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Capucine Legentil, Jeanne Pellerin, Paul Cupillard, Algiane Froehly, Guillaume Caumon. Testing scenarios on geological models: Local interface insertion in a 2D mesh and its impact on seismic wave simulation. *Computers & Geosciences*, 2022, 159, pp.105013. 10.1016/j.cageo.2021.105013. hal-03602649

HAL Id: hal-03602649

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Submitted on 16 Feb 2023

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Testing scenarios on geological models: local interface insertion in a 2D mesh and its impact on seismic wave simulation

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ARTICLE INFO

Keywords:

Triangle meshing
Gas-water contact
Geomodeling
Uncertainties
Inverse problem

ABSTRACT

In this work, we propose a local updating method to test different contact depth scenarios and assess their impact on wave propagation in the subsurface. We propose to locally modify a 2D geological model and run time-dependent elastic simulations. The input model triangulation is conforming to geological structures. The 2D meshed model is locally updated, which means that only the reservoir compartment is modified. Several model geometries are generated by inserting a new interface, in this paper a gas-water contact that is defined by a scalar field. We quantitatively evaluate the impact of the gas-water contact depth on elastic wave propagation. We run the numerical simulations with Hou10ni2D code, which is based on a Discontinuous Galerkin method. The simulation results are compared to a reference depth by computing the L2-norm at a set of seismic receivers. Results show a consistent behavior: we observe a positive correlation between the depth difference and global L2-norm for all receivers. This approach could therefore be integrated into an inversion loop to determine the position of the fluid contact and reduce uncertainties in the reservoir model from a few seismic sensors.

CRedit authorship contribution statement

Capucine Legentil: Conceptualization, Investigation, Methodology, Software, Writing (original draft). **Jeanne Pellerin:** Conceptualization, Methodology, Supervision, Funding Acquisition, Resources, Validation, Writing (reviewing and editing). **Paul Cupillard:** Conceptualization, Supervision, Funding Acquisition, Validation, Writing (reviewing and editing). **Algiane Froehly:** Software. **Guillaume Caumon:** Conceptualization, Methodology, Supervision, Funding Acquisition, Validation, Writing (reviewing and editing).

1. Introduction

Geological models have been used for decades to make forecasts about the behavior of subsurface reservoirs (e.g., Ringrose and Bentley, 2015), seismic hazards (Shaw et al., 2015) or underground waste disposal (e.g., Allen et al., 2018; Mari and Yven, 2014; Mulrooney et al., 2020). These multi-material models can be seen as three dimensional geological maps identifying the geometry of interfaces such as horizons, faults and intrusions. In general, these models are created from the observations at hand (e.g., borehole and seismic reflection data), then meshed and filled with petrophysical properties before solving the relevant physical equations (Ringrose and Bentley, 2015; Wellmann and Caumon, 2018). However, ambiguities and lack of observations often raise uncertainties about the location or even the existence of some interfaces (see Wellmann and Caumon (2018) and references therein). Additionally, temporal evolution at geological time scales or during subsurface engineering projects may involve geometric changes in the

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72 geomodel features. Assessing the impact of the uncertainties and/or the temporal changes on the model forecast is
73 a complex computational challenge, which can be partly addressed by deforming an existing mesh to avoid model
74 reconstruction (Abrahamsen, 1993; Tertois and Mallet, 2007). However, remeshing is needed when interfaces are
75 displaced along faults (Caumon et al., 2003), or when new surfaces are introduced in the model (Suter et al., 2017).

76 However, the repeated updating of a geological model is a difficult task. There are several strategies to edit an
77 interface or add a new one in an unstructured mesh. The most reliable is to rebuild the model from scratch, compute
78 the connectivity of the layers and fault blocks, and generate a new mesh of the whole model (Bidmon et al., 2004).
79 Another strategy is to directly modify the meshed model (Tertois and Mallet, 2007), but the output mesh validity must
80 be guaranteed.

81 The insertion of a polygonal line in a 2D triangulated surface is an example of such a mesh updating operation.
82 Although it seems simple at first sight, it can be challenging to implement in a robust way because of limited floating
83 point precision. For example, sequential insertion of several tangential lines representing close stratigraphic surfaces or
84 fractures in a fracture corridor may then raise invalid mesh features such as inverted triangles. A good test of robustness
85 is, therefore, to check that results are consistent and stable when inserting the same interface several times. Exact
86 geometric computations (Lévy, 2016; Li et al., 2005; Shewchuk, 1997) do address these issues in combination with
87 constrained remeshing algorithms (e.g., Si, 2015). However, they involve algorithmic complexity and a computational
88 overhead in terms of memory and time, which makes the problem « solved but practically challenging » (Li et al.,
89 2005). Therefore, we consider in this work a subclass of mesh updating problems where the element to insert is
90 represented implicitly by a level-set of a piecewise linear function defined on the mesh. This considerably simplifies
91 the management of geometric accuracy, as the target interface and the mesh to update are represented on the same
92 data structure. This implicit interface representation has been used in geosciences for structural modeling for several
93 years (e.g., Chilès et al., 2004; Houlding, 1994; Mallet, 1988), and it can be applied both on two-dimensional and
94 three-dimensional models.

95 In the mesh updating examples mentioned above, the modification is often done locally in areas of particular interest
96 (e.g., subdomains where the uncertainty is large, where sensitivity of the model features is important, or areas where
97 new data have been acquired and the model needs to be edited). Therefore, *local* mesh editing approaches, where
98 the vast majority of the mesh is kept intact, are essential for computational efficiency. However, they raise specific
99 challenges to ensure the mesh conformity between the intact and the edited parts (Caumon et al., 2003; Suter et al.,
100 2017).

101 If the insertion of lines in a 2D triangle mesh may be implemented efficiently, the corresponding 3D problem, i.e.,
102 inserting a triangulated surface or discretized lines in a tetrahedral mesh is more difficult. The steps are the same than
103 in 2D: for each element to insert, compute the intersection with the mesh, then insert the intersected elements in the

104 mesh.

105 In this paper, we propose a first step to address the above challenges by taking advantage of recent advances of
106 mesh updating libraries developed for shape optimization. Because implementing robust and efficient mesh generation
107 and mesh modification algorithms is extremely challenging, and since open-source implementations are available for
108 triangle and tetrahedral meshes (Dapogny et al., 2014; Geuzaine and Remacle, 2009; Si, 2015), we focus on the inte-
109 gration of one of these libraries in an automatic uncertainty evaluation workflow. Relying on the level-set formulation
110 of the interface to be inserted in the model, we consider the problem of monitoring the evolution of a gas-water contact
111 in a subsurface reservoir using seismic data. We start from an input meshed model conforming to the main geological
112 structures that is locally updated by inserting the fluid contact in the reservoir layer. For simplicity, we keep these
113 structures frozen and we only assume that the contact depth is unknown. Because fluid content directly affects the
114 effective elastic properties of the porous medium, fluid substitution can, in principle, be detected using seismograms
115 as done for instance in time-lapse seismic surveys (e.g., Arts et al., 2003; Landrø et al., 2003). We quantitatively eval-
116 uate the impact of contact depth on the simulation results. Under a simple assumption of horizontal fluid contact, we
117 show that our approach can be integrated into an inversion loop to determine the depth of the fluid contact and reduce
118 uncertainties in the reservoir model from only a few seismic sensors.

119 The contributions of this paper are the following: a practical approach to automatically update triangular meshes for
120 physical simulations is proposed. A special care is taken to the mesh validity, so that the model is suitable for physical
121 simulations. An assessment of the detectability of fluid contacts changes in reservoirs is presented. The algorithms
122 are available on Github and distributed under a GPL license, allowing reproducibility.

123 The geological models we consider are meshed with triangles that are conforming to geological interfaces such
124 as horizons and faults (Figure 1). We propose to modify a single layer (or region of the model) using the remeshing
125 platform Mmg (Dapogny et al., 2014) (Section 3.2.1). This library, initially developed by Dobrzynski and Frey (2008),
126 has been extended with level-set functionalities and is used to solve shape optimization problems (e.g., Allaire et al.,
127 2004; Osher and Santosa, 2001; Sethian and Wiegmann, 2000). A few adaptations enable to use this library to modify
128 geological models (Section 3.2) and a careful choice of options enables to obtain meshes of good quality, for numerical
129 simulations (Section 4). The approach is demonstrated on a simple synthetic model. Seismic wave propagation sim-
130 ulations are computed using the Discontinuous Galerkin approach implemented in Hou10ni2D [Hu-dee-nee] (Barucq
131 et al., 2014; Barucq, 2016) (Section 4.1). The simulation results for different gas-water contact depth are compared to
132 a reference one in order to solve a Bayesian inverse problem, leading to a probabilistic estimation of the contact depth
133 (Section 4.2).

