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Nonlinear Three-Tank System Fault Detection and Isolation Using Differential Flatness

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Abstract

Fault detection and isolation on hydraulic systems are very important to ensure safety and avoid disasters. In this paper, a fault detection and isolation method, based on the flatness property of nonlinear systems, is experimentally applied on the three-tank system, which is considered as a popular prototype of hydraulic systems. Specifically, fault indicators, called residues, are generated using flat output measurements, and for the purpose of fault isolation, a definition of the isolability is introduced. This definition allows the characterization of flat outputs that are useful for fault isolation. Multiplicative faults are considered on sensors and actuators.

Keywords— Hydraulic system, Flat system, Fault detection and isolation, Three-tank system

1 Introduction

The three-tank system is considered as a representative process of the aircraft fuel tank system, used by researchers to test various fault detection and isolation (FDI) methods. The fuel storage in the tanks and fuel consumption have a significant impact on the CG position of the aircraft [7]. In particular, the position of the center of gravity (CG) of an aircraft is very important for its stability and safety. Therefore, controlling fuel levels in each tank is essential to providing the desired CG position and guarantee the safety of the aircraft. To do this, it is important to have correct information about the fuel level in each tank and to have fuel flow supervision. Accordingly, any fault on the aircraft fuel system sensors or actuators may affect the aircraft's CG control and cause disasters. For this purpose, the application of a FDI technique on the aircraft fuel tank system is important to detect and isolate faults on sensors and actuators.

The three-tank system has been considered previously for validation of FDI methodologies. The authors in [6] tested a robust fault detection filter in order to detect and isolate faults on the three-tank sensors and actuators, and in [15], a nonlinear observer has been

designed, based on the nonlinear model of the three-tank system, in order to detect a leakage from a pipe.

FDI methods are classically based on the notion of redundant measurements which can be obtained either by multiple sensors or analytical components generating fault indicators, called residues. They represent the gap between each measurement (physical or analytical). For survey papers on FDI see [22, 17]. In the ideal case of noise free observations, if all the residues are equal to zero, then there is no fault on the system. However, if at least one residue is different from zero, then a fault is detected. In practice, due to the presence of noise on the system, the residues are compared next to fixed thresholds. If at least one residue exceeds its threshold, then a fault is detected, otherwise, there is no fault on the system. Studies on tuning thresholds can be found in [1, 5].

Recently, FDI methods based on the flatness property of nonlinear systems have also been shown to be effective in detecting and isolating faults on sensors and actuators (see [9, 16, 10]). Roughly speaking, we recall that a nonlinear system is said to be flat if there exists a variable z , called flat output, such that all the system states, inputs and outputs can be expressed in function of z and a finite number of its successive time derivatives. In [9], the flatness-based FDI method is used to estimate actuator faults only. The developed method in [16] is applied to linear systems and takes into account only sensor faults. In [10], the proposed flatness-based FDI method can be applied on both linear and nonlinear systems and takes into account sensor and actuator faults. In this method, the measurement of the flat output is used to calculate the redundant variables. However, due to the presence of noise, the flat output derivatives may not be defined, then they are estimated using a high-gain observer coupled with a low-pass filter to improve its performance. The fault detection is common with the other FDI methods: if a residue exceeds its threshold, then a fault is detected. However, the isolability is more complex and depends on the chosen flat output, and sometimes multiple flat outputs are needed to isolate all faults [18]. Nevertheless, the choice of these flat outputs is not arbitrary,

i.e. there are flat outputs that, when used together, increase the isolability of faults and others that do not.

Recently, in [13], an open-loop characterization of the flat outputs is briefly presented. It allows the choice of flat outputs that are independent, which is useful for the isolability. In the present work, a full presentation of this flat output characterization to the case of open or closed-loop systems is presented. This approach is applied experimentally on the three-tank benchmark. In this context, this paper is organized as follows: Section 2 describes the benchmark. The flatness-based fault detection and isolation is presented in Section 3. Section 4 presents the extension of the characterization of the flat outputs in closed-loop system. The experimental results are given in Section 5. Finally, Section 6 concludes the paper.

2 Description of the Three-Tank System

In this paper, all the experiments were performed on the three-tank system represented in Figure 1.

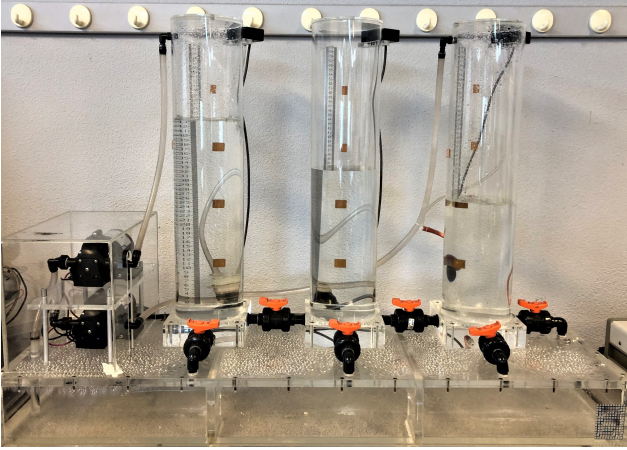


Figure 1: Three-tank system.

The three-tank system is about three cylindrical tanks of cross-sectional area S , connected to each other by means of pipes of section S_n . Each tank is also connected to the central reservoir through a pipe. Three piezo-resistive pressure transducer are installed on the top of each tank to measure the corresponding water level. The water is pumped from the central reservoir into tanks T_1 and T_2 with the help of two pumps P_1 and P_2 . The incoming flows, by unit of surface S , into tanks T_1 and T_2 are denoted by $u_1(t)$ and $u_2(t)$, respectively and correspond to the voltage command sent by the computer multiplied by a constant gain. The actuator's dynamics are negligible with respect to the system's dynamics. The water level in tank T_i is denoted by $x_i(t) \geq 0$, $i = 1, 2, 3$. The maximum water

level in any tank is denoted by h_{max} and the maximum incoming flow rate is denoted by u_{max} . A descriptive scheme of the system is presented in Figure 2.

