

# Checked Simplified Results Presented According to their Pattern

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# 1 Collected patterns for $R_{4,0}$

## 1.1 No derivative of $u$

File: SIMPLIFICATIONS/SPLIT-0/RTS+u.tex

$$R = \frac{1}{180} |R|^2 X_1[u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-0/RiccS+u.tex

$$R = -\frac{1}{180} |\text{Ric}|^2 X_1[u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-0/hlaplacianSC+u.tex

$$R = \frac{1}{30} (\widehat{\Delta} \mathfrak{R}) X_1[u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-0/SCS+u.tex

$$R = \frac{1}{72} \mathfrak{R}^2 X_1[u] \quad (\#1)$$

## 1.2 One first order derivative of $u$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)\*hnabla1(nu2)SC+hnabla1(nu1)u.tex

$$\begin{aligned} R &= -\frac{1}{12} (d-2) g^{\nu_1 \nu_2} (\widehat{\nabla}_{\nu_2} \mathfrak{R}) X_2[u \otimes (\widehat{\nabla}_{\nu_1} u)] \quad (\#1) \\ &+ \frac{1}{2} (d+2) g^{\nu_1 \nu_2} (\widehat{\nabla}_{\nu_2} \mathfrak{R}) X_4[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u)] \quad (\#2) \end{aligned}$$

## 1.3 Two first order derivatives of $u$

File: SIMPLIFICATIONS/SPLIT-0/RicU(nu1nu2)+hnabla1(nu1)u\*hnabla1(nu2)u.tex

$$\begin{aligned} R &= -\frac{1}{3} d \text{Ric}^{\nu_1 \nu_2} X_3[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#1) \\ &+ \frac{1}{3} (d+2)(d+3) \text{Ric}^{\nu_1 \nu_2} X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#2) \\ &- \frac{1}{12} d (d+2) \text{Ric}^{\nu_1 \nu_2} X_4[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#3) \\ &- (d+2)(d+4) \text{Ric}^{\nu_1 \nu_2} X_5[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#4) \\ &+ \frac{1}{4} (d+2)(d+4) \text{Ric}^{\nu_1 \nu_2} X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#5) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)\*SC+hnabla1(nu1)u\*hnabla1(nu2)u.tex

$$\begin{aligned} R &= -\frac{1}{6} d g^{\nu_1 \nu_2} \mathfrak{R} X_3[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#1) \\ &+ \frac{1}{12} d (d+2) g^{\nu_1 \nu_2} \mathfrak{R} X_4[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#2) \end{aligned}$$

## 1.4 Four first order derivatives of $u$

$$G(g)^{v_1 v_2 v_3 v_4} := \frac{1}{4} (g^{v_1 v_2} g^{v_3 v_4} + g^{v_1 v_3} g^{v_2 v_4} + g^{v_1 v_4} g^{v_2 v_3})$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)\*gi(nu3nu4)+hnabla1(nu1)u\*hnabla1(nu2)u\*hnabla1(nu3)u\*hnabla1(nu4)u-part1.tex

$$\begin{aligned} R = & -2(d+4)(d^2+10d+28)G(g)^{v_1 v_2 v_3 v_4} X_5[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#1) \\ & -4(d+4)(d^2+8d+28)G(g)^{v_1 v_2 v_3 v_4} X_6[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#2) \\ & -2(d+2)(d+4)(d+6)G(g)^{v_1 v_2 v_3 v_4} X_6[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#3) \\ & +2(d+4)^2(d+6)G(g)^{v_1 v_2 v_3 v_4} X_6[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#4) \\ & +2(d+4)(d+6)^2G(g)^{v_1 v_2 v_3 v_4} X_6[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes u \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#5) \\ & +4(d+4)(d+6)G(g)^{v_1 v_2 v_3 v_4} X_6[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u) \otimes u] \quad (\#6) \\ & +24(d+4)(d^2+10d+32)G(g)^{v_1 v_2 v_3 v_4} X_7[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#7) \\ & +16(d+4)(d^2+10d+28)G(g)^{v_1 v_2 v_3 v_4} X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#8) \\ & +8(d+4)(d^2+10d+32)G(g)^{v_1 v_2 v_3 v_4} X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#9) \\ & +4(d+4)(d^2+6d+16)G(g)^{v_1 v_2 v_3 v_4} X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes u \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#10) \\ & -32(d+4)^2G(g)^{v_1 v_2 v_3 v_4} X_7[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u) \otimes u] \quad (\#11) \\ & +8(d+4)^2(d+6)G(g)^{v_1 v_2 v_3 v_4} X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#12) \\ & +4(d+4)^2(d+6)G(g)^{v_1 v_2 v_3 v_4} X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#13) \\ & +2d(d+4)(d+6)G(g)^{v_1 v_2 v_3 v_4} X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes u \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#14) \\ & -16(d+4)(d+6)G(g)^{v_1 v_2 v_3 v_4} X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u) \otimes u] \quad (\#15) \\ & +2(d+4)(d+6)(d+8)G(g)^{v_1 v_2 v_3 v_4} X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u) \otimes u] \quad (\#16) \\ & +2(d+4)(d+6)(d+8)(d+10)G(g)^{v_1 v_2 v_3 v_4} X_8[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes u \otimes (\widehat{\nabla}_{v_4} u) \otimes u] \quad (\#17) \\ & +4(d+4)(d+6)(d+8)(d+10)G(g)^{v_1 v_2 v_3 v_4} X_8[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u) \otimes u \otimes u] \quad (\#18) \\ & +(d+4)(d+6)(d+8)(d+10)G(g)^{v_1 v_2 v_3 v_4} X_8[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes u \otimes (\widehat{\nabla}_{v_4} u) \otimes u] \quad (\#19) \\ & +2(d+4)(d+6)(d+8)(d+10)G(g)^{v_1 v_2 v_3 v_4} X_8[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u) \otimes u \otimes u] \quad (\#20) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)\*gi(nu3nu4)+hnabla1(nu1)u\*hnabla1(nu2)u\*hnabla1(nu3)u\*hnabla1(nu4)u-part2.tex

