When bonds matter: home bias in goods and assets

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When Bonds Matter: Home Bias in Goods and Assets*

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Abstract

Recent models of international equity portfolios exhibit two potential weaknesses. First, the structure of equilibrium equity portfolios is determined by the correlation of equity returns with real exchange rates and non financial income; yet empirically domestic equities don't appear to be a good hedge against either risk factors; Second, equity portfolios are highly sensitive to preference parameters. This paper addresses both issues. It shows that in more general and realistic environments, (a) the hedging of real exchange rate risks occurs through international bond holdings, since relative bond returns are strongly correlated with real exchange rate fluctuations; (b) domestic equities can provide a good hedge against non-financial income risk, *conditionally on bond returns*. The model delivers equilibrium portfolios that are well-behaved as a function of the underlying preference parameters. Empirically, we find reasonable empirical support for the theory for G-7 countries. We are able to explain short positions in domestic currency bonds for all G-7 countries, as well as significant levels of home equity bias for the US, Japan and Canada.

Keywords : International risk sharing, International portfolios, Home equity bias
JEL codes: F30, F41, G11

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1 Introduction

The current international financial landscape exhibits two critical features. First, the last twenty years have witnessed an unprecedented increase in cross-border financial transactions. Second, despite this massive wave of financial globalization, international portfolios remain heavily tilted toward domestic assets (French and Poterba (1991), Tesar and Werner (1995) and Ahearne, Griever and Warnock (2004); see appendix A.7 for recent evidence). The importance of these two features has not gone unnoticed and has generated renewed interest for theories of optimal international portfolio allocation.[1]

An important strand of literature, launched into orbit by the influential contribution of Obstfeld and Rogoff (2000), sets out to explore the link between the allocation of consumption expenditures and optimal portfolios in frictionless general equilibrium models with stochastic endowments à la Lucas (1982).[2] One popular approach, initially developed by Baxter et al. (1998), and extended by Coeurdacier (2009), Obstfeld (2007) among others, consists in characterizing the constant equity portfolio that –locally– reproduces the complete market allocation through trades in claims to domestic and foreign equities.

As emphasized by Coeurdacier (2009) and Obstfeld (2007), the structure of these optimal portfolios reflects the hedging properties of relative equity returns against real exchange rate fluctuations.[3] For instance, with Constant Relative Risk Aversion (CRRA) preferences, the optimal equity position is related to the covariance between the excess return on domestic equity (relative to foreign equity), and the rate of change of the real exchange rate. When the coefficient of relative risk aversion exceeds unity, home equity bias arises when excess domestic equity returns are positively correlated with an appreciation of the real exchange rate. In that case, efficient risk sharing requires that domestic consumption expenditures increase as the real exchange rate appreciates. If domestic equity returns are high precisely at that time, domestic equity provides the appropriate hedge against real exchange rate risk, and investors will tilt their portfolio towards domestic equity. Seen in this light, most of the theoretical literature mentioned above represents a search for models that generate the '*right*' correlation between relative equity returns and real exchange rate fluctuations.

This line of research faces two serious challenges. First, as shown convincingly by van Wincoop and Warnock (2006), the empirical correlation between excess equity returns and the real exchange rate is low, too low to explain observed equity home bias. Further, most of the fluctuations in the real exchange rate represent movements in the nominal exchange rate, so once forward currency markets are introduced, the conditional correlation between equity returns and real exchange rates disappears. This casts a serious doubt on

[1] See Adler and Dumas (1983) for a review of the earlier literature.

[2] A chronological but non-exhaustive list of contributions –some of which precedes Obstfeld and Rogoff (2000)– includes Dellas and Stockman (1989), Baxter and Jermann (1997), Baxter, Jermann and King (1998), Coeurdacier (2009), Obstfeld (2007), Kollmann (2006), Heathcote and Perri (2007a), Coeurdacier, Kollmann and Martin (2007) and Collard, Dellas, Diba and Stockman (2007).

[3] A result also emphasized in the earlier, partial equilibrium literature. See Adler and Dumas (1983).

the ability of this class of models to quantitatively explain the home equity bias. Second, as shown initially by Coeurdacier (2009) and Obstfeld (2007), the equilibrium equity portfolios are extremely sensitive to the values of preference parameters. Whether the coefficient of relative risk aversion is smaller, bigger than or equal to unity, whether domestic and foreign goods are substitute or complements, equity portfolios can exhibit home, foreign, or no bias. In other words, this class of models delivers equity portfolios that are *unstable*.

This paper addresses both issues simultaneously. We argue that many of the results in the previous literature are not robust to the introduction of bonds denominated in different currencies. Of course, bonds are redundant in the previous set-up since locally efficient risk-sharing is obtained by trading equities only. This creates an obvious and uninteresting indeterminacy. This indeterminacy is lifted once we allow for additional and realistic sources of risk. That the economic environment is subject to more than one source of uncertainty strikes us as eminently realistic. These additional risk factors can take many forms that cover many cases of interest: redistributive shocks, fiscal shocks, investment shocks, preference shocks, nominal shocks, etc.... In presence of these additional risks, optimal portfolio allocation will typically require simultaneous holdings of equities and bonds.

The important economic insight is that in many models of interest, as well as in the data, relative bond returns are strongly positively correlated with the real exchange rate. As a consequence, it is optimal for investors to use bond positions to hedge real exchange rate risks. All that will be left for equities is to hedge the impact of additional sources of risk on investors' total wealth. Of course, the precise form of the additional sources of risk matters for optimal portfolio holdings. We explore this question systematically using a simple extension of Coeurdacier (2009)'s model. We begin by adding only one additional risk factor, so that risk sharing remains -locally- efficient. This simple extension delivers two important results. First, equilibrium equity holdings take a very simple form; Unlike the previous literature, these holdings do *not* depend on the correlation between equity returns and the real exchange rate. Second, this optimal equity portfolio does *not* depend upon the preferences of the representative household.[4]

These simple results have important empirical implications. First, since equity positions are not driven by real exchange rate risk, home equity bias can only arise from hedging demands *other* than the real exchange rate. This simultaneously validates van Wincoop and Warnock (2006)'s result and establishes its limits. Moreover, we show that home equity bias arises if the correlation between the return on non-financial wealth and the return on equity, *conditional on bond returns*, is negative (a generalization of both Baxter and Jermann (1997), and Heathcote and Perri (2007b)).[5] In simultaneous and independent work, Engel and Matsumoto (2008) develop similar results in a specific model with nominal rigidities. These authors also draw the connection between the impact of forward trades (or bond trading) and optimal equity positions.

[4]Equivalently, optimal equity positions coincide with the equity positions of a *log-investor* who doesn't care about hedging the real exchange rate risk.

[5]see also Bottazzi, Pesenti and van Wincoop (1996).

The model also provides tight predictions about equilibrium bond holdings. First, we show that while these bond portfolios typically vary with investors' preferences, they do so smoothly. In other words, the portfolio instability of earlier models is not simply transferred to bond portfolios. Second, the model predicts that the overall domestic bond position reflects the balance of two effects: the optimal hedge for fluctuations in real exchange rates (for non-log investors), as well as a hedge for the implicit real exchange rate exposure arising from equilibrium equity holdings and non-financial wealth. In other words, households want to hold bonds in their own currency since local bonds have higher returns when the price of consumption goods is higher (real exchange rate hedging). However, if returns on their equity portfolio and their non-financial wealth are also higher (resp. lower) in those states, investors optimally undo this exposure by shorting the domestic currency bond (resp. going long in the domestic currency bond). We find that for plausible values, it is possible for a country to have short or long domestic currency debt positions. In line with our findings, recent empirical evidence (Lane and Shambaugh (2007) and Lane and Shambaugh (2008)) suggests large heterogeneity across countries in the currency denomination of external bond holdings. On average, advanced countries hold long (but small) domestic currency debt positions but some large countries, most notably the US, are short in their own currency debt.

Are these results relevant in practice? The answer is yes. First, we show that our general specification with one additional source of risk fits (or extends) a number of models that have been used in previous literature. We explore the case of redistributive shocks, fiscal shocks, investment shocks, or nominal shocks in the presence of price rigidities. Second and more importantly, we show how equilibrium portfolios can be computed from observable data on bond and equity returns together with data on real exchange rate and non-financial income. In particular, we show how simple regressions of real exchange rate and returns on non-financial wealth on asset returns (bonds and equity) help us to back out equilibrium portfolios from the data. This makes an important link between recent theoretical work on international portfolios and data on asset prices.

We also evaluate the robustness of our results in presence of more than one source of risk. By allowing for multiple sources of risks, markets are effectively incomplete (even locally). Importantly, we show that our results remain robust in that case and that the empirical counterpart of the theoretical portfolios can also be computed using the exact same empirical methodology.

We then confront our results to the data. We use quarterly data on equity, bond returns as well non-financial income for the G-7 countries since 1970 to estimate the parameters of the models. In particular, we ask wether data on asset prices are theoretically consistent with observed portfolios. For all countries, we show that the presence of bonds is key to obtaining more reasonable equity positions: without bond trading, 'the international diversification puzzle is indeed worse than you think' (Baxter and Jermann (1997)). With bond trading, we find reasonable estimates of home equity bias for the US, Japan and Canada. Finally, the model predicts currency exposure of international portfolios in line

with recent empirical evidence on industrialized countries (see Lane and Shambaugh (2007) and Lane and Shambaugh (2008)).

Finally, we explore the robustness of our theory in two important cases. First, we introduce non-traded goods as in Obstfeld (2007) and Collard et al. (2007). In presence of non-traded goods, real bonds still load on the real exchange rate while domestic equities (in traded and nontraded goods) still hedge the remaining sources of risks. We show that the overall home equity bias (across traded and non-traded equities) is independent of preferences. However, the optimal holdings of traded and non-traded domestic equity depend upon their hedging properties of movements in the terms of trade.

Second, we discuss cases where bond returns do not provide a good hedge for fluctuations in the (welfare-based) real exchange rate. This arises in two situations: in the presence of preference/variety shocks similar to Coeurdacier et al. (2007) or Pavlova and Rigobon (2003), and with nominal shocks as in Lucas (1982), or in Obstfeld (2007)'s version of Engel and Matsumoto (2006)'s sticky price model. In both cases, the new source of risk simply perturbates bond returns, leaving equities, consumption expenditures and non-financial income unchanged. It is then optimal not to hold bonds in equilibrium, which brings us back to the results of the equity-only models. While theoretically restoring the results from the earlier literature, the case of nominal shocks cannot be relevant in practice. Indeed nominal and real bonds returns are strongly correlated in industrial economies, limiting the extent to which nominal bonds are unable to hedge fluctuations in total nominal expenditures.

Section 2 follows Coeurdacier (2009) and develops the basic model with equities only. Section 3 constitutes the theoretical core of the paper. It introduces bonds and an additional source of risk, then characterizes the efficient equity and bond positions under different risk structures. Section 4 extends the model to incomplete markets. Section 5 presents our empirical results. Section 6 discusses some extensions and some potential caveats of our framework.

2 A Benchmark Model.

2.1 Goods and preferences.

Consider a two-period ($t = 0, 1$) endowment economy similar to Coeurdacier (2009). There are two symmetric countries, Home (H) and Foreign (F), each with a representative household. Each country produces one tradable good. Agents consume both goods with a preference towards the local good. In period $t = 0$, no output is produced and no consumption takes place, but agents trade financial claims (stocks and bonds). In period $t = 1$, country i receives an exogenous endowment y_i of good i. Countries are symmetric and we normalize $E_0(y_i) = 1$ for both countries, where E_0 is the conditional expectation operator, given date $t = 0$ information. Once stochastic endowments are realized at period 1, households consume using the revenues from their portfolio chosen in period 0 and their endowment received in period 1.

The country i household has the standard CRRA preferences, with a coefficient of relative risk aversion σ:

$$U_i = E_0 \left[\frac{C_i^{1-\sigma}}{1-\sigma} \right], \tag{1}$$

where C_i is an aggregate consumption index in period 1. For future reference, we consider in what follows the plausible case where $\sigma \geq 1$.

For $i, j = H, F$, C_i is given by:

$$C_i = \left[a^{1/\phi} c_{ii}^{(\phi-1)/\phi} + (1-a)^{1/\phi} c_{ij}^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)}; \quad \text{with } i \neq j \tag{2}$$

where c_{ij} is country $i's$ consumption of the good from country j at date 1. ϕ is the elasticity of substitution between the two goods and $1 \geq a \geq 1/2$ represents preference for the home good (mirror-symmetric preferences).

The ideal consumer price index that corresponds to these preferences is for $i = H, F$:

$$P_i = \left[a p_i^{1-\phi} + (1-a) p_j^{1-\phi} \right]^{1/(1-\phi)}; \quad \text{with } i \neq j \tag{3}$$

where p_i denotes the price of the country $i's$ good in terms of the numeraire.

Resource constraints are given by:

$$c_{ii} + c_{ji} = y_i; \quad \text{with } i \neq j \tag{4}$$

We denote Home terms of trade, i.e. the relative price of the Home tradable good in terms of the Foreign tradable good, by q:

$$q \equiv \frac{p_H}{p_F} \tag{5}$$

An increase in q represents an improvement Home's terms of trade.

2.2 Financial markets.

Trade in stocks and bonds occurs in period 0. In each country there is one stock à la Lucas (1982). A share δ of the endowment in country i is distributed to stockholders as dividend, while a share $(1-\delta)$ is not capitalized and is distributed to households of country i. At the simplest level, one can think of the share $1-\delta$ as representing 'labor income', but more general interpretations are also possible. More generally, $1-\delta$ represents the share of output that cannot be capitalized into financial claims. This could be due to domestic financial frictions, capital income taxation or poor property right enforcement. In our symmetric setting, δ is common to both countries. The supply of each type of share is normalized at unity. We assume also that agents can trade a CPI-indexed bond in each country denominated in the composite good of country i. Buying one unit of the Home (Foreign) bond in period 0 gives one unit of the Home composite (Foreign) good at $t = 1$. Both bonds are in zero net supply.

Initially, each household fully owns the local stock equity, and has zero initial foreign assets. Country i household thus faces the following budget constraint at $t = 0$:

$$p_S S_{ii} + p_S S_{ij} + p_b b_{ii} + p_b b_{ij} = p_S, \quad \text{with} \quad j \neq i \tag{6}$$

where S_{ij} is the number of shares of stock j held by country i at the end of period 0, and b_{ij} represents claims (held by i) to future unconditional payments of the good j. p_S is the share price of both stocks, while p_b is the price of the both countries real bond, identical due to symmetry.

Market clearing in asset markets for stocks and bonds requires:

$$S_{ii} + S_{ji} = 1; \quad b_{ii} + b_{ji} = 0; \quad \text{with } i \neq j \tag{7}$$

Symmetry of preferences and distributions of shocks implies that equilibrium portfolios are symmetric: $S_{HH} = S_{FF}$, $b_{HH} = b_{FF}$, and $b_{FH} = b_{HF}$. In what follows, we denote a country's holdings of local stock by S, and its holdings of bonds denominated in its local composite good by b. The vector $(S; b)$ thus describes international portfolios. $S > \frac{1}{2}$ means that there is equity home bias on stocks, while $b < 0$ means that a country issues bonds denominated in its local composite good, and simultaneously lends in units of the foreign composite good.

2.3 Characterization of world equilibrium.

We characterize first the equilibrium with locally complete markets (see appendix A.1 for a precise definition of locally complete markets). As shown below, markets are locally complete in our model when the number of shocks is at least equal to the number of assets. In a world with just endowment shocks, markets will be complete (locally) but portfolios will be indeterminate (i.e. the number of assets is larger than the dimension of the shocks).

2.3.1 Goods market equilibrium

After the realization of uncertainty in period 1, the representative consumer in country i maximizes $\frac{C_i^{1-\sigma}}{1-\sigma}$ subject to a budget constraint (for $j \neq i$):

$$P_i C_i = p_i c_{ii} + p_j c_{ij} \leq I_i \qquad (\lambda_i)$$

where I_i represent the (given) total income of the representative agent in country i and λ_i is the Lagrange-Multiplier associated with the budget constraint.
The intratemporal equilibrium conditions are as follows:

$$c_{ii} = a \left(\frac{p_i}{P_i}\right)^{-\phi} C_i \; ; \; c_{ij} = (1-a) \left(\frac{p_j}{P_i}\right)^{-\phi} C_i; \quad \text{with } i \neq j \tag{8}$$

Using equations (8) for both countries and market-clearing conditions for both goods (4) gives:

$$q^{-\phi}\Omega_a\left[(\frac{P_F}{P_H})^{\phi}\frac{C_F}{C_H}\right]=\frac{y_H}{y_F} \tag{9}$$

where $\Omega_u(x)$ is a continuous function of two variables (u,x) such that: $\Omega_u(x)=\frac{1+x(\frac{1-u}{u})}{x+(\frac{1-u}{u})}$. As emphasized by Obstfeld (2007), the term $\Omega_a(.)$ captures the Keynesian transfer effects due to consumption home-bias.

2.3.2 Budget constraints.

Recall that country i household holds shares S of the local stock with dividend $\delta p_i y_i$, and shares $1-S$ of the foreign stock, with dividend $\delta p_j y_j$. In addition, it holds b bonds denominated in the local good, with payment P_i and $-b$ bonds in the foreign good, with payment P_j. The period 1 budget constraints are thus:

$$P_iC_i=S\delta p_i y_i+(1-S)\delta p_j y_j+P_ib-P_jb+(1-\delta)p_i y_i;\quad \text{with } i\neq j \tag{10}$$

where the last term represents non-financial income.

These constraints imply:

$$P_HC_H-P_FC_F=[\delta\,(2S-1)+(1-\delta)](p_Hy_H-p_Fy_F)+2b(P_H-P_F) \tag{11}$$

which says that the difference between countries' consumption expenditures equals the difference between their incomes.

2.3.3 Log-linearization of the model and locally complete markets.

Denote $y\equiv y_H/y_F$ the relative output. We log-linearize the model around the symmetric steady-state where y equal unity, and use Jonesian hats ($\widehat{x}\equiv log(x/\bar{x})$) to denote the log-deviation of a variable x from its steady state value $\bar{x}$. Define the Home country real exchange rate as the foreign price of the domestic good, $RER=P_H/P_F$, so that an increase in the real exchange rate represents a real appreciation. Using (3) we can write:

$$\widehat{RER}=\widehat{\frac{P_H}{P_F}}=(2a-1)\widehat{q}. \tag{12}$$

so that the real exchange rate always appreciates when the terms of trade improve.

As shown in appendix A.1, if a rank and spanning conditions are satisfied, one can replicate the efficient risk-sharing allocation up-to the first order.[6] This implies that, abstracting from second-order terms, the equilibrium allocation is the one that prevails in a world with locally complete markets. This property turns out to simplify the portfolio problem: one

[6]The spanning condition states that the dimensionality of the shocks is smaller than the number of independent available assets. The rank condition states that shock innovations do not leave asset pay-off unaffected.

just needs to find the portfolio that replicates *locally* the efficient allocation.[7] In particular, when these two conditions are verified, the ratio of Home to Foreign marginal utilities of aggregate consumption is linked to the consumption-based real exchange rate by the familiar Backus and Smith (1993) condition (in log-linearized terms):

$$-\sigma(\widehat{C_H}-\widehat{C_F})=\widehat{\frac{P_H}{P_F}}=(2a-1)\,\widehat{q} \tag{13}$$

Hence, any shock that raises Home aggregate consumption relative to Foreign must be associated with a Home real exchange rate depreciation.

Log-linearizing (9) and substituting (13) gives:

$$\widehat{y}=-\phi\widehat{q}+(2a-1)(\phi-1/\sigma)\widehat{\frac{P_H}{P_F}} \tag{14}$$

Substituting (12) implies:

$$\widehat{y}=-\lambda\widehat{q} \tag{15}$$

where $\lambda\equiv\phi\left(1-(2a-1)^2\right)+(2a-1)^2/\sigma$ represents the equilibrium terms of trade elasticity of relative output. Note that $\lambda>0$ as $1/2\leq a\leq 1$: a relative increase in the supply of the home good ($\hat{y}>0$) is always associated with a worsening of the terms of trade ($\hat{q}<0$) with an elasticity $-1/\lambda$. Without home bias in preferences ($a=1/2$), λ is simply the elasticity of substitution between Home and Foreign goods (ϕ). When $a>1/2$, there are deviations from PPP. An increase in relative output triggers a fall in the relative price level. Under locally complete markets, this requires an increase in domestic consumption expenditures (at a rate $1/\sigma$) that increases relative demand for the home good.[8]

Note also that from equation (15), relative equity returns $\widehat{R}_e$ are equal to:

$$\widehat{R}_e=\widehat{q}+\widehat{y}=(1-\lambda)\widehat{q}=\left(1-\frac{1}{\lambda}\right)\hat{y} \tag{16}$$

When $\lambda>1$, an increase in relative output is associated with an improvement in relative equity returns. Conversely, when $\lambda<1$, an increase in Home relative output is associated with a relative decrease in Home equity returns. This happens when either the elasticity of substitution between goods is low ($\phi<1$) or the preference for the home good is sufficiently strong.[9]

We next log-linearize equation (11) using (13) to obtain:

$$\widehat{P_HC_H}-\widehat{P_FC_F}=\left(1-\frac{1}{\sigma}\right)(2a-1)\,\widehat{q}=\left[\delta\,(2S-1)+(1-\delta)\right](\widehat{q}+\widehat{y})+2b\,(2a-1)\,\widehat{q} \tag{17}$$

The first equality is simply the Backus-Smith condition. It records the response of relative consumption spending to a change in the real exchange rate. This response depends on the

[7] Appendix A.1 shows that such a portfolio is the one chosen by our utility-maximizing investors.

[8] See Obstfeld (2007).

[9] Specifically, when $\phi>1$ and $\sigma>1$ (the empirically plausible case), we need: $a>\frac{1}{2}\left[1+\left(\frac{1-\phi}{\frac{1}{\sigma}-\phi}\right)^{1/2}\right]$

coefficient of relative risk aversion σ. In a locally-efficient equilibrium, a shock that leads to an appreciation of the real exchange rate induces an increase in relative consumption expenditures when $\sigma \geq 1$. The expression to the right of the second equality in (17) shows the change in relative income necessary to obtain this locally-efficient allocation of relative consumption expenditures. The efficient portfolio has to be such that a real appreciation is associated with an increase in relative spending and income.

