

Computed Results

Presented According to their Pattern

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Notations

Let

$$F_{\mu\nu} := [\widehat{\nabla}_\mu, \widehat{\nabla}_\nu] = \widehat{\nabla}_\mu \widehat{\nabla}_\nu - \widehat{\nabla}_\nu \widehat{\nabla}_\mu$$

$$\widehat{\Delta}_{\mu\nu} := \frac{1}{2}(\widehat{\nabla}_\mu \widehat{\nabla}_\nu + \widehat{\nabla}_\nu \widehat{\nabla}_\mu)$$

so that

$$\widehat{\nabla}_{\mu\nu}^2 := \widehat{\nabla}_\mu \widehat{\nabla}_\nu = \widehat{\Delta}_{\mu\nu} + \frac{1}{2}F_{\mu\nu}$$

$$\widehat{\Delta} = g^{\mu\nu} \widehat{\nabla}_{\mu\nu}^2 = g^{\mu\nu} \widehat{\Delta}_{\mu\nu}$$

1 Collected patterns for $R_{4,0}$

1.1 No derivative of u

File: ../SPLIT-0/RTS+u.tex

$$R = \frac{1}{180} |R|^2 X_1[u] \quad (\#1)$$

File: ../SPLIT-0/RiccS+u.tex

$$R = -\frac{1}{180} |\text{Ric}|^2 X_1[u] \quad (\#1)$$

File: ../SPLIT-0/hlaplacianSC+u.tex

$$R = \frac{1}{30} (\widehat{\Delta} \mathfrak{R}) X_1[u] \quad (\#1)$$

File: ../SPLIT-0/SCS+u.tex

$$R = \frac{1}{72} \mathfrak{R}^2 X_1[u] \quad (\#1)$$

1.2 One first order derivative of u

File: ../SPLIT-0/gi(nu1nu2)*h nabla(nu2)SC+h nabla1(nu1)u.tex

$$\begin{aligned} R &= 8 g^{v_1 v_2} (\widehat{\nabla}_{v_2} \mathfrak{R}) X_5[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u)] \quad (\#1) \\ &\quad - (d-4) g^{v_1 v_2} (\widehat{\nabla}_{v_2} \mathfrak{R}) X_5[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u] \quad (\#2) \\ &\quad - (d-2) g^{v_1 v_2} (\widehat{\nabla}_{v_2} \mathfrak{R}) X_5[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u] \quad (\#3) \\ &\quad - \frac{1}{2} (d-2) g^{v_1 v_2} (\widehat{\nabla}_{v_2} \mathfrak{R}) X_5[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes u] \quad (\#4) \end{aligned}$$

1.3 Two first order derivatives of u

File: ../SPLIT-0/RicU(nu1nu2)+h nabla1(nu1)u*h nabla1(nu2)u.tex

$$\begin{aligned} R &= -4(3d+2) \text{Ric}^{v_1 v_2} X_6[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#1) \\ &\quad + \frac{1}{2} (d-2)(d-4) \text{Ric}^{v_1 v_2} X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#2) \\ &\quad + (d^2-2d+4) \text{Ric}^{v_1 v_2} X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#3) \\ &\quad - 2(d-2) \text{Ric}^{v_1 v_2} X_6[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#4) \\ &\quad + \frac{1}{6} (5d^2+10d+48) \text{Ric}^{v_1 v_2} X_6[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#5) \\ &\quad + \frac{1}{3} 2(d^2-d+6) \text{Ric}^{v_1 v_2} X_6[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u] \quad (\#6) \\ &\quad - \frac{1}{2} d(d+6) \text{Ric}^{v_1 v_2} X_6[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#7) \\ &\quad + \frac{1}{6} (d^2+2d+24) \text{Ric}^{v_1 v_2} X_6[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#8) \\ &\quad + \frac{1}{3} (d^2+2d+12) \text{Ric}^{v_1 v_2} X_6[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u] \quad (\#9) \\ &\quad - 2d \text{Ric}^{v_1 v_2} X_6[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u \otimes u] \quad (\#10) \end{aligned}$$

File: ../SPLIT-0/gi(nu1nu2)*SC+h nabla1(nu1)u*h nabla1(nu2)u.tex

$$\begin{aligned} R &= -4d g^{v_1 v_2} \mathfrak{R} X_6[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#1) \\ &\quad - 3d g^{v_1 v_2} \mathfrak{R} X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#2) \\ &\quad + \frac{1}{2} d(d-4) g^{v_1 v_2} \mathfrak{R} X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#3) \\ &\quad - 2d g^{v_1 v_2} \mathfrak{R} X_6[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#4) \\ &\quad + \frac{1}{3} d(d-4) g^{v_1 v_2} \mathfrak{R} X_6[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#5) \\ &\quad + \frac{1}{3} 2d(d-1) g^{v_1 v_2} \mathfrak{R} X_6[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u] \quad (\#6) \end{aligned}$$

$$-d g^{\nu_1 \nu_2} \Re X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#7)$$

$$+ \frac{1}{6} d(d-4) g^{\nu_1 \nu_2} \Re X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#8)$$

$$+ \frac{1}{3} d(d-1) g^{\nu_1 \nu_2} \Re X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u] \quad (\#9)$$

$$+ \frac{1}{2} d^2 g^{\nu_1 \nu_2} \Re X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u \otimes u] \quad (\#10)$$

1.4 Four first order derivatives of u

File: ../SPLIT-0/gi(nu1nu4)*gi(nu2nu3)+hnabla1(nu1)u*hnabla1(nu2)u*hnabla1(nu3)u*hnabla1(nu4)u.tex

[illegible]

File: ../SPLIT-0/gi(nu1nu2)*gi(nu3nu4)+hnabla1(nu1)u*hnabla1(nu2)u*hnabla1(nu3)u*hnabla1(nu4)u.tex

[illegible]

1.5 One second order derivative of u

File: ../SPLIT-0/SC+hlaplacianu.tex

$$\begin{aligned}
R &= 4\Re X_5[u \otimes u \otimes u \otimes u \otimes (\widehat{\Delta}u)] \quad (\#1) \\
&\quad - \frac{1}{2}(d-6)\Re X_5[u \otimes u \otimes u \otimes (\widehat{\Delta}u) \otimes u] \quad (\#2) \\
&\quad - \frac{1}{3}2(d-3)\Re X_5[u \otimes u \otimes (\widehat{\Delta}u) \otimes u \otimes u] \quad (\#3) \\
&\quad - \frac{1}{2}(d-2)\Re X_5[u \otimes (\widehat{\Delta}u) \otimes u \otimes u \otimes u] \quad (\#4)
\end{aligned}$$

1.5.1 Substitutions of double derivations

Simplifications of terms in:

File: SPLIT-0/RicU(nu1nu2)+h nabla2(nu1nu2)u

produce:

File: ../SPLIT-0-DOUBLE-DERIVATIONS-1-5-1/RicU(nu1nu2)+Sh nabla2(nu1nu2)u.tex

$$R = 8 \text{Ric}^{v_1 v_2} X_5[u \otimes u \otimes u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u)] \quad (\#1)$$

$$- (d-2) \text{Ric}^{v_1 v_2} X_5[u \otimes u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes u] \quad (\#2)$$

$$- \frac{1}{3} 2d \text{Ric}^{v_1 v_2} X_5[u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes u \otimes u] \quad (\#3)$$

1.6 Two first order derivatives of u, one second order derivative of u

File: ../SPLIT-0/gi(nu1nu2)+h laplacianu* h nabla1(nu1)u* h nabla1(nu2)u.tex

$$R = -48(d+3) g^{v_1 v_2} X_7[u \otimes u \otimes u \otimes u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#1)$$

$$+ 3(d^2 - 4d - 20) g^{v_1 v_2} X_7[u \otimes u \otimes u \otimes (\widehat{\Delta} u) \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#2)$$

$$+ 3(d-2)(d+2) g^{v_1 v_2} X_7[u \otimes u \otimes u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#3)$$

$$+ 6(2d^2 + 13d + 26) g^{v_1 v_2} X_7[u \otimes u \otimes u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#4)$$

$$+ 4(d^2 + 2d - 2) g^{v_1 v_2} X_7[u \otimes u \otimes (\widehat{\Delta} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#5)$$

$$+ 3d(d+2) g^{v_1 v_2} X_7[u \otimes u \otimes (\widehat{\Delta} u) \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#6)$$

$$- \frac{1}{2}(d^3 - 2d^2 - 32d - 80) g^{v_1 v_2} X_7[u \otimes u \otimes (\widehat{\Delta} u) \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#7)$$

$$+ 2(d^2 + 2d + 4) g^{v_1 v_2} X_7[u \otimes u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#8)$$

$$- \frac{1}{2}(d+2)(d^2 + 4d + 24) g^{v_1 v_2} X_7[u \otimes u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#9)$$

$$- (d^3 + 8d^2 + 36d + 56) g^{v_1 v_2} X_7[u \otimes u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u] \quad (\#10)$$

$$+ 3(d+2)^2 g^{v_1 v_2} X_7[u \otimes (\widehat{\Delta} u) \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#11)$$

$$+ 2(d+1)(d+2) g^{v_1 v_2} X_7[u \otimes (\widehat{\Delta} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#12)$$

$$- \frac{1}{2}(d+2)(d^2 + 4d + 12) g^{v_1 v_2} X_7[u \otimes (\widehat{\Delta} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#13)$$

$$+ (d-2)(d+2) g^{v_1 v_2} X_7[u \otimes (\widehat{\Delta} u) \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#14)$$

$$- \frac{1}{4}(d+2)(d^2 + 2d + 16) g^{v_1 v_2} X_7[u \otimes (\widehat{\Delta} u) \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#15)$$

$$- \frac{1}{2}(d+2)(d^2 + 4d + 12) g^{v_1 v_2} X_7[u \otimes (\widehat{\Delta} u) \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u] \quad (\#16)$$

$$- 6(d+2) g^{v_1 v_2} X_7[u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#17)$$

$$+ (d-2)(d+2) g^{v_1 v_2} X_7[u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#18)$$

$$+ 2(d+1)(d+2) g^{v_1 v_2} X_7[u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u] \quad (\#19)$$

$$+ 3(d+2)^2 g^{v_1 v_2} X_7[u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u \otimes u] \quad (\#20)$$