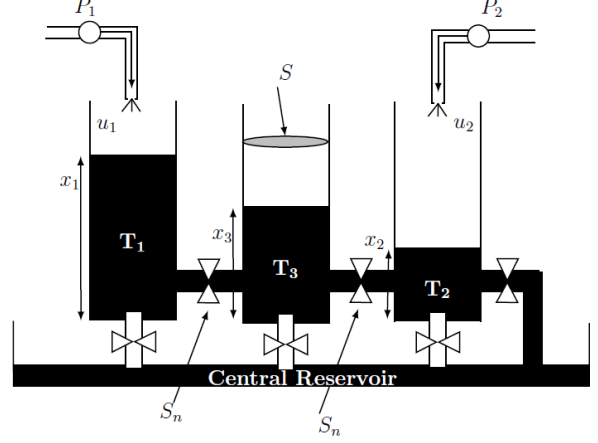


Figure 2: Scheme of the three-tank System, Source: [12]

The explicit system of equations that describes the dynamic of the three-tank model is given by:

$$\dot{x}_1 = -Q_{10}(x_1) - Q_{13}(x_1, x_3) + u_1 \quad (1)$$

$$\dot{x}_2 = -Q_{20}(x_2) + Q_{32}(x_2, x_3) + u_2 \quad (2)$$

$$\dot{x}_3 = Q_{13}(x_1, x_3) - Q_{32}(x_2, x_3) - Q_{30}(x_3) \quad (3)$$

where Q_{i0} , $i = 1, 2, 3$ represents the outflow by unit of surface S between each tank and the central reservoir, Q_{13} is the outflow between tank T_1 and tank T_3 and Q_{32} the outflow between tank T_3 and tank T_2 . In this study, the valves that linking tanks T_1 and T_3 to the central reservoir are considered closed, *i.e.* $Q_{10} \equiv 0$ and $Q_{30} \equiv 0$. The outflows by unit of surface S are given by:

$$Q_{13}(x_1, x_3) = \mu_{13} \operatorname{sgn}(x_1 - x_3) \sqrt{|x_1 - x_3|} \quad (4)$$

$$Q_{20}(x_2) = \mu_{20} \operatorname{sgn}(x_2) \sqrt{|x_2|} \quad (5)$$

$$Q_{32}(x_2, x_3) = \mu_{32} \operatorname{sgn}(x_2 - x_3) \sqrt{|x_3 - x_2|} \quad (6)$$

where μ_{13} , μ_{20} and μ_{32} are the flow coefficients and sgn is the sign function given by:

$$\operatorname{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases} \quad (7)$$

The parameters' values of the three-tank system are given in Table 1.

The experimental setup is composed of a DTS200 three-tank benchmark connected to a Windows7 PC using a MF624 plug-in card. Matlab/Simulink version R2008b is available on the development PC.

Parameters	Symbol	Value
Tank sectional area	S	0.0154 m^2
Pipes sectional area	S_n	$5 \times 10^{-5} \text{ m}^2$
Outflow coefficient	μ_{13}	8.5273×10^{-5}
Outflow coefficient	μ_{32}	8.5563×10^{-5}
Outflow coefficient	μ_{20}	1.5901×10^{-4}
Maximum water level	h_{max}	0.62 m
Maximum flow rate	u_{max}	$10^{-4} \text{ m}^3/\text{s}$

Table 1: Parameters' values of the three-tank benchmark.

For the FDI purpose, the three-tank model provides the application of any type of fault on both sensors and actuators:

- (i) Multiplicative faults: sensor and actuator gains may be reduced from 100% (total measurement) to 0% (complete measurement failure);
- (ii) Additive faults: sensors and actuators may present biases on their measurements.

Therefore, the sum of sensor and actuator faults can be expressed mathematically by [14]:

$$\begin{aligned} S_i^f(t) &= \alpha_i S_i(t) + S_{i0} \\ A_j^f(t) &= \beta_j A_j(t) + A_{j0} \end{aligned}$$

where $S_i^f(t)$ and $S_i(t)$ (resp. $A_j^f(t)$ and $A_j(t)$) denote faulty and unfaulty i^{th} sensor (resp. j^{th} actuator) respectively, S_{i0} and A_{j0} are the biases of i^{th} sensor and j^{th} actuator respectively, and $0 \leq \alpha_i \leq 1$ and $0 \leq \beta_j \leq 1$ are gain loss factors.

Hypothesis: In the sequel, we assume that there is only one fault at a time affecting sensors or actuators.

In the next section, we recall the flatness-based FDI method developed in [10] and we fully define the characterization of flat outputs to the case of a closed-loop system, in order to isolate faults.

3 Flatness-Based FDI

3.1 Flatness-based residual generation

Consider the following nonlinear system

$$\begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases} \quad (8)$$

where $x = (x_1, \dots, x_n)^T$ is the state vector, belongs to an n -dimensional manifold X , $u = (u_1, \dots, u_m)^T \in \mathbb{R}^m$ is the input vector, $y = (y_1, \dots, y_p)^T \in \mathbb{R}^p$ is the measured output, $m \leq n$, $p \geq m$ and $\text{rank}(\frac{\partial f}{\partial u}) = m$. In the sequel, we denote by $\bar{\xi} = (\xi, \dot{\xi}, \dots) \in \mathbb{R}_{\infty}^m$ the sequence of infinite order jets of a vector ξ and

$\bar{\xi}^{(\alpha)} \triangleq (\xi, \dot{\xi}, \dots, \xi^{(\alpha)})$ the truncation at the finite order $\alpha \in \mathbb{N}$. Let $(x, \bar{u}) \triangleq (x, u, \dot{u}, \ddot{u}, \dots)$ be a prolongation of the coordinates (x, u) to the manifold of jets of infinite order $\mathfrak{X} \triangleq X \times \mathbb{R}_{\infty}^m$ [8, Chapter 5].

Definition 1 ([2]). The system (8) is said to be flat at a point $(x_0, \bar{u}_0) \in \mathfrak{X}$ if, and only if, there exist a vector $z = (z_1, \dots, z_m)^T \in \mathbb{R}^m$ and two applications ψ , defined on a neighbourhood \mathcal{V} of $(x_0, \bar{u}_0) \in \mathfrak{X}$, and $\varphi = (\varphi_0, \varphi_1, \dots)$, defined on a neighbourhood \mathcal{W} of $\psi(\mathcal{V})$ of $\bar{z} = (z, \dot{z}, \dots) \triangleq \psi(x_0, \bar{u}_0)$ such that:

1. z is a function of x , u and successive derivatives of u up to a finite order ν :

$$z = \psi(x, u, \dot{u}, \dots, u^{(\nu)}); \quad (9)$$

2. In turn, x and u are functions of z and its successive derivatives up to a finite order ρ :

$$(x, u) = (\varphi_0(z, \dot{z}, \dots, z^{(\rho)}), \varphi_1(z, \dot{z}, \dots, z^{(\rho+1)})), \quad (10)$$

hence, the expression of the output y is given by:

$$y = h(\varphi_0(z, \dot{z}, \dots, z^{(\rho)}), \varphi_1(z, \dot{z}, \dots, z^{(\rho+1)})); \quad (11)$$

3. The differential equation $\frac{d\varphi_0}{dt} = f(\varphi_0, \varphi_1)$ is identically satisfied.

The vector z is called flat output of the system and its components z_1, \dots, z_m and their successive derivatives are linearly independent. The mappings ψ and φ are called isomorphisms of Lie-Bäcklund and are inverse of one another.

