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Fabio Acerbi

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# BYZANTINE RECENSIONS OF GREEK MATHEMATICAL AND ASTRONOMICAL TEXTS: A SURVEY

### FABIO ACERBI\*

CNRS, UMR8560 Centre Alexandre Koyré 27, Rue Damesme 75013 Paris (France) fabacerbi@gmail.com

#### Abstract

This article contains a fairly complete survey of Byzantine recensions of Greek mathematical and astronomical texts, along with an outline of their main stylistic features. Lists of such recensions are provided and discussed, keyed on two different parameters: the manuscript containing the revision and the revised work. Byzantine scholars likely or certainly to be associated with any of the recensions are briefly introduced. A final appendix offers a full analysis of the way a specific theorem was modified by a number of revisers. The article also includes extensive discussions of the two main mathematical encyclopaedias assembled during the Palaiologan period.

Key-words: Greek Science, Byzantine Recensions, Mathematics, Astronomy, Manuscripts

### Resumen

Este artículo contiene una panorámica que se prentende completa de las recensiones bizantinas de textos matemáticos y astronómicos griegos junto con un esbozo de sus rasgos estilísticos principales. Se proporcionan y discuten listas de tales recensiones organizadas según dos criterios: el manuscrito que contiene la revisión y la obra revisada. Se presenta brevemente a los estudiosos bizantinos que pueden ser asociados con cierta probabilidad a alguna de estas recensiones. Un apéndice final ofrece un análisis completo del modo en que un teorema específico fue modificado por cierto número de revisores. El artículo incluye asimismo discusiones amplias de las dos enciclopedias matemáticas principales reunidas durante la época paleóloga.

Metadata: Ciencia griega, Recensiones bizantinas, Matemáticas, Astronomía, Manuscritos

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# BYZANTINE RECENSIONS OF GREEK MATHEMATICAL AND ASTRONOMICAL TEXTS: A SURVEY

### FABIO ACERBI

#### **1. INTRODUCTION**

Classical scholars approached Byzantine recensions of Greek mathematical works as a text-critical problem: some editors were deluded by the better text provided by a specific branch of the tradition and regarded such text as nearer to the original, whereas in fact it was the result of a revision. Once perceived, the problem transformed into an interesting methodological challenge confronting philologists, since it is not obvious whether and how to accommodate the variant readings of a recension in a critical edition. Finally, studying Byzantine recensions has been recognized quite recently as an unexpectedly effective tool to get a more complete picture of the practice of science in Byzantium, as well as to outline more fully the intellectual profile of some of the scholars involved in such a practice<sup>1</sup>.

<sup>1</sup> The following sigla will be used throughout the article: AGE = Apollonii Pergaei quaeGraece exstant cum commentariis antiquis, ed. J. L. Heiberg, 2 vol., Lipsiae 1891-3; AOO = Archimedis opera omnia cum commentariis Eutocii, ed. J. L. Heiberg, 3 vol., Lipsiae 1910-15; AP = Autolycus de Pitane, ed. J. Mogenet, Louvain 1950; DOO = Diophanti Alexandrini opera omnia cum Graeciis commentariis, ed. P. Tannery, 2 vol., Lipsiae 1893-5; EOO = Euclidis opera omnia, edd. J. L. Heiberg, H. Menge, 8 vol., Lipsiae 1883-1916; GC = Le «Grand Commentaire» de Théon d'Alexandrie aux Tables Faciles de Ptolémée, ed. J. Mogenet, A. Tihon, 3 vol. (Studi e Testi 315, 340, 390), Città del Vaticano 1985-99; HOO = Heronis Alexandrini opera quae supersunt omnia, 5 vol., edd. J. L. Heiberg, L. Nix, W. Schmidt, H. Schöne, Lipsiae 1899-1914; iA = Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste, ed. A. Rome, 3 vol. (Studi e Testi 54, 72, 106), Città del Vaticano 1931-43; MGM = Mathematici Graeci minores, ed. J. L. Heiberg, København 1927; PC = Le "Petit Commentaire" de Théon d'Alexandrie aux Tables Faciles de Ptolémée, ed. A. Tihon, Città del Vaticano 1978; PoH = Porphyrios Kommentar zur Harmonielehre des Ptolemaios, ed. I. Düring, Göteborg 1932; POO = Claudii Ptolemaei opera quae exstant omnia, edd. J. L. Heiberg, W. Hübner, F. Lammert, 3 vol. in 5 tomes, Lipsiae 1898-1998; PtH = Die Harmonielehre des Klaudios Ptolemaios, ed. I. Düring, Göteborg 1930; PLP = E. Trapp – R. Walter – H.V. Beyer (eds.), Prosopographisches Lexikon der Palaiologenzeit, 12 vol., Wien 1976-96; RGK I-III = Repertorium der griechischen Kopisten 800-1600, I. Handschriften aus Bibliotheken Großbritanniens, A. Verzeichnis der Kopisten, E. Gamillscheg, D. Harlfinger (erst. von), B. Paläographische Charakteristika, H. Hunger (erst. von), C. Tafeln, II. Handschriften aus Bibliotheken Frankreichs

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The phenomenon of Byzantine recensions of Greek mathematical works has been dealt with in the best possible way by the Louvain philological school. The editions of A. Rome, J. Mogenet, A. Tihon -especially those of the latter- pay full and careful attention to singling out the several recensions featuring in the manuscript tradition of the edited work, and to presenting in a convenient way both their main characteristics and, in a separate apparatus, their variant readings. The present article could not have been written without the ground-breaking work of these scholars. Still, there is need for an overall assessment outlining the main characters of such recensions, the way they were actually done, the Byzantine scholars and the manuscripts involved. My aim has been to take a first step towards filling this lacuna; this I have done by collecting a fair amount of raw material, that I have organized as follows. In Sect. 2 the general stylistic characters of the recensions of Greek mathematical writings are introduced, keyed on the crucial notion of "saturation" of a mathematical text. Sect. 3 contains a list of such recensions made in Late Antiquity, followed by a short discussion. Sect. 4 offers a reasoned catalogue of manuscripts containing several recensions of ancient (and possibly Byzantine) treatises. Sect. 5 presents the extant documentary record adopting the approach "by ancient authors and their writings". Sect. 6 provides a list of scholars or scholarly circles certainly or probably to be associated with well-defined mathematical manuscripts. The Appendix will explain in detail, on the basis of a specific (and quite technical) example, how a theorem perceived as mathematically unsatisfactory was modified by Byzantine scholars. The article also includes extensive discussions of the two main mathematical encyclopaedias assembled during the Palaiologan period.

The structure of the article needs a few words of justification. Since its main goal is to present facts concerning (Byzantine) recensions, it does not have a "conclusion" and relegates a case study of the way a theorem was revised in an appendix. Still, the material I have collected needs being organized. Several categorizations can be used to order such a variegated documentary record:

• It can be organized as a list of ancient authors and their writings. This choice is natural since information on possible recensions of any Greek mathematical treatise can typically be found in the *prolegomena* of a critical edition of that very treatise. Still, this choice is ill-suited to providing usable information on the kind of scholarship practiced in Byzantium.

und Nachträge zu den Bibliotheken Großbritanniens, A. Verzeichnis der Kopisten, E. Gamillscheg, D. Harlfinger (erst. von), B. Paläographische Charakteristika, H. Hunger (erst. von), C. Tafeln, III. Handschriften aus Bibliotheken Roms mit dem Vatikan, A. Verzeichnis der Kopisten, E. Gamillscheg (erst. von), D. Harlfinger, P. Eleuteri (unter Mitarbeit von), B. Paläographische Charakteristika, H. Hunger (erst. von), C. Tafeln, Wien 1981-97.

- It can be organized as a list of authors of recensions. This choice faces an obvious shortcoming: the recensions are almost always anonymous. What is more, as we shall see in greater detail later, the modifications a strictly mathematical text can undergo were of a standard kind, and as impersonal as mathematical style itself is. Still, when confronted with the discursive portions of a mathematical writing, a reviser whose original writings in technical matters are otherwise known may reveal himself through his stylistic habits. This, and information coming from the manuscripts in which a given recension is contained will allow us to draw a fairly complete picture as to who, and for what purposes, engaged in revising a mathematical text coming from Greek antiquity. A couple of warnings are in order. First, as is usual when information is incomplete, one must repress the tendency to assign anonymous recensions to outstanding scholars with a conspicuous mathematical penchant, such as Maximus Planudes, Theodorus Metochites, Isaac Argyros, John Chortasmenos, etc. Even if it turns out that a number of recensions were actually redacted by one or the other of these celebrated personalities, no ascription should be put forward on purely inferential grounds. Second, one must repress the tendency to think that all recensions were done in the Palaiologan period, which is obviously the most productive period of Byzantine mathematics itself; this may well be true, and probably is, but it is not certain (see item 1 of Sect. 5 for a likely counter-example).
- It can be organized as a list of manuscripts containing several recensions of ancient (and possibly Byzantine) treatises. The existence of such manuscripts, while being by no means obvious, is a fact. May we confidently assume that recensions contained in one and the same manuscript were redacted by one and the same scholar? We may not. Or, at least, may we confidently assume that such recensions come from one and the same scholarly circle? This may happen, but certainly not always. Maybe the safest attitude is to think that manuscripts containing several recensions show that specific scholarly circles were interested in collecting the "best possible texts" of a given scientific discipline, either by directly producing them or by making eclectic choices from the available material. Note, finally, that a "best possible text" in the sense of Byzantine scholars almost never coincides with the "best text" of modern philology, that is, with a text as close as possible to the "original".

These considerations explain the structure of the present article, whose core sections 4-6 more or less (but not exactly) deal with the same raw material, only arranged in different ways. I have tried to keep repetitions to a strict minimum, but they could not be avoided. The choice of presenting the data in different fashions is not only dictated by the above-mentioned necessity to look at them from different perspectives, but also

by the fact that the present paper lies somehow midway between history of science and palaeography, and is in fact intended, to the best of my ability, to meet the expectations of both readerships, either of which may be more interested in one specific view of the data –that is, in one specific section of the article– than in another. For the same reason, the bibliography I shall mention is quite selective, sometimes even trivial-looking (cf. the references to the entries of *PLP*), and sometimes quite specific. It is intended to give palaeographers an orientation on technical literature and on editions they may not be accustomed to peruse, and historians of science an orientation on palaeographical literature they may happen not to know; readers interested in going more deeply into specific issues will easily complete the bibliographical record. Data pertaining to any manuscript that are not accompanied by bibliographical indications are drawn from the appropriate standard catalogue of the library preserving the manuscript; such catalogues will not be mentioned in the bibliography.

# 2. The Philological problem. General characters of the recensions of Greek mathematical texts

In principle, editing an ancient mathematical or astronomical text is easy. On the one hand, mathematical consistency usually allows the solution of textual problems arising from mechanical accidents or from a copyist's mistakes. On the other hand, since Greek mathematical style is lexically very restricted and syntactically rigid, the variant readings affecting such features of a text are usually immaterial to the mathematical sense and grammatically trivial. For these reasons, the variants that mainly interest an editor of a mathematical text have the size of fully developed deductions or of entire propositions: if any variant involves the simple presence of such large-scale textual units, we are surely facing an interpolation or an omission. But this implies that our text was deliberately manipulated at some stage of its transmission: if interventions of this kind feature abundantly and systematically in a branch of the tradition, we say that this branch carries a recension of the text. Recensions of technical texts are frequently met for an obvious reason: such writings were read to be used, appropriated, and taught by technically skilled authors.

A philologist is confronted with three main problems when editing a Greek mathematical work affected by a recension:

• First and foremost, how to identify a specific branch of the tradition as carrying the recension and not a particularly good –and hence closer to the "original"– version of the text. Such blind faith in the superiority of the best text, a pernicious principle when dealing with ancient mathematical works, has claimed illustrious victims:

Schöne (edition of Damianus, *Opticae hypotheses*; see item 22 of Sect. 5), Manitius and De Falco (Hypsicles, *Anaphoricus*; see item 8 of Sect. 5), even Heiberg (*Elementa* XI.36-XII.17: see item 1 of Sect. 3; Theodosius, *Sph.* III.15).

- Second, where to set the threshold beyond which a text is so tampered with as to give rise to a fully-fledged recension, and how to distinguish a recension from the set of variant readings due to an "inventive" copyist-scholar, or arising from his collating several exemplars. In fact, many textual interventions characterizing a Byzantine recension (they are described in items 1-3 below) are within easy reach of a "simple" copyist-scholar, or may result from collation of other models. What counts are the *motivations* of the reviser, on which, of course, we have no knowledge. These crucial points of method are not at all easy to deal with *a priori*, and will recur time and again in the present paper.
- Third, whether and how to present the documentary record pertaining to the recension: whether not to present it at all, or to set it out in a separate apparatus, or as a list of variant readings placed in the *prolegomena* or in an appendix. What to do if there happens to exist several recensions of the same base text?

A recension does not necessarily manifest itself by means of large-scale interpolations. Other reasons may motivate a revision of a Greek mathematical or astronomical writing. First, some such writings did not benefit from a final redaction (in ancient terms an  $\xi\kappa\delta\sigma\sigma\iota\varsigma$ ) or, if they did, they were disfigured by early copyists: such writings may happen to be (and usually are) unsatisfactory in terms of syntax. Second, late writings were redacted by resorting to a lexicon and to a morphology that were likely to be regarded as unacceptable by "purist" readers. Third, not every Greek technical text adheres to the strictly mathematical style that we read, for instance, in *Elementa*. Later works resorted to at least two stylistic registers: a strictly technical register (geometrical proofs, formulation of algorithms) and a discursive register (observational reports, points of method), the latter frequently used as a framework for justifying and finalizing the former. In all cases of astronomical treatises based on observational data (as for instance Ptolemy's *Almagestum*) this superposition of registers occurs necessarily.

Of course, there was no instruction manual as to how to redact a mathematical text: this is the result of a shared practice and of the redactor becoming deeply acquainted with Greek mathematical works and with the way they are written; and in the perception of the Byzantine scholars who authored the revisions we shall deal with, such a shared stylistic practice had not suffered any intermission from Euclid to their own times. In a sense, what was perceived as canonical in a Greek mathematical text was its style, not the text itself: hardly anyone would dare to extensively correct Plato or Pindarus, while any scholar could feel entitled to correct even such a mathematical genius as Apollonius in points of style. As a consequence of this, and of the fact alluded to above that such writings were read, used and appropriated by technically skilled authors, in the tradition of Greek mathematical writings the phenomenon of recensions is much more conspicuous, in terms of its pervasiveness, than analogous phenomena in other literary genres.

That said, even in the case of strictly mathematical works that have benefited from a final redaction, Greek mathematical style is so conceived as typically to produce texts that are not "saturated" from the lexical, syntactical, or deductive point of view –still, they can be "saturated" fairly easily and almost univocally. Since that of "saturation" of a Greek mathematical text is a crucial notion, it is necessary to explain what does it mean. It means in the first place that the text is written according to a canonical code: this is a lexically poor, strictly formulaic, syntactically rigid idiolect, some elements of which may be omitted without harm to the logic of the argument<sup>2</sup>. But if such elements may safely be omitted, they may also safely be restored; how to do that was in most cases univocally dictated by the canon, which provides, for every element of an "abbreviated" expression, a first, and in a strong sense paradigmatic, occurrence in which the element is formulated in full words. An example will help to understand what happens. Consider the formulaic expression

ό λόγος τοῦ ὀρθογωνίου παραλληλογράμμου περιεχομένου ὑπὸ τῶν ΑΒ ΒΓ εὐθειῶν γραμμῶν πρὸς τὴν ΑΒ εὐθεῖαν γραμμήν.

The ratio of the rectangular parallelogram contained by straight lines AB, B $\Gamma$  to straight line AB.

This expression will never be read in any Greek mathematical text: what we read is, for instance,

ό τοῦ ὑπὸ τῶν ΑΒ ΒΓ πρὸς τὴν ΑΒ

in which all designations by means of substantives and their modifiers are understood, or even

ό τοῦ ὑπὸ ΑΒΓ πρὸς ΑΒ

<sup>2</sup> The notion of "saturation" was first introduced in Vitrac 1990-2001 III, 398, with reference to structural adjustments, occurring in the Arabic tradition of *Elementa*, of the kinds mentioned in item 3 of the list below. I have enlarged the extension of this notion to a series of characteristics of Greek mathematical writings in Acerbi 2010, 66-9. The crucial feature of Greek mathematical style of resorting to canonically "abbreviated" expressions is thoroughly studied in Federspiel 2003 (which contains references to important previous studies of the same author).

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in which two articles and one denotative letter are also eliminated. The omitted words can always, and with absolute certainty, be restored *on mere grounds of style* (that is, without looking at the context) and by referring to basic "first occurrences" like the definition, in *El*. II.def.1, of the predicate "to be contained by the two straight lines that contain the right angle" applied to a rectangular parallelogram. The "mere grounds of style" are that a designation made of a feminine article followed by two letters can only be a straight line, that a masculine article within whose scope features a  $\pi \rho \delta \varsigma$  can only be a ratio, that the syntagm "non-feminine article +  $\delta \pi \delta$  + genitive" can only designate a rectangle. Even the articles and the denotative letter B omitted in the third instance of the formula can be restored with certainty: if the article is non-feminine, the designation "article +  $\delta \pi \delta$  AB B $\Gamma$ " is enough to identify its relatum as a rectangle whose adjacent sides are AB and B $\Gamma$ ; in this case, AB must be a straight line, so that putting the article  $\tau \eta v$  in front of AB becomes non necessary. Finally, to simplify AB B $\Gamma$  to AB $\Gamma$  *within a designation of a rectangle* does not introduces ambiguities.

So, why restore the elements omitted in the basic formula above if their presence is, in a strong sense, irrelevant? Because mathematical style had been perceived as a rigid canon represented by Euclid's *Elementa*; departures from the canon, either of authorial or of traditional origin, are likely to be perceived as (stylistic) faults. Still, if the omission of the substantives is canonical, simply because it is consistently operated throughout *Elementa*, that of the article  $\tau \tilde{\omega} v$  and of letter B in the designation of the rectangle is not canonical, again because it is operated in a strict minority of cases, if at all, in *Elementa*. To eliminate the article  $\tau \hat{\eta} v$  in front of AB can even be regarded as a violation of the stylistic code, according to the basic principle that every designation that includes denotative letters must also include an article.

A second occasion for "saturation" comes from the fact that a complex mathematical argument may take for granted a result without presenting a proof of it, or may only implicitly refer to a previously proven result. Such "missing" or "concealed" steps in Greek mathematical works were the hobbyhorse of revisers and commentators of all times; they were restored in a variety of ways, often carrying a strong metamathematical connotation, sometimes even in the form of fully-fledged theorems. A widespread, simple, and non-invasive specimen of such restorations is the so-called παραγραφαί: very short, non-argumented *marginal* annotations explaining a mathematical passage either by an operative indication (for example, "because a tangent cuts a circle at exactly two points"), or, much more frequently, by a reference to a canonical text (*Elementa*, *Data*, *Conica*...), such as  $\delta i \dot{\alpha} \tau \dot{o} i \theta' \tau \tilde{\omega} \beta' \tau \tilde{\omega} v \sigma \tau ot \chi \epsilon i \omega v:$  number of proposition, book, treatise. *No* primary Greek mathematical treatise (that is, excluding commentaries) *ever* contains indications of this kind in the main text; they may sometimes be counted by the thousands in the margins of Greek mathematical manuscripts; Arabic revisions of Greek mathematical writings frequently put them within the main text. Finally, a mathematical text must of necessity be corrected when it is unsatisfactory or even faulty from the deductive point of view. As said above, all of this labouring on revision was the consequence of the fact that Greek mathematical texts were used, and generations of scholars tried to understand them and to put them in good order when they were not, or at least when they were perceived not to be: these texts were written in a canonical style but were not really canonical.

The following list organizes the attested types of interventions on a strictly mathematical text according to the size of the linguistic unit involved. This categorization can naturally be modified to a grading according to the logical import of each intervention: syntactical, deductive, or structural level; object language or metalanguage, etc. In this connection, one should note that recensions of all periods tend to increase the frequency of metalinguistic expressions. Many of the listed kinds of interventions will be further discussed in the *Appendix*, which offers a full analysis of the way a specific theorem was modified by a number of revisers.

- *Style*. This type of "saturation" of a text assumes a variety of forms (see the example 1. above): adding articles (in particular within the designations of angles:  $\dot{\eta} \dot{\upsilon} \pi \dot{\upsilon} \tau \tilde{\omega} v$ ABF, and of rectangles:  $\tau \dot{\delta} \, \dot{\tau} \, \tilde{\omega} \nu$  AB BF, or before the second term of a ratio:  $\dot{\eta}$  AB  $\pi\rho\delta\varsigma \tau \eta\nu$   $\Gamma\Delta$ ), particles (in particular the connectors  $\delta\epsilon$  and  $\alpha\sigma\alpha$ , and conjunctions such as  $\kappa \alpha i$  before  $\dot{\epsilon} \pi \epsilon i$  or before the conclusion of a deduction by manipulation of ratios:  $\underline{\kappa \alpha i}$   $\dot{\omega} \zeta \ddot{\alpha} \rho \alpha$ ), and words such as  $\dot{\alpha} \rho i \theta \mu \dot{\rho} \zeta$ ,  $\sigma \eta \mu \epsilon i \delta \nu$ ,  $\epsilon \dot{\nu} \theta \epsilon i \alpha$ ,  $\gamma \omega \nu i \alpha$ ,  $\tau \rho i \gamma \omega \nu \sigma \nu$ , τετράγωνον, παραλληλόγραμμον or (very frequently) forms of the verb εἶναι; regularizing conjunctions (for instance,  $\dot{\epsilon}\pi\epsilon i$  must be preferred to  $\dot{\epsilon}\pi\epsilon i\delta\eta$ ; a thorny issue was regularizing the use of adverbial  $\kappa \alpha i$  and connectors (most notably, inserting ἄρα in the principal clause of a paraconditional, see the *Appendix* below); agreeing specific parts of a proposition with each other (for instance, the conclusion of a theorem must be identical with the enunciation, the setting-out must be a rewriting of the enunciation according to well-defined, yet nowhere expressly stated, rules); reorganizing denotative letters (for instance, assigning them in alphabetic order according to their order of occurrence); normalizing verbal forms and formulaic expressions (for instance, using the canonical denotation of an angle  $\dot{\eta}$   $\dot{\upsilon}\pi\dot{\upsilon}$ ABF instead of  $\eta \pi \rho \delta \zeta \tau \tilde{\psi}$  B); redrawing diagrams. The *prolegomena* of Heiberg and Menge to their own editions of *Elementa* and *Data* offer detailed surveys of this kind of interventions (EOO V, LI-LXXV, and EOO VI, XXXII-XLVIII).
- 2. *Logic*. Inserting deductive steps that are perceived as missing; changing the status of deductive steps (for an example see the *Appendix*); inserting expressions or words

aiming to make mathematical generality explicit (these are qualifiers such as forms of  $\tau\iota\varsigma$  or  $\delta\sigma\sigma\sigma\delta\eta\pi\sigma\tau\sigma\tilde{\nu}\nu$ , and participial forms of  $\tau\nu\gamma\chi$ άνειν); correcting faulty mathematical arguments, even by rewriting entire propositions; adding metamathematical clauses of the following kinds: postponed explanations of statements originally taken for granted, instantiated and non instantiated citations of propositions and definitions (that is, with or without denotative letters), references to assumptions, references to the "obvious" or to the diagram, "analogical" or "potential" proofs when multiple cases must be dealt with (introduced by  $\delta\iota\dot{\alpha}$   $\tau\dot{\alpha}$   $α\dot{\sigma}\tau\dot{\alpha}$   $\delta\dot{\eta}$  and  $\dot{\delta}\mu oi\omega\varsigma$   $\delta\dot{\epsilon}$   $\delta\epsiloni\xi$ ομεν, respectively), or identifications of objects (that is, syntagms preceded by τουτέστι).

3. *Structure*. Adding further cases of a proposition when only one case is proven; splitting one proposition into two or merging two into one; changing the status of a statement (for instance from unproven corollary to fully-fledged proposition); adding alternative proofs or replacing one proof by another; adding or eliminating cases; adding or shifting definitions; adding lemmas proving results tacitly assumed within a proof.

The types of interventions outlined here are well known; they have been thoroughly studied with the aim of detecting interpolations in Greek mathematical texts transmitted by the entire tradition<sup>3</sup>. Such a study allows the following to be singled out as the main features of Byzantine recensions of Greek mathematical texts:

- They usually intervene in the middle of a textual tradition whose main branches are fairly well known. As a consequence, they can in principle be identified after a complete examination of the variant readings with respect to a witness putatively "uncontaminated" by Byzantine interventions: for instance, such a witness may simply be a manuscript of the 9<sup>th</sup> century, assuming that no activity on earlier mathematical texts took place during the 7<sup>th</sup> and the 8<sup>th</sup> century; actually, my experience suggests that this period of reduced or absent scholarly activity on Greek mathematical texts can be extended as far as the 11<sup>th</sup> century.
- In cases where strictly mathematical texts are revised, and contrary to the more "aggressive" recensions redacted in (Late) Antiquity, Byzantine recensions usually do not venture into proposing structural adjustments (third category above, but see just below).
- Their authors appear to be more interested in astronomical works, whose discursive structure is far more developed than strictly mathematical texts. Interventions on

<sup>&</sup>lt;sup>3</sup> See Vitrac 2001, 38-60, for a methodical assessment.

these texts usually fall outside the categories outlined above; for obvious reasons, these interventions cannot be easily categorized.

- They often adapt language and prose style, either to contemporary usage or, much more frequently, to a Greek language perceived as "classical" and that was definitely not the one used by mathematical authors of Late Antiquity.
- A limiting case of structural interventions arises when whole chapters or books are redacted replacing lost portions of a treatise: cases in point are the brand new commentary on book III of Ptolemy's *Almagestum* composed by Nicholas Cabasilas to replace the corresponding book of Theon's commentary, at that time considered to be lost; or two chapters at the end of of book III of Ptolemy's *Harmonica*, (re)written by Nicephorus Gregoras and in their turn fiercely refuted by Barlaam of Seminara.
- Finally, of course, there exist Byzantine recensions of Byzantine mathematical texts. Studying such texts is at the same time simpler and more complicated. It is simpler because their manuscript witnesses are close to the time of redaction; in several cases, we even have access to autographs (some works of Georges Pachymeres, Maximus Planudes, Theodorus Meliteniotes, Isaac Argyros) or to copies "authorized" by the author himself (Theodorus Metochites). It is more complicated because the textual tradition of such texts may be so intricate (existence of several authorial redactions, wild contamination) as to make even outlining a stemma impossible. As these works frequently happen to be contained in manuscripts featuring ancient Greek mathematical treatises, we shall also deal with them occasionally in what follows.

Before approaching Byzantine recensions, it is necessary to provide some basic information about recensions of Greek mathematical texts redacted in (Late) Antiquity. This is done in the next section.

### 3. Recensions of Greek mathematical texts in Late Antiquity

As said above, the recensions of Greek mathematical texts redacted in (Late) Antiquity are more "aggressive" than the Byzantine recensions of the same kind of texts. We know of a number of ancient recensions, involving a good deal of the Greek mathematical *corpus*:

1. Euclid. *Elementa* I-XIII<sup>4</sup>: recension authored by the mid 4<sup>th</sup>-century mathematician Theon of Alexandria. All independent manuscripts of *Elementa* carry this recension, with the sole exception of Vat. gr. 190, first half of 9<sup>th</sup> century (*EOO* V, LI-LXX-

 $^4~$  The number of books comprising a given Greek mathematical treatise will be provided only on the first mention of the treatise.

vI)<sup>5</sup>. Note that Bonon. A 18-19, 10<sup>th</sup> century, presents an "aberrant" version of *Elementa* XI.36-XII.17 (ed. *EOO* IV, 385-423, discussion Heiberg 1903, 193-201, who regarded it as a later, and clumsy, recension), that probably belongs to textual strata even earlier than those witnessed by the Vatican manuscript (Knorr 1996). *Data*: recension authored again by Theon. Only seven manuscripts of *Data* carry at least a part of this recension, which is Menge's class  $\beta$ ; they all depend on Bonon. A 18-19. The only complete witness of this class is Laur. Plut. 28.1, 3<sup>rd</sup> decade of 14<sup>th</sup> century<sup>6</sup>.

<sup>5</sup> For the *codices vetustissimi* Vat. gr. 190, Vat. gr. 204, Laur. Plut. 28.18 (all of them first half of 9<sup>th</sup> century), see the forthcoming descriptions in the *Codices Graeci Antiquiores* project, authored by M. Menchelli, P. Orsini, D. Speranzi, respectively. Contents of these manuscripts: Vat. gr. 190: ff. 3r-13v *sch*. I.1 *in Elementa* (*EOO* V, 71.1-108.18); ff. 14r-247v Euclid, *Elementa*; ff. 248r-249v Marinus, *in Data*; ff. 250r-281v Euclid, *Data*; f. 282r-v *scholia in Data* (*sch*. 45, 55, 101, 23, 19, 46, 56, 175 in *EOO* VI, *dein prop*. 87 *vulgo et subsequens lemma*, ed. *ibid.*, 220.19-224.21) ff. 283r-292v [Euclid, *Elementa*] XIV-XV; ff. 293r-340v Theon, "great" commentary on Ptolemy's *Tabulae manuales* (incomplete); Vat. gr. 204: ff. 1r-135v treatises listed under item 5 below, and after them ff. 136r-145r Euclid, *Catoptrica*; ff. 146r-173r Eutocius, *in Conica*; ff. 173v-194v Euclid, *Data*; ff. 195r-v *scholia in Data* (the same sequence as in Vat. gr. 190; the beginning of *sch*. 45, at *EOO* VI, 269.10-20, is missing); ff. 196r-198r Marinus, *in Data*; ff. 199r-206v *scholia in Elementa* (a subset of the so-called *scholia vaticana*, *inc. sch*. I.88 *des. sch*. X.352, and after them *sch*. VI.2; on the collections of scholia to *Elementa*, see most recently Acerbi 2014, 117-27); Laur. Plut. 28.18: ff. 1r-203v Theon, *in Almagestum* I-IV; ff. 204r-258v Theon, *in Almagestum* V-VI.

