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# A NEW LOGISTIC TEXT OF NICHOLAS RHABDAS

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*Résumé.* Le manuscrit Paris, Bibliothèque nationale de France, grec 2381 contient un court texte logistique de Nicholas Rhabdas, un fonctionnaire de l’administration fiscale byzantine qui est bien connu, depuis une édition capitale de Paul Tannery, comme mathématicien et éditeur d’écrits mathématiques de Maxime Planude et de Manuel Moschopulos. Ce texte de Rhabdas, qui demeurait inédit, est ici mis en contexte, présenté en édition critique, traduit et commenté.

## INTRODUCTION

On November 16, 1322, St Matthew’s day, the brilliant civil servant Nicholas Artabasdos Rhabdas and his friends spent a pleasant night at “The Purple Lion”, the trendiest beer hall in Constantinople. Among the parlour games played that night, Rhabdas proposed some wowing pieces of recreational mathematics. When strolling back home, he conceived a “procedure by means of which every arithmetic and geometric means can be found, either being of a double ratio or of a triple or of a multiple or of a multiple-epimoric or of a multiple-epimeric, or epidimeric, and in general of whatever it is of all ratios” (the effects of beer can also be detected beyond this title, as we shall see). He set out to write down his *Procedure* as soon as he arrived at home. He filled a couple of pages, where he also noted the riddles with which he had amused his friends, then he fell asleep on his desk. Maybe he was dreaming about purple lions. The sheet containing the *Procedure* was filed among Rhabdas’ foul papers; it was not thrown in the waste after his death; it was recovered, about 50 years later, by a scholar who was fond of disseminating chunks of scientific treatises in pages left blank in the books of his own library, and who also collected less exploded *adversaria* for his own use. Part of these *adversaria* have been handed down to us in the form of a codex, now preserved in Paris. Rhabdas’ *Procedure* lay buried in that codex for about 640 years, until it was recorded in a little-read yet epoch-making catalogue series. About 110 years later, I fell on the relevant page of that catalogue. The endpoint of this partly fictionalized trajectory is the present study, where Rhabdas’ *Procedure* is presented, edited, and translated.

## BASICS ABOUT NICHOLAS RHABDAS

Nicholas Artabasdos Rhabdas from Smyrna (*PLP*, nr. 1437) was a high-brow scholar and functionary of the imperial fiscal administration in Constantinople around 1320-42; he had connections with Nikephoros Gregoras and the circle of Maximus Planudes’ pupils.<sup>1</sup> His administrative role, mentioned

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\* Online reproductions of most manuscripts mentioned in this article can be found through the website <https://pinakes.irht.cnrs.fr/>, which also provides additional bibliography. I am grateful to S. Di Mambro and to G. Pausillo for her logistic support.

above, has been revealed by a recent finding, which has also led to the identification of his handwriting. The finding involves a document of the Chilandar monastery dated 1323 and redacted by an imperial land-surveyor who signs himself as Nicholas Rhabdas.<sup>2</sup> Once the identification with our Rhabdas was confirmed by R. Estangüi Gómez, and with the assistance of I. Pérez Martín, his hand could also be found in the manuscripts Heidelberg, Universitätsbibliothek, Pal. gr. 129 (mainly 14<sup>th</sup> century; *Diktyon* 32460), ff. 11v-12r, Leeds, University Library, Brotherton Coll. MS 31/2-3 (first half of the 14<sup>th</sup> century; *Diktyon* 3761),<sup>3</sup> and Paris, Bibliothèque nationale de France, grec 2650 (*Diktyon* 52285), ff. 147r-150v. All these manuscripts preserve autograph works of Rhabdas.<sup>4</sup>

These manuscripts also provide two new dates. Leeds, UL, Brotherton Coll. 31/3, ff. 64r-69r, contains an autograph Easter Computus, to be dated to 1342 (nay, to AM 6840) because its examples are worked out for that year, which is stated to be the current year. One of the algorithms set out in the Computus is identical to the algorithm presented in the short computistical section of Rhabdas' *Rechenbuch*, namely, the so-called *Letter to Tzavoukhes* (see below). This section also calculates the date of Easter for a year that is stated to be the current year; consequently, the *Letter to Tzavoukhes* can be dated to 1341.

A second date comes from the fact that Nikephoros Gregoras (died ca. 1358-61; *PLP* nr. 4443) recycled a square root table Rhabdas had sent to him (see below), and which is now preserved in Gregoras' notebook Heid. Pal. gr. 129, ff. 11v-12r. Gregoras copied this table, among his own preliminary annotations to Ptolemy's *Handy Tables*, in the manuscript Venezia, Biblioteca Nazionale Marciana, gr. Z. 325 (coll. 518; ca. 1325; *Diktyon* 69796), f. 89v. These annotations are dated (f. 86r marg. sup.) to AM 6834 [= AD 1325/6]. Consequently, Rhabdas composed his table in 1326 at the latest.

The *Procedure* here edited also provides a new date: in its title, as anticipated in my fictionalized introduction, Rhabdas asserts that this mini *Rechenbuch* was conceived and set out in the middle of the night of St Matthew's day of AM 6831, that is, on November 16, 1322.

Evidence on Rhabdas's life and activities nearly contemporary with the one provided by the *Procedure* comes from a letter addressed to Andronikos Zarides (died after 1327; *PLP*, nr. 6461), in which Rhabdas informs his addressee that a partial solar eclipse and a lunar eclipse will occur on June 26, 1321 and on July 10, 1321, respectively.<sup>5</sup> The style and content of this letter suggest that Rhabdas was

<sup>1</sup> On Rhabdas' life and works see most recently the synthesis in F. ACERBI – D. MANOLOVA – I. PÉREZ MARTÍN, *The Source of Nicholas Rhabdas' Letter to Khatzykes: An Anonymous Arithmetical Treatise in Vat. Barb. gr. 4*, in *JÖB*, 68 (2018), pp. 1-37: 2-6, on which I partly rely here.

<sup>2</sup> This document will be published in O. DELOUIS – M. ŽIVOJINOVIĆ, *Actes de Chilandar. II. De 1320 à 1335 (ArchAth, 24)*, Paris, forthcoming, nr. 90. This shows that the epithet γεωμέτρης applied to Rhabdas in the title of Moschopoulos' treatise on magic squares (see n. 6 below) and in the titles of his *Letter to Khatzykes* and *Letter to Myrsiniotes* (see below) must be translated "land-surveyor".

<sup>3</sup> The Leeds manuscripts are described in detail in F. ACERBI, *The "Third Letter" of Nicholas Rhabdas: an Autograph Easter Computus*, in *Estudios Bizantinos*, 9 (2021), pp. 1-52: 1-5.

<sup>4</sup> In particular, the said folios of the Paris manuscript are the only extant witness of the grammatical compendium Rhabdas composed for his son Paul Artabasdos (*PLP*, nr. 1438). The aim of the compendium is to expound the appropriate use of words, in order to avoid barbarisms and solecisms. The exposition is based on analytical divisions of the main grammatical arguments, treated by means of "some short notes" (μικροὺς τινὰς ὑπομνηματισμούς).

<sup>5</sup> In both cases, Rhabdas also provides the hour in which the eclipse will occur. The astronomical data and a part of the Greek text were first presented in A. TIHON, *Nicolas Eudaimonoioannes, réviseur de l'Almageste?*, in *Byz*, 73 (2003), pp. 151-161: 153-154; the entire letter is edited in A. RIEHLE, *Epistolographie und Astronomie in der frühen Palaiologenzeit*, in *JÖB*, 65 (2015), pp. 243-252: 251 (see also p. 246 n. 23-24 for the data of the real eclipses). On John Zarides (*PLP*, nr. 6462), the

a young man when he composed it, even if he apparently had reached a high level of astronomical expertise. On these grounds, and taking into account that Manuel Moschopoulos dedicated his treatise on magic squares to Rhabdas,<sup>6</sup> A. Riehle has proposed that Rhabdas was born *ca.* 1295.<sup>7</sup>

Through Andronikos Zarides and Manuel Moschopoulos, both pupils of Maximos Planudes (he died *ca.* 1305; *PLP*, nr. 23308), Rhabdas was connected with the latter's circle. This is confirmed by the fact that Rhabdas prepared a revision—including a couple of long additions—of Planudes' *Great Calculation according to the Indians*.<sup>8</sup>

#### THE LITERARY FORM OF THE *PROCEDURE*: BASICS ABOUT BYZANTINE *RECHENBÜCHER*

Rhabdas' scientific production focused on logistic, the branch of arithmetic in which a unit can be divided and that deals with counting numbers and with computations.<sup>9</sup> Logistic as an autonomous discipline developed in Late Antiquity as a support to mathematical astronomy,<sup>10</sup> and partly retained this

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brother of Andronikos, see A. COHEN-SKALLI – I. PÉREZ MARTÍN, *La Géographie de Strabon entre Constantinople et Thessalonique: à propos du Marc. gr. XI.6*, in *Scriptorium*, 71 (2017), pp. 175-207: 181-183, 187, 195-197, and S. MARTINELLI TEMPESTA, *Tricliniana et 'Planudea'*. Alcune osservazioni sul Demostene *Paris. Coislin.* 339, in M. CRONIER – B. MONDRAIN (eds), *Le livre manuscrit grec : écritures, matériaux, histoire. Actes du IX<sup>e</sup> Colloque international de Paléographie grecque. Paris, 10-15 septembre 2018 (TM, 24/1)*, Paris, 2020, pp. 247-278: 265-266.

<sup>6</sup> On Moschopoulos, who died after *ca.* 1306, see *PLP*, nr. 19373, and C. CONSTANTINIDES, *Higher Education in Byzantium in the Thirteenth and Early Fourteenth Centuries* (1204 - *ca.* 1310) (*Texts and Studies of the History of Cyprus*, 11), Nicosia, 1982, pp. 103-108. The title of Moschopoulos' treatise gives more prominence to Rhabdas than to its author: the former is qualified "arithmetician and land-surveyor" (ἀριθμητικὸς καὶ γεωμέτρης), whereas Moschopoulos is simply "most learned and most happy" (λογιώτατος καὶ μακαριώτατος)—and thus he was already dead—and he redacted the treatise "spurred on" (βιασθεῖς) by Rhabdas. All of this suggests that Rhabdas himself took care of the edition after Moschopoulos' death. On Moschopoulos' treatise, see P. TANNERY, *Le traité de Manuel Moschopoulos sur les carrés magiques. Texte grec et traduction*, in *Annuaire de l'Association pour l'encouragement des études grecques en France*, (1886), pp. 88-118, repr. ID., *Mémoires scientifiques*, publiés par J. L. Heiberg & H. G. Zeuthen, 17 vols., Toulouse – Paris, 1912-50, vol. 4, 1920, pp. 27-60: 32.1-4. An analysis of the treatise is found in J. SESIANO, *Les carrés magiques de Manuel Moschopoulos*, in *AHES*, 53 (1998), pp. 377-397; see also P. TANNERY, *Manuel Moschopoulos et Nicolas Rhabdas*, in *Bulletin des Sciences mathématiques*, 2<sup>e</sup> série, 8 (1884), pp. 263-277, repr. ID., *Mémoires scientifiques*, vol. 4, pp. 1-19.

<sup>7</sup> RIEHLE, *Epistolographie* [see n. 5], pp. 246-248.

<sup>8</sup> That Rhabdas revised the *Great Calculation* is borne out by the title carried in the only independent witness of this version of Planudes' treatise, namely, the manuscript Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. gr. 1411 (end 14<sup>th</sup> century – beginning 15<sup>th</sup> century; *Diktyon* 68042), f. 122r, and by the fact that Rhabdas' additions are expressly marked as such in the same manuscript (ἐκ τῆς προσθήκης τοῦ Ῥαβδᾶ Νικολάου on ff. 123r and 125v, which shows that the version copied in the Vatican manuscript is not Rhabdas' revision, but a later and carefully conceived compilation, in which two adjuncts, each marked in the margin by τοῦτο ἡμέτερον, enrich Rhabdas' additions). The title downplays Planudes' contribution, alluding to the fact that a source must be understood: *Great Calculation According to the Indians. This formulation of it is by the most scholarly scholar and most honourable monk Maximos Planudes and by Nicholas Rhabdas* (Ψηφηφορία κατ' Ἰνδοῦς ἡ λεγομένη μεγάλη. ταύτης ἡ φράσις τοῦ φιλοσοφώτατου ἐν φιλοσόφοις καὶ τιμιωτάτου ἐν μοναχοῖς κυροῦ Μαξίμου τοῦ Πλανούδη καὶ τοῦ Ῥαβδᾶ Νικολάου). The edition of Rhabdas' additions is in A. ALLARD, *Maxime Planude, Le grand calcul selon les Indiens*, Louvain-la-Neuve, 1981, pp. 203-211; for the title just read see *ibidem*, p. 25 *app.* Planudes' treatise in Vat. gr. 1411 is incomplete, ending in the middle of f. 126v at *ibidem*, p. 61.8 εἴρηται.

<sup>9</sup> According to Eutocius (see below), logistic and arithmetic are distinguished by allowing the unit to be divided or not (J. L. HEIBERG, *Archimedis opera omnia cum commentariis Eutocii* [BSGRT], 3 vols., Lipsiae, 1910-15, vol. 3, p. 120.28-30). A different account—echoing Plato, *Laws* 819B-C: P. TANNERY, *L'éducation platonicienne. Second article*, in *Revue Philosophique de la France et de l'Étranger*, 11 (1881), pp. 283-299: 286, repr. ID., *Mémoires Scientifiques*, vol. 7, pp. 21-44: 24-25—most likely to be ascribed to Geminus (a 1<sup>st</sup> century BC mathematically-minded philosopher and polymath), can be read at pseudo-Hero, *Def.* 135.5-6 (J. L. HEIBERG – L. NIX – W. SCHMIDT – H. SCHÖNE, *Heronis Alexandrini opera quae supersunt omnia* [BSGRT], 5 vols., Lipsiae, 1899-1914, vol. 4, pp. 98.12-100.3) and, by a different line of tradition, as a scholium to Plato, *Chrm.* 165E6, edited in W. C. GREENE, *Scholia Platonica* (*Monograph Series*, 8). Haverford, 1938, p. 115 = Scholium 27 in D. CUFALO, *Scholia Graeca in Platonem, Volumen I, scholia ad dialogos tetralogiarum I-VII continens* (*Pleiadi*, 5.1), Roma, 2007, p. 173; see also Proclus' account at G. FRIEDLEIN, *Procli Diadochi In Primum Euclidis Elementorum Librum Commentarii* (BSGRT), Lipsiae, 1873, p. 40.2-9.

