Byzantine Easter Computi: An Overview with an Edition of Anonymus 892

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ABSTRACT: The Easter Computus in Par. suppl. gr. 920, dated to year 892, is the earliest known Byzantine Easter Computus that is not embedded in a discursive framework but is a collection of bare procedures and examples. After an outline of the tradition of Easter Computi, I present four approaches to this Computus: an edition of the Greek text that preserves all linguistic features of the original; a faithful translation; a transcription of the involved algorithm in a mildly symbolic formalism; and a discursive elucidation of the same algorithm. The symbolic transcriptions will prove more useful in comparing the procedures set forth in different Computi than the algebraic formulas usually used to formalize them.

KEYWORDS: Algorithm, Byzantine Mathematics, Easter Computus, Southern Italy

INTRODUCTION

Easter Computi are a body of chronological writings whose subject-matter is the determination of the date of Easter. Most modern accounts of Easter Computi are focused on their history in Western Europe, where they were a major component of the mathematical lore available in the Early Middle Ages, and whose literature-theoretical categorization underwent an evolution that eventually made them general scientific encyclopedias[1]. After an initial period that Western and Easter Computi share (say *ca*. AD 150 to *ca*. AD 550) and that—with its controversies on the Paschal terms and its competing Easter cycles—has been thoroughly studied[2], a branching occurs in the

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[*] The present edition is an extract from a complete record of the sources (about 300 Greek manuscripts contain computistical material) and from an exhaustive discussion of them. I have collated Par. suppl. gr. 920 at the Bibliothèque nationale de France on November 28, 2019 (the manuscript is also available online: search in the website https://pinakes.irht.cnrs.fr/ by using the *Diktyon* number); I warmly thank Christian Förstel for his logistic support. I am extremely grateful to Immo Warntjes for a series of enlightening discussions, to Leofranc Holford-Strevens for a critical reading, and to the referees for the suggestions. The pieces of mathematical notation I use will be explained in a footnote on their first occurrence. With the crucial exception of the "modulo reduction", all these pieces of notation are nothing more than convenient shorthands.

[1] Byzantine Computi took a more mathematically-oriented turn, as we shall see.

[2] The reader will profit from the following studies: F. RÜHL, Chronologie des Mittelalters und der Neuzeit. Berlin 1897, 63–208; B. MAC CARTHY, Annals of Ulster, 1155–1541. IV. Introduction. Dublin 1901, XIV–CLXXVII; E. SCHWARTZ, Christliche und jüdische Ostertafeln (*Abhandlungen der königlichen Gesellschaft der Wissenschaften zu Göttingen, Philologisch-historische Klasse, n.f.,* VIII.6). Berlin 1905; CH. W. JONES, Bedae Opera de Temporibus. Cambridge (MA) 1943, 6–77; V. GRUMEL, La Chronologie (*Traité d'Études Byzantines* 1). Paris 1958, 1–128, which contains a wealth of material; A. STROBEL, Ursprung und Geschichte des frühchristlichen Osterkalenders (*Texte und Untersuchungen zur Geschichte der altchristlichen Literatur* 121). Berlin 1977; G. DECLERCQ, Anno Domini. The Origins of the Christian Era. Turnhout 2000, and G. DECLERCQ, Dionysius Exiguus and the Introduction of the Christian Era. *Sacris Erudiri* 41 (2002) 165–246, to be checked against L. HOLFORD-STREVENS, review of DECLERCQ, Anno Domini. *Peritia* (2001) 401–410; C. LEONHARD, The Jewish Pesach and the Origins of the Christian Easter (*Studia Judaica* 35). Berlin – New York 2006; L. HOLFORD-STREVENS, Paschal Lunar Calendars up to Bede. *Peritia* 20 (2008) 165–208; A. A. MOSSHAMMER, The Easter Computus and the Origins of the Christian Era. Oxford 2008, filled with detail and reconstructions that dilute the main line of the argument, but extremely useful for this very reason (for instance, the analytic refutations of other scholars' hypotheses constitute useful summaries of them); C. PH. E. NOTHAFT, Dating the Passion. The Life of Jesus and the Emergence of Scientific Chronology (200–1660) (*Time, Astronomy, and Calendars* 1). Leiden – Boston 2012, and C. PH. E. NOTHAFT, Scandalous Error. Calendar Reform and Calendrical Astronomy in Medieval Europe. Oxford 2018, which cover the period 200–1600 and which I have found informative and particularly clear; S. STERN, Calendars in Antiquity. Oxford 2012, 380–424, in a strictly socio-political perspective; I. WARNTJES, The Mechanics of Lunar Calendars and the Modes of Calculating Easter, AD 400-1100: Contexts and Perspectives, in: La conoscenza scientifica nell'Alto Medioevo (*Settimane di Studio*

sources and in scholarship: on the one side, a proliferation of Latin sources and a continuous flow of studies of them[3], enriched by remarkable findings; on the other, a desert punctuated with scattered Greek-speaking oases: the *Chronicon Paschale*, the computists of the middle-7th century, the need for a reform voiced in fourteenth-century Constantinople[4]. As for Byzantium, the recent contributions from the Louvain philological school were preceded by wide-ranging excavations in manuscripts coordinated by a nearly-forgotten yet fascinating figure of scholar, Otmar Schissel, whose work was interrupted by an untimely, Walserian death during a walk on December 28, 1943.

Apart from the different abundance of sources, the reason for such a polarization is evident at once: the intellectual freedom Western Medieval computists manifested in critically appropriating the received lore is outlandish by Byzantine standards[5]. On the other hand, it can be argued—but I shall not do that here—that the set of algorithms used by Western computists was less mathematically connotated[6], and poorer from the stylistic point of view, than the set of algorithms we may gather from Byzantine sources. Of course, these two facts are interrelated.

The main goal of the present article is to provide an edition, a translation, and a commented paraphrase of the Easter Computus I shall call *Anonymus* 892, from the year in which it was composed. I selected this Computus for the following reason: according to my survey, it is the earliest specimen of a complete Byzantine Computus (1) which is not embedded in any discursive or doctrinal frame, (2) which is shaped as a collection of bare prescriptions formulated in algorithmic style and (3) whose data are not set out in tables. The provenance of the manuscript that carries *Anonymus* 892 (Southern Italy), its material features (small size and bad parchment), the fact that the text is anonymous and exhibits an obvious vernacular tinge, show that this Computus was written for personal use. With *Anonymus* 892, and as far as our documentary record goes, Computi

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della Fondazione Centro Italiano di Studi sull'Alto Medioevo 67). Spoleto 2020, 273–310. A complete and technically informed account of Alexandrian Computi, a key character in the history of Computus, has been reached only thanks to the masterly studies of Ethiopic sources carried out in O. NEUGEBAUER, Ethiopic Easter Computus. *OC* 63 (1979) 87–102, O. NEUGEBAUER, Ethiopic Astronomy and Computus (*Sitzungsberichte der Österreichischen Akademie der Wissenschaften* 347). Wien 1979, and O. NEUGEBAUER, Abu Shaker's Chronography (*Sitzungsberichte der Österreichischen Akademie der Wissenschaften* 498). Wien 1988. An interest in technical features comparable to Neugebauer's can more recently be found in the writings of Leofranc Holford-Strevens: see B. BLACKBURN – L. HOLFORD-STREVENS, The Oxford Companion to the Year. Oxford 1999, 661–692, 708–711, 762–832, 858–875; HOLFORD-STREVENS, Paschal Lunar Calendars; the book reviews in *Peritia* 15 (2001) 401–410, 19 (2005) 359–371, and 22–23 (2011) 356–368.

[3] A state of research is found in I. WARNTJES, Introduction: State of Research on Late Antique and Early Medieval Computus, in: Late Antique Calendrical Thought and Its Reception in the Early Middle Ages, ed. I. Warntjes – D. Ó Cróinín (*Studia Traditionis Theologiae* 26). Turnhout 2017, 1–41, which also see for references to editions of some Latin texts more recent than B. KRUSCH, Studien zur christlich-mittelalterlichen Chronologie. Der 84jährige Ostercyclus und seine Quellen. Leipzig 1880, and B. KRUSCH, Studien zur christlich-mittelalterlichen Chronologie. Die Entstehung unserer heutigen Zeitrechnung. I. Victorius. Ersatz der fehlerhaften Ausgabe Mommsens in den M.G. II. Dionysius Exiguus, der Begründer der christlichen Ära (*Abhandlungen der Preußischen Akademie der Wissenschaften. Jahrgang 1937. Phil.-hist. Klasse* 8). Berlin 1938. To give an idea of the figures involved, Bede's *De Temporum Ratione* is witnessed by some 245 manuscripts, a number which far exceeds the number of Greek manuscripts that contain fully-fledged Byzantine Computi.

[4] For the *Chronicon Paschale*, see GRUMEL, La Chronologie 73–84; J. BEAUCAMP – R.-CL. BONDOUX – J. LEFORT – M.-F. ROUAN – I. SORLIN, Temps et Histoire I: Le prologue de la Chronique pascale. *TM* 7 (1979) 223–301; MOSSHAMMER, The Easter Computus 286–311; and CH. GASTGEBER, The So Called Chronicon Paschale, the Vatican Chronological Compilation with Computus, in: A Companion to Byzantine Chronicles, ed. R. Tocci. Leiden – Boston 2021, XXX–XXX. For the computists of the of the middle-7th century, see A. TIHON, Le calcul de la date de Pâques de Stéphanos-Héraclius, in: Philomathestatos. Studies in Greek and Byzantine Texts Presented to Jacques Noret for his Sixty-Fifth Birthday, ed. B. Janssens – B. Roosen – P. Van Deun (*Orientalia Lovaniensia Analecta* 137). Leuven 2004, 625–646; and J. LEMPIRE, Le calcul de la date de Pâques dans les traités de S. Maxime le Confesseur et de Georges, moine et prêtre. *Byz* 77 (2007) 267–304. For the reforms, see A. TIHON, Barlaam de Seminara. Traité Sur la date de Pâques. *Byz* 81 (2011) 362–411. These references also provide earlier bibliography.

[5] See the in-depth survey in NOTHAFT, Scandalous Error.

[6] Conversely, the tool-box of Western Computists abounded with reckoning shortcuts: *concurrentes*, several types of *regulares*, *litterae punctatae*, *numeri aurei*, *claves terminorum*, tables and calendars of all kinds.

entered Byzantine low-brow intellectual production. Thus, there are several motives of interest in publishing this Computus.

However, a bare Computus, even translated, is as next to a totally opaque piece of writing as possible. To complete my edition with adequate context, I shall outline the evolution and the main actors in the tradition of Byzantine Easter Computi; I shall also explain the structure of the tradition and the stylistic codes adopted in Computi. A computistical glossary and a thematic word index prelude to four approaches to *Anonymus* 892: these are an edition of the Greek text that preserves all linguistic features of the original; an English translation; a discursive elucidation of the algorithms employed in the text; and, embedded in the latter, a transcription of these algorithms in a mildly symbolic formalism. I am convinced that the symbolic transcriptions are the real highlight of my approach: they are more faithful to the syntactical structure and to the "mathematical content" of the original procedures than algebraic formulas can be summarizing entire procedures in one single equality, thereby erasing their operational structure. Algorithms will certainly prove more accurate than the "static" algebraic formulas, if our goal is to compare the procedures set forth in different Computi.

THE TRADITION OF EASTER COMPUTI: EVOLUTION AND MAIN ACTORS

The early history and evolution of Easter Computi can be summarized as follows. The date of Easter, the most important Christian festival, depends, in a way that underwent early variations, on the date of the Jewish festival Passover, for the Gospels relate that Jesus had his Last Supper in the evening of that day (then a Thursday)[7], died on Friday, and resurrected on Sunday. Passover corresponds to the 14th day of a schematic lunar month and must occur on or straight after the Spring equinox, whose date was fixed, as far as computistical matters are concerned, to March 21. Easter is the first Sunday after Passover; if Passover falls on Sunday, Easter is celebrated on the Sunday next thereafter. Since Passover occurs on a fixed day of a specific lunar month, its date and the date of Easter vary from year to year. The dates of all other festivals in the annual Christian calendar that depend on Easter must vary with it, which explains the reason why the Easter date must be calculated in advance. In order to determine this date, it is necessary to know—for the given year and possibly for a string of years—the date of the beginning of each lunar month, which the Jews set at first crescent visibility. This can be ascertained by observation (and thus with no advance), by means of geometric models of the motions of the Sun and of the Moon that may even be very refined but difficult to use, or by means of reasonably accurate approximations of these motions. In the latter two cases, what is computed is the instant, or the date, of the new Moon. The only viable solution for such a widespread community as Christendom was the third. The relevant approximations of the motions of the Sun and of the Moon are called "cycles": in the case of the Moon, a cycle is a time interval after which the sequence of new Moons repeats itself on the same dates. Some of these cycles were well-known in Babylonian and in Hellenistic astronomy[8]. In Computi, and throughout the present paper as well, "Passover" is therefore the 14th day of a *schematic* lunar month in a lunar cycle[9].

The first cycle that was devised for the purpose of computing the date of Easter comprises 8 calendar years, and was employed by some Hippolytus and by someone called "Computist of 243". Our sources for it are a stone chair of "Hippolytus" (it was carved in the 2th century, engraved in

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[7] The Last Supper took place the day before, according to John.

[8] Cycles are of crucial importance for their Christological import: as all Christian eras were devised in strict correlation with Easter cycles, different cycles entailed different dates for Christ's birth and for the Passion. Further constraints came from the Genesis account that the Moon came to existence as a full Moon on the fourth day of Creation, and from the numerological requirement that the Incarnation had to occur 5500 years after Creation.

[9] Since the lunar phases depend on the position of the Sun, a lunar cycle should more properly be called a "lunisolar cycle", but I shall use the shorter denomination.

AD 233–235, excavated in 1551), carrying engraved the Passover and Easter dates for an iteration of the 8-year cycle to a period of 112 years that begins in AD 222, and the text of an anonymous computist of 243—in fact the earliest computistical work—respectively[10].

This cycle soon proved to be inadequate. It was replaced in Rome by a cycle of 84 years. There are two versions of the 84-year cycle, denoted 84(12) and 84(14) according to the position of the *saltus lunae* (see below). The former is a table whose beginning is set at AD 298 and is called *Supputatio Romana*, the latter is the table called *latercus*, which was conceived by Sulpicius Severus and started in AD 354 but was used much later in the Irish and British Churches[11].

Alexandrian scholars were in a better position to exploit the resources of (Babylonian-)Hellenistic astronomy. They adapted the well-known lunar cycle of 19 years to computistical purposes, and this adaptation was sanctioned by the Alexandrian Church. Before reaching its definitive form, the 19-year cycle had to undergo some adjustments, needed to give a stable seat to specific, and controversial, Passover and Easter dates. However, major points of detail that could not be settled by simply adopting a lunar cycle had to be fixed: whether Passover was allowed to fall before the Spring equinox or not (eventually it was not: the heretics are the Protopaschites) and what date had to be retained for the equinox (eventually the date was March 21)[12]; if Passover fell on a Sunday, whether Easter was allowed to coincide with Passover or not (eventually it was not: some of the heretics were the Quartodecimans, whose Easter squarely coincided with Passover); whether Easter

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[10] Sources for Hippolytus, who also composed a chronography now lost, are Eusebius, *HE* VI.22, and the *Chronography* of Elias of Nisibis (dated 1019), in E. W. BROOKS, Eliae Metropolitae Nisibeni Opus Chronologicum (*CSCO* 63). Lipsiae 1910, 120–121; the engraved tables are transcribed in *PG* X 875–884, and in M. GUARDUCCI, La statua di «Sant'Ippolito» in Vaticano. *RPAA* 47 (1974–1975) 163–190, and M. GUARDUCCI, Epigrafia Greca. I–IV. Roma 1967–1978 IV 535–545. Guarducci showed that the statue, heavily restored as a male character in 1564–5, originally represented a female character and did not bear any inscription (see E. CASTELLI, La cattedra della Chiesa e il trono del vescovo tra II e III secolo a Roma: ricerche sul contesto storico della "statua d'Ippolito". *ASE* 27 [2010], 35–50, for un updated bibliography). The Computist of 243 is edited as Pseudo-Cyprian, *De Pascha computus*, in *PL* IV 939–972 (this work is mainly a chronography). On the 8-year cycle see SCHWARTZ, Christliche und jüdische Ostertafeln 29–40; F. K. GINZEL, Handbuch der Mathematischen und Technischen Chronologie. I–III. Leipzig 1906–1914 II 236–238; M. RICHARD, Comput et chronographie chez saint Hippolyte. *MSR* 7 (1950) 237–268; M. RICHARD, Notes sur le cycle de cent douze ans. *REB* 24 (1966) 257–277; M. RICHARD, Le comput pascal par octaétéris. *Muséon* 87 (1974) 307–333; HOLFORD-STREVENS, Paschal Lunar Calendars 167–172; MOSSHAMMER, The Easter Computus 109–129; NOTHAFT, Dating the Passion 38–56; A. A. MOSSHAMMER, Towards a new edition of the Computus of AD 243, in: Late Antique Calendrical Thought and Its Reception in the Early Middle Ages, ed. I. Warntjes – D. Ó Cróinín (*Studia Traditionis Theologiae* 26). Turnhout 2017, 43–70. A further source on the 8-year cycle is Quintus Julius Hilarianus (*ca*. 397), *Expositum de die Paschae et mensis* XIII, in *PL* XIII 1105–1114, 1113, whose short computation is also found, applied to the *ogdoas* constitutive of a 19-year cycle, in the letter of Dionysius Exiguus to Boniface and Bonus, in KRUSCH, Studien (1938) 84.20–24. The 8-year cycle was described, and shown to be inadequate, already in Geminos, *Isagoge* VIII.27–49. See also O. SCHISSEL, Neue Zeugnisse für die ΟΚΤΑΕΤΗΡΙΣ. *Hermes* 72 (1937) 317–333, for possible Byzantine traces of the 8-year cycle.

[11] On the 84-year cycles see KRUSCH, Studien (1880), who edits the then known sources; SCHWARTZ, Christliche und jüdische Ostertafeln 40–104; HOLFORD-STREVENS, Paschal Lunar Calendars 173–187; MOSSHAMMER, The Easter Computus 204–239; I. WARNTJES, The Munich Computus and the 84 (14)-year Easter reckoning. *Proceedings of the Royal Irish Academy* 107C (2007) 31–85, I. WARNTJES, The Munich Computus: Text and Translation (*Sudhoffs Archiv* 59). Stuttgart 2010, and WARNTJES, The Mechanics. The *latercus* was recovered in a Padua manuscript: D. MC CARTHY – D. Ó CRÓINÍN, The 'Lost' Irish 84-year Easter Table Rediscovered. *Peritia* 6–7 (1987–1988) 227–242 (discovery), D. MC CARTHY, Easter Principles and a Fifth-Century Lunar Cycle Used in the British Isles. *JHA* 24 (1993) 204–224 (reconstruction of the principles underlying the table), BLACKBURN –HOLFORD-STREVENS, The Oxford Companion to the Year 870–875 (overview), HOLFORD-STREVENS, Paschal Lunar Calendars 178–187 (lunar calendar). Reformed avatars of the 84(12) cycle are the *laterculus of Augustalis* (for which see A. A. MOSSHAMMER, The Computus of 455 and the *Laterculus* of Augustalis, with an Appendix on the Fractional Method of Agriustia, in: The Easter Controversy of Late Antiquity and the Early Middle Ages, ed. I. Warntjes – D. Ó Cróinín [*Studia Traditionis Theologiae* 10]. Turnhout 2011, 21–47) and the Zeitz table (AD 447), for which see TH. MOMMSEN, Chronicorum Minorum saec. IV. V. VI. VII. I–III (*Monumenta Germaniae Historica, Auctorum Antiquissimorum Tomus* IX). Berlin 1892–1998 I 501–510; B. KRUSCH, Neue Bruchstücke der Zeitzer Ostertafel vom Jahre 447. *Sitzungsberichte der Preußischen Akademie der Wissenschaften. Jahrgang 1933. Philos.-hist. Kl*. Berlin 1933, 981–997; E. OVERGAAUW – F.-J. STEVING, Die Zeitzer Ostertafel aus dem Jahre 447. Petersberg 2005.

[12] See NOTHAFT, Scandalous Error 26–34 on competing conventions in Latin West.

was allowed to fall on the day immediately after Passover or not (eventually it was: in the middle of the 5th century, this point sparked a controversy between Alexandria and Rome, which championed the losing view)[13].

Names of major characters in the early evolution of the 19-year cycle are as follows. Anatolius, bishop of Laodicea until AD 283, first devised this cycle and clearly stated the "rule of the equinox". Theophilos, patriarch of Alexandria until AD 412, constructed a 100-year table for AD 380–479. His nephew and successor patriarch of Alexandria until AD 444, Cyril, set out a recalculated 95-year table for AD 437–531 adapted to the Roman calendar[14]. Annianus (*ca*. AD 400), in the framework of a general chronography, created the Alexandrian world era by synchronizing a proleptic era Diocletian with the 19-year cycle and devised a 11×532-year table accompanied by now-lost operational rules[15]. Victorius of Aquitaine, in an attempt to solve the above-mentioned controversy, proposed in AD 457 a table of 532 years that unsuccessfully tried to reach a compromise between Roman and Alexandrian conventions[16]. Dionysius Exiguus continued in AD 525 Cyril's tables for the ensuing 95 years AD 532–626, crucially introduced the Incarnation era that grounds the *anno domini* era still in use, was decisive in establishing the view that the rules for computing Easter were fixed during the Council of Nicaea[17], and, translating Greek sources, composed an extant—and extremely successful as to diffusion—set of algorithms for computing the quantities required to construct a 19-year cycle[18]. These algorithms are called *argumenta*, which

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[13] In the terminology I shall explain in the glossary, the Alexandrian Easter limits ranged from *luna XV* to *luna XXI*, the limits endorsed by the Roman Church from *luna XVI* to *luna XXII*. In addition, the Roman Church deemed them unacceptable Easter dates after April 21, the day celebrating the foundation of the town. In the 5th century, the Roman Church still used the 84(12)-year cycle. The 84(14) cycle in the *latercus* was supplemented with limits from *luna XIV* to *luna XX* and the equinoctial term was March 25, the traditional date in Rome.

[14] The prologues to the two tables, edited already in KRUSCH, Studien (1880) 220–226 and 337–349, have been recently re-edited in A. A. MOSSHAMMER, The Prologues on Easter of Theophilus of Alexandria and [Cyril]. Oxford 2017. A version of Theophilos' prologue has been very recently recovered in the partial Alexandrian Computus I shall call *Anonymus* 487 (its structure is characteristically much looser than the structure of any Byzantine Computus; it only contains algorithms for computing the epacts and the age of the Moon) dated Diocletian 203 [= AD 486/7] and witnessed in the manuscript Milano, Biblioteca Ambrosiana, A 45 sup. (gr. 1; 12th century, Southern Italy; *Diktyon* 42172) ff. 1r–8v: see CH. GASTGEBER, Neue texte zum Computus byzantinischer Zeit im Codex Ambrosianus A 45 sup. *JÖB* 71 (2021) XXX–XXX: XXX–XXX and XXX–XXX; this manuscript also contains an adaptation of Cyril's table for the years Diocletian 191–229 [= AD 475–513] (XXX–XXX) and a Byzantine Computus (XXX–XXX) I shall call *Anonymus* 830. The significance of the 95-year time interval is explained in NEUGEBAUER, Abu Shaker's 96–97 (and see also NEUGEBAUER, Ethiopic Astronomy, *s.v.*); MOSSHAMMER, The Easter Computus 55. The 100 years of Theophilos' table are a rounding of 95.

[15] As 11×532 = 5852 and Annianus set the beginning of his era in BC 5492 March 25, the end of the eleventh cycle falls in AD 360/1. The epoch of the Alexandrian era was eventually reset to BC 5493 August 29 (the first day of the Alexandrian calendar year). This shifts back the end of the eleventh cycle to AD 359/60.

[16] Victorius is also known for his arithmetical tables: see A. M. PEDEN, Abbo of Fleury and Ramsey: Commentary on the Calculus of Victorius of Aquitaine. Oxford 2003. Despite the mathematical abilities testified to by these tables, in his Easter table Victorious got involved in a series of mistakes: see JONES, Bedae Opera 63, and MOSSHAMMER, The Easter Computus 239–244.

[17] This belief was finally reduced to its historical measure in L. DUCHESNE, La question de la Pâque au Concile de Nicée. *Revue des Questions Historiques* 28 (1880) 5–42, and again in F. DAUNOY, La question pascale au concile de Nicée. *Échos d'Orient* 24 (1925) 424–444, but see already A. DE MORGAN, On the Ecclesiastical Calendar. *Companion to the Almanac* (1845) 1–36: 6–8.

[18] Sources for the authors mentioned in this list are edited in MOMMSEN, Chronicorum Minorum I 667–756, and KRUSCH, Studien (1938). A recent overview, with discussion of previous scholarship, is found in MOSSHAMMER, The Easter Computus 130–203, 239–277, and 339–356 (on the era of Dionysius). The dates preceding Dionysius and that I have given in the era AD should more accurately be given in the era Diocletian. What KRUSCH, Studien (1938) 75–81 edits as Dionysius' *argumenta* is a reworking to be dated to AD 675: I. WARNTJES, The *Argumenta* of Dionysius Exiguus and Their Early Recensions, in: Computus and Its Cultural Context in the Latin West, AD 300–1200, ed. I. Warntjes – D. Ó Cróinín (*Studia Traditionis Theologiae* 5). Turnhout 2010, 40–111. The earliest reworking, dated AD 562, is edited in P. LEHMANN, Cassiodorstudien. *Philologus* 71 (1912) 278–299. Studies of the *argumenta* that derive from Dionysius Exiguus' rules are also found in O. NEUGEBAUER, On the Computus Paschalis of "Cassiodorus". *Centaurus* 25 (1982) 292–302, and in MOSSHAMMER, The Easter Computus 97–106.

Dionysius further qualifies as *Aegyptiorum*. Owing to the principle of marginal areas, a major witness to the early Alexandrian 19-year cycle are the Computi in Ethiopic sources; the same cycle, in Dionysius' version, was also sanctioned in Bede's *De Temporum Ratione*.

At the end of the whole process a definitive list of Passover and Easter dates[19] and a stable set of rules was arrived at, and the tradition of Eastern Computi branched off from the mainstream of Latin Computi, which culminated in Bede's masterpiece (AD 725) before entering a period of critical re-evaluation of the whole method. Accordingly, our story reaches a branching point, and takes the nearly deserted route to Byzantium. One might ask how the story should not also end, if it is true that a definitive list of Passover and Easter dates was established. Well, part of the answer will be given in the following Section. The remaining part of the answer is that first, the Byzantines adopted a world era different from the Alexandrian world era, a fact that entailed a reorganization of the lunar cycle[20]; second, computistical techniques were an integral part of scientific and notarial lore and were therefore transmitted as such[21]; and third, the mathematical notions and the algorithms for computing the quantities required to construct a 19-year cycle underwent an evolution. This evolution *is* the history of Byzantine Easter Computi, which ended with the clear perception that a reform of the cycles was necessary.