2.4 The Instability of Optimal Equity Portfolios.

Financial markets are locally complete when there exists a portfolio (S, b) such that (15) and (17) both hold for arbitrary realizations of the relative shocks $\widehat{y}$. Clearly, here portfolios are undetermined since the dimension of 'relative' shocks exceeds the dimension of 'relative assets.[10] Much of the literature focuses on the case where bonds are not available and efficient risk sharing is implemented with equities only (Coeurdacier (2009), Obstfeld (2007), Kollmann (2006)).

Substituting $b = 0$ into (17) and using (15), we solve for the equilibrium equity portfolio position:

$$S = \frac{1}{2}\left[\frac{2\delta - 1}{\delta} - \frac{\left(1 - \frac{1}{\sigma}\right)(2a - 1)}{\delta\left(\lambda - 1\right)}\right] \tag{18}$$

When $\delta = 1$, this expression coincides with the equilibrium equity position of Coeurdacier (2009) and Obstfeld (2007). In the more general case where $\delta < 1$, the optimal equity portfolio has two components. The first term inside the brackets represents the position of a log-investor ($\sigma = 1$). As in Baxter and Jermann (1997), the domestic investor is already endowed with an implicit equity position equal to $(1 - \delta)/\delta$ through non-financial income. Offsetting this implicit equity holding and diversifying optimally implies a position $S = (2\delta - 1)/2\delta < 1/2$ for $\delta < 1$. As is well known, this component of the optimal portfolio impart a *foreign* equity bias.

The second component of the optimal equity portfolio represents a hedge against real exchange rate fluctuations. It only applies when $\sigma \neq 1$, i.e. when total consumption expenditures fluctuate with the real exchange rate. Looking more closely at the structure of this hedging component calls for a number of observations. First, this hedging demand is a complex and non-linear function of the structure of preferences summarized by the parameters σ, ϕ and a. As Obstfeld (2007) and Coeurdacier (2009) note, for reasonable parameter values, this hedging demand can contribute to home equity bias only when $\lambda < 1$, i.e. when the terms of trade impact of relative supply shocks is large.[11] Finally, using (16) and (12), this hedge component can be rewritten as $(1 - 1/\sigma)/\delta \; cov\left(\hat{R}_e, R\hat{E}R\right)/var\left(\hat{R}_e\right)$, a function of the covariance-variance ratio between excess equity returns and the real exchange rate.

[10] In other words, while the spanning condition is verified, the rank condition is not verified: bond and equity excess returns are perfectly correlated.

[11] When $\lambda = 1$, this component is indeterminate since the relative return on equities is independent of the real exchange rate (and constant). This case is similar to Cole and Obstfeld (1991).

This model faces three main problems. First, the non-linearity in (18) implies that small changes in preferences can have a large impact on this hedging demand. This is most apparent if we consider the optimal portfolio in the neighborhood of $\lambda = 1$. As figure 1 makes clear, small and reasonable changes in σ, ϕ or a have a large and disproportionate impact on optimal portfolio holdings, from large foreign bias ($S < 0$) to unrealistically high domestic bias ($S > 1$). To the extent that we don't know precisely what value these parameters take, one is left with the inescapable conclusion that this model does not provide enough guidance to pin down equity portfolios, or a-fortiori, explain the home portfolio bias. As emphasized by Obstfeld (2007), and as the figures make clear, things are even worse since the benchmark model cannot deliver home equity holdings between $S = 1 - 1/2\delta < 0.5$ and $S = 1$, thus excluding the relevant empirical range.

Second, given the constant income sharing rule δ, the model predicts a perfect correlation between equity returns and non-financial income. This tilts portfolios towards foreign equities (the first term in (18)), as emphasized by Baxter and Jermann (1997). While this correlation might be positive, it is hard to believe that it is perfect and many papers found it pretty low (see Fama and Schwert (1977) for earlier work and Bottazzi et al. (1996), Julliard (2003, 2004), Lustig and Nieuwerburgh (2005)).

Third, the extent to which the model can deliver home equity bias depends on the hedging properties of equities for real exchange risk, as captured by the covariance-variance ratio $cov\left(\hat{R}_e, R\hat{E}R\right)/var\left(\hat{R}_e\right)$. In the case of the US, van Wincoop and Warnock (2006) show that relative equity returns are poorly correlated with the real exchange rate. They find a covariance-variance ratio $cov\left(\hat{R}_e, R\hat{E}R\right)/var\left(\hat{R}_e\right)$ equal to 0.32, unable to account for the observed home portfolio bias.[12]

3 Equity and Bond Equilibrium Portfolios: the case of locally-complete markets.

This paper's main objective is to characterize both equity and bond portfolios once additional sources of uncertainty are allowed. Of course, introducing bonds in the model of the previous section yields an uninteresting indeterminacy since markets are already locally complete. In this section, we allow for exactly one additional source of uncertainty in the model so that the markets remain locally complete with both equities and bonds. We can then use an extension of the previous method to characterize optimal portfolio holdings.

This calls for three remarks. First, since relative endowment or supply shocks are unlikely to represent the only source of uncertainty in the economy, adding other sources of uncertainty is quite realistic and general. Second, adding only one source of additional uncertainty is mostly done for tractability. Section 4 will cover the more general case where markets are incomplete (even locally)[13] and show that our portfolio characterization holds

[12] They find essentially a zero correlation once controlling for forward markets.

[13] This means that spanning condition defined in Appendix A.1 is not verified: the number of shocks is

in that more general case. Lastly, going from the general to the particular, we show how to map our results in specific models where the additional source of risk arises from redistributive shocks, shocks to government expenditures or investment, from demand shocks, or from nominal shocks.

3.1 A general representation with one additional source of risk.

Assume that a shock ε_i affects country i in period $t = 1$. Denote $\varepsilon = \varepsilon_H/\varepsilon_F$ the relative shock and assume $E_0(\varepsilon) = 1$. The only assumption we make is that the stochastic properties of ε_i are symmetric across countries and that $\widehat{\varepsilon} = \ln \varepsilon$ is not perfectly correlated with $\widehat{y}$. To characterize optimal portfolio, we only need to specify how this additional shock impacts equity returns $\hat{R}_e$, bond returns $\hat{R}_b$ and the return on non-financial wealth $\hat{R}_n$.[14] That is, we assume the following:

$$\begin{cases} \hat{R}_e &= (1-\bar{\lambda})\widehat{q} + \gamma_e \widehat{\varepsilon} \\ \hat{R}_b &= (2a-1)\widehat{q} + \gamma_b \widehat{\varepsilon} \\ \hat{R}_n &= (1-\bar{\lambda})\widehat{q} + \gamma_n \widehat{\varepsilon} \end{cases} \tag{19}$$

where $\bar{\lambda}$ is a positive number ($\bar{\lambda}$ is model dependant but will be closely related to the previous λ and reflect preference parameters; see the examples below). The parameters γ_k can be positive or negative. They represent the impact of $\hat{\varepsilon}$ on equity returns, bond returns and non-financial income, respectively. Different models will have different implications on what γ_k and $\bar{\lambda}$ should be, and will be explored in more details in subsection 3.2. For this section, the only restriction we impose on the model is $\gamma_e \neq 0$, that is, the new shock affects equity returns.[15]

3.1.1 Equilibrium Portfolios.

Under the assumption -verified below- that markets remain *locally*-complete, the budget constraint (17) can be rewritten as follows:

$$(1 - \frac{1}{\sigma})(2a-1)\widehat{q} = \delta\,(2S-1)\,\hat{R}_e + (1-\delta)\hat{R}_n + 2b\hat{R}_b \tag{20}$$

Financial markets are *locally*-complete since one can always find a portfolio (S, b) such that (20) holds for arbitrary realizations of the shocks $\widehat{y}$ and $\widehat{\varepsilon}$. Clearly, here portfolios are uniquely determined since the dimension of 'relative shocks' equals the dimension of 'relative

larger than the number of available assets.

[14] In the static model, the return on non-financial wealth equals simply non-financial income. In anticipation of our empirical treatment, we adopt a dynamic terminology.

[15] This will prove to be the relevant case empirically. For completeness, we explore the case $\gamma_e = 0$ in details in section 6.2.

assets'. The unique portfolio (S^*, b^*) that satisfies (20) for all realization of shocks is[16]:

$$\begin{cases} b^* &= \frac{1}{2}\frac{(2a-1)\left(1-\frac{1}{\sigma}\right)+(1-\delta)\left(1-\bar{\lambda}\right)(\gamma_n/\gamma_e-1)}{(2a-1)-\gamma_b/\gamma_e\left(1-\bar{\lambda}\right)} \\ S^* &= \frac{1}{2}\left[1-\frac{1-\delta}{\delta}\frac{\gamma_n}{\gamma_e}-\frac{\gamma_b}{\gamma_e}\frac{2b^*}{\delta}\right] \end{cases} \tag{21}$$

While this expression may look forbidding, we show next that it can be reinterpreted in terms of simple factor loadings.

3.1.2 Equilibrium Loadings

To start with, let's rewrite the equilibrium bond and equity portfolios in terms of the equilibrium asset return loadings on the real exchange rate $R\hat{E}R = (2a-1)\,\hat{q}$ and non-financial income $\hat{w}$. To do this, let's first manipulate equations (19) to eliminate $\hat{\varepsilon}$:

$$\begin{aligned} R\hat{E}R &= (2a-1)\,\hat{q} = (2a-1)\,\psi\hat{R}_b - (2a-1)\,\psi\frac{\gamma_b}{\gamma_e}\hat{R}_e \qquad (22)\\ &\equiv \beta_{RER,b}\hat{R}_b + \beta_{RER,e}\hat{R}_e \\ \hat{R}_n &= \left(1-\bar{\lambda}\right)\left(1-\frac{\gamma_n}{\gamma_e}\right)\psi\hat{R}_b + \left(\frac{\gamma_n}{\gamma_e} - \left(1-\bar{\lambda}\right)\left(1-\frac{\gamma_n}{\gamma_e}\right)\psi\frac{\gamma_b}{\gamma_e}\right)\hat{R}_e \qquad (23)\\ &\equiv \beta_{n,b}\hat{R}_b + \beta_{n,e}\hat{R}_e \end{aligned}$$

where $\psi = \left[(2a-1) - \left(1-\bar{\lambda}\right)\gamma_b/\gamma_e\right]^{-1}$.

The advantages of the above formulation are twofold. First, the loadings $\beta_{n,i}$ and $\beta_{RER,i}$ have the interpretation of covariance-variance ratios. It is immediate to see that they can be expressed as:

$$\beta_{n,i} = \frac{cov\left(\hat{R}_n, \hat{R}_i|\hat{R}_j\right)}{var\left(\hat{R}_i|\hat{R}_j\right)}; \beta_{RER,i} = \frac{cov\left(R\hat{E}R, \hat{R}_i|\hat{R}_j\right)}{var\left(\hat{R}_i|\hat{R}_j\right)}$$

where $i, j = e, b$. Importantly, these loadings are conditional on the other asset return.

Second, since these loadings are expressed in terms of observables, they have an intuitive empirical counterpart, independently of the specifics of the model and of the source of the shock $\hat{\varepsilon}$: They can be readily estimated from a multivariate regression. This formulation will motivate our empirical analysis in section 5.

We can now express the optimal portfolios in terms of these equilibrium loadings:

$$\begin{cases} b^* &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\beta_{RER,b} - \frac{1}{2}\left(1-\delta\right)\beta_{n,b} \\ S^* &= \frac{1}{2}\left[1-\frac{1-\delta}{\delta}\beta_{n,e} + \frac{1-\frac{1}{\sigma}}{\delta}\beta_{RER,e}\right] \end{cases} \tag{24}$$

Let's consider the equilibrium bond portfolio b^* first. Equation (24) indicates that it contains two terms. The first term represents the hedging of real exchange rate risk. When

[16]Note that we must assume: $(2a-1)\,\gamma_e \neq \gamma_b\left(1-\bar{\lambda}\right)$. This condition makes sure that our rank condition is satisfied (see appendix A.1), i.e equity and bond excess returns are not collinear.

$\sigma > 1$, the household's relative consumption expenditures increase when the real exchange rate appreciates. If domestic bonds deliver a high return precisely when the currency appreciates, then domestic bonds constitute a good hedge against real exchange rate risk. Since we expect the conditional correlation between relative bond returns and real exchange rates to be positive, this term should be positive. The second term represents the hedging of non-financial income risk. When domestic bonds and relative non-financial income are positively correlated ($\beta_{n,b} > 0$) conditionally on the equity return, investors want to short the domestic bond to hedge the implicit exposure from their non-financial income. Equation (24) indicates that investors will go long or short in their domestic bond holdings depending on the relative strength of these two effects.

Let's now turn to the equilibrium equity position S^* in (24). The first term inside the brackets represents the symmetric risk-sharing equilibrium of Lucas (1982): $S = 1/2$. The second term determines how this symmetric equilibrium is affected when non-financial income and equity returns are conditionally correlated. In the case of Baxter and Jermann (1997), $\beta_{n,e} = 1$ and the equilibrium equity position becomes $S = (2\delta - 1)/2\delta < 1/2$. In general, the correlation between non-financial income and equity returns is less than perfect. In particular, home equity bias can arise if $\beta_{n,e} < 0$. Importantly for the empirical exercises we conduct below, what matters is the covariance-variance ratio between non-financial income and equity returns *conditional on the bond returns* $\hat{R}_b$. To our knowledge, this condition has not yet been empirically investigated in the literature.[17]

Finally, the last term inside the brackets is the term that has been emphasized in the literature so far. It represents the demand for domestic equity that arises from the correlation between equity returns and the real exchange rate, $\beta_{RER,e}$. If this correlation is positive, domestic equities represent a good hedge against movements in real exchange rates that affect relative consumption expenditures when $\sigma \neq 1$.

To summarize, our model indicates that equity home bias can arise, even if equities are a poor hedge for exchange rate risk, as long as non-financial income and equity returns are negatively conditionally correlated: $\beta_{n,e} < 0$. The model can also potentially account for short positions in domestic bond market if we find that $(1 - 1/\sigma)\,\beta_{RER,b} < (1 - \delta)\,\beta_{n,b}$ for plausible values of the intertemporal elasticity of substitution σ.

3.1.3 The case $\gamma_b = 0$.

We know from van Wincoop and Warnock (2006) that $\beta_{RER,e}$ is empirically close to zero. In fact, these authors show that the covariance-variance ratio is very close to zero precisely after conditioning on the excess bond returns, or equivalently on forward rates. Going back to equation (22), $\beta_{RER,e} = 0$ requires that $\gamma_b = 0$. In words, bond returns are unaffected by the $\hat{\varepsilon}$ risk factor. In this case, equilibrium portfolio holdings simplify further. Substituting $\gamma_b = 0$ in (21), we obtain:

[17] Engel and Matsumoto (2006) also note that this is the relevant condition in presence of bond holdings, or forward exchange contracts.

$$\begin{cases} b^* &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right)+\frac{1}{2}\left(2a-1\right)^{-1}\left(\bar{\lambda}-1\right)\left(1-\delta\right)\left(1-\frac{\gamma_n}{\gamma_e}\right) \\ S^* &= \frac{1}{2}\left[1-\frac{1-\delta}{\delta}\frac{\gamma_n}{\gamma_e}\right] \end{cases} \tag{25}$$

Let's concentrate on each term in turn. The optimal equity portfolio S^* presents a number of interesting characteristics. First, and contrary to most of the literature, it does not depend on the 'tradability' of goods in consumption, as measured by a. Second, it is also independent of preference parameters such as the elasticity of substitution across goods ϕ or the degree of risk aversion σ. Hence the complex and non-linear dependence of equilibrium equity portfolios on preferences parameters *disappears* once we introduce trade in bonds.

This independence of equity positions from preference parameters implies that the optimal equity portfolio would be the same for a log-investor ($\sigma = 1$). Since we know that log-investors do not care about fluctuations in the real exchange rate, what determines optimal equity holdings is *not* the correlation between equity returns and the real exchange rate. Instead, equity holdings insulate total income (both financial and non-financial) from the $\widehat{\varepsilon}$ shocks. The domestic investor is endowed with an implicit equity exposure through the impact of $\hat{\varepsilon}$ on nonfinancial income, equal to $\gamma_n\left(1-\delta\right)/\delta$. Offsetting this implicit conditional equity position and diversifying optimally implies an equity position $S^* = 0.5\left(1-\gamma_n/\gamma_e\left(1-\delta\right)/\delta\right)$. When $\gamma_n/\gamma_e = 0$, so that the implicit conditional exposure is zero, the optimal equity portfolio is perfectly diversified: $S^* = 0.5$.

More generally, for equity portfolio holdings to exhibit home bias requires a negative γ_n/γ_e, i.e. a negative covariance between non-financial and financial income, conditional on bond returns. This result echoes the partial equilibrium finding above since when $\gamma_b = 0$, we can check that $\beta_{n,e} = \gamma_n/\gamma_e$.

A couple of remarks are necessary at this stage. First, as we will show shortly, a negative γ_n/γ_e arises naturally when the additional shock reallocates income between its financial and nonfinancial components. This occurs with redistributive shocks, but also with shocks to government or investment expenditures (as in Heathcote and Perri (2007b)). Second, and more importantly, it is obvious that this invalidates the results of much of the previous literature that emphasized the hedging properties of equity returns for real exchange rate risk. In particular, in our model, home portfolio bias can arise independently of the correlation between equity returns and the real exchange rate. The finding that $\beta_{RER,e} = 0$, as emphasized by van Wincoop and Warnock (2006), has no bearing on the optimal portfolio holdings. Instead, equity portfolio bias arises only when $\beta_{n,e} < 0$, a condition that has not been investigated in the empirical literature.

Since our results are so different from the previous literature, one is entitled to wonder why the optimal equity portfolio in (25) is independent from real exchange rate changes? After all, (19) shows that relative equity returns fluctuate with the terms of trade, or equivalently with the real exchange rate. The answer is that real exchange rate risk is best taken care of through bond holdings, since the latter load perfectly on the real exchange rate, and not on $\widehat{\varepsilon}$. Intuitively, real bond trading is equivalent here to trading in forward real exchange contracts that remove perfectly real exchange rate risk. Hence, once the $\widehat{\varepsilon}$ shocks have been

hedged by equity positions, the bond portfolio will be structured such that financial and non-financial income have the appropriate exposure to real exchange rate changes.

Looking at the bond position in (25), we can decompose the optimal bond portfolio as the sum of two components. The first term on the right hand side of (25) is the optimal hedge for fluctuations in total consumption expenditures when $\sigma \neq 1$ (the term $\frac{1}{2}\left(1-\frac{1}{\sigma}\right)$). Investors more risk averse than the log-investor want to have a positive exposure of their incomes to real exchange rate changes. They do so by increasing their holding of Home bonds (and decreasing their holdings of Foreign bonds) since Home bonds have higher pay-offs when the real exchange rate appreciates.[18]

The second term on the right hand side represents the bond portfolio of the log-investor (term $(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\gamma_n/\gamma_e)$). This term represents a hedge for the *implicit* real exchange rate exposure arising from both the equilibrium optimal equity position and non-financial income. The log-investor wants to neutralize the exposure of his total income to real exchange movements. It does so by structuring his bond portfolio such that any capital gains on financial and non-financial incomes are offset by capital losses on the bond portfolio. To understand this result, consider a combination of shocks that leads to a 1% increase in the Home terms-of-trade.[19] Given (19) and (20), relative equity returns and non-financial incomes changes are equal to $(1-\bar{\lambda})\%$. At the optimal equity portfolio (S^*), capital gains/losses on equity positions and non-financial incomes for the Home investor (relative to the Foreign one) are equal to $(\bar{\lambda}-1)[\delta(2S^*-1)+(1-\delta)]\% = (\bar{\lambda}-1)(1-\delta)(1-\gamma_n/\gamma_e)\%$. In these states of the world, Home bond excess returns over Foreign bonds are equal to $(2a-1)\%$. Then, holding $b = \frac{1}{2}(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\gamma_n/\gamma_e)$ Home bonds and $(-b)$ Foreign bonds generates capital gains/losses on the bond position necessary to insulate relative incomes from real exchange rate changes.

This intuition helps understand why the model predicts a specific relationship between domestic equity and bond holdings. Expressing γ_n/γ_e in terms of S^* and substituting the result into (25) one obtains:

$$\begin{aligned} b^* &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right) - \frac{1}{2}(2a-1)^{-1}\left(1-\bar{\lambda}\right)\left(1-\delta+\delta\left(2S^*-1\right)\right) \\ &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right) - \frac{1}{2}\frac{\beta_{n,b}}{1-\beta_{n,e}}\left(1-\delta+\delta\left(2S^*-1\right)\right) \end{aligned} \tag{26}$$

The slope of this relationship is controlled by the sign of $\beta_{n,b}/\left(1-\beta_{n,e}\right)$, which can be estimated empirically. In the empirically plausible case where $\beta_{n,b} > 0$ and $\beta_{n,e} < 1$, we would expect a negative relationship between home equity bias $(2S^*-1)$ and domestic bond holdings: the investor optimally hedges the real exchange risk implicit in holdings of domestic equity holdings and nonfinancial income, by shorting the domestic currency bond.

[18] This result is closely related to Adler and Dumas (1983) and Krugman (1981).

[19] Of course in this model, terms-of-trade are endogenous but it is always possible to find a combination of shocks that leads to a 1% increase in the Home terms-of-trade.

Finally, notice that the bond portfolio depends upon preference parameters σ, a and potentially $\bar{\lambda}$ in a complex and non-linear way. A natural question then, is whether this bond portfolio inherits the instability of the equity portfolio of the previous model. To answer this question requires that we flesh out some of the details of the model, as we do next.

