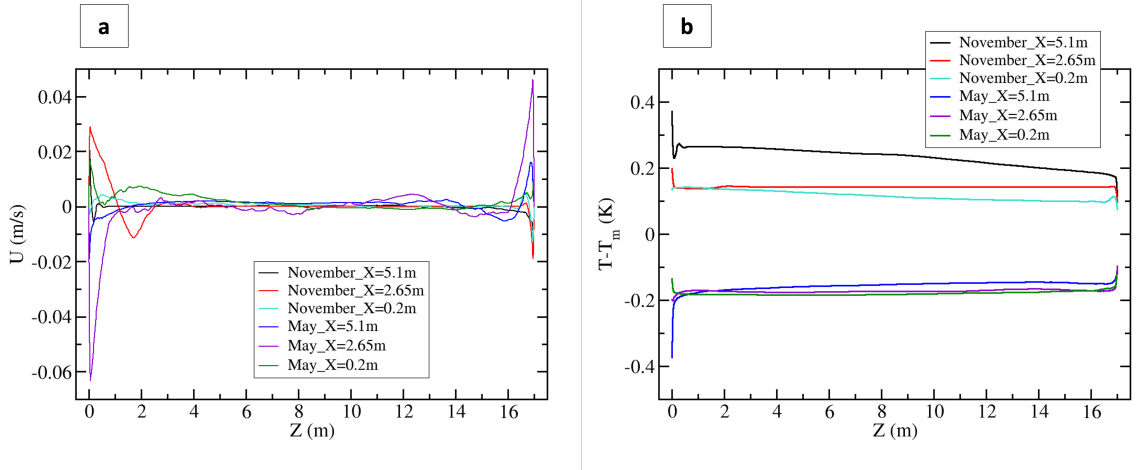


Figure 9: Horizontal profiles of the vertical velocity component  $U$  (a) and temperature (b) in the  $Y$  mid-plane for three different heights ( $X = 0.2$  m  $\approx 0.04L_X$ ,  $X = 2.65$  m  $= 0.5L_X$  and  $X = 5.1$  m  $\approx 0.96L_X$ ). The vertical walls are located at  $Z = 0$  and  $Z = 17$  m. May (one-cell regime) and November (multiple-cell regime) are presented.

460 for both months, though the main mean dynamics happen in the  $X$ - $Z$  plane.

#### 461 4.2. Near wall regions

462 To get a better insight into the flow structure in the near-wall region, we  
 463 discuss in this section the temperature and velocity profiles along vertical or  
 464 horizontal lines.

465 Fig. 9 presents horizontal profiles of the vertical velocity component  $U$   
 466 and the temperature in the  $Y$  mid-plane for three different heights ( $X =$   
 467  $0.2$  m  $\approx 0.04L_X$ ,  $X = 2.65$  m  $= 0.5L_X$  and  $X = 5.1$  m  $\approx 0.96L_X$ ), for May  
 468 (one-cell regime) and November (multiple-cell regime). Velocity boundary  
 469 layers are observed in Fig. 9 (a), ascending or descending according to the  
 470 sign of the horizontal temperature gradients in the thermal boundary layers  
 471 (see Fig. 9 (b)). It can be noted that in May, the horizontal temperature and  
 472 velocity gradients change sign in the left bottom part ( $X = 0.2$  m,  $Z \approx 0$ )  
 473 due to the presence of a small vortex in the corner (see Figure 5). The  
 474 thickness of the velocity and thermal boundary are estimated as follows. The  
 475 thickness of the velocity boundary layer,  $\delta_u$ , is referred as the distance from  
 476 the wall to a point where the velocity reaches an extremum. The thickness  
 477 of the thermal boundary layer,  $\delta_\theta$ , is referred as the distance from the wall  
 478 where the temperature reaches 90 % of its value in the core. We find that