<sup>6</sup> See EOO VI, XXXI-XLIX -see also item 7 of Sect. 4. Contents of Bonon. A 18-19 (for which see first and foremost EOO V, XXXIII-XXXIV; for its date, see Acerbi, Pérez Martín 2017), which was owned and annotated by Theodorus Cabasilas (cf. note at f. 148r): ff. 1r-35v Elementorum et Datorum principia propositionesque (sine demonstrationibus); ff. 36r-45v prooemium ad Elementa (immo excerpta e Proclo; this is a cento that can also be found in several other manuscripts: Acerbi 2014, 118); f. 45v of tome A 18- f. 152v of tome A 19 Euclid, Elementa (incomplete: *des*. XIII.18, *EOO* IV, 330.26  $\Gamma\Delta$ ), ff. 153r-176v Euclid, *Data* (incomplete). The other manuscripts of the Theonine recension are the recentiores Barb. gr. 260 (olim II.81; watermark dated 1575), Par. gr. 2352 (copied by John Rhosos in 1488: *RGK* II, nr. 237; cf. I, nr. 178, III, nr. 298), Berolin. Phillipps 1542 (16<sup>th</sup> century; maybe a copy of Par. gr. 2352), Scorial. X.I.4 (copied by Valeriano Albini in 1542; cf. RGK I, nr. 336, II, nr. 452, III, nr. 530), Toletanus Bibl. capitul. 98-13. The last two codices were not collated by Menge; their subscriptions assigns the text to Theon's recension (Εὐκλείδου δεδομένα τῆς Θέωνος ἐκδόσεως). Only starting from Data 80 aliter, EOO VI, 220.11 τοῦ ἀπὸ, the first three codices in this list do are in the Theonine recension. As for what precedes of Data, these manuscripts derive from Monac. gr. 361a (non-Theonine; see item f of Sect. 4 for this manuscript), whose support changes exactly at that point, from oriental to occidental paper (the latter is a Renaissance restoration; see *ibid.*, XXXI). Laur. Plut. 28.1 was copied from Bonon. A 18-19 before the latter had lost two quires (nr. 36-7), between the very end of Elementa and the beginning of Data, and a number of folios at its end: what we read of Data in the Bologna manuscript is exactly contained in quires 38-40 (Bonon. A 19, ff. 153r-176v; f. 176 is the last leave of the codex), *inc. Data* 38 (certainly numbered  $\lambda\theta'$  in the manuscript), EOO VI, *Optica* recension  $\mathbf{B}^7$ . *Phaenomena* recension  $\mathbf{b}^8$ . The last two recensions are anonymous; the most ancient witness is in both cases Vat. gr. 204, first half of 9<sup>th</sup> century. *Optica* recension  $\mathbf{A}$  and *Phaenomena* recension  $\mathbf{a}$  are instead transmitted by Vindob. phil. gr. 31 and (as for *Optica*  $\mathbf{A}$  only) Bodl. Auct. F 6 23 (both 11<sup>th</sup> century).

- 2. Archimedes. *De sphaera et cylindro* I-II: school of the architect and engineer Isidorus of Miletus, early 6<sup>th</sup> century (Heiberg 1880, 384-98; *AOO* III, xCIII). It is attested in all manuscripts (that is, the "original" version is irremediably lost). *Dimensio circuli*: we read this treatise in a severely abridged (and in several respects mathematically unsatisfactory) version, redacted before early 6<sup>th</sup> century. It is attested in all manuscripts; W. Knorr's contention (1989, Part III) that Hypatia had something to do with the radical modifications the Archimedean tract has undergone is pure speculation. *De planorum aequilibriis*: we certainly do not read the text in its original form (*AOO* III, xC-XCII; Berggren 1977).
- 3. Apollonius. *Conica* I-IV: not simply a recension, but a true edition redacted by collating a number of divergent witnesses, authored by the early 6<sup>th</sup>-century Neopla-

64.22 ἀχθεῖσα (*marg. sup. m.* 2 title Εὐκλείδου δεδομένα τῆς Θέωνος ἐκδόσεως and annotation λείπει ἡ ἀρχή), *des. Data* 87, *ibid.*, 172.20 τῷ μεγέθει. All Theonine manuscripts derive from a model affected by a mechanical accident extending over about 36 lines of Bonon. A 19; these range from f. 155r9 *ab imo* τῆ θέσει to f. 156r1 δοθὲν ἄρα: *ibid.*, 56.14-58.10, parts of props. 33 and 34, including prop. 33 *aliter* (to be read *ibid.*, 196.17-198.11), which replace *ibid.*, 76.15 τῷ εἴδει-80.8 τὴν AΓ, parts of props. 43 and 44. This is in fact the second half of a partial transposition which also involves the portion of *Data* now lost in the Bologna manuscript; we may detect it in Laur. Plut. 28.1, ff. 330v19-11 *ab imo* (in this case, *ibid.*, 76.13 ἐὰν τριγώνου-80.8 τὴν AΓ κάθετος, prop. 43 and part of 44, replace prop. 33 *aliter* followed by part of prop. 34, *ibid.*, 56.22-58.10) and ff. 331v7 *ab imo*-332r10 (same replacement as in Bonon. A 19). Menge's description of the accident as a simple transposition *ibid.*, vII, is therefore inaccurate. Note that both Vat. gr. 190 and Vat. gr. 204 carry the non-Theonine recension of *Data*.

<sup>7</sup> See EOO VII, xvI-xxXI, in particular xxx-xXXI; Acerbi 2007, 587-610, with references to previous literature: scholarly debate has arisen as to which is prior between recensions **A** and **B**. The former was discovered by Heiberg in Vindob. phil. gr. 31, ff. 254v-271v; see EOO V, xXIX-XXXIII, for a very detailed analysis of this manuscript, 11<sup>th</sup> century, parchment and then oriental paper from f. 243 on, one of the main witnesses of the Theonine recension of *Elementa*; it contains ff. 1r-245v Euclid, *Elementa*; ff. 246r-254r [Euclid, *Elementa*] XIV-XV; ff. 254v-271v Euclid, *Optica* **A**, ff. 272r-282r Euclid, *Phaenomena* **a**; ff. 283r-292v scholia in Elementa (a subset of the scholia vaticana, from sch. X.2 to sch. XIII.44). I take it for almost certain (Acerbi 2007, 609) that recension **A** took its final form in Byzantine times.

<sup>8</sup> See *EOO* VIII, XXIV-XXXIV; Acerbi 2007, 666-7 and 673-5. Recensions **a** and **b** present marked structural divergencies, and have interfered with each other. They are both later than Pappus (early 4<sup>th</sup> century), who comments on specific points of Euclid's *Phaenomena* in book VI of his *Collectio*. It is not to be excluded, but cannot be proved, that recension **a** took its final form in Byzantine times.

tonic philosopher, and leader of the Alexandria school, Eutocius (Decorps 1998, 1999, 2000, 61-144; Acerbi 2012). It is attested in all manuscripts, possibly containing further, Byzantine and Renaissance, revisions of the text established by Eutocius (see the discussion after item 2 of Sect. 4, and item 7 of Sect. 5). The entire tradition depends on Vat. gr. 206, second half of 12<sup>th</sup> century<sup>9</sup>.

- 4. Hypsicles. A recension of the original tract on the comparison between dodecahedron and icosahedron is handed down as "book XIV" of *Elementa*. It is attested in all manuscripts (Vitrac – Djebbar 2011, 63-77); the most ancient witness is Vat. gr. 204.
- 5. The so-called "little astronomy". All treatises in this thematic *corpus* have been lightly (?) and consistently edited by drawing on a lost commentary by Theon on the whole *corpus* (Hultsch 1883; Tannery 1887; Acerbi 2014, 141-51). It is attested in all manuscripts. The most ancient witness, Vat. gr. 204, orders the treatises in the *corpus* according to decreasing theoretical character: ff. 1r-37v Theodosius, *Sphaerica* I-III; ff. 38r-43v Autolycus, *De sphaera mota*; ff. 43v-59r Euclid, *Optica* B; ff. 59r-77v Euclid, *Phaenomena* b; ff. 77v-83v Theodosius, *De habitationibus*; ff. 84r-109v Theodosius, *De diebus et noctibus* I-II; ff. 109v-118v Aristarchus, *De magnitudinibus et distantiis solis et lunae*; ff. 119r-133r Autolycus, *De ortibus et occasibus* I-II; ff. 133v-135v Hypsicles, *Anaphoricus*.
- 6. Ptolemy. *Almagestum* I-XIII: a very light recension, made in the school of the Neoplatonic philosopher Ammonius, of whom Eutocius was a pupil. It is attested in the branch of the tradition whose independent witnesses are Vat. gr. 1594, second half of 9<sup>th</sup> century, and Marc. gr. 313, early 10<sup>th</sup> century<sup>10</sup>.
- 7. Pappus. *Collectio* II-VIII: a literary executor published Pappus' "foul papers" shortly after his death (Jones 1986, 24-6); book I is lost, books II and VIII are incomplete. Some of the books of *Collectio* circulated separately (for instance book VIII). It is

<sup>9</sup> See *AGE* II, XV-LVI for the relations among the manuscripts, and LVII-LXVIII for the tradition as far as the 12<sup>th</sup> century. Decorps 2001 does not add new elements. Contents of Vat. gr. 206: ff. 1r-160v Apollonius, *Conica*; ff. 161r-193v Serenus, *De sectione cylindri*; ff. 194r-239v Serenus, *De sectione coni*.

<sup>10</sup> See POO II, XXVI-XXXIII (specific features) and XXXIV-XXXVII (characters of their common model); Acerbi 2017 for a very detailed analysis of the Vatican manuscript. Contents of Vat. gr. 1594: ff. 1r-8v Prolegomena ad Almagestum et adnotationes variae; ff. 9r-263v Ptolemy, Almagestum, ff. 264r-272r Phaseis, ff. 272v-276v De iudicandi facultate et animi principatu, ff. 278r-283r De hypothesibus planetarum I. Contents of Marc. gr. 313: ff. 1r-30v Prolegomena ad Almagestum et adnotationes variae; ff. 31r-370v Ptolemy, Almagestum (des. POO I.2, 593.23 öπου). impossible to determine the extent of the interventions of the literary executor; the mathematical inconsistencies marring the text we read suggest that he did not revise it. The entire tradition depends on Vat. gr. 218, early 10<sup>th</sup> century<sup>11</sup>.

- 8. Theon of Alexandria. Commentary *in Almagestum* III: proofread by his daughter, Hypatia, as we read in the title<sup>12</sup>. The only witness of the third book of Theon's commentary is Laur. Plut. 28.18, first half of 9<sup>th</sup> century (*iA*, xx1). Since this manuscript did not circulate in Byzantium, most manuscripts of Theon, *in Almagestum*, contain a new commentary on book III, redacted by Nicholas Cabasilas.
- 9. Eutocius. Commentaries *in De sphaera et cylindro I-II* and *in Dimensionem circuli*: proofread by Isidorus of Miletus, early 6<sup>th</sup> century, and published within his school<sup>13</sup>. They are attested in all manuscripts; the tradition is the same as that of the Archimedean *corpus*.
- 10. Commentaries on Nicomachus, *Introductio arithmetica* I-II: there are at least four redactions of a course held by Ammonius on Nicomachus' treatise; one of these redactions was published under the name of Philoponus (ed. Hoche 1864-65 and 1867), another one under the name of Asclepius (ed. Tarán 1969); the others still remain unpublished. The relations between these redactions remain unclear.

We may thank these recensions, and so many others which we are unable to recognize as such, if we can read most of the Greek scientific production in a text which is stable and syntactically correct: they got rid of all corruptions that arose in early transcriptions. It is enough to read works that did not enjoy Late Antiquity revisions to appreciate the extent to which scribal mistakes can mar a Greek mathematical text, which, because of its massive use of abbreviations and denotative letters, must have looked like a huge meaningless string of signs to unskilled copyists. Cases in point are Euclid's *Sectio canonis* (see item 6 of Sect. 5) and a recently discovered scrap of papyrus containing 76 characters

<sup>11</sup> See Treweek 1957. The manuscript only contains ff. 1r-2v Anthemius of Tralles, *De par-adoxis mechanicae*; ff. 3r-202v Pappus, *Collectio*.

<sup>12</sup> See *iA*, CXVI-CXXI and 317 n. 1; Cameron 1990 for Hypatia revising the entire *Almagestum*; Jones 1999, 168-72, for a rebuttal of Cameron's hypothesis. The title of book III is at *iA*, 807.1-5; the verb for "proofreading" there is παραναγιγνώσκειν: on its meaning hinges the Cameron-Jones debate, that in fact almost exactly repeats the terms of a forgotten debate, hinging on the same verb, between Heiberg (1880, 359, recantation at *AOO* III, xCIII) and Tannery (1884, 119 of the reprint) about whether Isidorus' school took charge of proofreading an edition of Archimedes or simply of Eutocius' commentaries thereon.

<sup>13</sup> See AOO III, XCIII. That Eutocius' commentaries were proofread and published within Isidorus of Miletus' school is borne out by the spurious subscriptions at AOO III, 48.28-31, 224.7-10, 260.10-12; cf. also 84.8-11. The subscriptions are discussed in Jones 1999, 168-72; see also the previous note.

(and three big mistakes) of Ptolemy's *Psephophoria*, a treatise whose medieval tradition is exceptionally stable (Acerbi – Del Corso 2014).

The recensions completed in (Late) Antiquity have been occasioned by a number of scholarly circumstances:

- Personal initiative of a scholar, not related to any teaching activity: Theon of Alexandria revising Euclid; Eutocius revising Apollonius.
- Work done within teaching institutions: the school of Isidorus of Miletus and the Neoplatonic school of Alexandria.
- Interaction with commentaries. Pappus, *Collectio* VI, and Theon's commentary (now lost) on the "little astronomy" as a whole have interacted both with the layout of each treatise and with the formation of the *corpus* itself.

The most extensive modifications to Greek mathematical works are transmitted by the entire tradition and, according to the available documentary record, impossible to connect with any well-defined scholarly enterprise. Still, very ancient interpolations can be detected<sup>14</sup>. As for Apollonius' *Conica*, Eutocius' commentary provides us with unusually detailed information; in particular, Eutocius attests to a massive exegetical work on the treatise preceding his own edition, which was devised precisely to put a stop to a messy textual tradition, maybe originated by Apollonius' double redaction of books I and II (Decorps 2001, 61-97; Acerbi 2012; but note that Eutocius emphasizes the variety of recensions of book III, connecting it with scholarly attentions: *AGE* II, 314.2-4).

A warning is in order when assessing the extent and the purposes of a (Late) Antique recension of a Greek mathematical treatise. The problem can best be seen by comparing Heiberg's and Menge's analyses of the relationships between the non-Theonine and the Theonine recension of *Elementa* and *Data*, respectively. The two scholars drew opposite conclusions: Heiberg's Theon tends to expand the text of *Elementa*, Menge's Theon tends to shorten the text of *Data*, despite the fact that *Data* was already redacted in a particularly terse style. Menge, who published his edition after Heiberg's and conformed to the latter's assessment of Theon's recension as essentially motivated by pedagogic aims, tried to explain this embarrassing behaviour by means of the allegedly advanced character of *Data*: Theon must expand for his younger students, but he can be more relaxed in more advanced classes (*EOO* VI, XLVIII). This view does not even have a shred of evidence in

<sup>&</sup>lt;sup>14</sup> See EOO V, LXXVI-XCIII, Acerbi – Vitrac 2014, 31-9, for *Elementa* (in particular the interpolations originating in Hero's commentary); the above-cited literature for Euclid's *Optica* **B** and *Phaenomena* **b**; *AGE* II, LVIII-LXVI, Decorps 2000, 43-59 and 99-128, for Apollonius' *Conica* before Eutocius' edition.

its favour: we have no reason to suppose that Theon ever gave courses on either work, on neither of which did he comment; *Elementa* and *Data* are simply not comparable. Maybe the really relevant parameter, over which we simply have no control, is the quality of the manuscripts used by Theon to produce his recensions.

### 4. How to do a recension. A manuscript-oriented survey

How was a recension done? Our documentary record suggests the following answer: directly on a manuscript containing the text to be revised (this I shall call a "recension manuscript"). "Directly" means erasing and rewriting words or clauses, or inserting them in the margins and between the lines. The modifications will feature in the main text of any subsequent copy, where they can no longer be identified as modifications by means of "material" criteria (this I shall call a "manuscript containing a recension"). Our documentary record also suggests that one is not entitled to postulate a fair copy of a recension as a model of any manuscript containing a recension. Therefore, it is unmethodical to suppose that an intermediate exemplar between a recension manuscript and any extant copy of it has existed in order to explain the presence of mistakes which, on the one hand, cannot be ascribed to the copyist and, on the other hand, seem unworthy of the author of an otherwise high-level scholarly recension. On the contrary, a manuscript containing a recension may present several mistakes, even involving mathematical content, simply because copying a recension manuscript correctly may prove very difficult. A final warning is in order: we know of at least one instance of a Byzantine scholar -namely, Isaac Argyros- who revised mathematical texts while copying them. This shows that a recension manuscript may look like a manuscript containing a recension. On the other hand, we do not know whether Argyros' practice was exceptional (as I would be inclined to think) or not.

The rest of this section comprises a list of recension manuscripts and a list of manuscripts containing a recension<sup>15</sup>. It so happens that these lists are almost disjoined. I shall also discuss in detail the recensions contained in the largest mathematical encyclopaedia assembled during the Palaiologan period, namely, the one transcribed by Malachias in the two-tome set made of Par. gr. 2342 and Vat. gr. 198.

a. Par. gr. 2390, oriental paper, almost certainly transcribed upon request of Manuel Bryennios (fl. *ca*. 1300)<sup>16</sup>. Contents: ff. 1r-14v *Prolegomena ad Almagestum et adnotationes variae*; ff. 14v-146v Ptolemy, *Almagestum*; ff. 147r-149v Ptolemy, *Psephophoria* (but

<sup>15</sup> An outline of the transmission of Greek mathematical and astronomical texts, with a detailed survey of the manuscripts involved, is Vitrac, in progress.

<sup>16</sup> For a detailed description of the manuscript, with identification of the hands involved, see Acerbi – Pérez Martín 2015, 108-13.

subscription at f. 150r); ff. 150r-155v *adnotationes variae*; ff. 156r-159r Ptolemy, *De hypothesibus planetarum* I; ff. 160r-165r Ptolemy, *Phaseis*; ff. 165r-167v Ptolemy, *De iudicandi facultate et animi principatu*; ff. 168r-235v Theon, *in Almagestum* I-II; ff. 236r-260v Theodosius, *Sphaerica*; ff. 261r-264v Autolycus, *De sphaera mota*; ff. 265r-275v Euclid, *Optica* **B**. *Prolegomena ad Almagestum* and *Almagestum* were thoroughly corrected by a hand which is the same as the one that apposed five long scholia to *Almagestum*, and which can be identified with that of Manuel Bryennios himself (*POO* II, XXXIX-XL; Acerbi – Pérez Martín 2015). Since Heiberg was only interested in the text of *Almagestum*, no trace of such corrections can be found in his *prolegomena* or in the critical apparatus of his edition. The copyist of most of Par. gr. 2390 is the same as that of ff. 1r-62v of Marc. gr. 321 (paper, watermarks dated from 1310 to 1320), the most ancient witness of Manuel Bryennios, *Harmonica* I-III<sup>17</sup>.

b. Vat. gr. 184, oriental paper, written some time about 1269-71<sup>18</sup>, contains scholia and corrections of John Pothos Pediasimos, John Catrarios, Nicholas Eudaimonoioannes; the latter extensively annotated *Almagestum*<sup>19</sup>. Contents: ff. 1r-2r *De astrolabio* (ed. Delatte 1939, 254-62); ff. 2r-8r *Psephophoria secundum Indos* (anonymous treatise dated 1252; ed. Allard 1977; see 59 and 94.8 for the date); ff. 8r-9v *adnotationes variae*<sup>20</sup>; ff. 10r-24v *Prolegomena ad Almagestum et adnotationes variae*; ff. 25r-80v selection of scholia copied from Vat.

<sup>17</sup> The copyist of Ptolemy, *Harmonica* I-III, and Porphyry, commentary on Ptolemy's *Harmonica* I.1-4 (and of ff. 131v-132v, that contain musical diagrams and where the manuscript ends), of Marc. gr. 321, named John, is a collaborator of Nicephorus Gregoras in other manuscripts; he was identified in Bianconi 2003, 550-1, exactly at ff. 65r-75r, 76r-98r9, 99r-132v. Cf. *RGK* II, nr. 271, III, nr. 328; Gamillscheg 1984, 96; Pérez Martín 1997, 80-3 (in particular 83 and n. 53, where John's hand is identified in Vat. gr. 1087, ff. 2r-4r, 28r-33v, 88r-91r, 97v-101v17, 102r18-122r, 148r-158v, 161v-171r, 172r-178r, 179v-182v, 183v-190r, 191v-221v, 223r-230v, 233r1-6, 279r-299v); Fonkič 2004, nr. I, 47; Mondrain 2007, 183-8 (with a list at 187-8). For John's hand in Par. Coislin 173, see note 23 below. On f. 63r-v of Marc. gr. 321, a sligthly later hand added Ptolemy, *Harmonica* III.14-16, originally missing in the Marciana codex and partly reconstructed by Nicephorus Gregoras (see item 12 of Sect. 5).

<sup>18</sup> Dated annotations, maybe of copyist A, at f. 9v (a supplementary folio to a quaternion): one of them indicates the current year as 1269/70 (= κατὰ δὲ τὸ νῦν ,ςψοη ἔτος), another as 1270/71 (= κατὰ τὸ ἐνεστὸς ,ςψοθ ἔτος).

<sup>19</sup> The copyists of Vat. gr. 184 are discussed in Bianconi 2004, 331 and n. 59: hand A: ff. 1r1-22, 9r-12r, 38r12-60r20, 82r-90r; B: ff. 1r22-8v, 90v-91v; C: ff. 12v-17v, 90v-91v *marg.*, 92r-220v; D: ff. 18r-38r12, 60r21-80v31. The revisers were first identified (but two of them only tentatively) in Tihon 2003. Bianconi (2004, 331 n. 60) has confirmed the identification, already suggested by Tihon, of two of these hands with Turyn's copyist R of Vat. gr. 191 (Turyn 1964, 89-97 and plate 54) and with John Catrarios. The identification of John Pothos Pediasimos with Turyn's R is in Pérez Martín 2010. See also notes 91 and 96 below, and the last remark of Sect. 6.

<sup>20</sup> Two of these texts are edited in Tihon 1968 and 1982.

gr. 1594<sup>21</sup>; f. 81r-v *adnotationes variae*; ff. 82r-220v Ptolemy, *Almagestum (des. POO I.2,* 589.7 ἀνωμαλίας). Recension manuscript as for *Prolegomena ad Almagestum*; *Almagestum* appears to have been corrected by collation of Vat. gr. 1594; these corrections are mainly introduced by the hand of the main copyist and do not amount to a fully-fledged recension (*POO II, CXIII, CXVII-CXX*).

c. Par. Coislin 173, paper (watermarks at ff. 263-312 dated from 1316 to 1340)<sup>22</sup>, written by several copyists. It contains many autograph notes and corrections by Nicephorus Gregoras<sup>23</sup> and, at f. 1r *marg. sup.*, a possession note of the Great Lavra, on Mount Athos. Contents: ff. 1r-2v Nicephorus Gregoras, *Prooemium in interpretationem orationis de insomniis*, completed by Gregoras himself (last 7 lines of f. 2v); ff. 3r-29v Synesius, *De insomniis*, with Gregoras' commentary (composed in 1328) partly *in margine*; ff. 30v-31r *adnotatio*; ff. 31v-111v Ptolemy, *Harmonica*, with Gregoras' notes and scholia; ff. 112r-147r Ptolemy, *Geographia* (incomplete), with Gregoras' notes and scholia (ff. 144-147 are between ff. 135 and 136); ff. 148r-205v Porphyry, commentary on Ptolemy's *Harmonica*; ff. 206r-211v Nicomachus, *Harmonices Encheiridion*; ff. 211v-212v [Domninus], *Quomodo ratio removeatur a ratione*<sup>24</sup>;

<sup>21</sup> The liminar collection of scholia is preceded by the title Θέωνος Ἀλεξανδρέως· σχόλια πάνυ χρήσιμα εἰς τὴν μεγάλην σύνταξιν Πτολεμαίου. The model is Vat. gr. 1594 since the collection contains several annotations which are unique to Vat. gr. 1594, there apposed by an anonymous scholar of 12<sup>th</sup> century. The rich apparatus of scholia in the margins of the main text of *Almagestum* in Vat. gr. 184, written again by the main copyists (for instance, hand A copied both the main text and the scholia to *Almagestum* I.10, ff. 86r-87v; hand C transcribed all scholia to *Almagestum* I.13-15, ff. 90v-93r), was instead copied from Marc. gr. 313. Concerning the text of *Almagestum* itself, see *POO* II, xxxII-xxXIII and CxVII-CXXI, as for the fact that Vat. gr. 1594 was collated with an ancestor of Vat. gr. 184 and vice versa, and *POO* II, cxXI, as for the model of Vat. gr. 184 having been collated with Marc. gr. 313.

<sup>22</sup> See the description at *GC* I, 7-9, improving on Devreesse 1945, 154-5. Düring erroneously makes the Paris manuscript a copy of Vat. gr. 198 as for Ptolemy's *Harmonica* (see Tihon 1987, 203-4, and item 3 below).

<sup>23</sup> Gregoras extensively annotated this manuscript: his *marginalia* can be found at ff. 1v, 2v18-24, 14v, 30v-31r, 31v-143v, 205r, 206v, 244v-245r, 222v-263r, and titles e.g. at ff. 1r, 57v, 85r, 112r-v, 113v, 198r (Bianconi 2005, 415 nr. 25). Of the outmost importance is the scholium at f. 32r, edited in *PtH*, LXXX-LXXXI, partly an autograph of Gregoras (from où µóvov: Mondrain 2002, 321 n. 26; 2005, 19, with a complete translation [but see also Guilland 1926, 273]; and 2007, 164 n. 10), in which he asserts that he has revised Ptolemy's *Harmonica* and added the chapters missing at the end. At f. 263r *marg. sup.*, Gregoras again asserts that the text of Theon's "great" commentary δεῖται πολλῆς καὶ παντοίας διορθώσεως. As for the copyists of Par. Coislin 173, John (see note 17 above) penned ff. 176r-196v8 and 206r-222v (Bianconi 2003, 550), an anonymous collaborator of Gregoras in other manuscripts transcribed ff. 1-2v17, 2r-29v, 31v, as well as the first part of the scholium at f. 32r completed by Gregoras: Mondrain 2002, 321; 2007, 163 n. 9; Pérez Martín 2008, 437 n. 182. See also item *d* below.

<sup>24</sup> Edited in Riedlberger 2013, in fact a Byzantine (?) recension of a scholium to Ptolemy's *Almagestum*: Acerbi – Riedlberger 2014.

ff. 212v-216v Ocellus Lucanus, *De universi natura*; ff. 217r-220v Bacchius Geron, *Introductio harmonica*; ff. 220v-222r [Bacchius], *Introductio harmonica*; f. 222v Hymns to Calliope, to the Sun and to Nemesis; ff. 223r-262r Manuel Bryennios, *Harmonica*; f. 262v *adnotatio* by Gregoras; ff. 263r-311r Theon, "great" commentary on Ptolemy's *Tabulae manuales* I-IV. Recension manuscript as for at least Ptolemy's *Harmonica*; autograph corrections of Gregoras (Mondrain 2005, 19). Par. Coislin 172 (parchment), ff. 1r-37r, is the fair copy of Par. Coislin 173 as for Ptolemy's treatise<sup>25</sup>.

- d. Par. gr. 2450, parchment, written about 1335; the copyist also penned Par. Coislin 173, ff. 1-2v17, 2r-29v, 31v, and two manuscripts of *Almagestum*: Neap. Borb. III C 19 (dated 1335; Turyn 1972, 184-5 and plate 149) and Ferrara, Biblioteca Comunale Ariostea II 178 (*POO* II, XXI). Contents: ff. 1r-90r Ptolemy, *Harmonica*; ff. 92r-131v Nicomachus, *Introductio arithmetica*; ff. 132r-178r Theon, "great" commentary on Ptolemy's *Tabulae manuales*; ff. 179r-207r Theon of Smyrna, *Expositio rerum mathematicarum ad legendum Platonem utilium*; f. 207v *adnotationes astronomicae*. Recension manuscript as for Theon's "great" commentary (*GC* I, 19-22 and 33-5; "un lecteur éclairé, amateur d'astronomie" at 35). The manuscript also contains Gregoras' recension of Ptolemy's *Harmonica* (*PtH*, XXVIII and LXV).
- e. Laur. Plut. 28.7, paper, second half of 14th century (a text at f. 32v is dated 1344; watermarks dated from 1336 to 1362), and Leiden, Bibliothek der Rijksuniversiteit, BPG 78, parchment, partly written during the reign of Leon V (813-820)<sup>26</sup>. Contents of Laur. Plut. 28.7: ff. 1r-29v Theon, "little" commentary on Ptolemy's *Tabulae manuales*; ff. 29v-32v *varia astronomica*; ff. 33r-40v Ptolemy, *Psephophoria*; ff. 40v-41v *adnotationes variae*; ff. 41v-48r Ptolemy, *De hypothesibus planetarum* I; f. 49r-v *adnotationes variae*; ff. 50r-109r Ptolemy, *Tabulae manuales*; ff. 112r-144v Proclus, *Hypotyposis*<sup>27</sup>; ff. 144v-145v *excerpta e Gemino*; ff. 146v-147r *iambica in Virginis gloriam*; ff. 148r-172v Paul of Alexandria, *Introductio in astrologiam*; ff. 173r-177v Rhetorius, *Thesaurus Antiochi*; f. 178r *tabula mensorum*; ff. 178v-179r *adnotatio astronomica*. Contents of Leiden, BPG 78: ff. 1r-2v *adnotationes et tabulae variae*; ff. 3r-49r Theon, "little" commentary; ff. 19v-50v *nota astronomica*; ff. 50v-155r Ptolemy, *Tabulae manuales*; f. 155v *nota astronomica*; ff. 156r-161v Theon, "little" commentary; ff. 162r-163r adnotationes variae.

<sup>25</sup> Par. Coislin 172, ff. 41r-268r, contains Ptolemy, *Almagestum* –this part, as well as its model Par. gr. 2391 (*POO* II, xxv; Pérez Martín 2008, 437-8 and n. 182-3; it only contains *Almagestum*), are copied by the scribe at work in Par. Coislin 173, ff. 1-2v17, 2r-29v, 31v, see note 23 above. Par. gr. 2391 and Par. Coislin 173 share a watermark dated 1334 (*ibid.*, 437 n. 182).

<sup>26</sup> See *PC*, 90-1, and Tihon 2011, 24-31, respectively, and also items 19cd of Sect. 5. Hands in Leiden, BPG 78: f. 1r, 13<sup>th</sup>-14<sup>th</sup> century; ff. 1v-2r, in majuscule, early 9<sup>th</sup> century; f. 2v, 9<sup>th</sup>-10<sup>th</sup> century; ff. 3r-10v, 11r-49r and 49v-51v, three hands of 13<sup>th</sup>-14<sup>th</sup> century; ff. 52r-152v, in majuscule, early 9<sup>th</sup> century; f. 53v, 14<sup>th</sup> century; ff. 153r-155v, end 13<sup>th</sup>-beginning 14<sup>th</sup> century; ff. 156r-161v, 9<sup>th</sup>-10<sup>th</sup> century, the same hand as f. 2v; ff. 162r-163r, probably 12<sup>th</sup> century.

<sup>27</sup> Not fully collated by the editor: Manitius 1909, VIII, XXIX, who, however, asserts at XXIX that this manuscript "haud spernendas praebere scripturas [...] quae ingenio librarii non ignari attribuendae esse videntur".