3.2 Examples

We now show how fully specified general equilibrium models are nested in the reduced-form model given by the system of equations (19). To do so, we specify the additional source of uncertainty necessary to pin-down bond and equity portfolios. We focus first on the case $\gamma_b = 0$ and $\gamma_e \neq 0$, i.e relative bond returns perfectly load on the real exchange rate. We do so because it turns out to be the relevant one empirically (see section 5).[20]

3.2.1 Redistributive shocks.

The distribution of total income between financial and non-financial income is controlled by the parameter δ. Variations in δ redistribute income from its financial to non-financial components or vice versa. If we interpret non-financial income as labor income, shocks to δ affect the labor share of total income. Fluctuations in the labor share can occur in a model where capital and labor enter into the production function with a non-unit elasticity in presence of capital *and* labor augmenting productivity shocks or in presence of *biased* technical change in the sense of Young (2004) (see also Rios-Rull and Santaeulalia-Llopis (2006)).

In terms of the previous set-up, we can interpret ε_i as shocks to the share that his distributed as dividend, with $E_0(\varepsilon_i) = \delta$. One can verify that financial and non-financial incomes satisfy:

$$\begin{cases} \hat{R}_e &= (1-\lambda)\widehat{q} + \widehat{\varepsilon} \\ \hat{R}_b &= (2a-1)\widehat{q} \\ \hat{R}_n &= (1-\lambda)\widehat{q} - \frac{\delta}{1-\delta}\widehat{\varepsilon} \end{cases} \tag{27}$$

This system of equation is a specific case of the general representation described above (equations (19)) where $\gamma_e = 1$ and $\gamma_n = -\frac{\delta}{1-\delta}$.

Then, the optimal portfolio can be easily derived from (25) with $\gamma_n/\gamma_e = -\delta/(1-\delta)$:

$$\begin{aligned} S^* &= 1 \\ b^* &= \frac{1}{2}(1-\frac{1}{\sigma}) + \frac{1}{2}(2a-1)^{-1}(\lambda-1) \end{aligned} \tag{28}$$

The implications for portfolios are similar to Coeurdacier et al. (2007). Since purely redistributive shocks only affect the distribution of total output, but not its size, the optimal hedge is for the representative domestic household to hold all the domestic equity. This

[20] We will explore the case where relative bond returns do not load on the (welfare) based real exchange rate in section 6.

perfectly offsets the impact of $\hat{\varepsilon}$ shocks on total income. The equity portfolio exhibits full equity home bias.[21]

The bond position is negative when $\lambda < 1 - (1 - \frac{1}{\sigma})(2a - 1)$ and positive otherwise. A negative bond position (borrowing in domestic bonds and investing in foreign bonds) is possible only for sufficiently low values for λ. This condition echoes the condition for home equity bias in the equity only model of section 2. However, unlike (25) inspection of (28) reveals that the optimal bond positions are nicely behaved as a function of the underlying preference parameters (for $\sigma > 1$ and $a > 0.5$). Figure 2 reports the variation in b^*. Hence, unlike equity positions in the equity only model (see figure 1), the portfolios positions vary smoothly with preferences parameters. This implies that uncertainty about the true preference parameters translates into uncertainty of the same order regarding optimal portfolio positions.

3.2.2 Nominal shocks with preset prices.

One can show that the model with redistributive shocks has the exact same portfolio implications (and the same reduced form) as Engel and Matsumoto (2006)'s two period model with preset prices and nominal shocks (shocks to money supply). In their model, productivity shocks act as redistributive shocks: firms cannot adjust their prices but modify their profit margin, redistributing income between labor and dividends. This point is made clear in a specific case of the model developed in section 6.2.1 along the lines of Engel and Matsumoto (2006): when prices are preset in period 0 (full price rigidity), the nominal exchange rate and the real exchange rate are the same and nominal bonds load perfectly on the real exchange rate. As a consequence,[22] the optimal portfolio is identical to (28).

3.2.3 Government expenditures shocks/Investment expenditures shocks.

Government expenditures constitute another potential source of uncertainty. They break the link between private consumption and output and can also affect net returns from both financial and non-financial incomes, depending on the way fiscal expenditures are financed. This will also severe the link between the real exchange rate and relative equity returns *net of taxes*.

Assume that in each country i, the government must finance period-1 government expenditures $E_{g,i}$ equal to $P_{g,i}G_i$, where G_i is the aggregate consumption index of the government and $P_{g,i}$ is the price index for government consumption, potentially different from the price index for private consumption. G_i is stochastic and symmetrically distributed, with $E_0(G_i) = \bar{G}$.

We denote $E_g = \frac{P_{g,H}G_H}{P_{g,F}G_F}$ the ratio of Home to Foreign government expenditures and $\widehat{E_g}$ the log deviation from its steady-state symmetric value of one.

[21] Notice that this result does not depend upon the size of the redistributive shock: even a very small amount of redistributive variation leads to full equity home bias, as long as fluctuations in nonfinancial income shares are of the first order.

[22] Set the degree of price rigidity ω to 1 in the model of section 6.2.1 for a proof of this result.

Preferences of the government are similar to that of the consumers:[23]

$$G_i = \left[a_g^{1/\phi} \left(g_{ii}\right)^{(\phi-1)/\phi} + \left(1-a_g\right)^{1/\phi} \left(g_{ij}\right)^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)} \tag{29}$$

where g_{ij} is country i government's consumption of the good from country j in period 1 and $a_g > 1/2$ represents the preference for the home good of the government (mirror-symmetric preferences) that may differ from the bias in household preferences ($a_g \neq a$).

Government expenditures in country $i = \{H, F\}$ are financed through taxes on financial income (for a share δ_g), $T_{R,i} = \delta_g E_{g,i}$, and through taxes on non-financial incomes (for a share $(1-\delta_g)$), $T_i^w = (1-\delta_g)E_{g,i}$, so as to ensure budget balance in period 1.[24]

Market-clearing conditions for both goods are now:

$$c_{ii} + c_{ji} + g_{ii} + g_{ji} = y_i. \tag{30}$$

Following similar steps as before, relative demand of Home over Foreign goods by governments ($y_g = \left(g_{HH} + g_{FH}\right) / \left(g_{HF} + g_{FF}\right)$) satisfies (in log-linearized terms):

$$\hat{y}_g = -\lambda_g \widehat{q} + (2a_g - 1)\widehat{E_g} \tag{31}$$

where $0 \leq \lambda_g = \phi(1-(2a_g-1)^2) + (2a_g-1)^2 \leq \phi$ represents the impact of fluctuations in the terms of trade on relative government consumption, after controlling for relative expenditures $\widehat{E_g}$.

Relative demand of Home over Foreign goods by consumers ($y_c = \left(c_{HH} + c_{FH}\right) / \left(c_{HF} + c_{FF}\right)$) still satisfies equation (15) since the private allocation across goods has not changed:

$$\hat{y}_c = -\lambda \widehat{q} \tag{32}$$

Equation (31) and (32) together with market clearing conditions of both goods (30) implies the following equilibrium on the goods market:

$$\widehat{y} = s_c \hat{y}c + s_g \hat{y}_g = -\bar{\lambda}\widehat{q} + s_g(2a_g - 1)\widehat{E_g} \tag{33}$$

where s_c (resp. $s_g = 1 - s_c$) is the steady-state ratio of consumption spending (resp. government spending) over GDP and $\bar{\lambda} = s_c\lambda + s_g\lambda_g$. Note that, intuitively, efficient terms-of-trade $\widehat{q}$ are decreasing with the relative supply of goods $\widehat{y}$ (with an elasticity $1/\bar{\lambda}$) and increasing with relative government expenditure shocks (due to the presence of government home bias in preferences a_g), which act as relative demand shocks in this set-up.

[23] One can also allow for a different elasticity of substitution between Home and Foreign goods for government consumption. This extension is straightforward and does not add much substance.

[24] We restrict ourselves to cases where the *marginal* and *average* shares of taxes on financial and non-financial income in total fiscal revenues are the same (and equal to δ_g and $1-\delta_g$ respectively). However, what matters for equity portfolios is how marginal changes in government expenditures are financed, not how they are financed on average. So δ_g must be understood as the contribution of taxes on financial income to finance a marginal increase in government expenditures.

This gives the following (net-of-taxes) relative returns:[25]

$$\begin{cases} \hat{R}_e &= (1-\bar{\lambda})\widehat{q} + s_g(2a_g - 1 - \frac{\delta_g}{\delta})\widehat{E_g} \\ \hat{R}_b &= (2a-1)\widehat{q} \\ \hat{R}_n &= (1-\bar{\lambda})\widehat{q} + s_g(2a_g - 1 - \frac{1-\delta_g}{1-\delta})\widehat{E_g} \end{cases} \tag{34}$$

Direct inspection of (34) reveals that in general markets are complete and that the system is similar to (19) with:[26]

$$\widehat{\varepsilon} = \widehat{E_g} \; ; \gamma_e = s_g(2a_g - 1 - \frac{\delta_g}{\delta}); \; \gamma_b = 0 \; ; \gamma_n = s_g(2a_g - 1 - \frac{1-\delta_g}{1-\delta}) \tag{35}$$

$$\frac{\gamma_n}{\gamma_e} = -\frac{\delta}{1-\delta}\left(1 - \frac{2\,(1-a_g)}{2\,(1-a_g)\,\delta - (\delta - \delta_g)}\right) \tag{36}$$

The impact of fiscal shocks on relative equity returns and non-financial incomes depends on the fluctuations in relative government expenditures $\widehat{E_g}$, as well as the government preferences for the home good a_g, the steady state share of government expenditures in output s_g, and the relative fiscal incidence of the shocks δ_g/δ. Importantly, the parameter γ_n/γ_e does not depend upon the preferences of the representative household. It is only a function of the preferences of the government in terms of consumption (a_g) and taxation (δ_g).

In this set-up, (20) needs to be slightly modified since private consumption in steady-state does not equal total consumption. (20) can be rewritten as follows where γ_e and γ_n are defined in (35):

$$s_c(1 - \frac{1}{\sigma})(2a-1)\widehat{q} = \delta\,(2S-1)\,\hat{R}_e + (1-\delta)\hat{R}_n + 2b\hat{R}_b \tag{37}$$

Equilibrium portfolios are given by:

$$\begin{cases} b^* &= \frac{1}{2}s_c(1-\frac{1}{\sigma}) + \frac{1}{2}(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\frac{\gamma_n}{\gamma_e}) \\ S^* &= \frac{1}{2}\left(1 - \frac{\gamma_n}{\gamma_e}\frac{(1-\delta)}{\delta}\right) \end{cases} \tag{38}$$

Once again, portfolios are uniquely determined and the equity portfolio is independent from consumer preferences (ϕ, σ and a). While optimal equity portfolio are independent from household preferences, they depend on government preferences through a_g and δ_g. Let's consider some specific cases.

[25] Note that the bond return is unaffected because bond returns are not taxed. This ensures that $\gamma_b = 0$.

[26] The exception is the very peculiar case where $2a_g = 1 + \delta_g/\delta$. In that case, government expenditures do not modify equity returns conditionally on bond returns, and thus cannot be hedged perfectly. This rules out the case where government expenditures fall entirely on the domestic good ($a_g = 1$) and the fiscal incidence is equally distributed on financial and non-financial income ($\delta_g = \delta$).

- When $a_g = 1$ (government expenditures are fully biased towards local goods), the equity portfolio is fully biased towards local stocks: $S^* = 1$.[27] The reason is simple: from (33), a 1% increase in Home government expenditures raises Home dividends and Home non-financial income before taxes by s_g%. With a portfolio fully biased towards local equity, Home taxes also increase by s_g%. Such a portfolio insulates completely consumption expenditures from changes in government expenditures (and taxes). Notice that in this case, government expenditures shocks act as redistributive shocks since $\gamma_n/\gamma_e = -\delta/(1-\delta)$.[28]

- When $a_g < 1$, the equity portfolio depends on the incidence of taxes. When $\delta_g = \delta$, *i.e* when increases in government expenditures fall on financial income proportionally to its share in gross GDP, $\gamma_n/\gamma_e = 1$ and the equity portfolio is the one of Baxter and Jermann (1997); in particular, investors exhibit foreign bias in equities.

$$S^* = \frac{1}{2}\frac{2\delta - 1}{\delta} \tag{39}$$

  The reason is simple. Conditionally on relative bond returns, shocks to Home government expenditures exactly decrease Home equity returns and Home labor incomes in the same proportion, making financial and non financial incomes perfectly correlated. Being over-exposed on government expenditures shocks due to their non-financial incomes, investors will reduce their holdings of local stocks and increase their holdings of foreign stocks.

- When $\delta_g = 1$, *i.e* changes in government expenditures are entirely financed by taxes on financial incomes, $\gamma_n/\gamma_e = (2a_g - 1)/\left(2a_g - 1 - \frac{1}{\delta}\right)$ and the equilibrium equity portfolio becomes:

$$S^* = \frac{1}{2}\left[1 + \frac{(2a_g - 1)(1-\delta)}{1 - \delta(2a_g - 1)}\right] \tag{40}$$

  The equity portfolio always exhibits Home bias when $a_g > \frac{1}{2}$. Holding bond returns constant, an increase in Home government expenditures decreases dividends net of taxes at Home and raises Home non-financial incomes by raising the relative demand for Home goods (see (34) for $\delta_g = 1$). Conditional on bond returns, relative equity returns and relative non-financial incomes move in opposite directions and investor favors local equities to hedge non-financial. incomes. In other words, because higher Home government expenditures increase Home non-financial income, the burden of taxes must primarily fall on Home households to preserve efficient risk-sharing.

That the mechanism described above is very similar to the one in Heathcote and Perri (2007b) and Coeurdacier, Kollmann and Martin (2008). Government expenditures play the same role as (endogenous) investment in these papers: increases in Home investment raise

[27] This is true except for the knife-edge case where equity and bonds have the same pay-offs, which occurs here when $\delta_g = \delta$. In that case, the portfolio is indeterminate. See footnote 26.

[28] Although in that case $\bar{\lambda} = s_c\lambda + (1 - s_c)\phi$.

Home wages (non-financial income) due to Home bias in investment spending, but decreases Home dividends (net of the financing of investment). This implies a negative covariance between relative nonfinancial income and relative equity returns (holding bond returns constant). Hence, to hedge fluctuations in nonfinancial income generated by changes in investment across countries, investors exhibit Home equity bias.

Because investment in these models is entirely financed by shareholders, it is isomorphic to our model of government expenditures with $\delta_g = 1$ (government expenditures are financed by shareholders). Hence, the equity portfolios of (40) is identical to the one described in Heathcote and Perri (2007b) and Coeurdacier et al. (2008) if we replace Home bias in government expenditures by the degree of Home bias in investment expenditures.

While equilibrium portfolios do depend on the assumptions regarding the additional source of uncertainty, some common features are robust across models:

- The equity portfolio is driven by the covariance between relative equity returns and non-financial income, conditional on bond returns.
- Optimal equity positions will be 'robust' to changes in household preferences:
  1. Changing the risk aversion induces a change in the exposure of total consumption expenditures to the real exchange rate (the left hand side of (20) but this is optimally taken care of by bonds holdings (term $\frac{1}{2}s_c(1-\frac{1}{\sigma})$).
  2. Changing the elasticity of substitution across goods changes the response of the real exchange rate to output shocks ($\bar{\lambda}$) but his will be also taken care of by optimal bond holdings (term $\frac{1}{2}(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\gamma_n/\gamma_e)$).

4 The General Case: Bond and Equity Holdings under Incomplete Markets.

We now consider multiple source of uncertainties. While markets are not locally complete anymore, one can use the Devereux and Sutherland (2006) approach (see also Tille and van Wincoop (2007)) to characterize the optimal equity and bond positions. Our key result is that the expression of optimal portfolios as a function of the loadings (24) still applies when markets are incomplete. This is important since this validates our empirical methodology of section 5 even in an environment where the number of assets is not sufficient to span all the risks.

4.1 A generic representation

We use a similar set-up similar to the one of section 2 and 3 but with more than one additional source of risk. The model can be summarized by the following relationships (log-linearized):

- Intratemporal allocation across goods:

$$\widehat{y} = -\phi\widehat{q} + (2a-1)[\widehat{P_H C_H} - \widehat{P_F C_F} - (1-\phi)R\hat{E}R]$$

- Budget constraint:

$$\widehat{P_H C_H} - \widehat{P_F C_F} = \delta\left(2S-1\right)\hat{R}_e + (1-\delta)\hat{R}_n + 2b\hat{R}_b$$

- Relative returns on equities, non-financial wealth and bonds are expressed in reduced form as follows:

$$\begin{cases} \hat{R}_e &= \widehat{q} + \widehat{y} + \boldsymbol{\gamma}_e'\hat{\boldsymbol{\varepsilon}} \\ \hat{R}_b &= (2a-1)\widehat{q} + \widehat{y} + \boldsymbol{\gamma}_b'\hat{\boldsymbol{\varepsilon}} \\ \hat{R}_n &= \widehat{q} + \widehat{y} + \boldsymbol{\gamma}_n'\hat{\boldsymbol{\varepsilon}} \end{cases}$$

where $\hat{\boldsymbol{\varepsilon}}$ is a N-dimensional vector of shocks and $\boldsymbol{\gamma}_i$ for $i = \{b, e, n\}$ is a $N \times 1$ vector that controls the impact of $\hat{\boldsymbol{\varepsilon}}$ on assets returns.

4.2 Equilibrium portfolios and equilibrium loadings.

Following Devereux and Sutherland (2006) approach (see also Tille and van Wincoop (2007)), we show in Appendix A.2 that the equilibrium portfolio (bond and equity) is unique and well-defined unless equilibrium equity returns and bond returns are perfectly correlated. As in previous cases, it is informative to rewrite the equilibrium bond and equity portfolios in terms of the equilibrium asset return loadings on the real exchange rate $R\hat{E}R$ and on non-financial income $\hat{w}$:

$$R\hat{E}R \equiv \beta_{RER,b}\hat{R}_b + \beta_{RER,e}\hat{R}_e + u_{RER} \quad (41)$$

$$\hat{R}_n \equiv \beta_{n,b}\hat{R}_b + \beta_{n,e}\hat{R}_e + u_n \quad (42)$$

where u_i for $i = \{RER, n\}$ is orthogonal to $\hat{R}_j$ for $j = \{b, e\}$: $E\left[u_i\hat{R}_j\right] = 0$ and $\beta_{RER,i} = cov\left(R\hat{E}R, \hat{R}_i|\hat{R}_j\right)/var\left(\hat{R}_i|\hat{R}_j\right)$, $\beta_{n,i} = cov\left(\hat{R}_n, \hat{R}_i|\hat{R}_j\right)/var\left(\hat{R}_i|\hat{R}_j\right)$ as before.

Define m as the difference between stochastic discount factor across countries: $m = C_H^{-\sigma}/P_H - C_F^{-\sigma}/P_F$. From the Euler equation of the investor problem, observe that m satisfies:

$$E\left[\hat{m}\hat{R}_i\right] = 0 \text{ for } i = e, b \quad (43)$$

Substituting (42) into the budget constraint and using the definition of m, we obtain:

$$\begin{aligned} \widehat{P_H C_H} - \widehat{P_F C_F} &= -\frac{1}{\sigma}\hat{m} + (1-\frac{1}{\sigma})R\hat{E}R \\ &= \delta\left(2S-1\right)\hat{R}_e + (1-\delta)[\beta_{w,b}\hat{R}_b + \beta_{w,e}\hat{R}_e + u_w] + 2b\hat{R}_b. \end{aligned}$$

Finally, use (43) to project the budget constraint on $\hat{R}_e$ and $\hat{R}_b$ to obtain:

$$\begin{cases} b^* &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\beta_{RER,b} - \frac{1}{2}\left(1-\delta\right)\beta_{n,b} \\ S^* &= \frac{1}{2}\left[1 - \frac{1-\delta}{\delta}\beta_{n,e} + \frac{1-\frac{1}{\sigma}}{\delta}\beta_{RER,e}\right] \end{cases} \tag{44}$$

This is the exact same expression as before (see equation (24)). The intuition is that the equilibrium bond and equity positions will hedge optimally the components of real exchange rate and non-financial income fluctuations with which they are correlated.

5 Empirical Analysis: What do we know about the hedging properties of bond and equity returns?

The results of the previous sections indicate that the pivotal factor for equilibrium equity positions is the ability of bond returns to hedge real exchange rate risk. When this is the case, equity portfolios are driven by the correlation between equity returns and non financial income, *conditional on bond returns*. This section provides prima facie evidence on the role of bond-hedging for equilibrium equity holdings for G-7 countries. We do so by estimating the reduced-form loading factors $\beta_{RER,i}$ and $\beta_{n,i}$ for $i = e, b$ defined in equations 22-23. Sections 3 and 4 showed that this is all we need to assess the empirical validity of the model.

5.1 Description of the data.

We collect and use quarterly data for G-7 countries over the period 1970:1-2008:3. To estimate the empirical counterpart of the general specification of section 3.1, we treat in turn each member of the G-7 as the Home country and aggregate the remaining members of the G-7 as the Foreign country. We also present the results for a Eurozone aggregate consisting of France, Germany and Italy. For each country, we build a measure of non-financial income, equity returns, bond returns and real exchange rate. We use National Income data from the OECD quarterly national income accounts, supplemented by annual data from the United Nations National Accounts Statistics; Data on equity total returns, 3-months bond returns and US dollar nominal exchange rates are obtained from the Global Financial Database; Local consumer price indices are obtained from the OECD Main Economic Indicators.[29]

5.1.1 Non-financial income.

We use the National Income Account decomposition of GDP by income to construct the empirical counterpart of nonfinancial income and of the average share of financial income in total income, δ. For each country i, we express income measures in a common currency (US Dollar), converting local currency measures with the current nominal exchange rates S_{it}, measured as the dollar price of foreign currency i so that an increase in S_{it} represents an appreciation of currency i relative to the US dollar.

[29] See the appendix for a detailed description of data sources.

Denote Y_{it} the GDP of country i at time t in US dollars. It can be decomposed in the following way:

$$Y_{it} = WL_{it} + M_{it} + \Pi_{it} + D_{it} + T_{it} \tag{45}$$

where WL_{it} denotes the aggregate compensation of employees, M_{it} denotes mixed income, Π_{it} represents net operating surplus, D_{it} is the consumption of fixed capital, and T_{it} represents taxes minus subsidies on production and imports.