$$\begin{aligned} R = & -\frac{1}{2}(d+2)(d+4)^2 g^{v_1 v_2} g^{v_3 v_4} X_5[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#1) \\ & +\frac{1}{2}(d+2)(d+4)(d+6) g^{v_1 v_2} g^{v_3 v_4} X_6[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes u \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#2) \\ & +\frac{1}{2}(d+2)(d+4)(d+6) g^{v_1 v_2} g^{v_3 v_4} X_6[u \otimes (\widehat{\nabla}_{v_1} u) \otimes u \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#3) \\ & +(d+2)(d+4)(d+6) g^{v_1 v_2} g^{v_3 v_4} X_6[u \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u)] \quad (\#4) \\ & +\frac{1}{4}(d+2)(d+4)(d+6)(d+8) g^{v_1 v_2} g^{v_3 v_4} X_7[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u \otimes (\widehat{\nabla}_{v_3} u) \otimes (\widehat{\nabla}_{v_4} u) \otimes u] \quad (\#5) \end{aligned}$$

## 1.5 One second order derivative of $u$

File: SIMPLIFICATIONS/SPLIT-0/SC+hlaplacianu.tex

$$\begin{aligned} R = & -\frac{1}{12}(d-2)\Re X_2[u \otimes (\widehat{\Delta}u)] \quad (\#1) \\ & +\frac{1}{6}d\Re X_3[u \otimes u \otimes (\widehat{\Delta}u)] \quad (\#2) \end{aligned}$$

### 1.5.1 Substitutions of double derivations

File: SIMPLIFICATIONS/SPLIT-0-DOUBLE-DERIVATIONS-1-5-1/RicU(nu1nu2)+Shnabla2(nu1nu2)u.tex

$$\begin{aligned} R = & -\frac{1}{3}d\text{Ric}^{v_1 v_2} X_3[u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u)] \quad (\#1) \\ & +(d+2)\text{Ric}^{v_1 v_2} X_4[u \otimes u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} u)] \quad (\#2) \end{aligned}$$

## 1.6 Two first order derivatives of $u$ , one second order derivative of $u$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+hlaplacianu\*hnabla1(nu1)u\*hnabla1(nu2)u.tex

$$R = \frac{1}{2}(d+2)^2 g^{v_1 v_2} X_4[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#1)$$

$$\begin{aligned}
& -\frac{1}{2}(d+2)(d+4)g^{\nu_1\nu_2}X_5[u\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#2) \\
& - (d+4)^2g^{\nu_1\nu_2}X_5[u\otimes u\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#3) \\
& -\frac{1}{4}(d+2)(d+4)(d+6)g^{\nu_1\nu_2}X_6[u\otimes(\widehat{\Delta}u)\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#4) \\
& + (d+4)(d+6)g^{\nu_1\nu_2}X_6[u\otimes u\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#5) \\
& - (d+2)(d+4)(d+6)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u] \quad (\#6) \\
& -\frac{1}{2}(d+2)(d+4)(d+6)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#7) \\
& + 3(d+4)(d+6)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\Delta}u)\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#8) \\
& + 9(d+4)(d+6)g^{\nu_1\nu_2}X_7[u\otimes u\otimes u\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#9)
\end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u\*hlaplacianu\*hnabla1(nu2)u.tex

$$\begin{aligned}
R = & - (d+2)g^{\nu_1\nu_2}X_4[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#1) \\
& + \frac{1}{2}(d+4)(3d+14)g^{\nu_1\nu_2}X_5[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#2) \\
& - 2(d+4)(d+6)g^{\nu_1\nu_2}X_6[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u] \quad (\#3) \\
& - (d+4)(d+6)g^{\nu_1\nu_2}X_6[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#4) \\
& + \frac{1}{2}(d+4)(d+6)g^{\nu_1\nu_2}X_6[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\Delta}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#5) \\
& + (d+4)(d+6)g^{\nu_1\nu_2}X_6[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#6) \\
& - \frac{1}{2}(d+4)(d+6)(d+8)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u] \quad (\#7) \\
& - \frac{1}{4}(d+4)(d+6)(d+8)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\Delta}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#8) \\
& - (d+4)(d+6)(d+8)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u] \quad (\#9) \\
& - \frac{1}{2}(d+4)(d+6)(d+8)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\Delta}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#10)
\end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u\*hnabla1(nu2)u\*hlaplacianu.tex

$$\begin{aligned}
R = & - (d+2)g^{\nu_1\nu_2}X_4[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)] \quad (\#1) \\
& + \frac{1}{2}(d+4)(3d+10)g^{\nu_1\nu_2}X_5[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)\otimes u] \quad (\#2) \\
& + \frac{1}{2}(d+2)(d+4)g^{\nu_1\nu_2}X_5[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes(\widehat{\Delta}u)] \quad (\#3) \\
& - (d+4)(d+6)g^{\nu_1\nu_2}X_6[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)\otimes u\otimes u] \quad (\#4) \\
& + \frac{1}{2}(d+4)(d+6)g^{\nu_1\nu_2}X_6[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes(\widehat{\Delta}u)\otimes u] \quad (\#5) \\
& - \frac{1}{2}(d+4)(d+6)(d+8)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes(\widehat{\Delta}u)\otimes u\otimes u] \quad (\#6) \\
& - \frac{1}{2}(d+4)(d+6)(d+8)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u\otimes(\widehat{\Delta}u)\otimes u] \quad (\#7) \\
& - \frac{1}{2}(d+4)(d+6)(d+8)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)\otimes u\otimes u] \quad (\#8) \\
& - \frac{1}{4}(d+4)(d+6)(d+8)g^{\nu_1\nu_2}X_7[u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes(\widehat{\Delta}u)\otimes u] \quad (\#9) \\
& - (d+4)(d+6)(d+8)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes(\widehat{\Delta}u)\otimes u\otimes u] \quad (\#10) \\
& - \frac{1}{2}(d+4)(d+6)(d+8)g^{\nu_1\nu_2}X_7[u\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes(\widehat{\Delta}u)\otimes u] \quad (\#11)
\end{aligned}$$