<sup>10</sup> The first known treatise of this kind is included in the *Prolegomena to the Almagest*, a computational primer to Ptolemy's treatise made of (non-redacted) lecture notes of a course held at the end of the 5<sup>th</sup> century in the circle of the Neoplatonic

role in Byzantine times.<sup>11</sup> Literary products pertaining to logistic include the following: arithmetical tables;<sup>12</sup> the chronological primers traditionally called Easter Computi;<sup>13</sup> computational primers giving theoretical grounds for, and explaining how to perform, the basic arithmetical operations in the decimal or in the sexagesimal system, including extraction of approximate square roots and composition and removal of ratios: such are Planudes' *Great Calculation According to the Indians* and its anonymous 1252 source (these use the decimal system with Hindu numerals), Rhabdas' *Letter to Khatzykes* (see below) and its anonymous source (decimal system in Greek notation), the computational primers to Ptolemy's *Almagest* (sexagesimal system),<sup>14</sup> and the primers embedded in the astronomical "way" of *Quadrivia*.<sup>15</sup> A prominent literary product belonging to the genre of logistic are the *Rechenbücher*.<sup>16</sup> These are collections of computational techniques and of arithmetical or metrological problems unrelated to each other, sometimes in daily-life guise, sometimes organized in sequences of almost identical items. As a matter of fact, the designation *Rechenbuch* refers to a constellation of more or less well-structured, highly sectional, logistic collections; these can sometimes prove difficult to delimit in a given manuscript, because of the simultaneous presence of geometric metrological material or of Easter Computi, which we might wish to attach to an intended *Rechenbuch* or not.

The tradition of *Rechenbücher* in the Greek language can be traced back to late antiquity. Diachronically, these corpora originate in core collections of metrological recipes (conversions of weights and currencies, but also measurement of geometric figures) accompanied by computational tools obviously relevant for solving these problems, such as resolution of common fractions into unit fractions. A purely mathematical setting for the arithmetical problems found in *Rechenbücher* is provided in Diophantus'

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philosopher Ammonius; see F. ACERBI – N. VINEL – B. VITRAC, *Les Prolégomènes à l'Almageste. Une édition à partir des manuscrits les plus anciens: Introduction générale – Parties I-III*, in *SCIAMVS*, 11 (2010), pp. 53-210. The most comprehensive introduction to Greek logistic is still K. VOGEL, *Beiträge zur griechischen Logistik. Erster Teil (Sitzungsberichte der Bayerischen Akademie der Wissenschaften, Mathematisch-naturwissenschaftliche Abteilung)*, München, 1936, pp. 357-472. For Byzantine logistic, see F. ACERBI, *Arithmetic and Logistic, Geometry and Metrology, Harmonic Theory, Optics and Mechanics*, in S. LAZARIS (ed.), *A Companion to Byzantine Science*, Leiden – Boston, 2020, pp. 105-159: 116-128.

<sup>11</sup> See the explicit statement opening the anonymous 1252 *Calculation according to the Indians*, in A. ALLARD, *Le premier traité byzantin de calcul indien: classement des manuscrits et édition critique du texte*, in *RHT*, 7 (1977), pp. 57-107: 80.2-4, and, in a smoother formulation, Planudes' *Great Calculation*: ALLARD, *Maxime Planude* [see n. 8], p. 27.1-5. See also the preface of Barlaam's *Logistikē*: P. CARELOS, *Βαρλαάμ τοῦ Καλαβροῦ, Λογιστική. Barlaam von Seminara, Logistikē (Corpus philosophorum Medii Aevi. Philosophi byzantini, 8)*, Athens – Paris – Bruxelles, 1996, p. 1.10-26.

<sup>12</sup> Such tables are already, and widely, attested in papyri; see the lists (exhaustive to that date) in D. FOWLER, *The Mathematics of Plato's Academy*, 2<sup>nd</sup> ed., Oxford, 1999, pp. 268-276. For arithmetical tables in Byzantine manuscripts see n. 30 below and ACERBI, *Arithmetic and Logistic* [see n. 10], pp. 123-124.

<sup>13</sup> On Byzantine Easter Computi, see F. ACERBI, *Byzantine Easter Computi: An Overview with an Edition of Anonymus 892*, in *JÖB*, 71 (2021).

<sup>14</sup> Byzantine primers of this kind include Book II of Theodoros Metochites' *Abridged Astronomical Elements* (unpublished, but see the study B. BYDÉN, *Theodore Metochites' Stoicheiosis Astronomike and the Study of Natural Philosophy and Mathematics in Early Palaiologan Byzantium* [Studia Graeca et Latina Gothoburgensia, LXVI], Göteborg, 2003); part of Book I of Theodoros Meliteniotes' *Three Books on Astronomy*, edited in R. LEURQUIN, *Théodore Méliénote, Tribiblos Astronomique. Livre I (CAB, IV)*, Amsterdam, 1990, pp. 92-154; a text of unknown author that plunders the previous two, edited in J. MOLL, *Étude sur un traité anonyme d'initiation à l'Almageste*, 2 vols., Mém. Louvain, 1965.

<sup>15</sup> See for instance the computational primer to the sexagesimal system in §§ 1-6 and 26 of the astronomical "way" of Pachymeres' *Quadrivium*, in P. TANNERY, *Quadrivium de Georges Pachymère (ST, 94)*, Città del Vaticano, 1940, pp. 330.33-363.11 and 451.15-454.16.

<sup>16</sup> On Byzantine *Rechenbücher*, see F. ACERBI, *Byzantine Rechenbücher: An Overview with an Edition of Anonymi L and J*, in *JÖB*, 69 (2019), pp. 1-57, which I use here, while adding some pieces of information, and F. ACERBI, *Struttura e concezione del vademecum computazionale Par. gr. 1670*, in *S&T*, 19 (2021), pp. 167-255.

*Arithmetica* and in a possibly lower-status tradition that surfaces in P.Mich. 620 (2<sup>nd</sup> century).<sup>17</sup> The earliest extant *Rechenbuch*, and a fully-fledged one, is the single-quire vademecum P.Math. (4<sup>th</sup> century);<sup>18</sup> the finest, but later, specimen from late antiquity is the Papyrus Akhmīn (7<sup>th</sup> century).<sup>19</sup> A passing remark by the Neoplatonic commentator Eutocius (early 6<sup>th</sup> century) attests to the existence of a treatise of logistic.<sup>20</sup> An important passage in Olympiodorus' commentary on Plato's *Gorgias* proves that arithmetical problems in daily-life guise were regarded as characteristic of logistic.<sup>21</sup> Contemporary with the Papyrus Akhmīn is the activity of Anania Širakac'i, an Armenian scholar trained in the Byzantine empire (Constantinople and Trebizond) who composed a collection of 24 arithmetical problems in daily-life guise and a set of 6 "banquet games", apparently as a part of the arithmetic "way", conceived

<sup>17</sup> This papyrus is edited in F. E. ROBBINS, *P. Mich. 620: A Series of Arithmetical Problems*, in *CPh*, 24 (1929), pp. 321-329; a technical discussion is found in K. VOGEL, *Die Algebräischen Probleme des P. Mich. 620*, in *CPh*, 25 (1930), pp. 373-375.

<sup>18</sup> This important papyrus is edited in R. S. BAGNALL, A. JONES, *Mathematics, Metrology, and Model Contracts. A Codex from Late Antique Business Education* (P.Math.), New York, 2019; see also my review in *Plekos*, 23 (2021), pp. 21-27. A typology of the contents is as follows: 3 model contracts (2 of them for loan of money); 5 metrological lists (= units of measurement of length, area, volume, or liquid capacity), 31 standard problems of geometric metrology, namely, measurement of areas (14 items) and volumes (17) of geometric figures or of objects measured as geometric figures; 6 problems of partition of common fractions into unit fractions; 3 problems of proportional partition (shipload of wheat; distribution of amount of wheat; the three θησαυροί problem, what is stored are "artabas"); 1 problem of pursuit; 1 application of the rule of three (pay); 2 problems are too fragmentary to allow any safe reconstruction.

<sup>19</sup> The Papyrus Akhmīn (edition J. BAILLET, *Le papyrus mathématique d'Akhmīn*, in *Mémoires publiés par les membres de la Mission Archéologique Française au Caire*, 9,1 [1892], pp. 1-89) contains 50 problems, sometimes very short; problems 3, 4, 10, 13, 17, 47-49 are formulated in terms of "units". A typology is as follows (cf. *ibidem*, pp. 32-33): calculation of volumes: probs. 1, 2, 5; proportional partition: 3, 4 (partition between associates), 10 (selling portions of a house), 11 (tax proportional to land surface), 47-49 (the three θησαυροί problem); iterative partition: 13, 17 (assigned ratios of a θησαυρός are successively removed); calculation of interests: 26-28, 33-37, 44-46; basic rule of three (if I give  $x$ , I take  $y$ ; what do I take if I give  $z$ ?): 41, 42; calculations with fractions: 6-9, 12, 14-16, 18-25, 29-32, 38-40, 43, 50. The problems are preceded by a table of resolution of common fractions into unit fractions.

<sup>20</sup> This is Magnes' *Logistic*, mentioned in the commentary on Archimedes' *Measurement of a circle*, at HEIBERG, *Archimedis opera omnia* [see n. 9], vol. 3, pp. 258.29-260.1. Eutocius, a pupil of Ammonius (cf. n. 10 above), was his successor as the scholarch of the Neoplatonic school of Alexandria, and the author of extant commentaries on works of Archimedes and on Apollonius' *Conics* I-IV.

<sup>21</sup> Olympiodorus was the successor of Eutocius as the scholarch of the Neoplatonic school of Alexandria; he lectured as far as AD 565; his extant works are commentaries on some of Plato's dialogues and on some of Aristotle's treatises. Olympiodorus is contrasting logistic, which is concerned with numerical matter, with arithmetic, which is concerned with numerical form: ἐπειδὴ δὲ ἐμνημονεύσαμεν ἀριθμητικῆς τε καὶ λογιστικῆς, δεῖ εἰδέναι ὅτι διαφέρουσι τῷ τὴν μὲν ἀριθμητικὴν περὶ τὰ εἶδη τῶν ἀριθμῶν καταγίνεσθαι, τὴν δὲ λογιστικὴν περὶ τὴν ὕλην. [...] ὕλη δὲ ἐστὶ τὸ πλῆθος τῶν μονάδων, οἷον ὁ πολλαπλασιασμός, ὃ ἐστὶ τετράκις δ καὶ πεντάκις ε καὶ τὰ τοιαῦτα. καὶ οὐ μόνον τοῦτο· οὕτω γὰρ ἂν εὐεπιβάτος ἦν πᾶσιν, εἴ γε καὶ οἱ μικροὶ παῖδες ἴσασιν τοὺς πολλαπλασιασμούς· ἀλλὰ καὶ γλαφυρά τινα διδάσκει, ὥς ἐπὶ τῶν ἀνδριάντων ἐν οἷς ἐπεγέγραπτο "ἔχω τὸν ἐξῆς καὶ τὸ τοῦ τρίτου τρίτον" καὶ ἐπὶ τοῦ β' "κάγω τὸν ἐξῆς καὶ τὸ τοῦ πρώτου τρίτον" καὶ ἐπὶ τοῦ γ' "ὀκτώ δ' ἐγὼ μᾶς καὶ τὸ τοῦ μέσου τρίτον"· καὶ ἐπὶ τῶν κρουσῶν τοῦ λέοντος τῶν εἰς τὴν δεξαμενὴν ἐμβαλλόντων, καὶ περὶ ἄλλων μεθόδων. The text is at in *Gorgiam* IV.8.29-49, in L. G. WESTERINK, *Olympiodori in Platonis Gorgiam commentaria*, Leipzig, 1970; a translation is found in R. JACKSON – K. LYCOS – H. TARRANT, *Olympiodorus, Commentary on Plato's Gorgias (Philosophia Antiqua, 78)*, Leiden, 1998, p. 89 (forget the explanatory footnote). Variants of the two problems mentioned by Olympiodorus are *AP* XIV, 51 and 7, 130-136 (see n. 23 below), respectively (for the former, see also ACERBI, *Byzantine Rechenbücher* [see n. 16], n. 22).

as a *Rechenbuch*, of a *Quadrivium*.<sup>22</sup> Finally, a long-standing Greek tradition of arithmetical riddles in the form of epigrams is collected in Book XIV of the *Palatine Anthology*.<sup>23</sup>

As for Byzantine high-brow literary products, the tradition of *Rechenbücher* surfaces in the manuscript Paris, Bibliothèque nationale de France, gr. 1670 (end of 12<sup>th</sup> century; *Diktyon* 51293),<sup>24</sup> a computational primer resulting from a conscious selection of texts, entrusted to an excellent copyist, and intended for conservation purposes. Par. gr. 1670 was designed to carry a complete technical record, both as regards the proposed material and on account of the possibility of a double level of use. Linguistic excellence, an inflexible formulaic rigidity, and the solutions of layout adopted in Par. gr. 1670 marked a turning point in the development of high-brow technical literature in Byzantium. As said, Easter Computi were included in *Rechenbücher* from the very outset (Par. gr. 1670 does contain a Computus): apparently, Computi were perceived as homogeneous material in point of style and insofar as they involve extensive calculations.