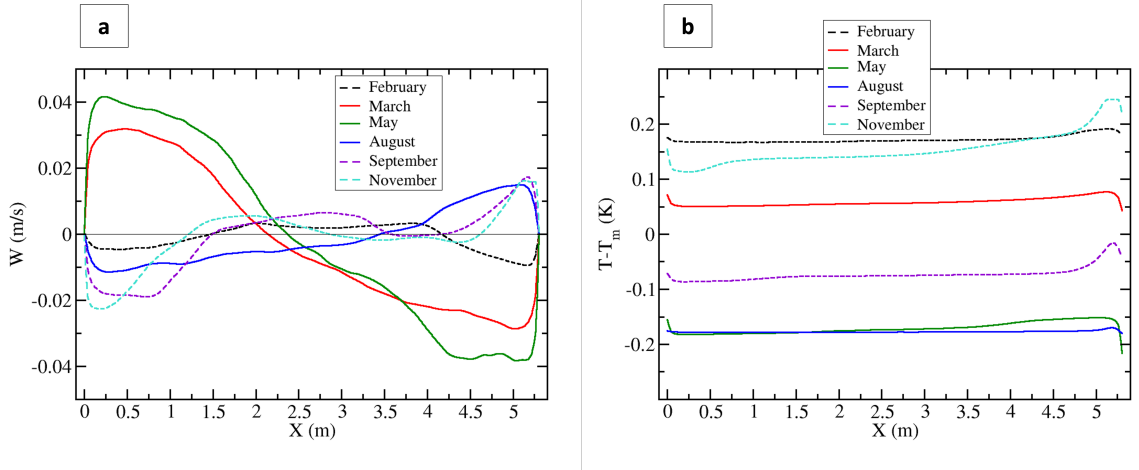


Figure 10: Vertical profiles of the horizontal velocity component  $W$  (a) and the temperature (b) along the vertical centerline ( $Y = 0.5L_Y$ ,  $Z = 0.5L_Z$ ) for the six months investigated. The horizontal walls are located at  $X = 0$  and  $X = 5.3$  m. The solid and dotted lines correspond to the one-cell and multiple-cell regime, respectively.

479  $\delta_u \sim \delta_\theta$  ranges from 4 cm to 8 cm, i.e., less than 0.5% of the cavity length  
 480 ( $L_Z = 17$  m).

481 As highlighted in subsection 4.1, Fig. 9 (b) shows that the core of the  
 482 cavity is nearly isothermal in May whereas a stable vertical thermal strati-  
 483 fication takes place in November, especially in the upper half of the cavity  
 484 where a temperature difference of about 0.1 K is observed between the air at  
 485 mid-height (red curve) and the air adjacent to the top wall (black curve).

486 Fig. 10 presents the horizontal velocity component  $W$  and the tempera-  
 487 ture profiles along the vertical centerline ( $Y = 0.5L_Y$ ,  $Z = 0.5L_Z$ ), in the  
 488 one-cell regime for March, May and August (solid lines) and in the multiple-  
 489 cell regime for February, September and November (dashed lines). In the  
 490 one-cell regime, Fig. 10 (a) exhibits the large-scale circulation extending over  
 491 the entire height of the cavity, with a reverse direction of rotation for Au-  
 492 gust compared to March and May. By contrast, the core is almost motion-  
 493 less in the multiple-cell regime, with however small variations around zero  
 494 that are the signature of the recirculations of low intensity. Temperature  
 495 profiles (Fig. 10 (b)) confirm that the temperature gradients are confined  
 496 in very thin layers (few centimeters thick, i.e., less than 1% of the cavity  
 497 height ( $L_X = 5.3$  m)) near the horizontal walls in the one-cell regime. In the  
 498 multiple-cell regime, the temperature variations are observed near the ceiling

499 for September and November, in a layer that extends over about 1 m with a  
 500 maximum temperature located at a few tens of centimeters from the wall.

## 501 **5. Heat transfer analysis**

### 502 *5.1. Wall conductive fluxes*

503 In this subsection, we analyse the local heat transfer rates at the walls  
 504 by quantifying the conductive wall heat flux  $q_{con}$  defined in Eq. (7). As the  
 505 underlying physical mechanisms are very diverse, depending on the location  
 506 on the wall or the month considered, we do not conduct this analysis in terms  
 507 of dimensionless quantities; we aim to characterize the local heat transfer  
 508 coefficient  $h$  in the Newton's law

$$q_{con} = h(T_{wall} - T_{gas}), \quad (12)$$

509 where  $T_{wall}$  is the local wall temperature and  $T_{gas}$  is the air temperature  
 510 averaged over the entire domain.