Among the earliest Greek sources of interest to our purposes figure the wheel (= table of circular format) IV in the *Chronicon Paschale*, which records the dates of Passover from AD 344 to 362[22]; an algorithm for computing the weekday of an assigned date in Paul of Alexandria, *Apotelesmatica* 19–20 (AD 378)[23]; *Anonymus* 487 mentioned above and an algorithm for computing the epacts of the Moon, dated Diocletian 239 [= AD 522/3][24]; two almost identical algorithms for finding the age of the Moon of a given date, found in Theon of Alexandria, "little commentary" on Ptolemy's *Handy Tables* 20 (AD 364)[25], and in Stephanus of Alexandria, *in Ptolemaei Tabulas Manuales* 12 (AD 617)[26]. In the latter treatise, sects. 28–30 (AD 623) were almost certainly authored by the emperor Heraclius, and contain a concise Easter Computus that will prove important in our perspective[27]. To the same period belong three fully-fledged treatises. The *Chronicon Paschale*

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[19] Maximus, *Enarratio* I.15, in *PG* XIX 1232, explicitly makes this point.

[20] On the transition from the Alexandrian era to the Byzantine era, see most recently MOSSHAMMER, The Easter Computus 278–316, with discussion of previous scholarship. See also the commentary on sect. **12** of *Anonymus* 892.

[21] However, no documents prove that this specific lore was taught in Byzantine schools or higher institutions; see also point **B** of the next Section. On scientific teaching in Byzantium, see most recently D. MANOLOVA – I. PÉREZ MARTÍN, Science Teaching and Learning Methods in Byzantium, in: A Companion to Byzantine Science, ed. S. Lazaris. Leiden – Boston 2020, 53–104. In Latin West, computistical techniques were an integral part of monastic education; see I. WARNTJES, Seventh-century Ireland: the Cradle of Medieval Science? in: Music and the Stars: Mathematics in Medieval Ireland, ed. M. Kelly – C. Doherty. Dublin 2013, 44–72.

[22] See the edition in L. DINDORF, Chronicon Paschale. I–II. Bonnae 1832 I 534. A reconstruction is put forward in GRUMEL, La Chronologie 232, and see also discussion at 77–84 (the wheel is reproduced on page 78). A different reconstruction is found in MOSSHAMMER, The Easter Computus 293–296.

[23] See the edition in E. BOER, Pauli Alexandrini elementa apotelesmatica. Lipsiae 1958, 39.17–41.16.

[24] *Anonymus* 523 is found in the manuscript Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. gr. 1291 (shortly after 811; *Diktyon* 67922) f. 47r, and in the apographs of Vat. gr. 1594 (late 9th century; *Diktyon* 68225) (most recent edition in F. ACERBI, Topographie du Vat. gr. 1594, in: La «collection philosophique» face à l'histoire. Péripéties et tradition, ed. D. Bianconi – F. Ronconi. Spoleto 2020, 239–321: 285 and n. 19). It accompanies a wheel of epacts of the Moon for the years Diocletian 30–257 [= AD 314–541]. The wheel (for the years Diocletian 30–238 [= AD 314–522]), but not the algorithm, is also found in the manuscript Leiden, Universiteitsbibliotheek, B.P.G. 78 (813–20; *Diktyon* 37735) f. 2r. This is Table C4 in the typology of A. TIHON, Πτολεμαίου Πρόχειροι Κανόνες, Les *Tables Faciles* de Ptolémée, volume 1a, Tables A1–A2 (*Publications de l'Institut Orientaliste de Louvain* 59a). Louvain-La-Neuve 2011, 61–66.

[25] See the edition in A. TIHON, Le "Petit Commentaire" de Théon d'Alexandrie aux *Tables Faciles* de Ptolémée (*StT* 282). Città del Vaticano 1978, 256.8–258.10 and 267.6–268.6.

[26] See the edition in J. LEMPIRE, Le commentaire astronomique aux *Tables Faciles* de Ptolémée attribué à Stéphanos d'Alexandrie. Tome I (*Corpus des Astronomes Byzantins* 11). Louvain-La-Neuve 2016, 154.2–158.19 and 176.1–15.

[27] See the edition in DINDORF, Chronicon Paschale II 210–219; H. USENER, De Stephano Alexandrino, in: H. USENER, Kleine Schriften III. Leipzig – Berlin 1914, 311–317. See also GRUMEL, La Chronologie 101; TIHON, Le calcul.

(AD 629/30) features a *Syntagma de Pascha* that provides sparse technical material, whereas the main body of the *Chronicon* includes a dozen of computistical calculations difficult to harmonize with each other[28]. In general, the *Chronicon* is a source of primary importance, but in our perspective it contributes little. Of the utmost importance are two complete Computi—contemporary with each other and possibly in explicit competition—that are embedded in a discursive framework and were quite obviously intended as reference works: first, the *Brevis Enarratio Christiani Paschatis* of Maximus the Confessor (AD 640/1)[29], for some time Heraclius' secretary, the champion of the Alexandrian era and our sole early source for the approach to Computus that goes under the name of πενταπλοῦντες καὶ ἑξαπλοῦντες[30]; second, the computistical portion of the treatise on heresies by George Presbyter (AD 638/9), the champion of the Byzantine era[31].

After the proto-Byzantine period and a couple of centuries marked by partial, isolated and still unpublished contributions, the Eastern Greek tradition surfaces in three different stylistic formats: tables, *paschalia*, and Computi proper, namely, a collection of rules for calculating the quantities involved in a 19-year cycle. The earliest Byzantine Easter Computus that is not embedded in a discursive and doctrinal frame[32], whose data are not set out in tables, and that is shaped as a collection of bare prescriptions formulated in algorithmic style was written at the end of the 9th century. It is dated to 892 and it is contained in the tenth-century manuscript fragment Paris, Bibliothèque nationale de France, suppl. gr. 920 (southern Italy, *Diktyon* 53604) ff. 2r–16r. I shall call it *Anonymus* 892[33]. This is the Computus which I shall edit in the present paper.

Nearly the same computistical material as we read in *Anonymus* 892 can be found, with the addition of two wheels and of their explanation, in *Anonymus* 830. This Computus is contained in folios 36r–43v of Ambr. A 45 sup., a twelfth-century collection of non-literary texts assembled in a middle-brow notebook that preserves excellent computistical material. The manuscript witnesses of both *Anonymus* 830 and *Anonymus* 892 come from Southern Italy—a further application of the principle of marginal areas—and show that a stable body of very early computistical lore was kept alive for centuries in a peripheral region of the Byzantine empire. *Anonymus* 830, however, exhibits a marked character of compilation: several algorithms are repeated in different sections of this

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[28] See the edition in DINDORF, Chronicon Paschale I 3–31 (*Syntagma de Pascha*) 368–415 (this page-range includes the computations). See also G. MERCATI, A Study of the Paschal Chronicle. *JThS* 7 (1906) 397–412; D. SERRUYS, De quelques ères usitées chez les chroniqueurs byzantins. *RPh* 31 (1907) 151–189: 158–172 O. SCHISSEL, Note sur un Catalogus Codicum Chronologorum Graecorum. *Byz* 9 (1934) 269–295: 278–285; GRUMEL, La Chronologie 73–84; BEAUCAMP *ET AL.*, Temps et Histoire; MOSSHAMMER, The Easter Computus 286–311; GASTGEBER, The So Called Chronicon Paschale.

[29] See the edition in D. PETAU, Uranologion. Lutetiae Parisiorum 1630, 313–355 = *PG* XIX 1217–1279. See also the discussion in LEMPIRE, Le calcul.

[30] For this approach, see also the commentary on sect. **8** of *Anonyms* 892.

[31] See the edition in F. DIEKAMP, Der Mönch und Presbyter Georgios, ein unbekannter Schriftsteller des 7. Jahrhunderts. *BZ* 9 (1900) 14–51. Two of the three wheels that are missing in Diekamp's reference manuscript—namely, Vat. gr. 2210 (10th century; *Diktyon* 68841), which contains an explanation of these wheels—have been recently recovered: GASTGEBER, Neue texte XXX–XXX; however, and contrary to Diekamp's assessment (50–51), I doubt that the explanation of the wheels was an integral part of George's treatise. On George's treatise, see also SCHISSEL, Neue Zeugnisse 322–323; M. RICHARD, Le traité sur les hérésies de Georges hiéromoine. *REB* 28 (1970) 239–269; LEMPIRE, Le calcul.

[32] This framework usually comprises a discussion of theological issues and of competing approaches to the determination of Easter. This is the structure of the treatises of George Presbyter and of Maximus the Confessor, of the *Syntagma de Pascha* preceding the *Chronicon Paschale*, and of the 8-book treatise—whose loss is much to be regretted—described in Photius, *Bibliotheca*, codex 116. Photius' description shows that this Computus included both a doctrinal exposition and computational procedures. See also *Bibliotheca*, codex 115. A comparison between codex 116 and Psellos' two-book treatise is carried out in G. REDL, Untersuchungen zur technischen Chronologie des Michael Psellos. *BZ* 29 (1930) 168–187: 172–176.

[33] The principle I have followed in assigning the denominations is to make the name of the author or the word *Anonymus* followed by the assumed current year. Between the two AD years corresponding to a given year in the Byzantine world era, the current year in a Computus is conventionally set to the year that includes the date of Easter; thus, the conversion shift is always 5508. As for *Anonymus* 892, the current year in sect. **7** of the treatise is AM 6396 [= AD 887/8], but I shall keep the year assumed in the rest of the Computus as its eponymous date. Of course, these different dates just show that *Anonymus* 892 is a compilation.

Computus (see the Appendix); the presence of the two wheels and of their explanation disrupts the stylistic continuity of the text. As only a handful of the sections of *Anonymus* 830 work out dated examples, it is clear that the (much later) copyist of Ambr. A 45 sup. ultimately depends on more disparate sources than the copyist of Par. suppl. gr. 920.

Two centuries later, two Computi (*Anonymus* 1079 and *Anonymus* 1092A–C)[34] drew on the same tradition as *Anonymus* 830 and *Anonymus* 892. To the same period belongs Michael Psellos' chronological primer (dated AD 1092)[35]: this is a well-conceived, well-argumented, and moderately verbose literary product organized in two books and in sections itemized as *quaestiones*; the algorithms and the long lists Psellos provides are embedded in a discursive frame. One century later, *Anonymus* 1183 was written to be included in Par. gr. 1670 (*Diktyon* 51293), a manuscript that is a computational primer resulting from a conscious selection of texts, entrusted to an excellent copyist, and intended for conservation purposes[36]. The manuscript was designed to carry a complete technical record, both as regards the proposed material and on account of the possibility of a double level of use. Linguistic excellence, an inflexible formulaic rigidity, and the solutions of layout adopted in Par. gr. 1670 marked a turning point in the development of high-brow technical literature in Byzantium.

After *Anonymus* 1183, Easter Computi abound in our manuscript sources[37]. Dozens of anonymous Computi and chronological primers[38], sometimes no more than few lines long and added in

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[34] *Anonymus* 1079 is witnessed in Par. gr. 854 (second half of 13th century; *Diktyon* 50441; on this manuscript, see Catalogus Codicum Astrologorum Graecorum. I–XII. Bruxelles 1898–1953 VIII.4 3–5; G. De Gregorio, Teodoro Prodromo e la spada di Alessio Contostefano (*Carm. Hist.* LII Hörandner). *Νέα Ῥώμη* 7 [2010] 191–295: 194–205; A.-L. Caudano, Cosmologies et cosmographies variées dans les manuscrits byzantins tardifs. *Byz* 85 [2015] 1–25: 10–12) ff. 168r–171r; for the edition, see A. Mentz, Beiträge zur Osterfestberechnung bei den Byzantinern. Dissertation Königsberg 1906, 76–100. *Anonymus* 1092A–C is witnessed in the manuscript Firenze, Biblioteca Medicea Laurenziana, Plut. 57.42 (12th century, southern Italy; *Diktyon* 16411) ff. 154ra–156rb, 156rb–161va, 161va–162vb; see the edition in F. P. Karnthaler, Die chronologischen Abhandlungen des Laurent. Gr. Plut. 57, Cod. 42. 154-162^{v}. *BNJ* 10 (1933) 1–64.

[35] Michael Psellos, Ποίημα περὶ τῆς κινήσεως τοῦ χρόνου, dated 1091/2. The tradition originates in Laur. Plut. 87.16 (end of 13th century; *Diktyon* 16833) ff. 324v–346v. The edition is G. Redl, La chronologie appliquée de Michel Psellos. *Byz* 4 (1927–1928) 197–236; 5 (1929–1930) 229–286. See also G. Redl, Studien zur technischen Chronologie des Michael Psellos. *BNJ* 7 (1930) 305–351; Redl, Untersuchungen; F. Zimmermann, review of Redl's works in *Philologische Wochenschrift* 51 (1931) 865–872. For the Florence manuscript see P. Moraux – D. Harlfinger – D. Reinsch – J. Wiesner, Aristoteles Graecus. Die griechischen Manuskripte des Aristoteles. Erster Band, Alexandrien–London. Berlin – New York 1976, 311–315.

[36] The manuscript is the only independent witness of the two *Logarikai*, the important treatises of fiscal accounting composed shortly after the death of Alexios I Komnenos in 1118; see C. E. Z. von Lingenthal, Jus Graeco-Romanum, Pars III, Novellae constitutiones. Lipsiae 1857, 385–400 (who resorts to a tabular set-up that destroys the original layout), M. F. Hendy, Coinage and Money in the Byzantine Empire 1091–1261 (*DOS* 12). Washington (DC) 1969, 50–64, and C. Morrisson, La *logarikè*: Réforme monétaire et réforme fiscale sous Alexis I^{er} Comnène. *TM* 7 (1979) 419–464, with a French translation. On this manuscript, see F. Acerbi, Struttura e concezione del vademecum computazionale Par. gr. 1670. *S&T* 19 (2021) XXX–XXX; the edition of *Anonymus* 1183 is in Appendix 5 of this study.

[37] See also O. Schissel – M. Ellend, Berechnung des Sonnen, Mond und Schaltjahrszirkels in der griechisch-christlichen Chronologie. *BZ* 42 (1942) 150–157.

[38] Published Computi of this kind include the following. *Anonymus* 1247, witnessed in Laur. Plut. 87.16, ff. 18r–20r and 21v, edited in O. Schissel, Chronologischer Traktat des XII. Jahrhunderts, in: Εἰς μνήμην Σπ. Λάμπρου. Ἀθῆναι 1935, 105–110. *Anonymus* 1273, on the basis of the manuscript Wien, Österreichischen Nationalbibliothek, phil. gr. 222 (second half of 14th century; *Diktyon* 71336; this is a copy of Laur. Plut. 87.16, ff. 324v–325v) ff. 3r–5r, edited in F. Buchegger, Wiener griechische Chronologie von 1273. *BNJ* 11 (1934–35) 25–54. *Anonymus* 1350, witnessed in Vindob. med. gr. 29 (*ca.* 1400; *Diktyon* 71054) ff. 124r–125r, edited in O. Schlachter, Wiener griechische Chronologie von 1350. Dissertation Graz 1934. *Anonymus* 1377, witnessed in Par. gr. 2509 (15th century; *Diktyon* 52141) ff. 152r–156v, edited in Petau, Uranologion 384–392 = *PG* XIX 1316–1329; see also O. Schissel, Die Osterrechnung des Nikolaos Artabasdos Rhabdas. *BNJ* 14 (1938) 43–59: 46–50. *Anonymus* 1379 or Pseudo-Andreas, 14th century, witnessed in Vat. gr. 578 (end of 14th century; *Diktyon* 67209) ff. 177r–187v, edited in Petau, Uranologion 393–395 = *PG* XIX 1329–1334. Μέθοδος ἀκριβεστάτη, 14th century, witnessed in Vat. Urb. gr. 80 (end of 14th–beginning 15th century; *Diktyon* 66547) ff. 33r–35v, edited in P. Collet, Deux textes relatifs à la fête de Pâques. Mémoire de Licence, Université Catholique de Louvain. Louvain 1969 (*non vidi*); studied and partly edited in Schissel, Neue Zeugnisse 323–333; see also O. Schissel, Niketas Seidos. Eine

blank spaces within manuscripts, prelude the authorial achievements of the 14th century: these are the Computi by Nicholas Rhabdas and Matthew Blastares[39], the latter embedded in a larger work; Nikephoros Gregoras' scattered expositions (*ca*. 1324) as well as his (self-)celebrated performance before the Emperor[40]; the monographs by Barlaam of Seminara and Isaak Argyros[41]; the computistical sections included in more comprehensive astronomical primers, such as Theodoros Meliteniotes' *Three Books on Astronomy*[42]. Gregoras and Barlaam, and Blastares drawing from the latter, discussed a possible reform; Gregoras even ventured to construct a revised Damascene-style table. They wisely concluded that, after all, they would not bother seriously to engage in such a reform: *exeunt* Byzantine critical computists[43].

Handschriftenstudie. *Divus Thomas* 15 (1937) 78–90: 89–90 (this includes a description of the chronological fragments in Αθήνα, Εθνικὴ Βιβλιοθήκη της Ελλάδος, 483; 16th century; *Diktyon* 2779); SCHISSEL – ELLEND, Berechnung 152 n. 3. On Vat. Urb. gr. 80, see P. CANART – G. PRATO, Les recueils organisés par Jean Chortasménos et le problème de ses autographes, in: Studien zum Patriarchatsregister von Konstantinopel I, ed. H. Hunger. Wien 1981, 115–178: 132–146, repr. in P. CANART, Études de paléographie et de codicologie. I–II (*StT* 450–451). Città del Vaticano 2008, 577–675: 594–608.

[39] Nicholas Rhabdas wrote a fully-fledged Computus (dated 1342), autograph in the manuscript Leeds University Library, Brotherton Coll. MS 31/3 (*Diktyon* 3761) ff. 64r–69r; it is edited in F. ACERBI, The "Third Letter" of Nicholas Rhabdas: an Autograph Easter Computus. *Estudios Bizantinos* 9 (2021); see also I. ΣΚΟΥΡΑ, Μια ανέκδοτη επιστολή του Νικολάου Ραβδά για τους εκκλησιαστικούς λογαρισμούς. *Νεύσις* 27–28 (2019–20) 353–399. One year earlier, a part of this Computus was included by Rhabdas in the so-called *Letter to Tzavoukhes*, dated 1341. The tradition of the *Letter to Tzavoukhes* originates in Vat. gr. 1411 (end of 14th–beginning of 15th century; *Diktyon* 68042) ff. 23r–25v (incomplete, *des*. 132.31 ἐστιν ὁ κε Tannery); the edition is P. TANNERY, Notice sur les deux lettres arithmétiques de Nicolas Rhabdas. *Notices et extraits des manuscrits de la Bibliothèque Nationale* 32 (1886) 121–252, repr. ID., Mémoires scientifiques IV. Toulouse – Paris 1920, 61–198: 134.23–138.28. See also O. SCHISSEL, Die Osterrechnung. Matthew Blastares included a computistical section in his Σύνταγμα, dated 1335. The earliest known witness of this treatise is the manuscript Moskva, Gosudarstvennyj Istoričeskij Muzej, Synod. gr. 149 (Vlad. 327), dated year 1342 (*non vidi*; *Diktyon* 43774); I have checked the manuscript Modena, Biblioteca Estense Universitaria, α.V.8.14 (Puntoni 190; *Diktyon* 43522) ff. 308v–316v, written before 1344; the edition is G. RHALLES – M. POTLES, Σύνταγμα τῶν θείων καὶ ἱερῶν κανόνων κατὰ στοιχεῖον. VI. Ἀθῆναι 1859, 404–425 = *PG* CXLV 65–104; see also MENTZ, Beiträge zur Osterfestberechnung 108–132.

[40] *Epist*. XX is edited in S. BEZDEKI, Nicephori Gregorae epistulae XC. *Ephemeris Dacoromana* 2 (1924) 239–377: 330–336; for *Historia Byzantina* VIII.13 see L. SCHOPEN, Nicephori Gregorae Byzantina Historia. I–II. Bonnae 1829–1830 I 364–373 = *PG* CXLVIII 548–558. Gregoras' Easter table is in *PG* XIX 1313–1316; as for manuscript evidence, this table can be found for instance in Vat. gr. 792 (*ca*. 1352; *Diktyon* 67423) f. 2r (but here beginning of the 15th century), Venezia, Biblioteca Nazionale Marciana, gr. Z. 328 (early 15th century; *Diktyon* 69799) f. 19r. The best discussion of Gregoras' contribution is TIHON, Barlaam 393–407; see also P. KUZENKOV, Correction of the Easter Computus: Heresy or Necessity? Fourteenth Century Byzantine Forerunners of the Gregorian Reform, in: Orthodoxy and Heresy in Byzantium, ed. A. Rigo – P. Ermilov. Roma 2010, 147–158.

[41] Barlaam of Seminara' short treatise is dated 1333. It is witnessed, with autorial corrections, in Marc. gr. Z. 332 (first half of 14th century; *Diktyon* 69803) ff. 67r–71v; the edition and a discussion are found in TIHON, Barlaam 362–393 and 402–411. On Marc. gr. Z. 332, see A. GIOFFREDA, Su scrittura, libri e collaboratori di Barlaam calabro. *S&T* 14 (2016) 361–378. Isaak Argyros' Computus is *De cyclis Solis et Lunae ad Andronicum*, dated 1372. This treatise is autograph in Laur. Plut. 28.13 (*Diktyon* 16194), ff. 91r–97v; see the editions in PETAU, Uranologion 359–383 = *PG* XIX 1279–1316; H. BAUFAYS, Isaac Argyre, Cycles solaire et lunaire. Comput pascal. Mémoire de licence, Université Catholique de Louvain. Louvain-La-Neuve 1981 (the editor used Marc. gr. Z. 328, ff. 1r–18v, which is in fact a recension). See also G. MERCATI, Notizie di Procoro e Demetrio Cidone, Manuele Caleca e Teodoro Meliteniota ed altri appunti per la storia della teologia e della letteratura bizantina del secolo XIV (*StT* 56). Città del Vaticano 1931, 229–236.

[42] See the edition in R. LEURQUIN, Théodore Méliténiote, Tribiblos Astronomique. Livre I; Livre II (*Corpus des Astronomes Byzantins* 4–6). Amsterdam 1990–1993 (Book III, which contains the computistical material dated 1352, is still unpublished; I have checked Meliteniotes' autograph Vat. gr. 792); see also the studies in R. LEURQUIN, La *Tribiblos astronomique* de Théodore Méliténiote (*Vat.gr*. 792). *Janus* 72 (1985) 257–282, and R. LEURQUIN, Un manuscrit autographe de la *Tribiblos astronomique* de Théodore Méliténiote: le *Vaticanus graecus* 792. *Scriptorium* 45 (1991) 145–162. There exists an anonymous version of Meliteniotes' Book III, traditionally called *Paradosis in Tabulas Persicas*; an edition is found in A. BARDI, Persische Astronomie in Byzanz. Ein Beitrag zur Byzantinistik und zur Wissenchaftsgeschichte. PhD thesis, Ludwig-Maximilians-Universität. München 2017.

[43] One finds the same attitude in Bede, *De Temporum Ratione* XLIII. The denomination "critical computists" has become a technical term to describe Western computists seeking a re-dating of the incarnation era since it was introduced in J. WIESENBACH, Sigebert von Gembloux, Liber decennalis (*Monumenta Germaniae Historica. Quellen zur Geistesgeschichte des Mittelalters* 12). Weimar 1986, 63–112.

THE TRADITION OF EASTER COMPUTI: GENERAL STRUCTURE

Sixty years ago, Alfred Cordoliani published two masterly studies, in which he established a typology of Western computistical literature, enriched by long lists of occurrences in manuscripts[44]. Cordoliani's database was huge and still very partial: on the one hand, the Western computistical tradition is incomparably richer than the parallel Byzantine tradition; on the other hand, he only sifted some libraries. Cordoliani's typology is threefold: mnemotechnic verse, tables, *argumenta*. These categories reduce to two in the case of Byzantine computistical literature, for I know of only one specimen of mnemotechnic verse: the short and incomplete 13th century text I call *Anonymus* F, in political verse[45].

As for tables, two subcategories can be identified: real tables and Paschal lists, called *paschalia*. The former comprise day-finding tables, the Damascene Easter table and Gregoras' Easter table, but also new and full Moon tables, epacts and Passover tables, tables of the motion of the Moon, *Horopodia*[46], calendars, chronological tables of any kind, some of which are firmly witnessed in the tradition of Ptolemy's *Handy Tables*. The *paschalia* list key chronological data for a specific string of years; such key data normally include the year in a suitable world era, the solar and lunar cycle years, the indiction, the dates of Passover, Easter, and Meat-Fare Sunday, the duration of the Apostles' Fast[47]. The *paschalia* may either take the form of mere lists or be organized as tables; more often, the data associated with a specific year are singled out by suitable iconic elements, usually circles. These *paschalia* may be richly illustrated; they present several motives of interest from the iconographic point of view. The *paschalia* were studied by Ferdinand Piper[48].

Following a long-standing tradition, Cordoliani calls my "algorithms" *argumenta*; their collections I call (Byzantine) "Easter Computi"[49], whose principal features are as follows.

(**A**) Easter Computi constitute themselves as a *tradition*: authors feel free to write their own compilation by drawing largely from previous compilations without bothering to mention them. From the material point of view, Easter Computi make a huge *corpus*, comparable in many respects to the geometric metrological *corpus* or to the *corpus* of *Rechenbücher*[50]. This *corpus* mainly comprises short, anonymous texts used as fillers of blank pages in manuscripts. However, as we have seen, almost all scientific personalities of the Palaiologan age—do not forget Michael Psellos well before them—set out to write on computistical matters: Nikephoros Gregoras, Matthew Blastares, Barlaam, George Chrysokokkes, Nicholas Rhabdas, Theodoros Meliteniotes, Isaak Argyros, all actively engaged in plagiarizing each other[51].

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[44] A. CORDOLIANI, Contribution à la littérature du comput ecclésiastique au moyen âge. *SM* 1 (1960) 107–137, and *SM* 2 (1961) 169–208.

[45] See SCHISSEL, Note sur un Catalogus 290–291, where the beginning of *Anonymus* F is published. This Computus is witnessed in Laur. Plut. 87.16, f. 17r–v.

[46] On *Horopodia* or "shadow-tables", see O. SCHISSEL, Antike Stundentafeln. *Hermes* 71 (1936) 104–117, and O. NEUGEBAUER, Über griechische Wetterzeichen und Schattentafeln. *Österreichischen Akademie der Wissenschaften. Philos.-hist. Kl. Sitzungsberichte* 240(2) (1962) 27–44; O. NEUGEBAUER, A History of Ancient Mathematical Astronomy (*Studies in the History of Mathematics and the Physical Sciences* 1). I–III. Berlin – Heidelberg – New York 1975, 736–748, and NEUGEBAUER, Ethiopic Astronomy 209–215.