<sup>22</sup> Anania's arithmetical problems are translated in S. KOKIAN, *Des Anania von Shirak Arithmetische Aufgaben*, in *Zeitschrift für die österreichischen Gymnasien*, 69 (1919/20), pp. 112-117, and see also the synthesis in K. VOGEL, *Anania of Shirak (Shiragooni)*, in *Scripta Mathematica*, 3 (1935), pp. 283-284, repr. ID. *Kleinere Schriften zur Geschichte der Mathematik (Boethius, 20)*, 2 vols., Stuttgart, 1988. On Anania, see F. C. CONYBEARE, *Ananias of Shirak (A. D. 600-650 c.)*, in *BZ*, 6 (1897), pp. 572-584; R. H. HEWSEN, *Science in Seventh-Century Armenia: Ananias of Sirak*, in *Isis*, 59 (1968), pp. 32-45; J.-P. MAHÉ, *Quadrivium et cursus d'études au VII<sup>e</sup> siècle en Arménie et dans le monde byzantin d'après le «K'nnikon» d'Anania Širakac'i*, in *TM*, 10 (1987), pp. 159-206 (the first banquet game is translated on p. 196: it is of the same kind as most of those contained in Rhabdas' *Procedure*); H. PETROSYAN, *Mathematics and the Feast: The Xraxčanakans of Anania Širakac'i*, in A. ORENGO (ed), *Sciences and Learning in Armenia between Anania Širakac'i and Grigor Magistros (OCP, 86)*, Roma, 2020, pp. 107-122 [the second (type: an unknown number plus given parts of itself is equal to a given number) and the sixth banquet game are translated]. The last two items list the relevant bibliography in Armenian, editions included. A typology of Anania's arithmetical problems is as follows: an unknown number minus given parts of itself yields a given number: probs. 1-7, 9, 10, 14, 15, 17, 18, 20; pursuit: 8, 16; iterative partition: 11, 13; an unknown number is the sum of given parts of itself and of given number: 12; an unknown number minus given parts of itself and of the successive remainders yields a given number: 21; partition of number 100 according to an arithmetic progression of 10 terms, such that the initial term  $x$  is equal to the common difference: 22; rule of three: 23; filling of a tank: 24. Problem 19 is corrupt (other problems are intelligible but the translation exhibits corrupt numbers).

<sup>23</sup> A typology of the mathematical epigrams in *AP* XIV is as follows: partition with a remainder, that is, an unknown number minus given parts of itself yields a given number: epigrams 1-4, 116-127, 137, 138 (116-120, 138 on distributing nuts or apples; 126, 127 on telling the age; 126 tells the age of Diophantus); the sum of given parts of an unknown number is a given number: 50; an unknown number plus a given part of itself yields a given number: 6, 139-142 (telling the hour), 128, 129, 143 (various settings; the latter with two given parts); filling of a tank: 7, 130-136; numbers in arithmetic progression with given ratio and sum, and unknown first term: 12; two or several unknown numbers satisfying specific relations: 11, 13, 48, 49, 51, 144 (the relations are 11, 13:  $x + y = k$  and  $x/a \pm y/b = h$ ; 48:  $ax = n(a + k)$  [ $n$  arbitrary; the solution is not unique]; 49:  $x + y + z + w = k$ ,  $x + y = ck$ ,  $x + z = bk$ ,  $x + w = ck$ ; 51:  $x = y + z/3$ ,  $y = z + x/3$ ,  $z = 10 + y/3$ ; on prob. 51 see also n. 21 above; 144:  $z + w = x$ ,  $2w = x$ ,  $z = 3y$  [indeterminate]); give-take problems: 145, 146. The epigrams carrying scholia and the scholia themselves are edited together, from the manuscript Paris, Bibliothèque nationale de France, suppl. gr. 384 (early-middle 10<sup>th</sup> century; *Diktyon* 53132), in P. TANNERY, *Diophanti Alexandrini opera omnia cum Graeciis commentariis (BSGRT)*, 2 vols., Lipsiae, 1893-95, vol. 2, pp. 43-72. See also, for an assessment, P. TANNERY, *Sur les épigrammes arithmétiques de l'Anthologie palatine*, in *REG*, 7 (1894), pp. 59-62, repr. ID., *Mémoires scientifiques*, vol. 2, pp. 442-446, and P. TANNERY, *Le calcul des parties proportionnelles chez les Byzantins*, in *REG*, 7 (1894), pp. 204-208, repr. ID., *Mémoires scientifiques*, vol. 4, pp. 283-287. Only problem V.33 in Diophantus' *Arithmetica* is conceived as the solution of a riddle set out in epigram form.

<sup>24</sup> This manuscript is the only independent witness of the two *Logarikai*, the important treatises of fiscal accounting composed shortly after the death of Alexios I Komnenos in 1118; see C. E. Z. VON LINGENTHAL, *Jus Graeco-Romanum, Pars III, Novellae constitutiones*, Lipsiae, 1857, pp. 385-400 (who resorts to a tabular set-up that destroys the original layout), M. F. HENDY, *Coinage and Money in the Byzantine Empire 1091-1261 (DOS, 12)*, Washington (DC), 1969, pp. 50-64, and C. MORRISON, *La logarikè: Réforme monétaire et réforme fiscale sous Alexis I<sup>er</sup> Comnène*, in *TM*, 7 (1979), pp. 419-464, with a French translation. On this manuscript, see ACERBI, *Struttura e concezione* [see n. 16], where most of the logistic material is edited. The contents of Par. gr. 1670 are: ff. 3r-13r *Palaia Logarikè*; 13r-21v *Nea Logarikè*; 21v-34v, multiples and submultiples of currency units; 35r-46v, a detailed collection of procedures for dividing numbers  $1 \dots n$  by  $n$ , with  $n = 5 \dots 12$ , followed (44v-46v) by a list of the results of the same divisions, ranging from 5 to 20; 46v-61v, Easter Computus and other chronological material, repeatedly assuming AM 6691 [= AD 1183] as the current year; 61v, measure of a stone solid. The final part of the manuscript (ff. 62r-130v) contains geometric metrological material, edited as part of the collection called *Geometrica* in HEIBERG *et al.*, *Heronis Alexandrini opera* [see n. 9], vol. 4.

*Rechenbücher* were an integral part of a mathematical culture disseminated in the entire Mediterranean basin, as is confirmed by Fibonacci's *Liber abaci* (which we read in the 1228 redaction).<sup>25</sup> If, as is obvious, transfers of lore and techniques must have occurred,<sup>26</sup> in particular between the Latin and the Greek world, the above survey shows that the Greek *Rechenbuch* tradition is, on the whole, perfectly self-contained. However, problems in daily-life guise, which seem to follow a separate line of tradition and are accordingly absent in Par. gr. 1670, appear to enter the corpus during the Nicene period (1204-61). For the beginning of the Palaiologan period sees a flourish of *Rechenbücher*. These come either from peripheral regions, as the very rich Cypriot *Rechenbücher* that make the entire manuscript El Escorial, Real Biblioteca del Monasterio de S. Lorenzo, X.IV.5 (gr. 400; 13<sup>th</sup> century; *Diktyon* 15016) and the core ff. 69r-97v of the manuscript Città del Vaticano, Biblioteca Apostolica Vaticana, Pal. gr. 367 (1317-20; *Diktyon* 66099), or from Constantinople, as a filler of blank pages in a capital witness of Iamblichus' works, the manuscript Firenze, Biblioteca Medicea Laurenziana, Plut. 86.3 (2<sup>nd</sup> half of 13<sup>th</sup> century; *Diktyon* 16789), ff. 165r-169v, as a struck through page in a recycled bifolium of Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. gr. 191 (2<sup>nd</sup> half of 13<sup>th</sup> century; *Diktyon* 66822), f. 261r, or as substantial parts of the scholarly notebook Paris, Bibliothèque nationale de France, suppl. gr. 387 (end 13<sup>th</sup> and beginning 14<sup>th</sup> century; *Diktyon* 53135), ff. 118v-140v and 148r-161v. These two groups (namely, *Rechenbücher* composed in Cyprus and *Rechenbücher* composed in the capital) must be referred to markedly homogeneous yet different campaigns of composition of this kind of collections. It is noteworthy that the two products coming from Cyprus are accorded a far higher literary and codicological status than those elaborated in the capital.

In the first half of the 14<sup>th</sup> century, Rhabdas contributed four logistic items: a computational primer in the Greek notational system, an Easter Computus, a fully-fledged *Rechenbuch* containing a capsule Easter Computus, and the *Procedure* edited in the present study. Rhabdas' *Procedure* is a remarkable example of a (mini) *Rechenbuch* insofar as all problems it presents are riddles—the kind of εὐωχητικά “banquet games” that can be properly termed “recreational mathematics”—some of which (sects. 7, 10, and 12 in the edition below) are of the standard kind of the riddle of the ring,<sup>27</sup> yet more refined in their (partly) using non-decimal multipliers. The presence of the *Procedure* in a notebook of a later scholar

<sup>25</sup> Read the preface in B. BONCOMPAGNI, *Scritti di Leonardo Pisano. Volume I. Liber abbaci*, Roma, 1857, p. 1 = E. GIUSTI, P. D'ALESSANDRO, *Leonardi Bigolli Pisani vulgo Fibonacci, Liber Abbaci*, Firenze, 2020, ch. 1.7-8.

<sup>26</sup> A claim of dependency on Latin sources concerning the *Rechenbuch* he publishes is made but not argued in K. VOGEL, *Ein byzantinisches Rechenbuch des frühen 14. Jahrhunderts* (WBS, 6), Wien, 1968, pp. 154-160 and the table there attached. For a different assessment, see J. HØYRUP, *Fibonacci – Protagonist or Witness? Who Taught Catholic Christian Europe about Mediterranean Commercial Arithmetic?*, in *Journal of Transcultural Medieval Studies*, 1 (2014), pp. 219-247: 236-238, who sees it as more likely a partial borrowing in the opposite direction, namely, “that the Italian and Iberian way to formulate alloying problems had its roots in a Byzantine money-dealers environment” (*ibidem*, p. 238, emphasis in the original). Recall that Fibonacci claims three times that one of his problems was proposed to him by a *magister constantinopolitanus* (BONCOMPAGNI *Liber abbaci* [see n. 25], pp. 188, 190, 249 = GIUSTI, D'ALESSANDRO, *Liber Abbaci*, [see n. 25], ch. 12.174, 193, 694).

<sup>27</sup> See H. HUNGER – K. VOGEL, *Ein byzantinisches Rechenbuch des 15. Jahrhunderts* (Österreichische Akademie der Wissenschaften, Philosophisch-historische Klasse, Denkschriften, 78.2), Wien, 1963, nr. 38 = the riddle edited in F. SPINGOU, *Πῶς δεῖ εὐρίσκειν τὸ δακτύλιον*. Byzantine Game or a Problem from Fibonacci's *Liber Abaci*? *Unpublished Notes from Codex Atheniensis EBE 2429*, in *Byz*, 84 (2014), pp. 357-369; *Anonymus L*, nr. 3, and *Anonymus J*, nr. e, in ACERBI, *Byzantine Rechenbücher* [see n. 16], and the commentaries thereon (the two *Anonymi* here edited are the above-mentioned *Rechenbücher* contained in Laur. Plut. 28.3 and Vat. gr. 191, respectively).

is well in keeping with the literary status, seen in the previous paragraph, of the earliest *Rechenbücher* of Constantinopolitan origin.

#### RHABDAS' SCIENTIFIC PRODUCTION OTHER THAN THE *PROCEDURE*

Rhabdas' wrote three logistic treatises in the form of a "letter" addressed to a friend:<sup>28</sup> the *Letter to Khatzykes*, the *Letter to Tzavoukhes*, and the *Letter to Myrsiniotes*.<sup>29</sup> Their contents can be summarized as follows.

The *Letter to Khatzykes* is a primer to the elementary operations in the Greek notational system. Its contents are as follows (the pages of Tannery's edition are referred to):<sup>30</sup> signs and denominations of numbers (myriads are denoted by pairs of superposed dots), and how to represent the integers from 1 to 9,999 on the fingers of the hands (86.1-96.12); abstract descriptions of the five elementary arithmetic operations on integers, extraction of an approximate square root included (96.13-102.9); denominations of the numerical orders of magnitude and their multiplication (102.10-110.5).<sup>31</sup> A structured set of tables of addition, subtraction, multiplication, and partition is found at the end of the treatise and was apparently meant to complete it; an introduction to the partition table is also provided (114.1-17).<sup>32</sup> No arithmetic operation is carried out; no instructions of use are provided for the tables. As proved by a recent finding, in his *Letter to Khatzykes* Rhabdas silently appropriated a previous, and shorter, treatise of an anonymous author, written some decades earlier.<sup>33</sup>

The *Letter to Tzavoukhes* is a fully-fledged *Rechenbuch* and contains the following:<sup>34</sup> multiplication and division (by reduction) of unit fractions (118.1-126.29); two methods of extraction of an approximate square root, the one a refinement of the other (128.1-134.22); an Easter Computus, assuming 1341

<sup>28</sup> After Rhabdas, this format of scientific writing was adopted by Isaak Argyros, in a short geometric metrological text (the *Letter to Kolybas*) and in his Easter Computus (the *Letter to Andronikos Oinaiotes*). See the discussion in I. PÉREZ MARTÍN, *Enseignement et service impérial à l'époque paléologue : à propos de la formation des serviteurs des empereurs*, in *TM*, 25 (2021).

<sup>29</sup> Nothing else is known of Theodoros Tzavoukhes of Klazomenai (*PLP*, nr. 27609) and of Theodoros Myrsiniotes (however, two Myrsiniotes are recorded as *PLP*, nr. 92694 and 92695). George Khatzykes (*PLP*, nr. 30724) served under Andronikos II as προκαθήμενος τοῦ κοιτῶνος (1305-10) and as ἐπὶ τῶν δεήσεων (until 1325); he corresponded with Manuel Gabalas (*PLP*, nr. 3309) and, like Zarides, with Michael Gabras (*PLP*, nr. 3372).

<sup>30</sup> This work is edited in P. TANNERY, *Notice sur les deux lettres arithmétiques de Nicolas Rhabdas*, in *Notices et extraits des manuscrits de la Bibliothèque Nationale*, 32 (1886), pp. 121-252, repr. ID., *Mémoires scientifiques*, vol. 4, pp. 61-198: 86-110, 114. Vat. gr. 1411, ff. 10r-13r, is the prototype of one branch of the tradition. The other branch, in which the treatise is anonymous (title Παράδοσις σύντομος καὶ σαφειστάτη τῆς ψηφοφορικῆς ἐπιστήμης), is represented by the manuscripts Venezia, Biblioteca Nazionale Marciana, gr. Z. 323 (*Diktyon* 69794), ff. 9r-13v + (greatly enriched) tables at 25r-37v (beginning 15<sup>th</sup> century), Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. gr. 1058 (*Diktyon* 67689), ff. 84r-86v + (the same enriched) tables at 33r-40r (the copyist is the same as the Venice codex), and, preceded by the standard tables, Paris, Bibliothèque Nationale de France, Coislin 338 (2<sup>nd</sup> half of 14<sup>th</sup> century; *Diktyon* 49479), ff. 1r-3v + 3v-8v.