### 1.6.1 First packet of 2 terms: substitutions of double derivations

File: SIMPLIFICATIONS/SPLIT-0-DOUBLE-DERIVATIONS-1-6-1/one+Shnabla2U(nu1nu2)u\*hnabla1(nu1)u\*hnabla1(nu2)u.tex

$$\begin{aligned}
R = & 2(d+4)(3d+16)X_5[u\otimes u\otimes(\widehat{\Delta}^{\nu_1\nu_2}u)\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#1) \\
& - 4(d+4)(d+6)X_6[u\otimes u\otimes(\widehat{\Delta}^{\nu_1\nu_2}u)\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#2) \\
& - 6(d+4)(d+6)X_6[u\otimes u\otimes(\widehat{\Delta}^{\nu_1\nu_2}u)\otimes u\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#3) \\
& - 18(d+4)(d+6)X_6[u\otimes u\otimes u\otimes(\widehat{\Delta}^{\nu_1\nu_2}u)\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)] \quad (\#4) \\
& - 2(d+4)(d+6)(d+8)X_7[u\otimes u\otimes(\widehat{\Delta}^{\nu_1\nu_2}u)\otimes(\widehat{\nabla}_{\nu_1}u)\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u\otimes u] \quad (\#5) \\
& - (d+4)(d+6)(d+8)X_7[u\otimes u\otimes(\widehat{\Delta}^{\nu_1\nu_2}u)\otimes(\widehat{\nabla}_{\nu_1}u)\otimes u\otimes(\widehat{\nabla}_{\nu_2}u)\otimes u] \quad (\#6)
\end{aligned}$$

### 1.6.2 Second packet of 2 terms: substitutions of double derivations

File: SIMPLIFICATIONS/SPLIT-0-DOUBLE-DERIVATIONS-1-6-2/one+hnabla1(nu1)u\*Shnabla2U(nu1nu2)u\*hnabla1(nu2)u.tex

$$\begin{aligned}
R = & -2(d+2) X_4[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#1) \\
& + (d+4)(3d+14) X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#2) \\
& -4(d+4)(d+6) X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u] \quad (\#3) \\
& -2(d+4)(d+6) X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#4) \\
& + (d+4)(d+6) X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#5) \\
& + 2(d+4)(d+6) X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#6) \\
& - (d+4)(d+6)(d+8) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u] \quad (\#7) \\
& - \frac{1}{2}(d+4)(d+6)(d+8) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#8) \\
& -2(d+4)(d+6)(d+8) X_7[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u] \quad (\#9) \\
& - (d+4)(d+6)(d+8) X_7[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#10)
\end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0-DOUBLE-DERIVATIONS-1-6-2/one+hnabla1(nu1)u\*comFUu(nu1nu2)\*hnabla1(nu2)u.tex

$$\begin{aligned}
R = & \frac{1}{2}(d+2)^2 X_4[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#1) \\
& - \frac{1}{2}(d+2)(d+4) X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#2) \\
& - (d+2)(d+4) X_5[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u)] \quad (\#3) \\
& - \frac{1}{4}(d+2)(d+4)(d+6) X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes [F^{\nu_1 \nu_2}, u] \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] \quad (\#4)
\end{aligned}$$

### 1.6.3 Third packet of 2 terms: substitutions of double derivations

File: SIMPLIFICATIONS/SPLIT-0-DOUBLE-DERIVATIONS-1-6-3/one+hnabla1(nu1)u\*hnabla1(nu2)u\*Shnabla2U(nu1nu2)u.tex

$$\begin{aligned}
R = & (d+4)(d+6) X_4[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u)] \quad (\#1) \\
& -4(d+4) X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes u] \quad (\#2) \\
& -2(d+4)(d+6) X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u)] \quad (\#3) \\
& +2(d+4)(d+6) X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u \otimes u \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u)] \quad (\#4) \\
& -2(d+4)(d+6) X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u)] \quad (\#5) \\
& -4(d+4)(d+6) X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u)] \quad (\#6) \\
& -6(d+4)(d+6) X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u)] \quad (\#7) \\
& - (d+4)(d+6)(d+8) X_7[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes u \otimes u] \quad (\#8) \\
& -2(d+4)(d+6)(d+8) X_7[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes u \otimes u] \quad (\#9)
\end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0-DOUBLE-DERIVATIONS-1-6-3/one+hnabla1(nu1)u\*hnabla1(nu2)u\*comFUu(nu1nu2).tex

$$\begin{aligned}
R = & \frac{1}{2}d(d+2) X_4[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes [F^{\nu_1 \nu_2}, u]] \quad (\#1) \\
& - \frac{1}{4}(d+2)(d+4)^2 X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes [F^{\nu_1 \nu_2}, u]] \quad (\#2) \\
& - (d+2)(d+4) X_5[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes [F^{\nu_1 \nu_2}, u]] \quad (\#3) \\
& + \frac{1}{2}(d+2)(d+4)(d+6) X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes [F^{\nu_1 \nu_2}, u]] \quad (\#4) \\
& + \frac{1}{2}(d+2)(d+4)(d+6) X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes [F^{\nu_1 \nu_2}, u]] \quad (\#5)
\end{aligned}$$

## 1.7 Two second order derivatives of $u$

File: SIMPLIFICATIONS/SPLIT-0/one+hlaplacianu\*hlaplacianu.tex

$$R = -\frac{1}{2}(3d+8) X_3[u \otimes (\widehat{\Delta} u) \otimes (\widehat{\Delta} u)] \quad (\#1)$$

$$\begin{aligned}
& + (d+4) X_4[u \otimes (\widehat{\Delta}u) \otimes u \otimes (\widehat{\Delta}u)] \quad (\#2) \\
& + (3d+10) X_4[u \otimes u \otimes (\widehat{\Delta}u) \otimes (\widehat{\Delta}u)] \quad (\#3) \\
& + \frac{1}{4}(d+4)(d+6) X_5[u \otimes (\widehat{\Delta}u) \otimes u \otimes (\widehat{\Delta}u) \otimes u] \quad (\#4) \\
& + 2(d+4) X_5[u \otimes (\widehat{\Delta}u) \otimes (\widehat{\Delta}u) \otimes u \otimes u] \quad (\#5) \\
& + (d+4)(d+6) X_6[u \otimes u \otimes (\widehat{\Delta}u) \otimes (\widehat{\Delta}u) \otimes u \otimes u] \quad (\#6)
\end{aligned}$$