Local mesh updating

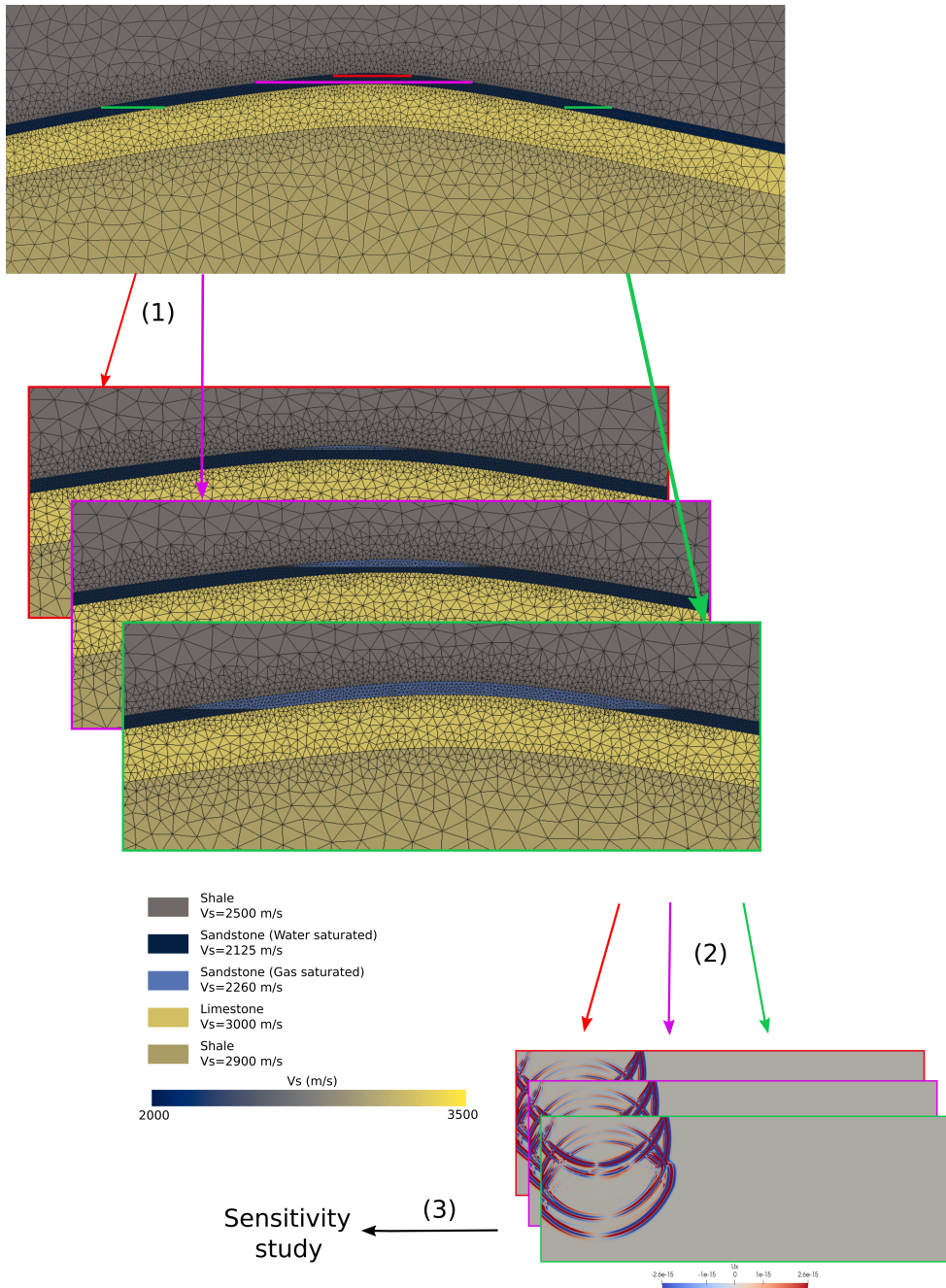


Figure 1: (1) From several level-sets defining the possible interface positions between water-saturated sandstone and gas-saturated sandstone, we use Mmg (Dapogny et al., 2014) to locally remesh a selected layer (dark blue) and (2) the Hou10ni2D software (Barucq, 2016) to simulate wave propagation and (3) compare the simulation results.

134 2. State of the art

135 *Definitions* Physical modeling in the subsurface generally involves the construction of a geological model represent-
 136 ing the interfaces between rocks that have different physical properties (such as porosity, permeability, density, Young

137 modulus, shear modulus) (Ringrose and Bentley, 2015; Wellmann and Caumon, 2018). These properties depend on
 138 multiple factors (e.g., facies, fluid saturation, compaction, temperature). Geological models, therefore, are multi-
 139 material models, a term used more generally in mechanics. The interfaces between different materials correspond to
 140 horizons, faults, unconformities, intrusions, cavities and fluid contacts. Among the various possibilities to represent
 141 geological media, unstructured meshes offer the interesting ability to honor complex geometries and to spatially vary
 142 the level of detail. In previous works, 3D models have been considered, but for simplicity here we consider triangulated
 143 two-dimensional geological models similar to those defined in Anquez et al. (2019) or Pellerin et al. (2017) (Figure 2).
 144 Indeed 2D models are most effective for proof of concept, as they allow to assess the impact of changes on physical
 145 simulation in acceptable time (Section 4.1) and they are commonly used for physical models in cylindrical domains.
 146 Generalization to 3D will be discussed in Section 5.

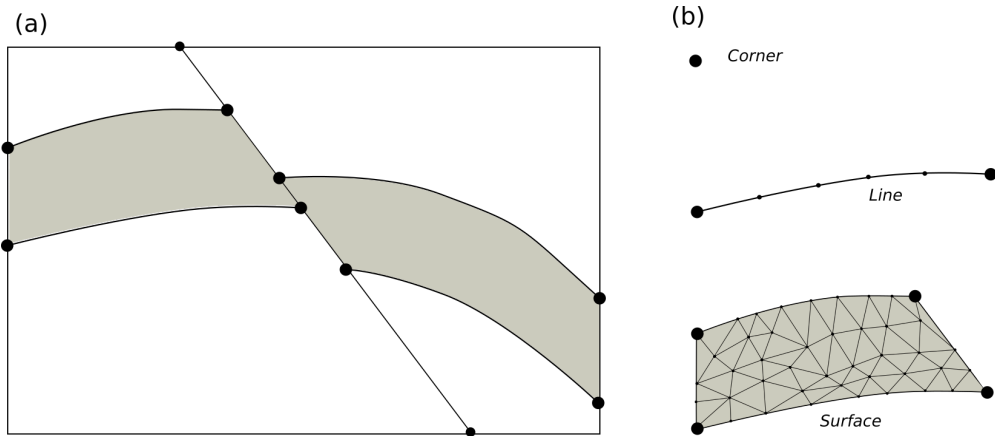


Figure 2: 2D triangulated models are defined by their Corners (black dots), Lines, and Surfaces (a) whose geometry is supported by points, segments and triangles (b)

147 Triangulated models are adequate to run wave simulations with a Discontinuous Galerkin method provided that the
 148 mesh is valid and of sufficient quality. By valid we mean that all intersections between two triangles of the mesh are
 149 either empty or a common edge/vertex to both triangles and that no element is empty. The quality of the mesh is related
 150 to the size and shape of the triangles. It has an impact on the accuracy and computation time, and its definition is specific
 151 to each solver (e.g., Loseille et al., 2010). The interfaces between two materials are represented by discretized lines
 152 that separate two regions of the model. An alternative representation of interfaces in geological modeling, and more
 153 generally in multi-material modeling, is the level-set representation (also called implicit). In geological modeling this
 154 representation has gained popularity and is the base of the implicit stratigraphic modeling. In this context, the scalar
 155 field representing stratigraphy in a conformal layer can be seen as relative geological time (Caumon et al., 2013; Collon
 156 and Caumon, 2017; Chilès et al., 2004; Houlding, 1994; Ledez, 2003; Mallet, 2004; Tertois and Mallet, 2007). In some
 157 cases, level-set interfaces can be directly integrated in physical solvers using Heaviside basis functions as in XFEM

158 (Moës et al., 2002) or to define specific boundary conditions (e.g., Durand-Riard et al., 2010). Level-set representation
 159 has also gained popularity to solve subsurface inverse problems (e.g., Cherpeau et al., 2012; Giraud et al., 2021; Guillén
 160 et al., 2008; Zheglova et al., 2018).

161 *Editing models* Different types of local modifications are described in the literature. Some authors propose to lo-
 162 cally deform the mesh using geometric methods (Laurent et al., 2015; Tertois and Mallet, 2007). These constrained
 163 deformation methods are based on the definition of a smooth deformation field interpolated from a few anchor points.
 164 One of the problems encountered when modifying the geometry of a set of vertices in a mesh is that the elements
 165 may become invalid, as some intersections may occur between the edges of the triangles or the faces of the tetrahedra
 166 (Tertois and Mallet, 2007).

167 However, general displacement, insertion and removal of boundaries lead to more substantial topological changes
 168 affecting all model entities (nodes, edges, regions). Sword Jr (1991) proposes methods to edit and update the object
 169 adjacencies and to verify the model topological validity at each modification (edge intersection verification, for exam-
 170 ple). In 3D, Euler et al. (1999) propose a general way to edit boundary representations which relies on intersections
 171 and border projections of triangulated surfaces before globally reconstructing a boundary representation. Although
 172 this approach is very flexible in principle, the use of projection raises instabilities and the approach does not provide
 173 a strict guarantee that the model integrity is maintained throughout the editing. As part of the update of geomodels
 174 during drilling monitoring, Suter et al. (2017) propose to modify the only geological block involved in the new data
 175 which does not correspond to the initial model. The region to modify is completely erased and meshed again taking
 176 into account the new data (Figure 3), but mesh conformity, required in most physical simulation codes, is not discussed.

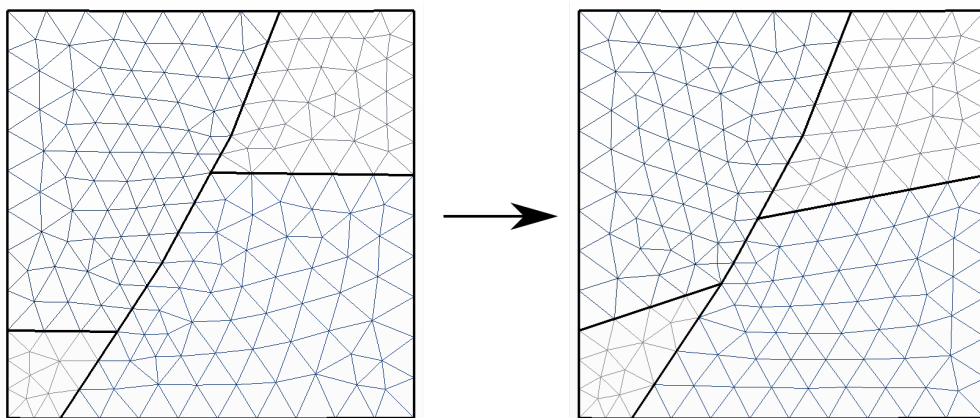


Figure 3: Example of a fault throw modification in which the model is completely erased and meshed again.