511 In Fig. 11,  $q_{con}$  is plotted versus  $T_{wall} - T_{gas}$  for the six months investi-  
 512 gated and for each wall (each point corresponds to a given spatial location).  
 513 Positive values correspond to heat transfer from the wall to the fluid. In  
 514 the one-cell flow regime (left panels), we observe that the dependence of the  
 515 conductive flux with  $T_{wall} - T_{gas}$  is roughly linear. For a given month, a linear  
 516 fit performed through the cloud of points (black line in the figures) allows to  
 517 estimate empirically a single heat transfer coefficient  $h$  for all walls, ranging  
 518 from  $0.45 \text{ W.m}^{-2}.\text{K}^{-1}$  (in August) to  $0.76 \text{ W.m}^{-2}.\text{K}^{-1}$  (in May), according  
 519 to the magnitude of the convection flow. For the multiple cell regime (right  
 520 panels), the conductive flux follows the same trend, except at the upper wall.  
 521 The linear fit associated to all the walls except the upper one provides values  
 522 of  $h$  ranging from  $0.33$  to  $0.68 \text{ W.m}^{-2}.\text{K}^{-1}$ . At the upper wall,  $q_{con} \neq 0$  when  
 523  $T_{wall} - T_{gas} = 0$  which means that  $T_{gas}$  is not the relevant reference temper-  
 524 ature for the surrounding air layer. The actual reference temperature  $T_{bulk}$   
 525 is such that  $q_{cond} = 0$  for  $T_{wall} = T_{bulk}$  in the Newton's law. It can be easily  
 526 deduced from the graphs in Fig. 11 that the averaged gas temperature  $T_{gas}$   
 527 underestimates  $T_{bulk}$  by approximately  $0.03 \text{ K}$  in February,  $0.1 \text{ K}$  in Septem-  
 528 ber, and  $0.15 \text{ K}$  in November. This is in line with the analysis presented in  
 529 the previous section. Indeed, we have shown that the air layer near the ceil-  
 530 ing is warmer than the core region at  $T_{gas}$ , which implies that the reference  
 531 temperature must be adjusted upwards.