### 1.7.1 Packet of 2 terms: substitutions of double derivations

File: SIMPLIFICATIONS/SPLIT-0-DOUBLE-DERIVATIONS-1-7/gi(nu1nu3)\*gi(nu2nu4)+hnabla2(nu1nu2)u\*Shnabla2(nu3nu4)u.tex

$$\begin{aligned}
R &= -4(d+4) X_5[u \otimes u \otimes (\widehat{\nabla}_{\nu_1 \nu_2}^2 u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes u] \quad (\#1) \\
& + 2(d+4)(d+6) X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1 \nu_2}^2 u) \otimes (\widehat{\Delta}^{\nu_1 \nu_2} u) \otimes u \otimes u] \quad (\#2)
\end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0-DOUBLE-DERIVATIONS-1-7/gi(nu1nu3)\*gi(nu2nu4)+hnabla2(nu1nu2)u\*comFu(nu3nu4).tex

$$\begin{aligned}
R &= \frac{1}{2}d(d+2) X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_1 \nu_2}^2 u) \otimes [F^{\nu_1 \nu_2}, u]] \quad (\#1) \\
& - (d+2)(d+4) X_5[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1 \nu_2}^2 u) \otimes [F^{\nu_1 \nu_2}, u]] \quad (\#2)
\end{aligned}$$

## 1.8 One first order derivative of $u$ , one third order derivative of $u$

$$\{\widehat{\Delta}, \widehat{\nabla}_{\nu}\} := \widehat{\Delta} \widehat{\nabla}_{\nu} + \widehat{\nabla}_{\nu} \widehat{\Delta}$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu3)\*gi(nu2nu4)+hnabla1(nu1)u\*hnabla3(nu2nu3nu4)u.tex

$$\begin{aligned}
R &= 2(d+2) g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2 \nu_3 \nu_4}^3 u)] \quad (\#1) \\
& - 2(d+4) g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2 \nu_3 \nu_4}^3 u)] \quad (\#2) \\
& - 2(d+4) g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_5[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2 \nu_3 \nu_4}^3 u)] \quad (\#3) \\
& + (d+4)(d+6) g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2 \nu_3 \nu_4}^3 u)] \quad (\#4) \\
& - (d+4)(d+6) g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2 \nu_3 \nu_4}^3 u) \otimes u] \quad (\#5) \\
& - (d+4)(d+6) g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2 \nu_3 \nu_4}^3 u) \otimes u] \quad (\#6) \\
& - (d+4)(d+6) g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2 \nu_3 \nu_4}^3 u)] \quad (\#7)
\end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu3)\*gi(nu2nu4)+hnabla3(nu1nu2nu3)u\*hnabla1(nu4)u.tex

$$\begin{aligned}
R &= 2(d+4) g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_5[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1 \nu_2 \nu_3}^3 u) \otimes (\widehat{\nabla}_{\nu_4} u)] \quad (\#1) \\
& - (d+4)(d+6) g^{\nu_1 \nu_3} g^{\nu_2 \nu_4} X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1 \nu_2 \nu_3}^3 u) \otimes (\widehat{\nabla}_{\nu_4} u) \otimes u] \quad (\#2)
\end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u\*hnabla(nu2)hlaplacianu.tex

$$\begin{aligned}
R &= \frac{1}{6}(d^2 + 4d + 24) g^{\nu_1 \nu_2} X_3[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u))] \quad (\#1) \\
& - \frac{1}{2}(d^2 + 10d + 32) g^{\nu_1 \nu_2} X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u))] \quad (\#2) \\
& + (d+2) g^{\nu_1 \nu_2} X_4[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u))] \quad (\#3) \\
& - \frac{1}{2}(d+4)(d+6) g^{\nu_1 \nu_2} X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u))] \quad (\#4) \\
& + \frac{1}{2}(d+4)(d+14) g^{\nu_1 \nu_2} X_5[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u))] \quad (\#5) \\
& - 2(d+4) g^{\nu_1 \nu_2} X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u)) \otimes u] \quad (\#6) \\
& - (d+4)(d+6) g^{\nu_1 \nu_2} X_6[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2}(\widehat{\Delta}u)) \otimes u \otimes u \otimes u] \quad (\#7)
\end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u\*hlaplacianhnabla(nu2)u.tex

$$R = -\frac{1}{12}(d+4)(d+6) g^{\nu_1 \nu_2} X_3[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2} u))] \quad (\#1)$$

$$\begin{aligned}
& + \frac{1}{2}(d+2)(d+4)g^{\nu_1\nu_2}X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u))] \quad (\#2) \\
& + (d+2)g^{\nu_1\nu_2}X_4[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u))] \quad (\#3) \\
& + \frac{1}{2}(d+4)(d+6)g^{\nu_1\nu_2}X_5[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u)) \otimes u \otimes u] \quad (\#4) \\
& - (d+2)(d+4)g^{\nu_1\nu_2}X_5[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u))] \quad (\#5) \\
& - (d+4)(d+6)g^{\nu_1\nu_2}X_6[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u)) \otimes u \otimes u \otimes u] \quad (\#6)
\end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+anticomHLHN(nu1)u\*hnabla1(nu2)u.tex

$$\begin{aligned}
R & = -(3d+10)g^{\nu_1\nu_2}X_4[u \otimes u \otimes (\{\widehat{\Delta}, \widehat{\nabla}_{\nu_1}\}u) \otimes (\widehat{\nabla}_{\nu_2}u)] \quad (\#1) \\
& + \frac{1}{2}(d+4)(d+6)g^{\nu_1\nu_2}X_5[u \otimes u \otimes (\{\widehat{\Delta}, \widehat{\nabla}_{\nu_1}\}u) \otimes (\widehat{\nabla}_{\nu_2}u) \otimes u] \quad (\#2) \\
& + 2(d+4)g^{\nu_1\nu_2}X_5[u \otimes u \otimes (\{\widehat{\Delta}, \widehat{\nabla}_{\nu_1}\}u) \otimes u \otimes (\widehat{\nabla}_{\nu_2}u)] \quad (\#3) \\
& + 8(d+4)g^{\nu_1\nu_2}X_5[u \otimes u \otimes u \otimes (\{\widehat{\Delta}, \widehat{\nabla}_{\nu_1}\}u) \otimes (\widehat{\nabla}_{\nu_2}u)] \quad (\#4) \\
& - (d+4)(d+6)g^{\nu_1\nu_2}X_6[u \otimes u \otimes u \otimes (\{\widehat{\Delta}, \widehat{\nabla}_{\nu_1}\}u) \otimes (\widehat{\nabla}_{\nu_2}u) \otimes u] \quad (\#5)
\end{aligned}$$