According to the System of National Accounts 1993, the net operating surplus represents the profits of incorporated entities. It is defined as "the surplus or deficit accruing from production before taking account of any interest, rent or similar charges payable on financial or tangible non-produced assets borrowed or rented by the enterprise, or any interest, rent or similar receipts receivable on financial or tangible non-produced assets *owned by the enterprise*." Mixed income M_{it} represents income from self-employment as well as proprietary income. It is defined as "the surplus or deficit accruing from production by *unincorporated enterprises owned by households*; it implicitly contains an element of remuneration for work done by the owner, or other members of the household, that cannot be separately identified from the return to the owner as entrepreneur but it excludes the operating surplus coming from owner-occupied dwellings."[30]

As we have argued earlier, we interpret nonfinancial income as that component of national income that cannot be capitalized into financial claims. A reasonable assumption is that the gross operating surplus $\Pi_{it} + D_{it}$, net of investment, measures the income on tradable financial claims, since the corresponding entities are incorporated. This is much less clear for mixed income. Most of self-employment and proprietary income cannot be capitalized into financial claims. Accordingly, we follow Gollin (2002) and construct our estimate of non-financial income, W_{it}, as the sum of the compensation of employees, WL_{it}, plus a share λ_{it} of mixed income:

$$W_{it} = WL_{it} + \lambda_{it} M_{it} \tag{46}$$

λ_{it} is set equal to the share of aggregate labor income WL_{it} in $WL_{it} + \Pi_{it}$.:[31]

$$\lambda_{it} = \frac{WL_{it}}{WL_{it} + \Pi_{it}}. \tag{47}$$

We calculate an equivalent measure for the rest of the world W_{-it} by summing non-financial incomes (in USD) for the 6 remaining countries:

$$W_{-it} = \sum_{j \neq i} WL_{jt} + \lambda_{jt} M_{jt} \tag{48}$$

We measure the average share of non-financial income in country i, $1 - \delta_i$, as the fraction of nonfinancial income in GDP measured at factor prices:

$$\delta_i = 1 - \frac{1}{T} \sum_{t=1}^{T} \frac{WL_{it} + \lambda_{it} M_{it}}{Y_{it} - T_{it}}$$

[30] See the description of SNA93 available at http://unstats.un.org/unsd/sna1993.

[31] We tried alternative measures but our results were essentially unaffected. In particular, we also attributed all mixed incomes to non-financial incomes ($\lambda_{it} = 1$, for all i, t) or all mixed incomes to financial incomes ($\lambda_{it} = 0$, for all i, t).

Table 1 summarizes our estimates of δ for the G7 countries. The table also reports a naive estimate using only labor compensation (naïve-$\delta = WL_{it}/(Y_{it} - T_{it})$). It is immediate that the partial inclusion of mixed income in nonfinancial income lowers significantly our estimates of δ, a point emphasized by Gollin (2002).

Finally, we compute relative nonfinancial income in US dollars as the log difference between nonfinancial income in country i and in the rest of the world: (in this section, the 'hat' notation denotes a relative variable)

$$\hat{w}_{it} = \log W_{it} - \log W_{-it}.$$

Figure 3 reports the relative nonfinancial income for each country in our sample. It exhibits marked fluctuations over the period and varies between 4 percent for Canada and 94 percent for the U.S. Looking at the figure, it is also apparent that relative nonfinancial income is correlated with the real exchange rate: it increases markedly between 1980 and 1985 for the US, then decreases, with opposite movements for European countries and Japan (see figure 6).

5.1.2 Bond and equity market returns.

Denote $\tilde{R}_{b,it}$ the quarterly local currency gross return on 3-month Treasury bills in country i at time t. We construct the corresponding dollar return as

$$R_{b,it} = \tilde{R}_{b,it}.\frac{S_{it+1}}{S_{it}}$$

We calculate an equivalent measure for the rest of the world using GDP-weights $\alpha^i_{jt} = Y_{jt}/\sum_{l\neq i} Y_{lt}$:

$$R_{b,-it} = \sum_{j\neq i} \alpha^i_{jt} R_{b,jt}$$

The relative log-bond return is simply $\hat{r}_{b,it} = \log\left(R_{b,it}/R_{b,-it}\right)$.

We perform similar calculations to construct relative equity returns $\hat{r}_{e,it}$.[32]

Figures 4 and 5 report the relative bond and equity returns for each country.

5.1.3 Real exchange rate

We define the real exchange rate of country i at date t as the ratio of the price level in country i, P_{it}, to the price level for the rest of the world P_{-it}:

$$\begin{aligned} RER_{it} &= \frac{P_{it}}{P_{-it}}, \\ \log P_{-it} &= \sum_{j\neq i} \alpha^i_j \log\left(S_{jt}P_{jt}/S_{it}\right). \end{aligned}$$

[32] We use market-capitalization weights to construct the equity return of the rest of the world

The rate of change of the real exchange rate is then:

$$\hat{\xi}_{it} = \log(\frac{RER_{it}}{RER_{it-1}})$$

Figure 6 reports the real exchange rate RER_{it} for all countries, normalized to 100 in 2001:1.

5.2 Estimating the loading on the real exchange rate.

A key implication of our theoretical model is that portfolios are drastically different depending on the financial asset that is used to hedge real exchange rate risk. As shown in section 3.1, the moments that matter for equilibrium portfolios are the loading factors of relative bond and equity returns on real exchange rate changes $\beta_{RER,j}$ for $j = e, b$. These unconditional moments can be estimated for each country by the following simple regression for each country i:

$$\hat{\xi}_{it} \equiv c + \beta^i_{RER,b}\hat{r}_{b,it} + \beta^i_{RER,e}\hat{r}_{e,it} + u_{it} \tag{49}$$

where u_{it} captures the fluctuations in the real exchange rate that are not spanned by relative bond and equity returns.

Results of the regression (49) for each countries are displayed in table 2. Our empirical results confirm the results of van Wincoop and Warnock (2006) for all the countries considered in the sample: relative bond returns capture most of the variations of the real exchange rate. Moreover, *conditional on bond returns*, the variance-covariance ratio for equity returns and the real exchange rate almost never statistically different from zero.[33] From a theoretical standpoint, the reduced-form model with $\gamma_b = 0$ provides a very reasonable approximation of the data.

5.3 Estimating the loading on the return on non-financial wealth.

5.3.1 Estimating the return to non-financial wealth

The previous results indicate that equities will not be used to hedge real exchange rate risks. Can they be used to hedge the returns on nonfinancial wealth? In order to answer that question, we need an estimate of $\hat{r}_n$, the relative return to non-financial wealth. In the static model we presented in section 3, this is equivalent to $\Delta\hat{w}_{it}$, the growth rate of relative non-financial income.

However, it is important to recognize that the world is not static, so that relative nonfinancial income $\hat{w}_{it}$ represents only the relative dividend and not the total return on nonfinancial wealth. We construct estimates of $\hat{r}_n$ following the method of Campbell and Shiller (1988)

[33]The exception is the U.K. Even in that case, $\beta_{RER,e}$ is not economically different from zero.

as detailed in Campbell (1996). A similar approach has been used by Baxter and Jermann (1997) and Julliard (2003).[34]

Denote $r_{n,it+1}$ the log of the gross simple return on non-financial wealth in country i between t and $t+1$. Following Campbell (1996), under the assumption that the dividend price ratio on human wealth is stationary, we can write the following approximation:

$$r_{n,it+1} \equiv \log\left(W_{it+1} + V_{n,it+1}\right) - \log V_{n,it} \approx k + z_{it} - \rho z_{it+1} + \Delta \log W_{it+1} \tag{50}$$

where V_{nit} measures nonfinancial wealth, $z_{it} = \log W_{it}/V_{n,it}$ is the log-dividend price ratio, ρ is a number slightly smaller than 1 and k is an unimportant constant.[35]

Solving (50) forward and imposing that $\lim_{t\to\infty} \rho^t \left(r_{n,it} - \Delta \log W_{it}\right) = 0$, we obtain (up to a constant):

$$z_{it} = \sum_{j=0}^{\infty} \rho^j \left(r_{n,it+1+j} - \Delta \log W_{it+1+j}\right) \tag{51}$$

This expression states that the dividend-price ratio is high today either when future returns are high (so that future nonfinancial wealth is high), or when future nonfinancial income growth is low (so that future nonfinancial income is low).

Using (51) and under the assumption that the conditional expected return on nonfinancial wealth equals the conditional expected return on equities ($E_t r_{n,it+1+j} = E_t r_{e,it+1+j}$) we can substitute into (50) to obtain the standard expression:[36]

$$r_{n,it+1} - E_t r_{n,it+1} = \left(E_{t+1} - E_t\right) \sum_{j=0}^{\infty} \rho^j \Delta \log W_{it+1+j} - \left(E_{t+1} - E_t\right) \sum_{j=1}^{\infty} \rho^j r_{e,it+1+j} \tag{52}$$

This expression makes clear that the innovation to the return on nonfinancial wealth depends upon revisions to the path of future expected nonfinancial income growth (the cash flow component represented by the first summation on the right hand side) as well as revisions to the path of future expected equity returns proxying for future expected nonfinancial wealth returns (the discount rate component represented by the second summation on the right hand side). To the extent that nonfinancial income growth and equity returns are not iid, both terms are potential important empirically in evaluating the return to nonfinancial wealth.

The last step consists in subtracting (52) for country i and for the rest of the world.

[34] Baxter and Jermann (1997) also estimates the returns to non-financial wealth using a VAR as we do. Compared to their paper, we do not need to impose a cointegration relationship. Since we measure all variables in relative terms, our variables of interest (relative non-financial incomes/relative equity returns) are all stationary.

[35] One can show that $\rho^{-1} = 1 + \exp(z)$ where z is the steady state value of the log dividend price ratio. We will use the value of $\rho = 0.98$ in line with standard estimates in the literature. Our results are robust to changes in the value of ρ.

[36] This assumption is not innocuous. Recent work by Lustig, Van Nieuwerburgh and Verdelhan (2008) using an affine-yield model estimated on bond yields and stock returns finds that the expected return on human wealth can be quite different from the expected return on equities.

Under the assumption that the discount rate ρ is the same in all countries, this yields:

$$\hat{r}_{n,t+1} - E_t\hat{r}_{n,t+1} = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\rho^j\Delta\hat{w}_{t+1+j} - (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j\hat{r}_{e,t+1+j} \tag{53}$$

so that the relative return to nonfinancial wealth depends upon revisions to the path of relative nonfinancial income growth as well as revisions to the path of relative future equity returns. This last term is important and merits to be discussed in detail. This term captures the predictability of the *excess* equity return (measured in dollars). In many models, parity conditions ensure that $E_t r_{e,it+k} = E_t r_{e,jt+k}$ so that the second summation on the right hand side disappears and the innovation to nonfinancial wealth comes entirely from revisions in future relative nonfinancial income growth. However, Gourinchas and Rey (2007) show that -at least for the US– excess equity returns contain a predictable component related to the intertemporal external constraint. It is therefore important to allow for this term in the empirical implementation of our tests.[37]

We construct the empirical counterpart of the left hand side of equation (53), using a VAR(1) of the following form:[38]

$$z_{t+1} = Az_t + \epsilon_{t+1}$$

where $z_t' = (\hat{r}_{e,t}, \Delta\hat{w}_t, x_t)$ and x_t represents other controls that helps to predict the (relative) growth rate of non-financial income and relative equity returns. In practice, we use relative log consumption expenditures, relative bond returns $\hat{r}_{b,t}$ as well as the nxa variable from Gourinchas and Rey (2007). This last variable proved to contain important information about future excess equity returns for the U.S.

With estimates of A and ϵ_{t+1} in hand, we construct an empirical counterpart to $\hat{r}_{n,t+1} - E_t\hat{r}_{n,t+1}$ as:

$$\hat{r}_{n,t+1} - E_t\hat{r}_{n,t+1} = \left(e'_{\Delta\hat{w}} - \rho e'_{\hat{r}_e}A\right)(I - \rho A)^{-1}\epsilon_{t+1}$$

These are the innovations to the returns on non-financial wealth that will be used to estimates the loadings of market returns on returns to non-financial wealth. Figure 7 reports the estimated innovations to nonfinancial wealth returns. Figure 8 reports the return to nonfinancial wealth $\hat{r}_{n,t+1} - E_t\hat{r}_{n,t+1}$ for the U.S., and relative nonfinancial income growth $\Delta\hat{w}$. The correlation between the two series is high (0.71), and the return innovation exhibits more volatility.

5.3.2 Market returns loadings non-financial wealth

We now use the returns $\hat{r}_{n,it}$ estimated for each country i to run the following equation:

$$\hat{r}_{n,it} = c + \beta^i_{n,b}\hat{r}_{b,it} + \beta^i_{n,e}\hat{r}_{e,it} + v_{it} \tag{54}$$

[37] Benigno and Nistico (2009) present estimates that differ from ours precisely becaus they omit the second summation in (53).

[38] Standard Akaike and Schwarz lag-selection criteria indicate that a VAR(1) is preferable for all countries.

where v_{it} is attributed again both to the measurement error in the construction of the return on nonfinancial wealth, and to the fluctuations in relative nonfinancial income risk not spanned by relative bond and equity returns.

This regression estimates directly the loadings of (relative) bond and equity returns for each country i of our sample. It gives insights about the hedging capacity of bonds and equities against fluctuations in non-financial wealth. Recall in particular that following our generic specification (and assuming $\gamma_b = 0$ in line with our previous estimates), $\beta_{n,e}$ is the empirical counterpart of γ_n/γ_e, which is the only parameter that has implications for the equity portfolio.

Results of the regression (54) for each countries are shown in table 3. The next to last row shows the estimate of the unconditional loading factor $\hat{\beta}_{n,e}^{i,unc} = cov\left(\hat{r}_{n,i}, \hat{r}_{e,i}\right)/var\left(\hat{r}_{e,i}\right)$. This coefficient is almost always positive and significant. The interpretation is the following: without hedging real exchange rate risk with bonds, nonfinancial income is positively correlated with relative equity returns. Hence, the international diversification puzzle is 'worse than you think'.

However, once we hedge real exchange rate risk with bonds, the remaining conditional covariance variance ratio $\beta_{n,e}^{i}$ is significantly smaller, even negative in four countries (U.S., U.K., Japan and Italy). For the U.S. and Japan in particular, the coefficients are significantly negative and economically relevant. Hence, our results differ strongly from Baxter and Jermann (1997) who do not condition on bond returns.[39]

The positive loadings of (relative) bond returns $\beta_{n,b}$ imply that shorting the local currency bond, and going long in the foreign currency bond, constitutes a good hedge against fluctuations in returns to non-financial wealth. Note that this result is not theoretically surprising. In our model, a (potentially large) part of relative non-financial and financial income comoves with the real exchange rate (see figures 5, 7 and 6), and we know that relative bond returns track almost perfectly the real exchange rate.

5.4 Implied bond and equity portfolios

The previous estimates allow us to back out the implied equity and bond positions using equations (24). Note however, that these equations hold for countries of symmetric size. Allowing for different country sizes, (24) must be rewritten in the following way (under the assumption verified empirically that $\beta_{RER,e} = 0$; see the appendix):

$$\left\{\begin{array}{lll} b^* & = & = \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\beta_{RER,b} - \frac{1}{2}\left(1-\delta\right)\beta_{n,b} \\ S^* & = & \omega_i - \frac{1-\delta}{\delta}\beta_{n,e}(1-\omega_i) \end{array}\right.$$

[39] An additional difference between Baxter and Jermann (1997) and our results is that we express everything in relative terms. In so doing, we make sure that aggregate shocks that affect Home and Foreign equity returns in the same way (and hence cannot be insured) are not affecting our results. Julliard (2003) argues that Baxter and Jermann (1997)'s implicit assumption that returns on financial and nonfinancial income are uncorrelated across countries is violated empirically.

where ω_i is the relative size of country i in world GDP (assumed to be equal to its relative wealth in the symmetric non-stochastic equilibrium).

The implied equity bias and bond portfolios are summarized in table 4. The model is successful in predicting a significant degree of equity home bias for the US, the U.K. and Japan, but not for the eurozone countries. For these countries, it predicts instead a sizeable foreign home bias, despite the hedging of real exchange rate risk through bonds. For the U.K., U.S. and Japan, the fraction of home equity bias is economically quite significant, indicating that hedging of nonfinancial income may be an important component for these countries.

The bottom part of the table looks at predictions for bond holdings. When $\sigma = 1$, bond holdings are determined by the hedging of nonfinancial income risk. Since the return on nonfinancial wealth is strongly positively correlated with bond returns, this implies a significant short position in domestic currency bonds, between 32% and 40%.[40] As investors become more risk averse, the second term, hedging real exchange rate risk on consumption expenditures becomes progressively more important. For $\sigma = 4$, we find relatively small bond positions, short in the U.S., but long in most other countries. This accord well with the evidence in Lane and Shambaugh (2008) suggesting large heterogeneity across countries in the currency denomination of external bond holdings. On average, they find that advanced countries hold long (but small) domestic currency debt positions but some large countries, most notably the US, are short in their own currency debt.

6 Extensions and discussions of the results

6.1 The Role of Nontradable

Recent work by Obstfeld (2007) and Collard et al. (2007) put forward the presence of non-traded goods as key to understand international equity portfolios.[41] But in these models, equity portfolios are also driven by the hedging of the real exchange rate coming from changes in the relative price of non-traded goods. Consequently their portfolios are also strongly affected by slight changes in preferences. Like in our benchmark model, their findings might be altered by trade in bonds in presence of an additional source of risk. For simplicity, we will focus on the case where relative bond returns load perfectly on the real exchange rate. It turns out that our previous findings are robust to the addition of non-traded goods. In particular, contrary to existing literature, the equity portfolio (aggregated across the traded and non-traded sector) will be independent on preferences and driven by the hedging of non-financial incomes conditional on bond returns. This result is shown in an extension of Obstfeld (2007)'s set-up with non-financial incomes. The details of the model and the

[40] Note that since β_{nb} is close to 1 empirically, this component is simply half of the nonfinancial income share $(1-\delta)$.

[41] See also Dellas and Stockman (1989), Baxter et al. (1998) for earlier work on the role of non-traded goods. See also Matsumoto (2007).

derivation of the results are described in Appendix A.3. We present briefly the set-up and the main equations.

We consider the same two-period ($t = 0, 1$) endowment economy with symmetric countries, Home (H) and Foreign (F). Each country now produces two goods, a tradable (T) and a non-tradable good (NT). At $t = 1$, country i receives an exogenous endowment y_i^T of the tradable good i and an exogenous endowment y_i^{NT} of the non-tradable good i such that $E_0(y_i^T) = E_0(y_i^{NT}) = 1$. As in the benchmark case, a share δ of the endowment in each sector is distributed to shareholders while a share $(1-\delta)$ is not capitalized and is distributed to households of country i. There is no output (and no consumption) at $t = 0$, but agents trade claims (stocks and bonds).

The aggregate consumption index C_i, for $i = H, F$ is given by:

$$C_i = \left[\eta^{1/\theta} \left(c_i^T\right)^{(\theta-1)/\theta} + (1-\eta)^{1/\theta} \left(c_i^{NT}\right)^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)} \tag{55}$$

where c_i^T is the consumption of a composite tradable goods using Home and Foreign tradable goods and c_i^{NT} is the consumption of non-tradable goods. θ is the elasticity of substitution between tradable and non-tradable goods.

Consumption of the tradable good is defined as in the benchmark model:

$$c_i^T = \left[a^{1/\phi} \left(c_{ii}^T\right)^{(\phi-1)/\phi} + (1-a)^{1/\phi} \left(c_{ij}^T\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \tag{56}$$

where c_{ij}^T is country i's consumption of the tradable good from country j.

Resource constraints over tradable and non-tradable goods for $i = H, F$ are given by:

$$c_{ii}^T + c_{ji}^T = y_i^T \tag{57}$$

$$c_i^{NT} = y_i^{NT} \tag{58}$$

6.1.1 Financial markets

There is trade in stocks and bonds in period 0. In both countries there are two different stocks, stocks of tradable and stocks of non-tradable. Each stock is a Lucas tree that gives a share δ of the future endowments (tradable or non-tradable). The supply of each type of share is normalized at unity. There is a CPI bond denominated in the Home composite good, and a CPI bond denominated in the Foreign composite good. Buying one unit of the Home (Foreign) bond in period 0 gives one unit of the Home (Foreign) composite good at $t = 1$. The CPI in each country is the standard CES aggregate over tradable and non-tradable goods (see appendix A.3). Using similar notations as previously, we denote a country's holdings of local stock of tradable (resp. non-tradable) by S^T (resp. S^{NT}), and its holdings of CPI bonds denominated in its local composite good by b. The vector $(S^T; S^{NT}; b)$ describes international portfolios.

6.1.2 Equity and bond portfolios with locally complete markets

Like in the benchmark model, portfolios are undetermined with endowment shocks in both sectors in presence of bonds but this knife-edge result breaks down when we add an additional source of risk which pins down the international bond and equity portfolios in both sectors. We do not show the resolution of the model and its log-linearization but presents directly the few key equations (see appendix for the derivation of the equations).

The (log-linearized) Home real exchange rate is defined as follows:

$$\widehat{RER} = \widehat{\frac{P_H}{P_F}} = \eta(2a-1)\widehat{q} + (1-\eta)\widehat{P^{NT}} \tag{59}$$

where $P^{NT} = P_H^{NT}/P_F^{NT}$ is the relative price of Home non-tradable goods over Foreign non-tradable goods, q denotes the terms-of-trade and η is the steady-state share of spending devoted to tradable goods[42]

Relative consumption expenditures are equal to relative financial and non-financial incomes; thus, assuming locally complete markets, we have:

$$\begin{aligned} \widehat{P_H C_H} - \widehat{P_F C_F} &= (1-\frac{1}{\sigma})\widehat{RER} && (60) \\ &= \eta\delta\left(2S^T-1\right)\widehat{R_e^T} + (1-\eta)\delta\left(2S^{NT}-1\right)\widehat{R_e^{NT}} + (1-\delta)\widehat{w} + 2b\widehat{RER} && (61) \end{aligned}$$

where $\widehat{R_e^k}$ denotes Home excess equity return in sector $k = \{T, NT\}$ and $\widehat{w}$ denote Home excess non-financial income (Home over Foreign aggregated over both sectors).