Remark 1 ([4]). The property of flatness is not defined globally on the state space \mathfrak{X} . It means that there may exist points on \mathfrak{X} where the system is not flat, or, in other words, where the isomorphisms of Lie-Bäcklund ψ and φ are not defined. The set of such points is called the set of intrinsic singularities. In [4] it is shown that the set of equilibrium points that are not first order controllable, is included in the set of intrinsic singularities.

Let us assume the system (8) is flat with $z = (z_1, \dots, z_m)^T$ as flat output. We also suppose that the full output y is measured by sensors S_1, \dots, S_p and we denote its measurement by

$$y^s = (y_1^s, \dots, y_p^s)^T. \quad (12)$$

Moreover, we assume that the values u_1, \dots, u_m of the input vector u , corresponding to the actuators A_1, \dots, A_m , are available at any moment.

In order to detect and isolate faults on physical sensors and actuators, their analytical measurements must first be computed. Equations (10) and (11) provide

an efficient way to construct these analytical measurements, as long as the measurement of the flat output is available during the system process. In the following, we suppose that the measurement of the flat output is available and we denote it by:

$$z^s = (z_1^s, \dots, z_m^s)^T. \quad (13)$$

According to (10), the analytical state x^z and input u^z , constructed via the flat output (13), read:

$$x^z = \varphi_0(\overline{z^s}^{(\rho)}) \quad \text{and} \quad u^z = \varphi_1(\overline{z^s}^{(\rho+1)}), \quad (14)$$

and the analytical output y_k^z , is given, according to (11), by:

$$y_k^z \triangleq h_k(\varphi_0(\overline{z^s}^{(\rho)}), \varphi_1(\overline{z^s}^{(\rho+1)})). \quad (15)$$

The following definition of residues is borrowed from [13]:

Definition 2. The k^{th} -sensor residue R_{S_k} , for $k = 1, \dots, p$, and the l^{th} -input residue R_{A_l} , for $l = 1, \dots, m$, are given by:

$$R_{S_k} = y_k^s - y_k^z, \quad R_{A_l} = u_l - u_l^z, \quad (16)$$

respectively.

Then, the full vector of residues, denoted by r , is of dimension $p + m$ and given by:

$$\begin{aligned} r &= (R_{S_1}, \dots, R_{S_m}, R_{S_{m+1}}, \dots, R_{S_p}, R_{A_1}, \dots, R_{A_m})^T \\ &= (r_1, \dots, r_m, r_{m+1}, \dots, r_p, r_{p+1}, \dots, r_{p+m})^T. \end{aligned} \quad (17)$$

Remark 2. We can consider, without loss of generality, that:

$$z^s = (y_1^s, \dots, y_m^s)^T. \quad (18)$$

In this case, the first m components of y^z are equal to the corresponding components of z^s , then

$$y^z = (z^s, \tilde{h}(\varphi_0(\overline{z^s}^{(\rho)}), \varphi_1(\overline{z^s}^{(\rho+1)})))^T \quad (19)$$

with

$$\begin{aligned} \tilde{h} &= h_{m+1}(\varphi_0(\overline{z^s}^{(\rho)}), \varphi_1(\overline{z^s}^{(\rho+1)}), \dots, \\ &\quad h_p(\varphi_0(\overline{z^s}^{(\rho)}), \varphi_1(\overline{z^s}^{(\rho+1)})). \end{aligned}$$

Hence, the first m residues are identically zero, and the vector (17) becomes:

$$r = (0, \dots, 0, r_{m+1}, \dots, r_p, r_{p+1}, \dots, r_{p+m})^T. \quad (20)$$

A zero residue means that even if a fault occurs on one sensor or actuator, this residue cannot be affected. Then, it is not useful either for detection or isolation of the fault and we eliminate it from (20), which will be truncated to the last p components:

$$\begin{aligned} r_\tau &= (R_{S_{m+1}}, \dots, R_{S_p}, R_{A_1}, \dots, R_{A_m})^T \\ &= (r_{\tau_1}, r_{\tau_2}, \dots, r_{\tau_p})^T. \end{aligned} \quad (21)$$

Remark 3. The components z_1^s, \dots, z_m^s of the flat output (13), must be derivated in order to calculate the values of y^z and u^z . However, these derivatives may not exist because of the presence of noise on system's sensors and actuators. So these derivatives have to be estimated. Many methods have been developed in the literature and can be used here, we cite among them the algebraic derivative estimation [11, 21] and high-gain observers [20]. In the three-tank model, the noise on the measurements are the water bubbles due to the water falling from the top of tanks T_1 and T_2 through pumps P_1 and P_2 . In our experiments on the real three-tank system, we use a Butterworth low-pass filter of order $N = 4$ and cutoff frequency $f_c = 0.35$ HZ to filter noise, together with discrete filtered derivative represented by the following transfer function [3]:

$$\left(\frac{K}{T}\right) \frac{z - 1}{z + T_s/T - 1} \quad (22)$$

where $K = 1$ is the gain, $T = 20$ s is the time constant and $T_s = 1$ s is the sample time.

3.2 Fault Detection and Isolation

In order to detect and isolate faults using the flatness-based approach, we use the notions of signature matrix and fault alarm signature introduced in [13]:

Definition 3 (Signature matrix). Given the vector of residue r defined in (17) and $\zeta = (y_1^s, \dots, y_p^s, u_1, \dots, u_m)^T \in \mathbb{R}^{p+m}$ the vector of available measurements, we define the signature matrix associated to z^s , the matrix \mathbf{S} given by:

$$\mathbf{S} = \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} & \dots & \sigma_{1,p+m} \\ \vdots & \vdots & \dots & \vdots \\ \sigma_{p+m,1} & \sigma_{p+m,2} & \dots & \sigma_{p+m,p+m} \end{pmatrix} \quad (23)$$

with

$$\sigma_{i,j} \triangleq \begin{cases} 0 & \text{if } \frac{\partial r_{\tau_i}}{\partial \zeta_j^{(\varrho)}} = 0 \quad \forall \varrho \in \{0, 1, \dots\}, \\ 1 & \text{if } \exists \varrho \in \{0, 1, \dots\} \text{ s.t. } \frac{\partial r_{\tau_i}}{\partial \zeta_j^{(\varrho)}} \neq 0. \end{cases} \quad (24)$$

Let Σ_j , for $j = 1, \dots, p + m$, be the j^{th} -column of the matrix \mathbf{S} . Σ_j indicates if a residue r_i is or is not functionally affected by a fault on the measurement ζ_j : $\sigma_{i,j} = 0$ means that r_i is not affected by a fault on ζ_j and $\sigma_{i,j} = 1$ if it is affected.