File: ../SPLIT-0/gi(nu1nu2)+h nabla1(nu1)u* h laplacianu* h nabla1(nu2)u.tex

$$R = -24(d+2) g^{v_1 v_2} X_7[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#1)$$

$$- 18(d+2) g^{v_1 v_2} X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#2)$$

$$+ 3(d^2 + 4d + 12) g^{v_1 v_2} X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#3)$$

$$+ 3(3d^2 + 20d + 44) g^{v_1 v_2} X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#4)$$

$$- 12(d+2) g^{v_1 v_2} X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#5)$$

$$+ 2(d^2 + 4d + 12) g^{v_1 v_2} X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\Delta} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#6)$$

$$+ 2(3d^2 + 20d + 44) g^{v_1 v_2} X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\Delta} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#7)$$

$$+ 2(d^2 + 4d + 12) g^{v_1 v_2} X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#8)$$

$$\begin{aligned}
& -\frac{1}{2}(d+2)(d^2+6d+32)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#9) \\
& - (d^3+10d^2+52d+88)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u] \quad (\#10) \\
& -6(d+2)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes u\otimes u\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#11) \\
& + (d^2+4d+12)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes u\otimes(\widehat{\Delta}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#12) \\
& + (3d^2+20d+44)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes u\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#13) \\
& + (d^2+4d+12)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\Delta}u)\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#14) \\
& -\frac{1}{4}(d+2)(d^2+6d+32)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\Delta}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#15) \\
& -\frac{1}{2}(d^3+10d^2+52d+88)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u] \quad (\#16) \\
& -6(d+2)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes u\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#17) \\
& + (d-2)(d+2)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#18) \\
& + 2(d+1)(d+2)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u] \quad (\#19) \\
& + 3(d+2)^2g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u\otimes u] \quad (\#20)
\end{aligned}$$

File: ../SPLIT-0/gi(nu1nu2)+h nabla1(nu1)u*h nabla1(nu2)u*hlaplacianu.tex

$$\begin{aligned}
R = & -24(d+2)g^{\nu_1\nu_2}X_7[u\otimes u\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)] \quad (\#1) \\
& -18(d+2)g^{\nu_1\nu_2}X_7[u\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)] \quad (\#2) \\
& +3(d-2)(d+2)g^{\nu_1\nu_2}X_7[u\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes(\widehat{\Delta}u)] \quad (\#3) \\
& +3(3d^2+16d+28)g^{\nu_1\nu_2}X_7[u\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)\otimes u] \quad (\#4) \\
& -12(d+2)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)] \quad (\#5) \\
& +2(d-2)(d+2)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes(\widehat{\Delta}u)] \quad (\#6) \\
& +2(3d^2+16d+28)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)\otimes u] \quad (\#7) \\
& +4(d+1)(d+2)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u\otimes(\widehat{\Delta}u)] \quad (\#8) \\
& -\frac{1}{2}d(d-2)(d+2)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes(\widehat{\Delta}u)\otimes u] \quad (\#9) \\
& -(d^3+8d^2+48d+104)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)\otimes u\otimes u] \quad (\#10) \\
& -6(d+2)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)] \quad (\#11) \\
& + (d-2)(d+2)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes(\widehat{\Delta}u)] \quad (\#12) \\
& + (3d^2+16d+28)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)\otimes u] \quad (\#13) \\
& + 2(d+1)(d+2)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u\otimes(\widehat{\Delta}u)] \quad (\#14) \\
& -\frac{1}{4}d(d-2)(d+2)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes(\widehat{\Delta}u)\otimes u] \quad (\#15) \\
& -\frac{1}{2}(d^3+8d^2+48d+104)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)\otimes u\otimes u] \quad (\#16) \\
& + 3(d+2)^2g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u\otimes u\otimes(\widehat{\Delta}u)] \quad (\#17) \\
& -\frac{1}{2}(d^3+6d^2+28d+56)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u\otimes(\widehat{\Delta}u)\otimes u] \quad (\#18) \\
& -\frac{1}{2}(d^3+4d^2+16d+40)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes(\widehat{\Delta}u)\otimes u\otimes u] \quad (\#19) \\
& + 6(d+2)(d+3)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)\otimes u\otimes u\otimes u] \quad (\#20)
\end{aligned}$$

1.6.1 First packet of 2 terms: substitutions of double derivations

Simplifications of terms in:

File: SPLIT-0/gi(nu1nu3)*gi(nu2nu4)+hnabla2(nu1nu2)u*hnabla1(nu3)u*hnabla1(nu4)u

File: SPLIT-0/gi(nu1nu4)*gi(nu2nu3)+hnabla2(nu1nu2)u*hnabla1(nu3)u*hnabla1(nu4)u

produce:

File: ../SPLIT-0-DOUBLE-DERIVATIONS-1-6-1/one+Shnabla2U(nu1nu2)u*hnabla1(nu1)u*hnabla1(nu2)u.tex

$$\begin{aligned}
R = & -48(d+4) X_7[u \otimes u \otimes u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#1) \\
& -24(d+4) X_7[u \otimes u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#2) \\
& +6(d+2)(d+4) X_7[u \otimes u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#3) \\
& +6(d+4)(3d+14) X_7[u \otimes u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#4) \\
& -8(d+4) X_7[u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#5) \\
& +2(d+2)(d+4) X_7[u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#6) \\
& +2(d+4)(3d+14) X_7[u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#7) \\
& +4(d+4)^2 X_7[u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#8) \\
& -(d+2)(d+4)^2 X_7[u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#9) \\
& -2(d+4)^3 X_7[u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u] \quad (\#10)
\end{aligned}$$

1.6.2 Second packet of 2 terms: substitutions of double derivations

Simplifications of terms in:

File: SPLIT-0/gi(nu1nu3)*gi(nu2nu4)+hnabla1(nu1)u*hnabla2(nu2nu3)u*hnabla1(nu4)u

File: SPLIT-0/gi(nu1nu2)*gi(nu3nu4)+hnabla1(nu1)u*hnabla2(nu2nu3)u*hnabla1(nu4)u

produce:

File: ../SPLIT-0-DOUBLE-DERIVATIONS-1-6-2/one+hnabla1(nu1)u*Shnabla2U(nu1nu2)u*hnabla1(nu2)u.tex

$$\begin{aligned}
R = & -48(d+2) X_7[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#1) \\
& -36(d+2) X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#2) \\
& +6(d^2+4d+12) X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#3) \\
& +6(3d^2+20d+44) X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#4) \\
& -24(d+2) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#5) \\
& +4(d^2+4d+12) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#6) \\
& +4(3d^2+20d+44) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#7) \\
& +4(d^2+4d+12) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#8) \\
& -(d+2)(d^2+6d+32) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#9) \\
& -2(d^3+10d^2+52d+88) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u] \quad (\#10) \\
& -12(d+2) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#11) \\
& +2(d^2+4d+12) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#12) \\
& +2(3d^2+20d+44) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#13) \\
& +2(d^2+4d+12) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#14) \\
& -\frac{1}{2}(d+2)(d^2+6d+32) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#15) \\
& -(d^3+10d^2+52d+88) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u] \quad (\#16) \\
& -12(d+2) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#17) \\
& +2(d-2)(d+2) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#18)
\end{aligned}$$

$$+ 4(d+1)(d+2) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u] \quad (\#19)$$

$$+ 6(d+2)^2 X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u \otimes u] \quad (\#20)$$

File: ../SPLIT-0-DOUBLE-DERIVATIONS-1-6-2/one+hnabla1(nu1)u*comFUu(nu1nu2)*hnabla1(nu2)u.tex

$$R = -24(d+2) X_7[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#1)$$

$$+ 3(d-2)(d+2) X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#2)$$

$$- 18(d+2) X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#3)$$

$$+ 3(d-2)(d+2) X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#4)$$

$$+ 4(d+1)(d+2) X_7[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#5)$$

$$+ 2(d-2)(d+2) X_7[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes [F^{\nu_1 \nu_2}, u] \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#6)$$

$$- \frac{1}{2}(d+2)(d^2 + 2d + 16) X_7[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#7)$$

$$- 12(d+2) X_7[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#8)$$

$$+ 2(d-2)(d+2) X_7[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#9)$$

$$+ 4(d+1)(d+2) X_7[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u] \quad (\#10)$$

$$+ 3(d+2)^2 X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes u \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#11)$$

$$+ 2(d+1)(d+2) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes [F^{\nu_1 \nu_2}, u] \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#12)$$

$$- \frac{1}{2}(d+2)(d^2 + 4d + 12) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#13)$$

$$+ (d-2)(d+2) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes [F^{\nu_1 \nu_2}, u] \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#14)$$

$$- \frac{1}{4}(d+2)(d^2 + 2d + 16) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes [F^{\nu_1 \nu_2}, u] \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#15)$$

$$- \frac{1}{2}(d+2)(d^2 + 4d + 12) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u] \quad (\#16)$$

$$- 6(d+2) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#17)$$

$$+ (d-2)(d+2) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#18)$$

$$+ 2(d+1)(d+2) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u] \quad (\#19)$$

$$+ 3(d+2)^2 X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u \otimes u] \quad (\#20)$$

1.6.3 Third packet of 2 terms: substitutions of double derivations

Simplifications of terms in:

File: SPLIT-0/gi(nu1nu3)*gi(nu2nu4)+hnabla1(nu1)u*hnabla1(nu2)u*hnabla2(nu3nu4)u

File: SPLIT-0/gi(nu1nu4)*gi(nu2nu3)+hnabla1(nu1)u*hnabla1(nu2)u*hnabla2(nu3nu4)u

produce:

File: ../SPLIT-0-DOUBLE-DERIVATIONS-1-6-3/one+hnabla1(nu1)u*hnabla1(nu2)u*Shnabla2U(nu1nu2)u.tex

$$\begin{aligned}
R &= 12(d+4)^2 X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u] \quad (\#1) \\
&+ 8(d+4)^2 X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u] \quad (\#2) \\
&+ 4(d+2)(d+4) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u] \quad (\#3) \\
&- 2(d+4)(d^2+8d+28) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes u] \quad (\#4) \\
&+ 4(d+4)^2 X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u] \quad (\#5) \\
&+ 2(d+2)(d+4) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u] \quad (\#6) \\
&- (d+4)(d^2+8d+28) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes u] \quad (\#7) \\
&- 8(d+4) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u] \quad (\#8) \\
&+ 2(d-2)(d+4) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes u] \quad (\#9) \\
&+ 6(d+2)(d+4) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes u \otimes u] \quad (\#10)
\end{aligned}$$

File: ../SPLIT-0-DOUBLE-DERIVATIONS-1-6-3/one+hnabla1(nu1)u*hnabla1(nu2)u*comFUu(nu1nu2).tex

$$\begin{aligned}
R &= -48(d+2) X_7[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes [F^{v_1 v_2}, u]] \quad (\#1) \\
&+ 6(d-2)(d+2) X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes [F^{v_1 v_2}, u]] \quad (\#2) \\
&+ 3(d+2)(d-8) X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes [F^{v_1 v_2}, u]] \quad (\#3) \\
&+ 3(d+2)(d-8) X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes [F^{v_1 v_2}, u] \otimes u] \quad (\#4) \\
&+ 8(d+1)(d+2) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes [F^{v_1 v_2}, u]] \quad (\#5) \\
&- \frac{1}{2}(d+2)(d^2-2d+24) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes [F^{v_1 v_2}, u]] \quad (\#6) \\
&- \frac{1}{2}(d+2)(d^2-2d+24) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes [F^{v_1 v_2}, u] \otimes u] \quad (\#7) \\
&+ 4(d-2)(d+2) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u \otimes [F^{v_1 v_2}, u]] \quad (\#8) \\
&+ 4(d-2)(d+2) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes [F^{v_1 v_2}, u] \otimes u] \quad (\#9) \\
&+ 4(d-2)(d+2) X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes [F^{v_1 v_2}, u] \otimes u \otimes u] \quad (\#10) \\
&+ 6(d+2)^2 X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes [F^{v_1 v_2}, u]] \quad (\#11) \\
&- \frac{1}{2}(d+2)(d^2+8) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes [F^{v_1 v_2}, u]] \quad (\#12) \\
&- \frac{1}{2}(d+2)(d^2+8) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes [F^{v_1 v_2}, u] \otimes u] \quad (\#13) \\
&- \frac{1}{2}(d+2)(d^2+2d+16) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u \otimes [F^{v_1 v_2}, u]] \quad (\#14) \\
&- \frac{1}{2}(d+2)(d^2+2d+16) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes [F^{v_1 v_2}, u] \otimes u] \quad (\#15) \\
&- \frac{1}{2}(d+2)(d^2+2d+16) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes [F^{v_1 v_2}, u] \otimes u \otimes u] \quad (\#16) \\
&+ 3d(d+2) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u \otimes u \otimes [F^{v_1 v_2}, u]] \quad (\#17) \\
&+ 3d(d+2) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes u \otimes [F^{v_1 v_2}, u] \otimes u] \quad (\#18) \\
&+ 3d(d+2) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes [F^{v_1 v_2}, u] \otimes u \otimes u] \quad (\#19) \\
&+ 3d(d+2) X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes [F^{v_1 v_2}, u] \otimes u \otimes u \otimes u] \quad (\#20)
\end{aligned}$$

