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Implementation of liquid crystal-based polarimeters: trade-off between speed and performance

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ABSTRACT

This work considers the implementation of polarimeters with liquid crystal (LC) cells as polarizing elements. Most works generally try to implement architectures with one or two pure retarding modulators such as nematic devices. In this case, rather thick LC devices able to provide a 2π retardation are generally used. Unfortunately, LC device switching speed is known to evolve as the inverse square of their thickness, which leads to practical implementations limited to a few tens of Hertz in the visible region. The alternative consisting in using much faster devices made of ferroelectric liquid crystals is not that obvious since these devices often operate in bistable mode. We show that using thinner, therefore faster nematic devices is possible with a minimal penalty in terms of performance. Therefore, several solutions can be considered. Performance evaluation will be performed through studying the system matrix condition number.

Keywords: polarimetry, imaging system, Stokes vector, light modulator, liquid crystal device

1. INTRODUCTION

Imaging polarimetry finds applications in many fields such as medical imaging¹, remote sensing², metrology³, material discrimination for target detection⁴, among others. Dynamic imaging polarimeters, also named polarimeteric cameras, have been allowing users to capture dynamic scenes for an approximate fifteen years. They often include liquid crystal (LC) modulators which act as tunable waveplates. The most popular liquid crystal devices for such a purpose are parallel aligned (PAL) nematic based devices. Excellent laboratory performance was reported for such systems^{5,6}, but they classically cannot operate faster than 50 Hertz. Faster solutions exist using ferroelectric liquid crystal devices,⁷ but the electrical control of these devices for such applications remains difficult. This article deals with the implementation of polarimeters with thin nematic LC devices. These devices are supposed to switch faster,⁸ but cannot produce a 2π phase retardation.

After reviewing basics of polarimetry in Section 2, polarization state analyzer formalism is described in Section 3. Then Section 4 reports simulation of polarimeters based on thin LC devices and their performance in terms of robustness and speed. Section 5 discusses the simulation results.

2. STOKES VECTOR AND MUELLER MATRIX DEFINITIONS

Under some assumptions, the 4-component Stokes vector \mathbf{S} totally describes polarization of light⁹:

$$\mathbf{S} = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} \tag{1}$$

where s_0 quantifies the total intensity of the light, s_1 is related to the vertical and horizontal polarizations, whereas s_2 is related to the polarizations at $\pm 45^{\circ}$. s_3 reflects the amount of left and right circular polarizations.

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The transfer function between two Stokes vector is a 4x4 matrix named Mueller matrix. Therefore, considering a incident beam with a Stokes vector \mathbf{S} , the emergent beam has a Stokes vector \mathbf{S}' defined by the relation:

$$\mathbf{S}' = \mathbf{M}.\mathbf{S} \tag{2}$$

3. POLARIZATION STATE ANALYZERS

Polarization state analyzers classically consist of a waveplate (a classical quartz waveplate or a liquid crystal device) followed by a fixed analyzer. The user combines several intensity measures in various conditions to estimate the Stokes vector. Change between measures can be a mechanical rotation (in the case of a waveplate) or a change in the control voltage (in the case of a LC device).

The Mueller matrices of a polarizer oriented at 0 $^\circ\,$ and of a waveplate exhibiting retardance φ oriented at an angle θ are:

$$\mathbf{M}_{\mathrm{WP}}(\theta,\varphi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2(2\theta) + \sin^2(2\theta) \cdot \cos(\varphi) & \cos(2\theta) \cdot \sin(2\theta) \cdot (1 - \cos(\varphi)) & -\sin(2\theta) \cdot \sin(\varphi) \\ 0 & \cos(2\theta) \cdot \sin(2\theta) \cdot (1 - \cos(\varphi)) & \sin^2(2\theta) + \cos^2(2\theta) \cdot \cos(\varphi) & \cos(2\theta) \cdot \sin(\varphi) \\ 0 & \sin(2\theta) \cdot \sin(\varphi) & -\cos(2\theta) \cdot \sin(\varphi) & \cos(\varphi) \end{bmatrix}$$
(4)

So the Mueller matrix of the PSA writes:

$$\mathbf{M}_{\rm PSA}(\theta,\varphi) = \mathbf{M}_{\rm Pol}.\mathbf{M}_{\rm WP}(\theta,\varphi) \\ = \frac{1}{2} \begin{bmatrix} 1 & \cos^2(2\theta) + \sin^2(2\theta).\cos(\varphi) & \cos(2\theta).\sin(2\theta).(1-\cos(\varphi)) & -\sin(2\theta).\sin(\varphi) \\ 1 & \cos^2(2\theta) + \sin^2(2\theta).\cos(\varphi) & \cos(2\theta).\sin(2\theta).(1-\cos(\varphi)) & -\sin(2\theta).\sin(\varphi) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
(5)