Recension manuscripts (different recensions) as for Theon, "little" commentary on *Tabulae manuales* (the version in Leiden, BPG 78, is that at ff. 3r-49r).

f. Monac. gr. 361a is a composite paper manuscript, written by eight hands, the earliest of which can be assigned to the end of 13<sup>th</sup> century<sup>28</sup>. Contents<sup>29</sup>: ff. 1r-6r mathematico-philosophical lucubrations; 7r-8r Euclid, Phaenomena b from a supplement to prop. 14, edited as sch. 129, to end (inc. EOO VIII, 156.20 καὶ ἡ ZH); ff. 8r-13v Euclid, Optica B, complete; ff. 13v-14v Euclid, Data from beginning to prop. 13 (des. EOO VI, 26.5 τὸ AH); ff. 15r-17r Euclid, Catoptrica, complete; ff. 17r-22v Euclid, Phaenomena b from beginning to a supplement to prop. 14, edited as sch. 129 (des. EOO VIII, 156.20 χρόνω καί); ff. 23r-30v Euclid, Data 13-80 aliter (inc. EOO VI, 26.5 καὶ λοιποῦ, des. 220.11 μετὰ); ff. 31r-36v Ptolemy, Harmonica II.2-15 (inc. PtH, 48.21 τοῖς λόγοις, des. end of chapter); ff. 37r-41v Ptolemy, Harmonica III.1-14 (inc. PtH, 85.19 τῆς τοῦ κανόνος, des. 109.11 τῶν γενομένων); ff. 42rv Ptolemy, Harmonica III.11-15 (complete chapters, including Gregoras' supplements); f. 43r-v Aristoteles, Problemata XIX.1-17; f. 44r-v Ptolemy, Harmonica II.16-III.1 (inc. PtH, 80.6 τὸ μὲν τοίνυν, des. 85.19 τοῦ ἑτέρου τρόπου sed reclamans alia manu marg. inf. τῆς τοῦ κανόνος); 45r-v Ptolemy, Harmonica II.1-2 (inc. PtH, 43.1 δύο λόγοι, des. 48.21 τὰς ἀκολούθους); ff. 46r-49v Euclid, Data 80 aliter to end (inc. EOO VI, 220.11 τοῦ ἀπὸ; this is a Renaissance restoration); ff. 50r-59v Ptolemy, Harmonica from beginning to II.1 (des. *PtH*, 42.17 ἐπίτριτος sed reclamans alia manu marg. inf. δύο λόγοι; this is a Renaissance restoration). Therefore, the (quite unusual, but the same as that in Vat. gr. 191: see just below in this very item, and cf. note 111 below) order of the Eucliden treatises in this manuscript was Catoptrica, Phaenomena b, Optica B, Data. The folia containing Ptolemy's Harmonica must instead be read in this order: 50r-59v, 45r-v, 31r-36v, 44r-v, 37r-41v, 42r-v. A recension (Düring's class f) of Ptolemy's treatise giving rise to a rich branch of the tradition stems from Monac. gr. 361a, in which we also read Gregoras' supplements (but f. 42 probably is a deliberate replacement), in principle belonging to class g. This manuscript changes its model (from Vat. gr. 191, itself a copy of Vat. gr. 204 ante correctionem, to an unknown copy of Vat. gr. 204 post correctionem) and type of paper (from oriental to occidental paper) in the final portion of Data, ff. 46-49 (EOO VI, XXI-XXIII). In these folios, in fact a Renaissance restoration as we have seen, the text integrates the corrections a later hand

<sup>28</sup> The hands are distributed as follows (I. Pérez Martín, *per litteras*). Copyist 1 (Manuel Glynzounios; cf. *RGK* I, nr. 248, II, nr. 341, III, nr. 409): f. 1r-v, second half of 16<sup>th</sup> century; copyist 2: ff. 2r-6r (f. 6v *vacuum*), second half of 16<sup>th</sup> century; copyist 3: ff. 7r-41v, 44r-45v, mimetic hand, end of 13<sup>th</sup> century; copyist 4 (Philotheos of Selymbria; cf. *PLP* 29896; he also copied ff. 13r-69v, 154r-160v, 179r-205r, 214r-219v and annotated ff. 162r-163r, 165v-166v, 177r-179r of Marc. gr. 309: Pérez Martín 2008, 445): f. 42r-v + *marginalia* ff. 34v, 38r, 40r *inf.*, 40v; copyist 5: f. 43r1-17v + *marg.* ff. 34r, 39v, 40r, 40v; copyist 6: f. 43r18-43v (hands 4-6 can be assigned to *ca.* 1350-75); copyist 7: ff. 46r-49v, 15<sup>th</sup> century; copyist 8: ff. 50r-59v, end 15<sup>th</sup>-beginning 16<sup>th</sup> century.

<sup>29</sup> See also Düring *PtH*, xx-xx1, whose description is however inaccurate.

had apposed on Vat. gr. 204 (*ibid.*, XXII; see also note 6 above and item 2 of Sect. 5). The Euclidean works on optics in Monac. gr. 361a are also copied from Vat. gr. 191 (*EOO* VII, XXIII); as for *Phaenomena* **b**, instead, the Munich codex is the prototype of an independent family, possibly carrying a light recension (*EOO* VIII, XXIX-XXXI). In the portion in oriental paper, it presents extensive corrections, interlinear and *in rasura*, to the text of *Data*.

Of course, many more manuscripts exist containing a recension. What is less obvious is that several manuscripts exist containing almost uniquely recensions of the Greek mathematical texts they collect. The reason for this is that in Palaiologan Byzantium there existed scholarly circles strongly interested in mathematics and astronomy, strongly motivated to study such texts seriously, and, finally, strongly inclined to collect homogeneous literary products in encyclopaedic collections. On the other hand, the mere existence of such encyclopaedic collections of texts is a clear sign of scholarly interests and activities aiming to appropriate (and hence, possibly to modify) these texts. What follows is a preliminary list of manuscripts containing recensions:

Par. gr. 2342, paper, written by Malachias, olim "anonymus aristotelicus" (most comprehen-1. sive account in Mondrain 2004, 278-90 and 292), toward the end of the 60s of 14<sup>th</sup> century (watermarks dated from 1355 to 1360). The quires are numbered from 23 to 48 (only one folio of the last quire has survived): the original codex had at least 384 folios. Contents: ff. 1r-92r Euclid, Elementa (inc. I.29, EOO I, 70.19: note that f. 1, severely damaged, has been wrongly placed on a paper frame: page 1v precedes 1r); ff. 92r-96r [Euclid, Elementa] XIV and XV; ff. 96r-97v Marinus, in Data; ff. 97v-108v Euclid, Data; ff. 109r-114r Euclid, Optica B; ff. 114r-115r Damianus, Opticae hypotheses; f. 115r-v Geminus, Excerpta optica; ff. 116r-118r Euclid, Catoptrica; ff. 118v-129r Theodosius, Sphaerica; ff. 129v-131r Autolycus, De sphaera mota; ff. 131r-137r Euclid, Phaenomena b; ff. 137r-139r Theodosius, De habitationibus; ff. 139r-147r Theodosius, De diebus et noctibus; ff. 147r-150r Aristarchus, De magnitudinibus et distantiis solis et lunae; ff. 150v-154v Autolycus, De ortibus et occasibus; f. 155r-v Hypsicles, Anaphoricus; ff. 156v-187r Apollonius, Conica (Eutocius' commentary starts at f. 155v); ff. 187r-195v Serenus, De sectione coni; ff. 195v-200v Serenus, De sectione cylindri (incomplete: des. prop. 33, Heiberg 1896, 114.10 σημεῖα). As noted above, the codex begins with quire 23; one may well wonder what might have preceded Euclid's Elementa<sup>30</sup>. Par. gr. 2342 is the first tome of a mathematical encyclopaedia (a quadrivium, in

<sup>30</sup> We may get an idea of the contents of these 22 lost quires from a surprising coincidence: ff. 21-196 of Vat. gr. 208, also transcribed by Malachias (see note 59 below) exactly amount to 22 quires numbered from α' to  $\kappa\beta$ '; they contain ff. 21r-132v Ptolemy, *Tabulae manuales*, interspersed with Theon, "little" commentary thereon; ff. 133r-186r Ptolemy, *Tetrabiblos*; ff. 187r-191r *varia astrologica*; ff. 192r-195v Ptolemy, *Centiloquium*; f. 196r-v *excerpta varia*. Note that, of these writings, only Theon's "little" commentary features in Vat. gr. 198, the second tome of the encyclopaedia of which Par. gr. 2342 is the first. Unfortunately, Par. gr. 2342 and ff. 21-196 of Vat. gr. 208 do not match at all from the codicological point of view (see note 104 below). Contents ancient terms), whose second tome is Vat. gr. 198 (item 3 below). Par. gr. 2342 is organized in three thematic units: Euclid and related scholarly material (scholia, extracts from tracts of other authors pertaining to Euclidean matters); "little astronomy" (without the Euclidean treatises, of course); conic sections (Apollonius and Serenus). The scholarly origin of this encyclopaedia is confirmed by the fact, first noted by Heiberg, Rome, and Mogenet<sup>31</sup>, that its conception and *mise en œuvre* (codicological features, lay-out, inks) are strictly homologous to an Aristotelian *corpus* contained in a series of manuscripts, among which Par. Coislin 161, 166, Par. gr. 1921, and Hierosolym. S. Sep. 150, penned by the very same copyist<sup>32</sup>. This Aristotelian *opera omnia* is endowed with commentaries transcribed in the margins, exactly as happens, in Par. gr. 2342, with Eutocius' commentary on Apollonius' *Conica* or with the imposing and multi-layered (several inks can be distinguished) scholiastic apparatus surrounding *Elementa*. This manuscript contains recensions of all treatises, maybe with the sole exception of Marinus' *in Data*, whose editor followed the principle of the best manuscripts (only three codices collated: *EOO* VI, VIII, and edition at 234-56).

Ambros. A 101 sup., paper, written by a professional copyist, first half of 16<sup>th</sup> century (watermarks dated from 1524 to 1548); diagrams are totally absent. Contents: ff. 1r-5v [Euclid, *Elementa*] XIV and XV; ff. 6r-7v Marinus, *in Data*; ff. 7v-25r Euclid, *Data*; f. 25v Geminus, *Excerpta optica*; ff. 26r-34v Euclid, *Optica* B; ff. 34v-35v Damianus, *Opticae hypotheses*; ff. 35v-39v Euclid, *Catoptrica*; ff. 40r-86v Apollonius, *Conica*; ff. 86v-100r Serenus, *De sectione coni*; ff. 100r-109r Serenus, *De sectione cylindri*; ff. 111r-138r Theodosius, *Sphaerica*; ff. 138r-142r Autolycus, *De sphaera mota*; ff. 142r-v *scholia in Sphaeram motam* (*sch.* 15, 19, 20, 22, 24, 26, 30, 35, 36, 40 in *AP*, sometimes with major variants); ff. 142v-154r Euclid, *Phaenomena* b; ff. 154r-158r Theodosius, *De magnitudinibus*; ff. 158r-174r Theodosius, *De diebus et noctibus*; ff. 174r-179v Aristarchus, *De magnitudinibus et distantiis solis et lunae*;

of Vat. gr. 208 others than those just mentioned: ff. 1r-4v notae astronomicae (ff. 3v-4r are in the hand of John Chortasmenos); ff. 5r-20r Isaac Argyros, opuscula astronomica duo (in Malachias' hand); f. 20v adnotationes astronomicae; ff. 197r-199v fragmenta astronomica; ff. 200r-205v Ptolemy, *De hypothesibus planetarum* I; ff. 206r-208v adnotationes et tabulae astronomicae; ff. 209r-219v Philoponus, *De usu astrolabii*; ff. 220r-225v Isaac Argyros, *De constructione astrolabii* (ff. 200-225 are in Malachias' hand); ff. 227v-230v scholia in Elementa (in the hand of John Chortasmenos; partial ed. Heiberg 1903, 350-2).

<sup>31</sup> *AGE* II, LXIX-LXX; Rome 1930; *AP*, 81-2, respectively. Mogenet mentions the same codex as Rome (Par. gr. 1921) without citing his master.

<sup>32</sup> An updated list, still not including Vat. gr. 208, of manuscripts transcribed by Malachias is in Mondrain 2004, 279-80. Note that this copyist can be found in at least another mathematical manuscript: Mutin. a.U.9.7 (*olim* III B 4), paper, ff. 1v-75r Nicomachus, *Introductio arithmetica*, with Philoponus' commentary in the margins; ff. 76r-77v *scholia in Elementa* (EOO V, 697.20-703.12, App. II *sch*. 6-7); ff. 76v-77r *nota numerologica cum tabula*; f. 77v *nota astrologica*; ff. 78r-292v Euclid, *Elementa*; only f. 77r is partly in Malachias' hand: Mondrain 2000, 22-3. The material at ff. 76r-77v can also be found in Par. gr. 2107, ff. 23r-25r; the non-mathemathical texts at ff. 76v-77v are edited in Acerbi, forthcoming. On Par. gr. 2107 see note 108 below.

ff. 180r-188r Autolycus, *De ortibus et occasibus*; ff. 188r-189v Hypsicles, *Anaphoricus*; ff. 190r-226r Theon, "great" commentary on Ptolemy's *Tabulae manuales*. This manuscript contains recensions of all treatises, maybe with the sole exception of Marinus' *in Data* (cf. item 1 above).

These two manuscripts deserve a careful discussion, because they have been studied in an article I shall later cite repeatedly (Decorps 1987). Decorps argues that all texts contained in both manuscripts have been revised, and that these recensions have been authored by one and the same scholar, forming thus a unitary *corpus*. If the former is unquestionable, I shall however argue that there is no reason to suppose that the latter has been the case.

The *corpus* is contained in three further manuscripts: Marc. gr. 301, Upsaliensis gr. 50, and Norimb. Cent. V, App. 8. As for the main features of them, Marc. gr. 301, paper, first half of 15<sup>th</sup> century (watermarks dated from 1401 to 1430), was owned by Bessarion<sup>33</sup>. Its contents are: ff. 1r-326v, Euclid, *Elementa*; ff. 328r-334r Marinus, *in Data*; ff. 334r-377v Euclid, *Data*; ff. 378r-435v Theodosius, *Sphaerica*; ff. 436r-464v Euclid, *Phaenomena* **b**; ff. 465r-466v three mathematical texts in the hand of George Gemistus Plethon (who also penned ff. 2 and 5); ff. 467r-480v Euclid, *Catoptrica*. Upsal. gr. 50 is a paper codex of 17<sup>th</sup> century containing ff. 3r-10v Marinus, *in Data*; ff. 10v-72v Euclid, *Data*; ff. 75r-236v Apollonius, *Conica*; ff. 238r-285v Serenus, *De sectione coni*; ff. 286r-315r Serenus, *De sectione cylindri*. Norimb. Cent. V, App. 8 will be described in item 4 below.

If we are to take these five manuscripts as avatars of a unitary *corpus* redacted and assembled by one single scholar, we must still note that no treatise is common to all five manuscripts, and that not all writings common to at least two manuscripts can be read in one and the same recension. In Decorps' reconstruction, the best representative of "the" recension is the Ambrosian codex, taken to be a direct copy of one single model, whereas in the Paris codex –whose copyist Malachias apparently worked by collation of different models, one of which allegedly coincides with one of the models of Ambros. A 101 sup.– some treatises are read in a different recension or in so slight a recension as hardly to be a recension at all (*Elementa*)<sup>34</sup>. That Malachias worked by collation of one of the models of Ambros. A 101 sup. is borne out by a series of notes of his, in which he indicates structural divergencies of the manuscripts he is transcribing that exactly match what we find in the

<sup>33</sup> On this codex, see Acerbi – Martinelli Tempesta – Vitrac 2016, that also contains an edition and an analysis of the texts redacted by George Gemistus Plethon.

<sup>34</sup> The treatises contained in the Nürnberg and in the Venice manuscripts are analyzed only indirectly by Decorps. The variant readings of Theon's "great" commentary in Norimb. Cent. V, App. 8, are derived from the apparatus' of *GC*, those of all treatises in Marc. gr. 301 are assumed to coincide with the corrections, listed by the relevant editors in their *prolegomena* or included by them in the critical apparatus, a later hand did on the same treatises in Vat. gr. 204. See Sect. 5 below for details.

Ambrosianus. Just to give some examples (compare the description of the Ambrosianus above): the title προθεωρία τῶν τοῦ Εὐκλείδου δεδομένων of Marinus' *in Data* we read in Ambros. A 101 sup. is transcribed by Malachias, in red ink, just after the title, accompanied by the indication ἐν ἄλλῷ (f. 96r); at the beginning of Geminus' *excerpta* he writes ταῦτα ἦν πρὸ τῶν ὀπτικῶν Εὐκλείδου κείμενα, as if it were a title (f. 115r); at the end of *Catoptrica* he notes in red ink μετὰ τὰ κατοπτρικὰ ἐν ἄλλοις βιβλίοις τὰ κωνικὰ τοῦ Ἀπολλονίου καὶ Σερίνου κωνικὰ καὶ κυλινδρικά (f. 118r marg. inf.)<sup>35</sup>.

Of course, this state of affairs did not pass unnoticed by the first editors of Greek mathematical texts. As said above under item 1, Heiberg both pointed out that Par. gr. 2342 contains a heavily revised text of Apollonius' Conica and of Serenus' treatises<sup>36</sup>, and suggested that Par. gr. 2342 was the mathematical representative of a huge philosophico-mathematical corpus enriched with ancient commentaries in the margins, written by the same copyist on behalf of a single intellectual milieu. On the basis of well-known testimonies in Theodorus Metochites<sup>37</sup> and of the fact that Par. Coislin 161, "prorsus gemellus" of Par. gr. 2342, was once in the Great Lavra of Mount Athos, Heiberg surmised that Par. gr. 2342 was also written at Mount Athos. Further, since this manuscript's revisions "apte conferri potest" with those contained in Par. Coislin 172 and 173 (see item c above), and since the latter revisions certainly are to be related to Nicephorus Gregoras, who was a pupil of Theodorus Metochites, Heiberg finally asserted "fortasse igitur dirthosis codicis [Par. gr. 2342] aut eius [scil. Metochites'] est aut saltim eo auctore facta"38. The main difference, and not a slight one, with respect to Decorps' reconstruction is that Heiberg, and all scholars after him except Mogenet, is convinced that Ambros. A 101 sup. is a (possibly further revised) copy of some known manuscript (Par. gr. 2342 or Marc. gr. 301)<sup>39</sup>. The other difference is that Decorps suggests Maximus Planudes as the author of the recension (1987, 52-3).

In short, Decorps resumes Heiberg's reconstruction, originally intended to apply only to Apollonius' and Serenus' treatises, and enlarges its scope to encompass all writings (that is, recensions) contained in Par. gr. 2342 *and* Ambros. A 101 sup. There are several reasons why Decorps' grand fresco cannot work.

<sup>35</sup> Analogous indications by Malachias in the twin codex Vat. gr. 198 (see item 3 below) are edited at *POO* II, XXII-XXIV.

<sup>36</sup> The variant readings of Par. gr. 2342 are first listed and discussed at *AGE* II, XXXI-LVI, Heiberg 1896, v-vIII, respectively, and then included in the apparatus of each edition –excluding those presented in the *prolegomena* in the case of *Conica*.

<sup>37</sup> These and other testimonies have been recently discussed in Acerbi – Pérez Martín 2015, 106-7.

<sup>38</sup> *AGE* II, LXVIII-LXX; cf. item 1 above. Heiberg later corroborated his thesis on the basis of the evidence coming from another "gemellus" of Par. gr. 2342, namely, Vat. gr. 198 (*POO* II, XXIII-XXIV), for which see item 3 below.

<sup>39</sup> *AGE* II, xx1; Heiberg 1896, 1x; and see also several items in Sect. 5 below: Heiberg took this almost for granted given the dates of the two manuscripts.

- First, that Planudes was the author of "the" recension is groundless illation, even if Planudes' wide-ranging mathematical interests are attested by a series of facts<sup>40</sup>. This speculation has already been disposed of by A. Tihon in the case of Theon's "great" commentary<sup>41</sup>. Tihon's counter-argument just amounts to show that the recension of the "great" commentary does not have the same stylistic characters as the recension of *Conica*. Even if the contrary were true<sup>42</sup>, we would still not be entitled to draw any inference from this.
- Second, one must insist that Ambros. A 101 sup. is the only witness of "the" recension. For if all treatises in it and in the Uppsala manuscript carry the text of the recension, only Apollonius' *Conica*, Serenus' and Damianus' writings and the excerpts from Geminus are read in this same text in Par. gr. 2342, a manuscript that elsewhere

<sup>40</sup> On Planudes' recension of Diophantus' *Arithmetica*, see item 14 of Sect. 5 (the fair copy of the recension manuscript, still an autograph of Planudes, is Ambros. & 157 sup.). Planudes also copied part of Par. gr. 2396, Theon, *in Almagestum*, and very likely had something to do with the text of Theon, *in Almagestum*, we read in Vat. gr. 1087 (see notes 78 and 79 below, respectively). Add to these the manuscripts annotated by Planudes, a list of which is in item 2 of Sect. 6. Finally, he redacted a treatise on logistic (but this is more or less a plagiarism of the anonymous *Psephophoria secundum Indos* of 1252). In his *epist.* 67, before describing Diophantus' manuscript he restored and which turns out to be Matr. 4678 (Pérez Martín 2006; see also item 2 of Sect. 6), Planudes mentions Theodosius' *Sphaerica* (Leone 1991, 102.3-7).

<sup>41</sup> Tihon 1992, 72 n. 59; GC I, 82-3; and III, 5-6; for details see item 18 of Sect. 5. Note also that Par. gr. 2342 was copied about 1360, whereas the base text of the recension of Catoptrica in Ambros. A 101 sup. derives from Vat. gr. 192, in its turn an apograph of Vat. gr. 191, written before 1296 (see item 5 of Sect. 5 and notes 19 above and 89 and 111 below). Vat. gr. 192, oriental paper, mm 335×245, written by several hands in the second half of 13th century, contains ff. 1r-2r Anonymi logica et quadrivium (only the musical part: Heiberg 1929, 65.8-72.16; unknown to the editor); f. 2v scholium in Elementa (EOO V, 700.12-703.12, App. II sch. 7 inc. ἰσόπλευρον); ff. 3r-94r Euclid, Elementa (sed ff. 24v-26v scholia in Elementa, in order sch. X.11, 13, 14, 15, 18, 21, 28, 9); ff. 95r-111v Euclid, Data; ff. 111v-112v scholia in Data (the same sequence as in Vat. gr. 204: see item 5 of Sect. 3); ff. 112v-114v Marinus, in Data; ff. 114v-124v scholia in Elementa (a subset of the so-called scholia vaticana, from sch. I.88 to sch. XIII.44); f. 124v Geminus, Excerpta optica; ff. 125r-126v Damianus, Opticae hypotheses; ff. 127r-138v + 143r Euclid, Optica B; ff. 139r-142v Euclid, Catoptrica; ff. 143r-145r Hypsicles, Anaphoricus; ff. 145r-150v Aristarchus, *De magnitudinibus et distantiis solis et lunae*; ff. 151r-163r Euclid, *Phaenomena* **b**; ff. 164r-194r Aristides Quintilianus, De musica I-III; f. 194r adnotatio ad musicam pertinens; ff. 194v-195v Theon of Smyrna, Sectio canonis (Hiller 1878, 87.4-93.9, 85.8-87.3); ff. 196r-214v Ptolemy, Harmonica; ff. 214v-221v [Plutarch], De musica; ff. 222r-227r adnotationes diagrammata et tabulae ad musicam pertinentes.

<sup>42</sup> Decorps tries to show that the stylistic characters of "the" recensions are the same for all treatises involved, but she does this by means of statements which are most of the time impressionistic, and in any case corroborated –whenever they are, and often they are not– just by a handful of examples. Still, one must concede that *Conica* and Theon's "great" commentary cannot be compared from the stylistic point of view. resorts to "the" recension only for some scholia to *Optica* and *Sphaerica* and for later corrections to "*Elementa*" XIV-XV. But it is obviously unmethodical to propose, as Decorps does twice (1987, 24 and 54), an undifferentiated *stemma*, in which *the* "manuscrit corrigé par le recenseur" is the common ancestor of a tradition only partly attested in the alleged apographs.

- Third, encyclopaedic attempts at devising a single *corpus* –the texts are revised and hence, in a strong sense, they are "in their best possible form"– have manifested themselves at least twice and independently: one project is the two-tome mathematico-astronomical encyclopaedia comprising Par. gr. 2342 and Vat. gr. 198, another project is the collection whose 16<sup>th</sup>-century avatar (if it is simply an avatar) is Ambros. A 101 sup. What must be emphasized is the eclectic approach of the copyists, in particular Malachias, who does not cease to search for better copies and frequently corrects *post factum* his own transcription by collation from a further exemplar. If there is a real one-man show in the whole affair, it is Malachias' construction of his manuscripts. Maybe he was also a scholar, maybe he was not: what is sure is that he had access to a well-endowed library and that he knew what to do with even the subtler details of the texts he transcribed.
- 3. Vat. gr. 198, paper, written by Malachias (POO II, XXIII-XXIV; Mondrain 2004, 292; Bianconi 2005, 401-3), in strict connection with Par. gr. 2342 (annotations by Malachias dated 1374, 1375 and 1377-78, the latter added later: PC, 146). Contents: f. 1r-v Photius, Bibliotheca, codex 186 (immo 187, Nicomachus)<sup>43</sup>; ff. 2r-33v Nicomachus, Introductio arithmetica with Philoponus' commentary in the margins; f. 34r-v diagrammata astrologica et musica; ff. 35r-88v Ptolemy, Harmonica with Porphyry's commentary in the margins, chapters III.14-16 (partly supplemented by Gregoras) being added in a different format (ff. 88v-89r); ff. 90r-93r Nicomachus, Harmonices enchiridion; f. 94v tabulae usque ad annum 1479; ff. 95r-125v Manuel Bryennios, Harmonica; f. 126r-v notae astronomicae; ff. 127r-136v Prolegomena ad Almagestum; ff. 137r-138v notae astronomicae, quarum una ad Theodorum Meliteniotem adscripta; ff. 138v-317v Ptolemy, Almagestum; ff. 318r-340r Nicholas Cabasilas, in Almagestum III; f. 340r-341r nota ad Cabasilam adscripta; ff. 341r-342v notae astronomicae; ff. 343r-406v Theon, in Almagestum I-II, IV; ff. 407r-421r Pappus, in Almagestum V; ff. 421v-424v fragment of Theon, in Almagestum V (headed by Malachias ταῦτα τὰ σχόλιά είσι οἶμαι τοῦ Θέωνος); f. 425r tabulae astronomicarum computationum; ff. 425v-468v and 479r-485v Theon, in Almagestum VI-X, XII-XIII; ff. 469r-478v and 485v-500v Theon, "little" commentary on Ptolemy's Tabulae manuales; ff. 501r-514v Proclus, Hypotyposis44; ff. 515r-516v varia astronomica. This is the second tome of the mathematical encyclopaedia whose first tome is Par. gr. 2342 (item 1 above); also in this case, Malachias apposed

<sup>43</sup> This folio was added later; with f. 2r starts the first quire (mark  $\alpha$ ) of the original codex.

<sup>44</sup> Not fully collated by the editor: Manitius 1909, VIII-IX, XXXIX-XLI, who, however, makes it and Vat. gr. 1059 one of the best representatives of a separate branch of transmission (class D).

notes attesting to his collations: we read these notes at POO II, XXII-XXIV. Vat. gr. 198 is organized in three thematic units: arithmetic (Nicomachus); harmonic theory; advanced astronomical matters (Ptolemy and related treatises). Düring (PtH, LXII-LXIII) was almost convinced that Vat. gr. 198 is the fair copy of Gregoras' recension of Ptolemy's Harmonica or an immediate copy of it: as a matter of fact, concerning the musical treatises, Vat. gr. 198 is an apograph of Par. Coislin 173 (item *c* above). Adopting the same strategy as in the twin manuscript Par. gr. 2342, Malachias endowed most of the treatises he transcribed in Vat. gr. 198 with commentaries transcribed in the margins; for instance, we read Philoponus' commentary in the margins of Nicomachus' Introductio arithmetica, Porphyry's commentary at ff. 35r-71v, in the margins of Ptolemy's Harmonica; an imposing and multi-layered scholiastic apparatus surrounds Almagestum. Such a strategy gave unexpected by-products: Tihon (1987) found the otherwise lost book V of Theon, in Almagestum, in Vat. gr. 198, ff. 200v-217v in the margins of Almagestum V, after Rome (1953) had identified a long extract included in the main text (ff. 421v-424v above). This manuscript contains Byzantine recensions of Prolegomena ad Almagestum (Acerbi – Vinel – Vitrac 2010, and item 21 of Sect. 5), Ptolemy, Harmonica (PtH, LX-LXV, LXXVIII-LXXXVIII) and Almagestum (POO II, XL-XLIII), Theon, in Almagestum (iA, XXI-XXIV and LXXXVIII-XC, but see already POO II, XLIII) and "little" commentary on Ptolemy's Tabulae manuales (PC, 153-6).

- 4. Norimb. gr. Cent. V, App. 8, paper, a composite manuscript whose first part, ff. 1r-59v, was copied in early 16<sup>th</sup> century (watermark dated 1512); the rest was almost entirely copied in late 14<sup>th</sup>-early 15<sup>th</sup> century (watermarks dated from 1369 to 1402; see Rome 1948; GC I, 16-19); ff. 102-6 and 238-47 belong to yet another manuscript. Autograph possession notes of Bessarion and Regiomontanus are apposed at f. 106v. Contents: ff. 1r-59v Ptolemy, Tetrabiblos; ff. 60r-101v Theon, in Almagestum I-II; ff. 102r-105r fragments of Prolegomena ad Almagestum; ff. 105r-v Barlaam, De eclipsi solari anni 1333 secundum Magnam Constructionem; ff. 107r-118v Theon, in Almagestum IV; ff. 119r-137r Pappus, in Almagestum V, including (ff. 121r-123r) a fragment ascribed to Theon by the copyist; ff. 137r-188v Theon, in Almagestum VI-XIII; ff. 189r-214v Nicholas Cabasilas, in Almagestum III; ff. 215r-237v, Theon, "great" commentary on Ptolemy's Tabulae manuales; ff. 238r-239v and 242r-244v fragments of Pappus, in Almagestum V; ff. 240r-241v theorems on the determination of shadows; ff. 245r-247v seven articles on philosophical matters. This manuscript contains Byzantine recensions of Ptolemy, Tetrabiblos (POO III.1, xIX-XX), Theon, in Almagestum (as in item 3 above) and "great" commentary on Ptolemy's Tabulae manuales (GC I, 16-19, 54-67, and 82-3; it is the same recension as that in Ambros. A 101 sup.), and of the fragments of Prolegomena ad Almagestum.
- Marc. gr. 310, paper, copied by Isaac Argyros (Mondrain 2007, 166) as far as f. 261v (watermarks dated from 1339 to 1370), thence in the hand of Bessarion. A copy of the astronomical portion of the encyclopaedia also contained in Vat. gr. 198. Contents: ff. 1r-13r *Prolegomena ad Almagestum*; ff. 13v-151v Ptolemy, *Almagestum*; ff. 153r-202v Theon, *in Almagestum* I-II, IV; ff. 202v-216v Pappus, *in Almagestum* V; ff. 216v-261v Theon, *in Al-*

*magestum* VI-X, XII; ff. 265r-286v Nicholas Cabasilas, *in Almagestum III*; ff. 287r-288r Barlaam, *De eclipsi solari annorum 1333 et 1337 secundum Magnam Constructionem*. This manuscript contains Byzantine recensions of *Prolegomena ad Almagestum*, Ptolemy, *Almagestum*, Theon, *in Almagestum*<sup>45</sup>, stemming from the same source as those in Vat. gr. 198 (item 3 above), still containing extensive variant readings, often simply involving syntactical features<sup>46</sup>. Since Marc. gr. 310 was penned by Isaac Argyros, all recensions there contained must be ascribed to him –in fact, as was usual with Argyros, his text is a correction *in scribendo* of the recension independently contained in Vat. gr. 198; for this reason, Marc. gr. 310 might also feature among the recension manuscripts.