1.7 Two second order derivatives of u

File: ../SPLIT-0/one+hlaplacianu*hlaplacianu.tex

$$R = 24 X_6[u \otimes u \otimes u \otimes u \otimes (\widehat{\Delta}u) \otimes (\widehat{\Delta}u)] \quad (\#1)$$

$$- 3(d-4) X_6[u \otimes u \otimes u \otimes (\widehat{\Delta}u) \otimes u \otimes (\widehat{\Delta}u)] \quad (\#2)$$

$$- 3(3d+4) X_6[u \otimes u \otimes u \otimes (\widehat{\Delta}u) \otimes (\widehat{\Delta}u) \otimes u] \quad (\#3)$$

$$- 4(d-1) X_6[u \otimes u \otimes (\widehat{\Delta}u) \otimes u \otimes u \otimes (\widehat{\Delta}u)] \quad (\#4)$$

$$+ \frac{1}{2}d(d-6) X_6[u \otimes u \otimes (\widehat{\Delta}u) \otimes u \otimes (\widehat{\Delta}u) \otimes u] \quad (\#5)$$

$$+ (d^2 + 2d + 12) X_6[u \otimes u \otimes (\widehat{\Delta}u) \otimes (\widehat{\Delta}u) \otimes u \otimes u] \quad (\#6)$$

$$- 3d X_6[u \otimes (\widehat{\Delta}u) \otimes u \otimes u \otimes u \otimes (\widehat{\Delta}u)] \quad (\#7)$$

$$+ \frac{1}{2}(d^2 + 8) X_6[u \otimes (\widehat{\Delta}u) \otimes u \otimes u \otimes (\widehat{\Delta}u) \otimes u] \quad (\#8)$$

$$+ \frac{1}{2}(d^2 + 8) X_6[u \otimes (\widehat{\Delta}u) \otimes u \otimes (\widehat{\Delta}u) \otimes u \otimes u] \quad (\#9)$$

$$- 3d X_6[u \otimes (\widehat{\Delta}u) \otimes (\widehat{\Delta}u) \otimes u \otimes u \otimes u] \quad (\#10)$$

1.7.1 Packet of 2 terms: substitutions of double derivations

Simplifications of terms in:

File: SPLIT-0/gi(nu1nu3)*gi(nu2nu4)+hnabla2(nu1nu2)u*hnabla2(nu3nu4)u

File: SPLIT-0/gi(nu1nu4)*gi(nu2nu3)+hnabla2(nu1nu2)u*hnabla2(nu3nu4)u

produce:

File: ../SPLIT-0-DOUBLE-DERIVATIONS-1-7/one+Shnabla2U(nu1nu2)u*Shnabla2(nu1nu2)u.tex

$$R = -12(d+4) X_6[u \otimes u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes u] \quad (\#1)$$

$$-4(d+4) X_6[u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes u] \quad (\#2)$$

$$+2(d+2)(d+4) X_6[u \otimes u \otimes (\widehat{\Delta}^{v_1 v_2} u) \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes u \otimes u] \quad (\#3)$$

File: ../SPLIT-0-DOUBLE-DERIVATIONS-1-7/one+comFUu(nu1nu2)*Shnabla2(nu1nu2)u.tex

$$R = -6(d+4) X_6[u \otimes u \otimes u \otimes [F^{v_1 v_2}, u] \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes u] \quad (\#1)$$

$$-2(d+4) X_6[u \otimes u \otimes [F^{v_1 v_2}, u] \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes u] \quad (\#2)$$

$$+(d+2)(d+4) X_6[u \otimes u \otimes [F^{v_1 v_2}, u] \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes u \otimes u] \quad (\#3)$$

File: ../SPLIT-0-DOUBLE-DERIVATIONS-1-7/one+Shnabla2(nu1nu2)u*comFUu(nu1nu2).tex

$$R = 2(d+2)(d-8) X_6[u \otimes u \otimes u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes [F^{v_1 v_2}, u]] \quad (\#1)$$

$$+2(d-2)(d+2) X_6[u \otimes u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes u \otimes [F^{v_1 v_2}, u]] \quad (\#2)$$

$$+2(d-2)(d+2) X_6[u \otimes u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes [F^{v_1 v_2}, u] \otimes u] \quad (\#3)$$

$$+d(d+2) X_6[u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes u \otimes u \otimes [F^{v_1 v_2}, u]] \quad (\#4)$$

$$+d(d+2) X_6[u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes u \otimes [F^{v_1 v_2}, u] \otimes u] \quad (\#5)$$

$$+d(d+2) X_6[u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u) \otimes [F^{v_1 v_2}, u] \otimes u \otimes u] \quad (\#6)$$

File: ../SPLIT-0-DOUBLE-DERIVATIONS-1-7/one+comFUu(nu1nu2)*comFu(nu1nu2).tex

$$R = (d+2)(d-8) X_6[u \otimes u \otimes u \otimes u \otimes [F^{v_1 v_2}, u] \otimes [F_{v_1 v_2}, u]] \quad (\#1)$$

$$+(d-2)(d+2) X_6[u \otimes u \otimes u \otimes [F^{v_1 v_2}, u] \otimes u \otimes [F_{v_1 v_2}, u]] \quad (\#2)$$

$$+(d-2)(d+2) X_6[u \otimes u \otimes u \otimes [F^{v_1 v_2}, u] \otimes [F_{v_1 v_2}, u] \otimes u] \quad (\#3)$$

$$+\frac{1}{2}d(d+2) X_6[u \otimes u \otimes [F^{v_1 v_2}, u] \otimes u \otimes u \otimes [F_{v_1 v_2}, u]] \quad (\#4)$$

$$+\frac{1}{2}d(d+2) X_6[u \otimes u \otimes [F^{v_1 v_2}, u] \otimes u \otimes [F_{v_1 v_2}, u] \otimes u] \quad (\#5)$$

$$+\frac{1}{2}d(d+2) X_6[u \otimes u \otimes [F^{v_1 v_2}, u] \otimes [F_{v_1 v_2}, u] \otimes u \otimes u] \quad (\#6)$$

1.8 One first order derivative of u , one third order derivative of u

File: ../SPLIT-0/gi(nu1nu3)*gi(nu2nu4)+hnabla1(nu1)u*hnabla3(nu2nu3nu4)u.tex

$$\begin{aligned}
R &= 16(d+1)g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_6[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2\nu_3\nu_4}^3u)] \quad (\#1) \\
&\quad - (d^2+8)g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\nabla}_{\nu_2\nu_3\nu_4}^3u)] \quad (\#2) \\
&\quad - (d^2+8)g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2\nu_3\nu_4}^3u) \otimes u] \quad (\#3) \\
&\quad - 8g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2\nu_3\nu_4}^3u)] \quad (\#4) \\
&\quad - (d^2+6d+16)g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\nabla}_{\nu_2\nu_3\nu_4}^3u) \otimes u] \quad (\#5) \\
&\quad + 4(d+2)g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2\nu_3\nu_4}^3u) \otimes u \otimes u] \quad (\#6) \\
&\quad + d(d+4)g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2\nu_3\nu_4}^3u)] \quad (\#7) \\
&\quad - 2(d+4)g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2\nu_3\nu_4}^3u) \otimes u] \quad (\#8)
\end{aligned}$$

File: ../SPLIT-0/gi(nu1nu3)*gi(nu2nu4)+hnabla3(nu1nu2nu3)u*hnabla1(nu4)u.tex

$$\begin{aligned}
R &= 8(d+4)g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_6[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1\nu_2\nu_3}^3u) \otimes (\widehat{\nabla}_{\nu_4}u)] \quad (\#1) \\
&\quad + 2(d+4)g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1\nu_2\nu_3}^3u) \otimes u \otimes (\widehat{\nabla}_{\nu_4}u)] \quad (\#2) \\
&\quad - (d+4)^2g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1\nu_2\nu_3}^3u) \otimes (\widehat{\nabla}_{\nu_4}u) \otimes u] \quad (\#3)
\end{aligned}$$

File: ../SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u*hnabla(nu2)hlaplacianu.tex

$$\begin{aligned}
R &= -8(d-2)g^{\nu_1\nu_2}X_6[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u))] \quad (\#1) \\
&\quad + \frac{1}{2}(d^2-6d+32)g^{\nu_1\nu_2}X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u))] \quad (\#2) \\
&\quad + \frac{1}{2}(d^2-18d+8)g^{\nu_1\nu_2}X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u)) \otimes u] \quad (\#3) \\
&\quad - 4(d-2)g^{\nu_1\nu_2}X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u))] \quad (\#4) \\
&\quad + (d^2-2d+8)g^{\nu_1\nu_2}X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u)) \otimes u] \quad (\#5) \\
&\quad + (d^2-2d+16)g^{\nu_1\nu_2}X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u)) \otimes u \otimes u] \quad (\#6) \\
&\quad - \frac{1}{2}d(d+10)g^{\nu_1\nu_2}X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u))] \quad (\#7) \\
&\quad + \frac{1}{2}(d^2-2d+8)g^{\nu_1\nu_2}X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u)) \otimes u] \quad (\#8) \\
&\quad + (d^2+2d+12)g^{\nu_1\nu_2}X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u)) \otimes u \otimes u] \quad (\#9) \\
&\quad - 6d g^{\nu_1\nu_2}X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u)) \otimes u \otimes u \otimes u] \quad (\#10)
\end{aligned}$$