The additional source of risk $\widehat{\varepsilon}$ is assumed to pertubate equity returns (in both sectors) and non-financial incomes and to leave bond returns unchanged.(see appendix). We denote (γ_e) the impact of the $\widehat{\varepsilon}$ shock on Home excess equity return $\widehat{R_e^k}$ in sector $k = \{T, NT\}$.[43]. The impact of the $\widehat{\varepsilon}$ shock on Home excess non-financial income $\widehat{w}$ is denoted (γ_w).

The method to solve for portfolios is essentially identical to our benchmark model: we derive the portfolio $(S^T; S^{NT}; b)$ that reproduces the complete market allocation for consumption locally. To do so, we make a projection of equation (61) on the three sources of risk.$(\widehat{y^T}; \widehat{y^{NT}}; \widehat{\varepsilon})$

The equity portfolio averaged across sectors satisfies (see appendix for a detailed equity positions across sectors)[44]:

$$\eta S^T + (1-\eta)S^{NT} = \frac{1}{2}\left(1 - \frac{\gamma_w}{\gamma_e}\frac{(1-\delta)}{\delta}\right) \tag{62}$$

[42]Here to simplify notations, we assume that the share of spending devoted to tradable goods is the same as the weight of tradable goods in the consumption index. This is true only if in the steady state tradable and non tradable goods have the same price: $p^{T*} = p^{NT*}$. This assumption is however irrelevant for equity portfolios (see Obstfeld (2007)).

[43](γ_e) is assumed to be identical across sectors. Extending the model with different γ_e^k for $k = \{T, NT\}$ is straightforward but testing empirically such a distinction will not be possible.

[44]The averaged equity bias is identical to section 4 when $\gamma_b = 0$ as we have assumed here that bond returns load perfectly on the real exchange rate.

As a consequence, the average equity portfolio is used to hedge the additional shock $\widehat{\varepsilon}$. Contrary to Obstfeld (2007) (see also Collard et al. (2007)), the overall equity bias in the economy does not depend on preferences (equation (62)).[45]

The bond position satisfies (with $\Omega = \frac{(\theta-1)(2a-1)}{\phi-1+(2a-1)^2(\theta-\phi)}$):

$$b = \frac{1}{2}(1-1/\sigma) - \frac{1}{2}\frac{(1-\gamma)(1-\delta)}{\eta\Omega+1-\eta}(1-\theta+(\theta-1/\sigma)(1-\eta+\eta(2a-1)\Omega)) \qquad (63)$$

As in our benchmark model, the bond portfolio is the sum of two terms: the first term is the optimal hedge for fluctuations in total consumption expenditures when $\sigma \neq 1$. The second term corresponds to the hedge of real exchange rate movements that are correlated with non-financial incomes and financial incomes arising from the optimal equity portfolios. Hence, the implications of the model with non-traded goods are very similar to our benchmark case.

6.2 Relative bond returns do not load on the (welfare-based) real exchange rate

Our previous results hinge on the key-assumption that bond returns differential across countries provide a good hedge for the fluctuations of the real exchange rate. This might not be true for at least two reasons: first, real bonds might not exist in practice.[46] Most bonds available to investors are nominal and nominal bonds returns differential across countries might not load perfectly on the real exchange rate in presence of nominal shocks. While nominal bonds may load pretty well on the real exchange rate in practice (see section 5 for empirical evidence), one might still want to know what are the predictions of our benchmark model in presence of nominal shocks. Second, even in the absence of nominal shocks, the bond return differential might not load perfectly on the *welfare-based* real exchange rate, the one that matters from the investor's point of view. This happens for instance in presence of shocks to the quality of goods (or equivalently changes in the number of varieties available to consumers) as in Corsetti, Martin and Pesenti (2005) or Coeurdacier et al. (2007).

We will explore these two cases sequentially. We do it in the benchmark model of section 3, i.e when markets are *locally* complete.

6.2.1 Nominal shocks in presence of price rigidities

Following Engel and Matsumoto (2006) and Engel and Matsumoto (2008), we add nominal shocks together with price rigidities in our model. We suppose that prices are set one period

[45] The way this equity bias is shared across sectors does depend on the two elasticities of substitution θ and ϕ. In appendix A.3, we show how the overall equity bias is shared between the two sectors. If θ and ϕ are both larger (or smaller) than unity (such that $(\theta-1)(2a-1)$ and $\phi-1+(2a-1)^2(\theta-\phi)$ have the same sign), then portfolio is well balanced across sectors. If $\theta < 1$ and $\phi > 1$, then equity portfolios will involve short position in one sector (see appendix). The equity positions across sectors never depend on σ.

[46] Note that hey do exist in most developed markets: US, Euro zone, UK, Sweden are some well known examples were inflation-indexed bonds have been created.

in advance for a share ω of firms. Firms do not know ex-ante if they will be able to adjust their price. In other words, all firms post a preset price and a share $(1-\omega)$ of firms will be able to adjust it. To preserve symmetry, we assume that the degree of price rigidities $\omega > 0$ in the same in both countries. Under this assumption, firms in both countries will post the same preset the same price in period 0,denoted $\bar{p}$.

We add money in our benchmark model by assuming that money enters directly the utility function. To simplify matters, we assume that consumption and real money balances are separable. The expected utility at date 0 of a representative agent in country i is now:

$$U_i = E_0\left[\frac{C_i^{1-\sigma}}{1-\sigma} + \chi log(\frac{M_i}{P_i})\right] \tag{64}$$

where M_i denotes money holdings in country i by agent i, χ is a positive parameter.[47]

One source of fluctuations will come from shocks to the money supply. We assume shocks to the money supply M_i in country i and introduce $M = M_H/M_F$ the relative money supply and $\widehat{M}$ its deviation from its mean value of one ($E_0(M) = 1$).

Bonds in country i are now nominal bonds that pay one unit of country i's currency. Due to the inflation risk, bonds returns differentials will not be perfectly correlated with real exchange rate changes. We introduce the nominal exchange rate s, defined as the number of Foreign currency units per unit of Home currency so that a rise in s represents a nominal appreciation of the Home currency. We express all variables in Home currency terms. We assume that $E_0(s) = 1$ and denotes $\widehat{s}$ the deviations of the nominal exchange rate from its steady-state value.

To make our results easily comparable with Engel and Matsumoto (2006), we assume that in the steady state the share of revenues going to shareholders δ is related to the mark-up in a monopolistic competition framework (Dixit and Stiglitz (1977)).[48] Output in country i is produced using labor l_i : $y_i = a_i l_i$ where a_i is a stochastic productivity shock, $E_0(a_i) = 1$ with symmetric distribution across countries. We denote the relative productivity shock $a = a_H/a_F$ and $\widehat{a}$ the deviation from its mean unitary value. This the second source of uncertainty. As in Engel and Matsumoto (2006), we have 'two relative shocks' ($\widehat{a}$ and $\widehat{M}$) and 'two relative assets': markets will be locally-complete.

Under Producer-Currency-Pricing (PCP), the Fisher-ideal price index in country $i = H, F$, expressed in the currency of country i is:[49]

$$P_H = \left[a\left(p_H\right)^{1-\phi} + (1-a)\left(\frac{p_F}{s}\right)^{1-\phi}\right]^{1/(1-\phi)} ; P_F = \left[(1-a)\left(sp_H\right)^{1-\phi} + a\left(p_F\right)^{1-\phi}\right]^{1/(1-\phi)}, \tag{65}$$

[47] We assume a log- utility for real money balance to simplify some of the calculations but our main results are essentially unaffected by this assumption. Supposing CRRA utility in money generates an additional hedging demand in the portfolio that goes towards zero once we converge towards a cashless economy.

[48] In such a set-up, the steady-state profit share δ is equal to the inverse of the elasticity of substitution between varieties within a given country

[49] We adopt Producer-Currency-Pricing but as shown by Engel and Matsumoto (2008) results are very similar with Local-Currency-Pricing.

where p_i is the aggregate price over all producing firms in country i in country i's currency. Given that some firms do adjust and some do not, p_i is the CES weighted sum of preset prices $\bar{p}$ and adjusted prices in country i (p_i^*).[50]

The log-linearization of the Home country's real exchange rate $RER \equiv \frac{sP_H}{P_F}$ gives:

$$\widehat{RER} = (2a-1)(\omega\widehat{s} + (1-\omega)\widehat{q^*}) \tag{66}$$

where $q^* = \frac{sp_H^*}{p_F^*}$ denotes the relative price of firms adjusting their price. Note that when $\omega = 1$, all prices are preset and the real exchange rate is perfectly correlated with the nominal exchange rate. When $\omega = 0$, the real exchange rate loads perfectly on the terms of trade equal to $\frac{sp_H}{p_F}$.

Under PCP, (15) still holds for relative private consumption $(\widehat{y_C})$, where one should note that terms-of-trade are now equal to $\frac{sp_H}{p_F}$. From (15) and market-clearing conditions, we get that relative total sales are equal to $(1-\lambda)(\omega\widehat{s} + (1-\omega)\widehat{q^*})$.

Due to price rigidities, output is (partly) demand determined and aggregate mark-ups will change in both countries: changes in mark-up are formally equivalent to changes in the profit share δ which will potentially lead to some home bias in equities. The intuition is clearly explained in Engel and Matsumoto (2006): due to price stickiness, a good productivity shock at Home leads a firm that cannot readjust prices to reduce its labor demand, reducing consequently labor revenues and increasing the profit share. Since Home workers want to hedge fluctuations in their labor incomes, they would rather hold Home equities.

One could solve the model to see how changes in productivity affect the profit share δ and relative flexible prices $\widehat{q^*}$. This step is however not necessary. To express portfolios, we just need to rewrite asset returns and labor incomes as a function the Home terms-of-trade $\widehat{q} = (\omega\widehat{s} + (1-\omega)\widehat{q^*})$ and changes in the profit share $\widehat{\delta}$. One can rewrite the system of equations as follows:

$$\begin{cases} \hat{R}_e &= (1-\lambda)\hat{q} + \hat{\delta} \\ \hat{w} &= (1-\lambda)\hat{q} - \frac{\delta}{1-\delta}\hat{\delta} \\ \hat{R}_b &= \hat{s} = \frac{1}{\omega}\left(\hat{q} + \frac{1-\omega}{\omega}\frac{\delta}{1-\delta}\hat{\delta}\right) \end{cases} \tag{67}$$

For $\omega > 0$, this last system allow us to express equilibrium portfolios as in section 2.[51] This leads to the following equilibrium portfolios:

$$S = \frac{1}{2}\left[1 - \frac{(1-\delta)(\lambda-1)(1-\omega)-\omega}{\delta(\lambda-1)(1-\omega)+(1-\delta)\omega} - \frac{(1-1/\sigma)(2a-1)(1-\omega)}{\delta(\lambda-1)(1-\omega)+(1-\delta)\omega}\right] \tag{68}$$

$$b = \frac{1}{2}\omega\left[(1-1/\sigma)(2a-1) + (2S-1)(\lambda-1)\right] \tag{69}$$

[50] The elasticity of substitution between varieties within a given country is equal to the inverse of the profit share δ.

[51] One should note here that the problem is slightly different from section 2 because nominal bonds loads on the nominal exchange rate and not the real exchange rate $\widehat{q} = \omega\widehat{s} + (1-\omega)\widehat{q^*}$

When, $\omega = 1$, all firms have preset prices: $S = 1$ and $b = \frac{1}{2}(2a-1)(1-\frac{1}{\sigma}) + \frac{1}{2}(\lambda - 1)$. The model is isomorphic to the one with redistributive shocks (where $\widehat{\varepsilon} = \widehat{\delta}$); indeed in that case bonds load perfectly on the real exchange rate.

When ω is close to zero, we get close to a flex-price model with nominal shocks. In this case, bond positions converge towards zero as investors want to minimize their exposure towards nominal risk.

In the extreme case of $\omega = 0$, nominal bonds do not load on the RER due to inflation risk but equities do. Then, the model is equivalent to our reduced form model of section 2 with $\gamma_e = \gamma_w = 0$ and $\gamma_b \neq 0$. In that case, the portfolio satisfies:

$$b = 0 \; ; \; S = \frac{1}{2}\left[\frac{2\delta - 1}{\delta} - \frac{\left(1-\frac{1}{\sigma}\right)(2a-1)}{\delta\,(\lambda - 1)}\right] \tag{70}$$

In particular, as might have been expected, the bond return differential fails to load on the real exchange because of the inflation differentials across countries. This additional source of risk on bond returns shifts bond position towards zero (to hedge inflation risk). It then remains for equities to hedge real exchange rate exposure efficiently. While potentially restoring the difficulties of previous literature, we can safely argue that this example is not relevant empirically. Indeed, it would contradict the empirical evidence provided by van Wincoop and Warnock (2006) (and confirmed in our empirical section; see section 5) as it would imply the correlation between equity returns and the real exchange rate, *conditional on nominal bond returns* ($\beta_{RER,e}$) is non-zero, while the correlation between bond returns and the real exchange rate, *conditional on equity returns* ($\beta_{RER,b}$) should be close to zero.

6.2.2 The case of changes in quality/preference shocks

We follow Coeurdacier et al. (2007) by adding preference shocks to the utility provided by Home goods and Foreign goods to the consumers of both countries. In that case, the aggregate consumption index C_i, for $i = H, F$, is now given by:

$$C_i = \left[a^{1/\phi}\left(\Psi_i c_{ii}\right)^{(\phi-1)/\phi} + (1-a)^{1/\phi}\left(\Psi_j c_{ij}\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \tag{71}$$

where Ψ_i, $i = H, F$ with $E_0(\Psi_i) = 1$ is an exogenous worldwide shocks to the (relative) preference for the country i good. Note that the shock Ψ_i can also have a more supply oriented interpretation, as a shock to the quality of good i. We denote $\Psi \equiv \frac{\Psi_H}{\Psi_F}$ the relative preference shocks and $\widehat{\Psi}$ its deviation from its steady-state value of one.

As shown by Coeurdacier et al. (2007), the welfare-based real exchange rate in this case is equal to (up to the first-order):

$$\widehat{RER} = (2a-1)\left(\frac{\widehat{p_H}}{p_F} - \frac{\widehat{\psi_H}}{\psi_F}\right) = (2a-1)\,\widehat{q} \tag{72}$$

where we adjust the terms-of-trade for quality/preference shocks by scaling q as follows: $q = \frac{p_H/\psi_H}{p_F/\psi_F}$.

As shown by Coeurdacier et al. (2007), under the assumption of (locally) complete markets, intratemporal allocation across goods imply:

$$\widehat{\psi y} = -\lambda \widehat{q} \tag{73}$$

where $\widehat{\psi y}$ are relative endowments adjusted for quality/preference shocks.

Relative equity returns and relative non-financial incomes still load perfectly on the real exchange rate adjusted for the quality/preference shocks (see also Coeurdacier et al. (2007)).

$$\widehat{R_e} = (1-\lambda)\widehat{q} \tag{74}$$
$$\widehat{w} = (1-\lambda)\widehat{q} \tag{75}$$

We assume that real bonds in each country pays one unit of the good **not** adjusted for quality:

$$\widehat{R_b} = (2a-1)\frac{\widehat{p_H}}{p_F} = (2a-1)\left(\widehat{q}+\widehat{\psi}\right)$$

Then, introducing $\widehat{\varepsilon} = (2a-1)\,\widehat{\psi}$, we are back to our reduced form model (with $\gamma_w = \gamma_e = 0$) and equity and bond portfolios will be the same as in (70)). Bond returns differential fails to load on the welfare-based real exchange rate because of relative change in quality/preference shocks $\widehat{\psi}$. This additional source of risk on bond returns shift bond position towards zero and risk-sharing is done by equities only.

7 Conclusion

[To be written]

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(percent)	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Euro	Average
δ	30.9	29.4	28.1	38.8	37.2	32.0	27.7	30.8	32.0
naïve-δ	39.9	39.4	40.8	50.9	43.4	36.7	38.2	42.7	41.3

Table 1: Estimates of the share of financial income in GDP, δ. Source: OECD Quarterly National Income and U.N. National Account Statistics. Authors' calculations.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Euro
$\hat{\beta}_{RER,b}$	1.00***	0.96***	0.99***	0.96***	1.00***	0.93***	0.98***	0.99***
	(0.02)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)
$\hat{\beta}_{RER,e}$	-0.01	0.00	-0.00	0.01**	0.00	0.03***	0.01	0.00
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)
R^2	0.97	0.98	0.98	0.96	0.97	0.93	0.98	0.99
#obs.	154	154	154	154	154	154	154	154

Table 2: Loadings on real exchange rate changes: $\xi_{it} = \beta^i_{RER,b} r_{b,it} + \beta^i_{RER,e} r_{e,it} + u_{it}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Constants are not reported. 1970:2 to 2008:3.

	Canada	France	Germany	Italy	Japan	U.K.	U.S.	Euro
$\hat{\beta}_{n,b}$	1.05***	0.79***	1.12***	1.03***	1.04***	1.04***	1.17***	1.13***
	(0.08)	(0.06)	(0.07)	(0.08)	(0.16)	(0.07)	(0.11)	(0.07)
$\hat{\beta}_{n,e}$	0.01	0.12***	0.11***	-0.01	-0.33***	-0.05*	-0.26***	0.05
	(0.04)	(0.03)	(0.04)	(0.03)	(0.07)	(0.03)	(0.07)	(0.04)
R^2	0.63	0.68	0.70	0.58	0.26	0.68	0.46	0.69
#obs.	136	136	136	136	136	136	136	136
$\hat{\beta}_{n,e}$, unc.	0.27***	0.26***	0.31***	0.09**	-0.06	0.17***	0.10	0.31***
	(0.05)	(0.03)	(0.06)	(0.04)	(0.07)	(0.04)	(0.08)	(0.06)
R^2	0.16	0.30	0.17	0.04	0.00	0.10	0.01	0.15

Table 3: Loadings on returns to nonfinancial wealth: $r_{n,it} = \beta^i_{n,b} r_{b,it} + \beta^i_{n,e} r_{e,it} + v_{it}$. Standard errors are in parenthesis. (***) (resp (**)) indicates significance at the 1% level (resp. 5%). Constants are not reported. 1970:3 to 2004:2.

		Canada	France	Germany	Italy	Japan	U.K.	U.S.	Euro
Output share	(ω_i)	4.18%	7.71%	10.63%	7.46%	16.64%	7.51%	45.88%	25.80%
Implied Equity Home Bias:									
$-\frac{1-\delta}{\delta}\beta_{n,e}(1-\omega_i)$		-2.1%	-26.6%	-25.2%	1.6%	**46.4%**	**9.8%**	**36.7%**	-7.9%
Implied Bond Positions:									
	$\sigma=1$	-36.3%	-27.9%	-40.3%	-31.5%	-32.7%	-35.4%	-42.3%	-38.4%
$\frac{1}{2}\left(1-\frac{1}{\sigma}\right)\beta_{RER,b}$	$\sigma=2$	-11.3%	-3.9%	-15.5%	-7.5%	-7.7%	-12.1%	-17.8%	-13.7
$-\frac{1}{2}(1-\delta)\beta_{w,b}$	$\sigma=4$	1.2%	8.1%	-3.1%	4.5%	**4.8%**	-0.5%	**-5.5%**	-1.3%

Table 4: Implied equity bias $(S_i^* - \omega_i)$ and bond positions b_i^* for each country i. Positions are calculated under the assumption that $\beta_{RER,e} = 0$. Calculations are done using the observed share of financial income δ of each country and for $\sigma = 1, 2, 4$.

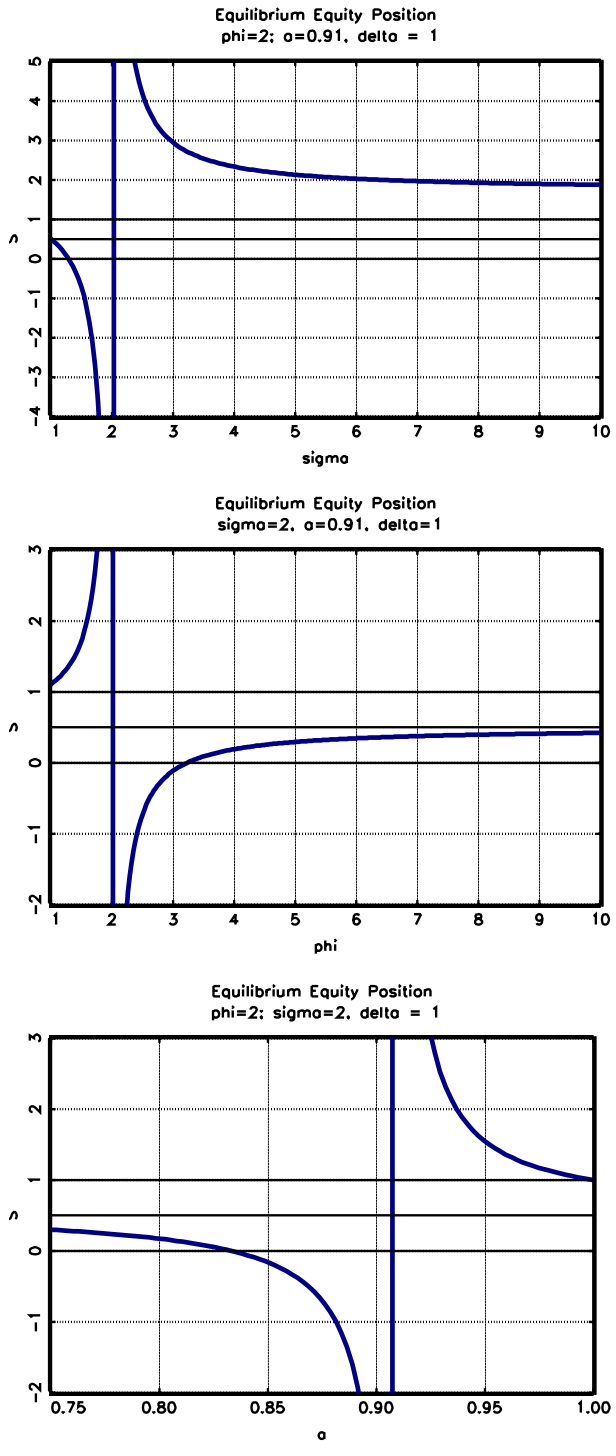

Figure 1: The *instability* of optimal equity position as a function of preference parameters.