## 1.9 One fourth order derivative of $u$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu3)\*gi(nu2nu4)+hnabla4(nu1nu2nu3nu4)u.tex

$$R = -2(d+4)g^{\nu_1\nu_3}g^{\nu_2\nu_4}X_5[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1\nu_2\nu_3\nu_4}^4u)] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+hnabla(nu1)hlaplacianhnabla(nu2)u.tex

$$\begin{aligned}
R & = (d+2)g^{\nu_1\nu_2}X_4[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}(\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u)))] \quad (\#1) \\
& - 2(d+4)g^{\nu_1\nu_2}X_5[u \otimes u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1}(\widehat{\Delta}(\widehat{\nabla}_{\nu_2}u)))] \quad (\#2)
\end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0/one+hlaplacianhlaplacianu.tex

$$\begin{aligned}
R & = -\frac{1}{2}dX_3[u \otimes u \otimes (\widehat{\Delta}(\widehat{\Delta}u))] \quad (\#1) \\
& + 2(d+2)X_4[u \otimes u \otimes u \otimes (\widehat{\Delta}(\widehat{\Delta}u))] \quad (\#2) \\
& - 2(d+4)X_5[u \otimes u \otimes u \otimes u \otimes (\widehat{\Delta}(\widehat{\Delta}u))] \quad (\#3)
\end{aligned}$$

## 1.10 No derivative of $u$ , no derivative of $q$

File: SIMPLIFICATIONS/SPLIT-0/SC+q.tex

$$R = \frac{1}{6}\Re X_1[q] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-0/one+q\*q.tex

$$R = X_2[q \otimes q] \quad (\#1)$$

## 1.11 Two first order derivatives of $u$ , no derivative of $q$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u\*hnabla1(nu2)u\*q.tex

$$\begin{aligned}
R & = \frac{1}{2}(d+2)^2g^{\nu_1\nu_2}X_4[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}u) \otimes q] \quad (\#1) \\
& - \frac{1}{2}(d+2)(d+4)g^{\nu_1\nu_2}X_5[u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes u \otimes (\widehat{\nabla}_{\nu_2}u) \otimes q] \quad (\#2) \\
& - (d+2)(d+4)g^{\nu_1\nu_2}X_5[u \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}u) \otimes q] \quad (\#3)
\end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+q\*hnabla1(nu1)u\*hnabla1(nu2)u.tex

$$\begin{aligned}
R & = -(d+2)g^{\nu_1\nu_2}X_4[u \otimes q \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}u)] \quad (\#1) \\
& - (d+2)g^{\nu_1\nu_2}X_4[q \otimes u \otimes (\widehat{\nabla}_{\nu_1}u) \otimes (\widehat{\nabla}_{\nu_2}u)] \quad (\#2)
\end{aligned}$$

$$+ \frac{1}{2}(d+2)(d+4)g^{v_1 v_2} X_5[u \otimes q \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#3)$$

$$+ \frac{1}{2}(d+2)(d+4)g^{v_1 v_2} X_5[q \otimes u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#4)$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u\*q\*hnabla1(nu2)u.tex

$$R = -(d+2)g^{v_1 v_2} X_4[u \otimes (\widehat{\nabla}_{v_1} u) \otimes q \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#1)$$

$$+ \frac{1}{2}(d+2)(d+4)g^{v_1 v_2} X_5[u \otimes (\widehat{\nabla}_{v_1} u) \otimes q \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#2)$$

### 1.12 Laplacian of $u$ , no derivative of $q$

File: SIMPLIFICATIONS/SPLIT-0/one+hlaplacianu-propagate-q.tex

$$R = \sum_{\ell=1}^3 (i_q^{(\ell)} X_3)[u \otimes (\widehat{\Delta} u)] \quad (\#1)$$

$$- \frac{1}{2}(d+2) \sum_{\ell=1}^4 (i_q^{(\ell)} X_4)[u \otimes (\widehat{\Delta} u) \otimes u] \quad (\#2)$$

### 1.13 One first order derivative of $u$ , one first order derivative on $q$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+hnabla1(nu1)q\*hnabla1(nu2)u.tex

$$R = 2g^{v_1 v_2} X_3[u \otimes (\widehat{\nabla}_{v_1} q) \otimes (\widehat{\nabla}_{v_2} u)] \quad (\#1)$$

$$- (d+2)g^{v_1 v_2} X_4[u \otimes (\widehat{\nabla}_{v_1} q) \otimes (\widehat{\nabla}_{v_2} u) \otimes u] \quad (\#2)$$

File: SIMPLIFICATIONS/SPLIT-0/gi(nu1nu2)+hnabla1(nu1)u\*hnabla1(nu2)q.tex

$$R = -(d+2)g^{v_1 v_2} X_4[u \otimes (\widehat{\nabla}_{v_1} u) \otimes (\widehat{\nabla}_{v_2} q) \otimes u] \quad (\#1)$$