File: ../SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u*hlaplacianhnabla(nu2)u.tex

$$\begin{aligned}
R &= -8(d+4)g^{\nu_1\nu_2}X_6[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u))] \quad (\#1) \\
&\quad + \frac{1}{2}(d-2)(d+8)g^{\nu_1\nu_2}X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u))] \quad (\#2) \\
&\quad + \frac{1}{2}(d+4)(d-10)g^{\nu_1\nu_2}X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u)) \otimes u] \quad (\#3) \\
&\quad + 4d g^{\nu_1\nu_2}X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u))] \quad (\#4) \\
&\quad - 8g^{\nu_1\nu_2}X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u)) \otimes u] \quad (\#5) \\
&\quad + (d+2)(d+4)g^{\nu_1\nu_2}X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u)) \otimes u \otimes u] \quad (\#6) \\
&\quad - \frac{1}{2}d(d-2)g^{\nu_1\nu_2}X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes u \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u))] \quad (\#7) \\
&\quad - \frac{1}{2}(d^2+2d+8)g^{\nu_1\nu_2}X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u)) \otimes u] \quad (\#8) \\
&\quad + 2(d+2)g^{\nu_1\nu_2}X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u)) \otimes u \otimes u] \quad (\#9)
\end{aligned}$$

File: ../SPLIT-0/gi(nu1nu2)+hnabla(nu1)hlaplacianu*hnabla1(nu2)u.tex

$$R = -4(d-2)g^{\nu_1\nu_2}X_6[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}(\widehat{\Delta}u)) \otimes (\widehat{\nabla}_{\nu_2}u)] \quad (\#1)$$

$$\begin{aligned}
& -4(d+1)g^{\nu_1\nu_2}X_6[u\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_1}(\widehat{\Delta}u))\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#2) \\
& +\frac{1}{2}(d^2-10d-32)g^{\nu_1\nu_2}X_6[u\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_1}(\widehat{\Delta}u))\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#3) \\
& -2(d+2)g^{\nu_1\nu_2}X_6[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}(\widehat{\Delta}u))\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#4) \\
& +\frac{1}{2}d(d+2)g^{\nu_1\nu_2}X_6[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}(\widehat{\Delta}u))\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#5) \\
& +(d+2)^2g^{\nu_1\nu_2}X_6[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}(\widehat{\Delta}u))\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u] \quad (\#6)
\end{aligned}$$

File: ../SPLIT-0/gi(nu1nu2)+hlaplacianhnabla(nu1)u*hnabla1(nu2)u.tex

$$\begin{aligned}
R &= -4(d-2)g^{\nu_1\nu_2}X_6[u\otimes u\otimes u\otimes u\otimes(\widehat{\Delta}(\widehat{\nabla}_{\nu_1}u))\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#1) \\
& -4(d+1)g^{\nu_1\nu_2}X_6[u\otimes u\otimes u\otimes(\widehat{\Delta}(\widehat{\nabla}_{\nu_1}u))\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#2) \\
& +\frac{1}{2}(d^2-10d-32)g^{\nu_1\nu_2}X_6[u\otimes u\otimes u\otimes(\widehat{\Delta}(\widehat{\nabla}_{\nu_1}u))\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#3) \\
& -2(d+2)g^{\nu_1\nu_2}X_6[u\otimes u\otimes(\widehat{\Delta}(\widehat{\nabla}_{\nu_1}u))\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#4) \\
& +\frac{1}{2}d(d+2)g^{\nu_1\nu_2}X_6[u\otimes u\otimes(\widehat{\Delta}(\widehat{\nabla}_{\nu_1}u))\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#5) \\
& +(d+2)^2g^{\nu_1\nu_2}X_6[u\otimes u\otimes(\widehat{\Delta}(\widehat{\nabla}_{\nu_1}u))\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u] \quad (\#6)
\end{aligned}$$

1.9 One fourth order derivative of u

File: ../SPLIT-0/gi(nu1nu3)*gi(nu2nu4)+hnabla4(nu1nu2nu3nu4)u.tex

$$R = -2(d+4)g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_5[u\otimes u\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_1\nu_2\nu_3\nu_4}^4u)] \quad (\#1)$$

File: ../SPLIT-0/gi(nu1nu2)+hnabla(nu1)hlaplacianhnabla(nu2)u.tex

$$\begin{aligned}
R &= 2d g^{\nu_1\nu_2}X_5[u\otimes u\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_1}(\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u)))] \quad (\#1) \\
& +(d+2)g^{\nu_1\nu_2}X_5[u\otimes u\otimes u\otimes(\widehat{\nabla}_{\nu_1}(\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u)))\otimes u] \quad (\#2)
\end{aligned}$$

File: ../SPLIT-0/one+hlaplacianhlaplacianu.tex

$$\begin{aligned}
R &= 8X_5[u\otimes u\otimes u\otimes u\otimes(\widehat{\Delta}(\widehat{\Delta}u))] \quad (\#1) \\
& -(d-4)X_5[u\otimes u\otimes u\otimes(\widehat{\Delta}(\widehat{\Delta}u))\otimes u] \quad (\#2) \\
& -dX_5[u\otimes u\otimes(\widehat{\Delta}(\widehat{\Delta}u))\otimes u\otimes u] \quad (\#3)
\end{aligned}$$

1.10 No derivative of u , no derivative of q

File: ../SPLIT-0/SC+q.tex

$$R = \frac{1}{3} \Re X_3[u \otimes u \otimes q] \quad (\#1)$$

$$+ \frac{1}{3} \Re X_3[u \otimes q \otimes u] \quad (\#2)$$

$$+ \frac{1}{3} \Re X_3[q \otimes u \otimes u] \quad (\#3)$$

File: ../SPLIT-0/one+q*q.tex

$$R = X_2[q \otimes q] \quad (\#1)$$

1.11 Two first order derivatives of u , no derivative of q

File: ../SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u*hnabla1(nu2)u*q.tex

$$R = -2(d+2) g^{\nu_1 \nu_2} X_5[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes q] \quad (\#1)$$

$$- (d+2) g^{\nu_1 \nu_2} X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes q] \quad (\#2)$$

$$+ \frac{1}{2}(d+2)^2 g^{\nu_1 \nu_2} X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes q] \quad (\#3)$$

$$+ \frac{1}{2}(d+2)^2 g^{\nu_1 \nu_2} X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes q \otimes u] \quad (\#4)$$

File: ../SPLIT-0/gi(nu1nu2)+q*hnabla1(nu1)u*hnabla1(nu2)u.tex

$$R = -2(d+2) g^{\nu_1 \nu_2} X_5[u \otimes u \otimes q \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#1)$$

$$-2(d+2) g^{\nu_1 \nu_2} X_5[u \otimes q \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#2)$$

$$- (d+2) g^{\nu_1 \nu_2} X_5[u \otimes q \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#3)$$

$$+ \frac{1}{2}(d+2)^2 g^{\nu_1 \nu_2} X_5[u \otimes q \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#4)$$

$$-2(d+2) g^{\nu_1 \nu_2} X_5[q \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#5)$$

$$- (d+2) g^{\nu_1 \nu_2} X_5[q \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#6)$$

$$+ \frac{1}{2}(d+2)^2 g^{\nu_1 \nu_2} X_5[q \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#7)$$

File: ../SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u*q*hnabla1(nu2)u.tex

$$R = -2(d+2) g^{\nu_1 \nu_2} X_5[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes q \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#1)$$

$$- (d+2) g^{\nu_1 \nu_2} X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes q \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#2)$$

$$- (d+2) g^{\nu_1 \nu_2} X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes q \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#3)$$

$$+ \frac{1}{2}(d+2)^2 g^{\nu_1 \nu_2} X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes q \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#4)$$

1.12 Laplacian of u , no derivative of q

File: ../SPLIT-0/one+hlaplacianu*q.tex

$$R = 2 X_4[u \otimes u \otimes (\widehat{\Delta} u) \otimes q] \quad (\#1)$$

$$- \frac{1}{2} d X_4[u \otimes (\widehat{\Delta} u) \otimes u \otimes q] \quad (\#2)$$

$$- \frac{1}{2} d X_4[u \otimes (\widehat{\Delta} u) \otimes q \otimes u] \quad (\#3)$$

File: ../SPLIT-0/one+q*hlaplacianu.tex

$$R = 2 X_4[u \otimes u \otimes q \otimes (\widehat{\Delta} u)] \quad (\#1)$$

$$+ 2 X_4[u \otimes q \otimes u \otimes (\widehat{\Delta} u)] \quad (\#2)$$

$$- \frac{1}{2} d X_4[u \otimes q \otimes (\widehat{\Delta} u) \otimes u] \quad (\#3)$$

$$+ 2 X_4[q \otimes u \otimes u \otimes (\widehat{\Delta} u)] \quad (\#4)$$

$$- \frac{1}{2} d X_4[q \otimes u \otimes (\widehat{\Delta} u) \otimes u] \quad (\#5)$$

1.13 One first order derivative of u , one first order derivative on q

File: ../SPLIT-0/gi(nu1nu2)+hnabla1(nu1)q*hnabla1(nu2)u.tex

$$R = 4 g^{\nu_1 \nu_2} X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} q) \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#1)$$

$$+ 2 g^{\nu_1 \nu_2} X_4[u \otimes (\widehat{\nabla}_{\nu_1} q) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#2)$$

$$- d g^{\nu_1 \nu_2} X_4[u \otimes (\widehat{\nabla}_{\nu_1} q) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#3)$$

File: ../SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u*hnabla1(nu2)q.tex

$$R = -(d+2) g^{\nu_1 \nu_2} X_4[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} q) \otimes u] \quad (\#1)$$

1.14 No derivative of u , one second order derivative of q

File: ../SPLIT-0/one+hlaplacianq.tex

$$R = X_3[u \otimes (\widehat{\Delta} q) \otimes u] \quad (\#1)$$

2 Collected patterns for $R_{4,2}$

File: ../SPLIT-2/gi(nu2nu3)+hnabla1(nu3)u*N(nu1).tex

$$\begin{aligned}
R &= 2g^{\nu_2\nu_3}X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_3}u) \otimes N^{\nu_1}]F_{\nu_1\nu_2} \quad (\#1) \\
&\quad - \frac{1}{2}dg^{\nu_2\nu_3}X_4[u \otimes (\widehat{\nabla}_{\nu_3}u) \otimes u \otimes N^{\nu_1}]F_{\nu_1\nu_2} \quad (\#2) \\
&\quad + \frac{1}{2}(d+4)g^{\nu_2\nu_3}X_4[u \otimes (\widehat{\nabla}_{\nu_3}u) \otimes N^{\nu_1} \otimes u]F_{\nu_1\nu_2} \quad (\#3)
\end{aligned}$$

File: ../SPLIT-2/gi(nu2nu3)+N(nu1)*hnabla1(nu3)u.tex

$$\begin{aligned}
R &= -2g^{\nu_2\nu_3}X_4[u \otimes u \otimes N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_3}u)]F_{\nu_1\nu_2} \quad (\#1) \\
&\quad + \frac{1}{2}dg^{\nu_2\nu_3}X_4[u \otimes N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_3}u) \otimes u]F_{\nu_1\nu_2} \quad (\#2) \\
&\quad + 2g^{\nu_2\nu_3}X_4[N^{\nu_1} \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_3}u)]F_{\nu_1\nu_2} \quad (\#3) \\
&\quad - \frac{1}{2}dg^{\nu_2\nu_3}X_4[N^{\nu_1} \otimes u \otimes (\widehat{\nabla}_{\nu_3}u) \otimes u]F_{\nu_1\nu_2} \quad (\#4)
\end{aligned}$$