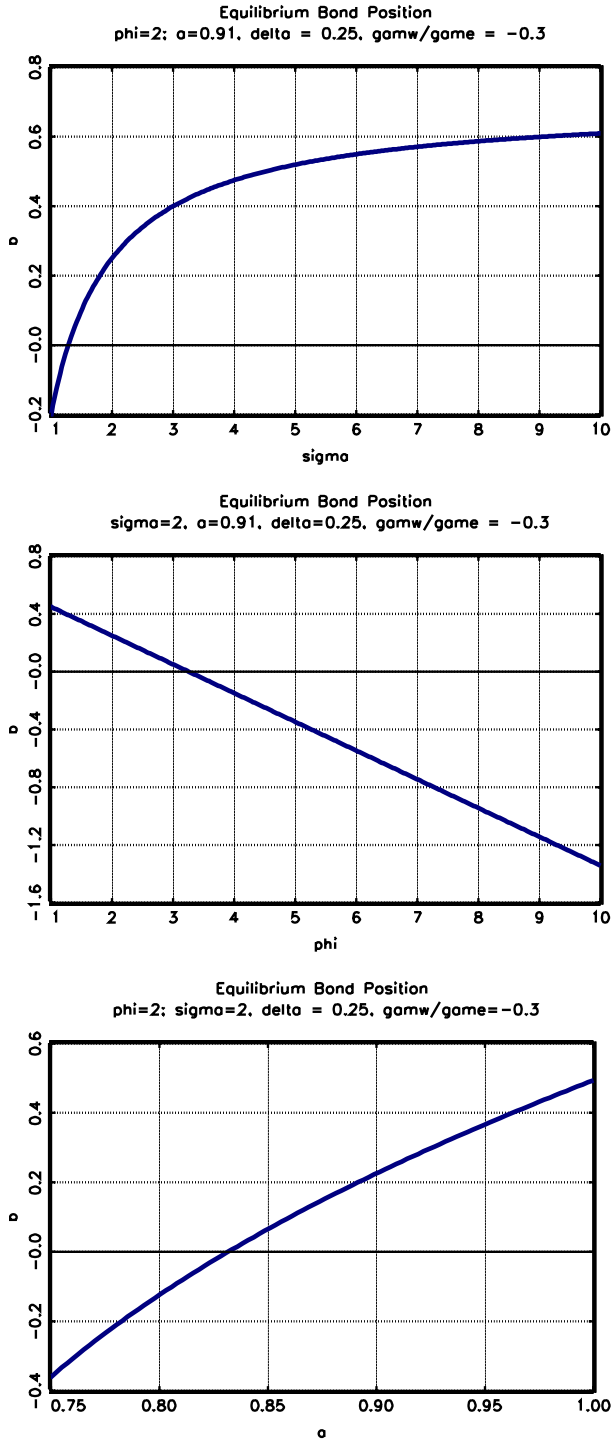

Figure 2: The *stability* of optimal bond positions as a function of preference parameters.

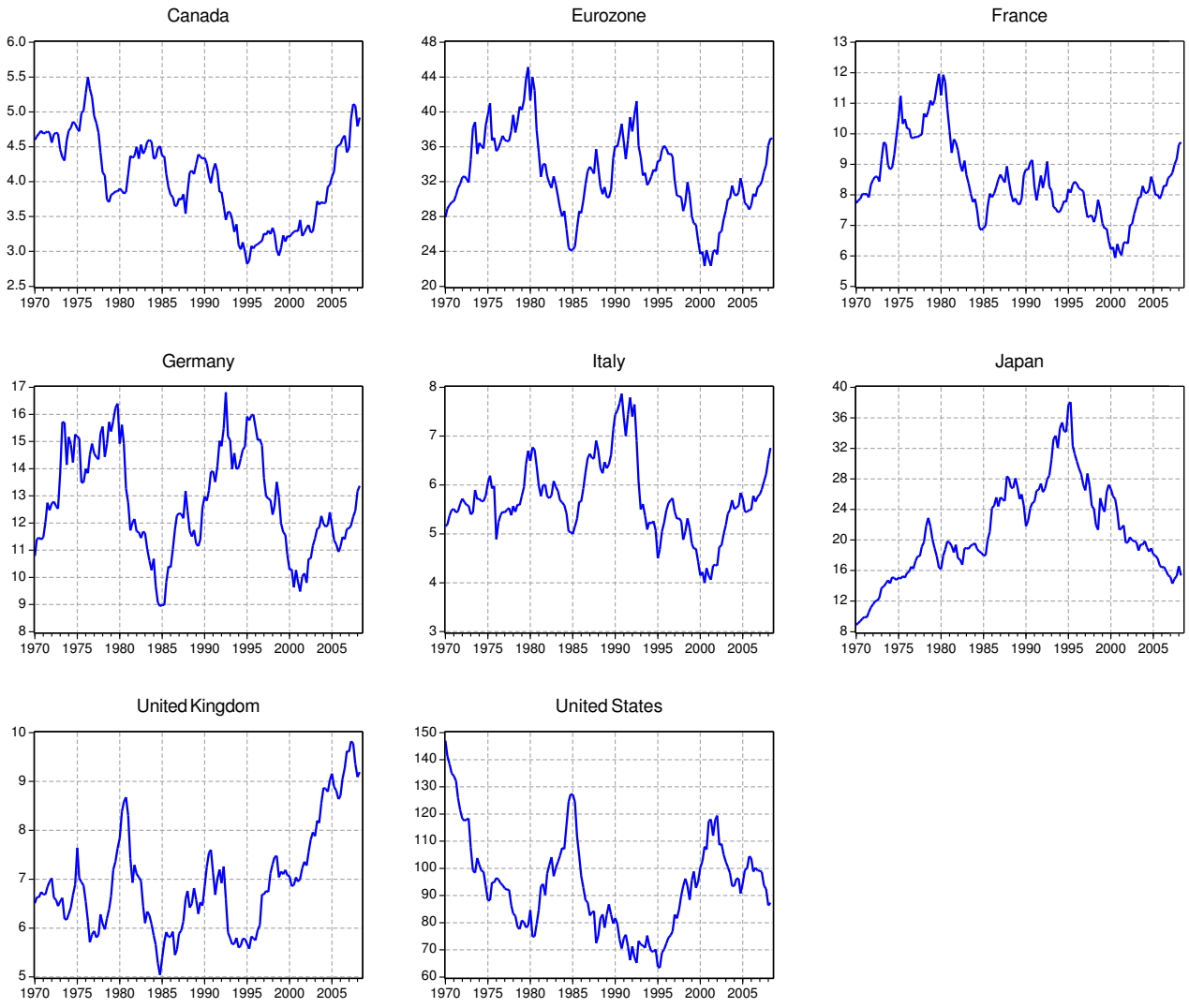

Figure 3: Relative nonfinancial income (percent), G7 countries (Eurozone represents France, Germany and Italy), 1970:1-2008:3. Data Sources: OECD Quarterly National Accounts and UN National Account Statistics. Authors calculations.

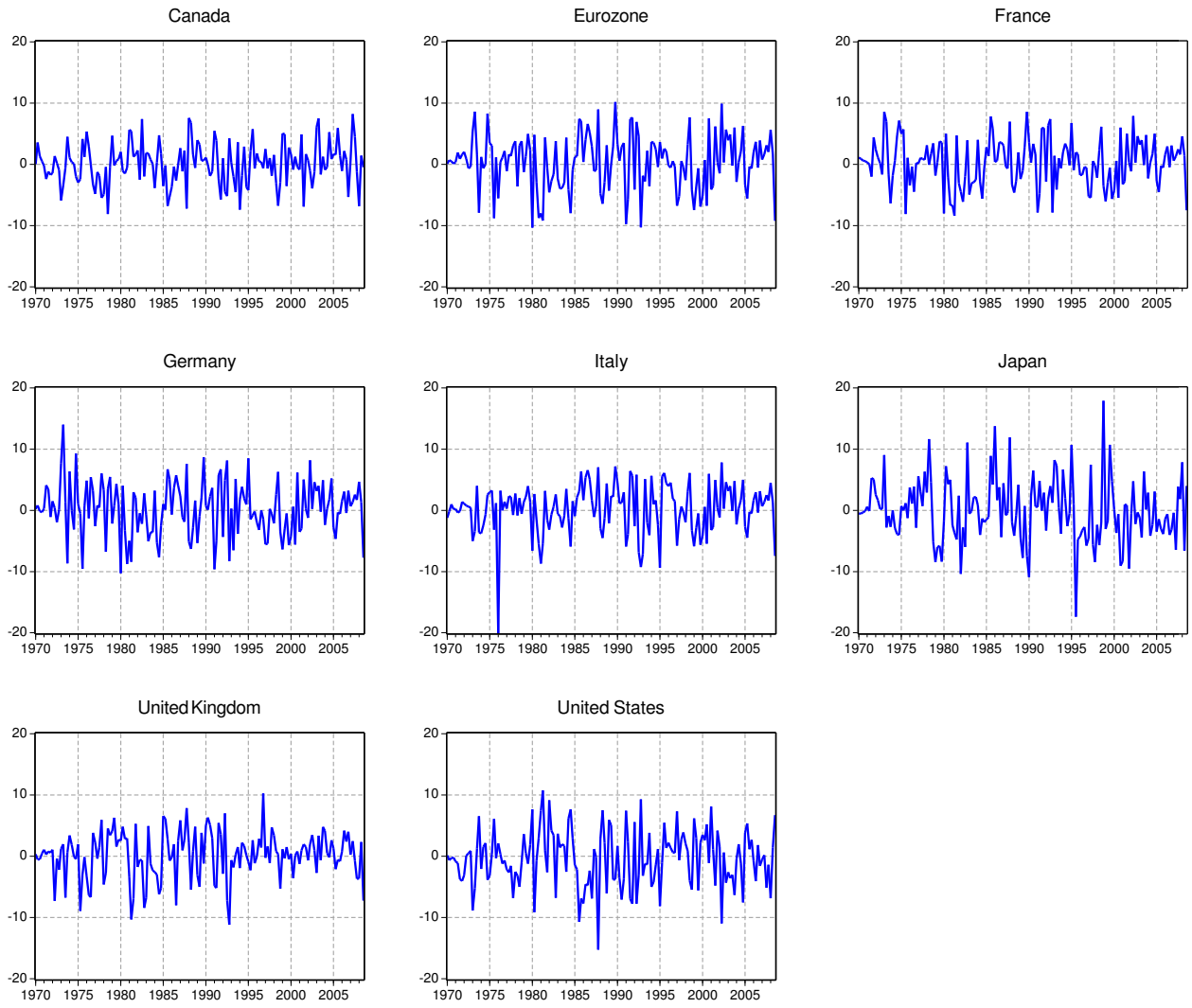

Figure 4: Relative Bond returns (percent), G7 countries (Eurozone represents France, Germany and Italy), 1970:1-2008:3. Data Sources: Global Financial Database. Authors calculations.

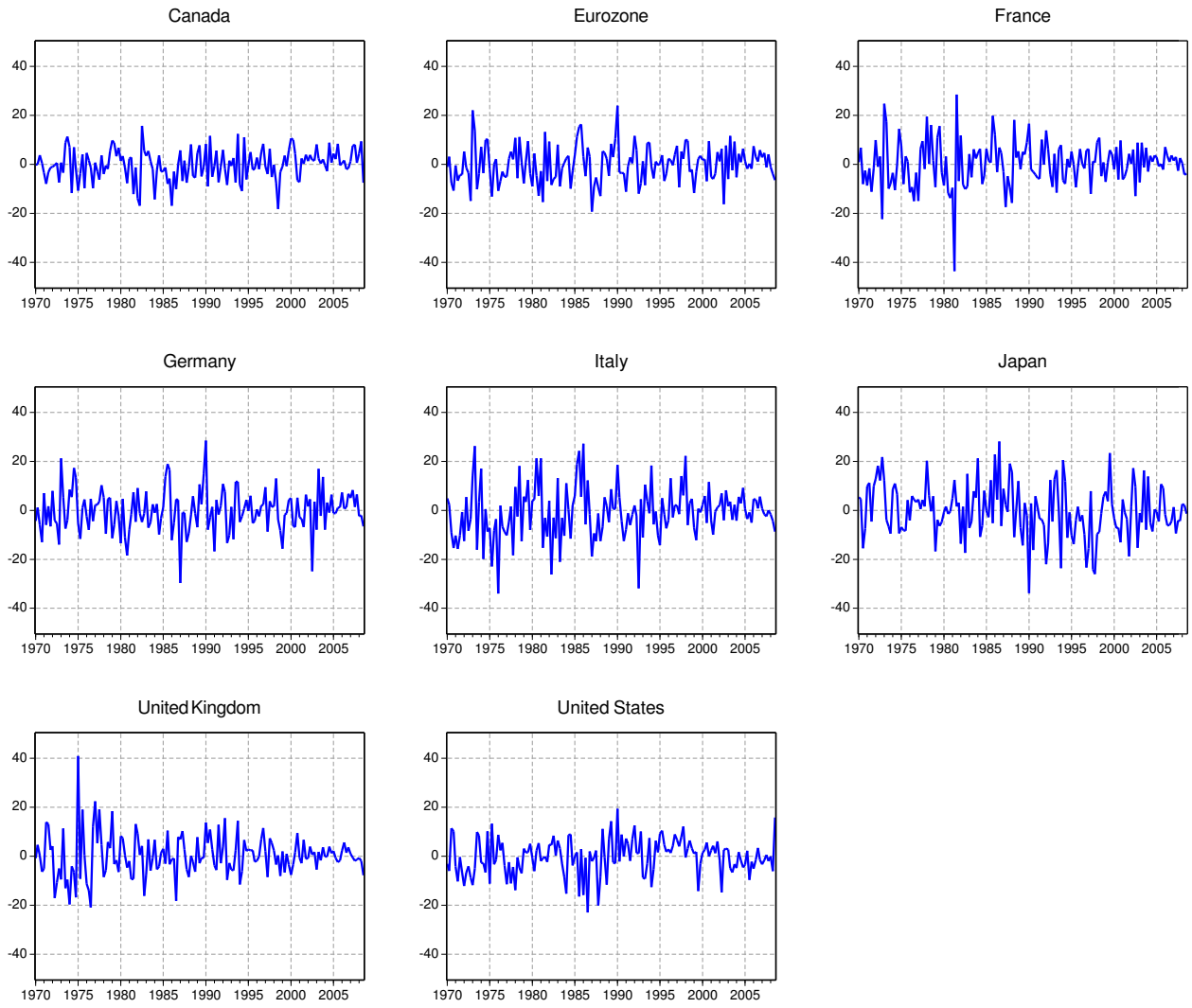

Figure 5: Relative Equity returns (percent), G7 countries (Eurozone represents France, Germany and Italy), 1970:1-2008:3. Data Sources: Global Financial Database. Authors calculations.

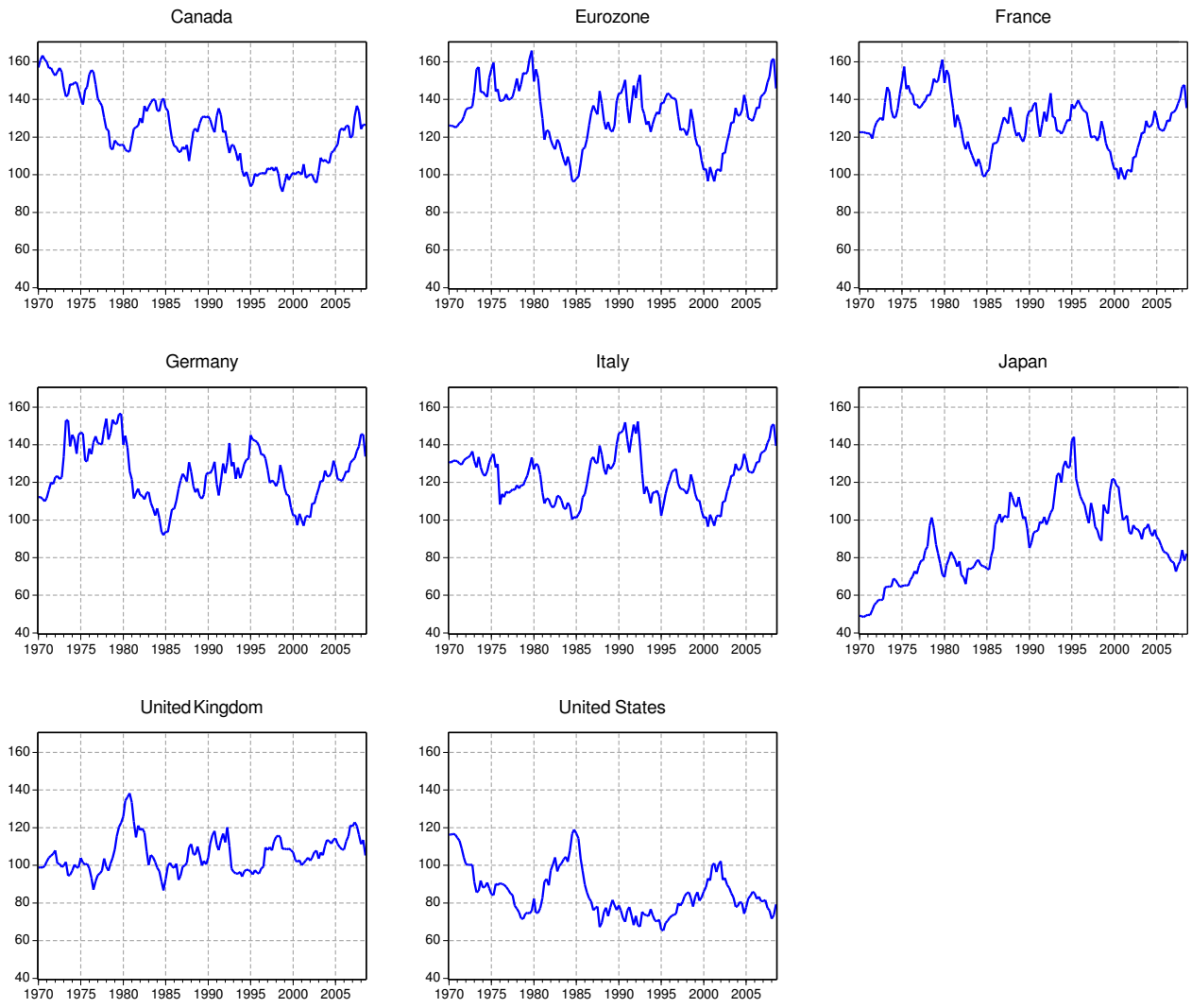

Figure 6: Real Exchange Rate, 100 in 2001:1, G7 countries (Eurozone represents France, Germany and Italy), 1970:1-2008:3. Data Sources: Global Financial Database and OECD Main Economic Indicators. Authors calculations.

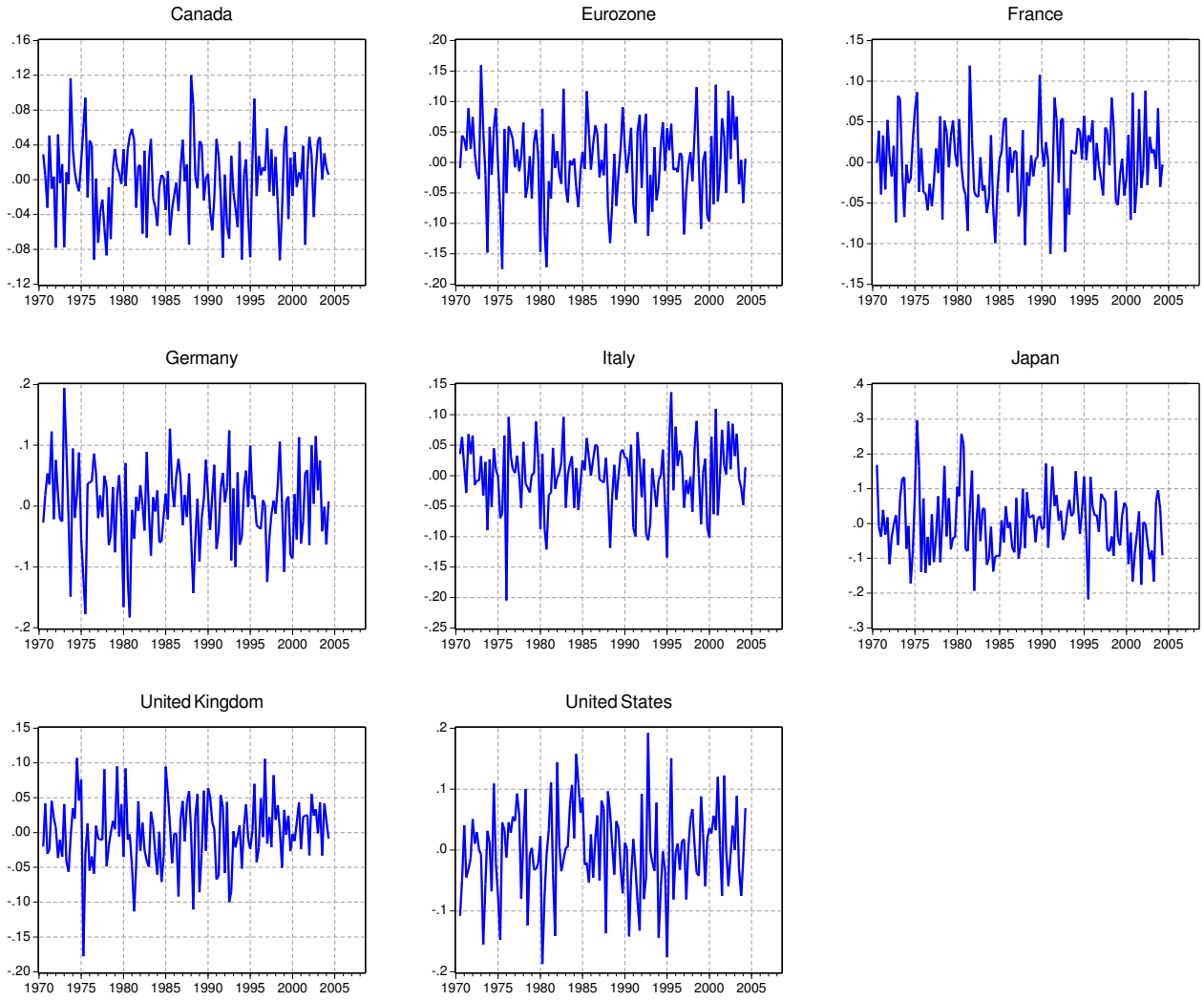

Figure 7: Innovations to the return on Nonfinancial Wealth, G7 countries (Eurozone represents France, Germany and Italy), 1970:1-2004:2.