<sup>31</sup> The "numerical orders of magnitude" are any of the monadic numbers, tens, hundreds, thousands, myriads, and so on.

<sup>32</sup> The tables carry the title "computational (set-out): Palamedes' discovery" (ψηφοφορικόν· εὔρημα Παλαμῆδους); they were only partly edited in TANNERY, *Notice* [see n. 30], pp. 110-116. Rhabdas refers to them at the end of the section on subtraction (*ibidem*, p. 96.25-27).

<sup>33</sup> See ACERBI – D. MANOLOVA – I. PÉREZ MARTÍN, *The Source* [see n. 1].

<sup>34</sup> This work is edited in TANNERY, *Notice* [see n. 30], pp. 118-186, but two problems at the end are omitted because they were already published in R. HOCHÉ, *Nicomachi Geraseni pythagorei Introductionis Arithmeticae libri II*, Lipsiae, 1866, pp. 152.4-154.10. Vat. gr. 1411, ff. 23r-25v, while incomplete (*des.* TANNERY, *Notice* [see n. 30], p. 132.31 ἐστιν ὁ κε), is the prototype of one branch of the tradition (there are complete apographs of the Vatican manuscript, though). The other branch, likewise incomplete (*des.* TANNERY, *Notice* [see n. 30], p. 140.1-172.15 πολυπλασίασον ταῦτα), is led by the manuscript Paris, Bibliothèque Nationale de France, gr. 2107, ff. 115v-122v (1425-1448; *Diktyon* 51736).

as the current year (134.23-138.28); a so-called *Procedure of Civil Life Calculations* (Μέθοδος πολιτικῶν λογαρισμῶν), namely: an exposition of the several species of the rule of three (140.1-144.9); generalities and some problems of conversion involving weight,<sup>35</sup> measure, and currency units of measurement, solved by application of the previous rules (144.10-154.5); the same for a problem involving alloying (154.6-24); twenty *Rechenbuch*-style problems,<sup>36</sup> with solutions and associated procedures (156.25-186.19). As is the rule for *Rechenbücher*, the contents of the *Letter to Tzavoukhes* are less homogeneous than those of the *Letter to Khatzykes*.

The *Letter to Myrsiniotes* is a standard Easter Computus; it assumes 1342 as the current year.<sup>37</sup> It explains how to calculate the following items: indiction, solar, and lunar cycle years (sects. 2-5); the “base” of the Moon (sect. 6); the age of the Moon on a specific date (sect. 7); the epacts of the Moon (sect. 8); knowing its age, the visibility of the waxing and waning Moon (sect. 9); the date of Passover (sect. 10); the weekday on which Passover falls, and, consequently, the date of Easter (sect. 11); what years are leap years (sect. 12); the date of Meat-Fare Sunday (sect. 13); the duration of Apostles’ Fast (sect. 14); and, finally, a Paschalion Meat-Fare Sunday – Easter – Apostles’ Fast, in this very order and, unlike the procedure in sect. 11, without using Passover (sect. 15). Rhabdas’ Computus is purely technical; only sects. 2 and 13 contain substantial discursive sequences, namely on the meaning and the origin of indiction (an excursus that tallies with Rhabdas’ role in the Byzantine administration) the former, and on the disagreement over the date of Easter among some regional Christian churches the latter. All sections of this Computus present worked-out examples; all calculations are correct. However, the Computus contains serious conceptual mistakes, thereby suggesting that the material for which Rhabdas claims original authorship was drawn from other sources.

It is plausible that the *Letter to Myrsiniotes* is the last in the series of three mathematical letters we know Rhabdas authored. What is certain is that this *Letter*, dated to 1342, is later than the *Letter to Tzavoukhes*, which is dated to 1341; moreover, sect. 15 of the former reproduces *verbatim* the core of the brief computistical section included in the latter. The reused passage provides an algorithm that allows one to calculate the date of Easter without having to compute that of Passover before. Rhabdas insists on this point, and is proud of his finding, allegedly prompted by an exchange with a Jew who reproached Christians for not being able to compute the date of Easter without using the date of Passover. However, the same procedure, without Rhabdas’ sagacious point about the algorithm not using Passover, is found in the 1335 Computus contained in Matthew Blastares’ Σύνταγμα.<sup>38</sup> Since Blastares’ treatise is a compilation, it is likely that Rhabdas and Blastares depend on a common source.

<sup>35</sup> The metrological portion at TANNERY, *Notice* [see n. 30], pp. 144.11-146.8, is reprinted in E. SCHILBACH, *Byzantinische metrologische Quellen* (*Βυζαντινά Κείμενα και Μελέται*, 19), Θεσσαλονίκη, 1982, pp. 135-136; see also *ibidem*, pp. 30-31.

<sup>36</sup> Some of these problems coincide with problems of the first *Rechenbuch* in Par. suppl. gr. 387, edited in K. VOGEL, *Ein byzantinisches Rechenbuch des frühen 14. Jahrhunderts* (WBS, 6), Wien, 1968: nr. 13 = example at TANNERY, *Notice* [see n. 30], pp. 142.26-144.9; nr. 14 = Rhabdas’ problem I; 18 = problem III; 20 = IV; 21 = VI; 22 = VII; 9 = X; 11 = XII; 24 = XIII; 35 = XVI. Algebraic formulations of the problems in this section can be found in TANNERY, *Manuel Moschopoulos* [see n. 6], p. 14.

<sup>37</sup> As said, the Computus is autograph in the manuscript Leeds, UL, Brotherton Coll. 31/3, ff. 64r-69r; it is edited in ACERBI, *The “Third Letter”* [see n. 3], whose partition into sections I adopt. See also I. ΣΚΟΥΡΑ, *Μια ανέκδοτη επιστολή του Νικολάου Παβδά για τους εκκλησιαστικούς λογαρισμούς*, in *Νεύσις*, 27-28 (2019/20), pp. 353-399.

<sup>38</sup> G. RHALLÉS – M. POTLES, *Σύνταγμα τῶν θείων καὶ ἱερῶν κανόνων κατὰ στοιχείου*, vol. 6, Ἀθήναι, 1859, pp. 418-419.

Isaak Argyros appropriated the same idea in his *Computus* dated 1372 and claimed that it was his own discovery, which is exactly what Rhabdas had claimed thirty-one years before.<sup>39</sup>

A fourth piece of evidence about Rhabdas' scientific activities is an autograph square root table preserved in Nikephoros Gregoras' notebook Heid. Pal. gr. 129, ff. 11v-12r.<sup>40</sup> The original inscription of the table, modified by a later hand, reads: "accept, wise Gregoras, inexpressible roots from Nicholas Artabasdos Rhabdas" (πλευρὰς ἀρρήτους, Γρηγορᾶ σοφέ, δέχου ἐξ Ἀρταβάσδου τοῦ Ῥαβδᾶ Νικολάου). The square root table ranges from 1 to 120 but its second half is empty. The table, whose values are provided in the sexagesimal system and are approximated to second minutes, is not calculated according to the procedure expounded in Rhabdas' *Letter to Tzavoukhes*; nor do the values are a rounding of those, approximated to third minutes, set out in a similar table that accompanies Isaak Argyros' treatise on the extraction of a square root.<sup>41</sup> As said, Gregoras copied this table among his preliminary annotations to Ptolemy's *Handy Tables* in Marc. gr. Z. 325, f. 89v. Gregoras enriched the table, here entitled "setting-out of expressible and inexpressible square roots" (ἔκθεσις πλευρῶν ἀριθμῶν ῥητῶν καὶ ἀρρήτων τετραγωνικῶν), by noting down the sides and the areas of some regular polygons (hexagon, pentagon, decagon, equilateral triangle) inscribed in a circle of standard radius of 60 parts.

It goes without saying that Rhabdas' institutional role as a functionary of the fiscal administration fits remarkably well the contents of the *Letter to Tzavoukhes*; this role, as we have seen, shows that the epithet γεωμέτρης applied to himself in the titles of his *Letter to Khatzykes* and *Letter to Myrsiniotes* and in the title of Moschopoulos' treatise on magic squares should be translated "land-surveyor" and not "geometer".<sup>42</sup> The same role is also fitted remarkably well by the fact that Matthew, the Saint celebrated on the day Rhabdas' *Procedure* was conceived, was a tax-gatherer. Unless we think that this is a coincidence, we may safely suppose that Rhabdas also contributed to the fictional character of my introduction.

#### MATHEMATICAL CONTENTS OF THE *PROCEDURE*<sup>43</sup>

The *Procedure* confirms Rhabdas' fondness for long, descriptive titles, where he also provides details on his own biography and on his acquaintances. The date of composition of this mini *Rechenbuch* confirms that the core of Rhabdas' activities must be set to the period 1320-45. On grounds of style, I take it that all pieces of mathematics on f. 62v of Par. gr. 2381 are to be ascribed to Rhabdas, and belonged to one and the same draft. I do not see any difficulties in the fact that such a draft was only a couple of

<sup>39</sup> See the discussion in O. Schissel, *Die Osterrechnung des Nikolaos Artabasdos Rhabdas*, in *BNJ*, 14 (1938), pp. 43-59.

<sup>40</sup> See A. BIEDL, *Der Heidelberger cod. Pal. gr. 129 — die Notizensammlung eines byzantinischen Gelehrten*, in *WJA*, 3 (1948), pp. 100-106: 104-106, and I. PÉREZ MARTÍN, *El Escorialensis XL13: una fuente de los extractos elaborados por Nicéforo Gregorás en el Palat. Heidelberg. gr. 129*, in *BZ*, 86-87 (1994), pp. 20-30. See also D. BIANCONI, *La biblioteca di Cora tra Massimo Planude e Niceforo Gregora. Una questione di mani*, in *S&T*, 3 (2005), pp. 391-438: 412. The table is set out in the inner pages of an additional bifolium, which was first appended to the body of the codex and then completed with disparate texts. On f. 11r, Gregoras copied several excerpts from Plutarch; f. 12v contains a rudimentary addition table. On f. 12v we can see the reinforcement flap added along the spine, with a struck through annotation in Gregoras' hand.

<sup>41</sup> See A. ALLARD, *Le petit traité d'Isaac Argyre sur la racine carrée*, in *Centaurus*, 22 (1978), pp. 1-43: 30-32.

<sup>42</sup> With hindsight, and as Rhabdas never exhibits his "geometric" skills, the cumulative and possibly redundant epithet ἀριθμητικὸς καὶ γεωμέτρης should have led us to suspect that the second qualifier does not refer to mathematical abilities.

<sup>43</sup> Technical details will be provided in the footnotes to the translation.

pages long. None of the contents of the *Procedure*—neither the procedure proper for finding a fourth proportional, nor the problems that follow it—was reused in the fully-fledged *Rechenbuch* that makes the *Letter to Tzavoukhes*.

The interpretation of some problems is made tricky by the fact that two characters are on stage and are both addressed by “you”: these characters are the leader of the riddle as staged in real life and the fellow-player who acts as a “stooge”.<sup>44</sup> The stooge thinks of a number, which is guessed by the leader after the latter has had the stooge and, secretly, himself carry out a suitable sequence of operations on the number thought of. These operations are normally conceived in such a way that those in the final segment of the sequence offset (most of) the operations performed in the initial segment.<sup>45</sup>

Referring to the partition into sections I have introduced, the contents of the *Procedure* are as follows. Sect. 1: How to find the ratio of two numbers that are in a multiple ratio. Procedure: seek a common divisor and this is the sought ratio (of course, this is false). Sect. 2: How to find the fourth proportional of three numbers if the ratio involved is a multiple ratio; both cases of greater-to-less and of less-to-greater ratio are discussed. Procedure: multiply two suitable terms among the given ones and divide by the third (the procedure is incorrectly expounded by Rhabdas, who is forced to tinker with his ratios in order to get the correct result). Sects. 3 and 4: The same as Sect. 2, but for multiple-epimoric ratios, cases less-to-greater and greater-to-less, respectively.<sup>46</sup> From Sect. 5 on, eight *Rechenbuch*-style problems are proposed and solved; these problems can be summarized as follows.

Sect. 5. A simple problem of finding a fourth proportional (the so-called “rule of three”), solved by means of a procedure different from the one adopted in the previous sections. A total allowance of 1000 golden staters was granted to 17 men. Another 50 men each got the same allowance as any of the former. Find the total allowance of the 50 men. Rhabdas first observes that  $\frac{50}{17} = 3 - \frac{3}{51}$ , and then applies this multiplicative factor to 1000, which yields  $3000 - \frac{3000}{51} = 3000 - 58\frac{14}{17} = 2941\frac{3}{17}$ . He finally remarks that the same result can be attained by computing  $(50 \times 1000)/17$ . The algorithm that, disregarding Rhabdas’ mistakes, should be applied in this case and in sects. 2-4 is  $(a, b, c, x) \rightarrow bc \rightarrow bc/a = x$ .<sup>47</sup>

Sect. 6. A riddle. The stooge keeps equal numbers of beans in his/her hands and transfers some of them from one hand to the other; the number of beans is asked. The text is far from clear, but the idea seems to be that the number of beans in each hand can be determined once one knows the number of beans in any of them after the give-take transfer and the number of transferred beans. *Equation*.  $(a, x) \rightarrow (x + a) + (x - a) = 2x$ , where  $a$  and  $x - a$  are known. The text is also spoiled by the unfortunate circumstance that the beans which, after the transfer, remain in the less equipped hand are equal to twice

<sup>44</sup> This character is variously denoted by “the fellow-player” (ὁ συμπαίζων), “the fellow” (ὁ ὀμιλῶν), “the conversation-fellow” (ὁ προσδιαλεγόμενος).

<sup>45</sup> For this kind of problems see ACERBI, *Byzantine Rechenbücher* [see n. 16], *Anonymus L*, nr. 3, and *Anonymus J*, nr. c, e.

<sup>46</sup> The distinction between greater-to-less and less-to-greater ratios is blurred by the fact that Rhabdas freely (and necessarily, when he has committed a mistake) inverts ratios in his examples. Offhand manipulations of ratios of the same kind also characterize Manuel Bryennios’ scholia on Ptolemy’s *Almagest*: see F. ACERBI – I. PÉREZ MARTÍN, *Gli scolii autografi di Manuele Briennio nel Par. gr. 2390*, in L. DEL CORSO – F. DE VIVO – A. STRAMAGLIA (eds), *Nel segno del testo. Edizioni, materiali e studi per Oronzo Pecere (Papyrologica Florentina, 44)*, Firenze, 2015, pp. 103-143: 120-122.