If we now consider an incident Stokes vector **S**, the emergent vector $\mathbf{S}'(\theta, \varphi)$ and the intensity $I(\theta, \varphi)$ of the emergent light are defined by the following equations:

$$\mathbf{S}'(\theta,\varphi) = \mathbf{M}_{\mathrm{PSA}}(\theta,\varphi) \cdot \mathbf{S} = \mathbf{M}_{\mathrm{PSA}}(\theta,\varphi) \cdot \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$
(6)

$$I(\theta) = \frac{1}{2} [s_0 + s_1 . (\cos^2(2\theta) + \sin^2(2\theta) . \cos(\varphi)) + s_2 . \cos(2\theta) . \sin(2\theta) . (1 - \cos(\varphi)) - s_3 . \sin(2\theta) . \sin(\varphi)]$$
(7)

Thus, if we measure four intensities I_1 , I_2 , I_3 , I_4 , for four different combinations (θ_1, φ_1) , (θ_2, φ_2) , (θ_3, φ_3) , (θ_4, φ_4) , we obtain the following system:

$$\begin{bmatrix} I_1\\I_2\\I_3\\I_4 \end{bmatrix} = \mathbf{A} \cdot \begin{bmatrix} s_0\\s_1\\s_2\\s_3 \end{bmatrix}$$
(8)

where the system matrix **A** writes:

$$\mathbf{A} = \frac{1}{2} \begin{bmatrix} 1 & \cos^2(2\theta_1) + \sin^2(2\theta_1) \cdot \cos(\varphi_1) & \cos(2\theta_1) \cdot \sin(2\theta_1) \cdot (1 - \cos(\varphi_1)) & -\sin(2\theta_1) \cdot \sin(\varphi_1) \\ 1 & \cos^2(2\theta_2) + \sin^2(2\theta_2) \cdot \cos(\varphi_2) & \cos(2\theta_2) \cdot \sin(2\theta_2) \cdot (1 - \cos(\varphi_2)) & -\sin(2\theta_2) \cdot \sin(\varphi_2) \\ 1 & \cos^2(2\theta_3) + \sin^2(2\theta_3) \cdot \cos(\varphi_3) & \cos(2\theta_3) \cdot \sin(2\theta_3) \cdot (1 - \cos(\varphi_3)) & -\sin(2\theta_3) \cdot \sin(\varphi_3) \\ 1 & \cos^2(2\theta_4) + \sin^2(2\theta_4) \cdot \cos(\varphi_4) & \cos(2\theta_4) \cdot \sin(2\theta_4) \cdot (1 - \cos(\varphi_4)) & -\sin(2\theta_4) \cdot \sin(\varphi_4) \end{bmatrix}$$
(9)

Knowing that I_i are measured quantities, and provided the angles θ_i are known and the matrix **A** is invertible, the components s_0 , s_1 , s_2 and s_3 , can be estimated by the equation:

$$\begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \mathbf{A}^{-1} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$
(10)

Parallel-aligned liquid crystal devices basically act as pure phase retarders and provide a system matrix **A** which is only rank-3. In order to be able to analyze the entire polarization of a light beam (getting a rank-4 **A** matrix), using two devices is mandatory. In this case, Eq. (5) is simply replaced by Eq. (11):

$$\mathbf{M}_{\mathrm{PSA}}(\theta_1, \varphi_1, \theta_2, \varphi_2) = \mathbf{M}_{\mathrm{Pol}} \cdot \mathbf{M}_{\mathrm{WP}_2}(\theta_2, \varphi_2) \cdot \mathbf{M}_{\mathrm{WP}_1}(\theta_1, \varphi_1)$$
(11)

To lower noise effects, more than 4 acquisitions can be performed and the pseudo-inverse of \mathbf{A} should be considered instead of its inverse.

The performance of PSA in terms of robustness is classically studied through the evaluation of the condition number¹⁰ or of the equally weighted variance $(EWV)^{11}$ of the system. In the following, we will arbitrarily focus on the condition number of the system. Considering the system matrix **A**, its L_2 norm condition number (CN) corresponds to the ratio of the highest singular value of **A** to the lowest singular value of **A**. CN should be minimized. Its theoretical minimum value is $\sqrt{M-1}$, where M is the number of Stokes parameters to evaluate.¹⁰ Performing laboratory setups were reported to exhibit CN at about 1.9 when analyzing the four Stokes parameters.⁶

4. SIMULATIONS

In the following, we consider a PSA consisting of two PAL LC devices and a fixed analyzer.⁵ The LC modulators are driven to get the states reported in Tab. 1.