Definition 4 (Fault alarm signature). Each column Σ_j of the signature matrix \mathbf{S} is called fault alarm signature or simply signature, associated to the sensor/actuator ζ_j .

Remark 4. Given that the flat outputs are measured by sensors (see Remark 2), the dimension of the signature matrix \mathbf{S} will be reduced to $p \times (p + m)$.

The following definitions of *detectability* and *isolability* in the flatness context are borrowed from [13]:

Definition 5 (Detectability). A fault on a sensor/actuator ζ_j is detectable if, and only if, there exists at least one $i \in \{1, \dots, p+m\}$ such that $\sigma_{i,j} = 1$.

Definition 6 (Isolability). A fault on a sensor S_k , $k = 1, \dots, p$, is said isolable if, and only if, its corresponding fault alarm signature Σ_k in the signature matrix \mathbf{S} is distinct from the others, *i.e.*

$$\Sigma_k \neq \Sigma_j, \quad \forall j = 1, \dots, p+m, j \neq k. \quad (25)$$

An isolable fault on the actuator A_l , for $l = 1, \dots, m$, is defined analogously:

$$\Sigma_{p+l} \neq \Sigma_j, \quad \forall j = 1, \dots, p+m, j \neq p+l. \quad (26)$$

This definition of isolability reflects the fact that if the signature matrix \mathbf{S} has two identical signatures $\Sigma_i = \Sigma_j$ with $i \neq j$, then a fault that affects the sensor/actuator ζ_i or ζ_j cannot be isolated. Therefore, in order to be able to isolate as many faults as possible, we need to increase the number of the residues by using several flat outputs. These flat outputs must be independent, in the sense that if a fault affects one flat output, not all the residues will be affected [19]. In the next section, a characterization of the relation between flat outputs is discussed.

4 Flat Output Characterization

We suppose that the flat system (8) admits different flat outputs whose measurements are available. We also define by μ the number of distinct signatures of the matrix \mathbf{S} , associated to a flat output, then μ is the number of isolated faults. So, in order to get more isolability of faults, we need to increase the number of distinct signatures μ .

In the following, we denote the i^{th} element of the set of q flat output vectors Z_i by

$$Z_i = (z_{i1}, \dots, z_{im})^T. \quad (27)$$

In order to characterize the flat outputs, the notion of augmented signature matrix is defined:

Definition 7 (Augmented signature matrix). Let Z_1, \dots, Z_q be q different measured flat outputs of the flat system (8). The augmented signature matrix $\tilde{\mathbf{S}}$ associated to Z_1, \dots, Z_q is defined by:

$$\tilde{\mathbf{S}} = \begin{pmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \\ \vdots \\ \mathbf{S}_q \end{pmatrix} \quad (28)$$

where \mathbf{S}_i is the signature matrix associated to the flat output vector Z_i .

Definition 8 (Independence). Let $\tilde{\mathbf{S}}$ be the augmented signature matrix associated to Z_1 and Z_2 :

$$\tilde{\mathbf{S}} = \begin{pmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \end{pmatrix},$$

μ_i , $i = 1, 2$, the number of distinct signatures of the matrix \mathbf{S}_i and $\tilde{\mu}$ the number of distinct signatures of the augmented matrix $\tilde{\mathbf{S}}$. We say that Z_1 and Z_2 are independent if, and only if,

$$\tilde{\mu} > \mu_1 \quad \text{and} \quad \tilde{\mu} > \mu_2. \quad (29)$$

According to Definition 8, the condition of full isolability is achieved if the augmented matrix

$$\tilde{\mathbf{S}} = \begin{pmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \\ \vdots \\ \mathbf{S}_q \end{pmatrix} \quad (30)$$

has $p+m$ distinct signatures, *i.e.* $\tilde{\mu} = p+m$.

In [13], this characterization of the flat outputs is applied on the three-tank system in the open loop case, and two flat outputs are needed to achieve full isolability. In the open loop case, a fault that affects a sensor has no impact on the actuators.

In this paper, we focus on the closed-loop case. As such, the control action depends on the measured outputs. Therefore, a fault in a closed-loop system is propagated with the feedback loop, increasing the difficulty of fault isolation. For the system (8), this implies that at least one input component u_l is related to an output measurement y_k^s . Then, the expression of the control input u_l is given by:

$$u_l = u_l^{ref} + C_{l,k}(y_k^{ref} - y_k^s) \quad (31)$$

where, u_l^{ref} and y_k^{ref} are the reference input and output trajectories, respectively, and $C_{l,k}$ denotes the discrete-time feedback controller between the l^{th} control input and the k^{th} measured output. In this case, the l^{th} input residue R_{A_l} is given by:

$$R_{A_l} = u_l - u_l^z \quad (32)$$

where u_l is replaced by (31). So a fault that appears on the sensor S_k and affects originally only the residue R_{S_k} , in the closed-loop case, affects both residues R_{S_k} and R_{A_l} .

5 Experimental Results

In this section, experiments on the three-tank system are presented in order to show the effectiveness of the flatness-based FDI method and the characterization of the flat outputs. First, subsection 5.1 represents the

flatness analysis of the three-tank model. Then, subsection 5.2 represents the path tracking, using the flatness property, and the the controller applied on the system. Finally, subsection 5.3 represents theoretical and experimental results of FDI.

5.1 Flatness analysis of the three-tank system

In the three-tank system, the only equilibrium point which is not first order controllable, or that represents an intrinsic singularity (see Remark 1) is where the water level is equal in all three tanks, *i.e.* $x_1 = x_2 = x_3$. To avoid this singularity, we consider the following configuration:

$$(C) : \quad x_1 > x_3 > x_2 > 0.$$

According to Definition 1, we can show that the three-tank system (1)-(2)-(3) is flat with

$$z = (x_1, x_3)^T = (z_1, z_2)^T \quad (33)$$

a flat output. In fact, from (3) and using (4) and (6), it is easy to express x_2 in function of z :

$$x_2 = z_2 - \frac{1}{\mu_{32}^2} \left(-\dot{z}_2 + \mu_{13}\sqrt{z_1 - z_2} \right)^2. \quad (34)$$

In addition, from (1) and (4), u_1 is given by:

$$u_1 = \dot{z}_1 + \mu_{13}\sqrt{z_1 - z_2}, \quad (35)$$

and from (2) and (34), u_2 is expressed by

$$u_2 = \dot{x}_2 + \mu_{20}\sqrt{x_2} - \mu_{32}\sqrt{z_2 - x_2} \quad (36)$$

and clearly $x_1 = z_1$ and $x_3 = z_2$, which proves that $z = (x_1, x_3)^T$ is a flat output of the three-tank system.