[47] On the meaning of most of these items see the computistical glossary below.

[48] F. PIPER, Karls des Grossen Kalendarium und Ostertafel. Berlin 1858, 124–162, with a database of 27 elements.

[49] I shall understand "Byzantine" henceforth. Suitable qualifiers will be added whenever I shall refer to other traditions.

[50] For the geometric metrological *corpus*, see volumes IV and V in J. L. HEIBERG – L. NIX – W. SCHMIDT – H. SCHÖNE, Heronis Alexandrini opera quae supersunt omnia. I–V. Lipsiae 1899–1914; F. ACERBI – B. VITRAC, Héron d'Alexandrie, Metrica (*Mathematica Graeca Antiqua* 4). Pisa – Roma 2014, 429–556; for *Rechenbücher*, see just below and F. ACERBI, Byzantine *Rechenbücher*: An Overview with an Edition of *Anonymi* L and J. *JÖB* 69 (2019) 1–57. On the peculiar features of Byzantine mathematics see now F. ACERBI, Arithmetic and Logistic, Geometry and Metrology, Harmonic Theory, Optics and Mechanics, in: A Companion to Byzantine Science, ed. S. Lazaris. Leiden – Boston 2020, 105–159.

[51] Most of these authors appropriated *verbatim* elaborations of their predecessors and claimed that these were original discoveries of their own. This was a widespread practice in the Palaiologan period: F. ACERBI – D. MANOLOVA – I. PÉREZ MARTÍN, The Source of Nicholas Rhabdas' *Letter to Khatzykes*: An Anonymous Arithmetical Treatise in Vat. Barb. gr. 4.

(**B**) As far as we know, Computi were not included in the curriculum taught in Byzantine schools and higher institutions[52]. Nor have Computi ever featured in the scientific encyclopedias called *Quadrivia* or in manuscript-based wide-ranging collections of primary sources as the one assembled in years 1360–70 by Malachias in the two-volume set Par. gr. 2342 (*Diktyon* 51974) and Vat. gr. 198 (*Diktyon* 66829)[53]. The point is that "logistic", the discipline encompassing all computational sciences, was not part of the Greek heritage, with the sole, and marginal, exception of the anonymous *Prolegomena to the Almagest*, a Late Antiquity primer on the elementary arithmetical operations in the sexagesimal system, intended to assist the student of the *Almagest* and thereby transmitted, in its earliest manuscript witnesses, as a preface to it[54]. Accordingly, only the kinds of techniques expounded in the *Prolegomena* might find a place in *Quadrivia*, as a minor subsection of the discipline headed "astronomy"[55]. Consequently, our knowledge of Byzantine logistic—the highlight of Byzantine science[56]—rests on scattered specimens, on anonymous or authorial treatises such as those I have just mentioned or shall mention below or such as Planudes' *Great Calculation According to the Indians*, and, most importantly, on dedicated "counting books" assembled for conservation or for personal purposes. An example of the former category of counting books is the above-mentioned Par. gr. 1670, an example of the latter is the early fourteenth-century manuscript Par. suppl. gr. 387 (*Diktyon* 53135)[57]. Computi were often included in these counting books, along with computational primers on the decimal system, usually featuring Indo-Arabic numerals, geometric metrological collections, and *Rechenbücher*. A miniature specimen of such a counting book is Nicholas Rhabdas' *Letter to Tzavoukhes*.

(**C**) Easter Computi have "patrimonial" purposes: they often compile different (and sometimes contradictory: check *Anonymus* 892, sects. **14**–**17** and **2** *vs*. **20**) algorithms for computing one and the same chronological item. No Computus known to me outlines even a sketch of a proof that the algorithms presented as alternatives are equivalent. *Almost* no Computus known to me outlines even a sketch of a justification of the algorithms presented and of the parameters they contain[58]. Any decently complete Computus counter-checks the algorithms by performing instance computations for a specific year; this is usually asserted to be the current year. Of course, such computa-

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JÖB 68 (2018) 1–37: 35–37, and ACERBI, Arithmetic and Logistic 107–108. The relevant part of George Chrysokokkes' *Syntaxis Persica* is edited in USENER, Ad historiam astronomiae symbola, in: H. USENER, Kleine Schriften III. Leipzig – Berlin 1914, 323–371: 369–370.

[52] It is possible that counting and the elementary operations were taught in schools or in dedicated apprenticeship curricula, and we do have faint traces of this: see A. TIHON, Enseignement scientifique à Byzance. *Organon* 24 (1988) 89–108, ACERBI, Arithmetic and Logistic 109, and MANOLOVA – PÉREZ MARTÍN, Science Teaching. There is a widespread yet unjustified tendency in scholarship to regard an educational context as the prime mover for technical writings of Byzantine scholars (*contra*, see F. ACERBI – A. GIOFFREDA, Un Aristotele di Massimo Planude. *REB* 77 [2019] 203–223). I contend that the educational perspective is unsuitable for a correct assessment of the ancient Greek and Byzantine scientific output: this comprises first and foremost literary products, and it must be evaluated in this perspective. Computi are no exception.

[53] On these manuscripts see F. ACERBI, Byzantine Recensions of Greek Mathematical and Astronomical Texts: A Survey. *Estudios bizantinos* 4 (2016) 133–213: 154–160, with bibliography. On Malachias, *olim* "anonymus aristotelicus", see B. MONDRAIN, La constitution de corpus d'Aristote et de ses commentateurs aux XIII^{e} et XIV^{e} siècles. *CodMan* 29 (2000) 11–33: 19–24; B. MONDRAIN, L'ancien empereur Jean VI Cantacuzène et ses copistes, in: Gregorio Palamas e oltre. Studi e documenti sulle controversie teologiche del XIV secolo bizantino, ed. A. Rigo (*Orientalia Venetiana* 16). Firenze 2004, 249–296: 278–290 and 292; and most recently T. MARTÍNEZ MANZANO, Malaquías mónaco, *alias anonymus aristotelicus*: filosofía, ciencias y exégesis bíblica en la Constantinopla de la controversia palamita. *Aevum* 93 (2019) 495–558.

[54] See F. ACERBI – N. VINEL – B. VITRAC, Les *Prolégomènes à l'Almageste*. Une édition à partir des manuscrits les plus anciens : Introduction générale – Parties I–III. *SCIAMVS* 11 (2010) 53–210, and the forthcoming edition F. ACERBI, Les Prolégomènes à l'Almageste (*Mathematica Graeca Antiqua* 5). Pisa – Roma 2022.

[55] For instance, a short computational primer is found in sects. 1–6 and 26 of the astronomical "way" of Pachymeres' *Quadrivium*, edited in P. TANNERY, Quadrivium de Georges Pachymère (*StT* 94). Città del Vaticano 1940.

[56] See again ACERBI, Arithmetic and Logistic 116–128. I would now include Computi in my survey.

[57] A detailed analysis of both manuscripts is carried out in ACERBI, Struttura e concezione.

[58] Exceptions are for instance Maximus the Confessor, Psellos, Matthew Blastares, and Isaak Argyros (who lifts much of his material from Blastares).

tions just confirm well-known and traditionally well-established dates. In a sense, *if* its goal is calculating the date of Easter in a given year, a Computus is a *useless* piece of scientific lore, and even more so because any Computus can be replaced by such a widespread tool as a Damascene Easter table and by the simple rules for converting years of the current era to lunar and solar cycle years, and for using the table[59]. On the other hand, this fact shows that Computi were conceived of as general chronological primers[60], confirms the formidable inertia of any scientific sub-genre constituting itself as a tradition, and in the long run had at least the merit of making discrepancies between schematic and actual lunar phases patent.

(**D**) Byzantine mathematics is "sectional" in its essence[61]: it mainly comprises works that do not display a tight deductive structure; consequently, these works can easily be—or actually are—subdivided into independent sections, or can easily be assembled to generate sectional texts: of this kind are logistic and geometric metrological writings, primers of any kind (including the primers on special astronomical "texts" like the *Handy Tables* and the *Persian Tables*)[62], scholia, isagogic compilations, compendia like the *Quadrivia*. Even such complex architectures as Metochites' *Abridged Astronomical Elements* and Meliteniotes' *Three Books on Astronomy* are sectional writings; a notable exception is Barlaam's *Logistic*[63]. An extreme example of sectional mathematics are the above-mentioned *Rechenbücher*: these are collections of computational techniques and of arithmetical or geometric metrological problems unrelated to each other, sometimes in (fictitious) daily-life guise, sometimes organized in sequences of almost identical items, and often formulated in a debased algorithmic code. Mid-way between the extreme of the *Rechenbücher* and more structured sectional texts lie Easter Computi: a fully-fledged Computus is made of a sequence of self-contained sections, each of which presents one or, less frequently, more algorithms for computing a specific chronological item. These items include: length and subdivisions of the solar year; reduction of years in a given era to indiction, lunar, and solar cycle years; epacts of the Moon and of the Sun; weekday of an assigned date; date and weekday of Passover, date of Easter; date of Meat-Fare Sunday; duration of the Apostles' Fast; age of the Moon at an assigned date in the year; embolismic months and years. Further chronological items can be computed. The sections of most Computi are usually very short, and contain only the algorithm and the examples.

(**E**) Easter Computi can be an integral part of several textual constellations. A Computus can be:
A treatise embedded in a discursive and doctrinal frame, as Maximus' *Enarratio* and George Presbyter's Computus.

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[59] See, for instance, the instructions written by John Pediasimos on f. 319r of Vat. gr. 191 (second half of 13th century; *Dikyon* 66822: see F. ACERBI – A. GIOFFREDA, Manoscritti scientifici della prima età paleologa in scrittura arcaizzante. *Scripta* 12 [2019] 9–52: 30–34 and 41–44)—or in GRUMEL, La Chronologie 311. On tables as a tool for someone *calculandi minus idoneus*, see Bede, *De Temporum Ratione* XIX.

[60] This characteristic is much more accentuated in Western Computi than in Easter Computi: just check the table of contents in Bede's *De Temporum Ratione*, in JONES, Bedae Opera 177–178, and see the comparative analysis of *De Temporum Ratione* and of early Irish Computi (*Computus Einsidlensis*, *De ratione conputandi* [edited in M. WALSH – D. Ó CRÓINÍN, Cummian's Letter *De controversia paschali* and the *De ratione conputandi* (*Studies and Texts* 86). Toronto 1988], Munich Computus), with a detailed description of the structure of the latter, in WARNTJES, The Munich Computus CVII–CXXII. Reading the twenty Computi magisterially edited in A. BORST, Schriften zur Komputistik im Frankenreich von 721 bis 818. I–III (*Monumenta Germaniae Historica. Quellen zur Geistesgeschichte des Mittelalters* 21). Hannover 2006, confirms this fact.

[61] I have introduced this notion in ACERBI, Arithmetic and Logistic 155.

[62] In the case of primers on tables, as for instance on Ptolemy's *Handy Tables* or on the *Persian Tables*, their sectional nature is obviously motivated by the nature of the reference text.

[63] A study, a (partial) edition, and a discussion of the manuscript tradition of the mentioned treatises is found in B. BYDÉN, Theodore Metochites' Stoicheiosis Astronomike and the Study of Natural Philosophy and Mathematics in Early Palaiologan Byzantium (*Studia Graeca et Latina Gothoburgensia* 66). Göteborg 2003; LEURQUIN, Théodore Méliténiote; P. CARELOS, Βαρλαὰμ τοῦ Καλαβροῦ, Λογιστική. Barlaam von Seminara, Logistiké (*Corpus philosophorum Medii Ævi. Philosophi byzantini* 8). Athens – Paris – Bruxelles 1996, respectively.

An adjunct to a more comprehensive treatise, as *Anonymus* 686, which complements Maximus' *Enarratio* in, among others, the manuscript Leiden, Universiteitsbibliotheek, Scal. 33 (11th century; *Diktyon* 37986) ff. 17r–26v.

An adjunct to (and a primer on) Paschal tables, as *Nicholas* 916, witnessed in the manuscripts Hamburg, Staats- und Universitätsbibliothek, in scrin. 50a, f. 1r–v, and London, British Library, Add. 18231, ff. 4v–5r, and *Theophylaktos* 956, again in Hamb., SUB, in scrin. 50a, f. 11v[64]. *Nicholas* 916 is written in the form of a letter.

A part of a florilegium, as the computistical sections of the *Florilegium Coislinianum*, for which I have checked the manuscripts Ambr. Q 74 sup. (gr. 681; end of 10th century; *Diktyon* 43158) f. 98r (an abridged version), and Par. gr. 924 (10th century; *Diktyon* 50513) ff. 286v–293v. An algorithm in the computistical part of the *Florilegium* is dated AD 716.

A section of a textbook of canonical law, as in Matthew Blastares' *Syntagma*. The contents of this treatise are organized alphabetically, as in the *Florilegium Coislinianum*.

A part of the material accompanying a psalter, as *Anonymus* 951, witnessed in the manuscript Oxford, Bodleian Library, Auct. D.4.1 (Misc. 5; *Diktyon* 46972) ff. 29v–34v, which ends with a mutilated *paschalion* dated AM 6459–6464 [= AD 951–956] (4 folios have been excised)[65]. Other early psalters that contain Computi include *Anonymus* 1021, witnessed in Vat. gr. 341 (*Diktyon* 66972) ff. 8r–13r, *Anonymus* 1095, in Vat. gr. 342 (*Diktyon* 66973) ff. 17r–23v (which includes a *paschalion* dated AM 6596–6667 [= AD 1088–1159] and features a passing reference to an 84-year cycle), and *Anonymus* 1105, in the manuscript Harvard University, Houghton Library, gr. 3 (*Diktyon* 12290) ff. 282r–288v, which includes a *paschalion* dated AM 6613–6632 [= AD 1105–1124].

A part of an instruction manual to a collection of astronomical tables, as the algorithms contained in Stephanus-Heraclius' *in Ptolemaei Tabulas Manuales* (sects. 12 and 28–30, respectively), in George Chrysokokkes' *Syntaxis Persica* (sects. 9–10), in Meliteniotes' *Three Books on Astronomy* (sects. II.19–20 and III.17, 23–24), and in the anonymous *Paradosis in Tabulas Persicas* (sect. 13), the latter three explaining how to use the "persian tables".

A part of an anthology; this can be a low-brow notebook intended for personal use, as the one containing *Anonymus* 892; a middle-brow notebook, as Ambr. A 45 sup., which features an Alexandrian Computus and *Anonymus* 830, and Par. gr. 854, where we find *Anonymus* 1079; or a high-brow anthology, as Par. suppl. gr. 690 (1075–85; *Diktyon* 53425), whose ff. 249r–v present computistical material[66].

A part of a philosophical encyclopedia, with special emphasis on logic and natural philosophy, as in Laur. Plut. 87.16. This manuscript, one of the most important witnesses of Nikephoros Blemmydes' *Epitome Isagogica*, contains four Computi, namely, *Anonymus* F, *Anonymus* 1247, *Anonymus* 1273, and Psellos' treatise.

A part of a *Rechenbuch*, as *Anonymus* 1183, *Anonymus* 1256, the isolated procedure in Par. suppl. gr. 387, and the computistical section in Rhabdas' *Letter to Tzavoukhes*.

An independent treatise dealing exclusively with technical issues, as the Computi composed by Psellos, Argyros, and Rhabdas, and many other anonymous works.

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[64] Theophylaktos also copied the Hamburg manuscript. See footnote 101 below for details.

[65] This is the earliest psalter with a Computus; see M. WALLRAFF, The Canon Tables of the Psalms: An Unknown Work of Eusebius of Caesarea. *DOP* 67 [2013] 1–14; G. R. PARPULOV, Towards a History of Byzantine Psalters ca. 850–1350 AD. Plovdiv 2014 *passim*.

[66] A detailed analysis of this manuscript can be found in G. ROCHEFORT, Une anthologie grecque du XIe siècle : le Parisinus suppl. gr. 690. *Scriptorium* 4 (1950) 3–17; see also I. PÉREZ MARTÍN, Les *Kephalaia* de Chariton des Hodèges (*Paris*, *BNF*, *gr*. 1630), in: Encyclopedic Trends in Byzantium? Proceedings of the International Conference held in Leuven, 6-8 May 2009, ed. P. van Deun – C. Macé (*Orientalia Lovaniensia Analecta* 212). Leuven – Paris – Dudley (MA) 2011, 361–386 (Par. suppl. gr. 690 was one of the sources of Chariton's notebook), and M. D. LAUXTERMANN, Byzantine Poetry from Pisides to Geometers. Texts and Contexts. Wien 2003 *passim* et in particular 329–333.

(**F**) Computi elude standard philological methods for establishing filiations among manuscript witnesses: as any highly sectional text, such collections can be assembled and disassembled very easily; moreover, any computistical algorithm or worked-out example can undergo (major) modifications in the process of transmission. Accordingly, hypotheses of filiation between versions of specific algorithms in different manuscripts can seldom be corroborated by uncontroversial textual evidence[67]. The only sensible attitude is to edit every Computus separately, even when there are—as there frequently are—overlaps with other Computi. This remark, and the sectional nature of Computi highlighted in the previous point, explain my choice of presenting *Anonymus* 892 as a sequence of disconnected chapters, each of which is followed by the translation and a commentary.

(**G**) Computi make exclusive use of the "procedural" and "algorithmic" stylistic codes, as we shall see in the next Section.

(**H**) Computi are firmly anchored to current linguistic practice; they frequently exhibit the vernacular and an "aberrant" morphosyntax. Special attention to these linguistic features was paid in the editions procured by the school of Otmar Schissel[68].

THE TRADITION OF EASTER COMPUTI: STYLISTIC FEATURES

The style adopted in Easter Computi requires an explanation. Greek and Byzantine mathematics adopted three stylistic codes: these are the demonstrative, procedural, and algorithmic codes[69]. The demonstrative code is the one in which ancient Greek geometry and number theory—in a word: Euclid's *Elements*—are written; it does not concern us here. In logistic treatises, the solution of a numerical problem, usually provided without any supporting "proof" in the strict sense, was encoded in two peculiar expository formats, which I have called the "procedural" and the "algorithmic" codes. These are two stylistic resources that formulate chains of operations on numerical entities, and such that the output of an operation is taken as the input of the operation next in order: they are the ancient counterpart of our computer programmes. In particular, the procedural code was aptly used to express in words operational sequences that we would summarize in an algebraic "formula". A description of these two codes runs as follows.

The procedural code formulates its prescriptions as a sequence of coordinated principal clauses with the verb in the imperative or in the first person plural, present or future; to each principal clause are subordinated one or more participial clauses coordinated with each other; the participle is a satellite and performs the function of modifier of the operating subject. There are, moreover, an initializing clause, which inserts the initial input into the procedure, and an end clause, which identifies the result of the chain of operations as the quantity to be calculated. This quantity is usually

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[67] These observations apply to most sectional writings, such as geometric metrological collections, *Rechenbücher*, and grammatical compendia; see, in this order, ACERBI – VITRAC, Héron d'Alexandrie sect. III; ACERBI, Byzantine Rechenbücher; the categorization and the examples in G. UCCIARDELLO, 'Atticismo', excerpta lessicografici e prassi didattiche in età paleologa. *AION* 41 (2019) 208–234. My approach to editing Computi collides with Borst's, best exemplified in BORST, Schriften: through his editorial approach, he suggests that there have been master texts, from which fragments and extracts were then distributed widely. For a criticism of Borst's approach, see I. WARNTJES, The *Computus Cottonianus* of AD 689: A Computistical Formulary Written for Willibrord's Frisian Mission, in: The Easter Controversy of Late Antiquity and the Early Middle Ages, ed. I. Warntjes – D. Ó Cróinín (*Studia Traditionis Theologiae* 10). Turnhout 2011, 173–212: 199 n. 82. I am grateful to Immo Warntjes for a discussion on this point; in collaboration with J. ter Horst and Th. Snijders, Warntjes is currently preparing a database centred on "computistical objects".

[68] See REDL, La chronologie appliquée, for Psellos; KARNTHALER, Die chronologischen Abhandlungen, for *Anonymi* 1092A–C; BUCHEGGER, Wiener griechische Chronologie, for *Anonymus* 1273; SCHLACHTER, Wiener griechische, for *Anonymus* 1350.

[69] These notions were first introduced in F. ACERBI, I codici stilistici della matematica greca: dimostrazioni, procedure, algoritmi. *QUCC* 101 (2012) 167–214. See also ACERBI – VITRAC, Héron d'Alexandrie sect. II.2, for the algorithmic code in Hero's *Metrica*, and my comprehensive discussion in F. ACERBI, The Logical Syntax of Greek Mathematics (*Sources and Studies in the History of Mathematics and the Physical Sciences*). Heidelberg – New York 2021, sects. 1.1–3.

declared in a clause that precedes the entire procedure. This code is used to formulate operatory prescriptions in the most general way; the verb forms—either finite or participial forms—represent the operations, and each verb form corresponds to exactly one operation; the involved mathematical objects, the "operands", are the complements of the verb forms and are designated by (sometimes extremely long) definite descriptions. The operations may be unary or binary.

Procedures prominently figure in the astronomical *corpus*; they expound how to use numerical tables for computing relevant astronomical quantities. Thus, we find procedures in Ptolemy's *Almagest*[70] and in his own instruction manual to the *Handy Tables*, in Pappus' and Theon's commentaries thereon, in Stephanus' commentary on the *Handy Tables*, and in all similar Byzantine primers like the eleventh-century manual best witnessed in Par. gr. 2425 (*Diktyon* 52057)[71], Metochites' *Abridged Astronomical Elements* (*ca*. 1316), George Chrysokokkes' *Syntaxis Persica* (1346), Meliteniotes' *Three Books on Astronomy* (1352), the anonymous *Paradosis in Tabulas Persicas* (1352). We also find procedures in the above-mentioned *Prolegomena to the Almagest*.

To become acquainted with this stylistic resource, let us read some examples in Byzantine writings. First, here is part of sect. 59 of the manual in Par. gr. 2425, which explains "how to find the true degree of the sought for syzygy"[72]:

> ζήτει πρῶτον τὰς πρὸ τῆς ζητουμένης συζυγίας μεσημβρινὰς ἐποχὰς τῶν δύο φωστήρων, καὶ ἀπόγραψε ἰδίᾳ ἑκάστην. εἶτα λαβὼν τὴν ὑπεροχὴν τοῦ ἡλίου πρὸς τὴν σελήνην τίθει ἐν δυσὶ τόποις, καὶ πενταπλασίασας τὸν ἕνα τὰ γινόμενα λεπτὰ προστίθει τῷ ἑτέρῳ, καὶ τὰ μὲν ἐκ τοῦ πενταπλασιασμοῦ λεπτὰ πρόσθες τῇ ἐποχῇ τοῦ ἡλίου, τὰ δὲ τοῦ ἑτέρου τόπου – ᾧ προσετέθη τὰ ἐκ τοῦ πενταπλασιασμοῦ λεπτὰ – πρόσθες τῇ ἐποχῇ τῆς σελήνης. καὶ ἐὰν ἀμφοτέρων τῶν φωστήρων αἱ ἐποχαὶ ἴσαι γένωνται, ἐψήφισας ἀκριβῶς.
>
> First seek for noon positions of the two luminaries before the sought for syzygy, and record each by itself. Then, taking the excess of the Sun over the Moon, put it in two places, and quintupling one add the resulting minutes to the other, and add the minutes from the quintuplation to the Sun's position, and add the figure of the other place—to which the minutes from the quintuplation were added—to the Moon's position. And if the positions of both luminaries turn out to be equal, you have computed correctly.

Second, let us read an extract from a geometric metrological compendium that contains only procedures; one is required to find the height of a pyramid[73]:

> πυραμίδος τήν τε κάθετον καὶ τὸ στερεὸν εὑρεῖν. ποίει οὕτως. καὶ τὴν μὲν κάθετον εὑρεῖν. πολλαπλασίασον μίαν τῶν πλευρῶν ἐφ' ἑαυτήν, καὶ τὸν γενόμενον δίπλωσον· εἶτα τοῦ γενομένου λάβε τὸ δ^{ov}, καὶ αὖθις ἀρίθμησον ἓν τῶν κλιμάτων ἐφ' ἑαυτό, καὶ ἐκ τοῦ γενομένου τὸ ῥηθὲν δ^{ov} ἀφελὼν τοῦ λοιποῦ λάβε πλευρὰν τετραγωνικήν, καὶ ἔξεις τὴν κάθετον.
>
> Find both the height and the volume of a pyramid. Do as follows. And find the height. Multiply one of the sides by itself, and double the result; then take a 4^{th} of the result, and reckon again one of the edges by itself, and removing the said 4^{th} from the result take the square root of the remainder, and you will have the height.

Finally, here is a Passover algorithm that will deserve a fuller discussion below; we read the formulation found in Meliteniotes' *Three Books on Astronomy* III.24[74]:

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[70] See *Almagest* II.9, III.8, III.9, V.9, V.19, VI.9–10, XI.12, XIII.6.

[71] See the edition in A. JONES, An Eleventh-Century Manual Of Arabo-Byzantine Astronomy (*Corpus des Astronomes Byzantins* 3). Amsterdam 1987. A preliminary study is in O. NEUGEBAUER, Commentary on the Astronomical Treatise Par. gr. 2425 (*Académie royale de Belgique. Classe des Lettres et des Sciences morales et politiques*. *Mémoires* 59.4). Bruxelles 1969.

[72] JONES, An Eleventh-Century 84 (text) and 85 (translation, slightly modified).

[73] The ancestor of the entire tradition is Vat. gr. 1411, ff. 17v–23r; our text is from f. 23r. A synopsis of the compendium is found in HEIBERG – NIX – SCHMIDT – SCHÖNE, Heronis Alexandrini V IC–CII.

[74] Vat. gr. 792, f. 350r. I retain the original accents for the enclitics.

ἔστι δὲ καὶ δεύτερος τρόπος τῆς τοῦ Πάσχα καταλήψεως τοιοῦτος. τὸν γὰρ τῆς σελήνης κύκλον ἑνδεκάκις ποιήσαντες καὶ τοῖς γενομένοις προσθέντες ἐπὶ μὲν τῶν ἄλλων κύκλων αὐτῆς ς ἐπὶ δὲ τοῦ ιζ^{ου} καὶ τοῦ ιη^{ου} καὶ τοῦ ιθ^{ου} προσθέντες ζ καὶ ἀπὸ τοῦ οὕτω συναχθέντος ἐκβαλόντες τὰς ἐμπεσούσας εἰ τύχοι τριακοντάδας, εἶτα τῷ λοιπῷ ἀριθμῷ τὸν λείποντα εἰς τὸν ν λαβόντες, εἰ μὲν ἐστὶν λα ἢ ἐλάττων τῶν λα, ἀπογραψόμεθα ἡμέρας τοῦ Μαρτίου, εἰ δὲ ἐστὶ πλείων, τὸν μετὰ ἀφαίρεσιν τῶν λα ἀπογραψόμεθα ἡμέρας τοῦ Ἀπριλλίου, καθ' ἃς τὸ νομικὸν ἐκτελεσθήσεται Φάσκα.