File: ../SPLIT-2/one+N(nu1)*N(nu2).tex

$$\begin{aligned}
R &= \frac{1}{2}X_3[u \otimes N^{\nu_1} \otimes N^{\nu_2}]F_{\nu_1\nu_2} \quad (\#1) \\
&\quad - \frac{1}{2}X_3[N^{\nu_1} \otimes u \otimes N^{\nu_2}]F_{\nu_1\nu_2} \quad (\#2) \\
&\quad + \frac{1}{2}X_3[N^{\nu_1} \otimes N^{\nu_2} \otimes u]F_{\nu_1\nu_2} \quad (\#3)
\end{aligned}$$

File: ../SPLIT-2/gi(nu2nu3)+hnabla1(nu3)N(nu1).tex

$$R = -g^{\nu_2\nu_3}X_3[u \otimes (\widehat{\nabla}_{\nu_3}N^{\nu_1}) \otimes u]F_{\nu_1\nu_2} \quad (\#1)$$

File: ../SPLIT-2/one+hnabla1(nu1)u*hnabla1(nu2)u.tex

$$\begin{aligned}
R &= -8(d-2)X_6[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}u)]F^{\nu_1\nu_2} \quad (\#1) \\
&\quad + (d^2-4d+16)X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\nabla}_{\nu_2}u)]F^{\nu_1\nu_2} \quad (\#2) \\
&\quad - 8(d-2)X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}u) \otimes u]F^{\nu_1\nu_2} \quad (\#3) \\
&\quad + (d^2-2d+16)X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2}u)]F^{\nu_1\nu_2} \quad (\#4) \\
&\quad + (d^2-4d+8)X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\nabla}_{\nu_2}u) \otimes u]F^{\nu_1\nu_2} \quad (\#5) \\
&\quad - 8(d-1)X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}u) \otimes u \otimes u]F^{\nu_1\nu_2} \quad (\#6) \\
&\quad - 6dX_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2}u)]F^{\nu_1\nu_2} \quad (\#7) \\
&\quad + (d^2+8)X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2}u) \otimes u]F^{\nu_1\nu_2} \quad (\#8) \\
&\quad + (d^2+8)X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\nabla}_{\nu_2}u) \otimes u \otimes u]F^{\nu_1\nu_2} \quad (\#9) \\
&\quad - 6dX_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}u) \otimes u \otimes u \otimes u]F^{\nu_1\nu_2} \quad (\#10)
\end{aligned}$$

2.1 Substitutions of double derivations

Simplifications of terms in:

File: SPLIT-2/one+hnabla2(nu1nu2)u

produce:

File: ../SPLIT-2-DOUBLE-DERIVATIONS/one+comFu(nu1nu2).tex

$$R = -(3d-4) X_5[u \otimes u \otimes u \otimes u \otimes [F_{\nu_1 \nu_2}, u]] F^{\nu_1 \nu_2} \quad (\#1)$$

$$-2(d-1) X_5[u \otimes u \otimes u \otimes [F_{\nu_1 \nu_2}, u] \otimes u] F^{\nu_1 \nu_2} \quad (\#2)$$

$$-d X_5[u \otimes u \otimes [F_{\nu_1 \nu_2}, u] \otimes u \otimes u] F^{\nu_1 \nu_2} \quad (\#3)$$

3 Collected patterns for $R_{4,3}$

File: ../SPLIT-3/gi(nu1nu3)*gi(nu2nu4)+hnabla1(nu4)u.tex

$$\begin{aligned}
R &= -16 g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_5[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_4} u)](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#1) \\
&+ 2(d+2) g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_5[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_4} u) \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#2) \\
&+ 8 g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_5[u \otimes u \otimes (\widehat{\nabla}_{\nu_4} u) \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#3) \\
&- 2(d-2) g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_5[u \otimes (\widehat{\nabla}_{\nu_4} u) \otimes u \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#4)
\end{aligned}$$

File: ../SPLIT-3/gi(nu1nu2)*gi(nu3nu4)+hnabla1(nu4)u.tex

$$\begin{aligned}
R &= 8 g^{\nu_1 \nu_2} g^{\nu_3 \nu_4} X_5[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_4} u)](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#1) \\
&- (d+2) g^{\nu_1 \nu_2} g^{\nu_3 \nu_4} X_5[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_4} u) \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#2) \\
&- 4 g^{\nu_1 \nu_2} g^{\nu_3 \nu_4} X_5[u \otimes u \otimes (\widehat{\nabla}_{\nu_4} u) \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#3) \\
&+ (d-2) g^{\nu_1 \nu_2} g^{\nu_3 \nu_4} X_5[u \otimes (\widehat{\nabla}_{\nu_4} u) \otimes u \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#4)
\end{aligned}$$

File: ../SPLIT-3/gi(nu1nu4)*gi(nu2nu3)+hnabla1(nu4)u.tex

$$\begin{aligned}
R &= 8 g^{\nu_1 \nu_4} g^{\nu_2 \nu_3} X_5[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_4} u)](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#1) \\
&- (d+2) g^{\nu_1 \nu_4} g^{\nu_2 \nu_3} X_5[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_4} u) \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#2) \\
&- 4 g^{\nu_1 \nu_4} g^{\nu_2 \nu_3} X_5[u \otimes u \otimes (\widehat{\nabla}_{\nu_4} u) \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#3) \\
&+ (d-2) g^{\nu_1 \nu_4} g^{\nu_2 \nu_3} X_5[u \otimes (\widehat{\nabla}_{\nu_4} u) \otimes u \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#4)
\end{aligned}$$

File: ../SPLIT-3/gi(nu1nu2)+N(nu3).tex

$$\begin{aligned}
R &= -g^{\nu_1 \nu_2} X_4[u \otimes u \otimes u \otimes N^{\nu_3}](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#1) \\
&+ g^{\nu_1 \nu_2} X_4[u \otimes u \otimes N^{\nu_3} \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#2) \\
&+ g^{\nu_1 \nu_2} X_4[u \otimes N^{\nu_3} \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#3) \\
&- g^{\nu_1 \nu_2} X_4[N^{\nu_3} \otimes u \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#4)
\end{aligned}$$

File: ../SPLIT-3/gi(nu2nu3)+N(nu1).tex

$$\begin{aligned}
R &= -g^{\nu_2 \nu_3} X_4[u \otimes u \otimes u \otimes N^{\nu_1}](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#1) \\
&+ g^{\nu_2 \nu_3} X_4[u \otimes u \otimes N^{\nu_1} \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#2) \\
&+ g^{\nu_2 \nu_3} X_4[u \otimes N^{\nu_1} \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#3) \\
&- g^{\nu_2 \nu_3} X_4[N^{\nu_1} \otimes u \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#4)
\end{aligned}$$

File: ../SPLIT-3/gi(nu1nu3)+N(nu2).tex

$$\begin{aligned}
R &= 2 g^{\nu_1 \nu_3} X_4[u \otimes u \otimes u \otimes N^{\nu_2}](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#1) \\
&- 2 g^{\nu_1 \nu_3} X_4[u \otimes u \otimes N^{\nu_2} \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#2) \\
&- 2 g^{\nu_1 \nu_3} X_4[u \otimes N^{\nu_2} \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#3) \\
&+ 2 g^{\nu_1 \nu_3} X_4[N^{\nu_2} \otimes u \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#4)
\end{aligned}$$

4 Collected patterns for $R_{4,4}$

File: ../SPLIT-4/one+u.tex

$$R = \frac{1}{12} X_1[u] F^{\nu_1 \nu_2} F_{\nu_1 \nu_2} \quad (\#1)$$

5 R_4 for u parallel

File: ../SPLIT-u-parallel/RTS+u.tex

$$R = \frac{1}{180} |R|^2 X_1[u] \quad (\#1)$$

File: ../SPLIT-u-parallel/RiccS+u.tex

$$R = -\frac{1}{180} |\text{Ric}|^2 X_1[u] \quad (\#1)$$

File: ../SPLIT-u-parallel/hlaplacianSC+u.tex

$$R = \frac{1}{30} (\widehat{\Delta} \mathfrak{R}) X_1[u] \quad (\#1)$$

File: ../SPLIT-u-parallel/SCS+u.tex

$$R = \frac{1}{72} \mathfrak{R}^2 X_1[u] \quad (\#1)$$

File: ../SPLIT-u-parallel/SC+q.tex

$$\begin{aligned} R &= \frac{1}{3} \mathfrak{R} X_3[u \otimes u \otimes q] \quad (\#1) \\ &+ \frac{1}{3} \mathfrak{R} X_3[u \otimes q \otimes u] \quad (\#2) \\ &+ \frac{1}{3} \mathfrak{R} X_3[q \otimes u \otimes u] \quad (\#3) \end{aligned}$$

File: ../SPLIT-u-parallel/one+q*q.tex

$$R = X_2[q \otimes q] \quad (\#1)$$

File: ../SPLIT-u-parallel/one+hlaplacianq.tex

$$R = X_3[u \otimes (\widehat{\Delta} q) \otimes u] \quad (\#1)$$

File: ../SPLIT-u-parallel/hnabla(nu1)SC+N(nu1).tex

$$\begin{aligned} R &= -(\widehat{\nabla}_{\nu_1} \mathfrak{R}) X_4[u \otimes u \otimes u \otimes N^{\nu_1}] \quad (\#1) \\ &+ \frac{1}{2} (\widehat{\nabla}_{\nu_1} \mathfrak{R}) X_4[u \otimes N^{\nu_1} \otimes u \otimes u] \quad (\#2) \\ &+ \frac{1}{2} (\widehat{\nabla}_{\nu_1} \mathfrak{R}) X_4[N^{\nu_1} \otimes u \otimes u \otimes u] \quad (\#3) \end{aligned}$$

File: ../SPLIT-u-parallel/one+hnabla1(nu1)q*N(nu1).tex

$$R = -X_3[u \otimes (\widehat{\nabla}_{\nu_1} q) \otimes N^{\nu_1}] \quad (\#1)$$

File: ../SPLIT-u-parallel/one+N(nu1)*hnabla1(nu1)q.tex

$$R = X_3[N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_1} q) \otimes u] \quad (\#1)$$

File: ../SPLIT-u-parallel/Ric(nu1nu2)+N(nu1)*N(nu2).tex

$$\begin{aligned} R &= \frac{1}{3} \text{Ric}_{\nu_1 \nu_2} X_4[u \otimes u \otimes N^{\nu_1} \otimes N^{\nu_2}] \quad (\#1) \\ &- \frac{1}{6} \text{Ric}_{\nu_1 \nu_2} X_4[u \otimes N^{\nu_1} \otimes u \otimes N^{\nu_2}] \quad (\#2) \\ &+ \frac{1}{3} \text{Ric}_{\nu_1 \nu_2} X_4[u \otimes N^{\nu_1} \otimes N^{\nu_2} \otimes u] \quad (\#3) \\ &- \frac{1}{3} 2 \text{Ric}_{\nu_1 \nu_2} X_4[N^{\nu_1} \otimes u \otimes u \otimes N^{\nu_2}] \quad (\#4) \\ &- \frac{1}{6} \text{Ric}_{\nu_1 \nu_2} X_4[N^{\nu_1} \otimes u \otimes N^{\nu_2} \otimes u] \quad (\#5) \\ &+ \frac{1}{3} \text{Ric}_{\nu_1 \nu_2} X_4[N^{\nu_1} \otimes N^{\nu_2} \otimes u \otimes u] \quad (\#6) \end{aligned}$$