5.2 Path tracking and control of the system

In control theory, the concept of path tracking consists in finding control input values allowing the system to follow a predefined reference trajectory. The flatness property ensures the calculation of the control variables in a very simple way. One needs only to calculate a trajectory $t \mapsto z_{ref}(t)$, sufficiently differentiable, for the flat output z then, since the variables x and u are functions of z and its successive derivatives, the reference trajectory $t \mapsto (x_{ref}(t), u_{ref}(t))$ of the system is deduced by differentiating $t \mapsto z_{ref}(t)$ a finite number of times. Concerning our three-tank system experiments, a reference trajectory of the system is generated using the flat output $z = (x_1, x_3)^T$. The initial and final conditions for the flat output are given by:

$$\begin{aligned} x_{1i} &= 0.2 \text{ m}, & x_{1f} &= 0.35 \text{ m} \\ x_{3i} &= 0.15 \text{ m}, & x_{3f} &= 0.25 \text{ m} \end{aligned} \quad (37)$$

The initial and final times are respectively $t_i = 0$ s and $t_f = 400$ s. The trajectory $t \mapsto z_{ref}(t)$, that does not satisfy any differential equation, is calculated using a fifth order polynomial interpolation [8].

In order to control the system against external faults and disturbances, the following PI controllers, that are already implemented on the system, are used to control the water level in tanks T_1 and T_2 [10]:

$$C_1(s) = -10^{-3} \frac{11.5s + 1}{11.5s} \quad (38)$$

$$C_2(s) = -10^{-3} \frac{12.5s + 1}{12.5s} \quad (39)$$

where s is the Laplace variable. The output of the system in the fault free case is represented in Figure 3.

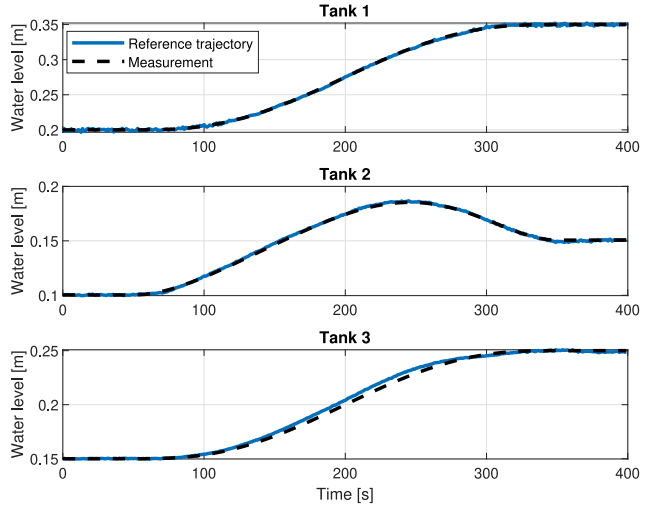


Figure 3: *Reference trajectories vs. measurements of the water level in each tank, in the fault free case.*

5.3 Results on FDI

Sensors on the three-tank system are S_1 , S_2 and S_3 and they measure the water level in each tank, respectively so the measured output is given by:

$$y^s = (x_1^s, x_2^s, x_3^s)^T \triangleq (y_1^s, y_2^s, y_3^s)^T. \quad (40)$$

Pumps P_1 and P_2 are the actuators of the system and we denote them by A_1 and A_2 and their outgoing flows are denoted by u_1 and u_2 , respectively.

In this paper, we show that a single flat output vector is not sufficient to isolate all possible faults on the three-tank system and that we need a second flat output to ensure full isolability. So two cases are represented: using one flat output and using two flat outputs.

Case A: using one flat output

As shown in section 5.1, the three-tank system is flat with $z = (x_1, x_3)^T$ as flat output. Components of the flat output are measured by sensors S_1 and S_3 and their measurements are denoted by:

$$z^s = (y_1^s, y_3^s)^T \triangleq (z_1^s, z_2^s)^T. \quad (41)$$

In order to construct the vector of residues, the redundant inputs and outputs are firstly computed using (14) and (15):

$$\begin{aligned} y_1^z &= z_1^s \\ y_2^z &= z_2^s - \frac{1}{\mu_{32}^2} \left(-\dot{z}_2^s + \mu_{13} \sqrt{z_1^s - z_2^s} \right)^2 \\ y_3^z &= z_2^s \\ u_1^z &= \dot{z}_1^s + \mu_{13} \sqrt{z_1^s - z_2^s} \\ u_2^z &= \dot{y}_2^z - \mu_{32} \sqrt{z_2^s - y_2^z} + \mu_{20} \sqrt{y_2^z}. \end{aligned} \quad (42)$$

Then, the vector of residues associated to z^s is given by:

$$r = \begin{pmatrix} R_{S_1} \\ R_{S_2} \\ R_{S_3} \\ R_{A_1} \\ R_{A_2} \end{pmatrix} = \begin{pmatrix} y_1^s \\ y_2^s \\ y_3^s \\ u_1 \\ u_2 \end{pmatrix} - \begin{pmatrix} y_1^z \\ y_2^z \\ y_3^z \\ u_1^z \\ u_2^z \end{pmatrix}. \quad (43)$$

Nevertheless, according to Remark 2, residues R_{S_1} and R_{S_3} are identically zero, hence the vector r is truncated to:

$$r_\tau = (R_{S_2}, R_{A_1}, R_{A_2})^T \triangleq (r_{\tau_1}, r_{\tau_2}, r_{\tau_3})^T. \quad (44)$$

The vector ζ of measured variables, introduced in Definition 3, is given by:

$$\zeta = (x_1^s, x_2^s, x_3^s, u_1, u_2) \in \mathbb{R}^5. \quad (45)$$

Therefore, the signature matrix \mathbf{S} , associated to z^s , is of dimension 3×5 :

$$\mathbf{S} = \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} & \sigma_{1,3} & \sigma_{1,4} & \sigma_{1,5} \\ \sigma_{2,1} & \sigma_{2,2} & \sigma_{2,3} & \sigma_{2,4} & \sigma_{2,5} \\ \sigma_{3,1} & \sigma_{3,2} & \sigma_{3,3} & \sigma_{3,4} & \sigma_{3,5} \end{pmatrix} \quad (46)$$

and constructed as follows:

- All the residues in (44) depend on the measurement of sensors S_1 and S_3 (see (42)), then the first and the third columns of the signature matrix contain only ones:

$$\sigma_{i,1} = \sigma_{i,3} = 1, \quad \forall i = 1, 2, 3.$$

- Only residues r_{τ_1} and r_{τ_3} depend on the measurement of sensor S_2 , then the second column will be such that:

$$\sigma_{1,2} = \sigma_{3,2} = 1 \quad \text{and} \quad \sigma_{2,2} = 0.$$

- Since r_{τ_2} depends only on u_1 and r_{τ_3} depends only on u_2 , column 4 and column 5 of \mathbf{S} are such that:

$$\sigma_{2,4} = 1 \quad \text{and} \quad \sigma_{i,4} = 0 \quad \forall i = 1, \dots, 3, i \neq 2$$

and

$$\sigma_{3,5} = 1 \quad \text{and} \quad \sigma_{i,5} = 0 \quad \forall i = 1, \dots, 3, i \neq 3$$

respectively.