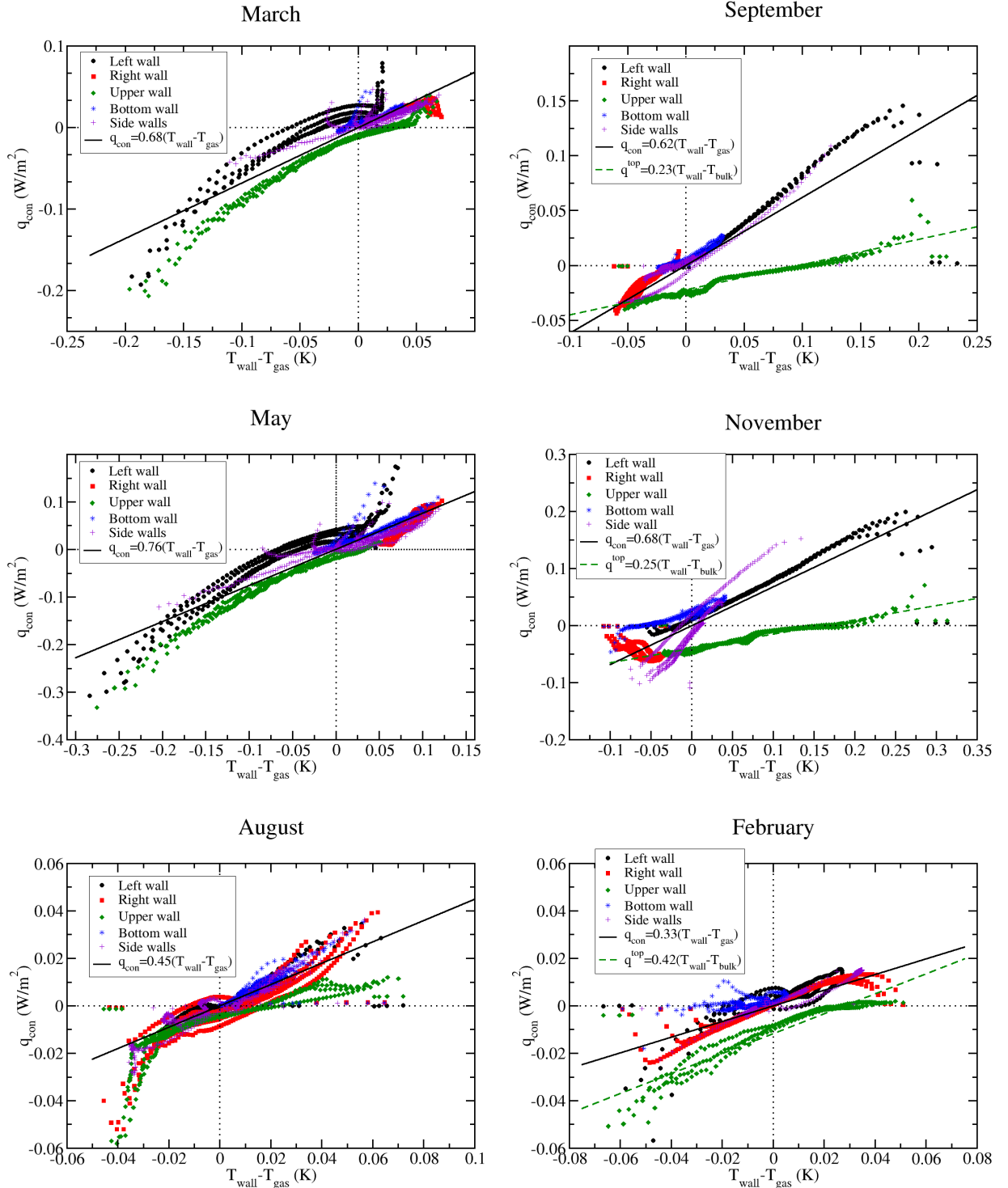


Figure 11: Cloud of points representing the local conductive heat flux at each wall versus the difference between the wall local temperature and the gas average temperature, for the six months investigated. Each point corresponds to a given spatial location on the wall. The black line represents the best linear fit of the cloud of points (with the exception of the top wall data in September, November and February). The green dashed line represents the best linear fit of the top wall in September, November and February, with  $T_{bulk}$  instead of  $T_{gas}$ .

532 By fitting both  $h^{top}$  and  $T_{bulk}$ , we get  $h^{top}$  ranging from  $0.23 \text{ W.m}^{-2}.\text{K}^{-1}$   
 533 (in September) to  $0.42 \text{ W.m}^{-2}.\text{K}^{-1}$  (in February). It is worthy to note that  
 534 all the values of the heat transfer coefficient  $h$  remain in a limited range,  
 535 from  $0.23$  to  $0.76 \text{ W.m}^{-2}.\text{K}^{-1}$ . This is not surprising given that the Nusselt  
 536 number usually scales as the Rayleigh number at the power  $0.25$  to  $0.33$   
 537 in natural convection [33], and thus increases slowly with the temperature  
 538 differences between the cavity walls. Therefore, these estimations of the heat  
 539 transfer coefficient might be used in a large-scale model coupling conduction  
 540 in the rock, radiative transfer in the cavity, and natural convection described  
 541 by the Newton's law (12). However, such approach raises the question of  
 542 how to define the reference gas temperature in the Newton's law. In the  
 543 case of complex non uniform temperature fields, relying on the averaged gas  
 544 temperature may result in large errors on wall convective heat fluxes.

## 545 5.2. Wall total fluxes

546 The aim of this section is to show the distribution of the conductive flux  
 547  $q_{con}$  and to compare it to the distribution of the radiative flux  $q_{rad}$  and the  
 548 total heat flux  $q_{tot} = q_{con} + q_{rad}$ . We focus here on the months with the  
 549 highest conductive fluxes, i.e., May and November.

550 Fig. 12 presents the distribution of conductive (a and b), radiative (c and  
 551 d), and total (e and f) fluxes at the walls for the month of May. The radiative  
 552 flux largely dominates the conductive flux but the latter remains significant at  
 553 several spots, especially in the left ceiling region, downstream the left vertical  
 554 boundary layer and near the right bottom edge. The maximum total heat  
 555 flux in the system reaches in absolute value  $1.15 \text{ W/m}^2$  and is concentrated  
 556 around the left upper edge. The conductive flux represents 40% of this value.