File: ../SPLIT-u-parallel/g(nu1nu2)*SC+N(nu1)*N(nu2).tex

$$R = -\frac{1}{6} g_{\nu_1 \nu_2} \mathfrak{R} X_4[u \otimes u \otimes N^{\nu_1} \otimes N^{\nu_2}] \quad (\#1)$$

$$-\frac{1}{6} g_{\nu_1 \nu_2} \Re X_4[u \otimes N^{\nu_1} \otimes u \otimes N^{\nu_2}] \quad (\#2)$$

$$-\frac{1}{6} g_{\nu_1 \nu_2} \Re X_4[u \otimes N^{\nu_1} \otimes N^{\nu_2} \otimes u] \quad (\#3)$$

$$-\frac{1}{6} g_{\nu_1 \nu_2} \Re X_4[N^{\nu_1} \otimes u \otimes u \otimes N^{\nu_2}] \quad (\#4)$$

$$-\frac{1}{6} g_{\nu_1 \nu_2} \Re X_4[N^{\nu_1} \otimes u \otimes N^{\nu_2} \otimes u] \quad (\#5)$$

$$-\frac{1}{6} g_{\nu_1 \nu_2} \Re X_4[N^{\nu_1} \otimes N^{\nu_2} \otimes u \otimes u] \quad (\#6)$$

File: ../SPLIT-u-parallel/g(nu1nu2)+q*N(nu1)*N(nu2).tex

$$R = -\frac{1}{2} g_{\nu_1 \nu_2} X_3[q \otimes N^{\nu_1} \otimes N^{\nu_2}] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu1nu2)+N(nu1)*q*N(nu2).tex

$$R = -\frac{1}{2} g_{\nu_1 \nu_2} X_3[N^{\nu_1} \otimes q \otimes N^{\nu_2}] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu1nu2)+N(nu1)*N(nu2)*q.tex

$$R = -\frac{1}{2} g_{\nu_1 \nu_2} X_3[N^{\nu_1} \otimes N^{\nu_2} \otimes q] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu1nu2)*g(nu3nu4)+N(nu1)*N(nu2)*N(nu3)*N(nu4).tex

$$R = \frac{1}{4} g_{\nu_1 \nu_2} g_{\nu_3 \nu_4} X_4[N^{\nu_1} \otimes N^{\nu_2} \otimes N^{\nu_3} \otimes N^{\nu_4}] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu1nu3)*g(nu2nu4)+N(nu1)*N(nu2)*N(nu3)*N(nu4).tex

$$R = \frac{1}{4} g_{\nu_1 \nu_3} g_{\nu_2 \nu_4} X_4[N^{\nu_1} \otimes N^{\nu_2} \otimes N^{\nu_3} \otimes N^{\nu_4}] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu1nu4)*g(nu2nu3)+N(nu1)*N(nu2)*N(nu3)*N(nu4).tex

$$R = \frac{1}{4} g_{\nu_1 \nu_4} g_{\nu_2 \nu_3} X_4[N^{\nu_1} \otimes N^{\nu_2} \otimes N^{\nu_3} \otimes N^{\nu_4}] \quad (\#1)$$

File: ../SPLIT-u-parallel/gi(nu1nu3)*Ric(nu2nu3)+h nabla1(nu1)N(nu2).tex

$$R = -g^{\nu_1 \nu_3} \text{Ric}_{\nu_2 \nu_3} X_4[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2})] \quad (\#1)$$

$$+ \frac{1}{3} g^{\nu_1 \nu_3} \text{Ric}_{\nu_2 \nu_3} X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes u] \quad (\#2)$$

$$+ \frac{1}{3} 2 g^{\nu_1 \nu_3} \text{Ric}_{\nu_2 \nu_3} X_4[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes u \otimes u] \quad (\#3)$$

File: ../SPLIT-u-parallel/SC+h nabla1(nu1)N(nu1).tex

$$R = -\Re X_4[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1})] \quad (\#1)$$

$$-\frac{1}{3} 2 \Re X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1}) \otimes u] \quad (\#2)$$

$$-\frac{1}{3} \Re X_4[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1}) \otimes u \otimes u] \quad (\#3)$$

File: ../SPLIT-u-parallel/one+q*h nabla1(nu1)N(nu1).tex

$$R = -X_3[u \otimes q \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1})] \quad (\#1)$$

$$-X_3[q \otimes u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1})] \quad (\#2)$$

File: ../SPLIT-u-parallel/one+h nabla1(nu1)N(nu1)*q.tex

$$R = -X_3[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1}) \otimes q] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu1nu3)+N(nu1)*h nabla1(nu2)N(nu2)*N(nu3).tex

$$R = \frac{1}{2} g_{\nu_1 \nu_3} X_4[u \otimes N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_2}) \otimes N^{\nu_3}] \quad (\#1)$$

$$+ \frac{1}{2} g_{\nu_1 \nu_3} X_4[N^{\nu_1} \otimes u \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_2}) \otimes N^{\nu_3}] \quad (\#2)$$

File: ../SPLIT-u-parallel/g(nu2nu3)+N(nu1)*hnabla1(nu1)N(nu2)*N(nu3).tex

$$R = -\frac{1}{2} g_{\nu_2 \nu_3} X_4[N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes u \otimes N^{\nu_3}] \quad (\#1)$$

$$-\frac{1}{2} g_{\nu_2 \nu_3} X_4[N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes N^{\nu_3} \otimes u] \quad (\#2)$$

File: ../SPLIT-u-parallel/g(nu1nu3)+N(nu1)*N(nu2)*hnabla1(nu2)N(nu3).tex

$$R = -\frac{1}{2} g_{\nu_1 \nu_3} X_4[N^{\nu_1} \otimes N^{\nu_2} \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_3}) \otimes u] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu1nu2)+N(nu1)*N(nu2)*hnabla1(nu3)N(nu3).tex

$$R = \frac{1}{2} g_{\nu_1 \nu_2} X_4[u \otimes N^{\nu_1} \otimes N^{\nu_2} \otimes (\widehat{\nabla}_{\nu_3} N^{\nu_3})] \quad (\#1)$$

$$+ \frac{1}{2} g_{\nu_1 \nu_2} X_4[N^{\nu_1} \otimes u \otimes N^{\nu_2} \otimes (\widehat{\nabla}_{\nu_3} N^{\nu_3})] \quad (\#2)$$

$$+ \frac{1}{2} g_{\nu_1 \nu_2} X_4[N^{\nu_1} \otimes N^{\nu_2} \otimes u \otimes (\widehat{\nabla}_{\nu_3} N^{\nu_3})] \quad (\#3)$$

File: ../SPLIT-u-parallel/g(nu1nu3)+N(nu1)*hnabla1(nu2)N(nu3)*N(nu2).tex

$$R = \frac{1}{2} g_{\nu_1 \nu_3} X_4[u \otimes N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_3}) \otimes N^{\nu_2}] \quad (\#1)$$

$$+ \frac{1}{2} g_{\nu_1 \nu_3} X_4[N^{\nu_1} \otimes u \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_3}) \otimes N^{\nu_2}] \quad (\#2)$$

File: ../SPLIT-u-parallel/g(nu2nu3)+N(nu1)*N(nu2)*hnabla1(nu1)N(nu3).tex

$$R = -\frac{1}{2} g_{\nu_2 \nu_3} X_4[N^{\nu_1} \otimes N^{\nu_2} \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_3}) \otimes u] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu2nu3)+hnabla1(nu1)N(nu2)*N(nu3)*N(nu1).tex

$$R = \frac{1}{2} g_{\nu_2 \nu_3} X_4[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes N^{\nu_3} \otimes N^{\nu_1}] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu2nu3)+hnabla1(nu1)N(nu2)*N(nu1)*N(nu3).tex

$$R = \frac{1}{2} g_{\nu_2 \nu_3} X_4[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes N^{\nu_1} \otimes N^{\nu_3}] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu2nu3)+hnabla1(nu1)N(nu1)*N(nu2)*N(nu3).tex

$$R = \frac{1}{2} g_{\nu_2 \nu_3} X_4[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1}) \otimes N^{\nu_2} \otimes N^{\nu_3}] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu2nu4)*gi(nu1nu3)+hnabla1(nu1)N(nu2)*hnabla1(nu3)N(nu4).tex

$$R = -g_{\nu_2 \nu_4} g^{\nu_1 \nu_3} X_4[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes (\widehat{\nabla}_{\nu_3} N^{\nu_4}) \otimes u] \quad (\#1)$$

File: ../SPLIT-u-parallel/one+hnabla1(nu1)N(nu1)*hnabla1(nu2)N(nu2).tex

$$R = 2 X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1}) \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_2})] \quad (\#1)$$

$$+ X_4[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1}) \otimes u \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_2})] \quad (\#2)$$

File: ../SPLIT-u-parallel/one+hnabla1(nu1)N(nu2)*hnabla1(nu2)N(nu1).tex

$$R = -X_4[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_1}) \otimes u] \quad (\#1)$$

File: ../SPLIT-u-parallel/one+N(nu1)*hnabla2(nu1nu2)N(nu2).tex

$$R = X_4[u \otimes u \otimes N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_1 \nu_2}^2 N^{\nu_2})] \quad (\#1)$$

$$-X_4[N^{\nu_1} \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1 \nu_2}^2 N^{\nu_2})] \quad (\#2)$$

$$-X_4[N^{\nu_1} \otimes u \otimes (\widehat{\nabla}_{\nu_1 \nu_2}^2 N^{\nu_2}) \otimes u] \quad (\#3)$$

File: ../SPLIT-u-parallel/one+N(nu1)*hnabla2(nu2nu1)N(nu2).tex

$$R = -X_4[u \otimes u \otimes N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_2 \nu_1}^2 N^{\nu_2})] \quad (\#1)$$

$$-X_4[u \otimes N^{v_1} \otimes (\widehat{\nabla}_{v_2 v_1}^2 N^{v_2}) \otimes u] \quad (\#2)$$

$$+X_4[N^{v_1} \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2 v_1}^2 N^{v_2})] \quad (\#3)$$

File: ../SPLIT-u-parallel/one+h nabla2(nu1nu2)N(nu2)*N(nu1).tex

$$R = X_4[u \otimes u \otimes (\widehat{\nabla}_{v_1 v_2}^2 N^{v_2}) \otimes N^{v_1}] \quad (\#1)$$

File: ../SPLIT-u-parallel/one+h nabla2(nu1nu2)N(nu1)*N(nu2).tex

$$R = X_4[u \otimes u \otimes (\widehat{\nabla}_{v_1 v_2}^2 N^{v_1}) \otimes N^{v_2}] \quad (\#1)$$