- A further recension of Prolegomena ad Almagestum can be found in the following manu-6. scripts. Laur. Plut. 89 sup. 48 (also partly copied by Isaac Argyros: D. Bianconi, per litteras, further specified to ff. 7r-17v and 137r-168r by A. Gioffreda, per litteras and 2018): ff. 1r-4v De confectione et usu astrolabii (ed. Mogenet 1958, non vidi); ff. 4v-6v Barlaam, De eclipsi solari anni 1337 secundum Magnam Constructionem; ff. 7r-19r Prolegomena ad Almagestum; ff. 20r-168r Ptolemy, Almagestum, ff. 170r-193r Barlaam, Logistica. Regin. gr. 90 (ff. 1-8 second half of 14th century, ff. 9-359 second decade of 15th century: I. Pérez Martín, per litteras): ff. 1r-8v Prolegomena ad Almagestum; ff. 9r-359v Ptolemy, Almagestum (the last folia are in disorder). Neap. Borb. III C 13 (f. 64r dated 1058, immo 1558; an obvious partial copy of Laur. Plut. 89 sup. 48): ff. 1r-23r Prolegomena ad Almagestum; ff. 24r-28v De confectione et usu astrolabii, ff. 28v-32v Barlaam, De eclipsi solari anni 1337 secundum Magnam Constructionem; ff. 33r-64r Barlaam, Logistica; ff. 65r-265r Ptolemy, Almagestum; ff. 266r-346v Manuel Bryennios, Harmonica (the editor did not know of this witness). These manuscripts contain Byzantine recensions of Prolegomena ad Almagestum and Ptolemy, Almagestum, stemming from the same source as those in Vat. gr. 198 and Marc. gr. 310 (items 3 and 5 above)<sup>47</sup>. Since the recension of *Prolegomena ad Almagestum* contained in Laur. Plut. 89 sup. 48 is beyond doubt derived from that in Marc. gr. 310, it follows that Argyros transcribed twice the same text, modifying it on both occasions.
- 7. Laur. Plut. 28.1, parchment, penned by "copyist F", 3<sup>rd</sup> decade of 14<sup>th</sup> century, owned and annotated by Demetrius Cydones<sup>48</sup>. This manuscript has two models: Par. gr. 2390 (item *a*

<sup>45</sup> Add to these the recension of Barlaam's tract on solar eclipses: Mogenet – Tihon – Donnet 1977, 37-40.

<sup>46</sup> See item 17 of Sect. 5, and Acerbi 2015 for a case study coming from Theon, *in Almagestum* VI, not edited in *iA*.

<sup>47</sup> As for Barlaam' *Logistica*, Carelos (1996, LIV-LV, LXXIV, LXXIX) surmises that the branch represented by Laur. Plut. 89 sup. 48 and its apograph Neap. Borb. III C 19 contains a sort of preliminary version of the treatise (dubbed "Barlaam 3"). The variants involved suggest instead that this redaction is the result of a recension, that we may safely ascribe to Argyros himself.

<sup>48</sup> For a detailed description of the manuscript see Acerbi – Pérez Martín 2015, 113-15; for "copyist F", see Pérez Martín 2000, 325-7 and Plate 5; Bianconi 2005a, 156-74 and 253; see also note 83 below.

above) and Bonon. A 18-19. Contents: ff. 2r-15v (= Par. gr. 2390, ff. 1r-14v) *Prolegomena ad Almagestum et adnotationes variae*; f. 16r ὄρια τῶν πέντε πλανωμένων κατὰ Πτολεμαῖον; ff. 16v-17r scholia manu Cydonii; ff. 17v-19v tabulae variae; ff. 21v-22v scholia manu Cydonii; ff. 23r-167r (= Par. gr. 2390, ff. 14v-146v; cf. *POO* II, XXXIX-XL) Ptolemy, *Almagestum*; ff. 168r-171v (= Par. gr. 2390, ff. 147r-149v; cf. *POO* II, CLXXV) Ptolemy, *Psephophoria*; ff. 171v-177r (= Par. gr. 2390, ff. 150r-155v) adnotationes variae; ff. 177v-180r (= Par. gr. 2390, ff. 150r-155v) adnotationes variae; ff. 177v-180r (= Par. gr. 2390, ff. 150r-155v) adnotationes variae; ff. 177v-180r (= Par. gr. 2390, ff. 160r-165r; cf. *POO* II, CLX and CLV) Ptolemy, *Phaseis*; ff. 184v-187r (= Par. gr. 2390, ff. 165r-167v<sup>49</sup>) Ptolemy, *De iudicandi facultate et animi principatu*; ff. 187v-243v (= Par. gr. 2390, ff. 168r-235v) Theon, *in Almagestum* I-II<sup>50</sup>; ff. 246r-327v Euclid, *Elementa<sup>51</sup>*; ff. 327v-338v Euclid, *Data<sup>52</sup>*. The ongoing edition of *Prolegomena ad Almagestum* shows that Laur. Plut. 28.1 integrates all corrections found in Par. gr. 2390 into the text; see the Appendix for a striking example.

8. Par. gr. 2448, paper, written in "chypriote bouclée", beginning 14<sup>th</sup> century. Contents: ff. 1-24v Anonymi logica et quadrivium (incomplete)<sup>53</sup>; ff. 25r-57r Euclid, Data (inc. prop. 24, EOO VI, 44.9); ff. 57r-59r Archimedes, Problema Bovinum; ff. 59r-70v, Euclid, Catoptrica; ff. 70v-78r Diophanes and other metrological material (= [Hero], Stereometrica 1.65-6; Geometrica 22); 78r-79v [Domninus], Quomodo ratio removeatur a ratione; ff. 79v-88r Autolycus, De sphaera mota; ff. 88v-140r Theodosius, Sphaerica (des. III.11, Czinczenheim 2000, 167.6 λοξός); ff. 140r-141v scholia in Data (unique to this manuscript; ed. MGM, 74-6); f. 141v alternative diagram of Sph. II.14 (cf. scholium f. 110r τὸ σχῆμα τοῦτο ζήτει βέλτιον ἐπὶ τοῦ τέλους τοῦ βιβλίου; it is in fact the diagram of II.13). A very interesting manuscript, still in need of a detailed study<sup>54</sup>. The texts of all writings in Par. gr. 2448 (maybe with the sole exception of Ratio) are Byzantine recensions not attested elsewhere<sup>55</sup>; some variant readings of De sphaera mota are in conspicuous agreement with the Arabic tradition of this treatise (AP, 179-81). The variant readings to

<sup>49</sup> Cf. Lammert at *POO* III.2, XIII: «L [*scil.* Laur. Plut. 28.1], codex Cydonii, e libro antecedenti amici [*scil.* Par. gr. 2390] descriptus esse videtur».

<sup>50</sup> One reads *in fine* ἐτελειώθη: this alludes to the end of the transcription from Par. gr. 2390. It is unclear whether Rome, *iA*, XXII and XCI, had entirely collated this manuscript.

<sup>51</sup> Here the model partly changes to Bonon. A 18-19: Heiberg 1903, 182 and 200-1, but only on the basis of a handful of variant readings. B. Vitrac (*per litteras*) finds that Heiberg's claim certainly holds for *Elementa* XI.36-XII.17 (see item 1 of Sect. 3), but that it is certainly false for III.31-X.33.

<sup>52</sup> The model is again Bonon. A 18-19: EOO VI, XXXI-XXXII, and see also item 1 of Sect. 3.

<sup>53</sup> Redacted Autumn 1007; ed. Heiberg 1929.

<sup>54</sup> See *AP*, 65-7; Czinczenheim 2000, 239-41; Riedlberger 2013, 101-2; Stefec 2014, for preliminary assessments.

<sup>55</sup> I know of only one apograph of Par. gr. 2448, and this as far as *De sphaera mota* is concerned (*AP*, 67-70): the composite Ambros. P 270 sup., ff. 78r-v, 68r-77v (15<sup>th</sup> century; it con-

Anonymi logica et quadrivium, Data, Catoptrica, Diophanes, supplementary metrological material, *Ratio*, *De sphaera mota*, *Sphaerica* are listed at Heiberg 1929, v-XII (very extensive interpolations) and *app.*; *EOO* VI, XXVI-XXX; *EOO* VII, XLVI-XLVIII; *MGM*, *app.*; *HOO* V, CXII-CXIII; Riedlberger 2013, *app.*; *AP*, *app*. and 283-4; Czinczenheim 2000, 241-58 and *app.*; respectively.

Marc. gr. 314, parchment, copied by Michael Clostomalles, alias "Metochitesschreiber" 9. (Bianconi 2005, 429-30), about 2<sup>nd</sup>-3<sup>rd</sup> decade of 14<sup>th</sup> century. Contents: ff. 1r-76v Ptolemy, Tetrabiblos; ff. 78r-181v Anonymous, in Tetrabiblon; ff. 182r-188r Porphyry, in Tetrabiblon; ff. 188r-195r scholia Demophili in Tetrabiblon; ff. 197r-201r Ptolemy, Centiloquium; ff. 201r-208r Achmet, in Centiloquium; ff. 209r-215r Ptolemy, Psephophoria; ff. 215r-223v adnotationes variae (the same as Par. gr. 2390, ff. 150r-155v); ff. 224r-229r Ptolemy, De iudicandi facultate et animi principatu; ff. 229v-234v Ptolemy, De hypothesibus planetarum I, ff. 235r-255r Prolegomena ad Almagestum; ff. 256r-286r Vettius Valens, Anthologiae I-IX (des. II.28.4). Ptolemy's minor astronomical works and Prolegomena ad Almagestum were copied directly from Vat. gr. 1594 (Acerbi 2017). The conspicuous variant readings to De iudicandi facultate et animi principatu and Anthologiae are discussed and listed at POO III.2, XI-XII and *app.*, and Pingree 1986, VII-IX and *app.*, respectively<sup>56</sup>; the variant readings of *Prolegomena ad Almagestum* will be analyzed in the ongoing edition<sup>57</sup>. The traditions of Tetrabiblos and Centiloquium are almost identical (cf. POO III.1, XI-XXV, POO III.2, XX-IV-XXXII); Marc. gr. 314 is one of the best representatives of a recension of both texts (class  $\beta^{58}$  of Boll-Boer-Hübner and Boer), probably of Byzantine origin; the texts of this branch

tains only this scientific writing). As for *Ratio*, Par. gr. 2448 is cognate to Par. gr. 2531 (copied by Michael Souliardos, end 15<sup>th</sup>-beginning 16<sup>th</sup> century).

<sup>56</sup> Let me quote the editor Lammert in *POO* III.2, XI-XII (**B** and **M** are Vat. gr. 1594 and Marc. gr. 314, respectively): «Quod attinet ad lectiones, **M** cum asseclis suis multo magis et gravioribus ad **B** quam ad **FLQ** [*scil*. Par. gr. 2390 and two of its apographs] codices correctos et interpolatos spectat. Attamen **M** ex ipso **B** descriptum esse minime putaverim. Vix animo fingi potest, quomodo tot quamvis partim exiguae varietates ortae sint in describendo e codice tam bene atque distincte scripto in codicem **M** ipsum quoque satis diligenter exaratum. [...] Marcianum illum ex **B** quidem pendere crediderim sed sive uno sive pluribus libris manuscriptis inter **B** et **M** insertis. Etiam de communi codicum **B** et **M** archetypo cogitari potest».

<sup>57</sup> The title of *Prolegomena ad Almagestum*, apposed by Nicephorus Gregoras and afterward corrected, is Θέωνος Ἀλεξανδρέως προλεγόμενα εἰς τὴν μεγάλην σύνταξιν τοῦ Πτολεμαίου; the corrector, who is not likely to be Bessarion (*pro* Mioni 1985, 27, *contra* D. Speranzi, *per litteras*), adds Διοφάντου above Θέωνος.

<sup>58</sup> The other best representatives of this branch are Laur. Plut. 28.20 (copied some time after 1332 by some Macarios, who also penned ff. 5r-27v of Vat. gr. 1087 –part of Theodorus Metochites, *Astronomike stoicheiosis*–: Pérez Martín 2004, 209-10, best description Caballero Sánchez 2013, 112-15) and Monac. gr. 419 (first quarter of 14<sup>th</sup> century; best descriptions Boll 1908, 25-7; *PC*, 14-16).

of the tradition was further revised by John Abramios (class  $\gamma^{59}$ ). We cannot exclude that some of the recensions in Marc. gr. 314 were authored by Clostomalles himself; the kind of variant readings of *De iudicandi facultate* and of *Prolegomena ad Almagestum* and the fact that they are unique to this manuscript suggest that this has been the case in these instances (see also items 11, 13, 19g, 21 of Sect. 5).

# 5. A list of Byzantine recensions of Greek mathematical and astronomical treatises

This chapter provides, for each author and treatise, the manuscripts containing the recension and a short discussion of its main characters. Nicomachus' *Introductio arithmetica* and Theon of Smyrna's *Expositio rerum mathematicarum ad legendum Platonem utilium* do not feature in the list since their editions are not grounded on a reconstruction of the manuscript tradition<sup>60</sup>. The Heronian writings are also excluded: *Metrica* has been transmitted only by Istanbul, Seragl. G.I.1 (written by Ephrem *ca.* 960), rediscovered in the Serail Library at the end of the 19<sup>th</sup> century. His other treatises fall outside the scope delimited by the designation "mathematical and astronomical", and in fact we have found none of them in the survey of the previous section: in Late Antiquity as well as in Byzantine times, technical *corpora* were assembled according to rigid divisions between disciplines. For the same reason I have excluded the isagogical treatises of Cleomedes and Geminus. As for the pseudo-Heronian metrological *corpus*, its structure and its manuscript tradition are so complex as to defy being treated by standard philological tools<sup>61</sup>.

<sup>59</sup> See Pingree 1971, 202. The best representatives of this branch are Laur. Plut. 28.16 (dated 1381/2, partly written by Abramios himself: Turyn 1972, 245-8 and plates 203-4 and 261e-f), Angelic. gr. 29 (a part of it, completed 24 July 1388, was copied by Eleutherius Eleus, an associate of Abramios: Turyn 1972, 250-1 and plates 208-9 and 262c-d; that Eleutherius was a pupil of Abramios is just Pingree's illation: 1971, 202), Vat. gr. 208 (paper, written about 1375-7 by Malachias: Mondrain 2005, 15 and 24, but ff. 197r-v, are not written by Malachias, and ff. 1r-4r, 227v-230r only contain annotations of John Chortasmenos: Hunger 1969, 22-3; see *PC*, 47 for the date; see already *POO* II, vi n. \*, and Heiberg 1903, 350 n. 1; the codex was personally owned by Abramios, who also apposed some notes; see also item 1 and note 30 above). That class γ depends on class β, contrary to the stemma in *POO* III.1, xxv –but according to the stemma in *POO* III.2, xxxII– is shown in Vuillemin-Diem – Steel 2015, 49-56.

<sup>60</sup> For the same reason I have excluded the Late Antiquity commentaries on Nicomachus' treatise, even if some manuscripts ascribe a revision (the so-called "Recension II") of it, and also some specific additions, to Isaac Argyros: Tannery 1888, 302-10 of the reprint, Tarán 1969, 5-20. Roueché (2002, 116-18) concludes that Recension II is not a simple reworking of Recension I.

<sup>61</sup> See Acerbi – Vitrac 2014, Étude complémentaire III, for a thorough, yet preliminary, analysis.

### 1) Euclid, Elementa

The situation is totally unclear since Heiberg almost completely neglected later manuscripts in his edition of *Elementa* (providing only very partial indications in his *paralipomena* of 1903), and a systematic collation of them has been undertaken only recently<sup>62</sup>: one does not find, in the tradition of *Elementa*, Byzantine recensions *strictu sensu*, but a great number of independent and very localized modifications, maybe simply introduced by well-informed copyists; this phenomenon can be already detected in the 11<sup>th</sup> and 12<sup>th</sup> centuries (Vindob. phil. gr. 31, Par. gr. 2344 and 2466<sup>63</sup>) and goes as far as Chortasmenos' own exemplar, Mutin. a.T.8.21. The discussion Marc. gr. 301 deserves in Heiberg's edition (*EOO* V, CIV-CVII) suggests that its text has undergone some changes: the adjective "deterrimus" used by him suffices to make us suspect that this has been the case. Heiberg deals with this codex in his *prolegomena* since it is, together with Par. gr. 2343 (16<sup>th</sup> century; ff. 1r-299r Euclid, *Elementa*; ff. 299r-311r [Euclid, *Elementa*] XIV-XV), the source of Grynaeus' *princeps* of 1533 and thereby "diu fons fundamentumque textus Graeci Elementorum mansit". Heiberg 1903, 183 and 190-1, also states that the Marcian codex is a copy of Mutin. a.U.9.7<sup>64</sup>, in its turn strictly related to Par. gr. 2342.

<sup>62</sup> *Elementa* is transmitted by over 100 manuscripts. We know of 96 witnesses earlier than year 1700; of these, more than 40 are complete or almost complete ("almost" means that at least 90% of the text is present); about 30 manuscripts only contain a portion of the treatise (between two and eleven books); the others only preserve fragments or collections of enunciations without proof. Twenty-five complete or almost complete witnesses and 15 partial copies have been written before 1450-60. Among the latter, a systematic survey has been done of all complete or almost complete manuscripts, of 6 incomplete manuscripts, and of 8 testimonies transcribed in the second half of 15<sup>th</sup> century or in the 16<sup>th</sup> century. Such a survey is preliminary to the new edition of the entire Euclidean *corpus* to appear in the Collection des Universités de France (Les Belles Lettres, Paris). The editor of *Elementa*, B. Vitrac, will provide there a reconstruction of the history of the text after 12<sup>th</sup> century more complete than the one only outlined by Heiberg in the *prolegomena* of his own edition and in the *Paralipomena* of 1903. I use in this item some of the results of such a survey, that Vitrac has kindly communicated to me. Some of these results have already been presented in Acerbi – Martinelli Tempesta – Vitrac 2016.

<sup>63</sup> Par. gr. 2344 is on parchment, written *ca*. 1120-40 by one single copyist. Contents: ff. 1r-16v *excerpta e Proclo, nota anthropologica et scholium in Elementa* (ed. of the latter *EOO* V, xxIV); ff. 17r-357v Euclid, *Elementa*; ff. 358r-366v *scholia in Elementa* (*El.* V.def.1-3 and *sch.* X.11, 13, 14, 15, 18, 21, 28, 9, 63, 128, 143, 157, 162, 164, 169, 193, 219, pertaining to the definitions and to the propositions as far as X.32). For a detailed description of this manuscript, with edition of the anthropological text, see Acerbi – Lami 2014. The other 12<sup>th</sup> century witness of *Elementa* is Par. gr. 2466, a parchment manuscript penned by two hands, ff. 1r-53r and 53v-239r. It contains only Euclid, *Elementa*.

<sup>64</sup> Olim III B 4; the subscription Θεόδωρος ἱερομόναχος γράψας transcribed by Heiberg is no longer readable; see also notes 30 above and 104 below. Marc. gr. 301 also presents a partition

As for Par. gr. 2342, Heiberg (1903, 59-65) shows that Malachias contaminated readings that are characteristic of a number of manuscripts<sup>65</sup>. In this case, then, the modified text we read in these manuscripts cannot be regarded as the result of a recension, but of systematic and deliberate contamination.

## 2) Euclid, Data

A Byzantine recension is attested in Ambros. A 101 sup. and Upsal. gr. 50 (Decorps 1987, 40-41). Decorps asserts that this recension is "exactement semblable" to that attested in Marc. gr. 301 and whose variant readings are listed, as a rich series of corrections apposed by a 15<sup>th</sup>-century hand to Vat. gr. 204, by Menge at *EOO* VI, xVIII-xXI<sup>66</sup>. Menge surmises that such corrections derive from Marc. gr. 301. Copies of Marc. gr. 301 are Marc. gr. 302 –first half of 15<sup>th</sup> century (watermarks dated from 1408 to 1469), partly written by Bessarion himself<sup>67</sup>– and Par. gr. 2467 and 2366, the latter written by John Honorius<sup>68</sup>; copies of Vat. gr. 204 *post correctionem* are Par. gr. 1981 and 2348 (*ibid.*, xXV-XXVI); copies of Vat. gr. 204 *ante correctionem* are Vat. gr. 191, 192, 202, Magliab. II.III.36, Mutin. a.V.7.14 (*ibid.*, xXI-XXIII). Note that, contrary to what Decorps claims, Menge asserts that Par. gr. 2342 also contains a recension, whose variant readings he does not give in detail, even if his discussion entails that they are different from those in Ambros. A 101 sup. The base text of Par. gr. 2342 appears to derive from Vat. gr. 190 (*ibid.*, xVII).

A further recension, not attested elsewhere, can be found in Par. gr. 2448 (variant readings in EOO VI, xxvI-xxx; see also item 8 of Sect. 4). A recension manuscript, whose variant readings have not been studied (only an allusion can be found in EOO VI, xxII-xXIII), is Monac. gr. 361a, ff. 46-49 excluded (see item *f* of Sect. 4).

of the principles opening book I of *Elementa* unique to this manuscript: Acerbi – Martinelli Tempesta – Vitrac 2016, which see also for the relations among the above-mentioned manuscrits.

<sup>65</sup> For Malachias' eclecticism, see the discussion under item 2 of Sect. 4.

<sup>66</sup> To be clear: Decorps has only collated Ambros. A 101 sup. and Upsal. gr. 50. In Menge's *prolegomena*, I have searched in vain for a discussion of the codex Ambrosianus.

<sup>67</sup> Marc. gr. 302 contains: ff. 1r-156v Euclid, *Elementa*; f. 157v the first of the three mathematical texts at Marc. gr. 301, ff. 465r-466v; ff. 161r-164r Marinus, *in Data*; ff. 164r-184r Euclid, *Data*; ff. 184v-208v Theodosius, *Sphaerica*; ff. 209r-220r Euclid, *Phaenomena* **b**; ff. 224r-228v Euclid, *Catoptrica*; ff. 232r-260v Barlaam, *Logistica* I-VI; ff. 260v-263v Barlaam, *Demonstratio arithmetica secundi Elementorum libri* (ed. EOO V, 725-38); ff. 265r-494r Ptolemaeus, *Almagestum*.

<sup>68</sup> See *RGK* II, nr. 232; cf. I, nr. 174, III, nr. 286. It is almost sure that Menge is wrong: as we shall see in items 3, 4, 8 below, Par. gr. 2366 is an obvious copy of Vat. gr. 204, whose contents it shares almost exactly, the treatises in the "little astronomy" being arranged by author. Note also that John Honorius himself restored ff. 1-2 of Vat. gr. 204 (Bianconi 2015, 247-8).

### 3) Euclid, Optica B

A Byzantine recension is attested in Ambros. A 101 sup. (Decorps 1987, 42-3). Decorps asserts that the variant readings typical of this recension are also attested as a rich series of corrections apposed by a 15<sup>th</sup>-century hand to Vat. gr. 204; Heiberg included such corrections in the apparatus. The Ambrosian codex is an indirect –that is, via the recension– apograph of Vat. gr. 202 (*EOO* VII, XXVI-XXVII). Copies of Vat. gr. 204 *post correctionem* are Par. gr. 2107 and 2366 (*ibid.*, XX-XXI). Par. gr. 2342 derives from Vat. gr. 204 *ante correctionem*, but Malachias also had in his hands a manuscript of the recension attested in Ambros. A 101 sup. (*ibid.*, XIX-XX and XXVI n. \*\*, and Decorps 1987, 42-3).

As suggested in note 7 above, it is not unlikely that also *Optica* recension **A** took its final form in Byzantine times; Vindob. phil. gr. 31 looks very much like both a recension manuscript and a manuscript carrying (previous) recensions.

### 4) Euclid, Phaenomena b

A Byzantine recension is attested in Ambros. A 101 sup. (Decorps 1987, 48). Decorps asserts again that "les mêmes innovations se retrouvent" in Marc. gr. 301, whose variant readings are also attested as a rich series of corrections apposed by a 15<sup>th</sup>-century hand to Vat. gr. 204; Menge listed such corrections at *EOO* VIII, XIX-XXI, and asserted that they derive form Marc. gr. 301. A copy of Marc. gr. 301 is Marc. gr. 302; a copy of the latter is Berolin. Phillipps 1544; copies of Vat. gr. 204 *post correctionem* are Par. gr. 2366 and 2388 (*EOO* VIII, XXVIII-XXIX). Menge asserts that the Ambrosian codex also is a copy of Marc. gr. 301 (*ibid.*)<sup>69</sup>, whereas Decorps mantains that it is an indirect –that is, via the recension– apograph of Vat. gr. 202 (1987, 48 and n. 120)<sup>70</sup>. Par. gr. 2342 derives from Vat. gr. 204 *ante correctionem* (*EOO* VIII, XXVII-XXVII).

A further, light recension is possibly contained in Monac. gr. 361a: *EOO* VIII, XXIX-XXXI. As suggested in note 8 above, it is not unlikely that also *Phaenomena* recension **a** took its final form in Byzantine times; see also the preceding item.

## 5) Euclid, Catoptrica

A Byzantine recension is attested in Ambros. A 101 sup. (Heiberg in *EOO* VII, XLIV; Decorps 1987, 44-5). Decorps asserts again that "nous retrouvons toutes les innovations

<sup>69</sup> He singled out this class of manuscripts since they have *sch*. 122 in the text.

<sup>&</sup>lt;sup>70</sup> Apart from the conjunctive variant readings, the fact of sharing characteristic scholia (*sch.* 114, 117, 118, 123) is used as a criterion of dependence. In particular, *sch.* 117 is unique to Vat. gr. 202, and Ambros. A 101 sup. has it in the text.

intégrées au texte" of Marc. gr. 301, whose variant readings are also attested as a rich series of corrections apposed by a 15<sup>th</sup>-century hand to Vat. gr. 204; Heiberg included such corrections in the apparatus; he asserts that they derive from Marc. gr. 301, even if the corrector "inde a p. 300, 18 [*scil.* with the end of prop. 8] taedio laboris inutilis ab incepto codici praestantissimo pulcherrimoque funesto destitit" (*EOO* VII, XLIII-XLIV). Copies of Marc. gr. 301 are Par. gr. 2013 (partly transcribed by Christopher Auer), a copy of which is Par. suppl. gr. 186, written by Angelo Vergezio; Vindob. suppl. gr. 9, penned by Camillo Zanetti; Berol. Phillipps 1543; and Marc. gr. 302; a copy of the latter is Berol. Phillipps 1544. Copies of Vat. gr. 204 *post correctionem* are Par. gr. 2107 and 2366 (*ibid.*, XX-XXI and XLIV). To Heiberg, the Ambrosian also is a copy of Marc. gr. 301, in its turn are indirect –that is, via the recension– apographs of Vat. gr. 192, in its turn a copy of Vat. gr. 191 (*ibid.*, XXIII, XLIII-XLIV), whereas Decoprs holds that Ambros. A 101 sup. indirectly derives from Vat. gr. 192. Decorps does not mention Par. gr. 2342, while Heiberg refers to his discussion of the tradition of *Optica* **B**.

A further recension, not attested elsewhere, can be found in Par. gr. 2448 (variant readings in *EOO* VII, XLVI-XLVIII; see also item 8 of Sect. 4).

#### 6) Euclid, Sectio canonis

A Byzantine recension is attested in Marc. gr. VI.3, 11<sup>th</sup> century (Acerbi – Pérez Martín 2015, 105 n. 13); the subscription reads in fact Εὐκλείδου κανόνος κατατομὴ Ζώσιμος διώρθου ἐν Κωνσταντινουπόλει εὐτυχῶς. The text was later heavily corrected by at least three, if not five hands. Zosimos' recension, supplemented with all layers of corrections, was very productive thanks to the abundant progeny of its direct apograph Marc. gr. 322, penned for Bessarion by John Rhosos and that the Cardinal regarded as one of the best codices of his (f. 1r *liber optimus et qui raro reperitur*). Another branch of the tradition is headed by a recension manuscript, namely, Vat. gr. 2338, a composite codex whose ff. 1r-22v are in imitative writing of the second half of 13<sup>th</sup> century (*ibid*.). One cannot exclude that the corrections in Vat. gr. 2338 are the result of collation with Marc. gr. VI.3. The only branch not affected by Byzantine recensions is represented by Matr. 4678 (see note 86 below), whose text is incomplete (*des. EOO* VIII, 170.28 τὸ AB ἄρα) and marred by almost incredible scribal mistakes. Three critical editions exist of *Sectio canonis*: Jan 1895, 148-66, reproduced by Menge in *EOO* VIII, 158-82, and Barbera 1991. None of them is reliable.

#### 7) Apollonius, Conica

We only read Apollonius' treatise in a series of recensions, a later one possibly modifying the text of an earlier one. As we have seen in item 3 of Sect. 3, the entire tradition de-

pends on Vat. gr. 206, that contains the edition, redacted by collating a number of divergent witnesses and limited to books I-IV, authored by the early 6<sup>th</sup>-century Neoplatonic philosopher Eutocius. The commentary on *Conica* Eutocius wrote in the margins of his own edition (Acerbi 2012) has followed a line of tradition totally different from that of *Conica*, since the main witness of the former is Vat. gr. 204, that of the latter Vat. gr. 206 (see also item 21 below). As seen in the discussion under item 2 of Sect. 4, a thorough Byzantine recension, included in a wide-ranging astronomico-mathematical *corpus*, is witnessed by Ambros. A 101 sup., Par. gr. 2342, Upsaliensis gr. 50 (Decorps 1987). Certainly redacted during Renaissance are a number of further recensions attested in very recent manuscripts, on which Latin translations as Memmus' (1537; Greek witness Bodl. can. gr. 106, 15<sup>th</sup> century) or magisterial editions such as Halley's *princeps* of 1710 are based (text drawn from apographs of Par. gr. 2356, 16<sup>th</sup> century, annotated by Pierre de Montdoré), or which constituted personal exemplars of renowned scientists (Greek witness Norimb. Cent. V, App. 6, 15<sup>th</sup> century, that has belonged to Regiomontanus).