### 1.14 No derivative of $u$ , one second order derivative of $q$

File: SIMPLIFICATIONS/SPLIT-0/one+hlaplacianq.tex

$$R = X_3[u \otimes (\widehat{\Delta} q) \otimes u] \quad (\#1)$$



## 2 Collected patterns for $R_{4,2}$

File: SIMPLIFICATIONS/SPLIT-2/one+hnabla1(nu1)u\*hnabla1(nu2)u.tex

$$\begin{aligned}
R &= -d X_3[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u)] F^{\nu_1 \nu_2} \quad (\#1) \\
&\quad - 4 X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u)] F^{\nu_1 \nu_2} \quad (\#2) \\
&\quad + \frac{1}{2}(d+2)(d+4) X_5[u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] F^{\nu_1 \nu_2} \quad (\#3) \\
&\quad + 2(d+4) X_5[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u) \otimes u] F^{\nu_1 \nu_2} \quad (\#4) \\
&\quad + 4(d+4) X_5[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes (\widehat{\nabla}_{\nu_2} u)] F^{\nu_1 \nu_2} \quad (\#5) \\
&\quad + (d+4)(d+6) X_6[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] F^{\nu_1 \nu_2} \quad (\#6) \\
&\quad + (d+4)(d+6) X_6[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_1} u) \otimes u \otimes (\widehat{\nabla}_{\nu_2} u)] F^{\nu_1 \nu_2} \quad (\#7)
\end{aligned}$$

### 2.1 Substitutions of double derivations

File: SIMPLIFICATIONS/SPLIT-2-DOUBLE-DERIVATIONS/one+comFu(nu1nu2).tex

$$\begin{aligned}
R &= -\frac{1}{2} d X_3[u \otimes u \otimes [F_{\nu_1 \nu_2}, u]] F^{\nu_1 \nu_2} \quad (\#1) \\
&\quad + (d+2) X_4[u \otimes u \otimes u \otimes [F_{\nu_1 \nu_2}, u]] F^{\nu_1 \nu_2} \quad (\#2) \\
&\quad - (d+4) X_5[u \otimes u \otimes u \otimes u \otimes [F_{\nu_1 \nu_2}, u]] F^{\nu_1 \nu_2} \quad (\#3)
\end{aligned}$$

## 3 Collected patterns for $R_{4,3}$

File: SIMPLIFICATIONS/SPLIT-3/gi(nu1nu2)\*gi(nu3nu4)+hnabla1(nu4)u+hnabla(nu1)F(nu2nu3).tex

$$\begin{aligned}
R &= \frac{1}{6}(d-2) g^{\nu_1 \nu_2} g^{\nu_3 \nu_4} X_2[u \otimes (\widehat{\nabla}_{\nu_4} u)] (\widehat{\nabla}_{\nu_1} F_{\nu_2 \nu_3}) \quad (\#1) \\
&\quad - d g^{\nu_1 \nu_2} g^{\nu_3 \nu_4} X_3[u \otimes u \otimes (\widehat{\nabla}_{\nu_4} u)] (\widehat{\nabla}_{\nu_1} F_{\nu_2 \nu_3}) \quad (\#2) \\
&\quad + 2(d+2) g^{\nu_1 \nu_2} g^{\nu_3 \nu_4} X_4[u \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_4} u)] (\widehat{\nabla}_{\nu_1} F_{\nu_2 \nu_3}) \quad (\#3)
\end{aligned}$$

## 4 Collected patterns for $R_{4,4}$

File: SIMPLIFICATIONS/SPLIT-4/one+u.tex

$$R = \frac{1}{12} X_1[u] F^{\nu_1 \nu_2} F_{\nu_1 \nu_2} \quad (\#1)$$

Number of terms so far: 180 in 43 expressions.

## 5 Collected patterns for $R_{4,2}, R_{4,3}, R_{4,4}$ with $N^\nu$

File: SIMPLIFICATIONS/SPLIT-2/gi(nu2nu3)+hnabla1(nu3)u\*N(nu1).tex

$$\begin{aligned} R &= \frac{1}{2}(d+4)g^{\nu_2\nu_3}X_3[u \otimes (\widehat{\nabla}_{\nu_3}u) \otimes N^{\nu_1}]F_{\nu_1\nu_2} \quad (\#1) \\ &\quad - (d+2)g^{\nu_2\nu_3}X_4[u \otimes (\widehat{\nabla}_{\nu_3}u) \otimes u \otimes N^{\nu_1}]F_{\nu_1\nu_2} \quad (\#2) \\ &\quad - (d+2)g^{\nu_2\nu_3}X_4[u \otimes u \otimes (\widehat{\nabla}_{\nu_3}u) \otimes N^{\nu_1}]F_{\nu_1\nu_2} \quad (\#3) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-2/gi(nu2nu3)+N(nu1)\*hnabla1(nu3)u.tex

$$\begin{aligned} R &= \frac{1}{2}d g^{\nu_2\nu_3}X_3[u \otimes N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_3}u)]F_{\nu_1\nu_2} \quad (\#1) \\ &\quad - \frac{1}{2}d g^{\nu_2\nu_3}X_3[N^{\nu_1} \otimes u \otimes (\widehat{\nabla}_{\nu_3}u)]F_{\nu_1\nu_2} \quad (\#2) \\ &\quad + (d+2)g^{\nu_2\nu_3}X_4[N^{\nu_1} \otimes u \otimes u \otimes (\widehat{\nabla}_{\nu_3}u)]F_{\nu_1\nu_2} \quad (\#3) \\ &\quad - (d+2)g^{\nu_2\nu_3}X_4[u \otimes u \otimes N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_3}u)]F_{\nu_1\nu_2} \quad (\#4) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-2/one+N(nu1)\*N(nu2).tex

$$\begin{aligned} R &= \frac{1}{2}X_2[N^{\nu_1} \otimes N^{\nu_2}]F_{\nu_1\nu_2} \quad (\#1) \\ &\quad - X_3[N^{\nu_1} \otimes u \otimes N^{\nu_2}]F_{\nu_1\nu_2} \quad (\#2) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-2/gi(nu2nu3)+hnabla1(nu3)N(nu1).tex

$$R = -g^{\nu_2\nu_3}X_3[u \otimes (\widehat{\nabla}_{\nu_3}N^{\nu_1}) \otimes u]F_{\nu_1\nu_2} \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-3/gi(nu1nu2)+N(nu3)+hnabla(nu1)F(nu2nu3).tex

$$\begin{aligned} R &= \frac{1}{6}g^{\nu_1\nu_3}X_1[N^{\nu_2}](\widehat{\nabla}_{\nu_1}F_{\nu_2\nu_3}) \quad (\#1) \\ &\quad - g^{\nu_1\nu_3}X_3[u \otimes N^{\nu_2} \otimes u](\widehat{\nabla}_{\nu_1}F_{\nu_2\nu_3}) \quad (\#2) \end{aligned}$$