File: ../SPLIT-u-parallel/g(nu1nu2)+h laplacianN(nu1)*N(nu2).tex

$$R = -\frac{1}{2} g_{v_1 v_2} X_4[u \otimes (\widehat{\Delta} N^{v_1}) \otimes u \otimes N^{v_2}] \quad (\#1)$$

$$-\frac{1}{2} g_{v_1 v_2} X_4[u \otimes (\widehat{\Delta} N^{v_1}) \otimes N^{v_2} \otimes u] \quad (\#2)$$

File: ../SPLIT-u-parallel/g(nu1nu2)+N(nu1)*h laplacianN(nu2).tex

$$R = -\frac{1}{2} g_{v_1 v_2} X_4[u \otimes N^{v_1} \otimes (\widehat{\Delta} N^{v_2}) \otimes u] \quad (\#1)$$

$$-\frac{1}{2} g_{v_1 v_2} X_4[N^{v_1} \otimes u \otimes (\widehat{\Delta} N^{v_2}) \otimes u] \quad (\#2)$$

File: ../SPLIT-u-parallel/gi(nu1nu3)+h nabla3(nu1nu2nu3)N(nu2).tex

$$R = 2 g^{v_1 v_3} X_4[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1 v_2 v_3}^3 N^{v_2})] \quad (\#1)$$

File: ../SPLIT-u-parallel/one+h nabla(nu1)h laplacianN(nu1).tex

$$R = -X_4[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1}(\widehat{\Delta} N^{v_1}))] \quad (\#1)$$

$$-X_4[u \otimes u \otimes (\widehat{\nabla}_{v_1}(\widehat{\Delta} N^{v_1})) \otimes u] \quad (\#2)$$

File: ../SPLIT-u-parallel/one+h laplacianh nabla(nu1)N(nu1).tex

$$R = -X_4[u \otimes u \otimes u \otimes (\widehat{\Delta}(\widehat{\nabla}_{v_1} N^{v_1}))] \quad (\#1)$$

$$-X_4[u \otimes u \otimes (\widehat{\Delta}(\widehat{\nabla}_{v_1} N^{v_1})) \otimes u] \quad (\#2)$$

File: ../SPLIT-u-parallel/one+N(nu1)*N(nu2).tex

$$R = \frac{1}{2} X_3[u \otimes N^{v_1} \otimes N^{v_2}] F_{v_1 v_2} \quad (\#1)$$

$$-\frac{1}{2} X_3[N^{v_1} \otimes u \otimes N^{v_2}] F_{v_1 v_2} \quad (\#2)$$

$$+\frac{1}{2} X_3[N^{v_1} \otimes N^{v_2} \otimes u] F_{v_1 v_2} \quad (\#3)$$

File: ../SPLIT-u-parallel/gi(nu2nu3)+h nabla1(nu3)N(nu1).tex

$$R = -g^{v_2 v_3} X_3[u \otimes (\widehat{\nabla}_{v_3} N^{v_1}) \otimes u] F_{v_1 v_2} \quad (\#1)$$

File: ../SPLIT-u-parallel/gi(nu1nu2)+N(nu3).tex

$$R = -g^{v_1 v_2} X_4[u \otimes u \otimes u \otimes N^{v_3}](\nabla_{v_1 v_2 v_3}^3 \mathbf{1}) \quad (\#1)$$

$$+g^{v_1 v_2} X_4[u \otimes u \otimes N^{v_3} \otimes u](\nabla_{v_1 v_2 v_3}^3 \mathbf{1}) \quad (\#2)$$

$$+g^{v_1 v_2} X_4[u \otimes N^{v_3} \otimes u \otimes u](\nabla_{v_1 v_2 v_3}^3 \mathbf{1}) \quad (\#3)$$

$$-g^{v_1 v_2} X_4[N^{v_3} \otimes u \otimes u \otimes u](\nabla_{v_1 v_2 v_3}^3 \mathbf{1}) \quad (\#4)$$

File: ../SPLIT-u-parallel/gi(nu2nu3)+N(nu1).tex

$$R = -g^{v_2 v_3} X_4[u \otimes u \otimes u \otimes N^{v_1}](\nabla_{v_1 v_2 v_3}^3 \mathbf{1}) \quad (\#1)$$

$$+g^{v_2 v_3} X_4[u \otimes u \otimes N^{v_1} \otimes u](\nabla_{v_1 v_2 v_3}^3 \mathbf{1}) \quad (\#2)$$

$$+ g^{\nu_2 \nu_3} X_4[u \otimes N^{\nu_1} \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#3)$$

$$- g^{\nu_2 \nu_3} X_4[N^{\nu_1} \otimes u \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#4)$$

File: ../SPLIT-u-parallel/gi(nu1nu3)+N(nu2).tex

$$R = 2 g^{\nu_1 \nu_3} X_4[u \otimes u \otimes u \otimes N^{\nu_2}](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#1)$$

$$- 2 g^{\nu_1 \nu_3} X_4[u \otimes u \otimes N^{\nu_2} \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#2)$$

$$- 2 g^{\nu_1 \nu_3} X_4[u \otimes N^{\nu_2} \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#3)$$

$$+ 2 g^{\nu_1 \nu_3} X_4[N^{\nu_2} \otimes u \otimes u \otimes u](\nabla_{\nu_1 \nu_2 \nu_3}^3 \mathbf{1}) \quad (\#4)$$

File: ../SPLIT-u-parallel/one+u.tex

$$R = \frac{1}{12} X_1[u] F^{\nu_1 \nu_2} F_{\nu_1 \nu_2} \quad (\#1)$$

6 R_2 for Non Commutative Torus: P_1

6.1 $k(\partial_{\nu_1} k) \otimes k(\partial_{\nu_2} k)$

File: ../SPLIT-NCT-R2-P1/gi(nu1nu2)+kdelta1(nu1)k*kdelta1(nu2)k.tex

$$\begin{aligned} R &= \frac{1}{2}(d+2)^2 g^{\nu_1 \nu_2} X_4[u \otimes k(\delta_{\nu_1} k) \otimes k(\delta_{\nu_2} k) \otimes u] \quad (\#1) \\ &\quad - (d+2) g^{\nu_1 \nu_2} X_4[k(\delta_{\nu_1} k) \otimes u \otimes k(\delta_{\nu_2} k) \otimes u] \quad (\#2) \\ &\quad - 2(d+2) g^{\nu_1 \nu_2} X_4[k(\delta_{\nu_1} k) \otimes k(\delta_{\nu_2} k) \otimes u \otimes u] \quad (\#3) \end{aligned}$$

6.2 $k(\partial_{\nu_1} k) \otimes (\partial_{\nu_2} k)k$

File: ../SPLIT-NCT-R2-P1/gi(nu1nu2)+kdelta1(nu1)k*delta1(nu2)kk.tex

$$\begin{aligned} R &= -2d g^{\nu_1 \nu_2} X_4[u \otimes u \otimes k(\delta_{\nu_1} k) \otimes (\delta_{\nu_2} k)k] \quad (\#1) \\ &\quad - (d-2) g^{\nu_1 \nu_2} X_4[u \otimes k(\delta_{\nu_1} k) \otimes u \otimes (\delta_{\nu_2} k)k] \quad (\#2) \\ &\quad + \frac{1}{2}(d^2 + 2d + 8) g^{\nu_1 \nu_2} X_4[u \otimes k(\delta_{\nu_1} k) \otimes (\delta_{\nu_2} k)k \otimes u] \quad (\#3) \\ &\quad + 4 g^{\nu_1 \nu_2} X_4[k(\delta_{\nu_1} k) \otimes u \otimes u \otimes (\delta_{\nu_2} k)k] \quad (\#4) \\ &\quad - (d-2) g^{\nu_1 \nu_2} X_4[k(\delta_{\nu_1} k) \otimes u \otimes (\delta_{\nu_2} k)k \otimes u] \quad (\#5) \\ &\quad - 2d g^{\nu_1 \nu_2} X_4[k(\delta_{\nu_1} k) \otimes (\delta_{\nu_2} k)k \otimes u \otimes u] \quad (\#6) \end{aligned}$$

6.3 $(\partial_{\nu_1} k)k \otimes k(\partial_{\nu_2} k)$

File: ../SPLIT-NCT-R2-P1/gi(nu1nu2)+delta1(nu1)kk*kdelta1(nu2)k.tex

$$R = \frac{1}{2}(d+2)(d+4) g^{\nu_1 \nu_2} X_4[u \otimes (\delta_{\nu_1} k)k \otimes k(\delta_{\nu_2} k) \otimes u] \quad (\#1)$$

6.4 $(\partial_{\nu_1} k)k \otimes (\partial_{\nu_2} k)k$

File: ../SPLIT-NCT-R2-P1/gi(nu1nu2)+delta1(nu1)kk*delta1(nu2)kk.tex

$$\begin{aligned} R &= -2(d+2) g^{\nu_1 \nu_2} X_4[u \otimes u \otimes (\delta_{\nu_1} k)k \otimes (\delta_{\nu_2} k)k] \quad (\#1) \\ &\quad - (d+2) g^{\nu_1 \nu_2} X_4[u \otimes (\delta_{\nu_1} k)k \otimes u \otimes (\delta_{\nu_2} k)k] \quad (\#2) \\ &\quad + \frac{1}{2}(d+2)^2 g^{\nu_1 \nu_2} X_4[u \otimes (\delta_{\nu_1} k)k \otimes (\delta_{\nu_2} k)k \otimes u] \quad (\#3) \end{aligned}$$

6.5 $(\partial_{\nu_1} k)(\partial_{\nu_2} k)$

File: ../SPLIT-NCT-R2-P1/gi(nu1nu2)+delta1(nu1)kdelta1(nu2)k.tex

$$R = -(d+2) g^{\nu_1 \nu_2} X_3[u \otimes (\delta_{\nu_1} k)(\delta_{\nu_2} k) \otimes u] \quad (\#1)$$

6.6 $(\partial_{\nu_1 \nu_2}^2 k)k$

File: ../SPLIT-NCT-R2-P1/gi(nu1nu2)+delta2(nu1nu2)kk.tex

$$\begin{aligned} R &= 2 g^{\nu_1 \nu_2} X_3[u \otimes u \otimes (\delta_{\nu_1 \nu_2}^2 k)k] \quad (\#1) \\ &\quad - \frac{1}{2}d g^{\nu_1 \nu_2} X_3[u \otimes (\delta_{\nu_1 \nu_2}^2 k)k \otimes u] \quad (\#2) \end{aligned}$$

6.7 $k(\partial_{\nu_1 \nu_2}^2 k)$

File: ../SPLIT-NCT-R2-P1/gi(nu1nu2)+kdelta2(nu1nu2)k.tex

$$\begin{aligned} R &= -\frac{1}{2}d g^{\nu_1 \nu_2} X_3[u \otimes k(\delta_{\nu_1 \nu_2}^2 k) \otimes u] \quad (\#1) \\ &\quad + 2 g^{\nu_1 \nu_2} X_3[k(\delta_{\nu_1 \nu_2}^2 k) \otimes u \otimes u] \quad (\#2) \end{aligned}$$