## 8) The astronomical treatises included in the "little astronomy"

In the order they are given in Vat. gr. 204, these are: Theodosius, *Sphaerica*; Autolycus, *De sphaera mota*; Euclid, *Optica* **B**; Euclid, *Phaenomena* **b**; Theodosius, *De habitationibus*; Theodosius, *De diebus et noctibus*; Aristarchus, *De magnitudinibus et distantiis solis et lunae*; Autolycus, *De ortibus et occasibus*; Hypsicles, *Anaphoricus*. Euclid's treatises have been discussed under items 3 and 4 above. Decorps 1987, 45-50, summarizes the information available in the standard critical editions; if no such data are available, she asserts to have proceeded to new collations. After her article, Noack 1992 and Czinczenheim 2000 have thoroughly studied the manuscript traditions of Aristarchus' treatise and of Theodosius' *Sphaerica*, respectively<sup>71</sup>. The results can be summarized as follows.

• Theodosius, *Sphaerica*. A Byzantine recension is attested in Ambros. A 101 sup. and Marc. gr. 301 (Decorps 1987, 45-7; Czinczenheim 2000, 550-603, with a final *stemma*). The variant readings are also attested as a rich series of corrections apposed by a 15th-century hand to Vat. gr. 204; Heiberg (1927) and Czinczenheim included such corrections in the apparatus. Czinczenheim's *stemma* shows that successive copies of Marc. gr. 301 are Marc. gr. 302, Vat. Palat. gr. 62, Bodl. Savil. 4, one of the other in this order; a further copy of Marc. gr. 302 is Berolin. Phillipps 1544; a copy of Vat. gr. 204 *post correctionem* is Par. gr. 2366. Both the Ambrosian and the Marcian codex are independent and indirect –that is, via the recension– apographs of

<sup>&</sup>lt;sup>71</sup> But note that Czinczenheim takes Decorps' reconstruction as an established fact, apparently conforming her own conclusions to Decorps'.

Vat. gr. 202. The variant readings of Par. gr. 2342, which belongs in the family whose earliest representative is Vat. gr. 204 and which appear to contain a different, lighter, recension, are listed and discussed in Czinczenheim 2000, 309-35; Malachias had in his hands a manuscript of the recension attested in Ambros. A 101 sup., from which he lifts a couple of scholia.

- A further recension of *Sphaerica*, not attested elsewhere, can be found in Par. gr. 2448 (variant readings *ibid.*, 241-58 and *app.*; see also item 8 of Sect. 4).
- Autolycus, *De sphaera mota* and *De ortibus et occasibus*. A Byzantine recension is attested in Ambros. A 101 sup., whose variant readings are listed in *AP*, 297-301 (*stemma* of the entire tradition at 156). Copy of this manuscript is Cesaraug. 1143, of the latter Monac. gr. 301 and Vindob. phil. gr. 268 (all three were written by Andreas Darmarius; all three only contain *De sphaera mota*). The codex Ambrosianus is an indirect –that is, via the recension- apograph of Vat. gr. 202 (*AP*, 108-15; Decorps 1987, 47-8). The variant readings of Par. gr. 2342, which belongs in the family whose earliest representative is Vat. gr. 204 (a copy of which is again Par. gr. 2366) and which appear to contain a different, lighter, recension, are discussed at *AP*, 78-84, and listed *ibid.*, 287-8; they also frequently offer a corrupted text.
- A further recension of *De sphaera mota*, not attested elsewhere, can be found in Par. gr. 2448 (variant readings in *AP*, *app*. and 283-4; see also item 8 of Sect. 4).
- Theodosius, *De habitationibus* and *De diebus et noctibus*. Decorps (1987, 47) asserts, without justifying her claim, that a Byzantine recension is attested in Ambros. A 101 sup. The variant readings of this manuscript are included, along with those of Vat. gr. 204, Vat. gr. 191, and of Par. gr. 2363, in the apparatus of Fecht 1927. The editor offers no discussion of the manuscript tradition.
- Aristarchus, *De magnitudinibus et distantiis solis et lunae*. A Byzantine recension is attested in Ambros. A 101 sup. Copy of this manuscript is Upsal. gr. 53. The Ambrosian codex is an indirect –that is, via the recension– apograph of Vat. gr. 202 (Noack 1992, 256-61, 270-6; *stemmata* of the entire tradition after 336: one *stemmat* for the scholia, one for the main text). The variant readings of Par. gr. 2342, which belongs in the family whose earliest representative is Vat. gr. 204, are discussed at Noack 1992, 97-104 and 110-12; they frequently offer a corrupted text.
- Hypsicles, *Anaphoricus*. A Byzantine recension is attested in Ambros. A 101 sup., whose variant readings are included in the apparatus of De Falco Krause 1966; De Falco, as Manitius before him, was deluded by the better quality of the text of the Byzantine recension, which he thereby took as deriving "recta via ab archetypo", even if he admits that it has been corrected (1966, 26). The codex Ambrosianus is

an indirect –that is, via the recension– apograph of Vat. gr. 202 (*ibid.*; Decorps 1987, 49-50). The variant readings of Par. gr. 2342, which belongs in the family whose earliest representative is Vat. gr. 204, are also included in De Falco's critical apparatus; they sometimes carry good readings but frequently offer a corrupted text.

# 9) Hypsicles, "Elementa XIV"

A treatise written *ca.* 150 BC, but transmitted in most manuscripts as book XIV of *Elementa*, along with a further supplement, redacted within the school of Isidorus of Miletus (*EOO* V, VIII and 50.21-2) and named "book XV". According to Decorps (1987, 37-40), a thorough revision of both writings is contained in Ambros. A 101 sup., whose readings are only introduced as corrections in Par. gr. 2342, which in its turn contains characteristic readings *ante correctionem* and seems to have used a model related to Bodl. Dorv. 301<sup>72</sup>: Heiberg 1903, 323-5. Heiberg did not collate these manuscripts in his critical edition (*EOO* V, 2-66), but later reported the variant readings of the Ambrosian codex, concluding that it was copied from Par. gr. 2342 but further corrections were introduced by collation from another manuscript (1903, 325-8). The base text of Ambros. A 101 sup., although marred by a number of lacunae, belongs to the rich family which presents "book XIV" to the Euclidean treatise, the other making the two separated by other Eucliden works (Vat. gr. 190). A second family, presenting "book XIV" alone and carrying a slightly different text, is represented by Monac. gr. 427, ff. 234r-240v<sup>73</sup>.

## 10) Ptolemy, Almagestum

As is natural, when editing Ptolemy's monumental treatise Heiberg was only interested in providing a reliable text; for this reason, he did not report in his *prolegomena* the variant readings possibly arising from Byzantine recensions. Still, he quite clearly indicates (*POO* II, XL-XLIII) that a carefully executed recension, stemming from an extensively corrected copy of Vat. gr. 1594, is contained in Vat. gr. 198 and Marc. gr. 310, whose co-

<sup>72</sup> Contents: ff. 1r-5v *adnotationes variae*; ff. 6r-370v (*immo* 360v) Euclid, *Elementa*; ff. 371r-387r (*immo* 361r-377r) [Euclid, *Elementa*] XIV-XV; ff. 387v-388v (*immo* 377v-378v) *adnotationes et diagrammata* (the numbering of folios leaps from 355 to 366). The copy was completed by Stephanus in September 888 (subscription f. 387v [*immo* 377v]); the codex was owned by Arethas (annotation *ibid*.), who annotated it extensively. Ff. 6-14 are a later restoration, *des*. I.14, *EOO* I, 38.17 ἄρα ὑπό.

<sup>73</sup> EOO V, v-vIII, Vitrac – Djebbar 2011, 85-8; see Noack 1990 for Monac. gr. 427, whose ff. 1r-233v, that contain Proclus, *in primum Elementorum*, are penned by a hand of the second half of 11<sup>th</sup> century, ff. 234r-244r by a hand of 12<sup>th</sup> century (ff. 240v-244r contain Marinus, in *Data*).

pyists, however, operated a selection of the interventions on Ptolemy's text, and maybe added further corrections (see items 17 and 21 below). Heiberg surmises that the author of the recension was Nicholas Cabasilas. As said under item *a* of Sect. 4, *Almagestum* was also corrected in Par. gr. 2390 by a hand which can be identified with that of Manuel Bryennios. The corrected text was transcribed in Laur. Plut. 28.1. No indication as of the extent and nature to such corrections can be found in Heiberg's *prolegomena*, who, however, shows that the recension manuscript used by the copyists of Vat. gr. 198 and Marc. gr. 310 had also left some traces in Par. gr. 2390 (*ibid.*, XXXVIII-XL and again XL).

#### 11) Ptolemy, De hypothesibus planetarum I

Book II of this treatise is lost, as well as a part from the beginning of book I. Heiberg argues (POO II, CLXVIII) in favor of two independent branches, but the textual data strongly suggest that the hyparchetype A he postulated for one branch is nothing but a tampered with apograph of Vat. gr. 1594, which also happens to be the ancestor of the other branch. In fact, Heiberg supports his *stemma* by noting that the text of the Vatican manuscript and of its apographs breaks off at POO II, 104.23 ἰσοταχῶς, whereas A also contains the end of Hyp. I. Now, the final part of the section on Saturn, which is missing in Vat. gr. 1594, is a word-by-word copy of that on Jupiter<sup>74</sup>, the only difference being that, in A, blank spaces take the place of all numerical values. Therefore, the end of the treatise has been completed by some reviser (unable to restore the correct numerical values), who very likely also was responsible of all other divergencies between the two branches. Since one of the main witnesses of class A is Vat. gr. 208, the reviser was very likely John Abramios: Pingree 1971, 202, and item 9 of Sect. 4 above. As for Ptolemy's other minor astronomical works, neither Phaseis nor Psephophoria seem to have attracted the attention of Byzantine scholars. Still, one may suspect the presence of extensive variant readings in Marc. gr. 314 as far as Psephophoria and De hypothesibus planetarum I are concerned (see again item 9 of Sect. 4 above, and also items 13, 19g, 21 below).

#### 12) Ptolemy, Harmonica

The manuscript tradition of this treatise is complex. A first recension (Düring's class **f**) gives rise to a rich branch of the tradition, entirely stemming from Monac. gr. 361a, for which see item *f* of Sect. 4. We also know of a recension authored by Nicephorus Gregoras about 1335<sup>75</sup>. Gregoras had access to manuscripts of class **m** (apparently unaffected

<sup>74</sup> Compare POO II, 104.23-106.8 with 100.22-102.8. I thank A. Jones for the suggestion.

<sup>75</sup> This is the likely date of a letter to Michael Kaloeidas (*epist.* 114, in Leone 1982, vol. 2, pp. 301.95-302.107) in which he speaks about having revised Ptolemy's treatise: see the whole

by any recension) and of class **f**; he also supplied part of chapter III.14 and the entire chapter III.15 (also contained in Monac. gr. 361a, see above), as is borne out by some scholia preserved in a number of manuscripts (see again *PtH*, LXXVIII-LXXXVIII). This is Düring's class **g**; the common source of this rich branch of the tradition is taken by Düring to be Vat. gr. 198, ff. 35r-89r, but, as we have seen under item 3 of Sect. 4, one must certainly invert the relation of dependency between the Vatican manuscript and Par. Coislin 173, ff. 31v-111v. As is usual with Vat. gr. 198, as well as with Par. gr. 2342, Porphyry's commentary on *Harmonica* is transcribed in the margins of Ptolemy's treatise (see items 15, 17, 23 below). A further recension of *Harmonica* was redacted by Isaac Argyros, whose fair copy is preserved (but recall that Argyros was used to correct *in scribendo*) in the autograph Vat. gr. 176, ff. 101r-159v<sup>76</sup>. See also, for the structure of the manuscript classes of the three recensions, *PtH*, LIV-LIX, LIX-LXV, LXVI, respectively, in the order just presented; the *stemma codicum* is at LXIX.

# 13) Ptolemy, Tetrabiblos and Centiloquium

The traditions of *Tetrabiblos* and *Centiloquium* are almost identical (cf. *POO* III.1, xIxxv; *POO* III.2, xXIV-XXXII). A recension of both texts (class  $\beta$  of Boll-Boer-Hübner and Boer; Marc. gr. 314 is one of its best representatives) probably is of Byzantine origin; the texts of this branch of the tradition was further revised by John Abramios (class  $\gamma$ ; see Pingree 1971, 202, and items 9 of Sect. 4 above and 11, 19g, 21 of this section). The manuscript Norimb. Cent. V, App. 8, whose first part was used in 1535 by Camerarius for the *princeps* of *Tetrabiblos* (Rome 1948), is contaminated with class  $\gamma$  and with Hephaestion's excerpts (*POO* III.1, xIX-XX).

# 14) Diophantus, Arithmetica

Maximus Planudes' recension, endowed with a commentary on books I-II, has survived in part in Ambros. & 157 sup. Its descendants, most notably Marc. gr. 308, contain the whole text. The Ambrosian manuscript, paper, is an autograph of Maximus Planudes written about 1293 (Turyn 1972, 78-81 and plate 57; Allard 1979). Only 23 folios remain of this codex, in a perturbed order. Contents: fragments of *Psephophoria secundum In-dos* by Planudes himself (6 folios and 1 page); fragments of [Iamblichus] *Theologumena arithmeticae* (5 folios and 1 page); 1 page of a text ascribed to Psellus; fragments amoun-

discussion at *PtH*, LXXVIII-LXXXVIII. In the same letter, Gregoras describes his works on Easter Computus and on the astrolabe (the latter apparently in two redactions; cf. item 10 of Sect. 6).

 $^{76}$  See PtH, xxxIII – the ascription is curiously downgraded in Mondrain 2007, 168 n. 20– and LXXXVIII-LXXXIX. ting to about one half of books I-II of Diophantus, *Arithmetica*, in Planudes' recension (10 folios: in correct order, these are ff. 13, 14, 8, 18, 20, 15, 9, 16, 17, 19), with his own commentary written in the margins and featuring authorial corrections. Marc. gr. 308, paper, is a composite manuscript made of two parts. The first six quires, ff. 1r-49v, were written by five different hands in the second half of 14<sup>th</sup> century (watermarks dated from 1342 to 1383); in particular, ff. 9r-34r (with the exception of f. 15, in fact a leaf that was added later) were penned by Isaac Argyros (Mondrain 2007, 166-7; Bianconi 2008, 357). The second part of the codex, ff. 50v-284r, was written by a single copyist at the very end of 13<sup>th</sup> century. Contents: ff. 1r-34v Cleomedes, *Caelestia* I-II; ff. 35r-46r pseudo-Aristoteles, *De mundo*; ff. 46v-49v *varia astronomica*; ff. 50v-263r Diophantus, *Arithmetica* (on two columns; Planudes' commentary is instead written on the full page, in a script of lesser size); ff. 263v-272v Diophantus, *De polygonis numeris* (incomplete, as in all manuscripts); ff. 273r-284r a fragment of Planudes, *Psephophoria secundum In-dos*. Marc. gr. 308 is the only direct and complete copy of the Ambrosian codex (Allard 1982-3, 100-2).

## 15) Porphyry, Commentary on Ptolemy's Harmonica

We know of a recension authored by Nicephorus Gregoras about 1335<sup>77</sup>. This is Düring's class **g**; the common source of this rich branch of the tradition is Par. Coislin 173, ff. 148r-205v, even if Düring (and Raffa 2016) take it to be Vat. gr. 198, ff. 35r-71v in the margins of Ptolemy's *Harmonica*, as is usual with this manuscript and with Par. gr. 2342 (see items 12, 17, 23 of this section). A further recension was redacted by Isaac Argyros, whose fair copy is preserved (but recall that Argyros was used to correct *in scribendo*) in the autograph Vat. gr. 176, ff. 160r-192v. A third recension, severely shortened *per capita*, is contained in a number of manuscripts; this is Düring's class **h**. For the structure of the manuscript classes of the three recensions, see *PoH*, XXI-XXVI, XXVI-XXVIII, respectively, in the order just presented; the *stemma codicum* is at XXIX.

#### 16) Serenus, De sectione coni and De sectione cylindri

A Byzantine recension of Serenus' treatises, with exactly the same characters as that of Apollonius' *Conica*, is contained in Ambros. A 101 sup. and Par. gr. 2342: Heiberg 1896, v-IX, who also included in the apparatus its variant readings, taken from the Paris codex. As usual, Heiberg held that the Ambrosian manuscript is a copy of Par. gr. 2342.

<sup>&</sup>lt;sup>77</sup> This is again the date of the letter in which he speaks about having revised Ptolemy's *Harmonica*; see the whole discussion at *PtH*, LXXVIII-LXXXVIII, and item 12 above.

## 17) Theon of Alexandria, commentary on Ptolemy's Almagestum

The manuscript tradition of this treatise, the longest Greek mathematical text and one of the longest of the entire Greek literary *corpus*, is complex. Books I-IV and VI are transmitted by the *vetustissimus* Laur. Plut. 28.18 (early 9<sup>th</sup> century; books I-IV are edited in *iA*); as said above, Tihon (1987) found book V, otherwise lost, in the margins of Vat. gr. 198, after Rome (1953) had identified a long extract included in the main text (ff. 421v-424v); book XI is lost; book VII can only be read in the Byzantine recension; the only witness of Theon *in Almagestum* VIII-X and XII-XIII *prima facie* not containing a Byzantine recension happens to be a portion, dated to the end of 13<sup>th</sup> century, of Vat. gr. 1087<sup>78</sup> –only available in a form in which, however, we have every reason to suspect interventions of Maximus Planudes<sup>79</sup>. We still read books VII-XIII in the 1538 Basel edition published by Camerarius, for which Walder' typesetters used Norimb. Cent. V, App. 8, a representative of the Byzantine recension. The first part of this codex was already used in 1535 by Camerarius for the *princeps* of *Tetrabiblos* (Rome 1948).

The main representatives of the Byzantine recension are Vat. gr. 198 (copyist Malachias) and Marc. gr. 310 (copyist Argyros); Rome also collated Par. gr. 2398, namely, the source of Halma's 1821 edition (only books I-II), and Norimb. Cent. V, App. 8, namely, the source of the *princeps* of 1538 (*iA*, XXI-XXIV and LXXXVI-XCII). Theon's commentary in this recension is the result of a wide-ranging edition project, achieved using several manuscripts as sources and presenting a thorough revision of the text, which was also completed with Pappus' and Cabasilas' commentaries on specific books of *Almagestum*. Rome describes what one finds in the main manuscripts in the following terms (E and J are the Marcian and the Vatican codex, respectively): "On voit l'éditeur à l'œuvre dans E et surtout dans J, qui présente des traces matérielles du travail, son encre ayant viré

<sup>78</sup> The relevant portion is at ff. 123-147. This manuscript must be completed (Rome 1927) with Par. gr. 2396, which contains Theon *in Almagestum* I-II, IV and whose ff. 3-76 date back to the Planudean period and actually are in part (ff. 33v-76v) an autograph of Planudes himself: Mondrain 2002. The copyist of Par. gr. 2396, ff. 77r-86v, and of the said folios of Vat. gr. 1087, was the anonymous collaborator of Nicephorus Gregoras known as "copyist G": *ibid.*, 318-9; Bianconi 2003, 545; Bianconi 2006, 147-51. Both codices also contain annotations by Gregoras (Mondrain 2002, 319-20; Bianconi 2005, 414-15 nr. 24 and 417 nr. 40). On Vat. gr. 1087, see most recently Menchelli 2013.

<sup>79</sup> A proof of this is a calculation that Theon performs at *in Almagestum XIII.3*, first διὰ τῶν ἐκ τῶν γραμμικῶν ἐφόδων ἐπιλογισμῶν, then διὰ τῆς τῶν διοφαντείων ἀριθμῶν ἀγωγῆς. We read this passage at Vat. gr. 1087, ff. 145r-v, where it is followed, in the main text, by a carefully arranged tabular set-up of the diophantine-style solution, identical in form to those displayed in Planudes' commentary on Diophantus' *Arithmetica* (Christianidis – Skoura 2013). Note that this manuscript does not contain the recension briefly described in the next paragraph.

autrement dans les additions postérieures que dans les choses qu'il à écrites d'abord. E et J ont travaillé de façon indépendante dans les mêmes bibliothèques, ou en tous cas, sur une série de manuscrits de mêmes types" (*iA*, LXXXIX; see also Rome 1930 and already, almost in the same terms, *POO* II, XLIII). In fact, Marc. gr. 310 contains a modified form of the recension (Acerbi 2015) –as was usual with Isaac Argyros, who wrote the entire manuscript, his text is a correction *in scribendo* of the recension independently attested in Vat. gr. 198. Contrary to Malachias, who appears to have used the recension manuscript only to correct a previous transcription of his (see also items 2 of Sect. 4 and 9 of this section), Argyros directly operated on such a manuscript. See also his annotation at Marc. gr. 310, f. 239v *imo marg. inf.*, justifying the permutation of books VII and VIII of Theon *in Almagestum* and edited at *POO* II, XLIII: μὴ θαυμάσῃς ἐνταῦθα εἰ πρὸ τοῦ ἑβδόμου κεῖται τὸ η'. διὰ γὰρ τὸ σποράδην ταῦτα συνάγειν ἡμᾶς ὡς μὴ ἐντυχόντας ἑνὶ βιβλίφ τὰ πάντα συνημμένως ἔχοντι τοῦτο γέγονεν.

# 18) Theon, "great" commentary on Ptolemy's Tabulae manuales

A revision is contained in the recension manuscript Par. gr. 2450, ff. 132r-178r (*GC* I, 19-22 and 32-5; cf. item *d* of Sect. 4). Two further, fully-fledged recensions are contained the one in Norimb. Cent. V, App. 8, ff. 215r-237v, Ambros. A 101 sup., ff. 190r-226r, and the several copies of the latter (*GC* I, 16-19, 4-6, respectively, 54-67, and 82-3), the other in Monac. gr. 419, ff. 110r-138v (*GC* I, 14-16, 50-52, and 83-4). Lighter revisions are contained in Laur. Plut. 28.12, ff. 41r-94v, and Par. Coislin 173, ff. 263r-312r (*PC*, 138-9 and *GC* I, 14, *GC* I, 7-9, respectively, 38-49, 83, and II, 3-6). *Stemma codicum* at *GC* I, 68, and, slightly modified, at II, 7.

 διορθώσεως (*GC* I, 8, 83). This suggests that the recension was made after Gregoras' annotation: Tihon tentatively proposes Nicholas Cabasilas as its author. As for the recension contained in Monac. gr. 419, Mogenet and Tihon set it in parallel with the recension of Theon's "little" commentary on *Tabulae manuales* redacted in 14<sup>th</sup>-century Thessalon-ike by John Catrarios (*GC* I, 83-4, and see item 19a just below).

## 19) Theon, "little" commentary on Ptolemy's Tabulae manuales

There is probably no Greek mathematical or astronomical text that has been modified so frequently, heavily, and independently, from Late Antiquity to Byzantine times, as Theon's "little" commentary. The reason is simple: Theon's treatise was the key to enter the most useful technical text from Greek Antiquity: Ptolemy's *Tabulae manuales*. As a consequence, Theon's treatise had a plethoric manuscript tradition, to a large extent made of Byzantine recensions; almost each of them also took care of modifying the morphology and the syntax of the "original", that were perceived as aberrant with respect to classical Greek language. The manuscript tradition of Theon's "little" commentary has been carefully studied in *PC*, whose results I summarize here<sup>80</sup>.

- a. A recension carried by nine manuscripts, whose prototype is Vat. gr. 175, written in part (ff. 1v-8v, 40r-79r, 81r-159r) by John Catrarios in 1321/22 (Turyn 1964, 124-30 and plates 97 and 190d). The recension certainly is to be ascribed to Catrarios himself; it is almost exclusively involved in "regularizing" morphology and syntax. Among the apographs of the Paris manuscript is Laur. Plut. 28.31. See *PC*, 15-41.
- b. A recension carried by six manuscripts (among which Vat. gr. 208), whose prototype is Marc. gr. 323. This is a manuscript made of codicological units to be referred to different periods. The earliest of such units, written by two copyists in the second half of 14<sup>th</sup> century<sup>81</sup>, contain autographs of Isaac Argyros (Mondrain 2007, 166;

<sup>80</sup> The families I do not mention do not give rise to fully-fledged recensions, even if the text of Theon's treatise has often been modified to some extent. At the end of each group of pages of *PC* in which a manuscript family is discussed, Tihon offers an outline of the main characteristics of the recension at issue.

<sup>81</sup> The copyist of the later sections (ff. 1r-22v, 25r-37v, 41r-70r, 71r-169v, 211r-212r3, 214rv, 222r-225v17, 226r-244v22, f. 245r-249v, 258r-263r11, 479r-487r14, 487v), to be referred to the early 15<sup>th</sup> century, also penned the entire Marc. gr. 335 (astrological miscellany); Vat. gr. 573, ff. 51r-214v (including Achmet's *Oneirocriticon*); Vat. gr. 792, ff. 1r-24v and 354r-360v (astronomical texts and tables; see Leurquin 1991 for this manuscript); Vat. gr. 1058 (for which see Heiberg 1899 and the masterful analysis in Neugebauer 1960), ff. 2r21 *ab imo*-3r, 4r-8v (*in tabulis, cum alia manu*), 9r-260v; Vat. gr. 1709, ff. 1r-196v (Euclid, *Elementa*; [Euclid, *Elementa*] XIV) and, after some blank pages and in a different layout, ff. 203r-210v (Nicomachus, *Introductio arith*- Bianconi 2008, 358), among which an important addition to Theon's treatise that characterises the whole manuscript family (the addition is at ff. 285r14-287v11: Bianconi *ibid.*; *PC*, 52). This family offers maybe the best text of the "little" commentary, but just because it contains a thorough recension; for most of the variant readings, Marc. gr. 323 is the recension manuscript. It is not clear whether the author of the corrections in the Venice manuscript is Argyros or not; Tihon, who was not aware that his hand is present in Marc. gr. 323, already surmised that this was the case (*ibid.*, 64-5). See *PC*, 42-65.

- c. A recension carried by seven manuscripts, whose prototype is Laur. Plut. 28.7, written some time after 1344. This manuscript both contains a recension (the one described in the following item) and is a recension manuscript (operating on the recension that constitutes the base text), whose corrections have passed on all its apographs. Tihon surmises that the corrections on the text were made in order to prepare for the copy of the *de luxe* apograph Ambros. H 57 sup. See *PC*, 88-101.
- d. A recension carried by seven manuscripts, whose prototype is Leiden, BPG 78, ff. 3r-49r. Theon's treatise was copied by two hands dated end 13<sup>th</sup>-beginning 14<sup>th</sup> century; ff. 52r-152v are a beautiful witness in majuscule, written during the reign of Leon V (813-820), of Ptolemy's *Tabulae manuales*; ff. 156r-161v contain a further fragment of the "little" commentary, written by a hand of 9<sup>th</sup>-10<sup>th</sup> century, in which we shall not be interested here. Leiden, BPG 78 is the recension manuscript of thorough revisions operated by different hands in different periods; these corrections marginal, interlinear and *in rasura* can be read in the main text of all its apographs, among which Laur. Plut. 28.7. See *PC*, 102-23.
- e. A recension carried by five manuscripts, whose prototype is Par. gr. 2399, end 13<sup>th</sup>-beginning 14<sup>th</sup> century, owned and annotated by John Chortasmenos (possession note at f. 1r). Among the apographs of the Paris manuscript are Vat. Urb. gr. 80 (a very short fragment, at f. 13r), partly written by Chortasmenos himself, and Vat. gr. 1059, entirely written by him (see Canart Prato 1981; see also *RGK* III, nr. 315; cf. I, nr. 191, II, nr. 252). Par. gr. 2399 is the recension manuscript, and its marginal and interlinear corrections can be read in the main text of all its apographs. In copying the version in Vat. gr. 1059, Chortasmenos availed itself of a further, unidentified manuscript. See *PC*, 124-35.
- f. A recension carried by only two manuscripts, Vat. gr. 198 and its apograph Vat. Ottob. gr. 26. The main intervention of the reviser has been changing the order of

*metica, des. mut.* Hoche 1866, 28.14 vη Δία τοῦ γ, καὶ); Vindob. suppl. gr. 75, ff. 29r-49v (Isaac Argyros, *Opuscula astronomica tria*): Mercati 1926, 51 n. 3; Acerbi – Gioffreda, forthcoming.

the chapters in such a way as to keep to the progression Sun-Moon-Planets; there is even a copyist's note to this effect at f. 476r. The only other major variant is the replacement of chapter 7 (*PC*, 221.14-222.5) with a text that has recently been identified as the corresponding chapter of the commentary by Stephanus of Alexandria on Ptolemy's *Tabulae manuales* (Lempire 2016, 309). See *PC*, 153-6.

g. A recension carried by only two manuscripts, Laur. Plut. 28.26 and its partial apograph Mutin. γ.J.2.19. Theon's treatise (ff. 2r-31v) has been copied by the Metochitesschreiber (Bianconi 2010); ff. 34v-128r are one of the most beautiful witnesses in majuscule, written during the reign of Leon VI (886-912), of Ptolemy's *Tabulae manuales*. As is to be expected, "le texte de [Theon's treatise in Laur. Plut. 28.26] est visiblement remanié" (see items 9 of Sect. 4 above and 11, 13, 21 of this section). See *PC*, 159-60.

## 20) Proclus, Hypotyposis astronomicarum positionum

The problem with Manitius' edition is that he did not completely collate most of the manuscripts. From what he writes, we may safely assert that certainly his class C and very likely his class D contain Byzantine recensions (1909, XXXV-XXXIX and XXXIX-XLI, respectively).

## 21) Anonymous, Prolegomena ad Almagestum

The ongoing edition of this computational primer to Ptolemy's work<sup>82</sup>, redacted in the early 6<sup>th</sup> century within Ammonius' school, shows that the text, mere course notes badly in need of a final redaction, was repeatedly corrected by Byzantine scholars. Laur. Plut. 28.1 integrates into the text all corrections found in the recension manuscript of Manuel Bryennios Par. gr. 2390; the same happens with the corrections typical of Vat. gr. 184, to be read in fair copy in all its apographs (Marc. gr. 311, ff. 2r-24r but only the restoration in occidental paper<sup>83</sup>, and Vat. Palat. gr. 95, ff. 24v-33v); as said under item *b* 

<sup>&</sup>lt;sup>82</sup> The edition of what preceds the computational primer in *Prolegomena ad Almagestum* is in Acerbi – Vinel – Vitrac 2010. Editions of the material attached to *Prolegomena* (called *adnotationes variae* in my descriptions of the manuscripts) are in *POO* II, XXXIV-XXXVII, and Jones 2005.

<sup>&</sup>lt;sup>83</sup> The copyist of a part of these restorations is Nicholas Triclines (Bianconi 2005a, 132; on Triclines see *ibid.*, 122-41; *PLP* 29315; *RGK* III, nr. 519; contents of Marc. gr. 311: ff. 2r-25r *Prolegomena ad Almagestum*; ff. 26r-342v Ptolemy, *Almagestum*; the original portions of this manuscript, in oriental paper, can be dated to the middle 13<sup>th</sup> century and amount to ff. 1, 3-11 26-58, 60-65, 67-112, 113 *pars superior*, 118-123, 125-165, 169-170, 172-184, 186-191), to whom

of Sect. 4, Vat. gr. 184 contains annotations and corrections of John Pothos Pediasimos, John Catrarios, Nicholas Eudaimonoioannes; the latter is the annotator of *Almagestum*. A fully-fledged Byzantine recension of *Prolegomena ad Almagestum* is contained in Vat. gr. 198; the same recension is found, in a still modified form, in Marc. gr. 310, written by Isaac Argyros. A further recension is contained in Laur. Plut. 89 sup. 48 (copies of which are Regin. gr. 90, Neap. Borb. III C 13, Norimb. Cent. V, App. 8), but this is beyond doubt derived from that in Marc. gr. 310: Argyros transcribed twice the same text, modifying it on both occasions (see items 5 and 6 of Sect. 4). Further recensions are contained in Marc. gr. 303, ff. 31r-38v (extensive corrections, maybe simply due to the copyist; the quality of the interventions is so low as to produce a disfigured text) and in Marc. gr. 314 (see items 9 of Sect. 4 above and 11, 13, 19g of this section). See also the *Appendix*.