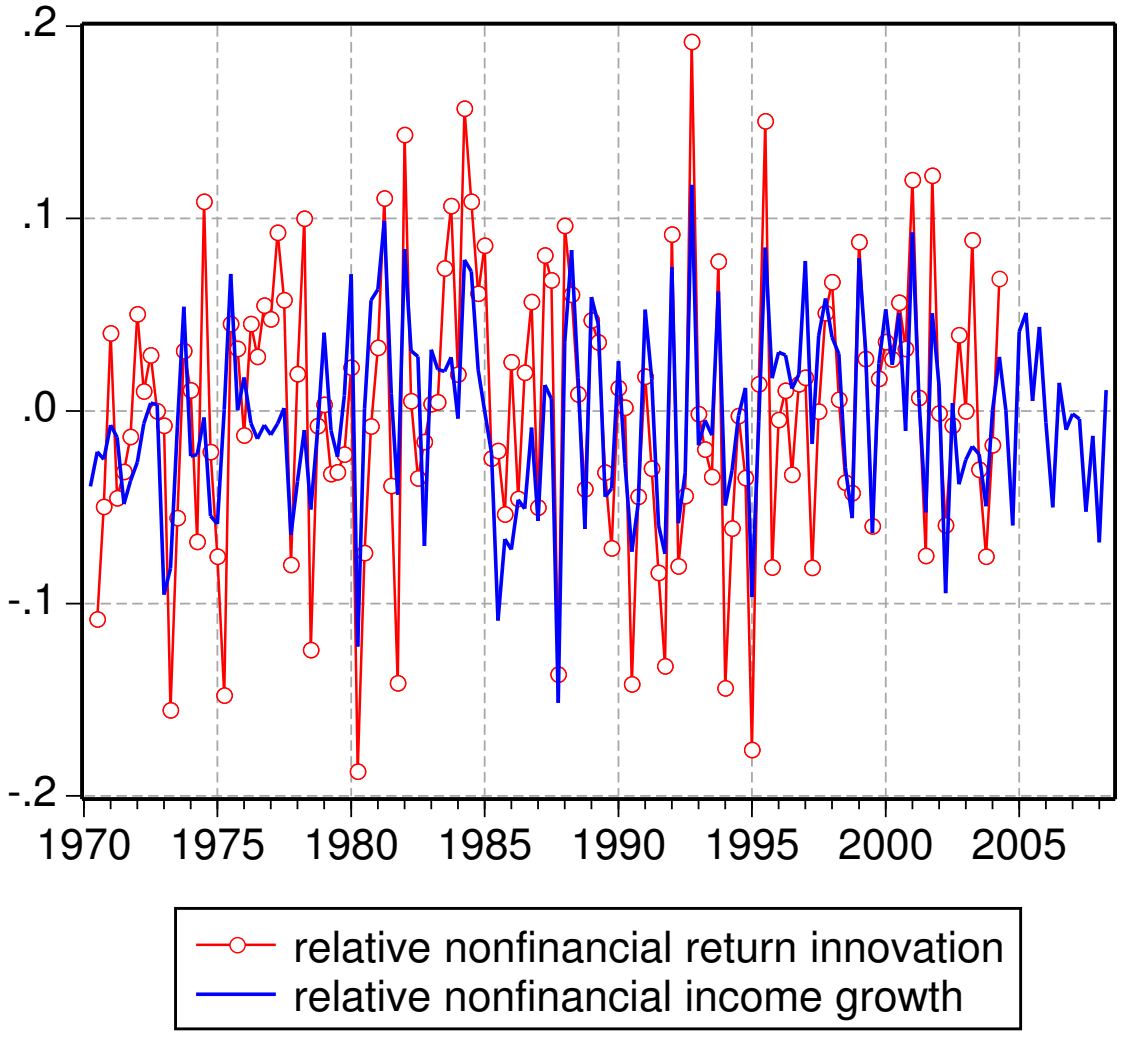

Figure 8: Returns on relative nonfinancial wealth, and relative nonfinancial income growth, U.S., 1970:1-2004:2.

A Appendix

A.1 Optimal portfolios with locally complete markets

We apply Devereux and Sutherland (2006) (see also Tille and van Wincoop (2007)) to characterize the equilibrium portfolio. We show that the zero-order (or static) portfolio is the one that locally replicates the efficient consumption allocation. The Devereux and Sutherland (2006) approach finds a portfolio that is consistent with 1) First-order approximation of non-portfolio equations (here intratemporal allocation across Home and Foreign goods and budget constraints in both countries) and 2) Second-order approximation of the Euler equations.

There are two non-portfolio equations. The first is the optimal intratemporal condition for the allocation of consumption across goods. In relative form this is (9). The log-linear first-order approximation is:

$$\widehat{y} = \left[-\phi + (2a-1)^2(\phi-1)\right]\widehat{q} + (2a-1)\widehat{PC} \tag{A.1}$$

where $\widehat{PC}$ denotes relative consumption expenditures $(\widehat{P_H C_H} - \widehat{P_F C_F})$.

The second non-portfolio equation is the budget constraint. The log-linear first order approximation is:

$$\widehat{P_H C_H} - \widehat{P_F C_F} = (1-\delta)\hat{R}_n + \delta\,(2S-1)\,\hat{R}_e + 2b\hat{R}_b \tag{A.2}$$

where $\hat{R}_n$ denotes the return on nonfinancial wealth, $\hat{R}_e$ denotes equity excess returns and $\hat{R}_b$ excess bond returns.

The Euler equations for equity holdings in country $i = H, F$ is:

$$\lambda_{i,0} = E_0[\frac{C_i^{-\sigma}}{P_i} R_{jH}] \; ; \; \lambda_{i,0} = E_0[\frac{C_i^{-\sigma}}{P_i} R_{jF}] \tag{A.3}$$

where $\lambda_{i,0}$ denotes the Lagrange-multiplier of the budget constraint in period $t = 0$ in country $i = H, F$ and $j = e, b$. In relative terms across countries:

$$E_0\left[\left(\frac{C_H^{-\sigma}}{P_H} - \frac{C_F^{-\sigma}}{P_F}\right) R_j\right] = 0 \tag{A.4}$$

The second-order approximation of equation (A.4) yields:

$$cov(\widehat{PC}, \hat{R}_j) = (1 - 1/\sigma) cov(\widehat{RER}, \hat{R}_j); \text{ for } j = e, b \tag{A.5}$$

The optimal first-order portfolio is the pair (S^*, b^*) such that the first order non-portfolio conditions (A.1) and (A.2) and the second-order portfolio conditions (A.5) are satisfied for all realizations of the shocks. It is immediate that a portfolio $(S^*; b^*)$ such that $\widehat{PC} = (1-1/\sigma)\widehat{RER} = (1-1/\sigma)(2a-1)\widehat{q}$ satisfies the two (second-order) Euler equation approximations. Let us a assume that it is possible to find such a portfolio. Then, this is the same thing as saying that relative consumption expenditures are linked to the real exchange by the expression (13) or equivalently that markets are (locally) complete.

If such a portfolio exists, it must also satisfy the first-order non-portfolio equations (A.1) and (A.2). These can be rewritten (see the equivalent expressions (15) and (17) in the benchmark model):

$$\begin{aligned} \hat{y} &= -\lambda \widehat{q} && \text{(A.6)} \\ (1-1/\sigma)(2a-1)\widehat{q} &= (1-\delta)\hat{R}_n + \delta\,(2S-1)\,\hat{R}_e + 2b\hat{R}_b && \text{(A.7)} \end{aligned}$$

The portfolio choice only affects equation (A.6) because of its impact on equity and bond excess returns (through its impact on $\widehat{q}$); so as long as asset returns are consistent with equation (A.6), then the first-order approximation of (A.1) is verified. The key question is wether one can verify (A.7) in all states of nature. Because we have two instruments (S and b), we must have at most two sources of risk. This is the **Spanning Condition**. Call $\widehat{\varepsilon}_1$ and $\widehat{\varepsilon}_2$ the two innovations (expressed in relative terms) arising from these two sources of risk and assume that our four endogenous variables $\{\widehat{q}; \hat{R}_n; \widehat{R}_e; \widehat{R}_b\}$ are driven by $\widehat{\varepsilon}_1$ and $\widehat{\varepsilon}_2$ according to the following expression in matrix form (where we assume that Home bond excess returns load perfectly on the real exchange rate, as in our benchmark case):[52]

$$\begin{pmatrix} \widehat{q} \\ \hat{R}_n \\ \widehat{R}_e \\ \widehat{R}_b \end{pmatrix} = \begin{pmatrix} a_{1,q} & a_{2,q} \\ a_{1,n} & a_{2,n} \\ a_{1,R} & a_{2,R} \\ (2a-1)a_{1,q} & (2a-1)a_{2,q} \end{pmatrix} \begin{pmatrix} \widehat{\varepsilon}_1 \\ \widehat{\varepsilon}_2 \end{pmatrix}$$

Then, (A.7) is verified for all possible realizations of the shocks if and only if the following equality holds in matrix form (obtained from projections on the set of shocks $(\widehat{\varepsilon}_1; \widehat{\varepsilon}_2)$):

$$(1-1/\sigma)(2a-1)\begin{pmatrix} a_{1,q} \\ a_{2,q} \end{pmatrix} = (1-\delta)\begin{pmatrix} a_{1,n} \\ a_{2,n} \end{pmatrix} + \underbrace{\begin{pmatrix} a_{1,R} & a_{1,q} \\ a_{2,R} & a_{2,q} \end{pmatrix}}_{M} \begin{pmatrix} \delta\,(2S-1) \\ 2b(2a-1) \end{pmatrix}$$

The second condition for the portfolio to be unique and determined is that $\det(M) = \left(a_R^1 a_q^2 - a_R^2 a_q^1\right) \neq 0$. This is the **Rank Condition**. This is equivalent to assuming that Home excess equity and Home bond excess returns are not perfectly correlated. In that case, the equilibrium portfolio $(S^*; b^*)$ is unique and determined as follows:

$$\begin{pmatrix} \delta\,(2S^*-1) \\ 2b^*(2a-1) \end{pmatrix} = \begin{pmatrix} a_{1,R} & a_{1,q} \\ a_{2,R} & a_{2,q} \end{pmatrix}^{-1} \begin{pmatrix} [(1-1/\sigma)(2a-1)]\,a_{1,q} - (1-\delta)a_{1,n} \\ [(1-1/\sigma)(2a-1)]\,a_{2,q} - (1-\delta)a_{2,n} \end{pmatrix}$$

One can rewrite the same proof by changing the basis of shocks as in our examples by using a projection on $\widehat{q}$ and $\widehat{\varepsilon} = \widehat{\varepsilon}_2$ (providing that $a_{1,q} \neq 0$, i.e. that $\hat{q}$ and $\widehat{\varepsilon}$ are not collinear); We obtain the results of section 3 with an evident change of notation:

[52] We do not need additional assumptions on the stochastic properties except that they are not perfectly correlated.

$$\begin{pmatrix} \widehat{q} \\ \hat{R}_n \\ \hat{R}_e \\ \hat{R}_b \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ (1-\bar{\lambda}) & \gamma_n \\ (1-\bar{\lambda}) & \gamma_e \\ (2a-1) & \gamma_b \end{pmatrix} \begin{pmatrix} \widehat{q} \\ \widehat{\varepsilon} \end{pmatrix}$$

and the optimal portfolio satisfies:

$$\begin{pmatrix} \delta\left(2S^*-1\right) \\ 2b^*\left(2a-1\right) \end{pmatrix} = \begin{pmatrix} -\frac{\gamma_b}{\gamma_e}\frac{(2a-1)\left(1-\frac{1}{\sigma}\right)+(1-\delta)\left(1-\bar{\lambda}\right)(\gamma_n/\gamma_e-1)}{(2a-1)-\gamma_b/\gamma_e\left(1-\bar{\lambda}\right)} - \frac{\gamma_n}{\gamma_e}(1-\delta) \\ \frac{(2a-1)\left(1-\frac{1}{\sigma}\right)+(1-\delta)\left(1-\bar{\lambda}\right)(\gamma_n/\gamma_e-1)}{1-\gamma_b/\gamma_e\left(1-\bar{\lambda}\right)(2a-1)^{-1}} \end{pmatrix}$$

where the **rank condition** takes the form: $(2a-1)\,\gamma_e \neq \gamma_b\left(1-\bar{\lambda}\right)$.

A.2 Optimal portfolios with incomplete markets

As above, we use the Devereux and Sutherland (2006) approach to characterize the optimal equity and bond positions. To do so, we use the first-order approximations of the non-portfolio equations (see reduced-form of the model below) and the second order approximation of the Euler equations. This pins down a unique equilibrium portfolio.

Non portfolio equations:

- Intratemporal allocation across goods:

$$\widehat{y} = -\phi\widehat{q} + (2a-1)[\widehat{P_H C_H} - \widehat{P_F C_F} - (1-\phi)R\hat{E}R]$$

- Budget constraint:

$$\widehat{P_H C_H} - \widehat{P_F C_F} = (1-\delta)\hat{R}_n + \delta\left(2S-1\right)\hat{R}_e + 2b\hat{R}_b$$

Relative returns on equities, bonds and nonfinancial wealth are expressed as follows:

$$\begin{cases} \hat{R}_e &= \widehat{q} + \widehat{y} + \boldsymbol{\gamma}_e'\hat{\boldsymbol{\varepsilon}} \\ \hat{R}_b &= (2a-1)\widehat{q} + \widehat{y} + \boldsymbol{\gamma}_b'\hat{\boldsymbol{\varepsilon}} \\ \hat{R}_n &= \widehat{q} + \widehat{y} + \boldsymbol{\gamma}_n'\hat{\boldsymbol{\varepsilon}} \end{cases},$$

where $\hat{\boldsymbol{\varepsilon}}$ is a N-dimensional vector of shocks and $\boldsymbol{\gamma}_i$ for $i = \{b, e, n\}$ is a $N \times 1$ vector that controls the impact of $\hat{\boldsymbol{\varepsilon}}$ on assets and non-financial wealth.

Portfolio equations: due to symmetry, we can write Euler equations in relative terms as follows for asset $i = \{e, b\}$:

$$E_0(mR_i) = 0 \text{ for } i = \{e, b\} \tag{A.8}$$

where m is the difference between stochastic discount factor across countries: $m = C_H^{-\sigma}/P_H - C_F^{-\sigma}/P_F$.

Using the budget constraint, the intratemporal condition can be rewritten as follows, where we introduce portfolio excess returns $\hat{\xi} = \delta\,(2S-1)\,\hat{R}_e + 2b\hat{R}_b = \begin{pmatrix} \delta\,(2S-1) & 2b \end{pmatrix}\begin{pmatrix}\hat{R}_e \\ \hat{R}_b\end{pmatrix}$:

$$\widehat{q} = q_y\widehat{y} + \mathbf{q}'_\varepsilon\hat{\varepsilon} + q_\xi\hat{\xi}$$

where $q_y = \frac{[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}[(2a-1)(1-\delta)-1]}{1-[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}$, $\mathbf{q}'_\varepsilon = \frac{[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}{1-[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}\gamma'_n$ and $q_\xi = \frac{[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}{1-[\phi\left(1-(2a-1)^2\right)+(2a-1)^2]^{-1}(2a-1)(1-\delta)}$

If we rewrite the reduced form model using Devereux and Sutherland (2006) notations, we get the following expression for the vector excess returns:

$$\begin{pmatrix}\hat{R}_e \\ \hat{R}_b\end{pmatrix} = \mathbb{R}_1\hat{\xi} + \mathbb{R}_2\begin{pmatrix}\widehat{y} \\ \hat{\boldsymbol{\varepsilon}}\end{pmatrix}$$

where $\mathbb{R}_2 = \begin{pmatrix} 1+q_y & \mathbf{q}'_\varepsilon + \boldsymbol{\gamma}'_e \\ (2a-1)q_y & (2a-1)\mathbf{q}'_\varepsilon + \gamma'_b \end{pmatrix}$ and $\mathbb{R}_1 = \begin{pmatrix} q_\xi \\ (2a-1)q_\xi \end{pmatrix}$

The first-order approximation of the difference between stochastic discount factor across countries gives:

$$\hat{m} = D_1\hat{\xi} + \mathbf{D}_2\begin{pmatrix}\widehat{y} \\ \hat{\boldsymbol{\varepsilon}}\end{pmatrix}$$

where D_1 is a scalar, $D_1 = 1 + [(1-\delta) + (2a-1)(1/\sigma - 1)]q_\xi$
and $\mathbf{D}_2 = \begin{pmatrix} (1-\delta)\,(1+q_y) + (2a-1)(1/\sigma-1)q_y & (1-\delta) + (2a-1)(1/\sigma-1)\mathbf{q}'_\varepsilon + (1-\delta)\gamma'_n \end{pmatrix}$ is a $1 \times N+1$ vector.

Following DS, we define $\tilde{\mathbb{R}}_2 = \mathbb{R}_1\tilde{\mathbf{H}} + \mathbb{R}_2$ and $\tilde{\mathbf{D}}_2 = D_1\tilde{\mathbf{H}} + \mathbf{D}_2$
with $\tilde{\mathbf{H}} = (\mathbf{1} - \begin{pmatrix} \delta\,(2S-1) & 2b \end{pmatrix}\mathbb{R}_1)^{-1}\begin{pmatrix} \delta\,(2S-1) & 2b \end{pmatrix}\mathbb{R}_2$

Then using the second-order approximation of the Euler equation, we get the following quadratic equation:

$$\tilde{\mathbb{R}}_2\Sigma\tilde{\mathbf{D}}'_2 = 0$$

where Σ is the (N+1) x (N+1) variance-covariance matrix of the vector of innovations $\begin{pmatrix}\widehat{y} \\ \hat{\boldsymbol{\varepsilon}}\end{pmatrix}$.

Rearranging terms, this equation simplifies into the following expression for portfolios:

$$\begin{pmatrix} \delta\,(2S-1) \\ 2b \end{pmatrix} = \left(\mathbb{R}_2\Sigma\mathbf{D}'_2\mathbb{R}'_1 - D_1\mathbb{R}_2\Sigma\mathbb{R}'_2\right)^{-1}\mathbb{R}_2\Sigma\mathbf{D}'_2$$

where we assume that the 2×2 matrix $[\mathbb{R}_2\Sigma\mathbf{D}'_2\mathbb{R}'_1 - D_1\mathbb{R}_2\Sigma\mathbb{R}'_2]$ is invertible (**Rank condition**). When this rank condition is satisfied, the equilibrium portfolio is unique and bond and equity excess returns are not collinear. Thus, there exists a unique decomposition such that:

$$\begin{aligned} \hat{RER} &\equiv \beta_{RER,b}\hat{R}_b + \beta_{RER,e}\hat{R}_e + u_{RER} \\ \hat{R}_n &\equiv \beta_{n,b}\hat{R}_b + \beta_{n,e}\hat{R}_e + u_n \end{aligned}$$

where u_i for $i = \{RER, w\}$ is orthogonal to $\hat{R}_j$ for $j = \{b, e\}$: $E\left[u_i \hat{R}_j\right] = 0$

A.3 Detailed derivation for the model with non-tradable goods

A.3.1 Set-up

The aggregate consumption index C_i, for $i = H, F$ is given by:

$$C_i = \left[\eta^{1/\theta} \left(c_i^T\right)^{(\theta-1)/\theta} + (1-\eta)^{1/\theta} \left(c_i^{NT}\right)^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)} \tag{A.9}$$

where c_i^T is the consumption of a composite tradable goods using Home and Foreign tradable goods and c_i^{NT} is the consumption of non-tradable goods. θ is the elasticity of substitution between tradable and non-tradable goods.

Consumption of the tradable good is defined as in the benchmark model:

$$c_i^T = \left[a^{1/\phi} \left(c_{ii}^T\right)^{(\phi-1)/\phi} + (1-a)^{1/\phi} \left(c_{ij}^T\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \tag{A.10}$$

where c_{ij}^T is country i's consumption of the tradable good from country j.

The consumer price index that correspond to these preferences is for $i = H, F$:

$$P_i = \left[\eta \left(P_i^T\right)^{(1-\theta)} + (1-\eta) \left(P_i^{NT}\right)^{1-\theta}\right]^{1/(1-\theta)} \tag{A.11}$$

where P_i^T is the price index over tradable goods in country i and P_i^{NT} is the price of non-tradable goods.

The tradable-goods price index in country $i = H, F$ is defined by:

$$P_i^T = \left[a \left(p_i^T\right)^{1-\phi} + (1-a) \left(p_j^T\right)^{1-\phi}\right]^{1/(1-\phi)}, \; j \neq i \tag{A.12}$$

where p_i^T is the price of the tradable good in country i.

We still denote Home terms of trade by q:

$$q \equiv \frac{p_H^T}{p_F^T} \tag{A.13}$$

We denote the Home price of non-tradable over the Foreign price of non-tradable by P^{NT}:

$$P^{NT} \equiv \frac{P_H^{NT}}{P_F^{NT}} \tag{A.14}$$

In both countries there are stocks of tradable and stocks of non-tradable where each stock is a Lucas tree that gives a share δ of the future endowments (tradable or non-tradable). The supply of each stock is normalized at unity. There is a CPI bond denominated in the Home composite good, and a CPI bond denominated in the Foreign composite good.

