

Generalized adaptive partition-based method for two-stage stochastic linear problems

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Mots-clés : *Stochastic Optimization, 2-stage stochastic linear problems, Adaptive partition-based methods*

1 Introduction

Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space and $\boldsymbol{\xi} = (\mathbf{T}, \mathbf{h}) \in L_1(\Omega, \mathcal{A}, \mathbb{P}; \Xi)$ an integrable random variable with values in $\Xi := \mathbb{R}^{l \times n} \times \mathbb{R}^l$. We consider the following problem 2-stage stochastic linear problem with fixed recourse (i.e. W is deterministic) :

$$\min_{x \in \mathbb{R}_+^n} \{c^\top x + \mathbb{E}[Q(x, \boldsymbol{\xi})] \mid Ax = b\} \quad (1)$$

where the recourse cost is

$$Q(x, \boldsymbol{\xi}) := \min_{y \in \mathbb{R}_+^m} \{q^\top y \mid Tx + Wy = h\} \quad (2)$$

Adaptive Partition-based Methods (APM) are numerical methods to solve two-stage stochastic linear problems (2SLP). The core idea is to iteratively construct an *adapted* partition of the space of alea in order to aggregate scenarios while conserving the true value of the cost-to-go for the current first-stage control. Relying on the normal fan of the dual admissible set, we extend the classical and generalized APM method by i) extending the method to almost arbitrary 2SLP, ii) giving a necessary and sufficient condition for a partition to be adapted even for non-finite distribution, and iii) proving the convergence of the method. We give some additional insights by linking APM to the L-shaped algorithm.

2 Contribution and literature review

Most results for 2SLP with continuous distributions rely on discretizing the distributions. The Sample Average Approximation (SAA) method samples the costs and constraints. It relies on probabilistic results based on a uniform law of large number to give statistical guarantees. Obtaining an approximation with satisfying guarantees requires a large number of scenarios. Otherwise, when the support of the random variables are simplices, we can leverage convexity inequalities (like Jensen's and Edmundson-Madansky's) or moments inequalities to construct finite scenario trees such that the discretized problem yields upper or lower bound of the continuous one.

In each of these approaches, we solve an approximate version of the stochastic program, with or without guarantees. In any case, the number of scenarios increase the numerical burden of 2SLP.

In order to alleviate the computations, we can use scenario reduction methods. Some are based on heuristics, aiming at matching properties of the underlying distribution (e.g. matching moments), others are based on adequate distances on the scenario tree. Recently Song and

Luedtke [SL15] proposed an exact scenario reduction method, which yields a reduced problem with the same value as the original. This adaptive partition-based method (APM) is designed for finitely supported, fixed recourse, problems. It consists in partitioning the space of alea, and replacing each subset of scenarios by their weighted mean. We say that a partition is *adapted* to a first-stage control x if the aggregated recourse problem has the same value as the recourse problem with the original distribution. APM iteratively refines the partition so that, at each step, the new partition is adapted to the current first-stage control.

Van Ackooj, de Oliveira and Song improved the performance of APM by combining it with level decomposition methods ideas. Finally, Ramirez-Pico and Moreno extended Song and Luedtke algorithm in [RPM21] to problems with continuous distribution for the right-hand side (and deterministic costs and matrices of constraints). Their GAPM algorithm relies on a sufficient condition on the optimal dual variables for the partition to be adapted. Nevertheless, finding a partition which satisfies this condition is left to the user, thus requiring an analytical approach for each particular problem.

Thanks to polyhedral geometry tools, we give a new necessary and sufficient condition for a partition to be adapted. It relies on the normal fan of the dual admissible set. This allows us to give a general algorithm, called G²APM, that can be used for any 2SLP with fixed recourse, including problems with a general stochastic cost. In addition, we get converging upper bounds for free.

G²APM can also be extended to cases with general cost distribution, leaving only the recourse matrix \mathbf{W} with a finite support.

Finally, we show that G²APM, GAPM and APM can be interpreted as acceleration of L-shaped methods where we approximate the expected cost-to-go function by an exact tangent cone instead of a cutting hyperplane. This allows us to prove the convergence of both GAPM and G²APM and provide an explicit bound on the number of steps.

Références

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