## 22) Damianus, Opticae hypotheses

This is a case study of an editor (Schöne 1897) who was deluded by the better quality of the text of the Byzantine recension, which he thereby took as the base text of his own edition (see already Hultsch 1898). Schöne used Par. gr. 2342, but the same recension of Damianus' tract is also attested in Ambros. A 101 sup. (Decorps 1987, 41-2), which Schöne, as Heiberg before him, assumed to be a copy of the Paris manuscript. The text is in need of a new edition, very likely to be based on Vat. gr. 192, which Schöne also collated. Most manuscripts of Damianus' tract are Renaissance copies; see the preliminary assessment in Todd 2003.

## 23) Eutocius, commentary on Apollonius' Conica

As it should be expected, Par. gr. 2342 also contains a thorough recension of Eutocius' commentary on Apollonius' *Conica: AGE* II, VI-VIII; the variant readings of the Paris

we also owe the copy of Laur. Plut. 28.31, ff. 1r-84v (he was the coordinator of the copy; the manuscript contains ff. 1r-3r *tabulae syzygiarum*; f. 5v *diagrammata duo*; ff. 6r-83v Ptolemy, *Tabulae manuales*; f. 84v *capita operarum sequentium*; ff. 85r-116v Theon, "little" commentary on Ptolemy's *Tabulae manuales*; ff. 117r-125v *adnotationes astronomicae*; ff. 126r-128r *De astrolabio* [ed. Delatte 1939, 254-62]; ff. 128v-141r Philoponus, *De usu astrolabii*; ff. 141v-143r *astronomica et astrologica varia*; cf. item 19a above), and of Vat. gr. 604, ff. 102r-152v (Proclus, *Hypotyposis*: Pérez-Martín 2000, 317; Bianconi 2005a, 128-9). The second copyist at work in Laur. Plut. 28.31 also transcribed Marc. gr. 317, ff. 1r-6v (contents: ff. 1r-49v Nicomachus, *Introductio arithmetica*; ff. 51r-71v Aratus, *Phaenomena*; ff. 72r-126r Cleomedes, *Caelestia*; ff. 128r-206v Euclid, *Elementa* I-VI). A third hand can be found in Laur. Plut. 28.31, that completes and corrects the *canon regius*: it is "copyist F": cf. item 7 of Sect. 4. The same hand also transcribed Par. gr. 2461, ff. 1r-149r (ff. 1r-93r Manuel Bryennios, *Harmonica*; ff. 94v-149r Ptolemy, *Harmonica*; f. 150r *tabula paschalis*; f. 151r *nota astronomica*; ff. 151v-187v George Chrysococces, *Syntaxis Persica*; ff. 188r-287v *tabulae Persicae*). See Bianconi 2012, and references therein, on all of this. manuscript are recorded in the critical apparatus. As is usual with this manuscript and with Vat. gr. 198, Eutocius' commentary is transcribed in the margins of Apollonius' treatise (see items 12, 15, 17 of this section); for this reason it is absent in Ambros. A 101 sup.

# 24) Philoponus, De usu astrolabii

As in so many other cases, Philoponus' treatise probably is the redaction of notes taken at a course given by Ammonius, whom Philoponus mentions at the beginning of his own treatise (Jarry 2015, 3.6-13). A branch of the tradition, called  $\Phi$ , contains a thorough recension, that one of the recent editors ascribes to Theodorus Meliteniotes (Jarry 2015, CXLII-CLXVI, and Jarry 2009 before it). Its main representatives are Marc. gr. 323, ff. 384r-393v (just preceding Argyros' autograph tract on the same subject: Bianconi 2008, 358), a copy of which seems to be Vat. gr. 208, ff. 209r-219v (just preceding Argyros' tract on the same subject); and independently Vat. gr. 792, ff. 9r-15r (for the most part an autograph of Meliteniotes himself, but here in a hand of 15<sup>th</sup> century: see note 81 above), and Ambros. E 104 sup., ff. 180r-189r. As a matter of fact, the variant readings strongly suggest that all manuscripts of this recension are a copy of Marc. gr. 323. For mysterious reasons, the recent editor insists on proposing Theodorus Meliteniotes as the author of the recension, whereas all evidence he adduces decidedly points towards Isaac Argyros.

# 6. A list of mathematical manuscripts certainly or probably to be associated with well-defined scholars or scholarly circles

This section presents in a different way a part of the documentary record already exploited in Sects. 4 and 5: it is an almost pure list of mathematical manuscripts, itemized according to the Byzantine scholars or scholarly circles with which any such manuscript certainly or probably is to be associated; in the cases of the copyist of Par. gr. 2448 and of Malachias (items 8 and 16 below) no such scholarly milieu has yet been identified. I have also included, under each item, the mathematical writings of the itemized scholar. Finally, I have added, more frequently than in the previous sections, bibliographical references and cross-references to other sections of the present paper.

It comes has no surprise that the list below emphasizes the filiation Bryennios-Metochites-Gregoras-Argyros, and by implication the library of the Chora monastery. Still, a word of caution is called for: it may well be that the orientation of recent palaeographical scholarship has introduced a strong bias towards this specific intellectual *milieu*. The section closes with a discussion of the origin and the purpose of the early-Palaiologan encyclopaedia in Vat. gr. 191, whose transcription is held, in recent palaeographic literature, to have been coordinated by John P. Pediasimos: I shall show that this contention does not stand a close scrutiny.

- George Pachymeres (*PLP* 22186). The autograph of his *Quadrivium* is Angelic. gr. 38<sup>84</sup>. He also redacted a very short commentary on some mathematical examples contained in Aristotle's treatises (ed. Heiberg 1904, 37-49, from Vindob. phil. gr. 150, ff. 199r-205v, a philosophical miscellany contemporary with Pachymeres).
- Maximus Planudes (*PLP* 23308). He copied the entire Ambros & 157 sup. (Turyn 1972, 78-81 and plate 57; Allard 1979; cf. item 14 of Sect. 5); Par. gr. 2396, ff. 33v-76v (Mondrain 2002, 314-18; cf. notes 40, 78 above). He annotated Laur. Plut. 28.2 (*Elementa*)<sup>85</sup>; Marc. gr. 312 (*Almagestum*; this almost certainly happened in 1293: Mondrain 2007, 160-1); Matrit. 4678<sup>86</sup>; Seragl. G.I.1, ff. 73v, 74r, 77r, 78v (metrological treatises and Hero, *Metrica*: Pérez Martín 2009, 65 n. 62); Vat. gr. 202<sup>87</sup>. He restored Matrit. 4678 (Pérez Martín 2006). He redacted a *Psephophoria secundum Indos* (ed. Allard 1981).

<sup>85</sup> At ff. 3v, 4r, 6r, 12r, 16r, 47r, 58r-v, 66r, 68r, 74r, 78v, 99v, 100v-101r, 130v-134v, 140r-142r, 162v, 221r, 229v: Pérez Martín 2010, 119 and n. 51; the scholia at ff. 12r, 131r (but this is not *sch*. X.98), 133r are ascribed to himself by Planudes. To Planudes are also ascribed *sch*. VI.6 (Par. gr. 2373, f. 123r) and X.223 (Vindob. phil. gr. 31, f. 144v) to *Elementa*: Heiberg 1888, 272-3. Laur. Plut. 28.2, for which see again Pérez Martín 2010, 117, paper, 12<sup>th</sup> century with a restoration (ff. 243-314) in imitative writing of end 13<sup>th</sup> century, contains ff. 2r-242r Euclid, *Elementa*, ff. 245r-301v Euclid, *Data*; ff. 301v-303r *scholia in Data* (the same sequence as in Vat. gr. 204: see note 5 above); ff. 303v-319r [Euclid, *Elementa*] XIV-XV.

<sup>86</sup> At ff. 58v, 65r-v, 67r, 68v, 70r, 76v, 78r-v, 79r-v, 89v: Pérez Martín 2006, 449-50, and the edition of the scholia in Allard 1983, *sch.* 1, 39, 52, 59, 76, 114-15, 122, 129, 146-7, 191 (scholia A2). Matrit. 4678, paper, mid 11<sup>th</sup> century, is written by two hands (ff. 9r-62r25 and 137r-143v; ff. 62r26-135v), the latter a beautiful *Perlschrift*. Contents: ff. 4r-57v Nicomachus, *Introductio arithmetica*; ff. 58r-130v Diophantus, *Arithmetica*; ff. 130v-135v Diophantus, *De polygonis numeris*; ff. 137r-142r Cleonides/[Euclid], *Introductio harmonica*; ff. 142r-143v Euclid, *Sectio canonis* (incomplete, *des.* prop. 10, *EOO* VIII, 170.28 ἄρα).

<sup>87</sup> Acerbi – Pérez Martín 2015, 111 n. 58: Planudes added titles and indications of end of text at ff. 50r, 82r, 95r, 95v, 132v, 133r, 177r, 220r, 269r, 299r, 305r. Short commentaries are at ff. 250r, 251r-v. On this basis he redacted the Greek *pinax* at f. 1r. Planudes' is not the only Palaiologan hand in Vat. gr. 202: cf. ff. 1r, 133r, 141v, 147r-v, 250r. Contents of Vat. gr. 202, oriental paper, second half of 13<sup>th</sup> century: ff. 1r-81v Theodosius, *Sphaerica*; ff. 82r-95r Autolycus *De sphaera mota*; ff. 95v-132v Euclid, *Optica* **B**; ff. 133r-176v Euclid, *Phaenomena* **b**; ff. 177r-191v Theodosius, *De habitationibus*; ff. 192r-249v Theodosius, *De diebus et noctibus*; ff. 250r-268v Aristarchus, *De magnitudinibus et distantiis solis et lunae*; ff. 269r-299r Autolycus *De ortibus et occasibus*; ff. 299r-305r, Hypsicles, *Anaphoricus*; ff. 305r-372r Euclid, *Data*; ff. 372v-381r Marinus, *in Data*; ff. 381v-398r, *scholia in Elementa* (a subset of the *scholia vaticana*, from *sch*. I.88 to *sch*. VII.40).

<sup>&</sup>lt;sup>84</sup> See *RGK* III, nr. 115; cf. II, nr. 89. Ed. Tannery 1940.

3. John Pothos Pediasimos (*PLP* 22235)<sup>88</sup>. He assembled, and annotated at least between 1296 and 1302/3, the early-Palaiologan mathematical encyclopaedia in Vat. gr. 191<sup>89</sup>. He annotated Bodl. Dorv. 301, f. 245v et *passim* (Mondrain, unpublished; it is Heiberg's hand B<sup>4</sup>); Laur. Plut. 28.2<sup>90</sup>; Vat. gr. 184<sup>91</sup>; Vat. gr. 2326, *passim<sup>92</sup>*. He redacted scholia to Cleomedes' *Caelestia* (ed. Caballero Sánchez 2016) and to Aristotle's *Analytica Posteriora* (among which a long section on the duplication of the cube: ed. De Falco 1926, 108.21-120.17); a so-called *Geometria* which is in fact a metrological compendium (ed. Friedlein 1866); a writing of *Specific Remarks* on music dealing mainly with musical intervals (ed. Vincent 1847, 290-314). Pediasimos also practiced the arithmological genre: we read of him short writings *Cur infantes VII* 

<sup>88</sup> For a first orientation on Planudes and Pediasimos, see Constantinides 1982, 66-89 and 116-28, respectively.

<sup>89</sup> See Turyn 1964, 89-97 and plate 54; Pérez Martín 2010 for the identification of Turyn's hand R with Pediasimos; see also the last remark of this section and notes 19 above and 111 below. Pediasimos' longest annotations are at ff. 1r (*pinax*; chronological calculation for year 1302/3), 29v (transcription of Marinus *in Data, des. EOO* VI, 238.24  $\pi$ ãv), 30v (note on some definitions of Euclid's *Data*), 108r-111v (two calculations of solar position for year 1302), 170v-172v (at f. 170r-v is a long scholium of copyist G whose calculations are carried out for April 14, 1298), 319r-v (among others, note on the earthquake of July 17, 1296), 359r-v, 397r-v.

<sup>90</sup> At ff. 57r, 58r, 120r, 122v, 129v, 134r, 160r, 246v-256v: Pérez Martín 2010, 117.

<sup>91</sup> At ff. 7r, 8r, 12v-13v, 14r, 18r-20r, 82r (see note 19 above). A scholium of Pedisimos to a clause located at the very beginning of *Almagestum* is in Vat. gr. 184, f. 82r, and in Marc. gr. 311, f. 26r: *POO* II, cxxx; Heiberg, however, only noted the latter occurrence.

<sup>92</sup> Bianconi 2014, 467-8 and n. 41. Pediasimos' interventions are at ff. 20r (κείμενον integration and scholium), 20v (two κείμενον integrations, a further short integration and a scholium), 21v (short integration), 22r (two short integrations), 22v (short integration), 24r (short integration and κείμενον integration), 27r (four κείμενον integrations), 29r (short integration), 30r (κείμενον integration), 31r (short integration and κείμενον integration), 31v (two integrations), 34v (marginal corrections), 36r (short integration), 36v (short integration), 37r (four short and one κείμενον integration), 39r (κείμενον integration), 39v (short integration), 40r (two integrations), 43r and 43v (κείμενον integration). The text of Prolegomena ad Almagestum is also corrected throughout. Vat. gr. 2326 is a composite manuscript, heavily mutilated and bound in disorder; it is a direct apograph of Vat. gr. 1594. After ff. 1-18 on parchment, containing musical matters, one finds, on oriental paper and written by a series of hands of end 13th century, Prolegomena ad Almagestum et adnotationes variae (correct order: 1 folio lost, ff. 26r-33v, 2 folios lost, 19r-24v, 43r-v, 34r-40v, 41r-v - the text is here deleted by pen strokes but it is same as that in the subsequent folio-, 25r-v; these leaves exactly correspond to three quires) and Almagestum: ff. 42, 44-60 = quires marked  $\kappa\alpha'$  and  $\kappa\gamma'$ , that are Almagestum V.5-13, POO I.1, 379.18 (inc.  $\kappa\alpha\lambda$  ėč άλλων δε)-414.1 (des. al δύο όρθαι τξ); V.19-VI.3, ibid., 450.3 (inc. και ΘΗ παραλλάξεων)-467; VI.3-5, *ibid.*, 470-481.19 (*des.* ὑποθώμεθα κατὰ τὸ Δ). The copyist of Vat. gr. 2326, ff. 26v-33v, coincides with copyist B of Vat. gr. 184 (see note 19 above): A. Gioffreda, per litteras.

*et IX mense nati vivant, VIII non vivant* (geometric variations are also found in the manuscripts) and *De novem musis* (ed. Cumont 1923 and De Falco 1923).

- Manuel Bryennios (*PLP* 3260). He annotated and corrected Par. gr. 2390. Scholia by Bryennios marked ἐμόν are at ff. 24v, 25v-26r, 26v, 26v-27v, 30r (Acerbi – Pérez Martín 2015; cf. items *a*, 7 of Sect. 4 and 10, 21 of Sect. 5; cf. also the *Appendix*). He redacted a treatise of *Harmonica* (ed. Jonker 1970).
- 5. Theodorus Metochites (PLP 17982). His Stoicheiosis Astronomike, including a summary of Ptolemy's Almagestum, was "published" in authorized copy during Metochites' lifetime<sup>93</sup>; this makes up Vat. gr. 182 and 181 (in this order; Vat. gr. 181 contains the second book of Stoicheiosis and, from f. 39r on, the summary; apographs include Marc. gr. 330, ff. 16r-169v, 176r-203v, 209r-286v, and its apograph Vat. gr. 1087, ff. 5r-122r and 148r-190r, 192r-221v, 223r-299v). The same copyist also penned Vat. gr. 303 (Bianconi 2005, 425-7; Bydén 2003, 385). A copy of Stoicheiosis Astronomike, within a project of Metochites' opera omnia coordinated by Nicephorus Gregoras and carrying autograph corrections of Metochites himself, is Vat. gr. 1365 (Bydén 2003, 383-430; for the copyist of this manuscript see the next item). The same copyist as Vat. gr. 181 and 182 also restored ff. 207-10, 255-70, 374-5 of Par. gr. 2389, that correspond to POO I.2, 10.5-28.8, 250.1-332.22, 599.5-608.10 (but 599.5-601.16 and 603.23-606.2 can also be read in the original transcription), see POO I.1, III-IV; II, XXXVII-XXXVIII; Pérez Martín 2008, 436 and n. 177. These portions are those in which Marc. gr. 312 depends on Vat. gr. 1594, with the only difference that the last of these portions starts at POO I.2, 481.2.
- Michael Clostomalles, *alias* "Metochitesschreiber" (*PLP* 11867)<sup>94</sup>. He transcribed Laur. Plut. 28.26, ff. 2r-31v, 51r-54v (Bianconi 2010, 39-52); Marc. gr. 314 (Bianconi 2005, 429-30; cf. item 9 of Sect. 4); Vat. gr. 1365 (Prato 1991, 127-8 of the reprint). He "revised" all mathematical treatises he transcribed in the first two manuscripts (cf. items 11, 13, 19g, 21 of Sect. 5).
- John Catrarios (*PLP* 11544). He copied and annotated Vat. gr. 175, ff. 1v-8v, 40r-79r, 81r-159r (Turyn 1964, 124-30 and plate 97; cf. item 19a of Sect. 5)<sup>95</sup>. He annotat-

<sup>93</sup> Very partial editions of the former in Bydén 2003: book I.1-5; and in Derycke 1985: book II.1-5. The first part of book II of *Stoicheiosis Astronomike* is in fact a logistic textbook very much in the style of *Prolegomena ad Almagestum*. On the summary of *Almagestum* being a part of *Stoicheiosis* see Ševčenko 1962, 284-6.

<sup>94</sup> References to previous literature in Bianconi 2010.

<sup>95</sup> Contents of Vat. gr. 175: f. 1r *notae astronomicae*; ff. 1v-8v *excerpta e Strabonis Geographia*; f. 8v *nota de astrolabio*; ff. 9r-38r Anonymus, *Hermippus siue de astrologia*; f. 39r *notae et diagrammata astronomici*; ff. 40r-68v Theon, "little" commentary on Ptolemy's *Tabulae manuales*;

ed Vat. gr. 184 (cf. Tihon 2003, 152; Bianconi 2004, 331 n. 60, 2005a, 151; note 19 above)<sup>96</sup>. He revised Theon, "little" commentary on Ptolemy's *Tabulae manuales* (see item 19a of Sect. 5). See the comprehensive synthesis in Bianconi 2005a, 141-56 and 250.

- 8. Copyist of Par. gr. 2448 (cf. items 8 of Sect. 4 and 2, 5, 8 of Sect. 5). The origin of the recensions contained in this manuscript transcribed in Cyprus remains a mystery.
- Barlaam of Seminara (*PLP* 2284). Autograph corrections to his *Logistica* in Athens, Bιβλιοθήκη τῆς Bouλῆς 5, ff. 6v, 36r, 49r-v (Carelos 1996, xxxIII-xxxv and Abb. 2-3), and to some of his own writings in Marc. gr. 332 (Mogenet –Tihon – Donnet 1977, 46-9; Tihon 2011a, 382-7)<sup>97</sup>. He redacted a treatise of *Logistica* (ed. Carelos 1996), a

ff. 68v-80v *notae et diagrammata astronomici*; ff. 81r-158v Ptolemy, *Tabulae manuales*; f. 159r diagram of the horizons. Catrarios added scholia at ff. 8v, 40r, 41r-v, 42r-v, 46r, 49v, 81r, 82r, 84v, 86r; he also transcribed blocks of short procedure texts at ff. 68v-76v, 78v-79r (these are items 79, 59, 61, 22, 81, 7, 6, 95, 83, 100, 88, 70 of the list in *PC*, 359-69).

<sup>96</sup> Titles are added at ff. 10v-11r, 15v, 16v, 20v-21v, 25r, 107v; other interventions at ff. 11rv (diagrams and one integration), 12v (indication τοῦ ὑπάτου heading Pediasimos' scholium), 13v (diagram), 15v (short annotation, tabular set-up of a multiplication), 16v (tabular set-up of a division), 18v (diagram), 19r-v (tabular set-up of a multiplication, a diagram, numerical corrections), 22r (tabular set-ups for composition of ratios, scholium), 22v (corrections, short annotations), 36r (very short annotation), 80v-81r (text on the structure of the sexagesimal system), 81v (epigram AG App. III.146 [title εἰς Πτολεμαῖον ἐπίγραμμα], followed by a series of items on disparate philosophical and mathematical matters, that can also be found in Ambros. D 137 sup., ff. 2-3, written by Anonymous G [see below]: Pasini 1994, 186-7 and tav. 1; Pérez Martín 2008, 435), 82r (he adds Ptolemy's epigram [= AG IX.577]), 85r (definition of "mean times"), 85v (a very short description of an astrolabe and a text on ratios between plane and solid figures and on incommensurable magnitudes), 86v (very short annotation), 87r (very short annotation), 87v (short story on Archimedes), 88v (correction of numerical values in a table of Alm. I.10), 139v (titles of a table supplied, scholium), 150v (short annotation [incomplete]), 183r, 184v, 187v (these three items: numbers and titles supplied in tables of Alm. IX.4), 207v-209r (titles supplied of tables of Alm. XI.11). Catrarios also drew most of the diagrams and provided infralinear corrections to specific loci.

<sup>97</sup> A detailed analysis of Barlaam's hand and of the activity of some of his collaborators is in Gioffreda 2016. In particular, Gioffreda's "copyist I" penned ff. 3r-4v9, 220v-247v3, 259r-288r of Laur. Plut 28.8 (f. 1r-v nota mathematica [if the sides of two similar triangles are one the triple of the other, the triangles are one nine times the other, cf. *El.* VI.19]; ff. 1v-2v notae philosophicae; ff. 3r-4v, scholia in Elementa [sch. VI.6, ascribed to Maximus Planudes, 4, 3, V.1 des. EOO V, 281.20 ὁ πολλαπλἁσιος immo οἱ πολλαπλἁσιοι, as in Par. gr. 2344, f. 87r –on the latter manuscript see note 63 above]; ff. 5r-6v notae philosophicae; f. 7r-v diagrammata ad El. XII.17 pertinentia; ff. 8r-247r Euclid, Elementa; ff. 248r-258r [Euclid, Elementa] XIV-XV; ff. 259r-288r Euclid, Data) and ff. 142r-152v of Marc. gr. 332 (ff. 1r-61r Barlaam, Logistica; ff. 61v-67r Barlaam, Demonstratio arithmetica ad Elementorum librum II pertinens; ff. 67r-71v Barlaam, De Paschate secundum Apostolorum canonem; ff. 73r-74v Ptolemy [immo Nicephorus Gregoras], *Computus paschalis* (ed. Tihon 2011a), two treatises *De eclipsi solari annorum 1333 et 1337 secundum Magnam Constructionem* (ed. Mogenet –Tihon – Donnet 1977), a tract in which he rewrote book II of *Elementa* in arithmetic language (ed. *EOO* V, 725-38, but just reprinting Dasypodius' of 1564, and without the proem, whose ed. is in Carelos 1999), a refutation of the chapters added by Gregoras to Ptolemy's *Harmonica* III (ed. *PtH*, 112-21), maybe a tract on the extraction of square root (only a small portion ed. Carelos 1999, 114; it is at least contained in Ambros. E 76 sup., ff. 108r-110v, and in its apograph Ambros. R 117 sup., ff. 152v-155v).

10. Nicephorus Gregoras (PLP 4443). Gregoras annotated or corrected the texts in the following scientific manuscripts (unless otherwise stated, I refer here to the items in the list in Bianconi 2005, 412-18, which entails that the identifications were proposed by other scholars: see the bibliographical references provided by Bianconi): Laur. Plut. 28.26, f. 1r-v (Bianconi 2010, 53-4); Marc. gr. 312 (nr. 8); Marc. gr. 314, f. 235r (ibid., 428-9); Marc. gr. 320 (ibid., 423-5); Marc. gr. 325 (nr. 9); Marc. gr. 330, f. 62r (nr. 10); Marc. gr. app. VI.10 (nr. 12: pinax, titles and numbers of chapters of book I of Ptolemy's *Harmonica*); Monac. gr. 361a (cf. items f of Sect. 4 and 2, 12 of Sect. 5; cf. also note 6 above); Monac. gr. 439, ff. 79r-83v (Bianconi 2008, 343-4); Par. Coislin 173 (nr. 25; cf. items *c*, *d*, 3 of Sect. 4 and 12, 15, 18 of Sect. 5; cf. also notes 23, 25 above); Par. gr. 2345 (nr. 23); Par. gr. 2396 (nr. 24); Par. gr. 2450, ff. 57r, 59r, 71v, 72v, 73r, 74v (Pérez Martín 2008, 438 and n. 184; Bianconi 2008, 340 and n. 7); Scorial. Ф.III.5 (second half of 13th century; Euclid, *Elementa*: Pérez Martín 2009, 67); Vat. gr. 182, f. 75r (Bydén 2003, 386; Bianconi 2005, 425); Vat. gr. 1087 (nr. 40; cf. item 17 of Sect. 5 and notes 17, 40, 58, 78, 79 above); Vat. gr. 1365, f. 1r (nr. 42); Vat. gr. 1594 (nr. 43; cf. notes 10, 21, 56, 92 above). Gregoras revised Ptolemy, Har*monica*, and Porphyry's commentary thereon. He redacted a tract on the astrolabe (ed. Delatte 1939, 195-212 and 213-235)<sup>98</sup> and one on the solar eclipse of July 16, 1330 (ed. Mogenet et al. 1983: transmitted with autograph corrections in Marc. gr. 325; see also the good survey of Gregoras' scientific activities in Mogenet – Tihon - Donnet 1977, 150-7); he restored two missing chapters at the end of book III of Ptolemy's Harmonica; a geometric construction is ascribed to him in Par. gr. 2345, f.

*Harmonica* III.14 and 16; ff. 75v-85r Barlaam, *Refutatio in capita addita Ptolemaei Harmonicae*; ff. 85r-92v and ff. 96r-140v Barlaam, *Epistolae*; ff. 142r-152v Barlaam, *De eclipsi solari annorum* 1333 et 1337 secundum Magnam Constructionem; f. 153r-v notae astronomicae (in Barlaam's hand); "copyist II" transcribed ff. 1r-140v of Marc. gr. 332.

<sup>98</sup> Delatte edited two writings on the astrolabe ascribed to Gregoras, which he denoted *A* and *B*. As a matter of fact, redaction *B* is a later abridgment; Gregoras redacted his own tract in two stages (cf. note 75 above); the final version, partly Gregoras' autograph, can be read in Vat. gr. 1087, ff. 312v-320v: Tihon 1995, 340.

118v (ed. *EOO* V, 723.13-724.21); at least a couple of scholia to Nicomachus' *Introductio arithmetica* are ascribed to him in a number of manuscripts (Acerbi, forthcoming); he redacted a short monography on the properties of "venerable" number seven (ed. Sbordone 1936). Noteworthy transcriptions by some of the copyists associated with Gregoras:

- a. John. He transcribed Marc. gr. 321, ff. 65r-75r, 76r-98r9, 99r-132v (Bianconi 2003, 550-1); Par. Coislin 173, ff. 176r-196v8 and 206r-222v (*ibid.*, 550); Vat. gr. 1087, ff. 2r-4r, 28r-33v, 88r-91r, 97v-101v17, 102r18-122r, 148r-158v, 161v-171r, 172r-178r, 179v-182v, 183v-190r, 191v-221v, 223r-230v, 233r1-6, 279r-299v (Pérez Martín 1997, 83 and n. 53).
- b. Anonymous G. He transcribed Ferrara, Biblioteca Comunale Ariostea II 178, and Neap. Borb. III C 19 (Turyn 1972, 184-5 and plate 149); Marc. gr. 312, paper restorations (Bianconi 2003, 552 n. 123); Monac. gr. 212, f. 25r (Mondrain 2012, 629); Par. Coislin 172, ff. 41r-268r (Mondrain 2002, 321; 2007, 164 n. 10; Pérez Martín 2008, 438 n. 183); Par. Coislin 173, ff. 1-2v17, 2r-29v, 31v (*ibid.*, 437-8 n. 182); Par. gr. 2391 (*ibid.*, 438 n. 183); Par. gr. 2396, ff. 77r-86v (Mondrain 2002, 319); Par. gr. 2450 (Turyn 1972, 184-5); Vat. gr. 1087, ff. 123-147 (Bianconi 2003, 545; 2006, 147-51).
- c. Macarius. He transcribed Laur. Plut. 28.20 (Pérez Martín 2004, 209-10); Marc. gr. 330, ff. 16r-286v (Menchelli 2013, 43); Vat. gr. 318 (A. Gioffreda, *per litteras* and 2018); Vat. gr. 1087, ff. 5r-27v (Pérez Martín 2004, 209-10).
- d. Anonymous. He transcribed Par. gr. 2345, ff. 118r-v, 121r-v (Bianconi 2003)<sup>99</sup>.
- Isaac Argyros (*PLP* 1285). He transcribed the entire Laur. Plut. 28.13 (Mondrain 2012, 630); Laur. Plut. 89 sup. 48, ff. 7r-17v and 137r-168r (A. Gioffreda, *per litteras* and 2018; cf. items 6 of Sect. 4 and 21 of Sect. 5; the hand at ff. 1r-6v, a later addition, also penned the incomplete Vat. gr. 1411; the hand at ff. 18r-136v also transcribed ff. 1r-8v of Marc. gr. 308 and ff. 105r-112v of Scorial. Y.III.21 [Euclid, *Elementa* I.1-14]: A. Gioffreda, *per litteras* and 2018; on the Vatican manuscript see also Acerbi, forthcoming); Marc. gr. 308, ff. 9r-34r (f. 15 excepted: Mondrain 2007, 166-7; Bianconi 2008, 357; cf. item 14 of Sect. 5); Marc. gr. 310, ff. 1r-261v

<sup>&</sup>lt;sup>99</sup> Par. gr. 2345, parchment, end of 13<sup>th</sup> century, ends exactly with the end of *Elementa*, but must be completed at least with ff. 337r-352v of Vat. gr. 1316, two quires containing Euclid, *Optica* **A** 1-58, *des. EOO* VII, 118.24 ή ὑπὸ AZE: Bianconi 2006, 141-7 (the folio with the end of *Optica* **A** missing in Vat. gr. 1316 is in fact misplaced as f. 223 of Par. gr. 2345; after the end of the Euclidean treatise, it contains a fragment of *Prolegomena ad Almagestum*: B. Vitrac, *per litteras*). On these quires of Vat. gr. 1316, see also *EOO* VII, v-vI and xIII; it is an apograph of Bodl. Auct. F 6 23 (11<sup>th</sup> century: ff. 1-265r Euclid, *Elementa*; ff. 266r-273v Euclid, *Optica* **A**, *des. EOO* VII, 60.17 ἀνήχθω).