<sup>47</sup> In my symbolic transcriptions, the input of an algorithm is enclosed in parentheses, its steps are linked by an arrow  $\rightarrow$ . A full stop indicates an algorithmic hiatus. The sign  $|$  separates the operations carried out by the stooge from what is communicated to the leader.

the number of transferred beans, that is, to the difference of the number of beans held by the two hands after the transfer.

Sect. 7. A riddle. A number (represented by the price of a garment) undergoes a series of operations; the leader guesses this number from the result. *Algorithm.*  $(a, x) \rightarrow 2x \rightarrow 2x + a \rightarrow 2a(2x + a) \rightarrow 2a(2x + a) - 2a^2 \mid = 4ax$ . The subsequent steps can be explained as follows. As there are  $x$  groups of  $4a$  units in the final number  $4ax$ ,  $4ax$  is made of  $x$  such groups, whence the prescription of taking 1 for each group in order to get  $x$ . This is the last problem whose solution is also worked out by means of a numerical example.

Sect. 8. A riddle. Several persons are seated around a table; the stooge secretly selects one of them and counts counterclockwise how many places away, say  $x$ , this person is seated. The stooge is also said to count clockwise an assigned number of places  $a$  from his/her own place. From the place so reached, the stooge must count clockwise a number of places equal to  $x + a$ , and s/he will get again at the persons s/he had selected. *Algorithm.*  $(a, x) \rightarrow -a + (x + a) \mid = x$ .

Sect. 9. A riddle.<sup>48</sup> A number of eggs  $x$  is put under each hen that belongs to a hen-set, purchased with 1 nomisma, of indeterminate multiplicity  $a$ ; each chicken is then sold at the same price as the purchase price of any hen; the total gain is asked. As the entire set of hens is purchased with 1 nomisma, the price of each hen is the inverse of their number. Therefore, the algorithm that represents the sequence of operations needed to solve the riddle amounts to multiplying and dividing the unknown by the same number. *Algorithm.*  $(a, x) \rightarrow ax \rightarrow (ax)/a \mid = x$ .

Sect. 10. A riddle. A simplified version of the riddle of the ring. A number (represented by the distance in sitting-place units from the stooge to the fellow who holds the ring) undergoes a series of operations; the leader guesses this number from the result. *Algorithm.*  $(x) \rightarrow 2x \rightarrow 2x + 11 \rightarrow 5(2x + 11) \rightarrow 5(2x + 11) - 55 \mid = 10x$ . The subsequent steps can be explained as in sect. 7. As there are  $x$  groups of 10 units in the final number  $10x$ ,  $10x$  is made of  $x$  such groups, whence the prescription of taking 1 for each group in order to get  $x$ .

Sect. 11. A riddle. Two numbers (represented by the prices of two garments) undergo a series of operations, and one must guess them from the result. *Algorithm.*  $(x, y) \rightarrow (x + y) \mid \rightarrow 3(x + y) \cdot (x, y) \rightarrow (4x, 3y) \rightarrow 4x + 3y \mid \rightarrow 4x + 3y - 3(x + y) = x \rightarrow (x + y) - x = y$ .

Sect. 12. A riddle. A number undergoes a series of operations, and one must guess this number from the result. *Algorithm.*  $(x) \rightarrow 3x \rightarrow 3x - 3x/2 \rightarrow 3(3x - 3x/2) \mid = 9(x/2)$ . The subsequent steps can be explained as in sects. 7 and 10. As there are  $x/2$  groups of 9 units in the final number  $9(x/2)$ ,  $9x$  is made of  $2 \times x/2 = x$  such groups, whence the prescription of taking 2 for each group in order to get  $x$ . If  $x$  is odd, dividing  $9(x/2)$  by 9 always leaves  $4 \frac{1}{2}$  as a remainder, and we must take an additional unit for this remainder in order to get  $x$ .

All in all, the *Procedure* is a poor piece of mathematics, marred by two blunders in applying a universally known, and *extremely* elementary, algorithm and by some infelicities of formulation. Conceptual

<sup>48</sup> The staging of this problem resembles that of HUNGER – VOGEL, *Ein byzantinisches Rechenbuch* [see n. 27], nr. 43.

mistakes also spoil a substantial part of Rhabdas' *Computus*, whose most characteristic algorithm he almost certainly did not conceive himself. Rhabdas sent Nikephoros Gregoras an unfinished square root table. Finally, in his *Letter to Khatzykes* Rhabdas appropriated a previous treatise. These recent findings urge us to reassess Rhabdas' scientific personality; I shall be charitable and will not pursue this reassessment further.<sup>49</sup> However, Rhabdas' *Procedure* raises interesting methodological issues. For, if on the one hand any serious historian must resist judging the texts s/he is studying and must publish any text that contributes to delineating a given socio-intellectual milieu, on the other hand it is a fact that the mistakes in the *Procedure* are so trivial that publishing this text may contribute to corroborating a negative assessment of Byzantine mathematics. My (partly) fictionalized introduction was also aimed at (partly) forestalling such a corroboration, while at the same time suggesting that a non-obvious contextualization is needed properly to assess the *Procedure*; very likely, this was an occasional composition, not intended for diffusion despite the long-winded title. The fact that the *Procedure* contains only real pieces of "recreational mathematics"—a unicum among *Rechenbücher*, as we have seen—appears to confirm this hypothesis.

#### THE MANUSCRIPT WITNESS OF THE *PROCEDURE*

The only witness of Rhabdas' *Procedure* I know of is the manuscript Paris, Bibliothèque nationale de France, gr. 2381 (*Diktyon* 52013), 295×220, 109 folios, a composite paper codex penned about 1371-73 (certainly before 1392: tables on ff. 100r and 101r; note of the main hand added on f. 104v; watermark range 1358-93).<sup>50</sup> Its composition is 1<sup>2</sup> (2, marked α' by a later hand, a1-a2), 1<sup>10</sup> (12, β', a3-a12) | 2<sup>8</sup> (28, γ'-δ', b1-b8 and c1-c8), 1<sup>10</sup> (38, ε', d1-d10), 1<sup>8</sup> (46, ζ', e1-e8) | 2<sup>8</sup> (62, ζ'-η', f1-f8 and g1-g8) | 0<sup>5</sup> (67, ιβ', h1-h5) | 0<sup>6</sup> (73, ιε', i1-i6) | 0<sup>4</sup> (77, h2-3, h7-8) | 0<sup>3</sup> (80, h9-11) | 1<sup>8</sup> (88, ιζ', i1-i8) | 0<sup>4</sup> (92) | 1<sup>8+2</sup> (102, ff. 100-101 being unwarrantedly added to a quire, ιη', m1-m10) | 0<sup>7</sup> (109, n1-n7), where 0<sup>x</sup> denotes *x* loose folios now bound together, | a junction, namely, a change of quire coinciding with a change of work; folios 89-92 are blank. The Greek quire numbers are placed in the middle of the lower margin of the verso of the last folio of a quire; the Latin quire and page markers are placed in the lower outer corner of the recto of each folio. Folios 100 and 103 are bound with recto and verso interchanged.

The contents of Par. gr. 2381 are as follows: ff. 1r-2v *notae chronologicae et metrologicae*; 3r-12v Maximus Planudes, *Psephophoria secundum Indos*; 13r-30v Barlaam, *Logistikē* I-VI;<sup>51</sup> 30v-32r

<sup>49</sup> One is reminded of the judgement Tannery passed on Rhabdas' works: "L'intérêt de ses écrits est surtout de montrer jusqu'où étaient tombés les héritiers dégénérés du nom hellène, ceux-là même qui avaient alors Diophante entre leurs mains" (TANNERY, *Manuel Moschopoulos* [see n. 6], p. 15). And Tannery had read only flawless mathematics!

<sup>50</sup> A very detailed description of this manuscript (here completed) is found in *Catalogus Codicum Astrologorum Graecorum*. 12 vols., Bruxelles, 1898-1953, vol. VIII.3 (P. BOUDREAUX), pp. 43-59. See also P. SCHREINER, *Die byzantinischen Kleinchroniken* (CFHB, 12), 3 vols., Wien, 1975-79, vol. 1, pp. 191-192; P. CABALLERO SÁNCHEZ, *El Comentario de Juan Pedíasimo a los «Cuerpos celestes» de Cleomedes* (Nueva Roma, 48), Madrid, 2018, pp. 107-110 (watermarks and identification of the main copyist). The editions of the works contained in Par. gr. 2381 are sometimes listed in clusters; to simplify the references, some of these editions are given in the final assessment of the manuscript.

<sup>51</sup> The editions of Barlaam's works contained in Par. gr. 2381 are as follows. *Logistikē* (despite the title, this is not a logistic text but a treatise of number theory in the style of Euclid, *Elements* VII-IX): CARELOS, *Βαρλαάμ* [see n. 11]; *Demonstratio* (this is an arithmetical rewriting of Euclid, *Elements* II.1-10): F. ACERBI, *Barlaam's Paraphrase of Euclid, Elements II.1-10. A Critical Edition*, submitted; *Refutatio* (it shows that the last three chapters of Ptolemy, *Harmonica* III handed down by a part

Barlaam, *Demonstratio arithmetica*; 32r-35r Barlaam, *Refutatio*; 35v-41v Gregorius Palamas, *Physica, theologica moralia et practica capita CL*;<sup>52</sup> 41v-46v Gregorius Palamas, *Pro Hesychastis Orationes duo*;<sup>53</sup> 46v *excerpta theologica*; 47r-62r Cleomedes, *Caelestia cum scholiis Pediasimi*; 55r marg. [Apollonius], *On finding two mean proportionals*;<sup>54</sup> 56r marg. Anatolius, *De generatione*; 56r marg. *nota astrologica*; 56r marg. [Melampus], *De divinatione ex naevis*;<sup>55</sup> 56v marg. *geographica et astronomica varia*; 62r Hermes Trismegistos, *De partibus hominis*; 62r *Oneirocriticon e Danielis psalmis*; 62r *excerptum e Galeni De dignotione ex insomniis*;<sup>56</sup> 62v Nicholas Rhabdas, *Methodus de arithmetica et geometrica medietatibus, et problemata arithmetica octo*; 63r-v *De Persici astrolabi usu capita XX-XXXIX*; 64r-77v Aratea, *astrologica et brontologica varia*;<sup>57</sup> 78r-79v Anonymous and Demetrius Triclinius, *De lunae schematismis*; 80r *De climatibus*;<sup>58</sup> 80v *excerpta ex Adamantii De ventis*;<sup>59</sup> 81r-85v Iohannes Pediasimos, *Geometria*; 85v-86r *notae metrologicae chronologicae astrologicae* (dated to 1371-73); 86r-88v [Aristotle], *De mundo*; 93r-96v Alexander of Aphrodisias medicus, *Quaestiones et solutiones physicae*;<sup>60</sup> 96v-99r [Philo], *De mundo*;<sup>61</sup> 99r-v [Aristotle], *De virtute*; 99v, 102r-v Theophylact Simocatta, *Dialogus de quaestionibus physicis*;<sup>62</sup> 100r-101v *tabulae (partim vacuae) et notae*

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of the manuscript tradition are inauthentic): I. DÜRING, *Die Harmonielehre des Klaudios Ptolemaios*, Göteborg, 1930, pp. 112-121; Düring's text is reprinted from J. FRANZ, *De musicis graecis commentatio*, Berolini, 1840, pp. 14-23, with sporadic corrections coming from a random sample of manuscripts, among which Par. gr. 2381; *De paschate*: A. TIHON, *Barlaam de Seminara. Traité sur la date de Pâques*, in *Byz*, 81 (2011), pp. 362-411.

<sup>52</sup> This text is edited in R. E. SINKIEWICZ, *Saint Gregory Palamas, The One Hundred and Fifty Chapters (Studies and Texts, 83)*, Toronto, 1988.

<sup>53</sup> These are parts 2 and 3 of the first Triad of Palamas' treatises edited in J. MEYENDORFF, *Grégoire Palamas. Défense des saints hésychastes (Spicilegium Sacrum Lovaniense. Études et documents, 30)*, Louvain, 1973.

<sup>54</sup> This is a method for finding two mean proportional lines of two given straight lines, witnessed in several sources and variously assigned to Hero of Alexandria or to Apollonius; see W. R. KNORR, *Textual Studies in Ancient and Medieval Geometry*, Boston – Basel – Berlin, 1989, pp. 11-28 and 41-61, and earlier, and paying attention to Byzantine authors, V. DE FALCO, *Sul problema delico*, in *Rivista Indo-Greco-Italica*, 9 (1925), pp. 41-56. The proof in Par. gr. 2381 is a concise version of Knorr's text PK, transmitted in late witnesses of Philoponus' commentary on Aristotle's *Analytica Posteriora*.

<sup>55</sup> The first of the last three texts is edited in CCAG, vol. VIII.3 [see n. 50], p. 188, the third is edited in J. FRANZ, *Scriptores physiognomoniae veteres*, Altenburgi, 1780, pp. 501-508, and now in S. COSTANZA, *Una versione bizantina e una metafrasi neogreca dello ps. Melampo De Naevis*, in *Byz*, 83 (2013), pp. 83-102.

<sup>56</sup> The first of the last three texts is edited in H. DIELS, *Beiträge zur Zuckungsliteratur des Okzidents und Orients. I. Die griechischen Zuckungsbücher (Melampus Περὶ πάλμων)*, in *Abhandlungen der königlich preussischen Akademie der Wissenschaften. Philosophisch-historische Klasse*, Berlin, 1907, Abh. IV, pp. 41-42, for the third see C. G. KÜHN, *Claudii Galeni opera omnia*, 20 vols., Lipsiae, 1821-1833, vol. 6, pp. 832-835.

<sup>57</sup> This sequence of extracts is very accurately described, and partly edited, in CCAG, vol. VIII.3 [see n. 50], pp. 47-53 and 189-191, to which I refer.

<sup>58</sup> This text is edited in J. A. CRAMER, *Anecdota Graeca e codicibus manuscriptis bibliothecae Regiae Parisiensis*, vol. 1, Oxonii, 1839, pp. 362.1-364.7 μέρεσι.