7 R_2 for Non Commutative Torus: $P_1 + P_2$

7.1 $k(\partial_{\nu_1} k) \otimes k(\partial_{\nu_2} k)$

File: ../SPLIT-NCT-R2/fi(nu1nu2)+kdelta1(nu1)k*kdelta1(nu2)k.tex

$$\begin{aligned} R &= \frac{1}{2} d f^{\nu_1 \nu_2} X_3 [u \otimes k(\delta_{\nu_1} k) \otimes k(\delta_{\nu_2} k)] \quad (\#1) \\ &\quad - f^{\nu_1 \nu_2} X_3 [k(\delta_{\nu_1} k) \otimes u \otimes k(\delta_{\nu_2} k)] \quad (\#2) \\ &\quad + \frac{1}{2} d f^{\nu_1 \nu_2} X_3 [k(\delta_{\nu_1} k) \otimes k(\delta_{\nu_2} k) \otimes u] \quad (\#3) \end{aligned}$$

File: ../SPLIT-NCT-R2/gi(nu1nu2)+kdelta1(nu1)k*kdelta1(nu2)k.tex

$$\begin{aligned} R &= -d g^{\nu_1 \nu_2} X_4 [u \otimes u \otimes k(\delta_{\nu_1} k) \otimes k(\delta_{\nu_2} k)] \quad (\#1) \\ &\quad - \frac{1}{2} (d-2) g^{\nu_1 \nu_2} X_4 [u \otimes k(\delta_{\nu_1} k) \otimes u \otimes k(\delta_{\nu_2} k)] \quad (\#2) \\ &\quad + (d^2 + 4d + 6) g^{\nu_1 \nu_2} X_4 [u \otimes k(\delta_{\nu_1} k) \otimes k(\delta_{\nu_2} k) \otimes u] \quad (\#3) \\ &\quad + 2 g^{\nu_1 \nu_2} X_4 [k(\delta_{\nu_1} k) \otimes u \otimes u \otimes k(\delta_{\nu_2} k)] \quad (\#4) \\ &\quad - \frac{1}{2} (3d + 2) g^{\nu_1 \nu_2} X_4 [k(\delta_{\nu_1} k) \otimes u \otimes k(\delta_{\nu_2} k) \otimes u] \quad (\#5) \\ &\quad - (3d + 4) g^{\nu_1 \nu_2} X_4 [k(\delta_{\nu_1} k) \otimes k(\delta_{\nu_2} k) \otimes u \otimes u] \quad (\#6) \end{aligned}$$

7.2 $k(\partial_{\nu_1} k) \otimes (\partial_{\nu_2} k)k$

File: ../SPLIT-NCT-R2/fi(nu1nu2)+kdelta1(nu1)k*delta1(nu2)kk.tex

$$\begin{aligned} R &= \frac{1}{2} d f^{\nu_1 \nu_2} X_3 [u \otimes k(\delta_{\nu_1} k) \otimes (\delta_{\nu_2} k)k] \quad (\#1) \\ &\quad - f^{\nu_1 \nu_2} X_3 [k(\delta_{\nu_1} k) \otimes u \otimes (\delta_{\nu_2} k)k] \quad (\#2) \\ &\quad + \frac{1}{2} d f^{\nu_1 \nu_2} X_3 [k(\delta_{\nu_1} k) \otimes (\delta_{\nu_2} k)k \otimes u] \quad (\#3) \end{aligned}$$

File: ../SPLIT-NCT-R2/gi(nu1nu2)+kdelta1(nu1)k*delta1(nu2)kk.tex

$$\begin{aligned} R &= -3d g^{\nu_1 \nu_2} X_4 [u \otimes u \otimes k(\delta_{\nu_1} k) \otimes (\delta_{\nu_2} k)k] \quad (\#1) \\ &\quad - \frac{1}{2} 3(d-2) g^{\nu_1 \nu_2} X_4 [u \otimes k(\delta_{\nu_1} k) \otimes u \otimes (\delta_{\nu_2} k)k] \quad (\#2) \\ &\quad + (d^2 + 3d + 8) g^{\nu_1 \nu_2} X_4 [u \otimes k(\delta_{\nu_1} k) \otimes (\delta_{\nu_2} k)k \otimes u] \quad (\#3) \\ &\quad + 6 g^{\nu_1 \nu_2} X_4 [k(\delta_{\nu_1} k) \otimes u \otimes u \otimes (\delta_{\nu_2} k)k] \quad (\#4) \\ &\quad - \frac{1}{2} 3(d-2) g^{\nu_1 \nu_2} X_4 [k(\delta_{\nu_1} k) \otimes u \otimes (\delta_{\nu_2} k)k \otimes u] \quad (\#5) \\ &\quad - 3d g^{\nu_1 \nu_2} X_4 [k(\delta_{\nu_1} k) \otimes (\delta_{\nu_2} k)k \otimes u \otimes u] \quad (\#6) \end{aligned}$$

7.3 $(\partial_{\nu_1} k)k \otimes k(\partial_{\nu_2} k)$

File: ../SPLIT-NCT-R2/fi(nu1nu2)+delta1(nu1)kk*kdelta1(nu2)k.tex

$$\begin{aligned} R &= \frac{1}{2} d f^{\nu_1 \nu_2} X_3 [u \otimes (\delta_{\nu_1} k)k \otimes k(\delta_{\nu_2} k)] \quad (\#1) \\ &\quad - f^{\nu_1 \nu_2} X_3 [(\delta_{\nu_1} k)k \otimes u \otimes k(\delta_{\nu_2} k)] \quad (\#2) \\ &\quad + \frac{1}{2} d f^{\nu_1 \nu_2} X_3 [(\delta_{\nu_1} k)k \otimes k(\delta_{\nu_2} k) \otimes u] \quad (\#3) \end{aligned}$$

File: ../SPLIT-NCT-R2/gi(nu1nu2)+delta1(nu1)kk*kdelta1(nu2)k.tex

$$\begin{aligned} R &= -d g^{\nu_1 \nu_2} X_4 [u \otimes u \otimes (\delta_{\nu_1} k)k \otimes k(\delta_{\nu_2} k)] \quad (\#1) \\ &\quad - \frac{1}{2} (d-2) g^{\nu_1 \nu_2} X_4 [u \otimes (\delta_{\nu_1} k)k \otimes u \otimes k(\delta_{\nu_2} k)] \quad (\#2) \\ &\quad + (d^2 + 5d + 8) g^{\nu_1 \nu_2} X_4 [u \otimes (\delta_{\nu_1} k)k \otimes k(\delta_{\nu_2} k) \otimes u] \quad (\#3) \\ &\quad + 2 g^{\nu_1 \nu_2} X_4 [(\delta_{\nu_1} k)k \otimes u \otimes u \otimes k(\delta_{\nu_2} k)] \quad (\#4) \\ &\quad - \frac{1}{2} (d-2) g^{\nu_1 \nu_2} X_4 [(\delta_{\nu_1} k)k \otimes u \otimes k(\delta_{\nu_2} k) \otimes u] \quad (\#5) \\ &\quad - d g^{\nu_1 \nu_2} X_4 [(\delta_{\nu_1} k)k \otimes k(\delta_{\nu_2} k) \otimes u \otimes u] \quad (\#6) \end{aligned}$$

7.4 $(\partial_{\nu_1} k)k \otimes (\partial_{\nu_2} k)k$

File: ../SPLIT-NCT-R2/fi(nu1nu2)+delta1(nu1)kk*delta1(nu2)kk.tex

$$\begin{aligned} R &= \frac{1}{2} d f^{\nu_1 \nu_2} X_3[u \otimes (\delta_{\nu_1} k)k \otimes (\delta_{\nu_2} k)k] \quad (\#1) \\ &\quad - f^{\nu_1 \nu_2} X_3[(\delta_{\nu_1} k)k \otimes u \otimes (\delta_{\nu_2} k)k] \quad (\#2) \\ &\quad + \frac{1}{2} d f^{\nu_1 \nu_2} X_3[(\delta_{\nu_1} k)k \otimes (\delta_{\nu_2} k)k \otimes u] \quad (\#3) \end{aligned}$$

File: ../SPLIT-NCT-R2/gi(nu1nu2)+delta1(nu1)kk*delta1(nu2)kk.tex

$$\begin{aligned} R &= -(3d+4) g^{\nu_1 \nu_2} X_4[u \otimes u \otimes (\delta_{\nu_1} k)k \otimes (\delta_{\nu_2} k)k] \quad (\#1) \\ &\quad - \frac{1}{2} (3d+2) g^{\nu_1 \nu_2} X_4[u \otimes (\delta_{\nu_1} k)k \otimes u \otimes (\delta_{\nu_2} k)k] \quad (\#2) \\ &\quad + (d^2+4d+6) g^{\nu_1 \nu_2} X_4[u \otimes (\delta_{\nu_1} k)k \otimes (\delta_{\nu_2} k)k \otimes u] \quad (\#3) \\ &\quad + 2 g^{\nu_1 \nu_2} X_4[(\delta_{\nu_1} k)k \otimes u \otimes u \otimes (\delta_{\nu_2} k)k] \quad (\#4) \\ &\quad - \frac{1}{2} (d-2) g^{\nu_1 \nu_2} X_4[(\delta_{\nu_1} k)k \otimes u \otimes (\delta_{\nu_2} k)k \otimes u] \quad (\#5) \\ &\quad - d g^{\nu_1 \nu_2} X_4[(\delta_{\nu_1} k)k \otimes (\delta_{\nu_2} k)k \otimes u \otimes u] \quad (\#6) \end{aligned}$$

7.5 $(\partial_{\nu_1} k)(\partial_{\nu_2} k)$

File: ../SPLIT-NCT-R2/gi(nu1nu2)+delta1(nu1)kdelta1(nu2)k.tex

$$R = -2(d+2) g^{\nu_1 \nu_2} X_3[u \otimes (\delta_{\nu_1} k)(\delta_{\nu_2} k) \otimes u] \quad (\#1)$$

7.6 $(\partial_{\nu_1 \nu_2}^2 k)k$

File: ../SPLIT-NCT-R2/gi(nu1nu2)+delta2(nu1nu2)kk.tex

$$\begin{aligned} R &= 2 g^{\nu_1 \nu_2} X_3[u \otimes u \otimes (\delta_{\nu_1 \nu_2}^2 k)k] \quad (\#1) \\ &\quad - (d+1) g^{\nu_1 \nu_2} X_3[u \otimes (\delta_{\nu_1 \nu_2}^2 k)k \otimes u] \quad (\#2) \end{aligned}$$

7.7 $k(\partial_{\nu_1 \nu_2}^2 k)$

File: ../SPLIT-NCT-R2/gi(nu1nu2)+kdelta2(nu1nu2)k.tex

$$\begin{aligned} R &= -(d+1) g^{\nu_1 \nu_2} X_3[u \otimes k(\delta_{\nu_1 \nu_2}^2 k) \otimes u] \quad (\#1) \\ &\quad + 2 g^{\nu_1 \nu_2} X_3[k(\delta_{\nu_1 \nu_2}^2 k) \otimes u \otimes u] \quad (\#2) \end{aligned}$$