177 We follow a similar strategy in this work, but we also capitalize on recent advances in shape optimization that
 178 regroups a set of methods for finding the “best shape” of an object so that it fulfills its functions. This optimization is

179 automated and the geometrical modeling also uses unstructured meshes. The management of heterogeneity as present
 180 in geological models, introduces specific difficulties compared to the optimization of homogeneous objects (Wang
 181 et al., 2015). Level-set methods are very interesting because they allow tracking fronts and free boundaries. Sethian
 182 and Wiegmann (2000) first introduced the level-set description into topology optimization. Level-set techniques were
 183 then developed in, e.g., Allaire et al. (2004), Osher and Santosa (2001), Sethian and Wiegmann (2000), Wang et al.
 184 (2003) to perform topological changes of the level-set components. An interface Ω is defined by an iso-value φ_0 of a
 185 level-set function (i.e, a scalar field) φ as

$$\Omega = \{\bar{x}, \varphi(\bar{x}) = \varphi_0\}. \quad (1)$$

186 This level-set representation is able to represent geometric interfaces between materials, such as geological interfaces,
 187 as shown in Figure 4. Allaire et al. (2014) propose to explicitly discretize the level-set at each iteration in a computa-
 188 tional domain equipped with a simplicial mesh. In doing so, a sub-mesh of the domain is created at each step.

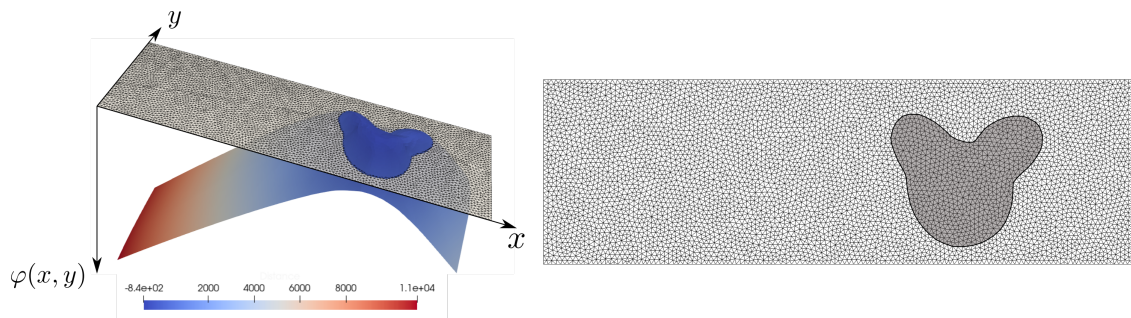


Figure 4: (Left) 2D level-set function; (right) a corresponding meshed description: the 2D domain is equipped with a mesh (composed of the white and grey elements). The iso-value 0 represents the interface between the white and grey regions.

189 In the shape optimization by a physical constraint, the accuracy of the border has a direct impact on the precision
 190 of the simulations. To build the obstacle boundaries, Santosa (1996) proposes an inverse approach involving a set of
 191 level-set functions that minimizes a residual to fit data. To deal with multi-material models, Wang et al. (2005) and
 192 Wang et al. (2015) introduce a “color” level-set method, where each material is defined by a level-set function. This
 193 approach is similar to a subsurface inverse problem considering, for example, the position of a horizon as a parameter
 194 set. To solve an inverse problem in full waveform case, Guo and de Hoop (2013) adapt the method of Santosa (1996)
 195 and use concepts from shape optimization and image segmentation to determine the position of a salt body. Giraud et al.
 196 (2021), and Zheglova et al. (2018) use multiple level-sets to represent multiple geological units in a model, therefore
 197 can recover the geometry of geological bodies using level-set inversion of geophysical data.

198 **3. Implicit local interface insertion**

199 In this section we propose to insert a level-set interface in a mesh by restricting the modifications to a prescribed
 200 region of the model. The chosen level-set iso-value is first discretized, and the modified region is remeshed conformably
 201 to prevent a steep decrease of the mesh quality that would slow down further computations or make them unstable.
 202 When merging the edited region with the other parts of the model, specific care to preserve the mesh conformity is
 203 taken.

204 **3.1. Input Data**

205 *Level-set function* To perform the local insertion of an interface in the geomodels, the input data is composed of
 206 the global mesh and the interface implicitly defined (e.g. a fault, a gas-water contact...). In our case, the level-set is
 207 approximated by the iso-values of a regular piecewise linear scalar field represented on the nodes of the triangular
 208 mesh. Typically, the scalar field represents the signed distance to the line, which is defined by the 0-valued segment
 209 in each intersected triangle (Figure 5b, Figure 6b). The insertion of the new points and edges is based on this implicit
 210 line definition.

211 *Model partitioning* In our method, the area to be modified (inner region) consists of one or several input surfaces
 212 representing geological units, but this can easily be changed to a list of triangles, e.g., to insert an internal fracture.
 213 The implicit line will be inserted in the inner region while maintaining the conformity to the lines and corners of that
 214 region, e.g., the blue layers (# A1, A4, B2 and B3) in Figure 6a. The rest of the model (outer region), is not affected
 215 by the modification, except a small set of triangles which needs to be modified to maintain the model conformity. In
 216 Figure 6a, all the triangles of the outer region that are in contact with the bold boundaries could be affected by this
 217 conformity requirement; in practice, as will be further explained in Section 3.2.3, only the triangles that are in contact
 218 with the level-set need to be considered.

219 **3.2. Local level-set discretization**

220 **3.2.1. Mmg software**

221 We use the Mmg code for the level-set insertion in the inner region. Mmg is an open source 2D and 3D remeshing
 222 software with two major features: level-set discretization and mesh adaptation. Mmg is dedicated to the processing of
 223 triangle and tetrahedral meshes. It is a meshing software mainly used for computational fluid dynamics applications
 224 and for shape optimization applications (Benard et al., 2016; Loseille et al., 2010). It has also recently been effectively
 225 integrated to adapt meshes in full waveform inversion problem to accelerate the computations (Jacquet, 2021).

226 The discretization of level-sets allows to insert a line or a surface that represents a scalar field iso-value in a mesh.
 227 The scalar field is defined at all the nodes of the mesh, forming a piecewise linear function. The software takes

Local mesh updating

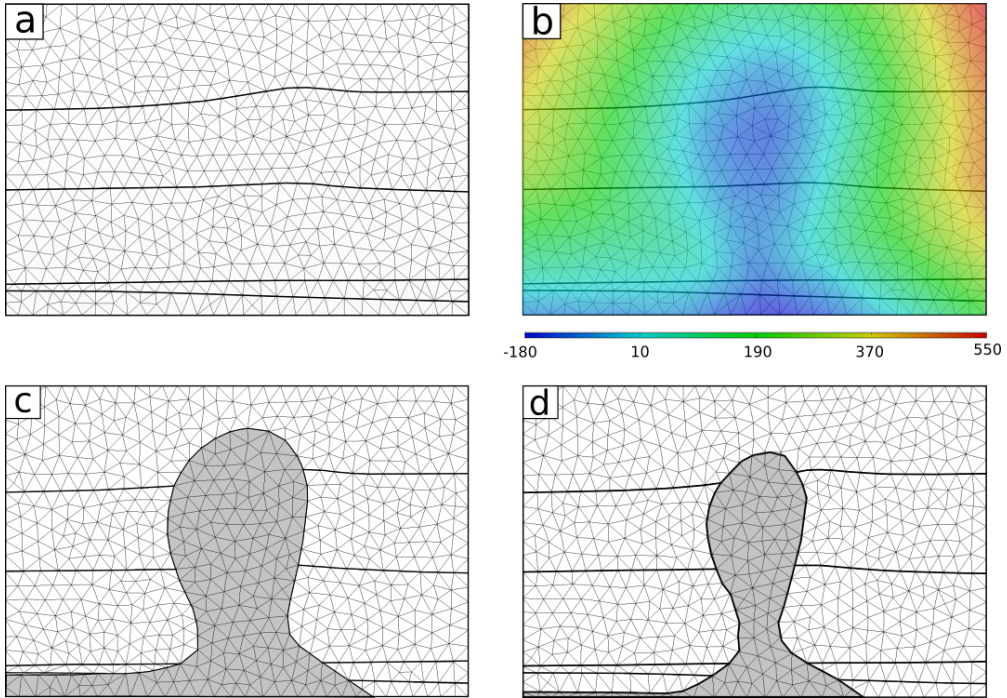


Figure 5: Insertion of an implicitly defined salt dome (b) in a geological model (a). The iso-values of the signed distance scalar field represent possible salt dome boundaries; (c) and (d) show two resulting models for scalar values 0 and -70, respectively.

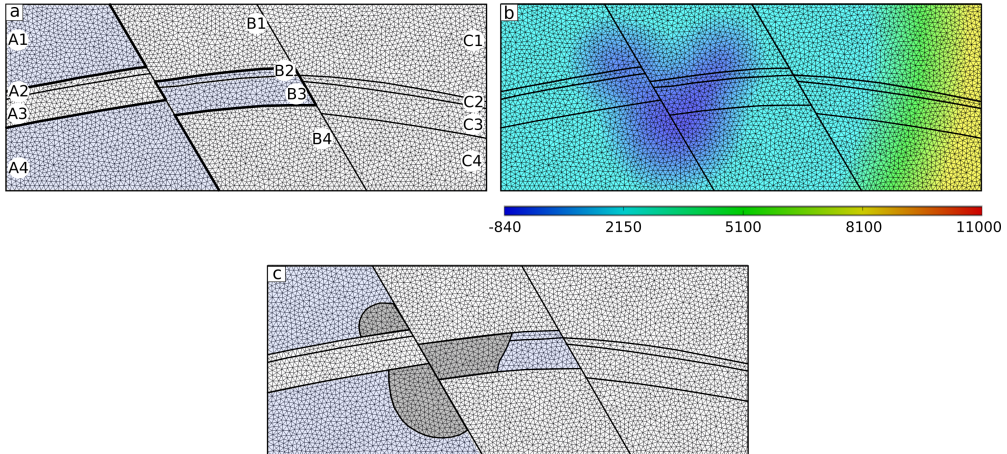


Figure 6: The flexibility of the proposed approach demonstrated on the insertion of an arbitrary shape in an edited region composed of surface units A_1, B_2, B_3 and A_4 (light blue).