There is also a second way of taking Easter, namely, the following one. For doing eleven times the cycle of the Moon and adding 6 to the result for the other cycles of it, and adding 7 for <cycles> 17, 18, and 19, and taking away the intervening thirties, if any, from the <number> gathered in this way, then taking what remains to 50 for the remaining number, if it is 31 or less than 31, we shall write it down as days of March; if it is more, we shall write down the <number> after removal of 31 as days of April, on which Passover will be accomplished.

In all commentaries or computational primers mentioned above, procedures precede paradigmatic examples presented in algorithmic form and are intended to validate them.

The algorithmic code features paradigmatic examples featuring specific numerical values. After the initializing clause, the algorithms are expressed as a sequence of principal clauses coordinated by asyndeton; each clause formulates exactly one step of the algorithm and comprises a verb form in the imperative (this is the operation) and a system of one or two objects[75]—a direct and an indirect object—in the form of demonstrative or (cor)relative pronouns or of numerals (these are the operands). The operation is often expressed by means of the preposition that introduces the indirect object, without any verb form: "these by 3" instead of "multiply these by 3". The result of each operation is identified as such in a dedicated clause, with the verb in the present indicative (forms of γίνομαι "to yield", "to result")[76], sometimes replaced by an adjective in predicative position (mainly λοιπός "as a remainder" after a subtraction); both syntactic structures are equivalent to our equality sign[77]. An end clause identifies the result of the chain of operations as the quantity to be calculated. This quantity was usually declared in a clause that precedes the entire algorithm. The main feature of an algorithm is the systematic use of parataxis by asyndeton: no coordinants, (almost) no connectors, no subordination. The algorithmic flow is usually one-step: any step (1) accepts a number that is the output of the immediately preceding step as input and (2) inserts new data by means of the second operand. Operations in which neither operand is the output of the immediately preceding step are less frequent. Such operations induce a hiatus in the algorithmic flow; the hiatus is often syntactically marked by the presence of particles or of specific verb forms.

Let us read a part of Hero, *Metrica* I.8, as an example of an algorithm[78]:

οἷον ἔστωσαν αἱ τοῦ τριγώνου πλευραὶ μονάδων ζ η θ.	For instance, let the sides of the triangle be of 7, 8, 9 units.
σύνθες τὰ ζ καὶ τὰ η καὶ τὰ θ· γίγνεται κδ·	Compose the 7 and the 8 and the 9: it yields 24;
τούτων λαβὲ τὸ ἥμισυ· γίγνεται ιβ·	take half of these: it yields 12;
ἄφελε τὰς ζ μονάδας· λοιπαὶ ε.	remove the 7 units: 5 as a remainder.
πάλιν ἄφελε ἀπὸ τῶν ιβ τὰς η· λοιπαὶ δ.	Again, remove the 8 from the 12: 4 as a remainder.
καὶ ἔτι τὰς θ· λοιπαὶ γ.	And further the 9: 3 as a remainder.
ποίησον τὰ ιβ ἐπὶ τὰ ε· γίγνονται ξ·	Make the 12 by the 5: they yield 60;
ταῦτα ἐπὶ τὰ δ· γίγνονται σμ·	these by the 4: they yield 240;
ταῦτα ἐπὶ τὰ γ· γίγνεται υκ·	these by the 3: it yields 720;
τούτων λαβὲ πλευράν,	take a side of these,
καὶ ἔσται τὸ ἐμβαδὸν τοῦ τριγώνου.	and it will be the area of the triangle.

[75] Accordingly, the operation is unary or binary, respectively.

[76] The former must be used to translate finite verb forms, the latter for participial forms.

[77] In mathematical papyri, γίνεται can be replaced by a vertical stroke: see, for instance, PMich. III.145, in J. G. WINTER, Papyri in the University of Michigan Collection. Miscellaneous Papyri (*Michigan Papyri* 3). Ann Arbor 1936, 34–52. This shows that the verb form is equivalent to our equality sign in a strong sense.

[78] ACERBI – VITRAC, Héron d'Alexandrie 174.3–7. This is "Hero's formula" for finding the area of a triangle once its sides are numerically given.

In ancient Greek sources, this code characterizes Hero's *Metrica*, and it is used exclusively in the geometric metrological *corpus*. In the *Metrica*, proofs using the "language of the givens" precede paradigmatic examples of computations in algorithmic form, and are intended to validate them. In all astronomical primers mentioned above, and more generally in all Byzantine texts of this kind, paradigmatic examples presented in algorithmic form are very frequent; they are systematically preceded by procedures; the latter are intended to validate the former. In these texts, algorithms are frequently replaced—or accompanied—by tabular arrangements of the performed operations; the tabular arrangements are nothing but an evolution of the algorithms in a more perspicuous format.

In Meliteniotes' *Three Books on Astronomy*, for example, each operation is frequently carried out three times: by means of a general procedure (called μέθοδος), by means of a procedure featuring actual numerical values (ὑπόδειγμα "example"), and finally by means of an algorithm in the sense just explained, often organized as a tabular set-up (ἔκθεσις τῶν ἀριθμῶν "setting-out of the numbers"). Let us read the second and third avatar of the procedure in Book III.24, read just above[79]:

> πάλιν κατὰ τὴν ἑτέραν ἔφοδον τὸν τῆς σελήνης κύκλον (ἤγουν τὸν β) ἑνδεκάκις ποιήσαντες καὶ τῷ γεγονότι ἀριθμῷ κβ προσθέντες ϛ καὶ τῷ συναχθέντι κη τὸν λείποντα εἰς τὸν ν λαβόντες, ὅς ἐστι ὁ κβ, τοῦτον ἀπεγραψάμεθα ἡμέρας τοῦ Μαρτίου, καθ' ἃς τὸ νομικὸν γενήσεται Φάσκα.
> […]
> σελήνης κύκλος δεύτερος· τοῦτον ἑνδεκάκις· γίνεται κβ· πρόσθες ϛ· γίνεται κη· ὁ λείπων εἰς τὸν ν· γίνεται κβ. καί εἰσιν ἡμέραι τοῦ Μαρτίου, καθ' ἃς τὸ νομικὸν ἔσται Φάσκα.
>
> Again, according to the second procedure, making eleven times the cycle of the Moon (namely, 2) and adding 6 to the number that has turned out to result and taking what remains to 50 for the gathered <number> 28, which is 22, we have written this down as days of March on which Passover will occur.
> […]
> Second cycle of the Moon; this eleven times: it yields 22; add 6: it yields 28; what remains to 50: it yields 22. And they are the days of March on which there will be Passover.

Both the procedural and the algorithmic code are adopted in Easter Computi, as some examples will show. I use an algorithm in *Anonymus* 1183 as my paradigmatic example. I have selected this Computus because, as said, it is contained in a manuscript redacted for conservation purposes as part of a multi-purpose notarial vade mecum; we may expect a highly regularized style from it. Let us read sect. 9, which presents the algorithms for the epacts and the base[80] of the Moon[81].

> εἰ θέλεις εὑρεῖν τοῦ ἐνισταμένου ἔτους τὴν ποσότητα τοῦ θεμελίου τῆς σελήνης καὶ τὰς ἐπακτὰς αὐτῆς, κράτησον τὰ ἀπὸ κτίσεως κόσμου ἔτη ἕως τοῦ ἐνεστῶτος, καὶ πολυπλασίασον αὐτὰ ἑνδεκάκις, τουτέστιν ἑνδεκαπλασίασον αὐτά· εἰθ' οὕτως πρόσθες καὶ τοῖς ἀπὸ τοῦ ἑνδεκαπλασιασμοῦ ἀναβιβασθεῖσι καὶ τὸ ἐννεακαιδέκατον μέρος τῶν ἀπὸ κτίσεως κόσμου ἐτῶν, καὶ ἑνώσας ταῦτα ὁμοῦ ὕφειλον ἐπὶ τῶν λ, καὶ τὰ καταλειφθέντα κάτωθεν τῶν λ εἰσὶν αἱ ἐπακταὶ τῆς σελήνης· τούτοις προστίθει ἀεὶ τὴν πρώτην τοῦ Ἰαννουαρίου μηνός, καὶ εὑρήσεις τὴν ποσότητα τοῦ θεμελίου τῆς σελήνης.

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[79] Vat. gr. 792, f. 352r and 353r, respectively.

[80] The "base" of the Moon is not used in *Anonymus* 892. For the "base" of the Moon, see footnote 95 below.

[81] The verb forms in the aorist tense must be translated with a present, unless they occur in first or second person verb forms or whenever a second-person subject is expressed. For in algorithms no temporal connotation can be present; the aorist tense there adopted simply intimates absence of temporal or aspectual connotations (the "pure action" expressed by the verb): this is possible only in the aorist, which is the less connotated pole of the aspectual opposition (see J. HUMBERT, Syntaxe grecque. Paris 1960, 133–181 *passim*, and again my discussion in ACERBI, The Logical Syntax, sects. 1.1–3). The reader will also note that there is an adverbial καὶ in excess in a clause (maybe both adverbial καὶ are in excess there); this feature is common in Computi; it is in fact a general trait of Greek mathematical style: ACERBI, The Logical Syntax, sect. 5.3.5.

If you want to find the quantity of the base of the Moon and its epacts for the present year, keep the years from the foundation of the world up to the present one, and multiply them eleven times, that is, undecuple them; then again also add the nineteenth part of the years from the foundation of the world to the <numbers> brought up from the undecuplation, too, and uniting them together remove by 30, and that which remains down from 30 are the epacts of the Moon; always add the first <day> of the month of January to these, and you will find the quantity of the base of the Moon.

Let us dissect this neatly formulated algorithm:

	conn.	gener.	operation	operands	phase
			keep *	* the years from the foundation of the world up to the present one	initialization
1	and		multiply * eleven times	* them *	algorithm
2	then again		add *(i) *(ii)	* (i) the nineteenth part of the years from the foundation of the world (ii) to the <numbers> brought up from the undecuplation	
3	and		uniting * together	* them *	
			remove by 30		
	and		* are *	that which remains down from 30 * the epacts of the Moon	identification of the result
1		always	add *(i) *(ii)	* (i) the first <day> of the month of January (ii) to these	algorithm
	and		you will find *	* the quantity of the base of the Moon	identification of the result

The first column numbers the truly operative steps of the algorithm. The second column sets out the "connector", which links two algorithmic steps; one of these steps can be the "initialization" clause or the "identification of the result". In the column "generality" one finds the linguistic units (usually adverbs) that mark operations or operands that are structural parameters of the algorithm. The "operation" column contains the verb forms that express the operations. Note step 3, where two operations are nested as a principal clause + participial satellite. The next-to-last column sets out the "operands", which are always designated by standard definite descriptions; I have conventionally included in this column the subject of the clause that identifies the result. The asterisks mark the position of the operands/operation omitted in the operation/operands column. Steps (1) and (3) of the first algorithm comprise unary operations, steps (2) of the first and (1) of the second algorithm are binary operations, to which two operands are accordingly associated. The last column identifies the "phase" of the entire algorithm. It is clear that two sequentially linked algorithms are at work here. If set against my categorization above, the text we have read is clearly a "procedure", even if I shall consistently use the denomination "algorithm". The procedural character is confirmed by the massive presence of definite descriptions, like "the nineteenth part of the years from the foundation of the world" above, not accompanied by any numerical exemplification.

According to the previous discussion, my symbolic transcription is as follows[82]:

$$(y) \overset{(1)}{\rightarrow} 11y \overset{(2)}{\rightarrow} 11y + y/19 \overset{(3)}{\rightarrow} (11y + y/19) \bmod 30 = e_m \overset{(1)}{\rightarrow} e_m + 1_J = b_m.$$

where y is the year in the Byzantine world era, e_m and b_m denote the epacts and the base of the Moon at lunar cycle m, respectively, and 1_J is January 1. The modulo reduction computes the remainder of the division by 30 of what precedes the "mod" sign[83].

[82] For clarity's sake, I have superimposed the numbers of the steps on the arrows. I shall never do this again.

As in the above example, the symbolic transcriptions I shall use throughout this article are intended faithfully to represent the computational flow. The initial input is the assumed quantity; it is enclosed in parentheses, thus: (*y*). A self-contained step of the transcription formalizes a complete "participial clauses + principal clause" sentence of the algorithm (thus, several operations may feature in it). Steps in which the output-input chain is not interrupted are linked by an arrow →. The operands in a given step are usually written in the same order as that in which they are introduced in the text. The sign | separates independent steps that follow one and the same step (that is, a branching has occurred). A full stop indicates an algorithmic hiatus or the end of an algorithmic branch. Levels of brackets go iteratively from parentheses to braces. The final output is preceded by the sign =.

A calculation that is standard in Computi finds the remainder of a reduction by repeatedly removing multiples of the modulus, as in *Anonymus* 892, sect. **10**, where it is shown that 16 ≡ 6400 (mod 28); this is an "algorithm" according to my categorization above:

> ὕφειλε τὰ αὐτὰ ἔτη ἐπὶ τῶν κη, καὶ εὑρήσεις τὸν κύκλον τοῦ ἡλίου· εἰκοσάκις διακώσιοι, ˏδ· ὀκτάκις σ, ˏαχ· λοιπὸν ἐνέμειναν ω· ὕφειλε καὶ ταῦτα οὕτως. εἰκοσάκις κ, υ· ὀκτάκις κ, ρξ· καὶ ἔμειναν σμ· ὀκτάκις κ, ρξ· ὀκτάκις ὀκτώ, ξδ· καὶ λοιπὸν ἔμειναν ις. καὶ γνώριζε ἓξ καὶ δέκατον κύκλον εἶναι τοῦ ἡλίου.
>
> Remove the same <6400> years by 28, and you will find the cycle of the Sun; twenty times two hundred, 4000; eight times 200, 1600: there remain 800 as a remainder; remove also these as follows. Twenty times 20, 400; eight times 20, 160: and there remain 240; eight times 20, 160; eight times eight, 64: and there remain 16 as a remainder. And recognize that it is the sixteenth cycle of the Sun.

A COMPUTISTICAL GLOSSARY

The computistical terms are in boldface when they are defined; they are in italics when they are mentioned. All the terms here defined are discussed in the commentary on the relevant section(s) of *Anonymus* 892.

The **indiction** (ἴνδικτος, ἰνδικτιών, ἐπινέμησις) is a 15-year **cycle** introduced in the late Roman empire for taxation purposes. There are several regional variants of the *indiction cycle*, and its initial history is complex;[84] AD 312/3 is year 1 of the most current *indiction cycle*. The Byzantine civil year and the *indiction* year begin on September 1.

An **era** is a non-cyclic count of *calendar years* starting from a year 1, called **epoch**. The *epoch* of the **Byzantine world era** (τὰ ἀπὸ κτίσεως κόσμου ἔτη "the years from the foundation of the world"; henceforth denoted AM) is BC 5509 September 1[85], a Saturday; years are *Julian years*[86]. The *epoch* of the **Alexandrian world era** is BC 5493 August 29 (BC 5492 March 25 according to Annianus), a Tuesday; years are *Julian years*. The *Byzantine era* is the *Alexandrian era* shifted 16

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[83] The crucial operation in a Computus is finding the remainder of the division of a number *x* by a number *n*. In modern terms, this is the "modulo" reduction, whose sign is "*x* mod *n*". We also write "$x \equiv y \pmod{n}$" (read "*x* is congruent to *y* modulo *n*") to signify that numbers *x* and *y*, once divided by *n*, yield the same remainder. As we shall see in *Anonymus* 892 (see the commentary on Sect. **3**), the division is carried out by removing suitable multiples of the divisor *n*. Again, in modulo *n* reductions in our text, if the dividend is a multiple of the divisor, the remainder is frequently taken to be *n*, and not 0 (or, to be accurate, "nothing").

[84] A detailed study is in S. BAGNALL – K. A. WORP, Chronological Systems of Byzantine Egypt. 2nd ed. Leiden – Boston 2004. GRUMEL, La Chronologie 192–206 provides a brief account and explains the regional variants. See also the account in MOSSHAMMER, The Easter Computus 20–24.

[85] To denote dates, I adopt the astronomical convention era – year – month – day.

[86] On eras, see the synopsis in GRUMEL, La Chronologie 207–226 and 279–296. See also NEUGEBAUER, HAMA 1143 *s.v.*, and especially 1064–1067 and 1074–1076 (with bibliography), and the dedicated sections in NEUGEBAUER, Ethiopic Astronomy, and NEUGEBAUER, Abu Shaker's.

years backwards. The shift was probably motivated by the requirement of synchronizing *lunar*, *solar*, and *indiction cycles* with each other and with the era: year 1 of the *Byzantine world era* is also year 1 of the *lunar*, *solar*, and *indiction cycles*[87]. This can be done because the beginning of the *solar* and *lunar cycles* is conventional[88], whereas the *indiction* is rigidly attached to the era of Diocletian—of which the *Alexandrian world era* is an avatar shifted 5776 years (= 304 lunar cycles of 19 years) back[89].

The **tropical year** is the time interval between two successive passages of the Sun through the same point of its own yearly circuit. A *tropical year* comprises 365 days and a fraction of a day that is very nearly approximated by ¼, that is, 6 hours. **Julian-style** calendar years take into account the fractional nature of the *tropical year* by introducing an **intercalary day** every fourth year and in a fixed place in the year; this year is called **leap year** (βίσεξτον, δίσεξτον, βίσεκτον)[90]. In this way, the *tropical year* is transformed into a calendrical entity, the **Julian year**, which comprises a number of days that is either 365 or 366. The Byzantine **calendar years** are *Julian-style* years; they employ Roman months endowed with a forward day-count from the first day of the month as the sole monthly *epoch*; the year begins on September 1; the additional day of a *leap year* is intercalated every fourth year as February 29[91].

Solar cycles of equal length exhibit the same sequence of pairings between dates and weekdays. As in *Julian years* an intercalary day is added every fourth year, the number of weekdays, 7, and 4 are prime to each other, and neither 365 nor 366 are multiples of 7, the shortest solar cycle consists of 7×4 = 28 *Julian years*. Byzantine **solar cycle years** begin on October 1.

The natural time interval associated with the motions of the Moon and of the Sun as seen from the Earth is the **synodic month**, which corresponds to the return of the Moon to the same position with respect to the Sun. The new Moon is traditionally taken as the boundary between two consecutive lunar months. A *synodic month* comprises 29 days and a fraction of a day that is very close to ½[92]. Hence, a *synodic month* of about 29 ½ days covers an interval of 30 days. The **age of the Moon** is the number of days elapsed since the immediately preceding new Moon; these days are traditionally denoted by the word *luna* followed by an ordinal number: the 14th day of a lunar

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[87] George, sect. II.1, in DIEKAMP, Der Mönch 24.20–31, claims that the synchronization of all cycles is the main virtue of the Byzantine era. As the years of the three cycles begin on different dates, synchronization is not exact: time intervals contained in two consecutive solar or lunar cycle years may belong to one and the same calendar year. However, Passover, Easter, and most movable feasts of the Christian calendar fall in the "safe" time segment bounded by January 1 and August 31.

[88] It is enough to call cycle year *X* of the old cycle "cycle year 1" of the new cycle and to rearrange the epacts in such a way that the Passover dates remain the same.

[89] The rule is as follows: $i = 1$ for Diocletian 14 = Alexandrian AM 5790 = Byzantine AM 5806 [= AD 297/8]. Since $5790 \equiv 0 \pmod{15}$, there is a crucial discrepancy of 1 unit between indiction cycle and Alexandrian era. On the other hand, $5806 \equiv 1 \pmod{15}$. To enforce synchronization while preserving the position of leap years, one must introduce a shift of $15k + 1 = 4p$ years, for some integers k and p. The smallest solution is $(k,p) = (1,4)$, and the shift is of 16 years.

[90] See STERN, Calendars in Antiquity 204–227, especially for a discussion of the problems with intercalation that affected the first decades of application.

[91] Unless otherwise stated, this is what I call the "Julian calendar". In counting the days in a year, the Romans used a backward day-count keyed to three monthly epoch: the calends (1st day of a month), the nones, and the ides (both variably located: 7th day in March, May, July, and October, 5th day in the other months; 15th day in the same four months, 13th day in the others, respectively); as a consequence, only the days counted from the nones and the ides carried the name of the month in which they were included. The intercalary day of leap years was located as a second February 24 = *VI Kal. Mar.* (counted backwards, that is, before our February 24), whence the denomination *bis-sextus* "twice-sixth". This system is tabulated in GRUMEL, La Chronologie 298–299, and in E. J. BICKERMAN, Chronology of the Ancient World. Ithaca (NY) 1980, 125. See also A. E. SAMUEL, Greek and Roman Chronology. Calendars and Years in Classical Antiquity (*Handbuch der Altertumswissenschaft* I.7). München 1972, 152–170. The structure of the Roman calendar is explained in the Computus contained in Par. suppl. gr. 690, sect. 6 (= TIHON, Le "Petit Commentaire" 364, text nº 55).

[92] The duration of the lunar month is in fact highly variable: there is a difference of more than 13 hours between the longest and the shortest lunations in the time span 1760–2200; see J. MEEUS, More Mathematical Astronomy Morsels. Richmond (VI) 2002, 19–31.

month is *luna XIV*. A **schematic lunar month** is the approximation of the *synodic month* to 29 ½ days, counted from one new Moon to the next and embedded in a *calendar year*. Such an embedding is usually put into effect by alternating *lunar months* of 30 or 29 days[93]. A **lunar cycle** is any period after which the sequence of pairings between calendar dates and *ages of the Moon* repeats itself. The **19-year lunar cycle** comprises 19 *calendar years* of 365 days, which equal 6935 days; these are organized as a sequence of 228 alternating *lunar months* of 30 and 29 days (= 6726 days) plus 7 **embolismic** (ἐμβόλιμοι) months of 30 days each (= 210 days) occurring in specific years[94] and resulting from the fact that 12 lunar months of 29 ½ days (a **lunar year**) correspond to only 354 days. The 11 days needed to complete a *calendar year* of 365 days accumulate (the quantity accumulated at each *lunar cycle year* is called **epacts** [ἐπακταί] of the Moon)[95] until they exceed 30 days; when this happens, an *embolismic* lunar month of 30 days is formed, and these days are subtracted from the cumulating *epacts*. In this case, a *calendar year* comprises 13 lunations, and the **lunar year** has 13 months. A *19-year lunar cycle* therefore comprises 228 + 7 = 235 *lunar months* of 30 or 29 days[96]. These 235 *lunar months* equal 6936 days: the discrepancy of 1 day between the 6935 days counted by 365-day *calendar years* and the 6936 days counted by *lunar months* is eliminated by increasing the *age of the Moon* by one day at some point of its cycle, an operation that is equivalent to deleting one lunar day: this is the ***saltus lunae***, the "leap of the Moon". Accordingly, the *lunar cycle year* that follows the year in which the *saltus lunae* is inserted carries 12 epacts and not 11[97]. In Byzantine Computi, the *saltus lunae* is normally inserted towards the end of the 16th *lunar cycle year*.[98]

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[93] The pattern of embedding is a **lunar calendar**, see HOLFORD-STREVENS, Paschal Lunar Calendars, and footnote 163 below. The new Moons that set the boundaries of these lunar months are fixed once and for all; accordingly, their sequence is also schematic.

[94] The periodic sequence in which the embolismic years are arranged in the 19-year cycle is CCECCECECCECCECCECE, where the sign "C" stands for a "common" year, "E" for an "embolismic" year, that is, a year that contains an embolismic month. The qualifier "periodic" in the previous sentence means that the first year of the cycle can be located anywhere in the sequence, according to the epacts assigned to this year. The sequence as given above can naturally be split in two subsequences: the first of them comprises 8 years; the second, 11 years; these were called *ogdoas* and *hendecas* in Western Computi (the *ogdoas* and the *hendecas* are also marked in the Greek 532-year tables mentioned in footnote 101 below; these tables are witnessed in manuscripts copied in Southern Italy; these manuscripts also contain the Computi *Nicholas* 916 and *Theophylaktos* 956, which expound Western computistical procedures): see, for instance, the letter of Dionysius Exiguus to Boniface, in KRUSCH, Studien (1938) 82–86, or Bede, *De Temporum Ratione* XLVI; more information on these designations in JONES, Bedae Opera 380–381. If our sources usually indicate the position of the embolismic years, it is less obvious to reconstruct where the embolismic month was exactly located, and according to what principles (see the main footnote to sect. **19** below).

[95] Since all numerical sequences related to cycles are periodic, a starting point must be selected for the epacts: see the discussion in the commentaries on sects. **12** and **14**. In a lunar cycle that is synchronized with January 1, the epacts coincide with the age of the Moon on December 31. As lunar days are counted from January 1, a **base** (θεμέλιος, θεμέλιον) of the Moon b_m was introduced such that b_m = epacts + 1, which is the age of the Moon on January 1; the "base" replaced the epacts in specific algorithms. A "base" adapted to the features of other algorithms and defined by b_m = epacts + 3, was also introduced. For this "base", see, for instance, *Anonymus* 1247, sect. 20, in SCHISSEL, Chronologischer 105–110; *Anonymus* 1256, sect. 18, in Vat. Pal. gr. 367 (*ca*. 1317–20; *Diktyon* 66099; this important manuscript is the paradigmatic example of the script type called "chypriote bouclée", see P. CANART, Un style d'écriture livresque dans les manuscrits chypriotes du XIVe siécle: la chypriote "bouclée", in: La paléographie grecque et byzantine. Actes du Colloque Paris, 21–25 octobre 1974, ed. J. Glénisson – J. Bompaire – J. Irigoin [*Colloques internationaux du C.N.R.S.* 559]. Paris 1977, 303–321, repr. in CANART, Études 341–359; an analysis of the manuscript, inclusive of the several datings occurring in it and of a rich bibliography, can be found in A. TURYN, Codices graeci Vaticani saeculis XIII et XIV scripti annorumque notis instructi. In Civitate Vaticana 1964, 117–124 and pl. 96) ff. 85r–88r; *Blastares* 1335, in RHALLES – POTLES, Σύνταγμα VI 414–415 and 416–417; *Argyros* 1372, sect. 7, in *PG* XIX 1293 (but he calls the bases "epacts"); *Anonymus* 1377, sect. 5, in *PG* XIX 1321; *Anonymus* 1379, in *PG* XIX 1334. See also the list of epacts and bases in GRUMEL, La Chronologie 54–55.

[96] There are 114 lunar months of 29 days and 114 + 7 = 121 lunar months of 30 days.