(Mondrain 2007, 166; cf. items 5 of Sect. 4 and 10, 17, 21 of Sect. 5); Marc. gr. 323, ff. 215r-v, 220r-221v, 285r14-288v16, 292v, 394r-400r, which he also annotated extensively (Bianconi 2008, 358; cf. item 19b of Sect. 5); Norimb. Cent. V, App. 38 (Ptolemy, Harmonica: Mondrain 2005, 20, and 2007, 167; it is an apograph of Vat. gr. 176); Par. gr. 2507 (Mondrain 2007, 167); Vat. gr. 176, ff. 11v29-27r, 100r-192v (Pérez Martín 2008, 446-7; Argyros also annotated ff. 1r-11v and 28-99; cf. items 12 and 15 of Sect. 5). He revised the following texts: Prolegomena ad Almagestum; Ptolemy, Almagestum, Harmonica; Theon, in Almagestum, Porphyry, in Harmonica; Barlaam, Logistica, probably De eclipsi solari annorum 1333 et 1337 secundum Magnam Constructionem (see items 21, 10, 12, 17, 15 of Sect. 5, note 47 above, in this order; Mogenet - Tihon - Donnet 1977, 37-40). He transcribed and annotated *Elementa*, and possibly other geometrical writings; the manuscript, once owned by Bessarion, is lost, but a part of Argyros' scholia to Elementa drawn from this exemplar were printed in Greek (only for book V) or in Latin translation (preliminary material, and books I-VI) by Konrad Rauchfuss (Cunradus Dasypodius) in 1573 and 1579, respectively: Labowsky 1979. The set of scholia translated by Rauchfuss appears to coincide almost exactly with the collection in Par. suppl. gr. 12, ff. 9r-35v (transcribed by Manuel Provataris: RGK II, nr. 350); the only other trace of scholia to Elementa ascribed to Argyros is the collection in Cantabr. Gg.II.33, ff. 153v-176r (this portion was penned by Constantinos Mesobotes: RGK I, nr. 224, much of the rest by Nikolaos Sophianos: *ibid.*, nr. 318; the manuscript also contains other writings of Argyros). Among the scientific works ascribed to him feature: a treatise on the calculation of approximate square root (ed. Allard 1978), a treatise on the astrolabe (ed. Delatte 1939, 236-53); a text on a stereographic projection used in Ptolemy's Geographia (ed. Laue - Makris 2002); a tract on De cyclis Solis et Lunae ad Andronicum, a Computus paschalis, a tract on the beginning of the year (ed. of the former two Petau 1630, 359A-370D and 370E-378B; ed. of all of them Baufays 1981, on the basis of Marc. gr. 328, ff. 1r-18v); two short treatises on new astronomical tables, including the new tables (Wampach 1978-79)<sup>100</sup>; a short treatise on the calculation of syzygies, so far discovered only in Urb. gr. 80, ff. 101r-108v, and Vat. gr. 1058, ff. 246r-254r (ed. Laurent 1969, on the basis of the former manuscript, of which the latter is a copy); a treatise on measurement of non-rectangular figures (sometimes transmitted in the form of a "letter to Colybas") and other metrological

<sup>100</sup> Argyros recalculated the tables of syzygies for the Julian calendar, taking as references the meridian through Byzantium and the date of September 1<sup>st</sup>, 1367. He did it twice, modifying in one treatise the corresponding table of *Almagestum* (*Alm.* VI.3) and, in the other treatise, that in *Tabulae manuales*.

material (detailed information in *HOO* V, xCVIII-CIII; "ed." Lefort *et al.* 1991, 154-66); a series of arithmetical problems (cf. item 14 below).

- Theodorus Meliteniotes (*PLP* 17851). Autograph copy of his *Tribiblos astronomica* in Vat. gr. 792 (Leurquin 1991; ed. of books I-II Leurquin 1990 and 1993, ed. of book III.1-12 Pêcheur 1972-73); he also transcribed Laur. Plut. 28.17 and annotated Monac. gr. 212 (Mondrain 2012, 628-9).
- 13. John Abramios (*PLP* 57). He owned, annotated and partly copied Laur. Plut. 28.16 and Vat. gr. 208 (cf. items 11, 19b of Sect. 5 and note 59 above).
- 14. Demetrius Cydones (*PLP* 13876). He owned and annotated Laur. Plut. 28.1 and Bonon. A 18-19<sup>101</sup>. He is credited to have annotated *Elementa: sch*. VI.58 is ascribed to some  $\Delta\eta\mu\eta\tau\rho$ iou in Bonon. A 18-19, f. 131v (*EOO* V, 355.7-14); however, the same scholium can be read, in the hand of the main copyist, in Bodl. Dorv. 301, f. 114r. Cydones' name is associated with that of Isaac Argyros in a short series of arithmetical problems<sup>102</sup>.
- 15. Nicholas Eudaimonoioannes. About the beginning of the 15<sup>th</sup> century he annotated Vat. gr. 184, ff. 122v-157r (some scholia are dated 1333/34), and also corrected the text of *Almagestum* in this manuscript (Tihon 2003).
- 16. Malachias. He transcribed Par. gr. 2342 and Vat. gr. 198 (cf. items 1-3 of Sect. 4, and 2-5, 7-9, 17, 21 of Sect. 5); Laur. Plut. 28.14 (Pérez Martín 2008, 450; this is a paper codex of 321 folios, mm 198×150, containing a huge astrological *corpus* described in Olivieri 1898, 20-37); Mutin. α.U.9.7, f. 77r (Mondrain 2000, 22-3; cf. item 1 of Sect. 5); Vat. gr. 208 (*POO* II, VI n. \*; cf. note 30 above). He annotated Vat. gr. 792 (A. Gioffreda, *per litteras*; Acerbi Gioffreda, forthcoming). Contrary to all other personalities featuring in this list, we have no reasons to suppose that Malachias was

<sup>101</sup> In Laur. Plut. 28.1, he completed the text with κείμενον integrations and apposed scholia at ff. 1r, 15v, 16v, 17r, 21v, 22r-v, 28r\*, 31v\*, 32r\*, 41v, 42v, 43r\*\*, 43v, 44r, 50v, 54r\*, 54v\*, 57v, 58r, 58v, 65r\*\*, 65v, 66r, 66v\*\*, 67r\*, 67v\*\*, 68r\*\*, 68v, 69r, 69v\*\*, 70r\*\*, 70v\*\*, 71v-74r, 75r, 76r, 78v, 79v, 81v\*, 90v\*, 91r\*, 93v, 94r, 94v, 95v, 114v\*, 115r\*, 133r\*, 136v\*, 148v, 149r, 149v\*, 191r\*, 193v\*, 194r\*, 198r\*, 204r\*, 215r\*, 217v\*, 229v, 231r, 238r\*, 240v\*, 244v\* (with an asterisk the diagrammatic scholia, with a double asterisk the tabular algorithms; an edition is in progress; cf. Acerbi – Pérez Martín 2015; items 7 of Sect. 4 and 10, 21 of Sect. 5). In Bonon. A 18-19, Cydones' *marginalia* are at ff. 8v (first line), 100v, 105v, 106r, 157v of tome A 18; the last three annotations are edited as *sch.* V.67, 68, VIII.30 in *EOO* V, 309.9-16 and 395.18-396.3: Acerbi – Pérez Martín 2017.

<sup>102</sup> Edited, on the basis of the sole Zeitz, Stiftsbibliothek 67, in Hoche 1866, 148.1-152.3. I am preparing a new edition of these problems (Acerbi, forthcoming); twelve manuscript witnesses are involved so far.

a scholar. If he was a "simple" copyist, he was perfectly at ease with mathematical texts and had at his disposal a rich library, and we may well wonder who and for what purpose might have asked him to prepare such obsessed masterpieces<sup>103</sup> as are the mathematical *quadrivium* in Par. gr. 2342 + Vat. gr. 198 and a complete Aristotelian *corpus*. Note also that Malachias produced, certainly on behalf of a different customer as the format suggests, Vat. gr. 208, a manuscript complementary to (the transmitted portion of) the said *quadrivium*<sup>104</sup>.

17. John Chortasmenos (*PLP* 30897). He transcribed Vat. gr. 1059 (cf. item 19e of Sect. 5), Vat. urb. gr. 80, Vindob. suppl. gr. 75<sup>105</sup>. He owned and annotated Matrit. 4678, *passim*<sup>106</sup>; Mutin. α.T.8.21 (*olim* II E 9), *passim*<sup>107</sup>; Par. gr. 2107, ff. 140r, 145r (Mondrain 2000, 17 n. 25; 2005, 15; cf. note 32 above, item 3 of Sect. 5, and note 108 below); Par. gr. 2399 (*POO* II, CXLVI n. 1; cf. item 19e of Sect. 5); Par. suppl. gr. 921 (*RGK* II, nr. 252; see also Mondrain 2008, 113); Vat. gr. 204, ff. 45v-49v, 136r-137r<sup>108</sup>; Vat. gr.

<sup>103</sup> They are obsessed since they are the mathematical manuscripts providing the maximum of information per unit surface: number of texts transcribed (primary treatises, infralinear annotations, commentaries and scholia thereon) and of manuscripts collated, mathematical consistency of the recensions proposed, size and density of the script. It is obvious that all of this is deliberate.

<sup>104</sup> Par. gr. 2342: mm 293×222; Vat. gr. 198: mm 294×200; Vat. gr. 208: mm 220×140, namely equal sheets of paper folded *in folio* or *in quarto*. Maybe also the latter manuscript was devised within a (lesser) unitary project.

<sup>105</sup> For Chortasmenos' manuscripts see in general Hunger 1969, 20-8, 51-3, and especially 54-63 for the Vienna codex; Canart – Prato 1981; *RGK* III, nr. 315; cf. I, nr. 191, II, nr. 252; Hunger 1994, 124-30. Vindob. suppl. gr. 75, ff. 234r-256v, 270v-281v, 303r-304r, are mathematical texts.

<sup>106</sup> *Marginalia* at ff. 4r, 57v-60r, 61r-v, 63r-66v, 67v-68r, 69v-70r, 72r, 73r-75v, 77r-79r, 82r-v, 83v-84v, 85v-87r, and almost everywhere *infra lineas* as far as *Arithm*. III.14: Pérez Martín 2006, 450, and the edition of the scholia in Allard 1983, *sch*. 148-200 (191 excluded) + 95, 100 (scholia A7). The hand was identified in Wilson 1996, 279. The most celebrated of Chortasmenos' scholia in this manuscript, *in Arithmetica II.7*, is studied in detail in Acerbi 2013.

<sup>107</sup> The text at f. 26r-v is a calculation of approximate square root according to *El*. II.4, the one at f. 131r is the possession note: Puntoni 1896, 390; *POO* II, CXLVI n. 1. The Modena manuscript is annotated by Chortasmenos much more extensively than indicated in *RGK* I, nr. 191; it contains ff. 2r-215r Euclid, *Elementa*; ff. 215v-216v *Phalaridis epistula*.

<sup>108</sup> See *RGK* III, nr. 315. The annotations at ff. 45v-49v are partly edited as *sch.* 4-6, 8-9, 11, 35, 43 (hand  $V^2$ ; *sch.* 43 can also be read included in the main text, between props. 23 and 24, in Par. gr. 2107, f. 34r-v, not in Chortasmenos' hand) *in Optica* **B**: *EOO* VII, 252.6-253.9, 253.15-254.4, 254.8-13, 264.9-15, 268.3-269.26, respectively; those at ff. 136r-137r are partly edited as *sch.* 2, 9 and 11 (hand V<sup>1</sup>) *in Catoptrica: EOO* VII, 347.4-14, 349.4-5, and 349.11-22, respectively; *sch.* 2 and 9 can also be read in Par. gr. 2107, ff. 48r and 49r, respectively, not in Chortasmenos' hand. In Vat. gr. 204, he also apposed several infralinear annotations, but he is not the corrector

208, ff. 227v-230v, in which some scholia to *Elementa* include propositions of the Euclidean treatise organized in deductive schemes<sup>109</sup>; Vat. gr. 1365, f. 1r (Bianconi 2005, 407, 426 n. 72) and *passim*, including a short preface to Metochites' *Stoicheiosis Astronomike*. Chortasmenos revised Theon, "little" commentary on Ptolemy's *Tabulae manuales* (see item 19e of Sect. 5). The first three manuscripts mentioned above are huge compilations containing transcriptions, calculations, scholia, exercises; some of the latter have been edited (Caudano 2003; Tihon 2006).

A few final remarks are in order.

• The same scholar may happen to annotate the same treatise several times, and on different manuscripts. This *prima facie* surprising phenomenon may be connect-

we have been dealing with in items 2-5, 8 of Sect. 5. The fact that Chortasmenos annotated Vat. gr. 204 makes it very unlikely the identification of the Vatican codex with one of the Greek manuscripts described in the 1295 and 1311 inventories of the Papal library (recensio Bonifatiana item 425 = Perusina 620): see Paravicini Bagliani 1983 for a first orientation on this major issue of the complex path of transmission of Greek manuscripts in the West. As for Par. gr. 2107 (a composite codex, ff. 59-113 of 14<sup>th</sup> century, the rest of  $15^{th}$  century), it contains: f.  $1r \mu \eta \nu \epsilon \zeta \epsilon \pi'$ Άθηναίων and corresponding Roman months, currency conversions, strength of bodies of soldiers; ff. 3r-10v initial portion of the so-called Anonymi Logica et Quadrivium (Heiberg 1929, des. mut. p. 12.8 καὶ τὰ λοιπὰ δὲ ἀπειράκις); ff. 12r-22r χρονικὸν κατ' ἐπιτομήν from Adam to John VIII Palaiologos (whose years of reign are not inserted; ruled 1425-48); ff. 23r-24v scholia in Elementa (EOO V, 697.20-703.12, App. II sch. 6-7); f. 24v\* nota numerologica cum tabula; f. 25r\* nota astrologica; f. 26r nota theologica (includes excerpts from Pseudo-Justinus, Quaestiones Christianorum ad Gentiles, 176c-D); f. 26v nota philosophica (these are reworked excerpts from Plato, Gorgias, 463A-465A); ff. 27r-47r Euclid, Optica B; ff. 48r-58v Euclid, Catoptrica; ff. 59r-112v Nicomachus, Introductio arithmetica; f. 113r-v\* adnotationes ad subsequentia pertinentes; f. 115r-v\* problems ascribed to Demetrius Cydones and Isaac Argyros; ff. 115v-122v Anonymus, μέθοδος πολιτικῶν λογαρισμῶν (this is in fact a part of the second letter of Nicholas Rhabdas: ed. Tannery 1886, 140.1-172.15 πολυπλασίασον {ταῦτα}); ff. 123r-127v\* geometrica quaedam; f. 129r John Pothos Pediasimos, Cur infantes VII et IX mense nati vivant, VIII non vivant; f. 129v\* αἴνιγμα ψηφικόν; ff. 130r-v diagrammata ad El. I.2 pertinentes; ff. 131r-137r Elementorum I-VI epitome cum figuris; ff. 138r-139v diagrammata ad El. I.47 pertinentes; f. 140r-v tabulae astronomicae; ff. 141r-226v Tabulae persicae, including Anonymus, Paradosis tabularum persarum; f. 229v diagramma astrologicum; f. 230r\* κανόνια παραλλάξεων νέα όρθωθέντα παρὰ Ἀνδρονίκω Δούκα τοῦ Σγούρου; f. 231v graphical representation of the twelve δάκτυλοι that measure eclipse magnitudes; ff. 232r-237v tabulae astronomicae; f. 240r\* a geometricoastronomical text ascribed to Andronicos Ducas Sgouros, with a diagram. A thorough study of this codex, with edition of a number of texts there contained (those marked with asterisk), is in Acerbi, forthcoming.

<sup>109</sup> See Hunger 1969, 22-3, 52. The scholia are edited in Heiberg 1903, 350-2; see also *ibid.*, 350, for a description of the deductive schemes. Syllogistic schemes of different kind can be found in Vat. gr. 193, ff. 9r-10v; they cover the proofs of *Elementa* I.1-17.

ed with temporarily limited availability of written resources, or with different, and temporally disjoint, editorial programmes.

- A number of scholars involved in the hesychastic controversy feature in the above list, and almost all of them side with the anti-Palamite party, even if they happened to be scientific rivals (think of Nicephorus Gregoras and Barlaam). A notable exception is the Palamite Malachias, in whose hand we read a note in Vat. gr. 208, f. 15v, offering a justification for his copying the two tracts on new astronomical tables by the anti-Palamite Isaac Argyros<sup>110</sup>.
- The Palaiologan period saw the conception of at least two mathematico-astronomical encyclopaedias. We have dealt with Malachias' enterprise throughout the article. Elaborating on Bianconi 2004, 330-3, I will briefly discuss the origin and the purpose of the early-Palaiologan encyclopaedia in Vat. gr. 191, whose transcription is held, in recent palaeographic literature, to have been coordinated by John P. Pediasimos<sup>111</sup>.

<sup>110</sup> Greek text in Mercati – Franchi de' Cavalieri 1923, 254, translation in Pérez Martín 2008, 451 n. 223.

<sup>111</sup> See already Heiberg in POO II, CXXVI. Vat. gr. 191, oriental paper, mm 365×237, written by sixteen copyists (see Turyn 1964, 89-97; Bianconi 2004, 324-30 and fig. 1, for the structure of the codex: I shall refer to the sigla assigned by Bianconi, that differ from Turyn's; best description, especially as far as astrological matters are concerned, in Kroll 1906, 3-23), original numbering of quires still visible at ff. 38v, 46v, 54v: α'-γ'; 188v, 196v, 204v, 212v, 220v, 221r, 228v:  $\beta'$ -ζ' (last page); 229r, 237r: α'-β'; 320r, 328r, 336r, 344r, 352r: α'-ε'; 360r, 368r: α'-β' (first page). Contents (marked by || the beginning of a block according to the original numbering of quires; quires referred to are those marked by Pediasimos; the block numbers assigned in Bianconi 2004, fig. 1, are also added): (1) ff. 2r-4v Euclid, Catoptrica; ff. 4v-11v Euclid, Phaenomena b; ff. 11v-17v Euclid, Optica B; ff. 18r-29v Euclid, Data; f. 30v adnotatio Pediasimi; ||(2) 31r-46r Theodosius, Sphaerica; ff. 46r-48v Theodosius, De habitationibus; ff. 48v-59v Theodosius, De diebus et noctibus; ff. 59v-63r Aristarchus, De magnitudinibus et distantiis solis et lunae; (3) ff. 64r-70v Autolycus, De ortibus et occasibus; ff. 70v-72r Hypsicles, Anaphoricus; ff. 72r-74v Autolycus, De *sphaera mota*; ff. 75r-88v Eutocius, *in Conica*; [only f. 88 survives of quire ιβ', quire ιγ' is lost;] (4) ff. 89r-104v Vettius Valens, Anthologiae (incomplete and in perturbed order: ff. 89 and 96 are the first and the last folio of quire  $\iota\delta'$ ; quire  $\iota\epsilon'$  has lost the first and the last folio; concordances between surviving folia and text in Pingree 1986, IX); ff. 105r-107v astrologica varia (ff. 105 and 106 are the first and the last folio of quire  $\zeta$ ; f. 107 goes with the subsequent binion 107-110); ff. 108r-111v adnotationes Pediasimi; [only f. 111 survives of quire  $i\theta'$ , quires  $\kappa'$  and  $\kappa\alpha'$  are lost;] (5) ff. 112v-127r tabulae astronomicae; (6) ff. 128v-169v Ptolemy, Geographia; ff. 170r-v astronomica varia; ff. 170v-172v adnotationes Pediasimi; ||(7) ff. 173r-189r Proclus, Hypotyposis; ff. 189r-194v Philoponus, De usu astrolabii; 194v-209v Aratea et astronomica varia; 210r-228v Hipparchus, in Arati et Eudoxi Phaenomena I-III; ||(8) 229r-286v astrologica varia; (9) 287r-291v Gaudentius, Introductio harmonica; ff. 292r-295r Cleonides/[Euclid], Introductio harmonica; ff. 295r-296v Euclid, Sectio canonis; ff. 297r-309r Aristoxenus, Elementa harmonica I-III; ff. 309r-314r Alypius, Isagoge musica; ff. 314r-316r Aristoxenus, Elementa rhythmica (fragment); ff. 317v-318v ta-

- i. Bianconi first remarked that *a*) copyist B of Vat. gr. 191 is the same as the one that transcribed the second codicological unit of Vat. gr. 203<sup>112</sup>, *b*) copyist K coincides with hand D of Vat. gr. 184 (item *b* of Sect. 4), *c*) the first hand of Vat. gr. 203 is the same as hand A of Oxford, Barocci 131 (a codex that does not contain technical writings), whose hands G and E coincide in their turns with hands C of Vat. gr. 184<sup>113</sup> and F of Vat. gr. 191, respectively. Add to this that copyist B of Vat. gr. 184 coincides with that of Vat. gr. 2326, ff. 26v-33v (the initial segment of *Prolegomena ad Almagestum*).
- ii. The three Vatican codices are on oriental paper and have the same dimensions. Vat. gr. 191 and 203 have a prominent block structure (see notes 111 and 112 above), whose original extent can be recovered from quire numbers, entirely in the case of Vat. gr. 203 (just one block of five quires plus three folios), only partly in the case of Vat. gr. 191: there are at least five block borders and hence at least six blocks (Bianconi introduces eleven).
- iii. Pediasimos annotated Vat. gr. 184 in a very selective way: all of his scholia are appended to *Prolegomena ad Almagestum*, plus an isolated annotation on the very first page of *Almagestum*, f. 82r. In the same way, he extensively annotated *Prolegomena ad Almagestum* in Vat. gr. 2326, but only once what remains of *Almagestum* in the same manuscript. Also his interventions in Vat. gr. 191 are much more selective and less extensive than usually assumed; most of them concern musical treatises<sup>114</sup>, and none

*bulae astronomicae*; f. 319r-v *tabula et adnotatio astronomica Pediasimi*; ||(10) ff. 320r-359r Ptolemy, *Harmonica*; ff. 359r-v *adnotationes Pediasimi*; ||(11) ff. 360r-390r Diophantus, *Arithmetica*; f. 390r-392v Diophantus, *De polygonis numeris*; ff. 393r-395v Cleonides/[Euclid], *Introductio harmonica*; ff. 395v-396v Euclid, *Sectio canonis* (incomplete: *inc. lac.* prop. 10, *EOO* VIII, 170.28 έστì, *des.* prop. 18, *ibid.*, 178.10 τρίτη; it is a subset of the lacuna in Matrit. 4678: the missing text corresponds exactly to 1 folio of the *Matritensis*); f. 397r-v *adnotationes Pediasimi*.

<sup>112</sup> Vat. gr. 203, oriental paper, mm  $344 \times 252$ , made of two different codicological units (ff. 1-55 and 56-98) written by two different hands. Contents: ff. 1r-15v Theodosius, *Sphaerica*; ff. 16r-18r Theodosius, *De habitationibus*; ff. 18r-29r Theodosius, *De diebus et noctibus*; ff. 29v-31v Autolycus, *De sphaera mota*; ff. 31v-38v Autolycus, *De ortibus et occasibus*; ff. 38v-39v Hypsicles, *Anaphoricus*; ff. 40r-44r Aristarchus, *De magnitudinibus et distantiis solis et lunae*; ff. 44r-55v Eutocius, *in Conica*; ff. 56r-84r Apollonius, *Conica*; ff. 84r-90r Serenus, *De sectione cylindri*; ff. 90r-98r Serenus, *De sectione coni*. The complete quires of the second codicological unit (+ ff. 96-8) are numbered from a' to  $\varepsilon'$  in the lower inner corner of the first and the last page of each quire. See already Heiberg in POO II, cxxv1, for a statement about the identity of "manus, charta, atramentum, totum genus codicis" in Vat. gr. 184 and in the first codicological unit of Vat. gr. 203.

<sup>113</sup> The same hand transcribed a fragment of *Almagestum* now contained in the composite Vat. gr. 1882, ff. 201-202 (*pars* XX): Pérez Martín 2013, 172.

<sup>114</sup> Apart from the long annotations on blank pages listed in note 89 above (*pinax* included) and from the quire numbers (?), Pediasimos' interventions are at ff. 2r (title), 4v-6r (title and extensive annotations), 12r (scholium), 17v-18r (extensive annotations), 59v (title), 89r (title), 130r-v (title and κείμενον integrations), 131v (κείμενον integration), 132v-134v (titles, short an-

of them can explicitly be connected with his supposed role of coordinator in the transcription of the manuscript. The frequent  $\gamma \rho \dot{\alpha} \phi \epsilon \tau \alpha i$  and  $\kappa \epsilon \dot{\mu} \epsilon \nu o \nu$  integrations, that constitute a conspicuous part of his annotations, may well come from collating other exemplars, not from checking the models actually used by the copyists.

- iv. The musical treatises of Cleonides and Euclid are copied twice in Vat. gr. 191 (ff. 292r-296v and 393r-396v), obviously from different models and without indications as to this doubling. This suggests that no coordination ever existed in the making of the codex, and that its several blocks were copied in different places and periods, and later assembled.
- v. We may perceive some self-contained thematic units in Vat. gr. 191:
  - ff. 2-88 three mimetic hands transcribing, in eleven quires + one leaf, the "little astronomy" and Eutocius *in Conica*: hands A, which penned ff. 2r-29v (Euclid), and C, ff. 64r-88v, are not found elsewhere in the codex; hand B, ff. 31r-63r, also copied ff. 105r-107v, 158v, 161r-169v (recall that hand B also penned ff. 56-98 of Vat. gr. 203, geometric material; hands B and C cannot easily be disentangled, even if there is a "junction" between them, because we are in the middle of the "little astronomy"; note, however, that one column of f. 63r and the entire f. 63v are blank; note also that after Euclid's treatises there is a "junction", whereas Eutocius begins in the middle of a quire) but, *contra*, just before Eutocius, that begins at the very beginning of f. 75r, we read, filling the final portion of the second column of f. 74v and deleted by pen strokes, the beginning of Euclid, *Optica* **B** that we have already read at f. 11v; note, finally, that Eutocius ends with the only surviving folio of a quire;
  - ff. 173-228 elementary astronomical matters: seven quires, only transcribed by hand J, to be found again in ff. 261r, 287r-312v, 314r-316r;
  - ff. 229-286 astrological matters: seven quires almost uniquely by hand K, not to be found elsewhere<sup>115</sup>;

notations and γράφεται integrations), 150r (short scholium), 153v (short scholium), 173r (scholium), 207v (scholium), 297r (title), 298r (κείμενον integration), 299v-301r (scholia, γράφεται and κείμενον integrations), 301v (title), 304r (short scholium), 307r (one γράφεται and one κείμενον integration), 307v (κείμενον integration), 308r (κείμενον integration), 317v (title of a table), 320r-324v (scholia), 325v-327v (scholia), 328v-329r (scholia), 331r-v (short scholia), 335r (scholium), 338v-339r (scholia and κείμενον integration), 340r (scholia), 347r-348r (scholia), 358v (short integration), 356r (scholium and integration), 357r-359r (scholia and integrations), 360r-361r (scholia), 362v (κείμενον integration), 366v (title), 370v (κείμενον integration and title), 371r (κείμενον integration), 373v (κείμενον integration and title).

<sup>115</sup> One also finds hand J, but only in f. 261r, and the text it writes is deleted by two long, crossed pen strokes (by Pediasimos?). The beginning of the text at f. 261v exactly fits the end of that at f. 260v. Maybe here is just a matter of recycling paper, but hands J and K obviously go together.

- ff. 320-359 Ptolemy, *Harmonica*: five quires by hand L, not to be found elsewhere;
- ff. 360-397 Diophantus: five quires by hands M-Q, one quire each; the hands are not to be found elsewhere<sup>116</sup>.

As is clear, such self-contained thematic units are also almost independent from the palaeographic and codicological point of view.

vi. Frankly speaking, Vat. gr. 191 is a thematic monster. It looks very much like something assembled by putting together every piece of scientific lore at hand. Of course, this scenario is perfectly compatible with any of the peculiar features of this manuscript (wild superposition of hands in some blocks, surprisingly high number of blank folios), and seems to me to be a better explanation of some of them than the scenario in which Pediasimos (or any other scholar) supervised a collective work of copying: very simply, there is no trace of intelligent coordination, either regarding the material features of the codex or its contents, in the making of Vat. gr. 191 as a whole. Maybe Pediasimos was simply the assembler of the manuscript from the membra disiecta of other codices (his pinax is fairly complete), quite reasonably chosen so as to be homogeneous from the codicological point of view, and not the coordinator of a collective work of copying. Maybe the real mathematical encyclopaedia no longer exists, partly lost, partly dismembered between ff. 2-88 of Vat. gr. 191 and ff. 56-98 of Vat. gr. 203<sup>117</sup>. I would, however, not give much weight to the fact that the hand that transcribed almost alone the astrological block of Vat. gr. 191 (and only this block) coincides with one of the hands of Vat. gr. 184<sup>118</sup>, were it not for the noteworthy fact that this manuscript contains notes dated 1269-71 (see note 18 above), but, on the other hand, do not forget that copyist G of Vat. gr. 191 transcribed a long scholium whose calculations are carried out for April 14, 1298<sup>119</sup>.

<sup>116</sup> I doubt about Bianconi's identification of the hand integrating f. 309r4-11 with Q.

<sup>117</sup> Other manuscripts of the same period in which one finds portions in an imitative writing of a very small module are Vat. gr. 192, ff. 3r-26v, 48v, 58v-163r, and Vat. gr. 2338, ff. 1-22 (ff. 1r-3r Cleonides, *Introductio harmonica*; ff. 3r-4v Euclid, *Sectio canonis*; ff. 5r-8r Gaudentius, *Introductio harmonica*; ff. 9r-12v *harmonica varia*; ff. 12v-21v Aristoxenus, *Elementa harmonica*; ff. 21v-22v *excerpta Neapolitana* (fragment); see Mathiesen 1988, 608-11). The latter, with its 60-3 lines on a full page, also has the same codicological characteristics as Vat. gr. 203 (mm  $344\times252$ ; Vat. gr. 2338 is  $345\times255$ ).

<sup>118</sup> Note that this hand ends its job at f. 80v, where the portion preliminary to *Almagestum* (that begins at f. 82r) also ends: the rest of f. 80v and f. 81r-v are occupied almost uniquely by annotations of John Catrarios, see item 7 above.