228 into account multi-material models, so the discretization of the iso-value can be limited to determined materials and
 229 maintain existing materials.

230 The other main feature of Mmg is the mesh adaptation: the user can set several parameters to control the size of
 231 the mesh elements. For example the $hmin$ and $hmax$ parameters are used to define the minimum and maximum size
 232 of the elements; the parameter $hgrad$ controls the ratio between the length of two adjacent edges. The user can set

233 several command line parameters or provide an input size map to control the size of mesh elements.

234 3.2.2. Level-set discretization

235 The mesh modifications are made with the level-set discretization method. The algorithm implemented in Mmg
 236 (Dapogny et al., 2014) consists of two main steps. First, the line is inserted in the defined region as simply as possible
 237 while keeping the mesh consistent: each triangle intersected by the level-set is cut into either two triangles (when the
 238 level-set goes exactly through one vertex) or one triangle on one side and two triangles on the other side (when the
 239 level-set cuts the interior of two edges) (Figure 7). However, most of the newly created triangles are of poor quality
 240 (Figure 8.a.), so remeshing is essential before running a seismic simulation.

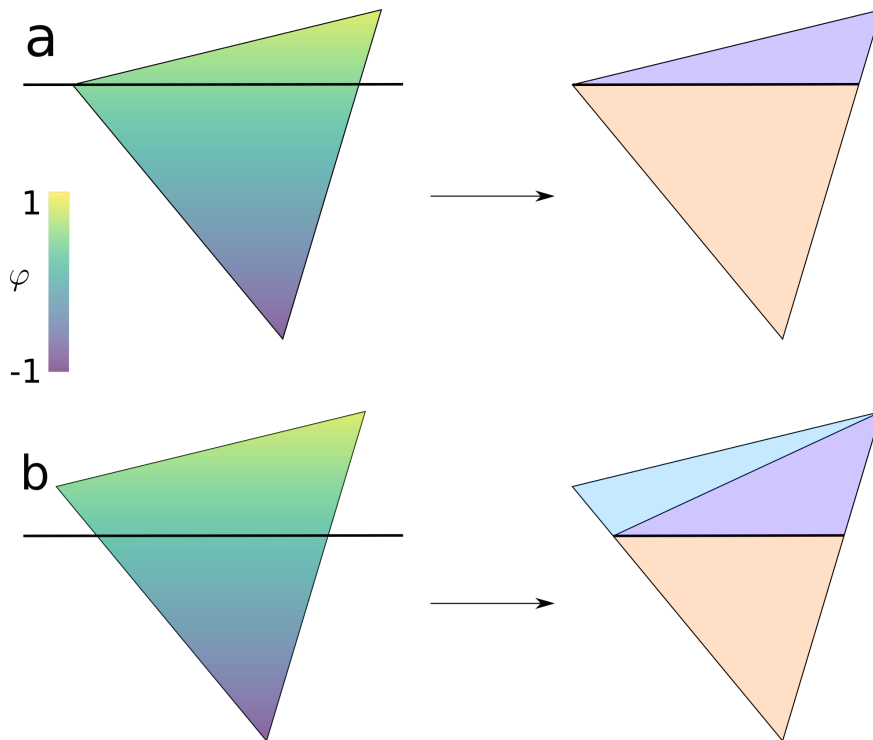


Figure 7: Two steps level-set discretization method in Mmg: (a) If the level-set goes exactly through one vertex, the triangle is cut into two triangles; (b) if the level-set cuts the interior of two edges, the triangle is cut into one triangle (red) on one side and two triangles (green and blue) on the other side

241 In the interface discretization, the major constraint is to guarantee the conformity of the entire mesh for both the
 242 inserted and pre-existing boundaries. To this end, only the triangles adjacent to the modified region and in contact
 243 with the ends of the implicit interface are modified. They are simply divided into two new triangles (Figure 9).

244 The inner region is then divided into two regions corresponding to the two sides of the inserted interface. The two
 245 parts of the mesh are remeshed separately to adapt the mesh to the constraints imposed by the user: the maximum
 246 or minimum element size and the gradation (i.e., the ratio of adjacent elements size). The operations performed on

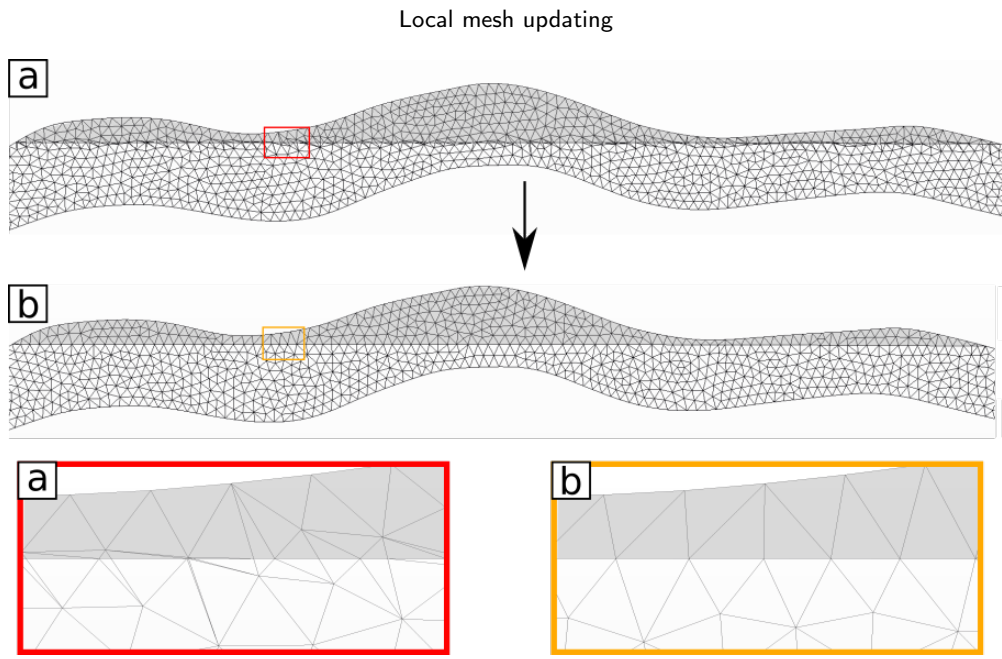


Figure 8: Two steps level-set discretization method in Mmg: (a) insertion of the line by creating new nodes and triangles; (b) remeshing of the region

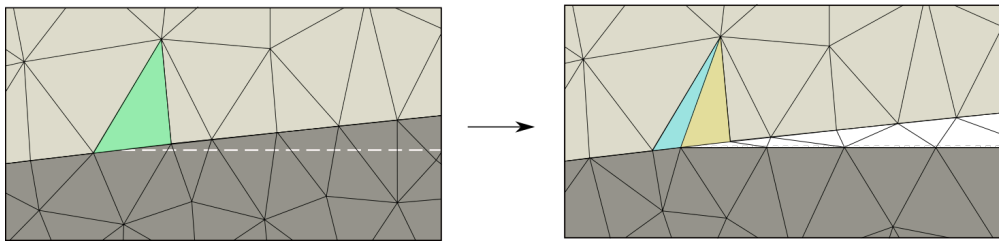


Figure 9: Division of a triangle at the edge of the modified region to maintain the mesh conformity

247 the mesh consist of insertion or displacement of nodes, edge swap or collapse. All decisions are made to adapt to the
 248 size map defined as input and to improve the mesh quality. To maintain the conformity of the overall mesh, the edges
 249 between sub-parts must not be modified (Figure 8b).

250 3.2.3. Merge

251 The last step consists in merging the modified and remeshed region and the preserved one. During this step some
 252 points are duplicated. The mesh cannot have points at the same position for our wave propagation simulation. Dupli-
 253 cated nodes are removed by using co-location detection from Gmsh¹. This API function is based on a R-Tree (Guttman,
 254 1984).

¹<https://gitlab.onelab.info/gmsh/gmsh>

255 3.3. Mesh quality

To evaluate the mesh quality we propose to compute the minimal height of the worst element among the n_t triangles:

$$Q = \min_{\forall k \in \llbracket 1; n_t \rrbracket} h_k, \quad (2)$$

256 where h_k is the lowest height of the triangle k . The quality Q has an important impact on elastic wave simulation time
 257 because of small time steps imposed by the Courant–Friedrichs–Lewy (CFL) condition (Section 4.2). To minimize
 258 the decrease of quality, if the distance between the level-set and a node located on another interface is less than a given
 259 tolerance ϵ , the scalar field is locally modified: the iso-value is set at this existing node so that no new node is created
 260 during the line insertion in Mmg (Figure 10). In Figure 10, the difference of the scalar field values of two vertices in a
 261 triangle $|\varphi_1 - \varphi_0|$ is normalized by the norm of gradient of the level-set function in the triangle $\|\vec{\nabla}\varphi\|$. This value is
 262 then compared to the tolerance ϵ that can be assimilated to a distance. By adjusting the parameter ϵ , one can limit the
 263 creation of very elongated triangles which otherwise appear at the intersection between the iso-value and the borders
 264 of the layer, at the expense of a small inaccuracy in the final geometry.