PSA state $\#$	LC #1 configuration	LC $#2$ configuration
1	$(heta_1,arphi_1)$	$(heta_1, arphi_1)$
2	$(heta_1, arphi_1)$	$(heta_2, arphi_2)$
3	$(heta_2,arphi_2)$	$(heta_2, arphi_2)$
4	(θ_2, φ_2)	(θ_1, φ_1)

Table 1. Configuration of our LC polarimeter when the two LC devices are driven in quadrature of phase.

This scheme has two main advantages: the polarimeter can be operated at twice the modulator speed and the determination of optimal parameters is simpler than if each modulator was driven with more than two control states.

It is well known that for this configuration, an optimal set of parameters is:⁵

$$\varphi_1 = 45^{\circ}, \varphi_2 = 225^{\circ} \\ \theta_1 = 27.4^{\circ}, \theta_2 = 72.4^{\circ}$$
(12)

We studied evolution of the system CN vs. maximum retardation for several differences in orientation $\theta_2 - \theta_1$. Results obtained from raw numerical simulations are reported Fig. 1. We first simulated with $\theta_2 - \theta_1$ ranging from 5° to 45° and the maximum phase retardation from 30° to 360°. As expected, the whole range of parameters is not of interest, and the rest of the study will focus on configurations able to provide reasonably low CN.



Figure 1. CN vs. maximum retardation for various orientation differences between the two LC devices (left: global simulation, right : close-up on the region of interest)

We can first notice that optimal performance (i.e. $CN=\sqrt{3}$) cannot be reached with systems exhibiting maximum retardation smaller than $\frac{5\pi}{4}$, whatever $\theta_2 - \theta_1$. The choice $\theta_2 - \theta_1 = 45^\circ$ is very popular, it leads to simple experimental configurations. When the LC modulators

The choice $\theta_2 - \theta_1 = 45^\circ$ is very popular, it leads to simple experimental configurations. When the LC modulators exhibit 2π phase modulation capability, as most commercial device do, it proves an excellent choice, leading to an optimal CN of $\sqrt{3}$. But if the modulator is too thin to produce a $\frac{5\pi}{4}$ phase, CN raises and the configuration may not be the best performing solution anymore. You can even notice that the classical configuration $\theta_2 - \theta_1 = 45^\circ$ provides rather poor results if the maximum retardation do not reach π . Other choices of $\theta_2 - \theta_1$ improve the polarimeter performance in case of thin LC devices, i.e. in case of devices whose maximum retardation is lower than 2π . From these simulations, it seems that there is no straightforward rule that relates the minimum possible CN with the maximum retardation for any orientation difference.

For orientation differences $\theta_2 - \theta_1$ higher than 30°, a maximum retardation higher than π clearly increases the system performance.

Since switching speed of an LC cell varies like the inverse square of its thickness⁸ and since, for a given wavelength, retardation maximum is directly related to the cell thickness, we also reported the evolution of the system CN vs. potential relative framerate (Fig. 2). This potential relative framerate is simply the ratio of the potential framerate to the framerate of the reference 2π thick cell. In order to get figures of maximum retardation smaller than 2π and even than π , the point is to use a custom LC cell, thinner than commercial ones which classically have a maximum retardation slightly above 2π . Please notice that the simulation model does not take into account the noise of the camera which may be higher for lower exposure times: in this case, CN is therefore not the only relevant figure of merit.

5. DISCUSSION

These simulations clearly show that if there is no speed issue, the classical configuration proposed by De Martino *et al.*⁵ should be used since its provides the highest robustness. Nevertheless, if speed is to be considered, alternative configurations may be more interesting, especially in low-noise environments where CN significantly differing from $\sqrt{3}$ may be considered without much damage. For instance, the configuration $\theta_2 - \theta_1 = 35^\circ$ can



Figure 2. CN vs. potential relative framerate for various orientation differences between the two LC devices

provide a CN of 2.6 with a maximum retardation range of 135° (which corresponds to a gain of 7x in speed). Since the basic operation rate for nematic LC devices is about 50 Hz and that the proposed driving scheme allows an update of the polarimetric information at a framerate twice as high as the LC switching speed, such a polarimeter running at 700 fps is possible.

Further work may include the study of the EWV as a figure of merit, consider other driving strategies for LC devices and also take int account the increase in noise due to a lower exposure time, as Goudail does.¹²

6. CONCLUSION

In this paper, we addressed the issue of performance vs. operating speed of nematic liquid crystal polarimeters. We showed that provided a slight loss in performance is acceptable, a significant increase in speed is possible. Clearly, a polarimeter based on two PAL nematic devices running at up 700 fps is possible. This open new possibilities for the study of dynamic systems or for embedded applications.

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