557 We show in Fig. 13 the distribution of  $q_{con}$  (a and b),  $q_{rad}$  (c and d),  
 558 and  $q_{tot}$  (e and f) for the month of November. It is worth noting that the  
 559 radiative flux distribution in November is opposite to that in May (tempera-  
 560 ture distributions are opposite), while this is not the case for the conductive  
 561 flux, sensitive to buoyancy. As in May, the system is mainly controlled by  
 562 radiative fluxes but there is still significant conductive heat flux in the upper  
 563 part of the left wall. Again, the maximum total heat flux reaches in absolute  
 564 value  $1.15 \text{ W/m}^2$  and is concentrated at the left upper edge. The conductive  
 565 flux represents about 35% of the total flux in the upper part of the left and  
 566 side walls and in the majority of the right wall.

567 Heat transfer at the cavity walls is thus mainly dominated by the radiative  
 568 heat flux, with the exception of some localized spots where both conductive



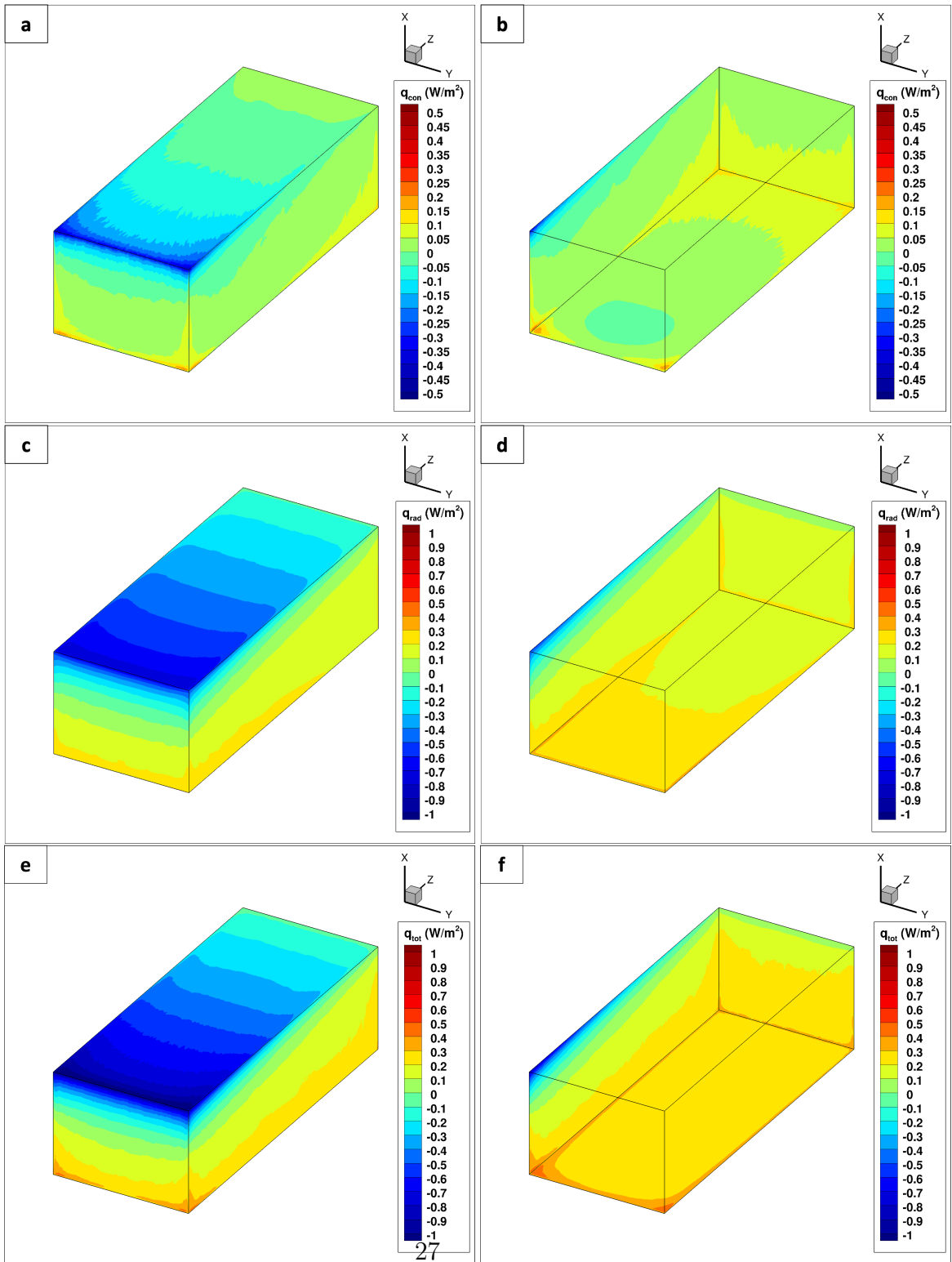


Figure 12: Spatial distribution of conductive (a and b), radiative (c and d) and total (e and f) heat fluxes at the walls for the month of May. The left part of the figure corresponds to the upper ( $X = L_X$ ), left ( $Z = 0$ ) and front ( $Y = L_Y$ ) cave walls. The right part of the figure corresponds to the bottom ( $X = 0$ ), right ( $Z = L_Z$ ) and back ( $Y = 0$ ) cave walls.