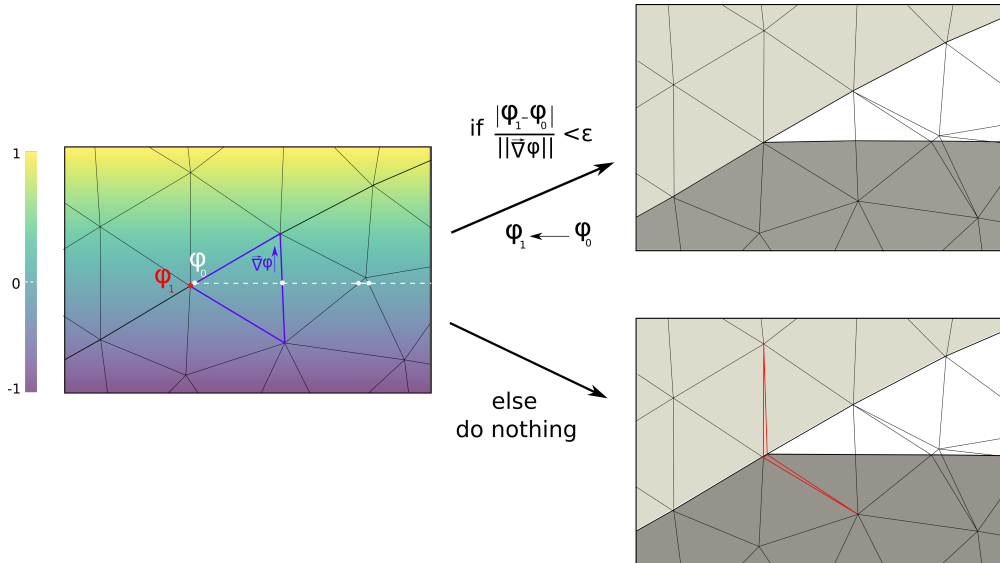


Figure 10: During the level-set discretization, if the distance between the iso-surface and an existing node is less than ϵ (2), the iso-value is approximated at this point and no other point is created when this approximation is not made, ill-shaped triangles are formed (in red).

265 3.4. Open-Source Implementation

266 *Process automation* We propose algorithms for the automation of the process. The code is available on Github²
 267 and depends on Mmg³ (Dapogny et al., 2014) and Gmsh⁴ (Geuzaine and Remacle, 2009). It allows to create the
 268 two sub-meshes to restrict the discretization to the modified region, and then to merge them. Duplicated nodes are
 269 removed by co-location detection from Gmsh. It is then possible to create models that correspond to the input format
 270 of the simulator. To prevent remeshing of the entire mesh, the options *-nomove*, *-noinsert*, *-noswap* are used on the
 271 command line when calling Mmg for level-set discretization. The *-nomove* option blocks the movement of vertices
 272 and the *-noinsert* one prevents the creation of new vertices. The *-noswap* option prevents the edge swapping between
 273 two adjacent triangles.

274 *Element quality improvement* A large part of finite element schemes solving boundary values problems imposes the
 275 absence of triangles whose edges are on two different borders of the model (Figure 11). In our application, the edges
 276 are not domain boundaries but only material boundaries, hence they do not carry boundary conditions. Therefore, we
 277 activate the *-nofem* option in Mmg to improve the mesh quality. The options *hmin* and *hmax* can also be used to set
 278 the mesh size to adapt the mesh to the physical simulator.

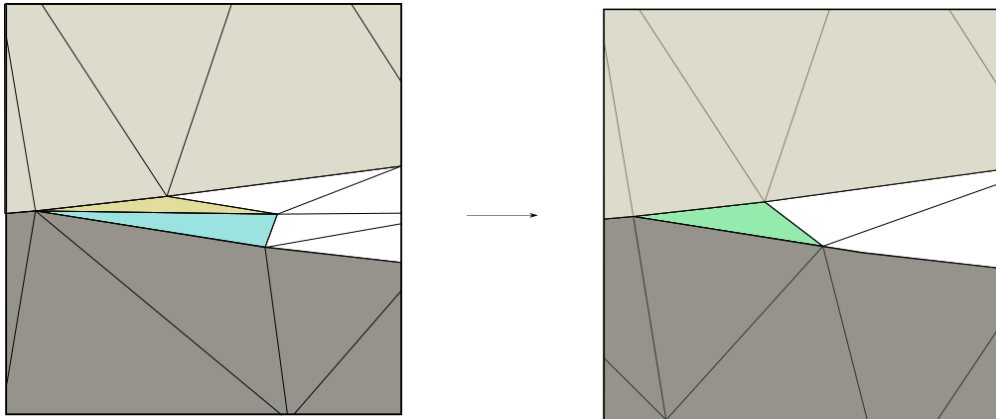


Figure 11: Results of MMG remeshing by default (left) and with the option *-nofem* activated (right). With this option, triangles with two constrained edges are allowed, yielding a higher quality mesh for our application.

279 However, there are very thin objects in geology, which inevitably yield poor quality triangles. During the physical
 280 simulations, such triangles show a very small value of the determinant of their Jacobian matrix, leading to potential
 281 inaccuracy in the simulation result. In dynamic problems, these triangles also imply a small time-step because of the
 282 CFL condition, which impacts the computational cost. Additionally, it could be combined with approaches to simplify
 283 models in order to reduce the simulation time. For example, during the level-set discretization, an option *-rmc* is

²<https://github.com/ring-team/LUMOS2D>

³<https://github.com/MmgTools/mmg>

⁴<https://gitlab.onelab.info/gmsh/gmsh>

284 available to remove small surface parts. The decision is taken by comparing the surface of the small component to the
 285 region surface.

286 For each new model created, the mesh quality (2) is computed. If the quality is too low (the choice of the quality
 287 threshold is discussed in Section 5), the process can be repeated after approximating the scalar field at several nodes
 288 by increasing ϵ to avoid problematic configurations (Figure 10, Figure 14). A new model is then created with less
 289 elongated triangles.

290 *File format* Mmg uses the Medit file format (.mesh). We use the tag that may be set on the elements to encode model
 291 entities. The scalar field and/or metric for remeshing values are defined on the vertices of the mesh and are stored in
 292 an other file format (.sol). The local parameters for multi-material models are defined in a 2 dimensional Mmg file
 293 (.mmg2d).

294 4. Gas-water contact uncertainty and elastic wave propagation

295 In this section, we apply our local level-set based model edition to evaluate the impact of a gas-water contact depth
 296 on seismic signals. We consider a synthetic model on which we quantitatively seek to evaluate the correlation between
 297 the contact depth and some seismic waveform misfits, demonstrating the usefulness of our approach to efficiently
 298 sample and reduce uncertainties on a material interface position.

299 Our synthetic geological model M_{ref} is an anticline overlying horsts and grabens (Figure 12). The anticline is
 300 composed of folded layers of shale and sandstone. After the insertion of the gas-water contact in the reservoir, the
 301 gas zone is represented in grey (layer 6 in Figure 12). The domain size is 16.2 km horizontally and 5 km vertically.
 302 The petrophysical values of the different layers of the model are summarized in Table 1. These values were chosen
 303 consistently with the case study of Hamada (2004). For simplicity, we use a constant density $d = 2600 \text{ kg/m}^3$.

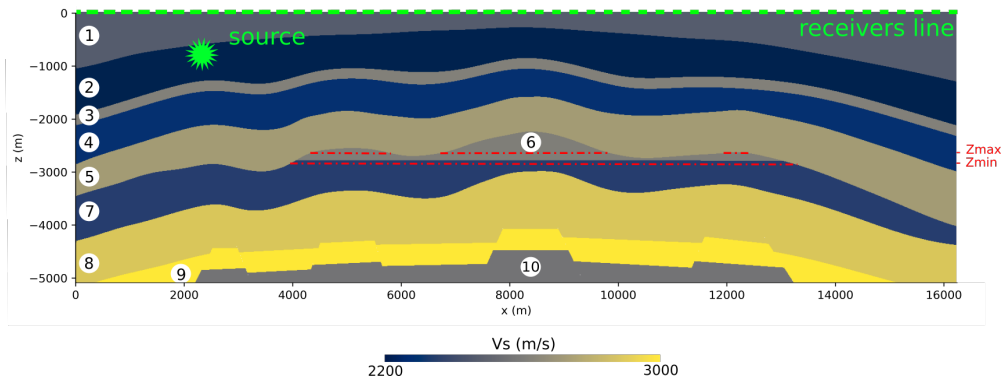


Figure 12: Reference model: the gas-water depth is $z = -2,785 \text{ m}$. The dashed green line represents a line of 150 receivers (one every 100 m) at the top of the domain. The lithology is presented in Table 1

Color map	Lithology	P-wave velocity (m/s)	S-wave velocity (m/s)
1	Shale	3000	2500
2	Sandstone	3400	2260
3	Shale	3200	2650
4	Sandstone	3500	2330
5	Shale	3300	2750
6	Sandstone (Gas sat.)	3700	2740
7	Sandstone (Water sat.)	3800	2375
8	Shale	3900	2600
9	Limestone	4000	2900
10	Sandstone	4500	3000

Table 1

Lithological and petrophysical characterization of the model Figure 12. Density is constant $d = 2600 \text{ kg/m}^3$.