## 6 $R_4$ for $u$ parallel

$$G(g)_{v_1 v_2 v_3 v_4} := \frac{1}{4} (g_{v_1 v_2} g_{v_3 v_4} + g_{v_1 v_3} g_{v_2 v_4} + g_{v_1 v_4} g_{v_2 v_3})$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/RTS+u.tex

$$R = \frac{1}{180} |R|^2 X_1[u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/RiccS+u.tex

$$R = -\frac{1}{180} |\text{Ric}|^2 X_1[u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/hlaplacianSC+u.tex

$$R = \frac{1}{30} (\widehat{\Delta} \mathfrak{R}) X_1[u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/SCS+u.tex

$$R = \frac{1}{72} \mathfrak{R}^2 X_1[u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/SC+q.tex

$$R = \frac{1}{6} \mathfrak{R} X_1[q] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+q\*q.tex

$$R = X_2[q \otimes q] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+hlaplacianq.tex

$$R = X_3[u \otimes (\widehat{\Delta} q) \otimes u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/hnabla(nu1)SC+N(nu1).tex

$$R = \frac{1}{12} (\widehat{\nabla}_{v_1} \mathfrak{R}) X_1[N^{v_1}] \quad (\#1)$$

$$- \frac{1}{2} (\widehat{\nabla}_{v_1} \mathfrak{R}) X_3[u \otimes u \otimes N^{v_1}] \quad (\#2)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/Ric(nu1nu2)+N(nu1)\*N(nu2).tex

$$R = \frac{1}{6} \text{Ric}_{v_1 v_2} X_2[N^{v_1} \otimes N^{v_2}] \quad (\#1)$$

$$- \frac{1}{2} \text{Ric}_{v_1 v_2} X_3[N^{v_1} \otimes u \otimes N^{v_2}] \quad (\#2)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu1nu2)\*SC+N(nu1)\*N(nu2).tex

$$R = -\frac{1}{12} g_{v_1 v_2} \mathfrak{R} X_2[N^{v_1} \otimes N^{v_2}] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu1nu2)+N(nu1)\*N(nu2)-propagate-q.tex

$$R = -\frac{1}{2} g_{v_1 v_2} \sum_{\ell=1}^3 (i_q^{(\ell)} X_3) [N^{v_1} \otimes N^{v_2}] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/gi(nu1nu2)\*gi(nu3nu4)+N(nu1)\*N(nu2)\*N(nu3)\*N(nu4)-tensor.tex

$$R = G(g)_{v_1 v_2 v_3 v_4} X_4[N^{v_1} \otimes N^{v_2} \otimes N^{v_3} \otimes N^{v_4}] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+hnabla1(nu1)q\*N(nu1).tex

$$R = -X_3[u \otimes (\widehat{\nabla}_{v_1} q) \otimes N^{v_1}] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+N(nu1)\*hnabla1(nu1)q.tex

$$R = X_3[N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_1} q) \otimes u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/gi(nu1nu3)\*Ric(nu2nu3)+hnabla1(nu1)N(nu2).tex

$$R = \frac{1}{3} g^{\nu_1 \nu_3} \text{Ric}_{\nu_2 \nu_3} X_2[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2})] \quad (\#1)$$

$$- g^{\nu_1 \nu_3} \text{Ric}_{\nu_2 \nu_3} X_3[u \otimes u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2})] \quad (\#2)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/SC+hnabla1(nu1)N(nu1).tex

$$R = -\frac{1}{6} \Re X_2[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1})] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+hnabla1(nu1)N(nu1)-propagate-q.tex

$$R = -\sum_{\ell=1}^3 (i_q^{(\ell)} X_3)[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1})] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu1nu2)+N(nu1)\*N(nu2)\*hnabla1(nu3)N(nu3).tex

$$R = \frac{1}{2} g_{\nu_1 \nu_2} X_3[N^{\nu_1} \otimes N^{\nu_2} \otimes (\widehat{\nabla}_{\nu_3} N^{\nu_3})] \quad (\#1)$$

$$- \frac{1}{2} g_{\nu_1 \nu_2} X_4[N^{\nu_1} \otimes N^{\nu_2} \otimes (\widehat{\nabla}_{\nu_3} N^{\nu_3}) \otimes u] \quad (\#2)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu1nu3)+N(nu1)\*N(nu2)\*hnabla1(nu2)N(nu3).tex

$$R = -\frac{1}{2} g_{\nu_1 \nu_3} X_4[N^{\nu_1} \otimes N^{\nu_2} \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_3}) \otimes u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu2nu3)+N(nu1)\*N(nu2)\*hnabla1(nu1)N(nu3).tex

$$R = -\frac{1}{2} g_{\nu_2 \nu_3} X_4[N^{\nu_1} \otimes N^{\nu_2} \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_3}) \otimes u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu1nu3)+N(nu1)\*hnabla1(nu2)N(nu2)\*N(nu3).tex

$$R = \frac{1}{2} g_{\nu_1 \nu_3} X_4[N^{\nu_1} \otimes u \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_2}) \otimes N^{\nu_3}] \quad (\#1)$$

$$+ \frac{1}{2} g_{\nu_1 \nu_3} X_4[u \otimes N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_2}) \otimes N^{\nu_3}] \quad (\#2)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu1nu3)+N(nu1)\*hnabla1(nu2)N(nu3)\*N(nu2).tex

$$R = \frac{1}{2} g_{\nu_1 \nu_3} X_4[N^{\nu_1} \otimes u \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_3}) \otimes N^{\nu_2}] \quad (\#1)$$

$$+ \frac{1}{2} g_{\nu_1 \nu_3} X_4[u \otimes N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_2} N^{\nu_3}) \otimes N^{\nu_2}] \quad (\#2)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu2nu3)+N(nu1)\*hnabla1(nu1)N(nu2)\*N(nu3).tex

$$R = -\frac{1}{2} g_{\nu_2 \nu_3} X_4[N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes N^{\nu_3} \otimes u] \quad (\#1)$$

$$- \frac{1}{2} g_{\nu_2 \nu_3} X_4[N^{\nu_1} \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes u \otimes N^{\nu_3}] \quad (\#2)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu2nu3)+hnabla1(nu1)N(nu1)\*N(nu2)\*N(nu3).tex