[97] The epacts of the Moon are a good example of an "incipient" quantity, namely, one that is attached to the beginning of a time interval and not to its end: the epacts attached to a given lunar cycle year record the advance accumulated at the end of the *previous* lunar cycle year. Incipient quantities have the advantage that they can be read off directly in tables: the age of the Moon of today, AD 2021 August 10, must be calculated by using the lunar advance from epoch accumulated up to

Leap years make no difference to the *lunar cycle*, whose sequence of months is supposed to fit to a *leap year* exactly as it fits to a non-leap year: this means that the date of the beginning of each *lunar month* is the same in *leap years* and in non-leap years[99]. The *lunar months* in which the *intercalary day* falls do have an additional day, but sometimes this is assigned an *age of the Moon*, sometimes it is not, sometimes it is assigned the same age as the previous day[100]. In Byzantine Computi, disregarding *leap years* amounts to assuming that February has always 28 days in *lunar cycle* computations. The adaptation of the *19-year cycle* to *Julian years* of 365 or 366 days is possible thanks to the fact that the duration of a *synodic month* is greater than 29 ½ days by a quantity that almost exactly offsets, after 19 years, the $19\times(\frac{1}{4}) = 4\frac{3}{4}$ mean additional days coming from the *leap years*. Accordingly, the sequence of *lunar months* is, by stipulation, extended to *Julian years* by assuming that it goes unchanged in *leap years*: and such an extension works remarkably well.

A **lunar cycle year** is a calendar year whose beginning can be shifted with respect to the beginning of the civil (*calendar*) *year*. A 19-year lunar cycle consists thus of 19 *calendar years*, 19 *lunar cycle years*, and 19 *lunar years* (the latter of variable length, since they can be either 12-*lunar-month* or 13-*lunar-month* sequences); these three 19-"year" periods overlap, but they differ from one another because different meanings of "year" are involved. Byzantine *lunar cycle years* begin on January 1.

Passover (τὸ νομικὸν Φάσκα / Πάσχα) is defined as the 14th day of a *schematic lunar month* and must occur on or straight after the Spring equinox, whose date was fixed to March 21 (this is the **rule of the equinox**).

Combining the lunar and the solar cycles, we obtain a **Period** (περίοδος or μέγας κύκλος) of 532 (= 19×28) years, at the end of which the same sequence of *Easter* dates recurs[101].

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end AD 2020 (which, *qua* incipient quantity, would be tabulated in front of AD 2021), and then counting the days contained in the months as far as July, and then counting 10 days. For this reason Ptolemy tabulated incipient quantities of their time-arguments in the *Handy Tables*, whereas the opposite is the case, with the notable exception of the mean syzygies, for the tables of the *Almagest*. Roughly speaking, using incipient quantities subsumes the epoch values in the tabulated values.

98 See again the list in GRUMEL, La Chronologie 54–55, and my discussion in the commentaries on sects. **12** and **14**.

99 In the 19-year lunar cycle, no assigned lunar cycle year is a leap year or a non-leap year *per se*: leap years may occur in any year of a 19-year lunar cycle. The reason is simply that 19 is not a multiple of 4.

100 See HOLFORD-STREVENS, Paschal Lunar Calendars *passim*. The "sometimes" also depends on the lunar cycle adopted.

101 There are at least two Greek manuscripts that set out an entire *Period* in tabular format. They are Hamburg, SUB, in scrin. 50a (end of 10th century; Southern Italy; a part of Laur. Conv. Soppr. 177 [*Diktyon* 15877]; *Diktyon* 32373; see M. MOLIN PRADEL, Katalog der griechischen Handschriften der Staats- und Universitätsbibliothek Hamburg [*Serta Graeca* 14]. Wiesbaden 2002, 32–37) ff. 2r–11r, and London, BL, Add. 18231 (971/2; *Diktyon* 38944; Southern Italy; see A. CATALDI PALAU, Manoscritti greci originari dell'Italia meridionale nel fondo 'Additional' della 'British Library' a Londra, in: A. CATALDI PALAU, Studies in Greek Manuscripts [*Testi, Studi, Strumenti* 24]. Spoleto 2008, 345–410: 386–390) ff. 5r–11v (these are Codex A and Codex C of Gregory of Nazianzus, respectively). In both of them, the period is AM 6385–6916 [= AD 877–1408]. The table is conceived like the one in GRUMEL, La Chronologie 266–277: the 532 Cycle years are listed sequentially; the lunar and solar cycle counts run independently on separate columns. A lengthy explanation of the properties of the 532-year cycle is found in the computistical section of the *Florilegium Coislinianum*, letter Π, nº 164; a more concise exposition is found in *Nicholas* 916, sect. 1, and in *Anonymus* 1172, sect. 1, a Computus first witnessed in Vat. gr. 432 (14th century; *Diktyon* 67063) ff. 139v–146r, a copy of which is Vat. gr. 509 (14th century; *Diktyon* 67140) ff. 312v–316v. See also the lists of 532-year cycles in GRUMEL, La Chronologie 137–139. BORST, Schriften 1106–1108 provides a list of 33 manuscripts containing 532-year Dionysiac Easter tables. The **Damascene Easter table**, which reduces the 532-year table to a manageable size without loss of information, is based on the observation that there are seven partially cyclical structures within a solar cycle of 28 years. In the Damascene table, the 28 solar cycle years in a cycle are subdivided into 7 groups of 4; to each group there corresponds a single Easter date for each lunar cycle year (the elements of each group are the "Evangelists" of the Ethiopic Computus: NEUGEBAUER, Ethiopic Astronomy 127–128). A mathematical analysis I shall present elsewhere shows that this structure formalizes the well-known 95-year imperfect subcycle of the 532-year table. Since the Passover dates are in any case the same after 95 years (for 95 = 19×5), the same *Easter* dates recur at the end of a 95-year period if and only if the intervening intercalary days are 24 and not 23, for $95 + 24 = 119 \equiv 0 \pmod 7$. In the Julian calendar, the unfavourable case occurs if and only if the first year of the 95-year period is a leap

Easter (τὸ Χριστιανῶν Πάσχα) is the first Sunday after *Passover*. If *Passover* falls on Sunday, *Easter* is celebrated on the Sunday next thereafter.

Meat-Fare (Ἀπόκρεως) is the third Sunday of preparation to Lent in the Byzantine liturgical calendar; it falls 8 weeks = 56 days before *Easter*.

The **terms** of a festival are the extremes of the interval in which it may occur. The *terms* for *Easter* are $22_M \leq r \leq 25_A$. The *terms* for *Passover* are $21_M \leq p \leq 18_A$. The former *terms* are an immediate consequence of the latter: *Easter* cannot coincide with *Passover*, whence the lower bound March 22; *Easter* is the first Sunday after *Passover*, whence the upper bound April 25. The *Passover terms* result from the facts that the lower bound is the Spring equinox (March 21) and that *Passover* is allowed to fall within 1 lunar month from that date.

THE COMPUTUS IN PAR. SUPPL. GR. 920: *ANONYMUS* 892

The tenth-century parchment manuscript Paris, Bibliothèque nationale de France, suppl. gr. 920 is made of 22 folios, written on 19 lines each; its dimensions are mm 150×110, its quire structure is 1^1, 2^8, 1^{6-1}. As for its contents[102], f. 1r is opened by a very short sequence from Herodianus' Περὶ καθολικῆς προσῳδίας[103]. This is followed, on f. 1r–v, by a subscription τέλος σὺν θεῷ τοῦ κειμένου εὐτυχίᾳ χρῶ, and by two lists of the alphabet letters accompanied by numbers. The lists carry the titles "How must one divide the 24 letters in three isopsephic parts?" and "How must one correctly pronounce the 24 letters for <completing> the number of a myriad?"[104]. The Computus I call *Anonymus* 892 is contained in ff. 1r–16r; the final segment of the manuscript, ff. 16r–22v, contains pieces of astronomical, geographical, and magical lore[105]. No graphic break demarcates these texts from the Computus. A Sicilian chronicle is copied in the margins of ff. 1v–3r, the period ranges from 827 to 982[106].

The final part of the manuscript is severely damaged and incomplete: a folio has been cut off after f. 20 (some letters can still be read in the stub); the last folio has a big hole in the middle; the last text is truncated in the middle of a sentence. If something is obviously missing at the end, it is not said, despite the presence of the subscription after the extract from Herodianus, that something is missing at the beginning. The first folio looks in fact like a guard-leaf for the three quires that follow; their early codicological continuity is warranted by the presence of the Sicilian chronicle. It is possible that our manuscript has never been a codex; it may well be a part of a multi-quire scrapbook of some moderately literate monk in Southern Italy[107]. The contents of the surviving three quires, which gradually shift from computistical themes to magic (but see sect. **23** of *Anonymus* 892) passing through basic astronomy, meteorology, and astrology, corroborate this hypothesis.

The copyist of Par. suppl. gr. 920 must have been moderately literate because misspellings are ubiquitous in *Anonymus* 892. Some examples are: ἀναβαίνις for ἀναβαίνῃς (**8**), ἀρχημινίαν and ἀρχημηνίαν for ἀρχιμηνίαν (**24**), ἐκὴ for ἐκεῖ (**12**), ἐνεστότος for ἐνεστῶτος (**4**), ἔτι for ἔτη (**16**), ἡ

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year, for the intercalary day precedes Easter, so that the first *intervening* intercalary day occurs in cycle year 5, and the last and 23rd intercalary day in cycle year 93.

[102] Descriptions of the manuscript are found in Catalogus Codicum Astrologorum Graecorum VIII.4 89–92, and CH. ASTRUC – M-L. CONCASTY, Bibliothèque Nationale. Catalogue des manuscrits grecs. Troisième Partie. Le Supplément Grec III. Paris 1960, 18–19.

[103] *Grammatici Graeci*. I–IV. Lipsiae 1867–1910 III.1 521.11 ἔχοντας–13.

[104] The material contained in f. 1r–v is studied in F. ACERBI, How to Spell the Greek Alphabet Letters. *Estudios bizantinos* 7 (2019) 119–130; I have recently found two more witnesses of these gematric computations: Leiden, Universiteitsbibliotheek, Voss. gr. Qº 20 (13th century; *Diktyon* 38127) f. 5r, and Bologna, Biblioteca Universitaria, 3632 (middle of 15th century; *Diktyon* 9761) f. 284r.

[105] They can partly be found also in *Anonymus* 1092B, sects. 11–15.

[106] See the edition in P. SCHREINER, Die byzantinischen Kleinchroniken. I–III (*Corpus Fontium Historiae Byzantinae* 12). Wien 1975–1979 I 326–340 (nr. 45).

[107] *Anonymus* 892 does not exhibit any explicit connection with Latin Computi.

for εἰ (**12**–**13**, **17**–**18**), ἤτι and ἤδε for εἴ τι and εἰ δέ[108] (**4**–**6**, **8**), καθέτος for κατ᾽ ἔτος (**19**), κράτη for κράτει (**8**, **12**, **14**, **17**–**18**), κτήσεως for κτίσεως (**9**), ὄντως for ὄντος (**12**), οὖτως for οὗτος (**22**), παρελθότων for παρελθόντων (**12**), πληρόσεις for πληρώσεις (**12**), ψήφησον for ψήφισον (12**4**), ὧν for οὖν (**23**).

Morphological peculiarities include ὐφείλῃς, ὕφειλε / ὕφειλον, ὐφεῖλαι and similar forms of non-indicative moods of the aorist tense in which the augment is retained (*passim*); ἐνεμείνασιν, an aorist with primary ending (sect. **14**); μένουν for μένουσιν (**3**); future ἀνοίεται (**20**); imperatives ἄρχου (**12**) and ἄρξε (**12**). Temporal determinations are formulated in the genitive or with εἰς + accusative. Note also the construction, widespread in all Computi I know of, ἀπό + accusative (**17**, **26**).

The copying mistakes and the misspelling in *Anonymus* 892 show that the scribe of Par. suppl. gr. 920 has slavishly copied a source. The material and methodological mistakes and the inconsistencies in *Anonymus* 892 show that this source was a compilation; it is less likely that the compilation originates with Par. suppl. gr. 920. One of the inconsistencies in *Anonymus* 892 is that AM 6396 is assumed as the current year in sect. **7**, whereas all other sections carry out the computations for AM 6400. On account of the presence of the gematric computations on f. 1r–v, it is possible that the choice of an end-of-century year as the current year was dictated by arithmological considerations.

A THEMATIC WORD INDEX TO *ANONYMUS* 892

The first and the last two sections of this index are organized discursively: the English terms are between quotation marks; they are followed by the Greek term they translate; each Greek term is followed by the numbers of the sections of *Anonymus* 892 in which it occurs. The other sections of this index are a list of words; each Greek word is followed by its translation and by the numbers of the sections of *Anonymus* 892 in which it occurs.

Chronological lexicon

A “cycle” (κύκλος: **1**–**2**, **4**–**5**, **7**–**8**, **10**–**12**, **14**–**15**, **18**–**20**, **22**, **26**–**27**) “begins” (ἄρχεται: **1**), “reaches” (ἀνέρχεται: **1**, **11**, **25** αὖθις πότε ἀνέρχεται “when it reaches anew”) “to” (ἕως: **1**) its last year, and “reverts back again” (πάλιν ὑποστρέφει: **1**, **11**). Temporal segments and computations go “from” (ἀπό, for instance the “years from the foundation of the world” [ἀπὸ κτίσεως κόσμου ἔτη]: **1**, **4**–**7**, **9**, **11**, **27**) the first item in a sequence “up to” (ἕως: **2**, **8**–**9**, **11**–**12**, **14**–**19**, **22**, **25**–**26**; μέχρι: **4**) the last item[109]. A numerical interval is identified by “within” (ἔσωθεν: **12**), its complement by “outside” (ἔξωθεν: **22**), its extremes by “beginning” (ἀρχή, lying “above” [ἄνω(θεν)]: **12**, **22**, **26**) and “end” (τέλος, lying “below” [κάτωθεν]: **22**). Past time segments are “past” (παρατρέχοντα: **7**) or “bygone” (παρελθόντα: **12**); the current month is “ongoing” (κατέχων: **12**); the current “year” (ἔτος: **4**, **16**, **24**, **27**), “period” (περίοδος: **7**; it “comprises” [συνίσταται διά] 532 years), “indiction” (ἴνδικτος: **9**; elsewhere **1**, **6**–**7**, **27**), or cycle (**11**) is “present” (ἐνεστώς); a year next in a sequence is “next” (μέλλων: **24**). The first “day” (ἡμέρα: **1**, **3**, **8**, **12**–**20**, **23**–**24**) of a “month” (μήν: **1**, **8**, **12**–**13**, **17**–**20**, **22**, **24**, **26**) is its “starting-day” (ἀρχιμηνία: **24**), that is, where a month “begins” (ἄρχεται: **24**); a month ends with its “last” (τελευταία: **13**) day. The “year” is ἔτος (**4**–**7**, **9**–**11**, **14**–**16**, **19**–**21**, **24**, **26**–**27**), χρόνος (**1**, **19**–**20**, **24**, **27**), or ἐνιαυτός (**3**); the “week” is ἑβδομάς (**1**, **12**–**13**, **17**–**20**, **23**–**24**; “holy” [μεγάλη]: **12**); a day is made of “hours” (ὧραι: **1**, **3**, **8**, **25**), which are made of “minutes” (λεπτά: **1**, **8**). The determination of the date of a festival is stressed by “there” (ἐκεῖ: **12**, **15**–**18**); a date “occurred” (κατήντησε: **18**) or “falls” (ἐστί: **12**, **24**) on a “weekday” (ἡμέρα τῆς ἑβδομάδος: **12**–**13**, **17**–**18**, **20**, **24**). The age of the “Moon” (σελήνη **1**, **5**, **7**–**8**, **11**–**12**, **14**–**15**, **17**–

[108] I keep faithful to the text in making δέ enclitic and in attaching or not attaching enclitics to the previous word.
[109] In most Computi, but not in *Anonymus* 892, a καί “also” is added if the last item is included.

20, **22–23**, **25–27**) is also its "daily quantity" (καθημερινὴ ποσότης: **8**); the age in days is expressed by means of ordinals but also by means of –ταῖος adjectives (**25**); the "lunar month" is also called φέγγος (**8**, **23**). The Moon "shines" (λάμπει: **25**) so many hours in a night. The Moon "is in advance with respect to" (προλαμβάνει: **19**) the solar year; this advance is accumulated in the "epacts" (ἐπακταί: **14–15**, **19**) of the Moon, which periodically consolidate an "embolismic" (ἐμβόλιμος: **19**) year or month. The "Sun" (ἥλιος: **1–2**, **4**, **7**, **10**, **12**, **18–20**, **27**) completes a year in 365 or 366 days; the latter occurs in a "leap-year" (βίσεκτον: **3**, **12**, **17**, **21**, **24**; this year includes a "bissextile <day>" [βίσεκτον]: **3**, **24**; such a day may "be impeding" [ἐπιφέρειν]: **24**); at each solar cycle year, the excess in days over a whole number of weeks is accumulated in the "epacts" of the Sun (ἐπακταί: **2**, **20**). A general meaning of ἐπακταί as anything "brought in" is also adopted (**8**, **12**).

Specific mathematical lexicon[110]

Investigation. γινώσκω: to know (**1**, **2**, **8**, **12**, **20**, **22**, **24–27**); εὑρίσκω: to find (**2**, **4–14**, **17–18**, **20**, **22**, **24**, **26–27**) and εὕρεσις: finding (**12**); ζητέω: to seek for (**14**, **18**).

Initializing an algorithm. βλέπε: look at (**8**); γίνωσκε / γνῶθι: know (**4**, **8**, **15**, **17** ἀκριβῶς: exactly, **18–19**, **22**, **24** / **12**, **16**); ζήτησον: seek for (**18**); ἔχε κατανοῦν: keep in mind (**13**); κάτεχε: hold (**7**, **18** ἐπὶ δακτύλων σου: in your fingers); κράτει: keep (**2**, **8**, **12**, **14**, **17**, **18**, **20**); κράτει εἰς τὰς χεῖρας σου / ἐπὶ χειρῶν / δακτύλων σου: keep in your hands / in your fingers (**8**, **14**, **18**)[111].

Counting and reckoning. ἀνέρχομαι: to reach (**12**, **17**, **18**); ἀπαρτίζω: to complete (**14**, **21**) and ἀπαρτισμός: completion (**19**); ἀριθμέω: to count (**15**) and ἀριθμός: number (**4**, **7**, **14–15**, **20**, **25–26**); ἄρχομαι: to begin (**12–13**, **15**, **22**); καταντάω: to arrive at (**15**, **24**); μετρέω: to determine (**8**); πλεονάζω: to exceed (**20**); πληρόω: to fill (**12**); φθάνω: to attain (**12**); ψηφίζω: to calculate (**12**, **17**, **26**) and ψῆφος / ψηφοφορία: calculation (**4–6**, **8–17**, **20**, **24–26** / **18**). The result of any operation can be indicated by ποιέω: to make (**15–17**).

Identification of the result of an operation as a chronological item. γινώσκω: to know (**12**, **14**, **17–18**, **24**, **26**); γνωρίζω: to recognize (**10**); δηλόω: to show (**7**, **13**); εὑρίσκω: to find (**8–10**); νοέω: to consider (**24–25**); σημαίνω: to signify (**25**).

Unknown quantities. ὅπου: wherever (**24**); ὅσος: what (**8**, **12–13**, **25**); ὅσπερ: that which (**25**); ποῖος: what, which (**8**, **12**, **17–18**, **24**); πόσος: how much (**2**, **4**, **16–17**, **25**); τοσοῦτος: such, so much (**2**, **4**, **8**, **12**).

Numerical sets. ἑβδομάς: week (that is, heptad) (**1**, **12–13**, **17–20**, **23–24**); ἑνδεκάς: hendecad (**19**); μυριάς: myriad (**1**); τετράς: tetrad (**2**); τριακοντάς: thirty (**12**); χιλιάς: thousand (**7**, **27**).

Operations

Addition. ἀναβαίνω: to mount (**2**, **8**, **17**); βάλλω: to put (**8**, **12–13**); ἑνόω ὅλους: to unite all of them (**8**, **12**); προστίθημι ἐν / εἰς: to add in / to (**7–8**, **12**, **14–15**, **26**); συνάπτω: to conjoin (**15**, **16**). The result is indicated by γίνομαι: to yield (**1**, **3**, **12**, **26**); σύναξις: gathering (**19**). The operation is called προσθήκη: addition (**19–20**, **24**, **26** where "remarkable cycles" [σεσημειώμενοι κύκλοι] may not "admit of it" [προσδέχεσθαι]).

Subtraction. ἀφίημι: to discharge (**7**); ὑφαιρέω: to remove (**8**, **12**, **14**, **16**, **20**, **26–27**). The "remainder" (λοιπός **16**; "remaining" **26–27**) is mainly indicated by predicative λοιπόν: as a remainder (**9–11**, **27**), but also by verbs μένω: to remain (**3–12**, **14**, **16**, **20**, **26–27**; ἐπὶ δακτύλων σου **14**) and

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[110] The technical lexicon of Computi overlaps with the technical lexicon of *Rechenbücher*. For the latter, see K. VOGEL, Ein byzantinisches Rechenbuch des frühen 14. Jahrhunderts (*Wiener Byzantinische Studien* 6). Wien 1968, 141–143, and the "thematic word index" in ACERBI, Byzantine Rechenbücher, whose translations I adopt.

[111] In logistic and astronomical texts, *Rechenbücher*, and Easter Computi, the verb κρατέω for "keeping" a number in order to use it in an operation is frequently found. However, this does not imply anything as to a possible application of a finger-notation, even if the verb is qualified by an expression like "in your hands".

περισσεύω: to remain over (**19**–**20**, **25**), and the related noun περισσεία: remainder (**19**). The operation is called ὑφειλμός: removal (**26**–**27**).

Multiplication. It is formulated by means of an –ακις adverb (**3**, **8**–**12**, **14**, **20**–**21**, **25**); this is systematically written, for instance, ἑπτάι or ἑπτάη or ἑπτάει for ἑπτάκις "seven times", a spelling common to many Computi and that I have normalized.

Taking multiples. Imperatives δίπλωσον: double (**1**, **3**); ἑνδεκαπλασίαζε (**14**) and ἑνδεκαπλασίασον (**12**, **20**): undecuple; ἑξάπλωσον: sextuple (**8**); πεντάπλου: quintuple (**25**). Subjunctives ἑξαπλόννῃς: sextuple (**8**); πενταπλόννῃς: quintuple (**8**). Participles ἑνδεκαπλασιάσας: undecupling (**12**) and ἑξαπλώμενον: sextupled (**8**).

Division. ἀναλύω εἰς: to resolve out into (**21**, **25**).

Modulo reduction. ὑφαιρέω ἐπί / διά: to remove by (**4**–**14**, **16**, **20**). The remainder is often indicated by (μένω) κάτωθεν: to remain down from (**4**–**8**, **12**–**14**, **16**), but the adverb may be absent.

Connectors and particles

Explanations of specific steps are introduced by "for" (γάρ: **14**, **19**–**20**, **23**), in some cases they are introduced by "since … really" (ἐπειδή: **14**, **20**) or "since" (ἐπεί: **14**). Synonyms are introduced by "namely" (ἤγουν: **7**, **19**–**20**) or "viz." (ἤτοι: **20**). A refreshed algorithm is introduced by "again" (πάλιν: **8**, **11**–**12**, **18**, **20**, **26**), a branching by "with the <following> exceptions" (πλήν: **14**). Operations that necessarily precede other steps may be introduced by "as soon as" (ἐπάν: **12**, **17**). There is just one occurrence of "then" (οὖν: **23**); the only occurrence of γε (**12**) is rendered by italicizing the lexical item the particle has scope over. Negation can be in the form οὐχί (**26**).

Metadiscourse

Universality of numerical parameters is conveyed by the adverb "always" (πάντοτε: **12**–**15**, **18**, **26**), genericity by the determiner "whatever" (οἱοσδηποτοῦν / οἱοσδήποτε: **12**, **14**, **24**). Iteration is formulated by "continuously" or "so on" (καθεξῆς: **14**, **20**, **24**–**25** or **12**, **14**, **18**). A shortened algorithm is marked by "easily" (εὐκόλως: **26**) or referred to as "concise" (σύντομος: **17**, **26**). Quantitative correlation is formulated by "how many … so many" ([π]όσος … τοσοῦτος: **2**, **8**). Examples are introduced by "for instance" (οἷον: **19**, **20**, **26**). Metamathematical markers include the modal operators "one must" (δεῖ: **1**, **2**, **27**) and "one has to" (χρή: **27**), the volition verbs "to want" (θέλω: **7**–**9**, **12**–**13**, **20**, **24**, **26**) and "to hesitate" (ὀκνέω: **7**), the modal verb "can" (δύναμαι: **12**). The verb form "there it is" (ἰδού: **19**, **26**–**27**) introduces a result. The verb forms "say" (λέγε: **21**–**22**; εἰπέ: **8**, **12**) and "do" (ποίει: **24** / ποίησον: **18**) initialize a computation. The adverb "as follows" / "in this way" (οὕτως: **2**, **9**–**11**, **14**, **18**, **24** / **3**, **12**, **19**–**20**, **24**–**25**) introduces an algorithm or refers to an algorithm just carried out. The adverb "how" (πῶς: **2**–**3**, **21**, **27**) presents an algorithm; the adverb "otherwise" (ἄλλως: **7**) presents an alternative algorithm; the adjective "further" (ἕτερος: **5**–**6**, **10**–**13**, **15**–**17**, **20**, **25**) introduces a new section. The adverbs "similarly" (ὁμοίως: **8**, **12**, **14**, **18**, **24**) and "likewise" (ὡσαύτως: **18**), and the syntagm "in the same way" (τῷ αὐτῷ τρόπῳ: **26**) replace an algorithm that is identical to an algorithm previously carried out. The expressions "exactly as we said / taught above" ([καθ]ὼς προείπαμεν / προεδιδάξαμεν: **8**, **12**) refer to algorithms previously carried out. A hiatus of an algorithm is indicated by "that's fine" (εὖ καὶ καλῶς: **12**); a restated rule is introduced by "the other way around" (ἀνάπαλιν: **12**). Note the two occurrences of the bewildering clause "for you do not find what is secure" (ἐπεὶ οὐκ εὑρίσκεις τὸ ἀσφαλές: **12**, **14**), stressing an unexpected branching in an algorithm.

PRELIMINARIES TO THE EDITION

The text of *Anonymus* 892 is divided into sections according to the titles in the manuscript. The subject-matters of the sections are as follows. Section **1**: subdivision of the year; **2**: epacts of the

Sun; **3**: leap years; **4**–**7** and **9**–**11**, **27** (examples): conversion from a world era year to a year within an assigned cycle; **8**: age of the Moon; **12**: date and weekday of Passover, dates of Easter and of Meat-Fare Sunday; **13**: weekday of an assigned date; **14**–**17**: alternative algorithms for the date of Passover; **18**: list of Passover dates in a lunar cycle; **19**: embolismic years; **20**: epacts of the Sun, names of the weekdays, and periods of the seven planets; **21**: leap years; **22**: terms for Passover; **23**: lucubrations about the phases of the Moon; **24**: weekday of the first day of any month; **25**: illumination of the waxing and waning Moon; **26**: how to compute the Passover dates in sequence. Each section presents the Greek text, its translation, and a commented paraphrase that is printed in reduced font size and is preceded by a title summarizing the contents of the section.