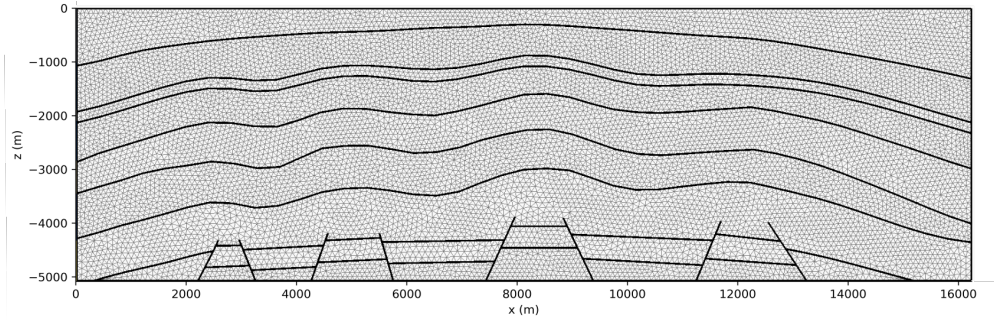


Figure 13: The initial mesh of our synthetic model generated with Gmsh and Mmg, contains 22,077 triangles.

304 4.1. Seismic simulation parameterization

305 In this study, we use the Hou10ni2D software (Barucq, 2016) which implements a Discontinuous Galerkin (DG)
 306 method (Reed and Hill, 1973) for solving the wave equation. More precisely, Hou10ni2D relies on Interior Penalty
 307 approach (Grote et al., 2006; Riviere, 2008) to simulate time-dependent elastic wave propagation. Hou10ni simu-
 308 lates seismic wave propagation in heterogeneous media and allows the use of different polynomial orders in the mesh
 309 elements (p-adaptivity in space) (Barucq et al., 2014). In addition, Hou10ni2D can run on parallel architectures.

310 We apply absorbing boundary conditions on the two lateral sides and the bottom of the cross-section, whereas the
 311 top of the domain is defined as a free surface. We choose the source time-function as a Ricker wavelet with a dominant
 312 frequency $f = 15 \text{ Hz}$. As a consequence, the minimum wavelength is

$$\lambda_{min} \simeq \frac{v_s}{3f}, \quad (3)$$

Local mesh updating

Depth (m)	Nb triangles	Quality	Time (s)	Nb CC
-2,645	21,997	8.54	17,321	3
-2,655	21,989	2.00	46,376	3
-2,665	22,007	6.37	16,973	3
-2,675	21,997	6.1	18,231	3
-2,685	21,995	2.35	42,651	3
-2,695	22,023	4.85	21,300	3
-2,705	22,003	4.76	21,759	2
-2,715	21,971	5.1	20,847	2
-2,725	22,003	7.99	18,469	2
-2,735	22,011	2.95	33,718	2
-2,745	21,991	6.01	19,620	1
-2,755	21,967	25.51	11,348	1
-2,765	21,997	2.77	35,271	1
-2,775	21,981	22.76	10,809	1
-2,785	21,959	7.36	20,298	1
-2,795	21,969	17.95	11,508	1
-2,805	21,971	24.84	11,514	1
-2,815	21,939	11.31	13,674	1
-2,825	21,955	6.89	17,883	1
-2,835	21,951	2.57	38,380	1
-2,845	21,959	22.55	11,015	1

Table 2

Model characteristics (Contact depth, number of triangles, worst element quality, simulation time and number of connected reservoir parts). The quality is the minimal triangle height (2). Simulations are run on a machine with a processor Intel(R) Xeon(R) CPU E5-1620 v2 @ 3.70GHz, 8 Cores.

313 where v_s is the S-wave velocity. The source is located at depth $z = -775$ m, two kilometers from the left side. We
 314 place a line of 150 receivers (one every 100 m) at the top of the domain (dashed green line in Figure 12). Simulations
 315 are run for $T = 4$ s after the shock. The displacement is computed for all the receivers at each time step.

316 **4.2. Comparison of simulation results with a varying gas-water contact depth**

317 *Set of models* We built 20 models by inserting the gas-water contact at several depths in the initial mesh Figure 13,
 318 using our method presented in Section 3. The process takes about 10 seconds per model. Depths vary from $z_{min} =$
 319 $-2,845$ m to $z_{max} = -2,645$ m (between the two dashed red lines in Figure 12), with a constant step δz equal to 10 m.
 320 Each model is characterized (Table 2) by the contact depth, number of triangles, mesh quality, simulation time, and
 321 number of connected reservoir parts (Nb CC in Table 2). To evaluate the quality, we chose the minimal of all triangle
 322 heights (2), as it is directly linked to the software heuristics for the determination of the time step and the level of the
 323 p-refinement. This value has a strong impact on the simulation run time of simulation as can be seen in Figure 14.

324 *Impact of quality on simulation time* The mesh quality depends on the model geometrical configuration. There is
 325 no direct link between the worst element quality, the depth of the contact or the number of connected components of the
 326 gas-saturated layer (#6 in Figure 12). Poor quality triangles are located around the ends of the gas-water contact line.
 327 They often are small and elongated triangles. The existence of these ill-shaped triangles is partly a consequence of

Local mesh updating

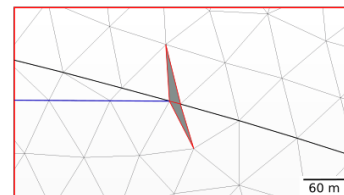
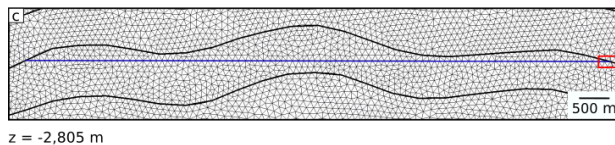
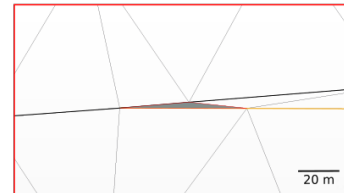
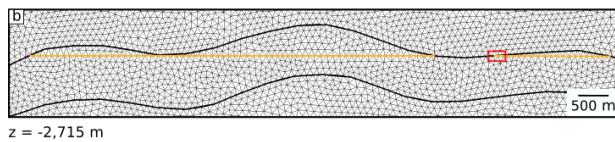
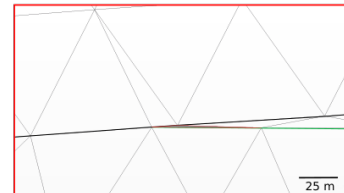
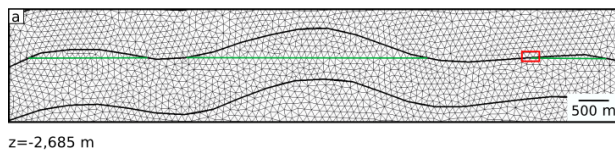
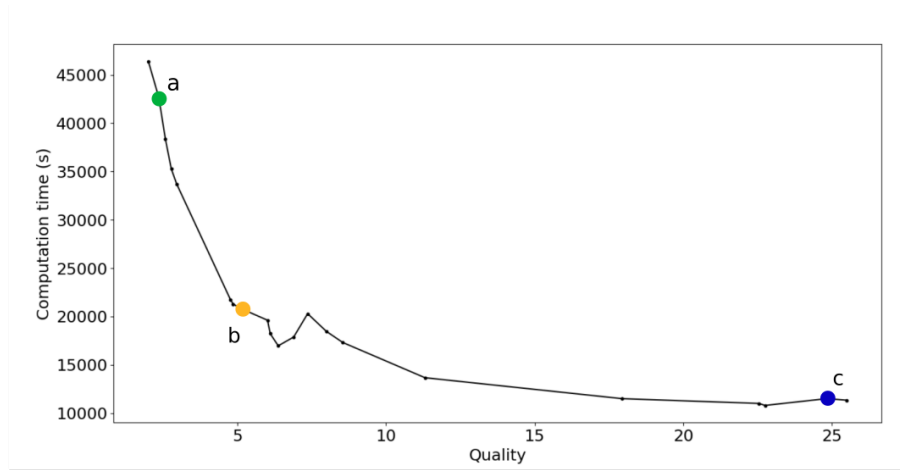


Figure 14: The mesh quality impacts the computation time: the highlighted triangles with the lowest quality (right) for 4 models (left) have elongated form, leading to decreasing time steps and increasing simulation run time (top).

328 low angles between the contact and stratigraphic boundaries (close-up views in Figure 14b and Figure 14c). However,
 329 the position of the new corners is the parameter that most impacts the mesh quality and the simulation time. If the
 330 corner is very close to a vertex of an adjacent triangle of the outer region (#5 in Figure 12), the quality of the triangles
 331 created is low (Figure 14a and b). The approximation of the scalar field described in Section 3.2 and Figure 10 avoids
 332 the creation of very poor quality triangles.

Local mesh updating

Depth (m)	Scalar field approximation	Quality	Time (s)
-2,705	$\epsilon = 0$	0.04	>2,000,000
	$\epsilon = 0.5 m$	4.76	21,759
-2,775	$\epsilon = 0$	0.74	129,102
	$\epsilon = 0.5 m$	22.76	10,809

Table 3

Impact of elongated triangles deletion: the simulation time is between 12 and 95 times shorter. Simulations are run on a machine with a processor Intel(R) Xeon(R) CPU E5-1620 v2 @ 3.70GHz, 8 Cores.

333 The scalar field has been locally approximated for two models: $z = -2,775 m$ and $z = -2,705 m$ with $\epsilon = 0.5 m$.
 334 These small modifications lead to a large quality improvement and a significant reduction of the computation time
 335 (Table 3).