<sup>59</sup> These excerpts are edited in V. ROSE, *Anecdota Graeca et Gaecolatina*, 2 vols., Berlin, 1864-70, vol. 1, pp. 49-52.

<sup>60</sup> This is a version of the compilation of problems edited in J. L. IDELER, *Physici et medici Graeci minores*, 2 vols., Berolini, 1841-42, vol. 1, pp. 3-80.

<sup>61</sup> This work is a compendium of Philo's *De aeternitate mundi*: see F. CUMONT, *Philonis De aeternitate mundi*, Berolini, 1891, p. xxvii; L. COHN – P. WENDLAND, *Philonis Alexandrini opera quae supersunt*, 6 vols., Berolini, 1896-1915, vol. 6, pp. xxxiv-xxxvi. The edition is found in K. E. RICHTER, *Philonis Judaei opera omnia*, 8 vols., Lipsiae, 1828-30, vol. 6, pp. 148-174.

<sup>62</sup> This text is edited in IDELER, *Physici et medici* [see n. 60], vol. 1, pp. 168.1-177.16, and L. MASSA POSITANO, *Teofilatto Simocatta. Questioni naturali*, Napoli, 1965, pp. 7.1-26.15.

*variae*;<sup>63</sup> 103v-r Barlaam, *De paschate*; 104r-105v line 2 Matthew Blastares, *Computus Paschalis*;<sup>64</sup> 105v line 3-107r line 3 Michael Psellos, *Opus chronologicum* (excerpts); 107r-v *notae physiognomonicae*; 108r line 14 *notae astronomicae*; 108r line 15-v *notae theologicae*; 108v line 12 a.i. *De astris*; 109r-v *notae astronomicae*.

Par. gr. 2381 is a high-brow manuscript written for personal use by a distinguished scholar who possibly assembled the quire comprising ff. 3-12 (Planudes) to his notebook (my doubts come from the fact that our scholar did not leave traces in this quire); he also briefly collaborated with another copyist on f. 88v. As first remarked by P. Caballero Sánchez, the main copyist also penned the so-called *textus tripartitus* of the *Harmonica* of Manuel Bryennius, whose three *membra disiecta* we read in the manuscripts Paris, Bibliothèque nationale de France, gr. 2549 (*Diktyon* 52181), ff. 43r-46v and 75v-78v, Madrid, Biblioteca nacional de España 4625 (*Diktyon* 40105), ff. 2r, 68r-71v, and 122v-123v, and München, Bayerische Staatsbibliothek, Cod. graec. 489 (*Diktyon* 44935), ff. 272r-289r.<sup>65</sup>

As is to be expected, the scientific texts contained in Par. gr. 2381, possibly by the intermediation of hyparchetypes, have very important witnesses as ancestors. This is the case for the treatises of the following authors: Planudes: the ancestor of Par. gr. 2381 is the manuscript Firenze, Biblioteca Medicea Laurenziana, Ashb. 1599 (14<sup>th</sup> century; *Diktyon* 15767); Barlaam: the ancestor is the manuscript Venezia, Biblioteca Nazionale Marciana, gr. Z. 332, corrected by Barlaam himself (coll. 643; watermark range 1335-1338; *Diktyon* 69803); and Cleomedes: the ancestor is the manuscript Edinburgh, National Library of Scotland, Adv. 18.7.15 (ca. 1290; copied for the most part by Maximus Planudes; *Diktyon* 13730).<sup>66</sup> Moreover, Par. gr. 2381 is an independent witness of Triclinius' treatise; it is the only known witness of Rhabdas' short work. The presence of Barlaam's *De paschate* in Par. gr. 2381 is not recorded in the edition of Barlaam' Easter Computus (the folio is bound with recto and verso interchanged), nor is the presence of the fragment from Psellos' chronological treatise.<sup>67</sup>

<sup>63</sup> The contents of these folios are as follows: f. 100v, day (Nov. 8 and 23, Dec. 8, Jan. 6, Feb. 5, Mar. 6, Mar. 11, Jun. 2, Dec. 19 AM 6881 [= AD 1372]), hour, longitude, distance from nodes of new and full Moons; definition of the base of the Moon; notes and expense reports for trips to Rhodes and back to Constantinople, and to Cyprus; f. 100r, incomplete (only the first row of tabulated values is inserted) table of the yearly mean longitude, mean anomaly, and double elongation of the Moon, years AM 6879-6900 [= AD 1371-90]; table of the mean longitude, mean anomaly, and double elongation of the Moon, for 1 to 10, 20 to 90, 100 to 300 days, 1 year of 365 days and 1 year of 366 days (I was unable to find this and the subsequent table, referred to long. 72°, among the *Persian Tables*); astrological *thema*; f. 101r, partly incomplete table of the yearly anomaly and apogee of the Sun, years AM 6879-6900 [= AD 1371-90]; table of the anomaly and apogee of the Sun, for 1 to 10, 20 to 90, 100 to 300 days, 1 year of 365 days and 1 year of 366 days; reason for taking (0)0;59,8 as the value of the mean daily motion in longitude of the Sun, with associated tabular computations; f. 101v, (incomplete) table 2b according to the list in R. LEURQUIN, *La Tribiblos astronomique de Théodore Méliénote* (Vat. gr. 792), in *Janus*, 72 (1985), pp. 257-282: 270-276, as in the manuscript Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. gr. 210 (middle of 14<sup>th</sup> century; *Diktyon* 66841), f. 50r (only the last two columns).

<sup>64</sup> This extract coincides with RHALLS – POTLES, *Σύνταγμα* [see n. 38], pp. 404-419.8.

<sup>65</sup> See G. H. JONKER, *De textu Bryennii tripartito*, in *Mnemosyne*, 19 (1966), pp. 399-400; G. H. JONKER, *Μανουήλ Βρυεννίου Ἀρμονικά. The Harmonics of Manuel Bryennius*, Groningen, 1970, pp. 36, 37, 40, 46-47; B. MONDRAIN, *Les écritures dans les manuscrits byzantins du XIV<sup>e</sup> siècle. Quelques problématiques*, in *RSBN*, 44 (2007), pp. 157-196: 194 and n. 70, who identified a fourth limb in the composite manuscript München, Bayerische Staatsbibliothek, Cod. graec. 489 (*Diktyon* 44937), f. 246r-v and provides references to other manuscripts where the hand of the copyist of Par. gr. 2381 can be found.

<sup>66</sup> See ALLARD, *Maxime Planude* [see n. 8], pp. 12-14; ACERBI, *Barlaam's Paraphrase* [see n. 51]; R. B. TODD, *Cleomedes Caelesta (Metora) (BSGRT)*, Leipzig, 1990, p. x.

<sup>67</sup> For Pediasimos' scholia, see CABALLERO SÁNCHEZ, *El Comentario* [see n. 50], pp. 139 (stemma) and 165-166. For the Anonymous and Triclinius, see A. WASSERSTEIN, *An Unpublished Treatise by Demetrius Triclinius on Lunar Theory*, in *JÖB*, 16 (1967), pp. 153-174 and F. ACERBI, *I problemi aritmetici attribuiti a Demetrio Cidone e Isacco Argiro*, in *Estudios Bizantinos*, 5 (2017), pp. 131-206: 136 n. 16 and Testo 2. The excerpts from Psellos' chronological treatise are sects. 1-3 and 21-22 (but other material is added); see G. REDL, *La chronologie appliquée de Michel Psellos*, in *Byz*, 4 (1927-1928), pp. 197-236

Μέθοδος δι' ἧς εὐρίσκεται πᾶσα ἀριθμητικὴ καὶ γεωμετρικὴ ἀναλογία, εἴτε τοῦ διπλασίου λόγου ἐστὶν εἴτε τοῦ τριπλασίου εἴτε τοῦ πολλαπλασίου εἴτε πολλαπλασιεπιμορίου εἴτε πολλαπλασιεπιμεροῦς ἢ ἐπιδιμεροῦς καὶ ἀπλῶς οὐτινοσοῦν ἐστὶ τῶν ἀπάντων, κατὰ τὸ μέσον τῆς εὐφρόνης τῆς μνήμης τοῦ μεγάλου ἀποστόλου καὶ εὐαγγελιστοῦ Ματθαίου τοῦ ,ζωλα<sup>ου</sup>

5 ἔτους εὐρεθεῖσα καὶ ἐκτεθεῖσα παρὰ Νικολάου Σμυρναίου Ἀρταβάσδου τοῦ Ῥαβδᾶ.

<1> Ὅτε συγκρίνειν ἐθέλεις λόγον πρὸς λόγον ἢ παραβαλεῖν ἢ ἀφελεῖν λόγον ἐκ λόγου, εἴτ' ἀριθμητικῶς δρᾷν ἐθέλεις εἴτε γεωμετρικῶς, ἐκτίθει πάντας τοὺς ὅρους τῶν ζητουμένων ἀριθμῶν ἢ τῶν χωρίων, καὶ ἔκτοτε ὅρα καθ' αὐτὸν πῶς ἐστὶν ἡ τῶν λόγων ζήτησις πρὸς δύο ἀριθμούς ἢ πρὸς δ<sup>ας</sup>. καὶ ἔστω<sup>69</sup> πρὸς β· οἷον φέρε εἰπεῖν ζητῶμεν τίνα λόγον ἔχει ὁ θ πρὸς τὸν κζ. ζήτησις

10 ἐστὶν ὁ κοινὸς ἀμφοτέρων ἀριθμὸς ὁ ἐνῶν ἀμφοτέρους, δηλαδὴ διὰ τοῦ πολυπλασιασμοῦ. καὶ ἐπεὶ<sup>70</sup> ὁ γ μετρεῖ τὸν θ τρις δι' ἑαυτοῦ καὶ τὸν κζ τρις διὰ τοῦ θ, λέγομεν οὖν ὅτι τριπλάσιός ἐστὶν ὁ κζ τοῦ θ, ὁ δὲ θ τοῦ κζ ὑποτριπλάσιος.

<2> ὅταν δὲ πρὸς δ<sup>ας</sup> ἀριθμούς ἢ τοῦ λόγου γίνεται ζήτησις - οἷον φέρε εἰπεῖν ὅτι λέγω ὄν λόγον ἔχει ὁ θ πρὸς τὸν κζ, τὸν αὐτὸν ὁ σε πρὸς ἕτερόν τινα, ἢ καὶ ἀνάπαλιν ὄν λόγον ἔχει ὁ κζ

15 πρὸς τὸν θ, τὸν αὐτὸν ὁ [[ο]] φε πρὸς ἕτερόν τινα - τότε μὴ κατὰ τὴν προτέραν ἐφοδὸν ποιεῖ τὸν κοινὸν ζητῶν ἀριθμόν, ἀλλὰ κατὰ τὴν παροῦσαν. οἷον τί λέγω, εἰ μὲν τὸν ὑπόλογον ζητεῖς, πολλαπλασιάσον τὸν ἥσσαν ἀριθμόν, ἥτοι τὸν α<sup>ον</sup>, μετὰ τοῦ μείζονος, ἥτοι τοῦ γ<sup>ον</sup>, καὶ μέρισον αὐτὸν παρὰ τὸν μέσον, ἥγουν καθ' ὑπόδειξιν τὸν θ μετὰ τοῦ σε, καὶ ποιεῖ τὸν χοε, καὶ παραβαλλόμενον εἴτουν μεριζόμενον παρὰ τὸν μέσον, ἥτοι τὸν κζ, τὸν κε εὐρίσκομεν· κ<sup>ις</sup> γὰρ καὶ

20 ζ<sup>ις</sup> συντιθέμενος ὁ κε τὸν χοε πλήρη συνάγει. καὶ οὕτως μὲν εἰ κατὰ τὴν τοῦ ὑπολόγου ζήτησιν ὁ λόγος γίνεται· ὄν γὰρ λόγον εἶχεν ὁ θ πρὸς τὸν κζ, τὸν αὐτὸν εὐρίσκομεν ὁ κε πρὸς τὸν σε. εἰ δὲ κατὰ τὴν τοῦ προλόγου, οὕτως μὲν καὶ πάλιν δοκεῖ γίνεσθαι· ὁ γὰρ α<sup>ος</sup> μετρεῖται μετὰ τοῦ [[.]] τρίτου, καὶ παρὰ τοῦ μέσου πάλιν μερίζεται, πλὴν ἐν μὲν τῷ προτέρῳ ὑποδείγματι ὁ θ ἐτύγχανεν α<sup>ος</sup> καὶ ὁ κζ β<sup>ος</sup>. νῦν δὲ ὁ κζ α<sup>ος</sup> ὁ δὲ θ μέσος· καὶ ὅρα πῶς καὶ οὗτος ὁ λόγος σῶζεται. ὁ γὰρ φε κ<sup>ις</sup>

25 καὶ ζ<sup>ις</sup> μετρούμενος τὸν ,βφε συνάγει ἀριθμόν, ὃς παρὰ τὸν θ διαιρούμενος ποιεῖ τὸν σπε τριπλάσιον ὄντα τοῦ φε· ὄν γὰρ λόγον εἶχεν ὁ κζ πρὸς τὸν θ, τὸν αὐτὸν ἔχει καὶ ὁ σπε πρὸς τὸν φε. τοῦτο γοῦν ἐπὶ παντὶ διδομένῳ λόγῳ.

and G. REDL, *La chronologie appliquée de Michel Psellos (suite)*, in *Byz*, 5 (1929-30), pp. 229-286. Pediasimos' *Geometria* is published in G. FRIEDLEIN, *Die Geometrie des Pediasimos*, Programm Ansbach, 1866.

<sup>68</sup> Since the copy bristles with abbreviations, accents are normalized to the conventions presently in use. I have punctuated the text anew, following the rules I use in editing Greek and Byzantine mathematical texts. These rules are expounded in F. ACERBI, *The Logical Syntax of Greek Mathematics (Sources and Studies in the History of Mathematics and the Physical Sciences)*, Heidelberg – New York, 2021, sect. 1.4. Whenever this occurs in Par. gr. 2381, ordinals and multiplicative adverbs (κ<sup>ις</sup> = εικοσάκις) are written as numeral letters with raised endings. Again following Par. gr. 2381, doubled ordinals represent the denominator of a fraction (lines 34 and 45). Note also the two occurrences of δ<sup>ας</sup> = τέσσαρας (lines 9 and 14) and the one of δ<sup>ας</sup> = τέσσαρας (line 75).