Edition. I have retained the original accents of proclitics and enclitics; otherwise, the accents are normalized to the conventions presently in use. After much hesitation, I have decided not to keep the original punctuation, for the following reasons. Most of the time, *Anonymus* 892 exhibits a beautiful "algorithmic" punctuation, made of comma and of a point located in a position that, however, covers the entire range between upper and lower. Because of this ambiguity, because the punctuation is not applied consistently in *Anonymus* 892—which almost uniquely comprises algorithmic texts—and because uniformity of punctuation is required if our aim is to study Computi as a corpus, I have punctuated the text anew, following the "algorithmic" rules I use in editing Greek and Byzantine mathematical texts[112]. In particular, such rules prescribe that consecutive steps of an algorithm are separated by an upper point; that an algorithmic hiatus is marked by a full stop; that commas separate the principal clauses of a procedure and the result of a multiplication from the two factors.

Lexical and morphological peculiarities of *Anonymus* 892 are kept unchanged: this Computus attests for a use of vernacular Greek that should not be erased. I have corrected the misspellings, but they are all recorded in the critical apparatus. Numeral letters standing for integers are not marked by an apex; ordinals that in the text are given as numeral letters are written with a raised ending; dates are always treated as counting numbers; I have normalized mixed numerals such as ἐννακαιιτου = ἐννεακαιδεκάτου or πεντικονταδ = πεντηκοντατέσσαρες. I have maintained adverbial expressions written in one single word as they appear, like κατέτος = κατ' ἔτος or κατανοῦν = κατὰ νοῦν.

Translation. Different Greek terms are normally translated with different English terms; the translations adopted for the main terms are listed in the thematic word index given in the previous section. As a rule, I do not translate δέ; otherwise, it is rendered by "and"; all other lexical items are translated. Within algorithms, aorist indicative is translated as a present. Words supplied in translation are put within angular brackets <…>; the reference of some pronouns is made explicit between square brackets […]. The translation of the algorithms is punctuated as follows: a colon precedes the statement of a result; a semicolon separates steps in which the output-input chain is not interrupted; a full stop indicates an algorithmic hiatus and precedes the final winding-up, where the outcome of the algorithm is identified as a specific chronological item.

Commented paraphrase. In my commented paraphrase, I provide a symbolic transcription of the algorithms set out in the text[113]. This kind of transcription is more faithful both to the syntactical

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[112] These rules are not rooted in the punctuation practice of any specific language; they are expounded in ACERBI, The Logical Syntax, sect. 1.4.

[113] The notation used in the commentary is as follows: 1_J, 1_F, 1_M, 1_A, 30_S, etc. = January 1, February 1, March 1, April 1, September 30, etc.; $a_m(x)$ = age of the Moon on day x in the calendar year at lunar cycle year m; $a_m(x_X)$ = age of the Moon on day x of month X at lunar cycle year m; b_m = base of the Moon at lunar cycle year m; e_m = epacts of the Moon at lunar cycle year m; e_s = epacts of the Sun at solar cycle year s; f = Meat-Fare Sunday; g = Period year; i = indiction cycle year; *J*, *F*, *M*, *A*, *Ma*, *Jn*, *Jl*, *Au*, *S*, *O*, *N*, *D* = January, February, March, April, May, etc. *or* counting number of their position in this sequence (the context always decides); l = leap year cycle year; m = lunar cycle year; p_m = date of Passover at lunar cycle year m; r_m = date of Easter at lunar cycle year m; s = solar cycle year; y = year in the Byzantine world era; $w_s(x_X)$ = weekday of day x of month X at solar cycle year s.

structure and to the "mathematical content" of the original algorithm than an algebraic formula summarizing the entire algorithm in one single equality can be. Nevertheless, such a formula can be found at the end of the algorithm (read the algorithm from right to left). Any symbolic transcription occupies no more than a handful of lines; it is followed by a commented paraphrase that combines rephrased sentences of *Anonymus* 892 and explanations of mine. The latter are sometimes intended to clarify the stepwise progression of the algorithm. These two components are easily disentangled with the help of the translation. In principle, the commented paraphrase is self-contained; readers who dislike the symbolic transcriptions may simply skip them[114].

EDITION AND TRANSLATION OF *ANONYMUS* 892, WITH A COMMENTARY

1

δεῖ γινώσκειν[a] ὅτι ἄρχεται ὁ κύκλος τοῦ ἡλίου ἀπὸ τῆς α τοῦ Ὀκτοβρίου, καὶ ἀνέρχεται ἕως κη, καὶ πάλιν εἰς πρῶτον ὑποστρέφει.

Ὁ δὲ τῆς σελήνης κύκλος ἄρχεται ἀπὸ τῆς α τοῦ Ἰαννουαρίου, καὶ ἀνέρχεται ἕως τῶν ιθ, καὶ πάλιν εἰς α ὑποστρέφει.

Ἡ δὲ ἴνδικτος ἀπὸ τῆς α τοῦ Σεπτεμβρίου μηνὸς ἄρχεται, καὶ ἀνέρχεται ἕως τῶν ιε, καὶ πάλιν εἰς α ὑποστρέφει.

Ὁ χρόνος ἔχει ἑβδομάδας νβ, ἡμέρας τξε δ^{ov}, ὥρας ˏδτπ – ταύτας δίπλωσον, καὶ γίνονται ὧραι ˏηψξ – λεπτὰ β μυριάδας ˏαϡ.

[a] γινώσκην

One must know that the cycle of the Sun begins on October 1, and reaches to <cycle> 28, and reverts back again to the first <cycle>

The cycle of the Moon begins on January 1, and reaches to 19, and reverts back again to 1.

The indiction begins on the 1st of the month of September, and reaches to 15, and reverts back again to 1.

The year has 52 weeks, 365 ¼ days, 4380 hours – double these, and they yield 8760 hours – 2 myriads and 1900 minutes.

Features of the solar, lunar, and indiction cycles; subdivision of the year. The solar cycle begins on October 1 and lasts 28 years. The lunar cycle begins on January 1 and lasts 19 years. The indiction cycle begins on September 1 and lasts 15 years. The subdivision of the year is: 1 year (χρόνος) = 52 weeks = 365 ¼ days = 4380 [double-]hours = 8760 hours = 21900 minutes. The third equality stated in the text is valid only for a year of 365 days; otherwise, one gets 365 ¼ days = 4383 [double-]hours = 8766 hours = 21915 minutes (see sect. **3**). I have introduced the qualifier "double" to distinguish between the two "hours" mentioned in the text; in principle, there is no connection with the "double hours" of the astronomical tradition[115], nor is there any connection with the algorithms that give the additional day of a leap year 12 hours (see sect. **3**). The above subdivision can be summarized as follows[116]:

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[114] I shall insert in footnotes references to analogous procedures or to similar sets of data found in early, unpublished, sources or in later Computi, as Psellos' or *Anonymus* 1092A–C; in the case of the latter, the relation with *Anonymus* 892 is obvious. The Appendix contains a concordance with *Anonymus* 830.

[115] For double-hours, see F. BÖLL, Sphaera. Neue griechische texte und Untersuchungen zur Geschichte der Sternbilder. Leipzig 1903, 311–319. For the subdivisions of an hour, see P. TANNERY, Sur les subdivisions des heures dans l'antiquité. *RA*, 3^{e} série, 26 (1895) 359–364, repr. ID., Mémoires scientifiques II. Toulouse – Paris 1912, 517–526.

[116] The subdivision in *Anonymus* 1092A, sect. 1, in Laur. Plut. 57.42, f. 154v, reads: 1 year = 52 weeks = 365 ¼ days = 4380 [double-]hours = 8760 hours = 43830 points = 21915 minutes. On account of sect. 2 of *Anonymus* 1092A, the text should be corrected as in KARNTHALER, Die chronologischen Abhandlungen 5.16–18: 1 year = 52 weeks = 365 ¼ days = 4383 [double-]hours = 8766 hours = 21915 points = 43830 minutes. Sect. **1** of *Anonymus* 892 should be corrected in the same way. Note the subdivision in *Anonymus* 1172, sect. 12: 1 year = 12 months = 52 weeks plus 1 day = 365 days = 4380 [double-]hours = 11900 [*lege* 21900] points.

year	day	[double-]hour	hour	minute
1	365 ¼	4380	8760	21900
	1	12	24	60
		1	2	5
			1	2 ½

Other subdivisions of the year are adopted in Byzanine Computi. A standard subdivision is as follows[117]:

year	day	hour	minute	point	momentum
1	365 ¼	8766	43830	175320	2103840
	1	24	120	480	5760
		1	5	20	240
			1	4	48
				1	12

whereas *Anonymus* 1256, sect. 16, sets out the following subdivision:

year	day	hour	point	momentum
1	365	8760	26280	105120
	1	24	72	288
		1	3	12
			1	4

The subdivision of the hour adopted by *Anonymus* 1092A, sect. 2, in KARNTHALER, Die chronologischen Abhandlungen 5.20–26, is as follows:

[double-]hour	point	minute	degree	momentum	indication	atom
1	5	10	150	1200	14400	864000
	1	2	30	240	2880	172800
		1	15	120	1440	86400
			1	8	96	5760
				1	12	720
					1	60

The "minutes" of *Anonymus* 892, and, more generally, of the Computistical tradition, do not coincide with our $^{1}/_{60}$-hour minutes. Two conventions were used, neatly differentiated by the context. The equivalence 1 hour = 5 minutes → 12 hours = 60 minutes, is usually found in the algorithms that compute the duration of visibility of the waxing and waning Moon (see sect. **25**)[118]. The equivalence 2 hours = 5 minutes → 1 day = 60 minutes, was used in other contexts, which in general are related to the age of the Moon[119].

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[117] See *Anonymus* 1041, sect. 1 (only the subdivisions of an hour), a Computus witnessed in the manuscript Napoli, Biblioteca Nazionale Vittorio Emanuele III, II.C.34 (beginning of 16th century; *Diktyon* 46080) ff. 100r–106v; *Psellos* 1092, sect. II.22, in REDL, La chronologie appliquée II 257.12–22; *Anonymus* 1183, sect. 1; *Anonymus* 1247, sect. 22, in SCHISSEL, Chronologischer 110.

[118] See *Anonymus* 830, sects. 24 and 33 (the fractions of an hour are called "points" [στιγμαί]), in GASTGEBER, Neue texte XXX and XXX; *Theophylaktos* 956 sect. 5–6 (πενταπλοῦντες καὶ ἑξαπλοῦντες and waxing and waning Moon, respectively); *Anonymus* 982, sect. 23; *Anonymus* 1041, sects. 1 and 19 (the latter waxing and waning Moon); *Anonymus* 1092B, sect. 5, in KARNTHALER, Die chronologischen Abhandlungen 9.159–170; *Anonymus* 1172, sect. 21; *Rhabdas* 1342, sect. 9; *Anonymus* 1377, sect. 7, in *PG* XIX 1324–1328. *Anonymus* 892, sect. **25**, does not mention minutes. In *Anonymus* 951, sect. 13 (an independent section) these fractions of an hour are called "points".

[119] See *Anonymus* 892, sect. **8** (age of the Moon computed according to the πενταπλοῦντες καὶ ἑξαπλοῦντες); *Anonymus* 1079, sect. 5, in MENTZ, Beiträge zur Osterfestberechnung 82, and the Computus contained in Par. suppl. gr. 690, sect. 1 (πενταπλοῦντες καὶ ἑξαπλοῦντες); *Anonymus* 1092B, sect. 7, in KARNTHALER, Die chronologischen Abhandlungen 11.234 (*saltus lunae* distributed among the lunar cycle years); *Rhabdas* 1342, sect. 8 (epacts of the Moon). In *Anonymus* 951, sect. 1 (*Horopodion*) these fractions of an hour are called "momenta" (ῥοπαί).

2

τὸ πῶς δεῖ εὑρίσκειν τὰς ἐπακτὰς τοῦ ἡλίου

Αἱ ἐπακταὶ τοῦ ἡλίου εὑρίσκονται οὕτως. κράτει τὸν κύκλον τοῦ ἡλίου, καὶ γνῶθι πόσας τετράδας ἔχεις, τοσαῦται ἐπα|$_{2v}$κταὶ εἰσὶν τοῦ ἡλίου[a]. αἱ ἐπακταὶ ἕως τῶν ἑπτὰ ἀναβαίνουσιν, καὶ πάλιν εἰς τὸν α ἀναστρέφουσιν.

[a] γνῶθι — ἡλίου corruptum cf. versionem et comm.

How one must find the epacts of the Sun

The epacts of the Sun can be found as follows. Keep the cycle of the Sun, and know how many tetrads you have, <and mount them, and remove by 7, and how many remain down from 7,> so many epacts of the Sun there are. The epacts mount up to seven, and revert back again to 1.

A computation for finding the epacts of the Sun[120]. The reconstructed algorithm is[121]:

$(s) \to s + \llbracket s/4 \rrbracket \to (s + \llbracket s/4 \rrbracket) \bmod 7 = e_s$.

See also sects. **12** and **20**. This algorithm computes the cumulative excess of the days, counted from the beginning of a solar cycle, of an assigned solar cycle year over a whole number of weeks. To explain the above algorithm, it should be kept in mind that a year of 365 days exceeds a whole number of weeks by 1 day (summand s in the algorithm; it also includes 365 of the 366 days of a leap year), a leap year exceeds it by 2 (further summand $\llbracket s/4 \rrbracket$; these are the "tetrads", that is, the number of leap years since the beginning of the cycle)[122]. Reducing the sum modulo 7 eliminates whole weeks. For example, one computes that $s = 1 \to e_s = 1$. As expressly stated in the text, the convention is that 7 mod 7 = 7. The transmitted algorithm is corrupt, since the solar cycle year and the tetrads must be added in order to find the epacts of the Sun.

3

τὸ πῶς γίνεται βίσεκτον

Ὁ ἐνιαυτὸς ἔχει ἡμέρας τξε δ^{ov}· ταύτας δίπλωσον· καὶ γίνονται ὧραι <ͅη>ψξ[a]· ἑπτάκις διακώσ<ιοι>, ͅαυ· μένουν τριακόσια ξ· ἑπτάκις ν, τν· μένουν ι· ἑπτάκις μίαν, ζ· καὶ μένουν γ ὧραι κατ' ἐνιαυτόν, καὶ εἰς τοὺς τεσσάρες[b] ἐνιαυτοὺς γίνονται ὧραι δώδεκα, καὶ οὕτως γίνεται τὸ βίσεκτον.

[a] corruptum cf. versionem et comm. [b] τεσσάρεις

How a bissextile <day> comes to be

The year has 365 ¼ days, <4380 hours;> double these: and they yield <8>760 hours[123]; <remove these by 7; seven times 1000, 7000: there remain 1760;> seven times two hundred, 1400: there remain three hundred 60; seven times 50, 350: there remain 10; seven times one, 7: and there

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[120] Other occurrences of this algorithm can be found in George, sect. II.3, in DIEKAMP, Der Mönch 26.3–14 (the epacts of the Sun are called ἐπακταὶ τῶν ἑβδομάδων "epacts of the weeks"); Maximus, *Enarratio* I.29, in *PG* XIX 1248; *Anonymus* 830, sects. 6 (identical wording) and 14, in GASTGEBER, Neue texte XXX and XXX; *Anonymus* 951, sect. 4; *Anonymus* 1092B, sect. 10 (identical wording), in KARNTHALER, Die chronologischen Abhandlungen 12.281–285; *Anonymus* 1172, sect. 19.

[121] The sign $\llbracket x \rrbracket$ denotes the floor (or integral part) of number x, namely, the nearest integer (0 included) less than or equal to x. The floor function is particularly effective in formalizing leap year computations: if y is a year in the Byzantine world era or in the era AD, $\llbracket (y \bmod 4)/4 \rrbracket$ singles out leap years—which in both eras are such that $y = 4k$ for some integer k—because $y \equiv 1, 2, 3$ or 4 (mod 4), and $\llbracket {}^1\!/_4 \rrbracket = \llbracket {}^2\!/_4 \rrbracket = \llbracket {}^3\!/_4 \rrbracket = 0$, $\llbracket 1 \rrbracket = 1$. As taking the floor of a division amounts to taking its quotient by disregarding the remainder, $\llbracket y/4 \rrbracket$ is the number of leap years since epoch. For any two integers m and n, by definition and with a slight abuse of notation, ${}^m\!/_n = \llbracket {}^m\!/_n \rrbracket + (m \bmod n)/n$, for the first addendum is the quotient of the division of m by n, and m mod n is the remainder of the same quotient. Alternatively, we may write $m = n\llbracket {}^m\!/_n \rrbracket + m \bmod n$.

[122] A solar cycle of 28 years contains by definition 7 leap years, whose 1-day-each overall contribution of 7 days is deleted, at the end of the cycle, by the modulo 7 reduction. Thus, the contribution of leap years to the epacts of the Sun can be restricted to the leap years occurring within a single solar cycle.

[123] The ensuing calculation makes it necessary to integrate the number 8 before 760, but the text also presents two lacunae; I fill them in my translation according to *Anonymus* 1092B, sect. 8, in KARNTHALER, Die chronologischen Abhandlungen 11.250–258. See also the commentary.

remain 3 hours per year, and they yield twelve hours in four years, and in this way the bissextile <day> comes to be.

The origin of leap years. The text is incomplete and the calculation mistaken, as it reads as follows: 365 ¼ days = 8760 hours; 8760 mod 7 = 3 hours; in 4 years, these 3 hours add up to 12 hours, namely, to 1 day. The text can be completed readily[124], but the (correct) modulo 7 calculation is a wrong attempt at coping with the mistake of setting 365 ¼ days = 8760 hours (see sect. **1**): the result 8760 mod 7 = 3 fits the number of exceeding double-hours per year only numerically and by coincidence[125]. To explain the presence of 366-day years, it suffices to remark that an excess of ¼ a day per year becomes an excess of one full day after 4 years.

The modulo reduction is equivalent to computing the remainder of a division between integers. Here, as elsewhere in *Anonymus* 892 and, more generally, in Byzantine Computi, the calculation is carried out by successively removing multiples of the divisor. This procedure is attested as early as Paul of Alexandria, *Apotel*. 19–20[126]. As only a few moduli are used in computistical reductions (namely, 7, 15, 19, 28, and 30), it is not necessary to suppose that multiplication tables were used to ease such calculations. Tables of this kind are never included in Byzantine Computi.

4

Σὺν θεῷ ψῆφος δι᾽ οὗ εὑρίσκεται ὁ κύκλος τοῦ ἡλίου

Γίνωσκε πόσα ἔτη εἰσὶν ἀπὸ κτίσεως κόσμου μέχρι τοῦ ἐνεστῶτος[a] ἔτους, καὶ ὕφειλε[b] αὐτὰ διὰ τῶν κη, καὶ εἴ τι[c] μένουσιν[d] κάτωθεν τῶν κη, ἔστιν ὁ κύκλος τοῦ ἡλίου· εἰ δε[e] ὕφειλον τοσοῦτον ἀριθμὸν τῶν ἐτῶν πάντων καὶ μέ|$_{3r}$νουσιν κη, εἰκο<σ>τὸς ὄγδοος κύκλος ἐστὶν τοῦ ἡλίου.

[a] ἐνεστότος [b] ὕφηλε [c] ἤτι [d] exp. μένει [e] ἤδε

God willing, calculation by means of which the cycle of the Sun can be found

Know how many years there are from the foundation of the world up to the present year, and remove them by 28, and if anything remains down from 28, this is the cycle of the Sun; but if <you> removed such a number of all the years and there remain 28, this is the twenty-eighth cycle of the Sun.

A prescription for finding the year of the solar cycle corresponding to a Byzantine world era year. The algorithm is $(y) \rightarrow y \bmod 28 = s$.

As expressly stated in the text, the convention is that 28 mod 28 = 28. The text writes "cycle of the Sun" and "twenty-eighth cycle", using κύκλος to name both the 28-year solar "cycle" and a solar "cycle year" within a solar cycle. Sections **4–6** allow converting a year in the Byzantine world era to a year within an assigned cycle. The indiction cycle is of no use in computing the date of Easter; its presence shows that an "Easter Computus" was conceived as a general chronological primer. As seen, in Byzantine Computi the solar cycle, the lunar cycle and the reference era are synchronized. For this reason, the reduction rules from world era years to solar and lunar cycle years are straightforward.

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[124] Other early occurrences of this algorithm can be found in the computistical section of the *Florilegium Coislinianum*, letter Π, nº 169; *Anonymus* 830, sect. 31, in GASTGEBER, Neue texte XXX; *Anonymus* 951, sect. 12; *Anonymus* 1041, sect. 14; *Anonymus* 1090, sect. 4, contained in the manuscript Berlin, Staatsbibliothek (Preußischer Kulturbesitz), Ham. 625 (*Diktyon* 9305) ff. 327r–329v; *Anonymus* 1172, sect. 22. An in-depth discussion of this algorithm (which is Dionysius' *argumentum* XVI: KRUSCH, Studien [1938] 80.17–81.6; see also K. SPRINGSFELD, Alkuins Einfluß auf die Komputistik zur Zeit Karls des Großen [*Sudhoffs Archiv* 48]. Stuttgart 2003, 203–214) in Western Computi is found in a study in preparation by T. Loevenich and I. Warntjes. On the tradition that the annual contribution to a leap-year was 3 hours (it is refuted by Bede, *De Temporum Ratione* XXXIX), see also JONES, Bedae Opera 372–374, and M. SMYTH, Once in Four: The Leap Year in Early Medieval Thought, in: Late Antique Calendrical Thought and Its Reception in the Early Middle Ages, ed. I. Warntjes – D. Ó Cróinín (*Studia Traditionis Theologiae* 26). Turnhout 2017, 229–264: 236–247.

[125] If hours are employed, 8760 should read 8766, and a modulo 24 reduction must be performed; if "[double-]hours" are employed, 4383 [double-]hours and a modulo 12 reduction are required, respectively.

[126] Alexandrian and Byzantine Early Computi may formulate the modulo reduction as a division: see for instance the Computus in Ambr. A 45 sup., ff. 7r–8r, in GASTGEBER, Neue texte XXX–XXX; George *passim*.

5

ἕτερος ψῆφος, δι᾽ οὗ εὑρίσκεται ὁ κύκλος τῆς σελήνης

Ὕφειλε ταῦτα τὰ ἔτη ἀπὸ κτίσεως κόσμου διὰ τῶν ιθ, καὶ εἴ τι[a] μένει κάτωθεν τῶν ιθ, ἔστιν ὁ κύκλος τῆς σελήνης.

[a] ἤτι

A further calculation, by means of which the cycle of the Moon can be found

Remove these years from the foundation of the world by 19, and if anything remains down from 19, this is the cycle of the Moon.

A prescription for finding the year of the lunar cycle corresponding to a Byzantine world era year. The algorithm is $(y) \rightarrow y \bmod 19 = m$.

6

ἕτερος ψῆφος, δι᾽ οὗ εὑρίσκεται ἴνδικτος

Ὕφειλε ταῦτα τὰ ἔτη ἀπὸ κτίσεως κόσμου διὰ τῶν ιε, καὶ εἴ τι[a] μένει κάτωθεν τῶν ιε, ἔστιν ἴνδικτος.

[a] ἤτι

A further calculation, by means of which the indiction can be found

Remove these years from the foundation of the world by 15, and if anything remains down from 15, this is the indiction.

A prescription for finding the year of the indiction cycle corresponding to a Byzantine world era year. The algorithm is $(y) \rightarrow y \bmod 15 = i$.

7

καὶ ἄλλως

[a]Εἰ δὲ ὀκνῇς ὑφεῖλαι ἀπὸ κτίσεως κόσμου, ἄφες τὰς ἓξ χιλιάδας τῶν ἐτῶν καὶ τὰ τ ἔτη, καὶ κάτεχε τὰ ἐνενήκοντα[b] ἓξ τὰ παρατρέχοντα ἄρτι[c], καὶ ὕφειλε αὐτὰ ἐπὶ μὲν τοῦ ἡλίου διὰ τῶν κη, καὶ τὰ μένοντα κάτωθεν τῶν κη δηλοῦσιν τὸν κύκλον τοῦ ἡλίου. ἐπὶ δὲ τῆς ἰνδίκτου |$_{3v}$ ὕφειλε δεκαπέντε, καὶ τὰ κάτωθεν τῶν ιε δηλοῦσιν αὐτήν. ἐπὶ δὲ τῆς σελήνης πρόσθες λ ἔτη εἰς τὰ παρατρέχοντα, καὶ ὕφειλε[d] διὰ τῶν ιθ, καὶ τὰ μένοντα κάτωθεν τῶν ιθ δηλοῦσιν τὸν κύκλον τῆς σελήνης. ἡ δὲ περίοδος συνίσταται διὰ πεντακοσίων τριακονταδύο ἐτῶν, ἤγουν διὰ τοῦ ἀριθμοῦ τοῦ ἄλφα. καὶ ἐὰν θέλῃς εὑρεῖν τὴν ἐνεστῶσαν περίοδον[e], ὕφειλε τὰ ἔτη τοῦ κόσμου διὰ τῶν πεντακοσίων λβ, καὶ τὰ μένοντα κάτωθεν τῶν φλβ δηλοῦσιν αὐτήν.

[a] ση(μείωσαι) marg. [b] ἐνενίκοντα [c] ἄρτη [d] ὕφηλε [e] περίωδον

And otherwise

If you hesitate to remove from the foundation of the world, discharge six thousand and 300 years from the years, and hold the ninety-six presently past, and for the Sun remove them by 28, and that which remains down from 28 shows the cycle of the Sun. For the indiction, remove <by> fifteen, and that which <remains> down from 15 shows it [*scil*. the indiction]. For the Moon, add 30 years to those past, and remove by 19, and that which remains down from 19 shows the cycle of the Moon. The Period comprises five-hundred-thirty-two years, namely, the number of *alpha*. And if you want to find the present Period, remove the years of the world by five hundred 32, and that which remains down from 532 shows it [*scil*. the Period].

Alternative algorithms, supposedly simpler, for finding the year of the solar, lunar, and indiction cycles, and of the Period. The algorithm for the solar cycle is:

$(y) \rightarrow y - 6300 \rightarrow (y - 6300) \bmod 28 = m$.

A computation of $y - 6300$ is carried out for current year AM 6396 [= AD 887/8]; the lunar cycle is not computed. This current year is different from the one assumed in the rest of *Anonymus* 892.

The algorithm for the indiction cycle is:

$(y) \rightarrow y - 6300 \rightarrow (y - 6300) \bmod 15 = i$.

The algorithm for the lunar cycle is:

$(y) \rightarrow y - 6300 \rightarrow (y - 6300) + 30 \rightarrow [(y - 6300) + 30] \bmod 19 = m$.

The algorithm for the Period (περίοδος) of 532 years, the number of ἄλφα[127], is:

$(y) \rightarrow y \bmod 532 = g$.

In these algorithms, the nearest end-of-century year is removed from the assigned world era year before the modulo reduction is carried out. Since $6300 \equiv 0 \pmod{28}$ and $6300 \equiv 0 \pmod{15}$, the algorithms for the solar and indiction cycles are modified by introducing only the shift $y \rightarrow y - 6300$. The addition of 30 in the computation of the lunar cycle results from the fact that $6300 \equiv 30 \pmod{19}$[128].