<sup>119</sup> There is much confusion about dating manuscripts by using annotations in which the date of the current year is included. One must not forget the following general facts (that is, coming *solely* from the presence of such annotations): what is certain is the date of *redaction* of the annotation; as for its date of transcription, the current year indicated in it can only be used as a *terminus post quem*, for we have plenty of examples of scholia of this kind that were copied

# Appendix. A case study: Revising a sentence of Prolegomena ad Almagestum

It is not easy to select an example of revision of a Greek mathematical passage. The text discussed in this appendix has some features that make it in several senses paradigmatic: it involves the interaction between a text and the associated diagram; it has been corrected by a number of scholars, a modern philologist included; it displays many of the typical interventions listed in Sect. 2; it is technical but elementary, as the deductive steps only involve basic results of *Elementa*.

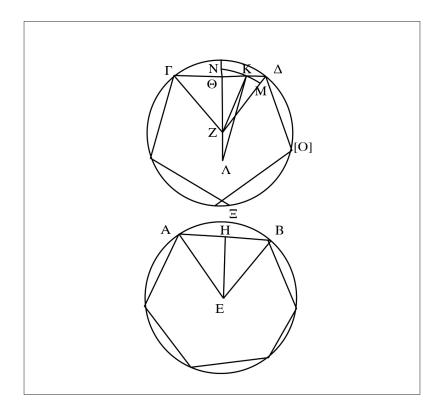
The first theorem of the short treatise *On isoperimetric figures* in the version contained in *Prolegomena ad Almagestum* is followed by a lemma, proving a result assumed in the proof of the theorem itself<sup>120</sup>. The proof of the lemma only requires slightly completing the geometric configuration of the theorem: therefore, it uses the same set of denotative letters introduced in the theorem, and refers to the same diagram. But the text of the theorem does not fully determine the associated diagram: since the figures involved are two regular polygons inscribed in circles, within each circle the geometric configuration assumed in the enunciation of the theorem has a left-right symmetry that is spontaneously broken only when the auxiliary construction required by the proof of the theorem is performed. Such a construction, to be attached to only one of the polygons, can indifferently be performed "on the left side" or "on the right side" of the geometric configuration represented by the diagram<sup>121</sup>, but once it is performed, the complete diagram is no longer symmetric (see **Fig. 1**, where the construction is performed on the right side).

Now, the lemma does not allow such a left–right degree of freedom: first, it does not set up a separate diagram but refers to the geometric configuration of the auxiliary construction of the theorem (just adding an arc  $\dot{\eta}$  MN which is irrelevant to our purpos-

decades after their redaction; *nothing* can be said on the date of transcription of the manuscript, that can be earlier, contemporary with, or later than the indicated current year (but which is of course earlier than the date of transcription of the annotation), and this *even if* the annotation is in the hand of one of the main copyists of the manuscript. In this case, it is only *very likely* that the transcriptions of the annotation and of the manuscript are near to each other.

 $^{\rm 120}$  A less complete version of the discussion in this Appendix is in Acerbi – Vinel – Vitrac 2010, 117-19 and 196.

<sup>121</sup> The auxiliary construction amounts to marking point  $\tau \delta$  K and joining straight lines  $\dot{\eta}$  KZ and  $\dot{\eta}$  KA. The choice between left and right side is made when marking point  $\tau \delta$  K: either on line segment  $\dot{\eta} \Gamma \Theta$  (left side) or on line segment  $\dot{\eta} \Theta \Delta$  (right side). I shall henceforth shortcut the expression "the geometric configuration represented by the diagram" to "the diagram", but the reader must keep in mind that the two must be carefully kept distinct: the former is a mathematical object, the latter a graphic object.



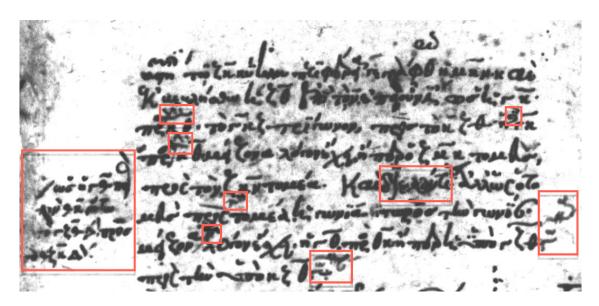
**Fig. 1**. A redrawing of the diagram of the lemma in *Prolegomena ad Almagestum* conformal to the one in Paris, Bibliothèque Nationale de France, gr. 2390, f. 1v

es); second, as we shall see, a key deductive step of the proof of the lemma requires, *if* one wants to use the diagram associated with the theorem along with the denotative letters there assigned, that the auxiliary construction of the theorem be performed on the left side of the diagram. An auxiliary construction performed on the right side is at variance with the text of the lemma. As a result, if one draws the diagram of the theorem without suspecting that it will also be of use in the lemma, the ensemble {text of the lemma + diagram of the theorem} will be inconsistent with a 50% probability.

But there is more to the issue. The ancestor of nearly all manuscripts of *Prolegomena ad Almagestum* is Vat. gr. 1594, a most celebrated manuscript of Ptolemy's treatise –alas, almost without diagrams... Only a skilled copyist, or a competent reviser, can reconstruct the missing diagrams of *Prolegomena ad Almagestum* and of *Almagestum*. But expecting any skilled copyist to check the ensemble {text of the lemma + diagram of the theorem} for consistency maybe is asking too much; still, such a check will hopefully be done by any reviser competent in mathematical matters. Let us see what happened in our case; the "original" text is that of Vat. gr. 1594; all texts are set out in a table just after the following list.

- 1. Vat. gr. 184, ff. 10v (text), 11r (diagram), and Par. gr. 2390, ff. 2r (text), 1v (diagram), the latter before correction. The diagram of the theorem is left unchanged, so that the auxiliary construction is performed "on the right side". Nor is the text of the lemma corrected. As a result, the ensemble {text of the lemma + diagram of the theorem} is inconsistent. Note that in Vat. gr. 184 the reference to *El*. I.post.3 is modified to its canonical form (= the form used in *Elementa*), namely with a  $\delta \epsilon$  replacing the original  $\kappa \alpha i$ . A note of caution: the inconsistency just mentioned partly arises from the fact that, both in Vat. gr. 184 and in Par. gr. 2390, the copyist was not identical with the person who drew the diagrams (note that text and diagram are on different pages).
- 2. Marc. gr. 303, ff. 31rb (text), 30vb (diagram). Only the diagram is different. The auxiliary construction is performed on the left side; as a further change, the diagram represents a square and a pentagon inscribed in circles, not a hexagon and a pentagon as in all other manuscripts (as said above, the text of the theorem only refers to "polygons")<sup>122</sup>. As a result, the ensemble {text of the lemma + diagram of the theorem} is consistent. Nor is in this case the diagram drawn by the copyist.
- Correctors of Par. gr. 2390 (two hands are at work; one of them is Manuel Bryennios: 3. see item a of Sect. 4). They modify the proof of the lemma; the diagram of the theorem is left untouched (but it had already been tampered with in an unfruitful attempt at modifying the proof of the theorem). As a result, the ensemble {text of the lemma + diagram of the theorem} is consistent, even if the resulting deductive progression is quite muddled. The crucial modifications to the proof of the lemma amount to the following (within frames in Fig. 2): erasing the original  $\sigma\nu\nu\theta\epsilon\nu\tau\iota$  (a dativum judicantis that denotes an operation on ratios; see below) and replacing it by  $\delta\iota\epsilon\lambda\delta\nu\tau\iota$  (see also below); putting a reference sign in the text and writing in the margin the conclusion of the διελόντι deductive step. Other variant readings in the text: an article  $\tau \eta v$  is added four times before the second term of a ratio; a connector  $\alpha \beta \alpha$  is added twice; a connector  $\delta \epsilon$  is added once; a noun ywvia is added twice (apparently by a different hand). Note that Laur. Plut. 28.1, a direct copy of Par. gr. 2390, has all the corrections of its model integrated in the main text (rectangular frames in Fig. 3), but adds a further article  $\tau \eta v$  (round frame in Fig. 3).
- 4. Byzantine recensions (they are identical in this case; the diagram in **Fig. 4** is that at Laur. Plut. 89 sup. 48, f. 7v). The auxiliary construction is performed on the left side.

<sup>&</sup>lt;sup>122</sup> Recall that, according to ancient conventions, a square is not a polygon, since no quadrilateral is. Therefore, the diagram of Marc. gr. 303 does not correctly represent the intended geometric configuration.



**Fig. 2**. The text of the lemma in *Prolegomena ad Almagestum*, with corrections. Paris, Bibliothèque Nationale de France, gr. 2390, f. 2r (detail)

Kai Stagniles Ti Too 2 K KURSOU TEPI O LOY & Feld Telpa OB W HLEKP Kair Ge GAN 200 12 - i To To Find owigivwenrk חדסס לאי אלי דס ראל דפו לשאטא אפי דסגלם ה לצירא אפיל דאיאל בעל לסיטאט וסיניויו i TEP OZLEK TOLLEW TOOTOVZKY TOLLE'S Kai NELOV TIWONTO TOOTHV &K outon TO TZOR TOOT To BZK Ter Ton HOH. D'ANCOC of OLLE TO OTTON TO LE DIN TONI a TO OTTAN TON UN AL Led ZOND D'A A TO TON יא אידם הרסילואים אי אי אדר אי הדידנם לסטי אני אי שיאי אנם למעומא -

**Fig. 3**. The text of the lemma in *Prolegomena ad Almagestum*. Firenze, Biblioteca Medicea Laurenziana, Ms. Plut. 28.1, f. 2v (detail). Su concessione del MiBACT. È vietata ogni ulteriore riproduzione con qualsiasi mezzo

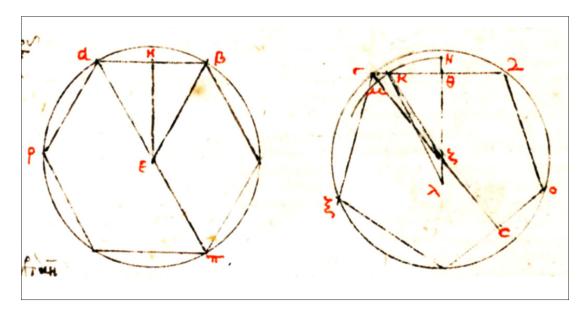
As a result, the ensemble {text of the lemma + diagram of the theorem} is consistent. As the diagram represents an hexagon and a pentagon, a collation of Marc. gr. 303 is to be excluded. Variant readings in the text: an article  $\tau\eta\nu$  is added twice before the second term of a ratio (this is a subset of the four  $\tau\eta\nu$  added by reviser 3); a connector  $\[mathcap{a}\]$  is added twice (the one in the final sentence was also added by reviser 3); two nouns are interchanged:  $\kappa\psi\kappa\lambda o\nu\pi\epsilon\rho\mu\phi\epsilon\rho\epsilon\iota\alpha \rightarrow \pi\epsilon\rho\mu\phi\epsilon\rho\epsilon\iota\alpha\kappa\psi\kappa\lambda o\nu$ ; the noun  $\[mathcap{a}\]$  of the adjective  $\[mathcap{a}\]$  of the adjective  $\[mathcap{a}\]$  is consistent.

5. The solution adopted by the first modern editor of the text is quite arbitrary but typical of a philologist (Hultsch 1876-78, 1142): he perceived the problem but decided to change neither the diagram nor the proof. Instead, he amended to  $\Delta$  three occurrences of the denotative letter  $\Gamma$  in the text, so that a crucial deductive step no longer refers to the left side of the diagram but to the right side of it. Hultsch also added a connector  $\alpha \alpha$  in the final sentence (the same as was added by revisers 3 and 4).

The Greek texts resulting from recensions 3-5 are set out in parallel in the following table, that also includes the base text of Vat. gr. 1594, f. 2r.

base text	correctors of Par. gr. 2390	Byzantine recension	Hultsch
ὅτι δὲ ἡ ΓΘ πρὸς ΘΚ μείζονα λόγον ἔχει ἤπερ ἡ ὑπὸ ΓΖΘ πρὸς τὴν ὑπὸ ΚΖΘ δέδεικται Θέωνι ἐν τῷ ὑπομνήματι τοῦ μικροῦ ἀστρονόμου, οὐδὲν δὲ ἦττον καὶ νῦν δειχθήσεται.	ὅτι δὲ ἡ ΓΘ πρὸς <u>τὴν</u> ΘΚ μείζονα λόγον ἔχει ἤπερ ἡ ὑπὸ ΓΖΘ πρὸς τὴν ὑπὸ ΚΖΘ δέδεικται Θέωνι ἐν τῷ ὑπομνήματι τοῦ μικροῦ ἀστρονόμου, οὐδὲν δὲ ἦττον καὶ νῦν δειχθήσεται.	ὅτι δὲ ἡ ΓΘ πρὸς <u>τὴν</u> ΘΚ μείζονα λόγον ἔχει ἤπερ ἡ ὑπὸ ΓΖΘ πρὸς τὴν ὑπὸ ΚΖΘ δέδεικται Θέωνι ἐν τῷ ὑπομνήματι τοῦ μικροῦ <u>ἀστρονομικοῦ</u> , οὐδὲν δὲ ἦττον καὶ νῦν δειχθήσεται.	ὅτι δὲ ἡ ΓΘ πρὸς ΘΚ μείζονα λόγον ἔχει ἤπερ ἡ ὑπὸ ΓΖΘ πρὸς τὴν ὑπὸ ΚΖΘ δέδεικται Θέωνι ἐν τῷ ὑπομνήματι τοῦ μικροῦ ἀστρονόμου, οὐδὲν δὲ ἦττον καὶ νῦν δειχθήσεται.
κέντρω γὰρ τῷ Ζ καὶ [καὶ om. Vat. gr. 184] διαστήματι [δὲ add. Vat. gr. 184] τῷ ΖΚ κύκλου περιφέρεια γεγράφθω ἡ ΜΚΝ, καὶ ἐκβεβλήσθω ἡ ΖΘ ἐπὶ τὸ Ν.	κέντρφ γὰρ τῷ Ζ καὶ διαστήματι τῷ ΖΚ κύκλου περιφέρεια γεγράφθω ἡ MKN, καὶ ἐκβεβλήσθω ἡ ΖΘ ἐπὶ τὸ Ν.	κέντρφ γὰρ τῷ Ζ καὶ διαστήματι τῷ ΖΚ <u>περιφέρεια κύκλου</u> γεγράφθω ἡ ΜΚΝ, καὶ ἐκβεβλήσθω ἡ ΖΘ ἐπὶ τὸ Ν.	κέντρφ γὰρ τῷ Ζ διαστήματι <u>δὲ</u> τῷ ΖΚ κύκλου περιφέρεια γεγράφθω ἡ ΜΚΝ, κα ἐκβεβλήσθω ἡ ΖΘ ἐπὶ τὸ Ν.
ἐπεὶ οὖν ἐστιν ὡς ἡ ΓΚ πρὸς ΚΘ τὸ ΓΚΖ τρίγωνον πρὸς τὸ ΚΖΘ, ἡ ΓΚ πρὸς ΚΘ μείζονα λόγον ἔχει ἤπερ ὁ ΖΜΚ τομεὺς πρὸς τὸν ΖΚΝ τομέα· καὶ συνθέντι·	ἐπεὶ οὖν ἐστιν ὡς ἡ ΓΚ πρὸς <u>τὴν</u> ΚΘ τὸ ΓΚΖ τρίγωνον πρὸς τὸ ΚΖΘ ἡ <u>δὲ</u> ΓΚ πρὸς <u>τὴν</u> ΚΘ μείζονα λόγον ἔχει ἤπερ ὁ ΖΜΚ τομεὺς πρὸς τὸν ΖΚΝ τομέα, καὶ <u>διελόντι</u> ὡς ἡ ΓΘ πρὸς τὴν ΘΚ οὕτως τὸ ΓΖΘ τρίγωνον	ἐπεὶ οὖν ἐστιν ὡς ἡ ΓΚ πρὸς ΚΘ τὸ ΓΚΖ τρίγωνον πρὸς τὸ ΚΖΘ, ἡ ΓΚ <u>ἄρα</u> πρὸς <u>τὴν</u> ΚΘ μείζονα λόγον ἔχει ἤπερ ὁ ΖΜΚ τομεὺς πρὸς τὸν ΖΚΝ τομέα· καὶ συνθέντι·	ἐπεὶ οὖν ἐστιν ὡς ἡ <u>Δ</u> Κ πρὸς ΚΘ τὸ <u>Δ</u> ΚΖ τρίγωνον πρὸς τὸ ΚΖΘ, ἡ <u>Δ</u> Κ πρὸς ΚΘ μείζονα λόγον ἔχει ἤπερ ὁ ΖΜΚ τομεὺς πρὸς τὸν ΖΚΝ τομέα· καὶ συνθέντι·
ἀλλ' ὡς ὁ τομεὺς πρὸς τὸν τομέα ἡ γωνία πρὸς τὴν γωνίαν· μείζονα λόγον ἔχει ἡ ΓΘ πρὸς ΘΚ ἤπερ ἡ ὑπὸ ΓΖΘ πρὸς τὴν ὑπὸ ΚΖΘ.	ἀλλ' ὡς ὁ τομεὺς πρὸς τὸν τομέα ἡ γωνία πρὸς τὴν γωνίαν· μείζονα <u>ἄρα</u> λόγον ἔχει ἡ ΓΘ πρὸς [τὴν add. Laur. Plut. 28.1] ΘΚ ἤπερ ἡ ὑπὸ ΓΖΘ <u>γωνία</u> πρὸς τὴν ὑπὸ ΚΖΘ <u>γωνίαν</u> .	ἀλλ' ὡς ὁ τομεὺς πρὸς τὸν τομέα ἡ γωνία πρὸς τὴν γωνίαν· μείζονα ἄρα λόγον ἔχει ἡ ΓΘ πρὸς ΘΚ ἤπερ ἡ ὑπὸ ΓΖΘ πρὸς τὴν ὑπὸ ΚΖΘ.	ἀλλ' ὡς ὁ τομεὺς πρὸς τὸν τομέα ἡ γωνία πρὸς τὴν γωνίαν· μείζονα <u>ἄρα</u> λόγον ἔχει ἡ ΓΘ πρὸς ΘΚ ἤπερ ἡ ὑπὸ ΓΖΘ πρὸς τὴν ὑπὸ ΚΖΘ.

The sentence in the third row of the table deserves a more detailed analysis, because it is a paradigmatic instance of the liberties one can take with Greek mathematical texts *salva ueritate*. The original sentence is made of a so-called paraconditional (= causal subordi-



**Fig. 4**. The diagram of the lemma in a Byzantine recension of *Prolegomena ad Almagestum*. Firenze, Biblioteca Medicea Laurenziana, Ms. Plut. 89 sup. 49, f. 7v (detail). Su concessione del MiBACT. È vietata ogni ulteriore riproduzione con qualsiasi mezzo

nate ἐπεὶ οὖν ἐστιν ὡς ἡ ΓΚ πρὸς KΘ τὸ ΓΚΖ τρίγωνον πρὸς τὸ KZΘ + main clause ἡ ΓΚ πρὸς KΘ μείζονα λόγον ἔχει ἤπερ ὁ ZMK τομεὺς πρὸς τὸν ZKN τομέα) and of a further principal clause, very elliptic as usual in Greek mathematical style: καὶ συνθέντι (transl. "and by composition"), that usually refers to taking both ratios in the preceding proportion and transforming each of them from *a:b* to (a + b):*b*. The presence of συνθέντι shows at once that, in order to have the ensemble {text of the lemma + diagram of the theorem} consistent, the auxiliary construction of the theorem had to be performed on the left side of the diagram associated with it. Only in this case, in fact, one gets, "by composition", from ratio ἡ ΓΚ πρὸς KΘ the ratio ἡ ΓΘ πρὸς ΘK featuring in the subsequent row of the table –since only in this case the line segment ἡ ΓΘ is the sum of line segments ἡ ΓK and ἡ KΘ, not their difference (compare **Fig. 1** and **4**).

As a matter of fact, a number of deductive steps are understood in the sentence just singled out (supposing of course that the auxiliary construction of the theorem was performed on the left side, as in **Fig. 4**).

The proportion stated by the causal subordinate of the paraconditional, ἐστιν ὡς ἡ ΓΚ πρὸς ΚΘ τὸ ΓΚΖ τρίγωνον πρὸς τὸ ΚΖΘ, is validated by *El*. VI.1. In principle, one might add the validating mathematical result, namely, the enunciation of *El*. VI.1 itself, to the deduction, but Greek mathematical style refrains from doing this. On the contrary, general (= without denotative letters) enunciations of

previous theorems validating istantiated (= with denotative letters) statements in a mathematical proposition are certainly interpolated (of course, this is an example of "saturation"). This holds true in all cases a theorem must be invoked to justify an application of it; this will happen for instance with *El.* V.13 in the next item.

- Now, since the inequality of ratios stated by the main clause of the paraconditional  $\dot{\eta}$ ΓΚ πρὸς ΚΘ μείζονα λόγον ἔχει ἤπερ ὁ ΖΜΚ τομεὺς πρὸς τὸν ΖΚΝ τομέα features circular sectors and not triangles, this replacement must be justified along with the change from a proportion to an inequality (note that the first ratio  $\dot{\eta} \ \Gamma K \ \pi \rho \dot{\delta} \varsigma \ K\Theta$ is the same both in the proportion and in the inequality). To get these, it is enough to show that triangle  $\tau \circ \Gamma KZ$  to triangle  $\tau \circ KZ\Theta$  has a greater ratio than sector  $\circ$ ZMK to sector  $\diamond$  ZKN, and to apply *El*. V.13 (if *a*:*b* = *c*:*d* and *c*:*d* > *e*:*f*, then *a*:*b* > *e*:*f*) to the proportion in the first statement and to this inequality between ratios. This inequality derives in its turn from an inequality which is obvious from the diagram (triangle  $\tau \circ \Gamma KZ$  to sector  $\circ ZMK$  has a greater ratio than triangle  $\tau \circ KZ\Theta$  to sector δ ZKN because triangle τδ ΓKZ includes sector δ ZMK, whereas triangle τδ KZΘ is included in sector o ZKN) and from an application of a proposition not attested in *Elementa*: if a:b > c:d, then  $a:c > b:d [*]^{123}$ . This operation is identified by the adverb έναλλάξ (transl. "by alternation"). Note that, in *Elementa*, alternation for proportions is proved in prop. V.16, namely, if a:b = c:d, then a:c = b:d, but what we need is alternation for inequalities of ratios.
- We must now justify the transition to the instantiated statement of the subsequent row. As said above, the transition from ratio ή ΓK πρòς KΘ to ratio ή ΓΘ πρòς ΘK is made by means of the operation "by composition", *a:b* → (*a* + *b*):*b*. This is harmless, the mathematically non-trivial point being to prove that such an operation preserves proportionality (this is done in *El*. V.18) and, as is required by our lemma, inequality of ratios: if *a:b* > *c:d*, then (*a* + *b*):*b* > (*c* + *d*):*d* [\*\*]<sup>124</sup>. We get as a result ή ΓΘ πρòς KΘ μείζονα λόγον ἔχει ἤπερ ὁ ZMN τομεὺς πρòς τὸν ZKN τομέα.
- We are almost done: it is enough to replace the circular sectors by the angles naturally associated with them<sup>125</sup>. This is done in the subsequent row of the table. First,

<sup>124</sup> We find a proof of this in Pappus, *Collectio*, VII.45-6, and in Eutocius, *in Sphaeram et cylindrum I.2 (AOO III*, 16.11-31); the result is again ascribed to Hero of Alexandria in *sch*. 65 *in prop*. 36 of Euclid, *Optica* **B**, mentioned in the previous note.

<sup>125</sup> This result is crucial from the philological point of view. We read it as a porism to a supplement to *El*. VI.33, but, along with the supplement itself, only in the Theonine manuscripts, and in all of them (*EOO* II, 183 n. 1 and 424.22-428.21, with the porism at 428.20-1; in Vat. gr.

<sup>&</sup>lt;sup>123</sup> But proven for instance by Pappus, *Collectio*, III.17 and VII.47, and ascribed to Hero of Alexandria in *sch*. 65 *in prop*. 36 of Euclid, *Optica* **B** (EOO VII, 275.3-22)

we find the general statement ἀλλ' ὡς ὁ τομεὺς πρὸς τὸν τομέα ἡ γωνία πρὸς τὴν γωνίαν, that we read in place of the instantiated clause we would have expected, and that would sound like ἀλλ' ὡς ὁ ZMN τομεὺς πρὸς τὸν ZKN τομέα ἡ ὑπὸ ΓΖΘ πρὸς τὴν ὑπὸ KZΘ. After it, we read the instantiation of the result μείζονα λόγον ἔχει ἡ ΓΘ πρὸς ΘΚ ἤπερ ἡ ὑπὸ ΓΖΘ πρὸς τὴν ὑπὸ KZΘ, again a consequence of applying *El.* V.13 to the proportion just established and to the inequality between rations we have obtained as a result at the end of the previous item.

This discussion shows that the deduction attested in Vat. gr. 1594 is far from being saturated. Adding the "missing" deductive steps and a couple of articles before the second term of some ratios, we get the following, (still not completely) saturated text as a result (the additions are within braces):

έπεὶ οὖν ἐστιν ὡς ἡ ΓΚ πρὸς {τὴν} ΚΘ τὸ ΓΚΖ τρίγωνον πρὸς τὸ ΚΖΘ [El. VI.1] {τὸ δὲ ΓΚΖ τρίγωνον πρὸς τὸ ΚΖΘ μείζονα λόγον ἔχει ἤπερ ὁ ΖΜΚ τομεὺς πρὸς τὸν ΖΚΝ τομέα· τὸ γὰρ ΓΚΖ τρίγωνον πρὸς τὸν ΖΜΚ τομέα μείζονα λόγον ἔχει ἤπερ τὸ ΚΖΘ τρίγωνον πρὸς τὸν ΖΚΝ τομέα, καὶ ἐναλλάξ [\*]}, ἡ ΓΚ πρὸς {τὴν} ΚΘ μείζονα λόγον ἔχει ἤπερ ὁ ΖΜΚ τομεὺς πρὸς τὸν ΖΚΝ τομέα [El. V.13]· καὶ συνθέντι [\*\*] {ἄρα ἡ ΓΘ πρὸς ΚΘ μείζονα λόγον ἔχει ἤπερ ὁ ΖΜΝ τομεὺς πρὸς τὸν ΖΚΝ τομέα}

190, f. 102r *marg.*, it is added by a later hand). Now, in *in Almagestum I.10*, Theon claims that he himself proved this result in his own edition of *Elementa*: ὅτι δὲ οἱ ἐπὶ ἴσων κύκλων τομεῖς πρὸς ἀλλήλους εἰσὶν ὡς αἱ γωνίαι δἑδεικται ἡμῖν ἐν τῇ ἐκδόσει τῶν στοιχείων πρὸς τῷ τέλει τοῦ ἕκτου βιβλίου at *iA*, 492.6-8. Heiberg rightly takes this testimony as the "cardo [...] quaestionis" of Vat. gr. 190 non containing Theon's recension of *Elementa*: EOO V, xxIv. Note that Theon does not "prove" exactly this result, but states that it holds as an immediate consequence of what is really proved, namely, that in equal circles sectors are to each other as the arcs naturally associated with them.

<sup>126</sup> As above with the operation "by composition", we find a proof of this in Eutocius, *in Sphaeram et cylindrum I.2* (AOO III, 18.1-22); the result is again ascribed to Hero of Alexandria in *sch.* 65 *in prop.* 36 of Euclid, *Optica* **B**.

- should also have inverted the steps ή δὲ ΓΚ πρὸς τὴν ΚΘ μείζονα λόγον ἔχει ἤπερ ὁ ΖΜΚ τομεὺς πρὸς τὸν ΖΚΝ τομέα and διελόντι ὡς ἡ ΓΘ πρὸς τὴν ΘΚ οὕτως τὸ ΓΖΘ τρίγωνον πρὸς τὸ ΘΖΚ τρίγωνον;
- should have noted that line segment  $\dot{\eta} \Gamma \Theta$  is equal to line segment  $\dot{\eta} \Delta \Theta$  and that triangle  $\tau \dot{\delta} \Gamma Z \Theta$  is equal to triangle  $\tau \dot{\delta} \Delta Z \Theta$  (recall Hultsch's correction);
- should finally have accordingly modified ή δὲ ΓΚ πρὸς τὴν ΚΘ μείζονα λόγον ἔχει ἤπερ ὁ ΖΜΚ τομεὺς πρὸς τὸν ΖΚΝ τομέα and got ἡ δὲ ΓΘ πρὸς τὴν ΘΚ μείζονα λόγον ἔχει ἤπερ ὁ ΖΜΝ τομεὺς πρὸς τὸν ΖΚΝ τομέα.

But there is more to the issue: how to deal with the paraconditional clause? As said, its meaning is: <u>Since</u>  $\dot{\omega}_{\zeta} \dot{\eta} \Gamma K \pi \rho \dot{o}_{\zeta} K \Theta$  to  $\Gamma K Z \tau \rho \dot{\gamma} \omega v ov \pi \rho \dot{o}_{\zeta} \tau \dot{o} K Z \Theta$ , <u>then</u>  $\dot{\eta} \Gamma K \pi \rho \dot{o}_{\zeta} K \Theta$   $\mu \epsilon \dot{\zeta} ov \alpha \lambda \dot{o} \gamma ov \, \check{\epsilon} \chi \epsilon i \, \check{\eta} \pi \epsilon \rho \, \dot{o} \, ZMK \tau o \mu \epsilon \dot{v}_{\zeta} \pi \rho \dot{o}_{\zeta} \tau \dot{o} v \, ZKN \tau o \mu \epsilon \dot{\alpha}$ . This is perfectly correct. But canonical style allows for adding  $\check{\alpha} \rho \alpha$  within the principal clause of the paraconditional (in an English translation of it, one may or may not add "then"): this was the choice of reviser 4. Further, one may also read the whole sentence in the third row of the table (namely, καὶ συνθέντι included) as a paraconditional, whose causal antecedent is made of two sub-clauses: <u>Since</u>  $\dot{\omega}_{\zeta} \dot{\eta} \Gamma K \pi \rho \dot{o}_{\zeta} K \Theta \tau \dot{o} \Gamma K Z \tau \rho \dot{\gamma} \omega v ov \pi \rho \dot{o}_{\zeta} \tau \dot{o} K Z \Theta <u>and</u> <math>\dot{\eta} \Gamma K \pi \rho \dot{o}_{\zeta} K \Theta \mu \epsilon \dot{\zeta} ov \alpha \lambda \dot{o} \gamma ov \, \check{\epsilon} \chi \epsilon i \, \check{\eta} \pi \epsilon \rho \, \dot{o} \, ZMK \tau o \mu \epsilon \dot{v}_{\zeta} \pi \rho \dot{o}_{\zeta} \tau \dot{o} v \, ZKN \tau o \mu \epsilon \dot{\alpha}$ , <u>then</u> καὶ διελόντι κτλ.: this was the choice of reviser 3<sup>127</sup>, who was thereby forced to add a coordinating  $\delta \dot{\epsilon}$  at the beginning of the second sub-clause in the causal antecedent of the paraconditional. Note that he did not add  $\check{\alpha} \rho \alpha$  to the new principal clause καὶ  $\delta \iota \epsilon \lambda \dot{o} v \tau \iota \kappa \tau \lambda$ .

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 $^{127}$  I also take into account that he replaced συνθέντι by διελόντι and made the result of this manipulation explicit –see above.

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