<sup>69</sup> scripsi : legi nequit cod.

<sup>70</sup> scripsi : legi nequit cod.

30 <3> ἵνα διὰ πλειόνων εἰδῶμεν τοῦτο βασάνων, ἔστω ἔτι καὶ διὰ πολλαπλασιασιμορίου. ζητῶ τοῖνυν, καὶ λέγω ὅτι ὃν λόγον ἔχουσι τὰ ιγ πρὸς τὰ οθ – ἔχουσι γὰρ τὸν ἑξαπλασιεπιτρискаιδέκατον – τὸν αὐτὸν τὰ κε πρὸς ἕτερόν τινα. ζητῶ τοιγαροῦν μαθεῖν ποῖος οὗτός ἐστι, καὶ λέγω κατὰ τὴν προτέραν μέθοδον· πολλα<πλα>σιάζω τὸν οθ μετὰ τοῦ κε, καὶ τὸν γινόμενον μερίζω παρὰ τὸν ιγ, καὶ εὐρίσκω τὸν ρνα μετὰ ιβ τρισκαιδεκάτων τὸν αὐτὸν ἔχειν λόγον πρὸς τὰ κε ὃν τὰ οθ πρὸς τὰ ιγ· ἑξάκις γὰρ τὰ κε, ρν γίνεται· καὶ τὸ ιγ<sup>ον</sup> τῶν κε, μονὰς μία καὶ ιβ ιγ<sup>α</sup>ιγ<sup>α</sup>, ἴσα ὄντα καὶ ταῦτα τῷ προρρηθέντι ἀριθμῷ.

35 <4> ἔτι καὶ ἄλλως ζητῶ εὐρεῖν ἐπιμόριον<sup>71</sup>, ὁ<sup>72</sup> δέ ἐστιν οὗτος· ὃν λόγον ἔχουσι τὰ οβ πρὸς τὸν ιδ, τὸν αὐτὸν τὰ ,γ πρὸς ἕτερόν τινα. καὶ μετρῶ αὐθις οὐ τὸν πρῶτον μετὰ τοῦ ἔσχατον, ἀλλὰ τὸν μέσον τὸν ιδ μετὰ τοῦ ,γ, καὶ ποιεῖ δ καὶ ,β· ταῦτα δὲ παραβάλλω εἰς τὸν οβ, καὶ εὐρίσκονται ἐκ τῆς παραβολῆς φπγ γ<sup>ον</sup>. οὗτος οὖν ὁ ἀριθμὸς ἔχει τὸν λόγον ὃν εἶχε τὰ ιδ πρὸς τὸν οβ.

40 <5> ἐὰν δὲ θελήσω προσθεῖναι ἢ ἀφελεῖν ἀριθμὸν πρὸς ἀριθμὸν ἢ ἐξ ἀριθμοῦ καὶ συγκρίνειν τούτους ἢ ὁμοιωσαὶ πρὸς μείζονα, ἄλλως χρῶμαι τῇ μεθόδῳ.

οἶον λόγου χάριν ἄνδρες ιζ εἶχον σιτηρέσιον ἀπὸ τῶν τοῦ δήμου χρημάτων στατῆρας χρύσεους ,α<sup>73</sup>. εἶτα προσετέθησαν αὐτοῖς δι' ἀρετὴν<sup>74</sup> ἄλλοι ν τοῖς χιλίοις ἀνδράσιον ἔχουσιν οὕτως ὥσπερ ἐκείνοις, καὶ κριθεῖσι ὁμοίως τοῖς ιζ. τί κατὰ λόγον ἀρμόττει προστεθῆναι; πάντως  
45 ἐρεῖ τις ἐχέφρων ὅτι ,βλμα καὶ τρία ιζ<sup>α</sup>ιζ<sup>α</sup>. τοῦτο δὲ μεθόδου δεῖται πρὸς εὐρεσιν· τί οὖν χρή ποιεῖν; δεῖ πάντως ἰδεῖν τίνα λόγον ἔχουσι πρὸς τὸν ν οἱ ιζ· καὶ ἔχουσι πάντως τὸν τριπλάσιον δέοντος ἑνὸς να<sup>ον</sup>. τὸ γὰρ ὑστεροῦν ἀεὶ πρὸς τὴν ὅλην ποσότητα τοῦ τριπλασίου λόγου ἢ οὐτινοσοῦν ἄλλου λαμβάνειν δεῖ. ἀνάγκη οὖν καὶ πρὸς τὰ ,α τοιοῦτον ἀριθμὸν εὐρεθῆναι καὶ τὸν αὐτὸν ἔχειν λόγον. πῶς δὲ εὐρεθήσεται; τριπλασιασθήτω ὁ ,α, καὶ ποιεῖ τὸν ,γ· εἶτα ἄφελε τὸ τούτων να<sup>ον</sup>  
50 μέρος, ὃ ἐστὶ μέρος νη καὶ ιδ ιζ<sup>α</sup>, ὧν ἀφαιρουμένων ἐκ τῶν ,γ ὁ δηλωθεὶς πρότερον ἀριθμὸς ,βλμα πρὸς τοῖς τρὶς ἑπτακαιδεκάτοις εὐρεθήσεται. ἔστι καὶ ἄλλως δυνατὸν εὐρεθῆναι· εἰ γὰρ ν<<sup>15</sup>> λάβω τὰ ,α καὶ μερίσω παρὰ τὸν ιζ, τὸ αὐτὸ καὶ αὐθις εὐρεθήσεται ἀναντιρρήτως.

<6> Ἔστω τις κρατῶν ἐν ταῖς χερσὶν αὐτοῦ ἀνὰ κύαμους 5 τυχὸν ἢ ἄλλους ὅσουσδηποτοῦν ἴσους. εἰ οὖν εἴπησ αὐτῷ, ἔκβαλε ἀπὸ τῆς μιᾶς χειρὸς β καὶ βάλε εἰς τὴν ἑτέραν, ἔξει ἡ μὲν η ἢ δὲ  
55 ἀποβαλοῦσα δ· τότε εἰπὲ τῷ συμπαίζοντι· ἴδε πόσα ἔχεις ἐν τῇ χειρὶ τῇ μιᾷ τῇ ἀποβαλούσῃ, καὶ εὐρεθήσονται διὰ πάντως. τοσαῦτα γοῦν βαλέτω εἰς αὐτὴν ἀπὸ τῆς ἐτέρας χειρὸς τῆς δεξαμένης, ἥτοι δ, καὶ τότε εἰπὲ ὅτι κατελείφθησαν ἐν ταύτῃ τῇ χειρὶ τῇ τελευταίᾳ ἀποβαλούσῃ δ, καὶ εὐρήσεις αὐτά.

<7> Εἰ βούλει εὐρεῖν τίμημα ὃ τις ἔδωκεν εἰς τι ἱμάτιον ἢ ἕτερον σκεῦος αὐτοῦ, λέγε αὐτῷ  
60 οὕτως· ὅσα ἔδωκας διπλασιάσον, καὶ μετὰ τὸ διπλασιάσαι ταῦτα εἰπὲ αὐτῷ λαβεῖν καὶ παρά σου τὸν τυχόντα ἀριθμὸν, ἢ δ τυχὸν ἢ ε ἢ ὃν ἂν ἐκείνῳ εἴπησ, καὶ εἰπὲ αὐτῷ ἐνῶσαι τὰ πάντα. εἶτα διπλασιάσας σὺ καθ' ἑαυτὸν τὸν παρά σου δοθέντα ἀριθμὸν ἐκείνῳ κέλευσον τῷ ὁμιλοῦντι<sup>75</sup>

<sup>71</sup> lege πολλαπλασιασιμόριον

<sup>72</sup> fort. lege ὅς

<sup>73</sup> τ cod.

<sup>74</sup> δι' ἀρετὸν cod.

<sup>75</sup> ὁμελοῦντι cod.

σοι πολλαπλασιάσαι τὸν παρ' αὐτῷ κατεχόμενον πάντα ἀριθμὸν κατὰ τὸ διπλάσιον τοῦ  
δοθέντος παρὰ σου ἀριθμοῦ. εἶτα πάλιν εἰς ἑαυτὸν πολυπλασιάσας καὶ τετραγωνίσας τὸν  
65 δοθέντα παρὰ σου ἀριθμὸν κέλευσον ἐκείνῳ ἀφελεῖν ἐκ τῆς ὅλης ποσότητος τοῦ κατεχομένου  
αὐτοῦ ἀριθμοῦ δις τὸν τετράγωνον. μετὰ δὲ ταῦτα τετραπλασιάσας τὸν παρὰ σου δοθέντα κατ'  
ἐκείνον τὸν τετραπλασιασθέντα κέλευε ἀφελεῖν ἐκ τοῦ καταλειφθέντος ἀριθμοῦ· ὡσάκις δ' ἂν  
ἐφέλῃ ἐκεῖνος, τοσαυτάκις συγκράτει ἐν τῇ χειρὶ μίαν, καὶ ὅταν εἴπῃ ἄλλην μὴ ἔχειν ἀφαίρεσιν, ἴδε  
σὺ πόσας μονάδας λεληθότως κρατεῖς, καὶ εἰπὲ αὐτῷ· κατ' ἐκείνας τοσάδε ἔδωκας.

70 οἷον λόγου χάριν ἔδωκεν<sup>76</sup> δ. ταῦτ' εἰπὲ αὐτῷ διπλασιασιάσαι, καὶ μετὰ τὸν διπλασιασμὸν  
ἐπίδος καὶ σὺ ἐκείνῳ γ· καὶ γίνεται ια· καὶ κατὰ τὸν διπλασιασμὸν τοῦ γ, τουτέστι ζ,  
πολλαπλασιάσαι αὐτῷ εἰπὲ τὸν ια· καὶ γίνεται ξς· εἶτα τὸν (τετράγωνον) τὸν ἀπὸ τοῦ γ,  
τουτέστι τὸν θ, δις ἀφελεῖν εἰπὲ ἀπὸ τῶν ξς· καὶ καταλειφθήσονται μη· καὶ πάλιν  
τετραπλασιάσας τὸν γ καὶ ποιήσας ιβ λέγε ἐκείνῳ ἀφελεῖν τὸν ιβ ἀπὸ τοῦ μη· δυνατόν δὲ ἀφελεῖν  
75 δ<sup>ς</sup>. δ<sup>α</sup> ἄρα ἦσαν τὸ τίμημα τοῦ ἱματίου.

<8> Ἔστωσαν λόγου χάριν ἄνδρες ι ἐν τραπέζῃ καθήμενοι ἢ ἰστάμενοι τυχερῶς ἢ ὡς ἂν  
γίνεσθαι συμβαίνειν. εἰπὲ τῷ ἐνὶ τούτων· ἐνθυμήθητι ὃν βούλῃ ἐξ ἡμῶν, καὶ εὐρήσης αὐτόν, καὶ ὅτε  
ἐνθυμηθῇ ὁ τοιοῦτος ὃν ἂν βουληθεῖν, μέτρησον πόσος ἐστὶν ἀριθμὸς ἀπὸ σου ἕως ἐκείνου  
δεξιόθεν, καὶ κράτει τὸν ἀριθμὸν ἐν τῇ χειρὶ σου κρυφίως. μεθὸ δὲ τοῦτο γένηται, τότε λέγε καὶ ὁ  
80 ἂν ἀποβάνῃ ἐν τῷ λάχει, κράτησον σὺ, καὶ μέτρησον ἀριστερόθεν ἀπὸ σου τοὺς καθημένους, καὶ  
εἰς ὃν ἂν ἀποβῇ ὁ τοῦ λάχους ἀριθμὸς, στῇθι, καὶ εἰπὲ τῷ συμπαίζοντι· πόσωσον ὅσα πρότερον  
εἶχες καὶ ταῦτα ὅσα ἐν τῷ λάχει ἐξῆλθον, καὶ μέτρησον ἀπὸ τούτου δεξιόθεν σου, καὶ εἰς ὃν ἂν  
πληρωθῇ ὁ ἀριθμὸς, ἐκεῖνός ἐστιν ὃν ἐνεθυμήθης, καὶ εὐρήσεις αὐτόν.

<9> Εἰπὲ ἐξωνήσασθαι τῷ νομίσματι ὄρνις ὅσας βούλεται, καὶ ὑποθεῖναι ἐκάστη ὅσα  
85 βούλεται ῥά, καὶ ταῦτα νεόττια γενόμενα πωλῆσαι καθὼς ἐπρίατο τὰς ὄρνις. γίνεται οὖν τὸ  
τίμημα ἴσον τῇ ποσότητι τοῦ κελευσθέντος παρὰ σου ἀριθμοῦ τῶν ῥῶν.

<10> Εἰ βούλει γινῶναι παρὰ τίνι ἐστὶν ὁ δακτύλιος, εἰπὲ τῷ προσδιαλεγόμενῳ μετρήσαι ἀπὸ  
σου καὶ μετὰ σου ἐπὶ δεξιᾷ ἄχρι τοῦ ἔχοντος τὸν δακτύλιον, καὶ αὐτοῦ διπλασιάσαι τὸν ἀριθμὸν,  
καὶ προσθεῖναι τούτῳ ια, καὶ τὸ συναγόμενον πολλαπλασιάσαι ἀνὰ ε, καὶ ἀναγγεῖλαί σοι τὸ  
90 ὅλον, ἐξ ὧν ὕφειλε νε· ἐκ τῶν λοιπῶν λάβε ἀνὰ ι μίαν, καὶ ἀπόλυσον τὸν ἀριθμὸν ἀπὸ σου· ἔνθα  
γὰρ καταντήσῃ, ἔστιν ὁ δακτύλιος.

<11> Κέλευσόν τινι ὠνήσασθαι μολχάδιον καὶ δίμιτον, καὶ ἐνῶσαι τὰ τούτων τιμήματα, καὶ  
ἀναγγεῖλαί σοι τὸ ὅλον, ὃ δὴ καὶ τριπλασιάσον καθ' ἑαυτόν, καὶ εἰπὲ αὐτῷ τετραπλασιάσαι μὲν  
τὴν τοῦ μολχαμίου τιμὴν τριπλασιάσαι δὲ τὴν τοῦ διμίτου, καὶ ἐνῶσαι τὰ πολυπλασιασθέντα,  
95 καὶ ἀναγγεῖλαί σοι, ἐξ ὧν ὕφειλε ἄ κατέχεις, καὶ τὰ καταλειφθέντα ἐστὶ τὸ τί{μη}μημα τοῦ  
μολχαμίου, ὃ δὴ καὶ ἀφαιρῶν ἐκ τῆς ἀναγγελθείσης ποσότητος τὸ πρῶτον καὶ εὐρήσεις καὶ τοῦ  
διμίτου.