336 *Comparison with the reference model* For each modified model \mathbf{M}_z , a simulation is run using the corresponding
 337 velocity model. Then the relative difference $\delta d_{i,z}$ at each receiver i between the seismogram $\bar{u}_{i,ref}$ of the reference
 338 model \mathbf{M}_{ref} (Figure 12) and the seismogram $\bar{u}_{i,z}$ simulated in each model \mathbf{M}_z is calculated (Geller and Takeuchi, 1995):

$$\delta d_{i,z} = \left(\frac{\int_0^T \left\| \bar{u}_{ref}(t) - \bar{u}_{i,z}(t) \right\|^2 dt}{\int_0^T \left\| \bar{u}_{ref}(t) \right\|^2 dt} \right)^{\frac{1}{2}} \quad (4)$$

339 The results are difficult to interpret because the simulated wavefield is complex. From Figure 15, we see that the
 340 largest waveform difference ($\delta d_{i,z} > 20\%$) is obtained between $x = 5,200 m$ and $x = 13,400 m$. On average, the
 341 seismogram error increases when the contact depth error Δz increases. Horizontally, however, Figure 15 shows that
 342 some receivers are not sensitive to contact depth uncertainty, e.g., from $x = 0 m$ to $x = 2000 m$ and from $x = 13,600 m$
 343 to $x = 15,000 m$. This corresponds to receivers for which the possible contacts are not illuminated by the wavefield.
 344 Interestingly, we also observe that the local error decreases with increasing depth differences, e.g. from $x = 6,500 m$
 345 and $x = 7,500 m$ with $\Delta z > 110 m$. This highlights the importance of the starting model to solve inverse problems
 346 from sparse seismic measurements to avoid being trapped in local minimum when only a few receivers are used. Due
 347 to the complexity of these results, a comparative analysis on a single receiver is delicate, and it is more relevant to take
 348 into account the seismograms at a large set of receivers to analyze the results.

349 To more easily compare simulations we choose a unique indicator, namely the mean μ_z of the errors on the n

Local mesh updating

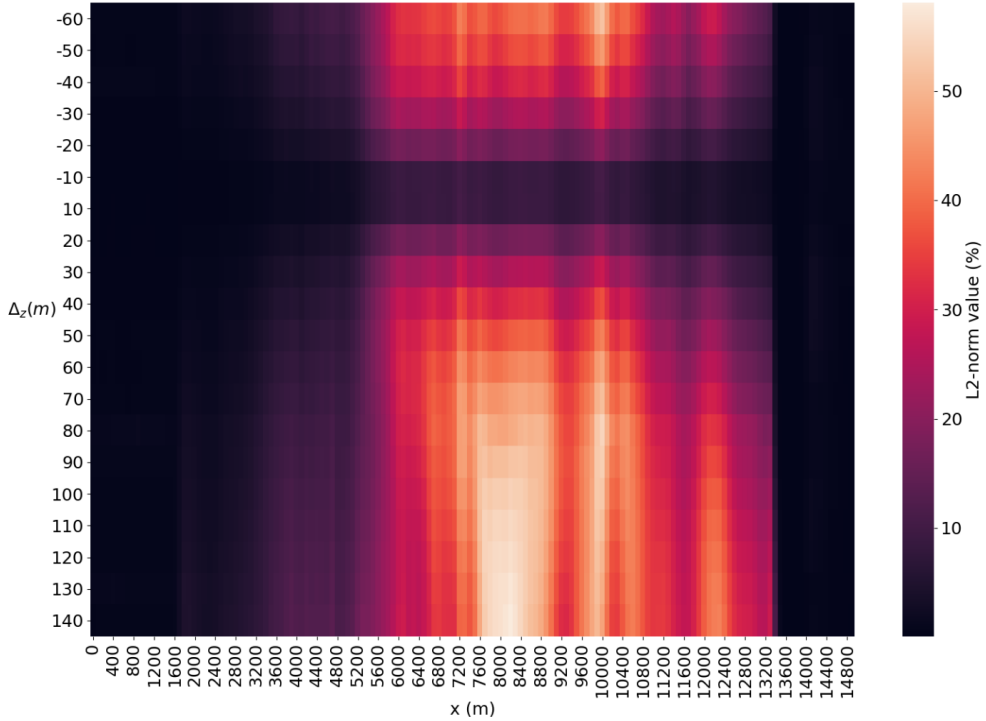


Figure 15: L2-norm waveform difference $\delta d_{i,z}$ (4) for each receiver i at position $x = i * 100 m$ as a function of contact depth difference Δ_z (results are in percentages). The color represents the L2-norm value at one receiver for one Δ_z . The Δ_z values are negative when the contact of a model is below the contact of the reference model.

350 receivers:

$$\mu_z = \frac{\sum_{i=0}^n \delta d_{i,z}}{n} \quad (5)$$

351 We observe in Figure 16 a correlation between the mean error and the depth error Δz : the larger the distance from
 352 the reference depth contact, the higher the mean of L2-norm.

353 The inverse problem we consider consists in finding a model \mathbf{M}_z containing a contact at the depth z , such that
 354 its seismic response $s(z)$ is “close” to a reference seismic response, obtained from the model \mathbf{M}_{ref} defined as $s(z) =$
 355 $s(z_{\text{ref}})$. The difference in seismic response is evaluated using the L2-norm (4 and 5). The inversion loop to determine
 356 the best model that fits the seismic response of the reference model is, in general, a difficult process. To solve a Bayesian
 357 inverse problem, many different models have to be generated and seismic simulations are required for each of them.
 358 Here, the inversion only concerns one parameter (the contact depth), so we use a simple grid search (brute-force)
 359 approach using the previous results (Figure 15 and Figure 16).

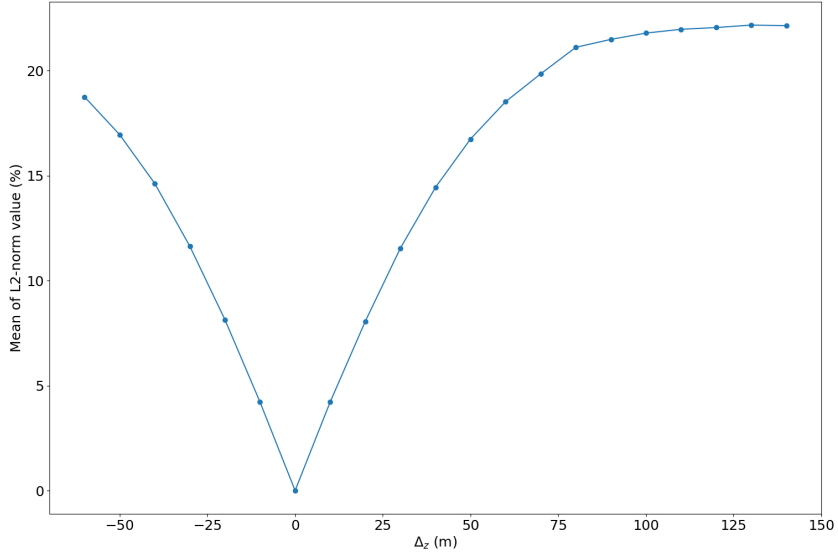


Figure 16: Mean of L2-norm μ_z (5) as a function of Δ_z

360 4.3. Quantification of uncertainties

361 To test the impact of the modifications on the simulations, and to study the feasibility of integrating our approach
 362 in an inversion loop, we compute the density of probability describing the depth of the contact. Bayes theorem (e.g.,
 363 MacKay, 2003) is used to combine prior information on the structural model with the observed data to give the posterior
 364 probability density function

$$\rho(z|\mathbf{D}) = \frac{\rho(z)L(\mathbf{D}, z)}{\int_z \rho(z)L(\mathbf{D}, z)dz} \quad (6)$$

365 where $\rho(z)$ is the prior probability density, i.e., the information we have on the contact depth before considering the
 366 data \mathbf{D} . In this study, we choose a non-informative (uniform) prior probability density

$$\rho(z) = \begin{cases} \frac{1}{z_{max}-z_{min}} & \text{if } z \in [z_{min}; z_{max}] \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

367 The likelihood function $L(\mathbf{D}, z)$ quantifies how equivalent a candidate model \mathbf{M} is to the reference model \mathbf{M}_{ref} by
 368 comparing their seismic simulation results: $\mathbf{D}(z)$ at all receivers for the candidate model and $\mathbf{D}_{ref} = \mathbf{D}(z_{ref})$ for the

reference model. In this work, we use a classical Gaussian assumption:

$$L(\mathbf{D}, z) = \rho(\mathbf{D}|z) = \frac{e^{-\frac{1}{2} \frac{\|\mathbf{D}(z) - \mathbf{D}_{ref}\|^2}{\sigma^2}}}{\sigma \sqrt{2\pi}}, \text{ with } \sigma = \|\mathbf{D}\| \quad (8)$$

This makes it possible to use the values of L2-norm introduced in the previous section to calculate the posterior density of probability. By using (4) to simplify (8) we obtain

$$\frac{\|\mathbf{D}(z) - \mathbf{D}_{ref}\|^2}{\sigma^2} = \sum_i (\delta d_{i,z})^2 = \Delta d_z^2, \quad (9)$$

$$(10)$$

and then

$$L(\mathbf{D}, z) = \rho(\mathbf{D}|z) = \frac{e^{-\frac{1}{2} \Delta d_z^2}}{\sigma \sqrt{2\pi}} \quad (11)$$

The posterior probability density function is computed for the set of models and we obtain the results presented in Figure 17. We can see that the density of probability is maximum around the reference depth. More generally, such a density of probability quantifies the posterior uncertainties on the depth of the contact.

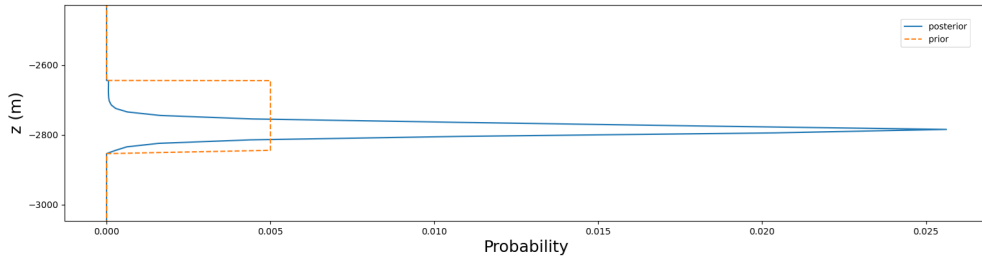


Figure 17: Posterior probability density function. The density is maximal around the reference depth $z = -2,785 \text{ m}$. The most probable gas-water contact depth can therefore be determined in the proposed settings.