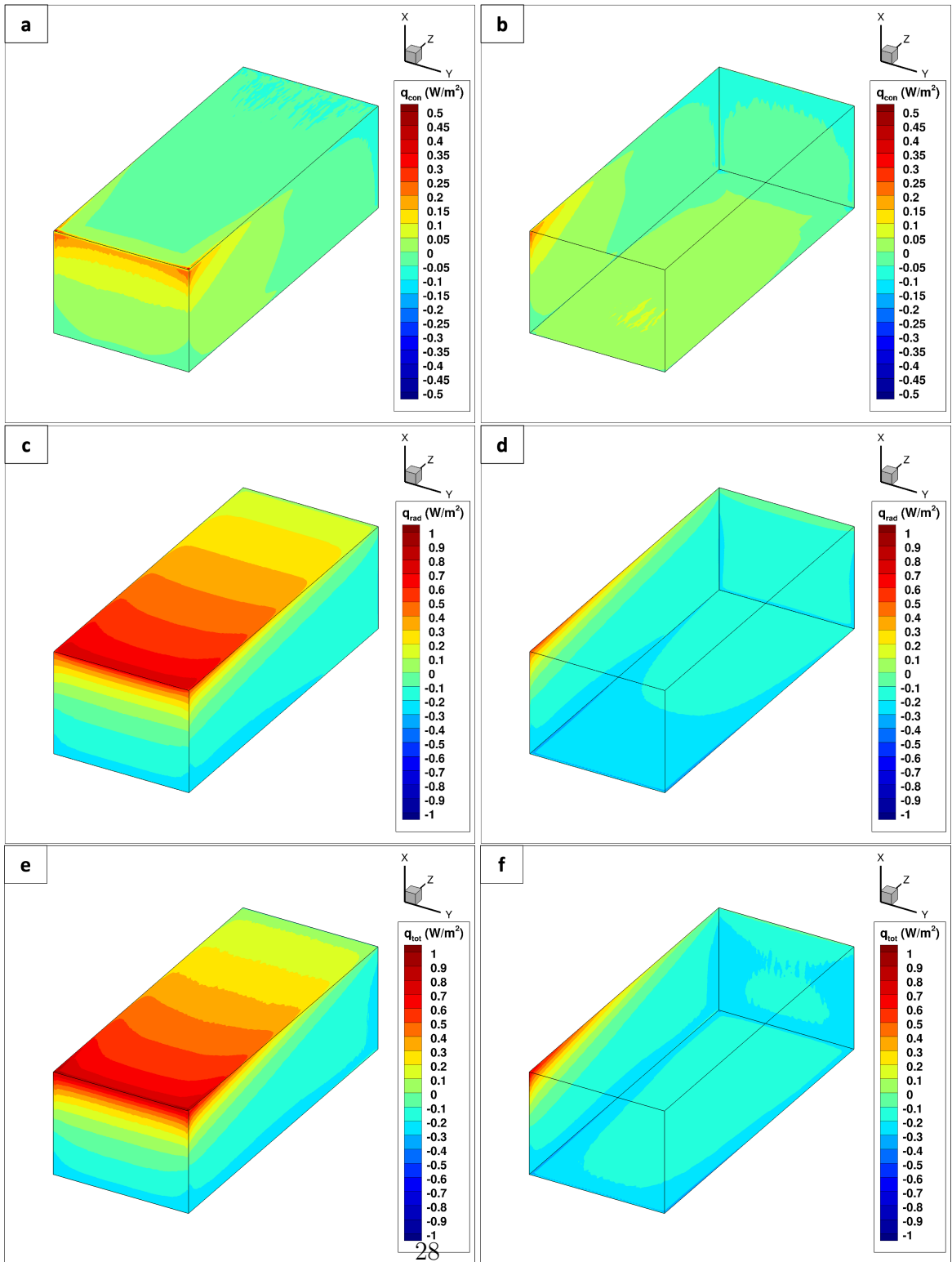


Figure 13: Spatial distribution of conductive (a and b), radiative (c and d) and total (e and f) heat fluxes at the walls for the month of November. The left part of the figure corresponds to the upper ( $X = L_X$ ), left ( $Z = 0$ ) and front ( $Y = L_Y$ ) cave walls. The right part of the figure corresponds to the bottom ( $X = 0$ ), right ( $Z = L_Z$ ) and back ( $Y = 0$ ) cave walls.

569 and radiative fluxes are on the same order. This conclusion must be tempered  
570 by the fact that dry air was considered. In humid caves, convection controls  
571 the heat flux associated with the latent heat of condensation and evaporation.  
572 This probably affects the total heat flux. Assuming that solutal buoyancy  
573 does not significantly modify the flow structure, the highest condensation  
574 flux should occur in May and the highest evaporation flux in November, in  
575 the region of the left upper edge in both cases. The effect of latent heat will  
576 be investigated in a future work.

## 577 **6. Conclusion**

578 In this paper, the effect of turbulent natural convection on heat transfer  
579 within a confined underground cavity was investigated using large eddy sim-  
580 ulations based on the spectral vanishing viscosity method. Non-uniform wall  
581 temperatures computed from a large-scale model and representative of exter-  
582 nal climate condition at six times of the year were used as thermal boundary  
583 conditions.

584 We identified two different flow regimes: (i) a one-cell flow regime asso-  
585 ciated with strong convection and unstable mean vertical temperature gra-  
586 dient, (ii) a multiple-cell flow regime associated with weak convection and  
587 stable mean vertical temperature gradient. For each regime the mean direc-  
588 tion of rotation of the flow is determined by the direction of the horizontal  
589 temperature gradient. The one cell flow regime (March, May, August) is  
590 characterized by a single-roll large-scale circulation, high turbulent fluctu-  