<12> Εἰπέ τινα κρατῆσαι ἀριθμὸν ὃν βούλεται, καὶ τριπλασιάσαι τοῦτον, καὶ ἀφελεῖν τὸ «,  
καὶ ἐὰν ἔχη κλάσμα, ὀρθῶσαι καὶ τοῦτο, καὶ πάλιν τριπλασιάσαι τὸ ὅλον, καὶ εἰπεῖν σοι τὴν

<sup>76</sup> ἔδωκας cod.

100 ποσότητα, ἥν καὶ ὕφειλε ἐπὶ τῶν θ, καὶ ὑπὲρ ἐκάστου ἐννεάδος λάβε δύο· εἰ δὲ καταλειφθῇ ἦττον τῶν θ ἀριθμῶν, λάβε ὑπὲρ τούτων μίαν, καὶ τὰ γινόμενα λέγε τοῦτον ἐνθυμηθῆναι.

# TRANSLATION OF THE *PROCEDURE*<sup>77</sup>

A procedure by means of which every arithmetic and geometric means can be found, either being of a double ratio or of a triple or of a multiple or of a multiple-epimoric or of a multiple-epimeric, or epidimeric,<sup>78</sup> and in general of whatever it is of all <ratios>, found and set out, in the middle of the night of the remembrance <day> of the great Apostle and Evangelist Matthew, in year 6831, by Nicholas Artabados Rhabdas from Smyrna.

<1> When you want to compare a ratio with a ratio or apply or remove a ratio from a ratio,<sup>79</sup> whether you want to do that arithmetically or geometrically, set out all terms of the sought numbers or regions, and thereafter check about it whether the search of the ratios is in two terms or in 4. Let it be in 2; for instance, granting that, let us seek what ratio 9 has to 27. Seek what is the number common to both that unites both of them, clearly by means of multiplication. And since 3 measures 9 three times by means of itself and 27 three times by means of 9, we then say that 27 is triple of 9, and 9 is subtriple of 27.<sup>80</sup>

<2> Whenever the search of the ratio comes about in 4 numbers—for instance, grant that I say what ratio 9 has to 27, the same <ratio> 75 has to some other <number>, or also inversely what ratio 27 has to 9, the same <ratio> 95 has to some other <number>—then do not do according to the previous procedure when you are seeking the common number, but according to the present one. What I mean is, if you seek the consequent, multiply the lesser number, namely, the 1<sup>st</sup>, with the greater one, namely, the 3<sup>rd</sup>, and divide it by the middle,<sup>81</sup> viz., according to the example at hand, <multiply> 9 with 75, and it makes 675, and once <this> applied or divided by the middle, namely, 27, we find 25; for 25, when composed 20 and 7 times, gathers 675 entirely. And <do> in this way if the argument comes about according to the search of the consequent; for what ratio 9 had to 27, we find that the same <ratio> 25 has to 75. If <it comes about> according to the <search> of the antecedent, it also appears to come about again in this way; for the 1<sup>st</sup> is measured with the third, and is divided again by the middle, except that in the previous example 9 happened to be the 1<sup>st</sup> and 27, the 3<sup>rd</sup>; now, on the contrary, 27 is the 1<sup>st</sup> and

<sup>77</sup> Different Greek terms are normally translated with different English terms; the translations adopted are those used in ACERBI, *Byzantine Rechenbücher* [see n. 16] and ACERBI, *Byzantine Easter Computi* [see n. 13]. Words supplied in translation are put within angular brackets <...>.

<sup>78</sup> In lowest terms, a multiple ratio is of the form  $\frac{n}{1} = n$ ; an epimoric ratio is of the form  $\frac{(n+1)}{n} = 1 + \frac{1}{n}$ ; a multiple-epimoric ratio is of the form  $k + \frac{1}{n}$ , with  $k > 1$ ; an epimeric ratio is of the form  $\frac{(n+m)}{n} = 1 + \frac{m}{n}$ , with  $1 < m < n$ ; a multiple-epimeric ratio is of the form  $k + \frac{m}{n}$ , with  $k > 1$  and  $1 < m < n$ . An epidimeric ratio in lowest terms is  $\frac{5}{3} = 1 + \frac{2}{3}$ . A “subtriple” ratio (see below) is the inverse of a triple ratio. The two terms of a ratio are called “antecedent” (πρόλογος) and “consequent” (ὑπόλογος).

<sup>79</sup> The “application” (παραβολή) and the “removal” (ἀφαίρεσις) of a ratio from a ratio correspond to the quotient of the corresponding fractions. On these notions, see F. ACERBI, *Composition and Removal of Ratios in Geometric and Logistic Texts from the Hellenistic to the Byzantine Period*, in M. SIALAROS (ed), *Revolutions and Continuity in Greek Mathematics*, Berlin, 2018, pp. 131-188.

<sup>80</sup> Of course, the fact that 3 is a common divisor of 9 and 27 has nothing to do with 27 and 9 being in a triple ratio.

<sup>81</sup> Given the assumed proportion, this is false, for what is required is to multiply 27 by 75 and to divide the result by 9. Rhabdas will cheat at the end of the argument, when he will write the proportion with one ratio inverted. Note that Rhabdas calls only the second term of a discontinuous proportion “middle”.

9 is the middle; and check how the same argument holds good. For 95, when measured<sup>82</sup> 20 and 7 times, gathers the number 2565, which, when divided by 9, makes 285, which is triple of 75; for what ratio 27 had to 9, the same <ratio> 285 also has to 95. This, then, for every given ratio.

<3> In order to understand this by means of several checks,<sup>83</sup> let it also further be by means of a multiple-epimoric <ratio>. Now then, I seek, and I say that what ratios 13 have to 79—for they have the sextuple-epithirteenth—the same <ratio> 25 has to some other <number>. Well then, I seek to learn what is this <number>, and I say according to the previous procedure: I multiply 79 by 25, and I divide the resulting <number> by 13, and I find that 151 plus  $\frac{12}{13}$  has to 25 the same ratio that 79 has to 13; for six times 25 yields 150, and  $\frac{1}{13}$  of 25, one unit and  $\frac{12}{13}$ , which are also equal to the number said above.

<4> Furthermore, I also seek to find in another way a multiple-epimoric <ratio>, and it is this: what ratio 72 have to 14, the same <ratio> 3000 has to some other <number>. And I measure anew, not the first with the last, but the middle, 14, with 3000, and it makes 42000; and I apply these to 72, and 583  $\frac{1}{3}$  are found from the application. Then, this number has <to 3000> the ratio that 14 had to 72.

<5> If I want to add or remove a number to or from a number and compare them or homogenize to a greater <number>,<sup>84</sup> I make use of the procedure in another way.

For instance, grant that, 17 men have got 1000 golden staters as an allowance from the budget of the township. Afterwards, other <staters> were added to the 1000 themselves, in favour of 50 men who, because of their services, were in the same situation as those <17 men><sup>85</sup> and were rewarded in a way analogous to the 17 <men>. What is fitting to be added in proportion? Smart people will immediately say 2941 and  $\frac{3}{17}$ . Yet, this requires a procedure of discovery. What, then, must one do? One must know in all instances what ratio 17 have to 50; and in all instances they have the triple <ratio> minus  $\frac{1}{51}$  <of it>;<sup>86</sup> for one must always take what is lacking with respect to the quantity of the triple ratio, or of whatever other.<sup>87</sup> Then, it is also necessary that such a number be also found with respect to 1000 and that it have the same ratio. How will it be found? Let 1000 be tripled, and it makes 3000; afterwards remove the 51<sup>th</sup> part of these, which part is 58 and  $\frac{14}{17}$ , which being removed from 3000, the number 2941 plus three seventeenths indicated above will be found. It is also possible that it is found in another way; for if I take 50 times 1000 and I divide <it> by 17, anew, the same will also be found incontrovertibly.

<6> Let there be someone who keeps in each of his hands 6 beans, say, or others, as many as we please, equal <in number>. Then, if you tell him “throw away 2 from one hand and throw in the other”, one <hand> will have 8 and the one that throws away <will have> 4; then tell your fellow-player: “know

<sup>82</sup> With the exception of sects. 8 and 9, in the *Procedure* the verb “to measure” (μετρέω) means “to multiply”.

<sup>83</sup> The uncommon word βάσανος (“check”) can also be found in Rhabdas’ additions to Planudes’ *Great Calculation*: see ALLARD, *Maxime Planude* [see n. 8], p. 207, and in Rhabdas’ autograph *Computus*, sect. 14, in ACERBI, *The “Third Letter”* [see n. 3].

<sup>84</sup> The meaning of “to homogenize to a greater <number>” (ὁμοιῶσαι πρὸς μείζονα) can only be a matter of speculation. Maybe it denotes the operation of raising a ratio to higher terms by making one of them equal to an assigned, greater number.

<sup>85</sup> The syntax is really awkward and the text might be slightly corrupt, apart from the obvious mistake τ for ,α.

<sup>86</sup> Again, Rhabdas is cavalier in his handling of ratios.

<sup>87</sup> This statement makes it clear that one must take  $\frac{1}{51}$  of 3, that is,  $\frac{1}{17}$ .

how many <beans> you have in the one hand that throws away, and they will be immediately found”. Then, let him throw so many (namely, 4) in that <hand> from the other hand, namely, from the receiving one, and then say that 4 were left out in that hand, namely, the one that last throws off, and you will find them.

<7> If you wish to find the price that someone paid for some garment or for another of his tools, tell him as follows: “double what you paid”, and after the doubling of these, tell him to take a random number from you too, either 4, say, or 5 or whatever you tell him, and tell him to unite all of them. Afterwards, doubling within yourself the number given by you, urge that fellow of yours to multiply the entire <number> held by him according to the double of the number given by you. Afterwards, again, multiplying within yourself and squaring the number given by you, urge him to remove twice the square from the whole quantity of the <number> held by him. After this, quadrupling the one given by you urge to remove the quadrupled <number> from the number held <by him>;<sup>88</sup> and how many times that fellow happens to remove, so many times keep a unit in your hand, and when he says that there is no other removal, know how many units you secretly keep, and tell him: you paid so-and-so, according to these <units>.

For instance, grant that, you paid 4. Tell him to double these, and after the duplication give him 3 in addition; and it yields 11; and tell him to multiply 11 according to the duplication of 3, that is, 6; and it yields 66; afterwards, tell to remove the square on 3, that is, 9, from 66; and 48 will be left out; and again, quadrupling 3 and doing 12 tell him to remove 12 from 48; and it is possible to remove <it> 4 times. Therefore the price of the garment was 4.

<8> Let there be, grant that, 10 men sitting at a table, or standing randomly or as it happens to occur. Tell one of them: “think of whomever you wish among us, and find him, and when such a fellow, whoever has been selected, has been thought of, count counterclockwise<sup>89</sup> what is the number from you up to that <fellow>, and keep the number hidden in your hand”. After this has happened, then also declare whatever has been cast by lot, keep it, and count clockwise the sitting people from you <position>, and stop at whatever fellow the number cast by lot has determined, and tell your fellow-player: “reckon what you had before and what came out by lot, and count from this <place> counterclockwise with respect to you, and at whatever fellow the number be completed, that fellow is the one you thought of, and you will find him”.

<9> Tell to buy with a nomisma how many hens one wishes, and to put under each of them as many eggs as one wishes, and sell these, once they have become chickens, <each> exactly <at the same price as> you purchased the hens.<sup>90</sup> Then the <total> price is equal to the quantity of the number of eggs urged by you.

<10> If you wish to know by whom the ring is, tell your conversation-fellow to count counterclockwise from you and after you up to the one who holds the ring, and to double his number, and to add 11

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<sup>88</sup> This removal is a division carried out by successive subtractions of the divisor.

<sup>89</sup> I have staged this problem and the one in sect. 10 around a table and accordingly replaced “towards the right” and “towards the left” with “counterclockwise” and “clockwise”, respectively.

<sup>90</sup> The meaning of Rhabdas’ sentence is clear, but the English translation needs extensive integrations.

to this, and to multiply the gathered <number> by 5, and to communicate the whole to you, from which remove 55; take one for each group of ten remainders, and discharge the number from you; for where it will arrive at, there is the ring.

<11> Urge someone to buy a coat and an overcoat, and to unite their prices, and to communicate the whole to you, which you really also triple within yourself, and tell him to quadruple the price of the coat and to triple that of the overcoat, and to unite what has been multiplied, and to communicate <the result> to you, from which remove what you hold, and what was left out is the price of the coat, also removing which, really, from the quantity communicated in the first place you shall also find <the price> of the overcoat too.

<12> Tell someone to keep the number he wishes, and to triple it, and to remove its  $\frac{1}{2}$ , and if there be a fraction, to get also rid of this,<sup>91</sup> and again to triple the whole, and to tell you the quantity, which remove by 9 too,<sup>92</sup> and take two for each group of nine; and if less than number 9 was left out, take one on behalf of them, and say that the result is that which was thought of.

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<sup>91</sup> The meaning of ὀρθῶσαι καὶ τοῦτο is not clear, but there are only two possibilities: it means either that fractions must be disregarded (this is the option I have adopted), or that they should not. In either case, the procedure is the same and gives the same result. The term Rhabdas uses for “fractions” is κλάσματα; according to the *TLG* and to the indices of VOGEL, *Ein byzantinisches Rechenbuch* [see n. 26], HUNGER – VOGEL, *Ein byzantinisches Rechenbuch* [see n. 27], and S. DESCHAUER, *Die große Arithmetik aus dem Codex Vind. phil. gr. 65. Eine anonyme Algorismusschrift aus der Endzeit des Byzantinischen Reiches. Textbeschreibung, Transkription, Teilübersetzung mit Fachsprache, Vokabular, Metrologie* (Österreichische Akademie der Wissenschaften, Philosophisch-historische Klasse, Denkschriften, 461), Wien, 2014, it is a semantic hapax.

<sup>92</sup> The operations of “removing by” (ὑφαίρειν ἐπὶ) amounts to finding the remainder of a division.