The three parameters $30 \equiv 11 \pmod{19}$, 0, and 0 are the values of m, s, and i for year 6300, respectively. Below is a table of the values of m, s, and i for the end-of-century years that are relevant to Byzantine Computi; these values are sometimes called the "bases" (θεμέλια) of the relevant cycles (see sect. **27**)[129].

	6200	6300	6400	6500	6600	6700	6800	6900
m	6	11	16	2	7	12	17	3
s	12	0	16	4	20	8	24	12
i	5	0	10	5	0	10	5	0

Since 100 is a multiple of 4 and of 5, the end-of-century year values of s and i have a period of 7 and 3 centuries, respectively. Such values of s and i can only be of the form $s = 4k$, with $k = 0, 1, \ldots, 6$, and $i = 5k$, with $k = 0, 1, 2$. Since 100 and 19 are mutually prime, no such periodicity exists for the lunar cycle.

8

ψῆφος δι' οὗ εὑρίσκεται ἡ καθημερινὴ ποσότης τῆς σελήνης

Γίνωσκε ποῖος κύκλος ἐστὶν τῆς σελήνης, καὶ εἰ μὲν ἐστὶν α^{ος}, εἰπέ· πεντάκις μίαν, ε· καὶ ἀνάβα τὰς ἡμέρας |$_{4r}$ τῶν μηνῶν ἀπὸ τοῦ Ἰαννουαρίου ἕως τοῦ μηνὸς οὗ^{a} θέλῃς εὑρεῖν τὴν ποσότητα, καὶ βάλε ὅσας ἔχει, καὶ ὕφειλε διὰ τῶν ξ, καὶ τὰ κάτωθεν τῶν ξ εἰσὶ τὰ λεπτὰ τοῦ φέγγου, καὶ βλέπε ὅσα ξ ὕφειλες, καὶ πρόσθες εἰς τὰ λεπτὰ τοσαύτας ἐπακτάς. γίνωσκε δὲ ὅτι χωρὶς τῶν ἐπακτῶν

[127] The "number" of a string of alphabet letters is the sum of their values as digits. Thus, the "number" of ἄλφα is 1(α) + 30(λ) + 500(φ) + 1(α) = 532: see ACERBI, How to Spell. The mention in *Anonymus* 892 is among the earliest occurrences of the mnemonic device relating the standard Period of 532 years to the "number" of letter ἄλφα. A similar statement is found in *Anonymus* 830, sect. 16 (see also sect. 7)—in GASTGEBER, Neue texte XXX (and XXX)—which claims that this computation was established by Hero and Athanasius (see GASTGEBER, Neue texte XXX–XXX for the edition and a discussion); *Anonymus* 1092A, sect. 1, in KARNTHALER, Die chronologischen Abhandlungen 4.2–3; *Anonymus* 1172, sect. 1, which mentions the view that, because of this relation, "the completion of the Period will also bring about the perfection of the world" (εἰς τὸ πλήρωμα τῆς περιόδου τότε καὶ ἡ συντέλεια τοῦ κόσμου γενήσεται).

[128] Of course, the most economical summand is 11.

[129] Other Computi in which end-of-century years are subtracted are *Anonymus* 830, sect. 22, in GASTGEBER, Neue texte XXX; *Anonymus* 982, sect. 4, contained in the manuscript Ambr. B 113 sup. (gr. 134; *Diktyon* 42357) ff. 210r–215r (this Computus is obviously related to *Anonymus* 892). *Anonymus* 1092A, sect. 1, and 1092B, sect. 1 (both in computing indiction), in KARNTHALER, Die chronologischen Abhandlungen 5.1–3 and 8.136–138, respectively; *Anonymus* 1172, sects. 2–4; *Anonymus* 1204, contained in the manuscript Marc. gr. Z. 528 (coll. 777; middle of 14th century; *Diktyon* 69999) ff. 5r–6v; *Anonymus* 1247, sects. 2, 6, 8, in SCHISSEL, Chronologischer 106–107; *Anonymus* 1256, sects. 5–7; *Anonymus* 1273, sect. 3, in BUCHEGGER, Wiener griechische Chronologie 29.19–27; *Blastares* 1335, in RHALLES – POTLES, Σύνταγμα VI 414–416; *Rhabdas* 1342, sects. 2, 4–5, and 12; *Anonymus* 1350, sects. 1–3, in SCHLACHTER, Wiener griechische 5.3–6.14; *Argyros* 1372, sects. 3 and 6, in *PG* XIX 1284–1285 and 1292; *Anonymus* 1377, sects. 1–2, 4, in *PG* XIX 1317 and 1321; *Anonymus* 1379, in *PG* XIX 1329.

με[[ροσ]]τροῦνται τὰ λεπτά, καὶ κράτει[b] εἰς τὰς χεῖρας σου τά τε λεπτὰ καὶ τὰς ἐπακτάς, καὶ ἐξάπλωσον πάλιν τὸν κύκλον τῆς σελήνης, τουτέστιν ἑξάκις μίαν, ἕξ· καὶ ἑνώσας ὅλας ὁμοῦ τάς τε ἐπακτὰς καὶ τά τε λεπτὰ καὶ τὸν κύκλον τῆς [[..]] σελήνης τὸν ἐξαπλώμενον ὕφειλε διὰ τῶν λ, καὶ τὰ κάτωθεν τῶν λ ἐστὶν ἡ ποσότης τῆς σελήνης. ὁμοίως καὶ εἰς τὸν β[ov] κύκλον, πεντάκις δύο, ι, καὶ ἀναβαίνῃς[c] τοὺς μῆνας ὡς προείπαμεν, |4v καὶ πάλιν ἐξαπλόννῃς τὸν κύκλον. ὁμοίως καὶ εἰς τὸν γ[ov] κύκλον ἕως τοῦ ἐννεακαιδεκάτου[d] κύκλου, πρῶτον πενταπλόννῃς, καὶ ὑφείλῃς[e] διὰ τῶν ξ, καὶ εὑρίσκεις τὰ λεπτὰ καὶ τὰς ἐπακτάς, καὶ πάλιν ἐξαπλώννῃς, καὶ ὑφείλῃς διὰ τῶν λ, καὶ εἴ τι[f] μένει κάτωθεν τῶν λ, ἔστιν ἡ ποσότης τῆς σελήνης. γίνωσκε δὲ καὶ τοῦτο, ὅτι πέντε λεπτὰ ὥρα μία, καὶ ἑξήκοντα λεπτὰ ἡμέρα μία.

[a] τοῦ [b] κράτη [c] ἀναβαίνις [d] ἐννακαιιτου [e] ὑφήλῃς [f] ἤτι

Calculation by means of which the daily quantity of the Moon can be found

Know to what the cycle of the Moon amounts, and if it is the 1st, say: five times one, 5; and mount the days of the months from January up to the month where you want to find the quantity <of the Moon>, and put how many <days> it has, and remove by 60, and that which <remains> down from 60 are the minutes of the lunar month, and look at how many 60s you removed, and add so many epacts[130] to the<se> minutes. Know that, beyond the epacts, the minutes are determined, and keep both the minutes and the epacts in your hands, and again sextuple the cycle of the Moon, that is, six times one, six; and uniting all of them together, both the epacts and the minutes and the cycle of the Moon, remove what has been sextupled by 30, and that which <remains> down from 30 is the quantity of the Moon. Similarly also in the 2nd cycle, five times two, 10, and mount the months as we said above, and again sextuple the cycle. Similarly also for the 3rd cycle up to the nineteenth cycle, quintuple first, and remove by 60, and you find the minutes and the epacts, and again sextuple, and remove by 30, and if anything <remains> down from 30, this is the quantity of the Moon. Know also this, that five minutes are one hour, and sixty minutes one day.

A computation of the age of the Moon (called ποσότης "quantity") on day x counted from January 1[131]. The algorithm is:

$(m,x) \to 5m \to 5m + x \to 5m + x + (5m + x) \bmod 60 \to 5m + x + [(5m + x) \bmod 60]/60 + \llbracket (5m + x)/60 \rrbracket \to$
$\to \{5m + x + [(5m + x) \bmod 60]/60 + \llbracket (5m + x)/60 \rrbracket\} + 6m \to$
$\to (\{5m + x + [(5m + x) \bmod 60]/60 + \llbracket (5m + x)/60 \rrbracket\} + 6m) \bmod 30 = a_m(x).$

The age of the Moon on day x in the year is calculated in several steps: (1) add to day x counted from January 1 a number obtained from writing the epacts of the Moon at lunar cycle m—namely, $11m$ (defined in sect. **14**, and see also the commentary on sect. **12**—as $5m + 6m$; (2*a*) first, operate only on summand $5m + x$, with a correction (see just below); (2*b*) subtract pairs of lunar months by reducing modulo 60 rather than modulo 59 (= two lunar months of 29 ½ days each); (2*c*) add 1 day for every removed set of 60 units (this is summand $\llbracket (5m + x)/60 \rrbracket$) in order for the modulo 60 reduction to be equivalent to subtracting pairs of lunar months; (3) second, add $6m$ to complete the epacts of the Moon; (4) reduce modulo 30, which is required since $6m$ has been added. The final reduction modulo 30 is the standard way of eliminating whole lunar months, see sect. **12** below.

To understand what are the "corrections" I have alluded to, let us rewrite the last line of the algorithm as follows (for simplicity's sake, the modulo 30 reduction is disregarded):

$a_m(x) = (11m + x) \bmod 30 + \llbracket (5m + x)/60 \rrbracket + [(5m + x) \bmod 60]/60.$

The first addendum is the canonical contribution "epacts [= the age of the Moon on December 31] + number of days counted from January 1" to the age of the Moon. The other two addenda (the "correction") are the value of $5m + x$ written not in units but in sixtieths (that is, in minutes): divide $5m + x$ units by 60 and you get by definition[132] a quotient made of the floor $\llbracket (5m + x)/60 \rrbracket$ and of the fractional part $[(5m + x) \bmod 60]/60$. This correction

[130] Note, here and in sect. **12**, the term ἐπακταί in its general meaning of something "brought in".

[131] The procedure is tersely phrased; its first formulation suggests replacing my x counted from 1_J with $\sum_{k=J}^{X-1} n_k + x$, where n_k is the length of month k in days and x is the assigned day in month X. I have adopted a simpler notation.

[132] See footnote 121 above, and sects. **12** and **17**.

is equivalent to adding $^{5}/_{60} = {}^{1}/_{12}$ a day for every lunar cycle year + $^{1}/_{60}$ a day for every day in the lunar cycle year. The "$^{1}/_{60}$ a day" addendum contributes 1 day every 2 months and therefore offsets, as explained above, the difference between the reduction modulo 60 performed in the algorithm and the required reduction modulo 59. The "$^{1}/_{12}$ a day" addendum not only triggers a *saltus lunae* in cycle year 12, but also makes the Moon generally older than the Alexandrian Computus does, and this occurs in particular for the Passover *luna XIV*[133]. In particular, there are cases in which, what the Alexandrian Computus sets as *luna XIV* and the supporters of this algorithm set as *luna XV* or *XVI*, falls on a Sunday. In such cases, either the supporters of this algorithm set Easter to coincide with (Alexandrian) Passover—which is forbidden—or they set it on (their) *luna XXII* or *XXIII*, which falls outside the Alexandrian Easter limits. In either case, the supporters qualify themselves as computistically heterodox.

This is the algorithm adopted by the so-called πενταπλοῦντες καὶ ἑξαπλοῦντες[134], and it dates back to early stages of the Alexandrian Computus. As is clear, it is not an algorithm for computing Passover, but an algorithm for checking whether a traditional date of Passover falls on *luna XIV* or not, on the supposition that January 1 of the year that precedes solar cycle year 1 is *luna I*. Nevertheless, considering that a Moon age algorithm is in a sense the inverse of a Passover algorithm, this algorithm is incompatible with the Passover algorithms *Anonymus* 892 sets forth in sects. **12**, **14**, and **15**: this fact shows that *Anonymus* 892 is a compilation.

Additional information on time-subdivisions, required in the present context, is finally provided in the text: 1 double-hour = 5 minutes; 1 day = 60 minutes (see also sect. **1**).

9

ψῆφος σὺν θεῷ εἰς τὸ εὑρεῖν τὴν ἴνδικτον

Εἰσὶ τὰ ἔτη ἀπὸ κτίσεως[a] κόσμου ἕως τῆς ἐνεστώσης[b] ι^{ης} ἰνδίκτου ϛυ ἔτη. ἐὰν θέλεις εὑρεῖν τὴν ἴνδικτον, ὕφειλε αὐτὰ οὕτως. δεκάκις [[..]] τετρακώσ<ιοι>, ϡδ· πεντάκις τετρακώσ<ιοι>, ϡβ· λοιπὸν ἔμεινεν υ· ὕφειλε καὶ ταῦτα οὕτως. δεκάκις κ, σ· πεντάκις κ, ρ· λοιπὸν ἔμειναν ρ· ὕφειλε καὶ αὐτὰ ἐπὶ τῶν ιε· ἑξάκις ιε, ϙ· |$_{5r}$ καὶ ἔμειναν ι. καὶ εὑρίσκεις ὅτι δεκάτη ἐστὶν ἡ ἴνδικτος.

[a] κτήσεως [b] ἐνεστῶσις

God willing, calculation for finding the indiction

The years from the foundation of the world up to the present 10th indiction are 6400 years. If you want to find the indiction, remove them as follows. Ten times four hundred, 4000; five times four hundred, 2000: there remain 400 as a remainder; remove also these as follows. Ten times 20, 200; five times 20, 100: there remain 100 as a remainder; remove also them by 15; six times 15, 90: and there remain 10. And you find that the indiction is the tenth.

A computation of the current indiction cycle year. A computation of the indiction cycle year is carried out for current year AM 6400 [= AD 891/2]; it yields *i* = 10. In sections **9–11**, the general algorithms of sections **4–6** are applied, but in a different order.

10

⊙ ἕτερος ψῆφος, εἰς τὸ εὑρεῖν τὸν κύκλον τοῦ ἡλίου

Ὁ δὲ τοῦ ἡλίου κύκλος εὑρίσκεται οὕτως. ὕφειλε τὰ αὐτὰ ἔτη ἐπὶ τῶν κη, καὶ εὑρήσεις τὸν κύκλον τοῦ ἡλίου· εἰκοσάκις διακώσ<ιοι>, ϡδ· ὀκτάκις σ, ϡαχ· λοιπὸν ἐνέμειναν ω· ὕφειλε καὶ ταῦ-

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[133] A complete list, keyed to lunar cycle years, of ages of the Moon according to this algorithm when the Alexandrian Computus sets them as Passover *luna XIV* is in Maximus, *Enarratio* II.1, in *PG* XIX 1255–1256.

[134] See also Maximus, *Enarratio* I.11–12, 16, in *PG* XIX 1228–1229, 1233, and the entire chapter II, in *PG* XIX 1252–1264; *Anonymus* 830, sect. 19, in GASTGEBER, Neue texte XXX; *Theophylaktos* 956, sect. 5 (the algorithm is incomplete); *Anonymus* 1041, sect. 2; *Anonymus* 1079, sect. 5, in MENTZ, Beiträge zur Osterfestberechnung 80–84, and also Mentz's discussion at 51–66; the Computus contained in Par. suppl. gr. 690, sects. 1–2; *Anonymus* 1090, sect. 3; *Anonymus* 1092A, sect. 3, and 1092B, sect. 2, in KARNTHALER, Die chronologischen Abhandlungen 5.29–38 and 8.142–150, respectively. *Anonymus* 1092A calls the algorithm χαρτουλαρικός "archive-keeper-style": KARNTHALER, Die chronologischen Abhandlungen 5.28; *Anonymus* 1172, sect. 13. None of these sources sets out a clear-cut algorithm. Assessing the aims of the πενταπλοῦντες καὶ ἑξαπλοῦντες is not simple: see SCHWARTZ, Christliche und jüdische Ostertafeln 81–88; GRUMEL, La Chronologie 54 and 117–124. Modulo 60 algorithms for computing the age of the Moon can also be found in *Anonymus* 1183, sect. 9; *Anonymus* 1204; *Rhabdas* 1342, sect. 7.

τα οὕτως. εἰκοσάκις κ, υ· ὀκτάκις κ, ρξ· καὶ ἔμειναν σμ· ὀκτάκις κ, ρξ· ὀκτάκις ὀκτώ, ξδ· καὶ λοιπὸν ἔμειναν ις. καὶ γνώριζε ἓξ καὶ δέκατον κύκλον εἶναι τοῦ ἡλίου.

☉ *A further calculation, for finding the cycle of the Sun*

The cycle of the Sun can be found as follows. Remove the same years by 28, and you will find the cycle of the Sun; twenty times two hundred, 4000; eight times 200, 1600: there remain 800 as a remainder; remove also these as follows. Twenty times 20, 400; eight times 20, 160: and there remain 240; eight times 20, 160; eight times eight, 64: and there remain 16 as a remainder. And recognize that it is the sixteenth cycle of the Sun.

A computation of the current solar cycle year. A computation of the solar cycle year is carried out for current year AM 6400; it yields $s = 16$.

11

☾ ἕτερος ψῆφος, δι' οὗ εὑρίσκεται ὁ κύκλος τῆς σελήνης

Ἀνέρχεται ὁ κύκλος τῆς σελήνης ἕως τῶν ιθ, καὶ πάλιν εἰς πρῶτον ὑποστρέφει. ἀπὸ κτίσεως κόσμου ἔτους ἑξακισχιλιοστοῦ τετρακοσιοστοῦ ἕως τοῦ ἐνεστῶτος ἓξ καὶ δεκάτου κύκλου τῆς σελήνης· ὕφειλε καὶ ταῦ|$_{5v}$τα τὰ ἔτη οὕτως. δεκάκις τ, ˏγ· ἐννάκις τ, δισχίλια ψ· λοιπὸν ἔμειναν ψ ἔτη· ὕφειλε καὶ ταῦτα οὕτως. δεκάκις λ, τ· ἐννάκις λ, σο· λοιπὸν ἔμειναν ρλ ἔτη· ὕφειλε δὲ καὶ ταῦτα οὕτως. δεκάκις ς, ξ· ἐννάκις ς, πεντηκοντατέσσαρες[a]· λοιπὸν ἐμείνασιν ις, ὃς ἐστὶν ις κύκλος τῆς σελήνης.

[a] πεντικονταδ

☾ *A further calculation, for finding the cycle of the Moon*

The cycle of the Moon reaches up to 19, and reverts back again to 1. <From> the six-thousand-four-hundredth year from the foundation of the world up to the present sixteenth cycle of the Moon; remove also these years as follows. Ten times 300, 3000; nine times 300, two thousand 700: there remain 700 years as a remainder; remove also these as follows. Ten times 30, 300; nine times 30, 270: there remain 130 years as a remainder; remove also these as follows. Ten times 6, 60; nine times 6, fifty-four: there remain 16 as a remainder, which is the sixteenth cycle of the Moon.

A computation of the current lunar cycle year. The lunar cycle lasts 19 years. A computation of the lunar cycle year is carried out for current year AM 6400; it yields $m = 16$.

12

ἕτερος ψῆφος τοῦ Πάσχα, δι' οὗ εὑρίσκεται ἑκάστου ἔτους

Γνῶθι ποῖος κύκλος ἐστὶ[a] τῆς σελήνης, καὶ ἑνδεκαπλασίασον αὐτόν, καὶ εἰ μὲν ἐστὶν α^{ος}, εἰπέ· ἑνδεκάκις μία, ἕνδεκα· καὶ βάλε τὰς <ς> ἐπακτὰς τῶν αἰώνων τῶν παρελθό<ν>των· {ς ἐπακτὰς} [[τῶν αἰώνων τῶν παρελθό<ν>των]] καὶ γίνονται ἡμέραι ιζ· καὶ ἄρξε ἀπὸ τῆς α τοῦ Μαρτίου ἕως οὗ πληρώσεις[b] ἡμέρας ν, καὶ ἐκεῖ[c] ἐστὶν τὸ νομικὸν Πάσχα· |$_{6r}$ καὶ εἰ μὲν πληροῖς ἔσωθεν τοῦ Μαρτίου τὰς ν, εὖ καὶ καλῶς· εἰ δὲ μή γε, βάλε καὶ ἐκ τοῦ Ἀπριλίου, ἵνα πληρωθῶσιν[d] ν. γνῶθι δὲ ὅτι τὸ νομικὸν Πάσχα εἰς οἱανδηποτοῦν[e] [[εὑρέσεως]] ἡμέραν τῆς μεγάλης ἑβδομάδος εὑρίσκεται. καὶ ἐπὰν εὕρῃς τὸ νομικὸν Πάσχα, ψήφισον[f] ἐν ποίᾳ ἡμέρᾳ τῆς ἑβδομάδος ἐστίν, καὶ ἀνελθὼν ἕως τῆς κυριακῆς εὑρήσεις τὸ ἡμέτερον Πάσχα. ἐπὰν δὲ εὕρῃς τὸ ἡμέτερον Πάσχα καὶ θέλῃς εὑρεῖν τὴν Ἀποκρέωσιν, πρόσθες ἐν αὐτῇ ἡμέρας τρεῖς – εἰ δὲ ἔστιν βίσεκτον, δ – καὶ εἰς τοσαύτας ἡμέρας εὑρίσκεις τὴν Ἀποκρέωσιν. ἀπὸ μὲν κη Μαρτίου τὴν ἄνω (ἄνω δὲ λέγω τὴν ἀρχὴν τοῦ μηνός) εἰς τὸν Ἰαννουάριον εὑρίσκεις τὴν Ἀποκρέωσιν· εἰ δὲ βίσεκτον ἐστίν, ἀπὸ τῶν κζ.

καὶ ἀνάπαλιν[g]

Βισέκτου ὄντος[h] ἀπὸ τῶν κη τοῦ Μαρτίου |6v δύναται εὑρεθῆναι ἐν τῇ πρώτῃ τοῦ Φευρουαρίου ἡ Ἀποκρέωσις, ἀπὸ δὲ τῶν κθ τοῦ Μαρτίου καθεξῆς ἐν τῷ Φευρουαρίῳ πάντοτε εὑρίσκεται ἡ Ἀποκρέωσις.

ἔστιν δὲ ὁ ψῆφος τῆς οἱασδηποτοῦν εὑρέσεως τῆς ἑβδομάδος οὗτος. γνῶθι[i] ποῖος κύκλος ἐστὶν τοῦ ἡλίου, καὶ βάλε τόν τε κύκλον τοῦ ἡλίου καὶ τὰ τέταρτα αὐτοῦ, καὶ ἀρξάμενος ἀπὸ Ὀκτωβρίου[j] μηνὸς λάβε τοῦ μηνὸς τοῦ ἔχοντος λα τρεῖς ἡμέρας, τοῦ δὲ ἔχοντος λ, β ἡμέρας, καὶ ἄνελθε ἕως τοῦ κατέχοντος μηνός, καὶ βάλε ὅσας ἔχει, καὶ ἑνώσας ὅλας ὁμοῦ ὕφειλε διὰ τῶν ἑπτά, καὶ ἐὰν μένῃ μία ἡμέρα, γίνωσκε ὅτι κυριακὴ ἐστίν, εἰ[k] δὲ δύο, β$^{\alpha}$, εἰ δὲ τρεῖς, γ$^{\eta}$, εἰ[l] δὲ τέσσαρες[m], δ$^{\eta}$, καὶ καθεξῆς ἕως τοῦ σαββάτου. εἰ δὲ μὴ ἔχεις κάτωθεν τῶν ἑπτά, γίνωσκε σάββατόν ἐστιν. |7r ὁμοίως καὶ εἰς τὸν δεύτερον κύκλον τῆς σελήνης εἰπέ· ἑνδεκάκις δύο, κβ· καὶ βάλε τὰς ἐπακτὰς τῶν αἰώνων ϛ· γίνονται κη· καὶ ἄρξε πάλιν ἐπὶ τῆς α τοῦ Μαρτίου ψηφίζειν καθὼς προείπαμεν. ἐπὰν δὲ φθάσῃς εἰς τρίτον κύκλον τῆς σελήνης, εἰπέ· ἑνδεκάκις γ, τριακοντατρεῖς· καὶ ὕφειλε τὰς τριακοστάς· καὶ μένουσιν τρεῖς· καὶ πρόσθες αὐταῖς τὰς ἐπακτὰς τῶν αἰώνων· καὶ γίνονται ἐννέα· καὶ ἄρξε πάλιν ἀπὸ τοῦ Μαρτίου μηνὸς ψηφίζειν[n] καθὼς προεδιδάξαμεν. ὁμοίως καὶ εἰς τὸν δ^{ov} καὶ ε^{ov} καὶ ἕως τοῦ ιθ^{ου} κύκλου ἑνδεκαπλασιάσας ὕφειλε τριακοντάδας ὅσας ἔχεις, καὶ τὰς μενούσας ἡμέρας κάτωθεν τῶν τριακοντάδων κράτει[o], καὶ πρόσθες αὐταῖς τὰς ἐπακτὰς τῶν αἰώνων, καὶ οὕτως ἄρχου πάντοτε ἀπὸ τῆς α τοῦ Μαρτίου, καθὼς προείπαμεν. |7v γίνωσκε δὲ καὶ τοῦτο, ὅτι ἀπὸ πρώτου κύκλου ἕως ἑξκαιδεκάτου ἓξ ἐπακτὰς προστίθει[p], ἀπὸ δὲ ἑπτὰ καὶ δεκάτου ἕως ιθ^{ου}, ἑπτά, ἐπεὶ οὐκ εὑρίσκεις τὸ ἀσφαλές.

[a] ἐστὶ [b] πληρόσεις [c] ἐκὶ [d] πληροθῶσιν [e] οἱανδιποτοῦν [f] ψήφησον [g] ἀνάπαλην [h] ὄντως [i] γνῶθη [j] Ὠκτοβρίου [k] ἡ [l] ἡ [m] τεσσάρεις [n] ψιφίζειν [o] κράτη [p] προστίθι

A further calculation of Passover, by means of which it can be found in each year

Know to what the cycle of the Moon amounts, and undecuple it, and if it is the 1st, say: eleven times one, eleven; and put the 6 epacts of the bygone eras: and they yield 17 days; and begin from March 1 until you will fill 50 days, and Passover is there; and if you will fill the 50 <days> within March, that's fine; but if you *do not*, put also <days> from April in order for 50 to be filled. Know that Passover can be found on whichever day of the Holy Week. And as soon as you have found Passover, calculate on what weekday it falls, and reaching up to Sunday you will find our Easter. As soon as you have found our Easter and you want to find Meat-Fare Sunday, add three days in it—if it is a leap year, <add> 4—and you find <that> Meat-Fare Sunday <falls> on such-and-such days. <If Easter falls> from March 28 towards above (I mean by "above" the beginning of the month) you find Meat-Fare Sunday in January; if it is a leap year, from <March> 27 <towards above>.