Hence, the signature matrix (46) becomes:

$$\mathbf{S} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}. \quad (47)$$

Theoretically, according to Definition 5, all faults on the system sensors and actuators are detectable. In addition, since fault alarm signatures Σ_2 , Σ_4 and Σ_5 are distinct, faults on sensor S_2 and actuators A_1 and A_2 are isolable, according to Definition 6. This reflects the fact that if, at some point during system operation, a fault alarm is launched with the signature Σ_2 then we conclude that the sensor S_2 is faulty. However, if we obtain a signature like Σ_1 , the fault could be on the sensor S_1 or S_3 , since signatures Σ_1 and Σ_3 are identical. Then, a fault on S_1 or S_3 cannot be isolated.

Experimentally, as mentioned in Section 2, both additive and multiplicative faults can be applied on the DTS200 three-tank system. In this paper, we add multiplicative faults on all the system sensors and actuators. Moreover, due to the presence of noise on the system, thresholds for each residue are fixed: several nominal experiments were run, *i.e.* without introducing any fault on system sensors and actuators. For each experiment, the initial and final values of the reference trajectory are modified. The maximum and minimum values of the residues are extracted in each experiment, and the amplitude of the threshold is fixed by choosing the worst case among all the calculated residues. A safety margin of 5% is added to avoid false alarms.

For multiplicative faults we consider a 20% failure for sensors and actuators. At time $t = 200$ s, sensors measure 80% of the actual water level measurements instead of 100%, and for actuators, a 20% failure is considered:

- Consider a fault that affects sensor S_2 , then the associated residue r_{τ_1} is triggered. In addition, when the sensor loses 20% of its measurement, the system reacts as there is a loss of water in tank T_2 , and then pump P_2 puts in more work to cover this loss, so the residue r_{τ_3} exceeds its threshold temporarily (see Figure 4), which confirms the signature Σ_2 of \mathbf{S} .
- When a fault affects the actuator A_1 , only its corresponding residue r_{τ_2} exceeds its threshold (see

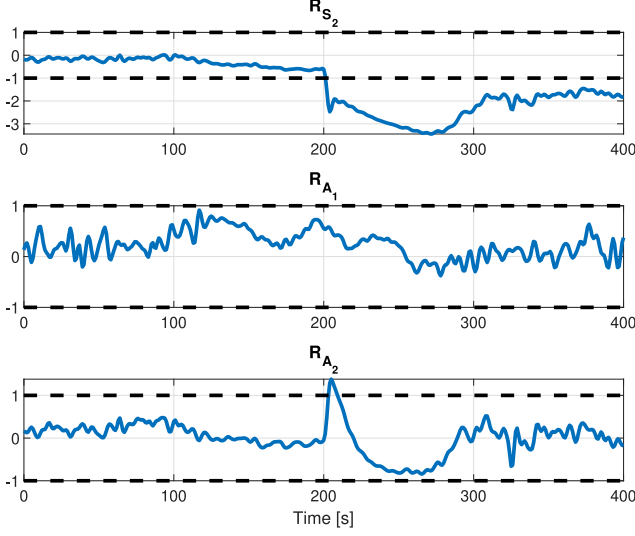


Figure 4: *Case A: residues responses to a fault on sensor S_2 .*

Figure 5) which confirms the signature Σ_4 of \mathbf{S} . Similarly, only the residue r_{τ_3} is triggered once a fault affects the actuator A_2 (see Figure 6) which confirms the signature Σ_5 of \mathbf{S} .

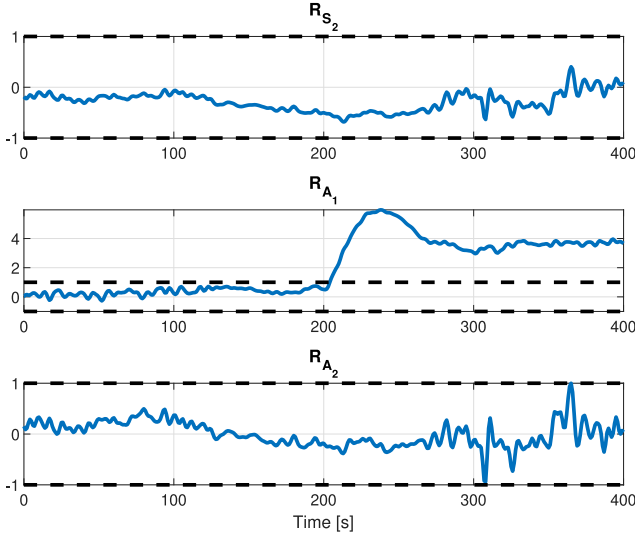


Figure 5: *Case A: residues responses to a fault on actuator A_1 .*

- When a fault affects sensor S_1 all the residues will be affected (see Figure 7), which confirms the signature Σ_1 of \mathbf{S}
- Finally, when a fault affects sensor S_3 , all the residues are expected to exceed their thresholds. However, even if the residue R_{A_1} changes its amplitude, it does not exceed its threshold only after 100s (see Figure 8). Then, the signature Σ_3 is