Both bonds are in zero net supply. Each household fully owns the local stock of tradable and the local stock of non-tradable, at birth, and has zero initial foreign assets. The country i household thus faces the following budget constraint, at $t = 0$:

$$p_S^T S_{ii}^T + p_S^T S_{ij}^T + p_S^{NT} S_{ii}^{NT} + p_S^{NT} S_{ij}^{NT} + p_b b_{ii} + p_b b_{ij} = p_S^T + p_S^{NT}, \quad \text{with} \quad j \neq i \tag{A.15}$$

where S_{ij}^k is the number of shares of stock of country j in sector $k = \{T, NT\}$ held by country i at the end of period 0, while b_{ij} represents claims (held by i) to future unconditional payments of the composite good j. p_S^k is the share prices of stock in sector $k = \{T, NT\}$, p_b is the bond price. Asset prices of each type are identical across countries due to symmetry.

Market clearing in asset markets for the for stocks and the two bonds requires:

$$S_{ii}^T + S_{ji}^T = S_{ii}^{NT} + S_{ji}^{NT} = 1 \tag{A.16}$$

$$b_{ii} + b_{ji} = 0 \tag{A.17}$$

Symmetry of preferences and shock distributions implies that equilibrium portfolios are symmetric: $S_{HH}^T = S_{FF}^T$, $S_{FH}^T = S_{HH}^T$, $S_{HH}^{NT} = S_{FF}^{NT}$, $S_{FH}^{NT} = S_{HH}^{NT}$, $b_{HH} = b_{FF}$ and $b_{FH} = b_{HF}$. In what follows, we denote a country's holdings of local stock of tradable (resp. non-tradable) by S^T (resp. S^{NT}), and its holdings of CPI bonds denominated in its local composite good by b. The vector $(S^T; S^{NT}; b)$ thus describes international portfolios.

A.3.2 Intratemporal allocation across goods

In period 1 (after the realization of productivity shocks), a representative consumer in country (i) maximizes:

$$\left[\frac{(C_i)^{1-\sigma}}{1-\sigma}\right]$$

subject to a budget constraint (for $j \neq i$):

$$\begin{aligned} p_i^T c_{ii}^T + p_j^T c_{ij}^T + p_i^{NT} c_i^{NT} &\leq I_i \qquad (\lambda_H) \\ P_i C_i &\leq I_i \qquad (\lambda_H) \end{aligned}$$

where I_i are total asset incomes of the representative agent in country (i), λ_i is the Lagrange-Multiplier associated to the budget constraint. At this point, I take portfolios chosen in period 0 as given.

The first-order conditions are:

For consumption:

$$1 = \lambda_i P_i C_i^{\sigma} \tag{A.18}$$

Intratemporal allocation across goods:

$$c_{ii}^T = a\left(\frac{p_i^T}{P_i^T}\right)^{-\phi} c_i^T \tag{A.19}$$

$$c_{ij}^T = (1-a)\left(\frac{p_j^T}{P_i^T}\right)^{-\phi} c_i^T \tag{A.20}$$

$$c_i^T = \eta\left(\frac{P_i^T}{P_i}\right)^{-\theta} C_i \tag{A.21}$$

$$c_i^{NT} = (1-\eta)\left(\frac{P_i^{NT}}{P_i}\right)^{-\theta} C_i \tag{A.22}$$

Using equations (A.19) and (A.20) for both countries and market-clearing conditions for tradable goods gives:

$$q^{-\phi}\Omega_a\left[(\frac{P_F^T}{P_H^T})^{\phi}\frac{c_F^T}{c_H^T}\right] = \frac{y_H^T}{y_F^T} \tag{A.23}$$

where $\Omega_u(x)$ is a continuous function of two variables (u,x) such that: $\Omega_u(x) = \frac{1+x(\frac{1-u}{u})}{x+(\frac{1-u}{u})}$

Then, using (A.21), we get:

$$q^{-\phi}\Omega_a\left[(\frac{P_F^T}{P_H^T})^{\phi-\theta}\frac{P_F^\theta C_F}{P_H^\theta C_H}\right] = \frac{y_H^T}{y_F^T} \tag{A.24}$$

Using equations (A.22) and market-clearing conditions for non-tradable goods for both countries, we get:

$$(\frac{P_H^{NT}}{P_F^{NT}})^{-\theta}\frac{P_H^\theta C_H}{P_F^\theta C_F} = \frac{y_H^{NT}}{y_F^{NT}} \tag{A.25}$$

A.3.3 Budget constraints

Recall that each household holds shares S^k and $1-S^k$ of local and foreign stocks in sector $k = \{T, NT\}$, respectively, while b denotes her holding of bonds denominated in her local composite good; also, 'tradable' stock $j's$ dividend is $p_j^T y_j^T$ and 'non-tradable' stock $j's$ dividend is $P_j^{NT} y_j^T$. The period 1 (relative) budget constraints of countries H and F are thus:

$$\begin{aligned} P_H C_H - P_F C_F = {} & \big(\delta(2S^T-1)+(1-\delta)\big)(p_H^T y_H^T - p_F^T y_F^T) \\ & + \big(\delta(2S^{NT}-1)+(1-\delta)\big)(P_H^{NT} y_H^{NT} - P_F^{NT} y_F^{NT}) + 2b(P_H - P_F) \end{aligned} \tag{A.26}$$

which says that the difference between countries' consumption spending equals the difference between their incomes.

A.3.4 Log-linearization of the model

Henceforth, we write $y^T \equiv \frac{y_H^T}{y_F^T}, y^{NT} \equiv \frac{y_H^{NT}}{y_F^{NT}}$ to denote relative outputs in both sectors. We log-linearize the model around the symmetric steady-state where y^T and y^{NT} equal unity, and use $\widehat{x} \equiv log(x/\bar{x})$ to denote the log deviation of a variable x from its steady state value $\overline{x}$.

The log-linearization of the Home country's real exchange rate $RER \equiv \frac{P_H}{P_F}$ gives:

$$\widehat{RER} = \widehat{\frac{P_H}{P_F}} = \eta(2a-1)\widehat{q} + (1-\eta)\widehat{P^{NT}}. \tag{A.27}$$

where $P^{NT} = P_H^{NT}/P_F^{NT}$ is the relative price of Home non-tradable goods over Foreign non-tradable goods and η is the steady-state share of spending devoted to tradable goods.[53] Note also that the relative price of the tradable composite in both countries verifies: $P_H^T/P_F^T = (2a-1)\widehat{q}$.

When markets are locally complete, the ratio of Home to Foreign marginal utilities of aggregate consumption is linked to the consumption-based real exchange rate by the following, familiar condition:

$$-\sigma(\widehat{C_H} - \widehat{C_F}) = \widehat{RER} = \eta(2a-1)\widehat{q} + (1-\eta)\widehat{P^{NT}}. \tag{A.28}$$

Log-linearizing (A.25) and using (A.28) implies:

$$\widehat{y^{NT}} = -\theta\widehat{P^{NT}} + (\theta - \frac{1}{\sigma})\widehat{RER} \tag{A.29}$$

Similarly, log-linearizing (A.24) and using (A.28) implies:

$$\begin{aligned}\widehat{y^T} &= -\left[\phi\left(1-(2a-1)^2\right) + (2a-1)^2\left((1-\eta)\theta + \eta\frac{1}{\sigma}\right)\right]\widehat{q} + (1-\eta)(\theta - \frac{1}{\sigma})(2a-1)\widehat{P^{NT}} \\ &= -\lambda\widehat{q} + (1-\eta)(\theta - \frac{1}{\sigma})(2a-1)\widehat{P^{NT}}\end{aligned} \tag{A.30}$$

where $\lambda \equiv \phi(1-(2a-1)^2) + (2a-1)^2\left((1-\eta)\theta + \frac{\eta}{\sigma}\right)$. Note that $\lambda > 0$ as $1/2 < a < 1$.

We next log-linearize equation (A.26); using (A.28) and we obtain:

$$\begin{aligned}\widehat{P_H C_H} - \widehat{P_F C_F} &= \eta\delta\left(2S^T - 1\right))\widehat{R_e^T} + (1-\eta)\delta\left(2S^{NT} - 1\right)\widehat{R_e^{NT}} + (1-\delta)\widehat{w} + 2b\widehat{RER} \\ &= (1 - \frac{1}{\sigma})\widehat{RER}\end{aligned} \tag{A.31}$$

where $\widehat{R^k}$ denotes Home excess return in sector $k = \{T, NT\}$ and $\widehat{w}$ denote relative non-financial income (Home over Foreign aggregated over both sectors).

[53] Here to simplify notations, we assume that the share of spending devoted to tradable goods is the same as the weight of tradable goods in the consumption index. This is true only if in the steady state tradable and non tradable goods have the same price: $p^{T*} = p^{NT*}$. This assumption is however irrelevant for equity portfolios (see Obstfeld [2007]).

Following the benchmark model and adding an additional source of uncertainty $\widehat{\varepsilon}$, we have the following relationships:

$$\begin{aligned} \widehat{R_e^T} &= \widehat{q} + \widehat{y^T} + \gamma_e \widehat{\varepsilon} \\ \widehat{R_e^{NT}} &= \widehat{P^{NT}} + \widehat{y^{NT}} + \gamma_e \widehat{\varepsilon} \\ \widehat{w} &= \eta \widehat{R^T} + (1-\eta)\widehat{R^{NT}} + (\gamma_w - \gamma_e)\widehat{\varepsilon} \end{aligned} \tag{A.32}$$

Using (A.29) and (A.30), this gives under efficient risk-sharing:

$$\begin{aligned} \widehat{R^T} &= (1-\lambda)\widehat{q} + (1-\eta)(\theta - \frac{1}{\sigma})(2a-1)\widehat{P^{NT}} + \gamma_e \widehat{\varepsilon} \\ \widehat{R^{NT}} &= (1-\theta)\widehat{P^{NT}} + (\theta - \frac{1}{\sigma})\widehat{RER} + \gamma_e \widehat{\varepsilon} \\ \widehat{w} &= \eta \widehat{R^T} + (1-\eta)\widehat{R^{NT}} + (\gamma_w - \gamma_e)\widehat{\varepsilon} \end{aligned} \tag{A.33}$$

The financial market is effectively complete (up to a first order approximation) when there exists a portfolio (S^T, S^{NT}, b) such that (A.29), (A.30) and relative budget constraint hold for arbitrary realizations of the relative shocks $\widehat{y^T}, \widehat{y^{NT}}$ and $\widehat{\varepsilon}$.

A.3.5 Equilibrium portfolios

Projection of equation (A.31) on $\widehat{\varepsilon}$ gives the averaged equity portfolio across sectors:

$$\eta S^T + (1-\eta) S^{NT} = \frac{1}{2}\left(1 - \frac{\gamma_w}{\gamma_e}\frac{(1-\delta)}{\delta}\right) \tag{A.34}$$

Projections on $\widehat{q}$ and $\widehat{P^{NT}}$ give the following relationships (assuming $a \neq 1/2$) and rearranging terms give the following sharing rule for the equity bias across sectors:

$$\frac{2S^T - 1 + (1-\delta)/\delta}{2S^{NT} - 1 + (1-\delta)/\delta} = \frac{(\theta-1)(2a-1)}{\phi - 1 + (2a-1)^2(\theta - \phi)} = \Omega \tag{A.35}$$

Solving further for equity portfolio using (62) and (A.35) gives:

$$\begin{aligned} S^T &= \frac{1}{2}\left(1 - \frac{1-\delta}{\delta} + \frac{1-\delta}{\delta}\frac{\Omega(1-\gamma)}{\eta\Omega + 1 - \eta}\right) \\ S^{NT} &= \frac{1}{2}\left(1 - \frac{1-\delta}{\delta} + \frac{1-\delta}{\delta}\frac{1-\gamma}{\eta\Omega + 1 - \eta}\right) \end{aligned}$$

The bond position satisfies:

$$b = \frac{1}{2}(1 - 1/\sigma) - \frac{1}{2}\frac{(1-\gamma)(1-\delta)}{\eta\Omega + 1 - \eta}(1 - \theta + (\theta - 1/\sigma)(1 - \eta + \eta(2a-1)\Omega)) \tag{A.36}$$

A.4 Estimation of returns to non-financial wealth: VAR results

[to be done]

A.5 Equilibrium Portfolios when $\gamma_e = 0$ and $\gamma_b \neq 0$

When $\gamma_e = 0$, the portfolio cannot be described by equations (21) as in section 2. Here, we solve for portfolios in a generic reduced-form model where relative equity returns load perfectly on the (welfare-based) real exchange rate ($\gamma_e = 0$) but bond returns do not ($\gamma_b \neq 0$).

We keep the same generic representation ignoring any additional source of risk on relative equity returns. This gives the following set of equations for the efficient terms-of-trade, relative equity returns and relative non-financial incomes (see section 2):

$$\widehat{R}_e = (1-\overline{\lambda})\widehat{q} \tag{A.37}$$

$$\widehat{R_b} = (2a-1)\widehat{q} + \gamma_b \widehat{\varepsilon} \tag{A.38}$$

$$\widehat{w} = (1-\overline{\lambda})\widehat{q} + \gamma_w \widehat{\varepsilon} \tag{A.39}$$

The real exchange rate is still defined by the following equation:

$$\widehat{RER} = (2a-1)\widehat{q} \tag{A.40}$$

Under the maintained hypothesis that markets are locally complete, the relative budget constraint (17) becomes:

$$(1-\frac{1}{\sigma})(2a-1)\widehat{q} = \delta\,(2S-1)\,(1-\overline{\lambda})\widehat{q} + (1-\delta)\left((1-\overline{\lambda})\widehat{q} + \gamma_w\widehat{\varepsilon}\right) + 2b((2a-1)\widehat{q} + \gamma_b\widehat{\varepsilon}) \tag{A.41}$$

Financial markets are still locally complete given that the representative investor still has two 'relative assets' to hedge two 'relative shocks'. Note that in this set-up, changes in relative incomes due to capital gains and losses on bond return differentials are not purely driven by changes in the real exchange rate. Since in turn relative equity returns load perfectly on the real exchange rate but bonds do not (due to $\widehat{\varepsilon}$), portfolios will be unique since the two 'relative assets' do not have the same pay-offs in all states of nature. Moreover, equities will be used to hedge changes in relative consumption expenditures and real exchange risk, contrary to bonds that will be used to hedge the shocks $\widehat{\varepsilon}$. The optimal portfolio then satisfies:

$$\begin{aligned} S^* &= \frac{1}{2}\left[\frac{2\delta-1}{\delta} - \frac{\left(1-\frac{1}{\sigma}-(1-\delta)\frac{\gamma_w}{\gamma_b}\right)(2a-1)}{\delta\left(\overline{\lambda}-1\right)}\right] \\ b^* &= -\frac{1}{2}(1-\delta)\frac{\gamma_w}{\gamma_b} \end{aligned} \tag{A.42}$$

The equity portfolio shares the same difficulties as in previous literature: it is highly dependent on preference parameters and involves for most parameter values shorting Home or Foreign equities. Note that in the specific case of $\gamma_w = 0$, the equity portfolio is identical to (18) and bonds are not used in equilibrium ($b = 0$) to insulate relative consumption expenditures from $\widehat{\varepsilon}$ shocks.

A.6 Countries of different sizes

[need to be generalized to the case $\gamma_b \neq 0$]

We extend our benchmark model by allowing different country sizes. We assume that expected production in period $t = 1$ is not equal across countries: $E_0(y_H) = \overline{y_H}$ and $E_0(y_F) = \overline{y_F}$. We denote by ω_i the relative size of country i: $\omega_i = \frac{\overline{y_i}}{\overline{y_i}+\overline{y_j}}$, with $\omega_H + \omega_F = 1$.

Both countries also differ in their consumption Home bias:

$$C_i = \left[a_i^{1/\phi} (c_{ii})^{(\phi-1)/\phi} + (1-a_i)^{1/\phi} (c_{ij})^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)}$$

We assume that is the non-stochastic equilibrium, the following relationship holds:

$$(1-a_H)\, y_H = (1-a_F) y_F$$

This ensure that in the trade balance is zero and terms-of-trade q are equal to unity in the non-stochastic equilibrium.

Following the benchmark model, we assume that the reduced-form of returns on equities and on non-financial income satisfies for $i = \{H, F\}$:

$$\begin{aligned} \widehat{R_{ei}} &= \widehat{p_i} + \widehat{y_i} + \gamma_e \widehat{\varepsilon_i} \\ \widehat{w_i} &= \widehat{p_i} + \widehat{y_i} + \gamma_w \widehat{\varepsilon_i} \end{aligned}$$

Keeping the same notations as in the symmetric case, log-linearization around the non-stochastic equilibrium implies the following relationships since one can easily verify that markets are still locally complete:

$$\begin{aligned} \widehat{RER} &= (a_H + a_F - 1)\widehat{q} \\ \widehat{PC} &= (1-\frac{1}{\sigma})\widehat{RER} = (1-\frac{1}{\sigma})(a_H + a_F - 1)\widehat{q} \\ \widehat{y} &= -\lambda \widehat{q} \end{aligned}$$

where λ is now such that: $\lambda = \phi(1-(a_H + a_F - 1)^2) + \frac{(a_H+a_F-1)^2}{\sigma}$.

Bond returns are still indexed on the country CPI such that Home bond excess returns $\widehat{R_b}$ are related to the real exchange rate as follows (assuming $\gamma_b = 0$):

$$\widehat{R_b} = \widehat{RER} = (a_H + a_F - 1)\widehat{q}$$

Log-linearization of the budget constraint in country i gives (using market clearing conditions in the asset market) for $i \neq j$:

$$\widehat{P_i C_i} = (1-\delta)\widehat{w_i} + \delta S_{ii} \widehat{d_i} + \frac{\omega_j}{\omega_i} \delta (1 - S_{jj})\, \widehat{d_j} + b_{ii} \widehat{P_i} - b_{jj} \widehat{P_j}$$

Taking the difference across countries, we get:

$$\begin{aligned}\widehat{PC} &= (1-\frac{1}{\sigma})\left(\widehat{P_H}-\widehat{P_F}\right)\\ &= (1-\delta)\left(\widehat{w_H}-\widehat{w_F}\right)+\delta\widehat{d_H}\left(S_{HH}-\frac{\omega_H}{\omega_F}(1-S_{HH})\right)\\ &\quad -\delta\widehat{d_F}\left(S_{FF}-\frac{\omega_F}{\omega_H}(1-S_{FF})\right)+2b_{HH}\widehat{P_H}-2b_{FF}\widehat{P_F}\end{aligned}$$

Projection on $\widehat{\varepsilon_H}$ gives the following holdings of Home stocks by Home households:

$$\begin{aligned}0 &= \omega_F(1-\delta)\gamma_w+\delta\gamma_e\left(\omega_F S_{HH}-\omega_H(1-S_{HH})\right)\\ S_{HH} &= \omega_H-\frac{1-\delta}{\delta}\frac{\gamma_e}{\gamma_w}(1-\omega_H)\end{aligned}$$

Similarly, holdings of Foreign stocks by Foreign households satisfy:

$$S_{FF}=\omega_F-\frac{1-\delta}{\delta}\frac{\gamma_e}{\gamma_w}(1-\omega_F)$$

Bond holdings must satisfy:

$$b^*=b_{HH}=b_{FF}=\frac{1}{2}(1-\frac{1}{\sigma})+\frac{1}{2}(1-\delta)(1-\frac{\gamma_e}{\gamma_w})(\lambda-1)(a_H+a_F-1)^{-1}$$

Because the model in reduced form can be rewritten as follows:

$$\begin{aligned}\widehat{R}_e &= (1-\bar{\lambda})\widehat{q}+\gamma_e\widehat{\varepsilon}\\ \widehat{R}_b &= (a_H+a_F-1)\widehat{q}\\ \widehat{w} &= (1-\bar{\lambda})\widehat{q}+\gamma_w\widehat{\varepsilon}\end{aligned} \tag{A.43}$$

We can rewrite the previous system in terms of loadings:

$$\begin{aligned}\widehat{w} &= (1-\frac{\gamma_w}{\gamma_e})(1-\bar{\lambda})(a_H+a_F-1)^{-1}\widehat{R}_b+\frac{\gamma_w}{\gamma_e}\widehat{R}_e\\ &= \beta_{w,b}\widehat{R}_b+\beta_{w,e}\widehat{R}_e\\ \widehat{q} &= \beta_{RER,b}\widehat{R}_b \text{ with } \beta_{RER,b}=1\end{aligned}$$

In terms of loadings this gives (under the maintained assumption that $\gamma_b=0$).

$$
\begin{aligned}
S_i^* &= \omega_i - \frac{1-\delta}{\delta}\beta_{w,e}(1-\omega_i) \\
b^* &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\beta_{RER,b} - \frac{1}{2}(1-\delta)\,\beta_{w,b}
\end{aligned}
$$

where S_i^* denotes the Holdings of stocks in country i by households of country i.

A.7 Home Bias in Equities

Source Country	Domestic Market in % of World Market Capitalization (1)	Share of Portfolio in Domestic Equity in % (2)	Degree of Home Bias $= HB_i$ (3)
Australia	1.9	83.6	0.832
Austria	0.3	58.5	0.583
Belgium	0.7	49.8	0.494
Canada	3.5	76.6	0.757
Denmark	0.4	62.7	0.625
Finland	0.5	63.3	0.631
France	4.2	68.8	0.674
Germany	2.9	57.5	0.562
Greece	0.3	93.4	0.933
Italy	1.9	57.1	0.562
Japan	13.2	91.9	0.906
Netherlands	1.4	32.1	0.311
New-Zealand	0.1	59.8	0.597
Norway	0.5	52	0.517
Portugal	0.2	77.8	0.777
Spain	2.3	86.3	0.859
Sweden	1	59.4	0.589
Switzerland	2.2	59.9	0.589
United Kingdom	7.3	65	0.622
United States	40.5	82.2	0.700
Average	3.67	70.58	0.70

Table 5: Home Bias in Equities in 2005 (from Sercu and Vanpee (2007); source CPIS).HB_i =1-(Share of Foreign Equities in Country i Equity Holdings)/(Share of Foreign Equities in the World Market Portfolio). By definition HB_i is equal to zero if the share of domestic equities in country i's portfolio is equal to the share of domestic equities in the world market portfolio and HB_i is equal to 1 if there is full equity home bias.