$$R = \frac{1}{2} g_{\nu_2 \nu_3} X_4[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_1}) \otimes N^{\nu_2} \otimes N^{\nu_3}] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu2nu3)+hnabla1(nu1)N(nu2)\*N(nu1)\*N(nu3).tex

$$R = \frac{1}{2} g_{\nu_2 \nu_3} X_4[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes N^{\nu_1} \otimes N^{\nu_3}] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu2nu3)+hnabla1(nu1)N(nu2)\*N(nu3)\*N(nu1).tex

$$R = \frac{1}{2} g_{\nu_2 \nu_3} X_4[u \otimes (\widehat{\nabla}_{\nu_1} N^{\nu_2}) \otimes N^{\nu_3} \otimes N^{\nu_1}] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+hnabla1(nu1)N(nu2)\*hnabla1(nu2)N(nu1).tex

$$R = -X_4[u \otimes (\widehat{\nabla}_{v_1} N^{v_2}) \otimes (\widehat{\nabla}_{v_2} N^{v_1}) \otimes u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu2nu4)\*gi(nu1nu3)+hnabla1(nu1)N(nu2)\*hnabla1(nu3)N(nu4).tex

$$R = -g_{v_2 v_4} g^{v_1 v_3} X_4[u \otimes (\widehat{\nabla}_{v_1} N^{v_2}) \otimes (\widehat{\nabla}_{v_3} N^{v_4}) \otimes u] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+hnabla1(nu1)N(nu1)\*hnabla1(nu2)N(nu2).tex

$$\begin{aligned} R &= X_4[u \otimes (\widehat{\nabla}_{v_1} N^{v_1}) \otimes u \otimes (\widehat{\nabla}_{v_2} N^{v_2})] \quad (\#1) \\ &+ 2 X_4[u \otimes u \otimes (\widehat{\nabla}_{v_1} N^{v_1}) \otimes (\widehat{\nabla}_{v_2} N^{v_2})] \quad (\#2) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+N(nu1)\*comFN(nu1nu2nu2).tex

$$\begin{aligned} R &= \frac{1}{2} X_3[u \otimes N^{v_1} \otimes [F_{v_1 v_2}, N^{v_2}]] \quad (\#1) \\ &- \frac{1}{2} X_3[N^{v_1} \otimes u \otimes [F_{v_1 v_2}, N^{v_2}]] \quad (\#2) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+N(nu1)\*Shnabla2(nu1nu2)N(nu2).tex

$$\begin{aligned} R &= -X_3[N^{v_1} \otimes (\widehat{\Delta}_{v_1 v_2} N^{v_2}) \otimes u] \quad (\#1) \\ &+ 2 X_4[N^{v_1} \otimes (\widehat{\Delta}_{v_1 v_2} N^{v_2}) \otimes u \otimes u] \quad (\#2) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+Shnabla2(nu1nu2)N(nu2)\*N(nu1).tex

$$R = 2 X_4[u \otimes u \otimes (\widehat{\Delta}_{v_1 v_2} N^{v_1}) \otimes N^{v_2}] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu1nu2)+hlaplacianN(nu1)\*N(nu2).tex

$$\begin{aligned} R &= -\frac{1}{2} g_{v_1 v_2} X_3[u \otimes (\widehat{\Delta} N^{v_1}) \otimes N^{v_2}] \quad (\#1) \\ &+ g_{v_1 v_2} X_4[u \otimes u \otimes (\widehat{\Delta} N^{v_1}) \otimes N^{v_2}] \quad (\#2) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/g(nu1nu2)+N(nu1)\*hlaplacianN(nu2).tex

$$\begin{aligned} R &= -\frac{1}{2} g_{v_1 v_2} X_3[N^{v_1} \otimes (\widehat{\Delta} N^{v_2}) \otimes u] \quad (\#1) \\ &+ g_{v_1 v_2} X_4[N^{v_1} \otimes (\widehat{\Delta} N^{v_2}) \otimes u \otimes u] \quad (\#2) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/gi(nu1nu3)+hnabla3(nu1nu2nu3)N(nu2).tex

$$R = 2 g^{v_1 v_3} X_4[u \otimes u \otimes u \otimes (\widehat{\nabla}_{v_1 v_2 v_3}^3 N^{v_2})] \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+anticomHLHN(nu1)N(nu1).tex

$$\begin{aligned} R &= -X_3[u \otimes u \otimes (\{\widehat{\Delta}, \widehat{\nabla}_{v_1}\} N^{v_1})] \quad (\#1) \\ &+ 2 X_4[u \otimes u \otimes u \otimes (\{\widehat{\Delta}, \widehat{\nabla}_{v_1}\} N^{v_1})] \quad (\#2) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+N(nu1)\*N(nu2).tex

$$\begin{aligned} R &= \frac{1}{2} X_2[N^{v_1} \otimes N^{v_2}] F_{v_1 v_2} \quad (\#1) \\ &- X_3[N^{v_1} \otimes u \otimes N^{v_2}] F_{v_1 v_2} \quad (\#2) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/gi(nu2nu3)+hnabla1(nu3)N(nu1).tex

$$R = -g^{v_2 v_3} X_3[u \otimes (\widehat{\nabla}_{v_3} N^{v_1}) \otimes u] F_{v_1 v_2} \quad (\#1)$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/gi(nu1nu2)+N(nu3)+hnabla(nu1)F(nu2nu3).tex

$$\begin{aligned} R &= \frac{1}{6} g^{v_1 v_3} X_1[N^{v_2}] (\widehat{\nabla}_{v_1} F_{v_2 v_3}) \quad (\#1) \\ &- g^{v_1 v_3} X_3[u \otimes N^{v_2} \otimes u] (\widehat{\nabla}_{v_1} F_{v_2 v_3}) \quad (\#2) \end{aligned}$$

File: SIMPLIFICATIONS/SPLIT-u-parallel/one+u.tex

$$R = \frac{1}{12} X_1[u] F^{v_1 v_2} F_{v_1 v_2} \quad (\#1)$$

**Number of terms so far: 55.**