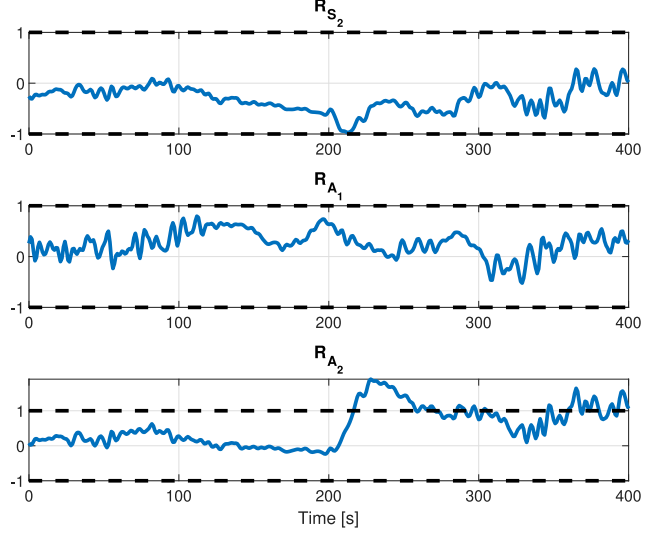


Figure 6: *Case A: residues responses to a fault on actuator A_2 .*

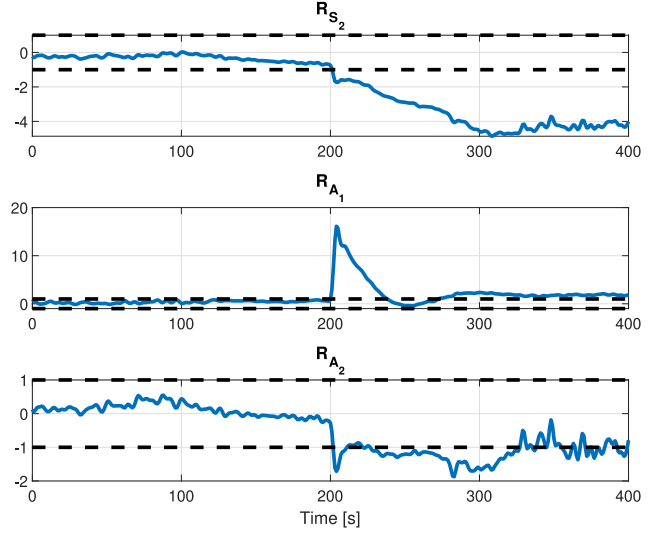


Figure 7: *Case A: residues responses to a fault on the sensor S_1 .*

given, experimentally, by

$$\Sigma_3 = (1 \ 0 \ 1)^T \quad (48)$$

and the signature matrix \mathbf{S} is given, experimentally, by:

$$\mathbf{S} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}. \quad (49)$$

It is clear that this unexpected behaviour in Figure 8 is due to the presence of uncertainties on the experimental platform. These uncertainties on the system force us to choose higher thresholds and therefore, these

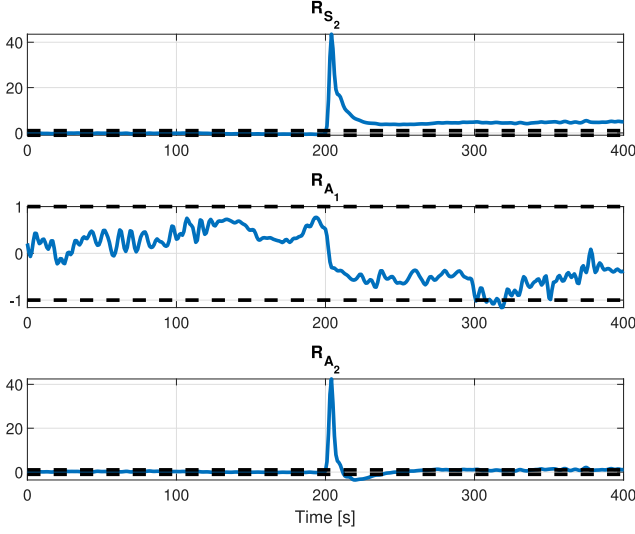


Figure 8: *Case A: residues responses to a fault on the sensor S_3 .*

thresholds may no longer be exceeded in the event of a fault.

Experimentally, the full isolability of faults is not achieved since signature Σ_2 and Σ_3 are identical. Then, the number of distinct fault alarm signatures is $\mu = 3 < p + m$.

Case B: using two flat outputs

In order to get more isolability on the three-tank system, a second flat output is needed. Therefore, we consider another flat output $z = (x_2, x_3)^T$. In the following, we denote by Z_1 the first flat output $Z_1 = (z_{11}^s, z_{12}^s)^T = (x_1^s, x_3^s)^T$ and by Z_2 the second flat output $Z_2 = (z_{21}^s, z_{22}^s)^T = (x_2^s, x_3^s)^T$. The signature matrix associated to Z_1 is given by (47) and we denote it by S_1 . The number of distinct signatures is $\mu_1 = 3 < p + m$.

In order to construct the signature matrix S_2 , associated to Z_2 , the redundant inputs and outputs are first computed using (14) and (15):

$$\begin{aligned} y_1^{Z_2} &= z_{22}^s + \frac{1}{\mu_{13}^2} \left(\dot{z}_{22}^s + \mu_{32} \sqrt{z_{22}^s - z_{21}^s} \right)^2 \\ y_2^{Z_2} &= z_{21}^s \\ y_3^{Z_2} &= z_{22}^s \\ u_1^{Z_2} &= \dot{y}_1^{Z_2} + \mu_{13} \sqrt{y_1^{Z_2} - z_{22}^s} \\ u_2^{Z_2} &= \dot{z}_{21}^s + \mu_{20} \sqrt{z_{21}^s - \mu_{32} \sqrt{z_{22}^s - z_{21}^s}} \end{aligned} \quad (50)$$

According to Remark 2, residues associated to sensors S_2 and S_3 are identically zero. Then, the vector of residues is truncated to:

$$r_{\tau}^{Z_2} = \begin{pmatrix} R_{S_2}^{Z_2} \\ R_{A_1}^{Z_2} \\ R_{A_2}^{Z_2} \end{pmatrix} = \begin{pmatrix} y_2^s \\ u_1 \\ u_2 \end{pmatrix} - \begin{pmatrix} y_2^{Z_2} \\ u_1^{Z_2} \\ u_2^{Z_2} \end{pmatrix}. \quad (51)$$

Theoretically, the signature matrix associated to Z_2 is given by:

$$S_2 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix} \quad (52)$$

and faults on sensors S_2 and S_3 are not isolable.

Experimentally, all signatures of the matrix S_2 are confirmed except signature Σ_3 which is given by

$$\Sigma_3 = (1 \quad 1 \quad 0)^T, \quad (53)$$

see Figure 9. Therefore, the signature matrix S_2 is given by:

$$S_2 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix} \quad (54)$$

and like case A, faults on sensors S_1 and S_3 are not isolable. The number of distinct signature is $\mu_2 = 3 < p + m$.