375 **5. Discussion and conclusion**

376 In this paper, we have proposed a method to locally modify a meshed geomodel by inserting a line implicitly
377 defined. This avoids to globally rebuild the structural model and create a new mesh, from scratch, which takes time
378 and effort (e.g., Pellerin et al., 2015; Zehner et al., 2015). Local editing techniques on geological models allow for
379 small adjustments to the models without having to go through the entire mesh construction process. Although the
380 principles are similar to those highlighted by Suter et al. (2017), we have proposed a strategy to maintain the mesh
381 conformity, which is a prerequisite for many physical modeling codes. The implicit formulation is interesting for mesh
382 simplifications and generalizations to more complex interfaces as long as they can be represented on the input mesh
383 by a piecewise linear scalar field. This formulation allowed us to demonstrate the practical impact of model updating
384 on wave propagation in a simple example of gas-water contact insertion into a cross-section. The simulation results
385 permit to solve a Bayesian inverse problem, leading to a probabilistic estimation of the contact depth.

386 Some of the modified models have poor quality triangles, often located near the newly created corners. These
387 triangles strongly impact the simulation time. In order to automate the reversal process, a quality threshold could be
388 defined to automatically adjust the tolerance ϵ used in Section 3.3 and avoid excessively long simulations (Figure 18).
389 Although this approach locally approximates the target geometry by projecting the level-set on nodes of the input mesh,
390 it allows to automatically and robustly reduce mesh quality issues. As compared to the simplification methodology
391 proposed by Anquez et al. (2019), this approach is more specific as it considers one interface at a time, but it is
392 significantly easier to implement. So whereas the initial target of this work was not global model simplification as in
393 the case of Anquez et al. (2019), we believe that a carefully designed incremental insertion of implicit interfaces could
394 provide an alternative way to robustly address this problem by locally merging interfaces which are too close one from
395 another. One may argue that such approximations should be avoided, but we consider that some of them are inevitable
396 to obtain physical solutions in acceptable time. As discussed by Anquez et al. (2019), due to the limited resolution of
397 the seismic wavefield, some mild interface modifications will only introduce negligible errors in the simulation results.

398 Coupling the local modifications with the evaluation of their impact by comparing to a reference model (by using
399 wave propagation simulations), leads to the fact that the integration in an inversion code could be done in more com-
400 plex settings (e.g., detecting fluid substitution during gas storage, CO₂ sequestration, or reducing the horizon position
401 uncertainty (Bodin and Sambridge, 2009)).

402 More generally, the local insertion of an implicit interface into an existing mesh opens interesting perspectives
403 to address other types of geological uncertainties, such as the simulations of undetected fractures and faults, or the
404 management of stratigraphic uncertainties. As faults and fractures do not necessarily terminate against pre-existing
405 boundaries, the definition of the region to be modified is essential. This particular region could be defined according to
406 the object that we want to modify or insert. In two dimensions, this could be achieved by defining a circle centered on

Local mesh updating

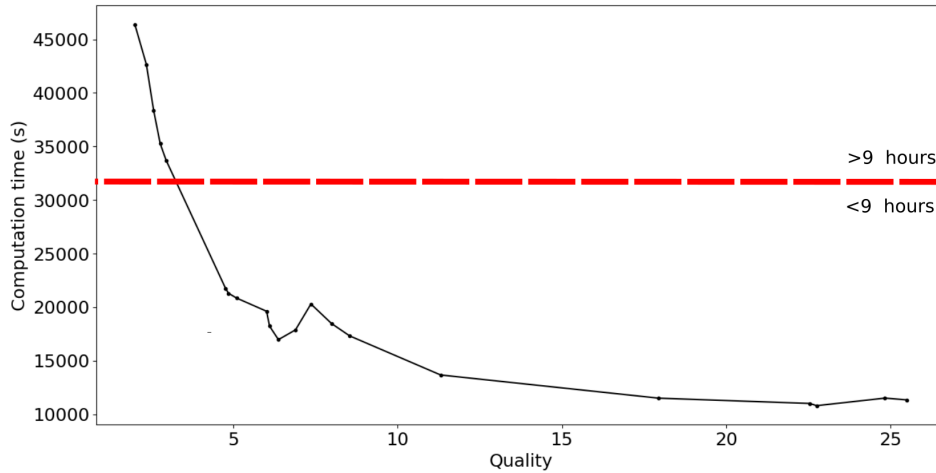


Figure 18: A quality threshold can be defined to avoid a high top simulation time. If the quality Q (2) is smaller than 3, the simulation time significantly increases (on a machine with a processor Intel(R) Xeon(R) CPU E5-1620 v2 @ 3.70GHz, 8 Cores).

407 the fracture to be inserted or more generally by a second level-set (Cherpeau et al., 2012; Moës et al., 2002). Finally,
408 the zone to be remeshed could be defined by a number of rows of triangles around the fracture, so as to preserve the
409 mesh quality during the insertion.

410 Although the principles discussed in this article for the local modification of a meshed geomodel in 2D could be
411 extended in 3D, the transition from 2D to 3D geometric operations is seldom trivial. Indeed, the 2D geometric model
412 editing operations are well understood and easily implemented, but the corresponding 3D operations often generate
413 more complex configurations which are difficult to handle. Nonetheless, we are confident that the use of piecewise
414 linear level-sets for local 3D mesh updating are very promising. As a matter of fact, it is already possible to discretize
415 surfaces in 3D multi-material models using a similar workflow relying on the *mng3d* code (Figure 19).

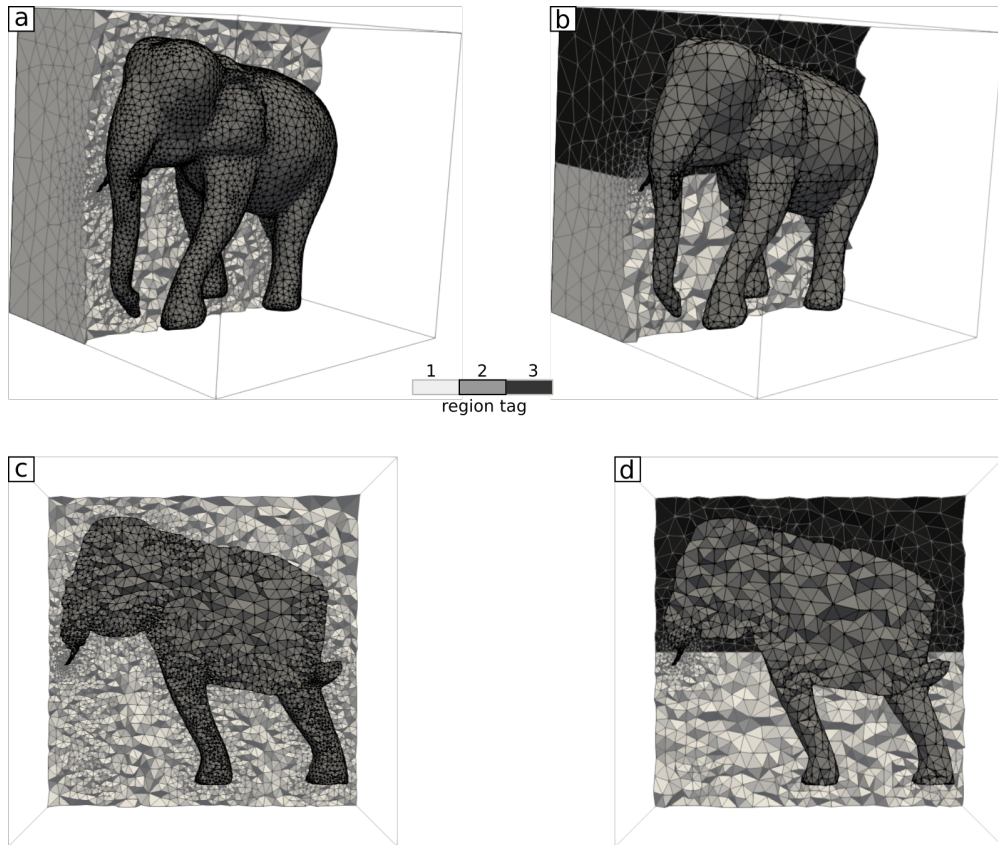


Figure 19: Insertion of an horizontal surface in multi-material model: (a, c) Initial multi material model; (b, d) The surface is inserted in the model only in the outer domain (1) of (a). The elephant is still composed of a unique region. The elephant surface is remeshed to maintain the conformity of the model.

416 Acknowledgments

417 This work was performed in the frame of the RING project
 418 (<http://ring.georessources.univ-lorraine.fr>) at Université de Lorraine. We would like to thank for their support the
 419 industrial and academic sponsors of the RING-GOCAD Consortium managed by ASGA. We also thank TOTAL for
 420 additional support for the PhD of Capucine Legentil. The software corresponding to this paper is available as a python
 421 and C++ code, which significantly relies on Mmg and Gmsh. We also acknowledge Inria and the Mmg Consortium
 422 for the hou10ni and Mmg libraries. The authors would also like to thank H el ene Barucq, Julien Diaz for their help.

423 Code availability section

424 Name of the code/library : LUMOS2D
 425 Contact: capucine.legentil@univ-lorraine.fr
 426 License: GNU GPL v.3
 427 Hardware requirements: Linux System
 428 Program language: Python
 429 Software required: python 3.6 or 3.7, Mmg, Gmsh, shapely, VTK, meshio (to run the jupyter notebook)
 430 Program size: 5Mo
 431 The source codes are available for downloading at the link: <https://github.com/ring-team/LUMOS2D>

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