591 ation level and strong mixing resulting in the homogenisation of the gas  
592 temperature. The multiple-cell flow regime (September, November, Febru-  
593 ary) is characterized by two counter-rotating large-scale structures. It corre-  
594 sponds to a flow of weak intensity with low turbulent fluctuation level and a  
595 significant vertical temperature gradient in the air near the ceiling.

596 The values of the heat transfer coefficient in the Newton's law were calcu-  
597 lated from the LES results. We found that the flow intensity and turbulence  
598 level have little influence on the heat transfer coefficient value. However, the  
599 definition of the reference gas temperature to be used in the Newton's law  
600 is a non trivial question in this problem with thermal boundary conditions  
601 defined from complex temperature fields. The choice of the average gas tem-  
602 perature is relevant when the air temperature is nearly uniform everywhere  
603 in the core of the cavity. Otherwise, significant errors in the prediction of  
604 wall heat fluxes may occur. This problem could likely be exacerbated by the

605 complex geometry of natural caves, making even more necessary the use of  
606 CFD approaches as presented here.

607 Our method allows to determine if heat transfer at the cavity walls is  
608 dominated by conducto-convective or radiative fluxes. Moreover, places in  
609 the cavity and times of the year corresponding to intense heat transfer can be  
610 identified. In future works, we will consider the coupling with mass transport  
611 of water vapour in order to predict the conditions leading to intense conden-  
612 sation, a problem of great importance for the conservation of painted caves.  
613 In addition, given the significance of radiative fluxes, it might be worth in-  
614 vestigating the effect of gas radiation associated with the presence of water  
615 vapour and carbon dioxide in the cave atmosphere.

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