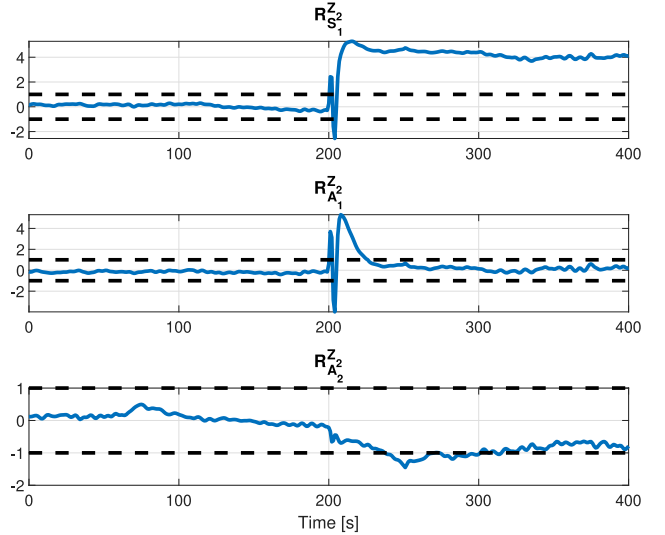


Figure 9: *The response of the residues associated to Z_2 to a fault on the sensor S_3 .*

In order to prove the independence of the flat outputs Z_1 and Z_2 , the following augmented signature matrix is constructed using the experimental results:

$$\tilde{S} = \begin{pmatrix} S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix}. \quad (55)$$

The number of distinct signature of \tilde{S} is $\tilde{\mu} = 5$, thus, even in the presence of uncertainties, the condition (29) is satisfied, then the flat outputs Z_1 and Z_2 are independent. In addition, $\tilde{\mu} = p + m$, then, according to

Definition 8, the flat outputs Z_1 and Z_2 provide full isolability of faults.

This result is demonstrated experimentally:

- Figure 10 shows that if a fault affects the sensor S_1 , all the residues exceed their threshold except $R_{A_2}^{Z_2}$ which confirms the signature Σ_1 of the matrix \tilde{S} .

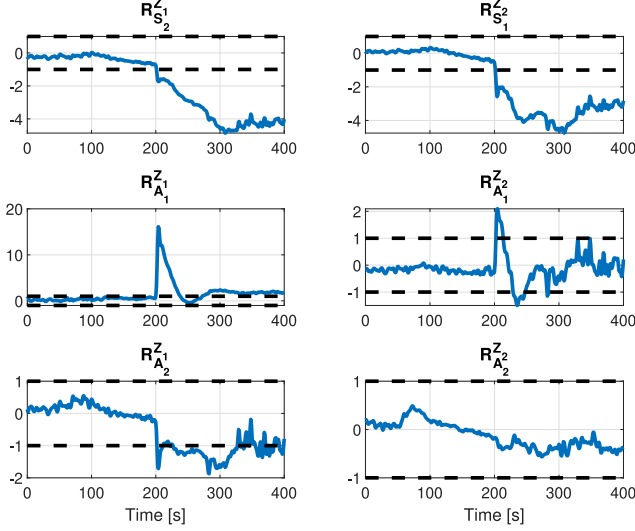


Figure 10: Case B: residues responses to a fault on sensor S_1 .

- Figure 11 shows that if a fault affects sensor S_2 , all the residues exceed their threshold except $R_{A_1}^{Z_1}$ which confirms the signature Σ_2 of the matrix \tilde{S} .

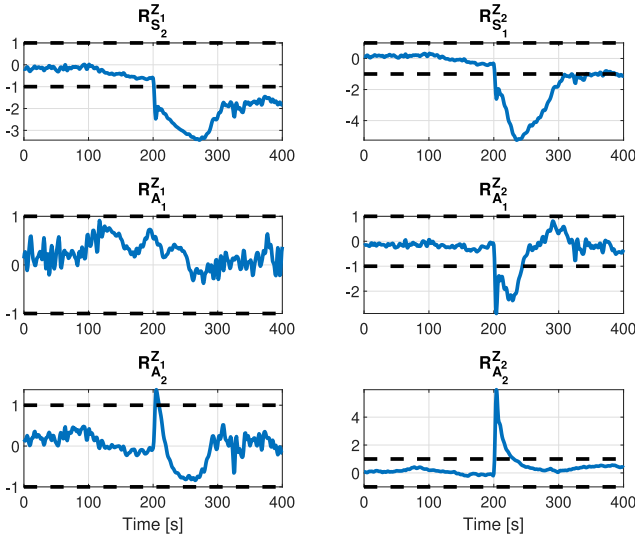


Figure 11: Case B: residues responses to a fault on sensor S_2 .

- Finally, Figure 12 shows that if a fault affects the sensor S_3 , then all the residues are affected, except

$R_{A_1}^{Z_1}$ and $R_{A_2}^{Z_2}$ which confirms the signature Σ_3 of the matrix \tilde{S} .

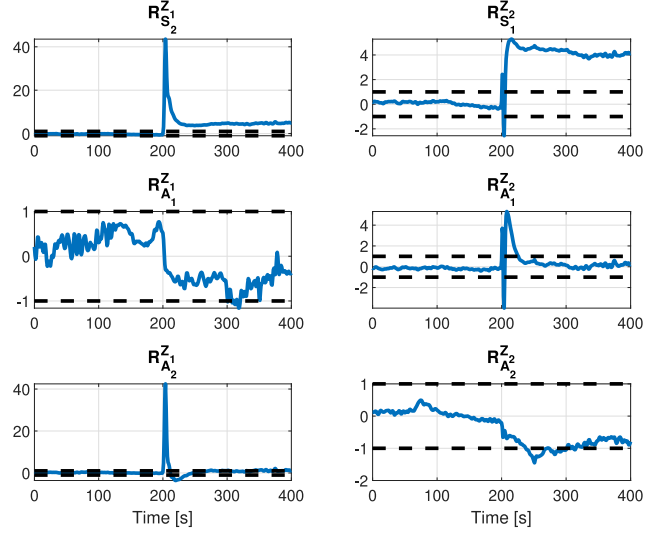


Figure 12: Case B: residues responses to a fault on sensor S_3 .

6 Conclusion

In this paper, a FDI method based on the flatness property of nonlinear systems is presented. The flat output measurement is used to calculate the redundant variables and then generate the residues. Moreover, it has been shown that sometimes, using a single flat output is not sufficient to ensure full isolability and several flat outputs may be needed. These flat outputs must be independent in the sense that by using them together the number of isolable faults increases. Therefore, a full presentation of this flat output characterization is provided. Finally, the validity of this method has been shown experimentally on the DTS200 three-tank benchmark. These experiments have shown that this method can be sensitive to uncertainties and hence requires further development in the future.

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