And the other way around

If it was a leap year, from March 28 <on> Meat-Fare Sunday was found on the first of February, whereas continuously from March 29 <on> Meat-Fare Sunday is always found in February.

The calculation of whatever finding of the week is this. Know to what the cycle of the Sun amounts, and put both the cycle of the Sun and its quarters, and beginning from the month of October take three days of the month that has 31, 2 days from the one that has 30, and reach up to the ongoing month, and put how many <days> it has, and uniting all of them together remove by seven, and if one day remain, know that <the intended day> is a Sunday, and if two, it is a Monday, if three, a Tuesday, if four, a Wednesday, and continuously up to Saturday. If you do not have <anything> down from seven, know that it is a Saturday. Similarly also in the second cycle of the Moon, say: eleven times two, 22; and put the 6 epacts of the eras: they yield 28; and again begin calculating on March 1 exactly as we said above. As soon as you have attained the third cycle of

the Moon, say: eleven times 3, thirty-three; and remove the thirtieths[135]: and there remain three; and add the epacts of the eras to them: and they yield nine; and again begin calculating from the month of March exactly as we taught above. Similarly also in the 4th and in the 5th <cycle> and up to the 19th cycle, undecupling remove how many thirties you have, and keep the days that remain down from the thirties, and add the epacts of the eras to them, and in this way always begin from March 1, exactly as we said above. Know also this, that from the first cycle up to the sixteenth add six epacts, from the seventeenth up to the 19th, seven, for you do not find what is secure[136].

A complete set of algorithms for computing Passover, Easter, and Meat-Fare Sunday. This is the core section of *Anonymus* 892. The algorithm for Passover is[137]:

$(m) \to 11m \to$
$\quad | \; m < 17,\ 11m + 6 \to (11m + 6) \bmod 30 \to 50 - [(11m + 6) \bmod 30] \text{ –: } 1_M = p_m.$
$\quad | \; 17 \le m \le 19,\ 11m + 7 \to (11m + 7) \bmod 30 \to 50 - [(11m + 7) \bmod 30] \text{ –: } 1_M = p_m.$

The first branch of the algorithm can be described as follows: multiply the lunar cycle year m by 11, add 6 units, reduce modulo 30, subtract the result from 50 (a parameter which I shall call **norm**) and count, from March 1, as many days as the remainder: the resulting day is the date of Passover. This day falls in April if the remainder is greater than 31.

This remarkable algorithm has two main features. First, the addendum $11m$ fits the definition of the epacts of the Moon given in sects. **14** and **19**, but the text—while using the general meaning of ἐπακταί (see sect. **8**) in the expression "epacts of the eras"—does not refer to the epacts of the Moon because the branching condition is formulated in terms of the lunar cycle years: therefore, the position of the *saltus lunae* is a feature of the algorithm, not of the epact sequence (see below and sect. **14** for a discussion). Second, this algorithm simplifies the fundamental algorithm expounded in early sources such as Heraclius and George Presbyter (see sect. **14**)[138], and which is a straightforward adaptation to the Byzantine era of the algorithm adopted in the early Alexandrian Church.

The adaptation of the Passover algorithm from the Alexandrian era to the Byzantine era can be explained as follows. Let us first consider the short Computus added by Heraclius as sect. 30 in Stephanus of Alexandria's *in Ptolemaei Tabulas Manuales*. Heraclius uses the era Maurice (here denoted y_M)[139], and gives the following algorithm for the epacts: $(y_M) \to 11y_M \to 11y_M \bmod 30 = e_m$. He then carries out a computation for April indiction 11 [= AD 623 April]; this yields $e_m = 15$ (no correlation is established by Heraclius between lunar cycle years and epacts, but I shall keep the subscript m). Heraclius' Passover algorithm computes the quantity "norm minus shifted epacts" $44 - (e_m + 8)$, and then counts as many days from 1_M or from 1_A as the value thus obtained; the resulting day is the date of Passover:

$(e_m) \to e_m + 8 \to 44 - (e_m + 8) \to$
$\quad | \; 20 \le 44 - (e_m + 8) \le 31,\ 44 - (e_m + 8) \text{ –: } 1_M = p_m \in M.$
$\quad | \; 44 - (e_m + 8) < 20,\ 44 - (e_m + 8) - 1 = [44 - (e_m + 8) - 1] \text{ –: } 1_A = p_m \in A.$
$\quad | \; 44 - (e_m + 8) > 31,\ [44 - (e_m + 8)] \bmod 31 \text{ –: } 1_A = p_m \in A.$

As $44 - 8 = 36$ and no modulo reduction overlaps with this subtraction, this is a norm 36 algorithm in disguise; as shown by O. Neugebauer, this norm characterizes the Alexandrian Computus[140]. As regards the three branches of the algorithm, they result from the fact that the relation $p_m + e_m \equiv 36 \pmod{30}$ typical of the Alexandrian Computus can be solved for p_m (as in the first branch of the algorithm), days being counted from 1_M, but one must not forget that (*a*) a modulo reduction is involved (consequently, one cannot simply write $p_m = 36 - e_m$), and (*b*) large epacts entail numerically small Passover dates, but these dates may be forbidden by the rule of the equinox if they fall in March. Therefore, for large epacts one must count from 1_A and not from 1_M. This requirement collides with the modulo 30 reduction constitutive of the Passover–epacts relation unless 1 unit is subtracted (second branch of

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[135] Here and in sect. **14**, note the ordinal αἱ τριακοσταί "the thirtieths", where "thirty" would normally be used.

[136] The same clause occurs in sect. **14**, and in *Anonymus* 830, sect. 25, in GASTGEBER, Neue texte XXX; *Anonymus* 1092A, sect. 4, in KARNTHALER, Die chronologischen Abhandlungen 6.68. See also *Anonymus* 982, sects. 9 and 15.

[137] The act of counting is denoted by the sign "–:". Thus, 50 –: 1_M are 50 days counted from March 1.

[138] See USENER, De Stephano Alexandrino 314–317; DIEKAMP, Der Mönch 30–31, respectively.

[139] The epoch of the era Maurice is AD 589 August 29, a Monday; this the 8th year of his reign, AD 582 August 14 – 602 November 23; years are Julian years. As $588 + 5493 - 1 = 6080 \equiv 0 \pmod{19}$, the Maurice era is an avatar of the Alexandrian era.

[140] NEUGEBAUER, Abu Shaker's 41–44 and 48–58.

the algorithm) or, what amounts to the same, a modulo 31 reduction is performed (third branch). Finally, there is no trace of *saltus lunae* in the algorithm. A feature of this algorithm, which is also found in other Computi, is that the branching conditions are given as inequalities involving the "norm minus shifted epacts" quantity (which in principle *does not straightforwardly* coincide with p_m, as just seen), and not simply the epacts or the lunar cycle year[141].

Why does Heraclius use norm 36 but computes with norm 44 and adds 8 to the epacts[142]? Why use a norm different from 36? A reason can be that 44 is the number of days nearest to 1 lunar month and one half (because 29 ½ + 14 ½ ¼ = 44 ¼), but George Presbyter will give us a complementary clue.

Before seeing this clue, recall that Heraclius computes the date of Passover for AD 623 (e_m = 15) with the above algorithm and finds $p_m = 21_M$. He also mentions the existence of a year without epacts and computes that in this case (last branch of the algorithm) $p_m = 5_A$. Thus, Heraclius' epact sequence coincides with the Alexandrian sequence[143]; moreover, according to the algorithm just seen, the *saltus lunae* (marked by a double vertical bar in the table below; Passover dates without a subscript fall in April) is located at the end of the cycle:

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
e_m	30	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26	7	18
p_m	5	25_M	13	2	22_M	10	30_M	18	7	27_M	15	4	24_M	12	1	21_M	9	29_M	17

Let us now turn to George Presbyter. He is the avowed champion of the Byzantine world era and epact sequence. His computations are set for current year Heraclius 29 = AM 6147 [= AD 638/9], i = 12.

George finds the lunar cycle as follows: $(y) \rightarrow y \bmod 19 = m$, as in sect. **5** of *Anonymus* 892. The algorithm for the epacts of the Moon is $(m) \rightarrow 11m \rightarrow 11m \bmod 30 = e_m$ (as in sect. **14** of *Anonymus* 892), but George offers an argument to justify the presence of the *saltus lunae* after cycle 16[144], and sets out all Passover days. The resulting list of epacts and Passover dates is as follows (the position of the *saltus lunae* is again marked by a double vertical bar):

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
e_m	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26	8	19	30
p_m	2	22_M	10	30_M	18	7	27_M	15	4	24_M	12	1	21_M	9	29_M	17	5	25_M	13

George gives the following Passover algorithm, which is a slightly modified, 2-branch version of Heraclius' algorithm:

$(e_m) \rightarrow 43 - e_m \rightarrow$
| $43 - e_m < 20$, $(43 - e_m) \bmod 30$ –: $1_A = p_m \in A$.
| $43 - e_m \geq 20$, $43 - e_m + 1 \rightarrow 43 - e_m + 1$ –: $1_M = p_m \in M$.

This norm 43 algorithm (a norm that is also operative in the second branch of Heraclius' algorithm) is straightforwardly equivalent to a norm 44 algorithm, for the reference epoch has to be 1_M and the actual norm in the second branch of the algorithm is 43 + 1 = 44. As in the case of Heraclius, no trace of *saltus lunae* is found in the algorithm; this confirms George's argument about the *saltus lunae* being a feature of the epact sequence.

The change of era, and Heraclius' epacts shift of 8 units, explain the transition from the Alexandrian era and epact sequence to the Byzantine era and epact sequence. To see this, note that the epacts shift of 8 units transforms Heraclius' cycle into

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
$e_m + 8$	8	19	30	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26
p_m	5	25_M	13	2	22_M	10	30_M	18	7	27_M	15	4	24_M	12	1	21_M	9	29_M	17

[141] For instance, $44 - (e_m + 8) < 20$ might simply be phrased $36 - e_m < 20$, or even $e_m > 16$. But it is not.

[142] Of course, 44 – 8 = 36, but the crucial point, let me stress it again, is that no modulo reduction separates the minuend and the subtrahend of this subtraction. This is not always the case, as the norm 50 algorithm shows. Thus, Heraclius applies in fact a norm 36 algorithm.

[143] The Alexandrian epact sequence and Passover dates are listed in GRUMEL, La Chronologie 54, column II.

[144] DIEKAMP, Der Mönch 27.27–28.5.

Of course, the *saltus lunae* remains at the end of the cycle. The 16-year backwards shift from the Alexandrian to the Byzantine world era makes the new cycle begin in (proleptic) Alexandrian $m = 4$. Therefore, since Heraclius' and George's Passover algorithms are equivalent, George's cycle is as follows (the *saltus lunae* after cycle year 16 characterizes the Byzantine epact sequence; the relocated cells are shaded gray):

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
e_m	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26	8	19	30
p_m	2	22_M	10	30_M	18	7	27_M	15	4	24_M	12	1	21_M	9	29_M	17	5	25_M	13

The argument just outlined explains the crucial transition from the Alexandrian era and epact sequence to the Byzantine era and epact sequence. Conversely, it cannot be a coincidence that Heraclius' additive shift of 8 units gives an epact sequence such that $e_1 = 11$ after the change of era.

Let us return to the norm 50 algorithm of *Anonymus* 892. The simplification of Heraclius' and George's Passover algorithms was carried out by writing 44 as the result of 50 – 6, with the norm 50 lying outside the modulo 30 reduction and the parameter 6 lying inside it: this rewriting allowed setting a branching condition much more transparent than the one in Heraclius' and George's algorithm[145]; it also allowed simplifying both these algorithms and the related, and unwieldy, algorithms we shall see in sects. **14** and **15**. This simplification is a consequence of the following fact: counting 50 days starting on March 1 one gets to April 19, which is the upper bound for Passover (see sect. **26**), hence no counting from April 1 is required for large epacts, contrary to what one finds in Heraclius' and George's algorithm.

The norm 50 algorithm of *Anonymus* 892 is attested in other sources; it is also called "notarial" (νοταρική)[146]. The 6 units to be added to $11m$ are called "epacts of the bygone eras" (ἐπακταὶ τῶν αἰώνων παρελθόντων)[147], which correspond to the 6 whole millennia elapsed since Creation: this is the basic mnemonic trick in the norm 50 computation of Passover[148]. In the second branch of the algorithm, the additional unit to be added to $11m$ in the cycle years from 17 to 19 (that is, 7 units are added instead of 6) is the *saltus lunae*. As usual, whole lunar months are removed by reducing modulo 30[149].

In this section of *Anonymus* 892, a complete set of prescriptions (Passover, Easter, Meat-Fare Sunday, weekday of an assigned date) is provided for the first year of the lunar cycle[150]. At the end of the section, however, the Passover algorithm is summarily retrieved for a representative sample of all lunar cycle years.

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[145] Just one branching remains in the norm 50 algorithm, and it occurs at a most natural place: the position of the *saltus lunae*. Moreover, the branching condition is formulated in terms of the lunar cycle, not in terms of the "norm minus shifted epacts" as Heraclius and George do.

[146] The denomination is used in *Anonymus* 1079, sect. 5, in MENTZ, Beiträge zur Osterfestberechnung 98. Other occurrences of this algorithm can be found in *Anonymus* 830, sects. 18, 25 (same wording as *Anonymus* 892), and 28, in GASTGEBER, Neue texte XXX, XXX and XXX; *Anonymus* 951, sect. 15; *Anonymus* 982, sect. 9; *Anonymus* 1041, sect. 8; *Anonymus* 1079, sect. 5, in MENTZ, Beiträge zur Osterfestberechnung 100; the Computus contained in Par. suppl. gr. 690, sect. 3; *Anonymus* 1090, sects. 1 and 5; *Anonymus* 1092A, sect. 4, and 1092B, sect. 6, in KARNTHALER, Die chronologischen Abhandlungen 5.40–6.47 and 9.191–10.198, respectively; *Anonymus* 1172, sect. 5; *Anonymus* 1183, sect. 6; *Anonymus* 1204; *Anonymus* 1247, sect. 3, in SCHISSEL, Chronologischer 106; *Anonymus* 1256, sect. 9; *Blastares* 1335, in RHALLES – POTLES, Σύνταγμα VI 416; *Rhabdas* 1342, sect. 10; *Meliteniotes* 1352, sect. III.24; *Anonymus* 1377, sect. 8, in *PG* XIX 1328; *Anonymus* 1379, in *PG* XIX 1329.

[147] This expression is also found in *Anonymus* 1092A, sect. 4, in KARNTHALER, Die chronologischen Abhandlungen 5.42, and in *Rhabdas* 1342, sect. 10.

[148] Matthew Blastares (in RHALLES – POTLES, Σύνταγμα VI 416) adduces another explanation of the parameter 6: because of the excess of 0;1,50 day of a true lunar month over 29 ½ days, which yields about 6 days in 1 year (*immo* 6;53 days).

[149] Thus, the convention assumes that lunar months are 30-day months. Unnecessary complications would arise from reducing modulo 29 ½; moreover, one would not let Easter coincide with Passover, and reducing modulo 30 rather than modulo 29 ½ shifts forward, and most conveniently, the schematic date of the computed Passover. There exist, however, algorithms that compute the age of the Moon (but *not* the date of Passover) by reducing modulo 29 ½: see, for instance, Maximus, *Enarratio* I.28 and the eighth algorithm compiled in III.8, in *PG* XIX 1245 and 1269; *Psellos* 1092, sect. I.15, in REDL, La chronologie appliquée II 237.1–11; *Anonymus* 1183, sects. 9 and 10; *Rhabdas* 1342, sect. 7; *Anonymus* 1377, sect. 6, in *PG* XIX 1323.

[150] In its first occurrence, the Passover algorithm is harmlessly incomplete: since $11m + 6 < 30$ for $m = 1$, the modulo 30 reduction is not operative and hence it is not mentioned.

A computation of Meat-Fare Sunday. A computation of the date of Meat-Fare Sunday (Ἀποκρέωσις) follows; it includes a clarification on the use of the adverb ἄνω "above" as a way to specify the time arrow (see sects. **22** and **26**). The algorithm is[151]:

$(r,y) \rightarrow r + 3 + [\![(y \bmod 4)/4]\!] \rightarrow (r + 3 + [\![(y \bmod 4)/4]\!]) \bmod 31 = t.$

If $r \geq 29_M$, then $t \in F$; if $r \leq 28_M - [\![(y \bmod 4)/4]\!]$, then $t \in J$.

In the Byzantine liturgical calendar, Meat-Fare is the third Sunday of the pre-Lenten period of preparation and repentance; it falls 8 weeks = 56 days before Easter. Since in non-leap years February plus March last 59 days, Meat-Fare Sunday falls numerically 3 days after Easter (summand $r + 3$) but 2 months before the month in which Easter falls. There is, however, a due adjustment in leap years (summand $[\![(y \bmod 4)/4]\!]$)[152], and one must keep in mind that if the date of Easter falls on a day after March 29 (28 in leap years), then Meat-Fare Sunday falls in February rather than in January. The modulo 31 reduction is not indicated in the algorithm of our text, but it must be introduced in order to take into account Easter dates in March shifting to April because of the addition of $3 + [\![(y \bmod 4)/4]\!]$.

A computation of the weekday of Passover, if this falls in month X. The algorithm is as follows, where n_k = length of month k in days[153]:

$(s,x,X) \rightarrow s + [\![s/4]\!] \rightarrow s + [\![s/4]\!] + \sum_{k=O}^{X-1}(n_k - 28) \rightarrow s + [\![s/4]\!] + \sum_{k=O}^{X-1}(n_k - 28) + x \rightarrow$

$\rightarrow [s + [\![s/4]\!] + \sum_{k=O}^{X-1}(n_k - 28) + x] \bmod 7 = w_s(x_X).$

This algorithm computes the weekday of any date x in month X. To this end, it suffices to count the days elapsed from a date falling on a known weekday and remove whole weeks. It should be kept in mind that a year of 365 days exceeds a whole number of weeks by 1 day (summand s in the above algorithm: recall that the Byzantine world era and the solar cycle are synchronized; this summand also includes 365 of the 366 days of a leap year); a leap year exceeds it by 2 days (further summand $[\![s/4]\!]$; the first two summands, after reduction modulo 7, are the epacts of the Sun, as seen in sect. **2**)[154]; a month exceeds it by its own length in days minus 28 days (= 4 weeks), namely, $n_k - 28$ in our notation. The sum $\sum_{k=O}^{X-1}(n_k - 28)$ is the excess over 28 days of the months from *O*(ctober) to the one preceding the given month X, denoted by $X - 1$ (the sum gives null values when the assigned month is October). The date x must then be added. Reducing the sum modulo 7 involves eliminating whole weeks. Since only months of 31 and 30 days are mentioned and because of the leap year contribution included in the term $[\![s/4]\!]$, February must be set to 28 days; given the fact that this summand is operative throughout the year, the month X must be a month coming after February. This restriction, however, is of no consequence as far as Passover or Easter computations are concerned. To check the consistency of the algorithm, recall that the weekday of the epoch date of the Byzantine world era is a Saturday = 7, so that $w(1_O) = 2$ for the first day of the solar cycle, which is the output for $s = 1$, $x = 1$.

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[151] Other occurrences of this algorithm can be found in *Anonymus* 951, sect. 17; *Anonymus* 982, sect. 9; *Anonymus* 1079, sect. 2, in MENTZ, Beiträge zur Osterfestberechnung 78; the Computus contained in Par. suppl. gr. 690, sect. 5; *Anonymus* 1041, sect. 8; *Anonymus* 1090, sects. 1 and 5; *Anonymus* 1172, sect. 10; *Anonymus* 1183, sect. 8; *Anonymus* 1256, sect. 12; *Blastares* 1335, in RHALLES – POTLES, Σύνταγμα VI 418; *Rhabdas* 1342, sect. 13; *Meliteniotes* 1352, sect. III.24; *Argyros* 1372, sect. 12, in *PG* XIX 1301–1304.

[152] For the meaning of this formula, see footnote 121 above.

[153] This algorithm is ubiquitous in Computi. See, for instance, Maximus, *Enarratio* I.24, in *PG* XIX 1244 (second algorithm); the computistical section of the *Florilegium Coislinianum*, letter Π, nº 168; *Anonymus* 830, sects. 15 and 28, in GASTGEBER, Neue texte XXX and XXX; *Anonymus* 951, sect. 14; *Anonymus* 982, sect. 9; *Anonymus* 1041, sect. 8; *Anonymus* 1079, sects. 1 and 7, in MENTZ, Beiträge zur Osterfestberechnung 76 and 90–92, respectively; *Anonymus* 1090, sect. 2; *Psellos* 1092, sect. I.13, in REDL, La chronologie appliquée II 229–232; *Anonymus* 1092A, sect. 5, in KARNTHALER, Die chronologischen Abhandlungen 6.69–77; *Anonymus* 1172, sect. 9; *Anonymus* 1183, sect. 7; *Anonymus* 1256, sect. 11 (one of the algorithms uses the "epacts of the months" [see just below], also computed in sects. 11–12); *Blastares* 1335, in RHALLES – POTLES, Σύνταγμα VI 418; *Rhabdas* 1342, sect. 11; *Meliteniotes* 1352, sect. III.23; *Anonymus* 1377, sect. 3, in *PG* XIX 1317–1320; *Anonymus* 1379, in *PG* XIX 1332. As for the "epacts of the months", they correspond to Western *regulares*: the epact of a given month is the weekday of the last day of the previous month (a further example of an "incipient" quantity), gauged to $w(1_O) = 2$: thus $e_O = 1$; see Maximus, *Enarratio* I.26, in *PG* XIX 1244–1245; *Anonymus* 830, sect. 1, in GASTGEBER, Neue texte XXX; *Anonymus* 1092B, sect. 11, in KARNTHALER, Die chronologischen Abhandlungen 12.286–13.1; *Anonymus* 1172, sect. 20; *Anonymus* 1204; *Anonymus* 1247, sect. 16, in SCHISSEL, Chronologischer 109; *Anonymus* 1256, sect. 11 and 14; *Anonymus* 1350 sect. 6, in SCHLACHTER, Wiener griechische 7; *Argyros* 1372, sect. 5, in *PG* XIX 1285–1288; *Anonymus* 1377, sect. 3, in *PG* XIX 1320 (but only the excess of each month over 4 weeks is tabulated); *Anonymus* 1379, in *PG* XIX 1332–1333.

[154] Solar cycle years are used only in computistical algorithms of this kind.

13

ἕτερος ψῆφος, δι' οὗ εὑρίσκεται ἑκάστη ἡμέρα τῆς ἑβδομάδος

Ἔχε κατανοῦν πάντοτε τὴν τελευταίαν[a] ἡμέραν τοῦ Σεπτεμβρίου μηνός, καὶ εἰ μὲν ἐστὶν κυριακή, ἔχε μίαν· εἰ δὲ ἐστὶν δευτέρα, ἔχε β· εἰ[b] δὲ τρίτη, τρεῖς· εἰ[c] δὲ τετάρτη, δ· εἰ[d] δὲ πέμπτη, ε· εἰ[e] δὲ ἕκτη, ς· εἰ[f] δὲ σάββατον, ζ· καὶ ἄρξαι ἀπὸ τοῦ Ὀκτωβρίου[g] μηνὸς λαμβάνειν τοῦ μηνὸς τοῦ ἔχοντος λα τρεῖς ἡμέρας, τοῦ δὲ ἔχοντος λ ἡμέρας β, τοῦ δὲ μηνὸς ἐν ᾧ θέλεις εὑρεῖν τὴν ἡμέραν βάλε ὅσας ἔχει, καὶ ὕφειλε διὰ τῶν ζ, καὶ τὰ κάτωθεν τῶν ζ δηλοῦσιν[h]. εἰ[i] δὲ μὴ ἔχεις κάτωθεν τῶν |$_{8r}$ ζ, σάββατόν ἐστιν.

[a] τελευτέαν [b] ἡ [c] ἡ [d] ἡ [e] ἡ [f] ἡ [g] Ὀκτοβρίου [h] pauca verba omissa vide versionem [i] ἡ

A further calculation, by means of which each weekday can be found

Always keep in mind the last day of the month of September, and if it is a Sunday, hold one; if it is a Monday, hold 2; if a Tuesday, three; if a Wednesday, 4; if a Thursday, 5; if a Friday, 6; if a Saturday, 7; and from the month of October begin taking three days of the month that has 31, 2 days of the one that has 30, and, of the month in which you want to find the day, put how many <days> it has, and remove by 7, and that which <remains> down from 7 shows <the weekday>. If you do not have <anything> down from 7, it is Saturday.

A computation of the weekday of date x in month X. The algorithm is as follows, where n_k = length of month k in days[155]:

$(w[30_S],x,X) \rightarrow w(30_S) + \sum_{k=O}^{X-1}(n_k - 28) + x \rightarrow [w(30_S) + \sum_{k=O}^{X-1}(n_k - 28) + x] \bmod 7 = w(x_X)$.

This algorithm for computing the weekday of date x in month X takes the weekday of September 30 (= the day before the beginning of a solar cycle) and then computes as in the previous algorithm. The solar cycle year begins on October 1; consequently, $w(30_S)$ takes the role of the epacts of the Sun. As the algorithm mentions only months of 31 and 30 days, and years are absent, we may suppose either that February has always 28 days, and the algorithm is accordingly valid if month X is not later than February, or that the case of February having 29 days has been omitted by negligence or mistake, and the algorithm is accordingly valid in general. All in all, the prescriptions for finding the weekday of a given date are carelessly formulated in *Anonymus* 892.

14

ψῆφος <εἰς> τὸ εὑρεῖν τὸ νομικὸν Πάσχα

Κράτει[a] ἐπὶ χειρῶν σου πάντοτε μγ, καὶ ὕφειλον ἐξ αὐτῶν τὰς ἐπακτὰς τῆς σελήνης ἑκάστου ἔτους, καὶ ὑφειλόντος δέ σου τὰς ἐπακτὰς τῆς σελήνης ἐνεμείνασιν ἐπὶ δακτύλων σου πλέον τῶν λ, τὰ μὲν λ ὕφειλε τὰ δὲ κάτωθεν τῶν λ, ἐὰν εἰσὶν ἀπὸ μιᾶς ἕως ιθ, γινώσκε ὅτι ἐν τῷ Ἀπριλίῳ ἐστὶν τὸ νομικὸν Πάσχα· εἰ δὲ μένουσιν[b] κ ἢ κα ἢ κβ ἢ κγ καὶ καθεξῆς, πάντοτε πρόσθες ἡμέραν μίαν, καὶ ζήτει τὸ νομικὸν Πάσχα ἐν τῷ Μαρτίῳ, πλὴν εἰς τὸν ιη^{ον} κύκλον μὴ προσθήσεις, εἰς δὲ τὸν ἑπτὰ καὶ δέκατον κύκλον τῆς σελήνης ἑπτὰ ἐπακτῶν μενουσῶν σοι ὀκτὼ ὕφειλε ἐκ τῶν τεσσαρακοντατριῶν. ὁμοίως[c] καὶ εἰς τὸν ιθ^{ον} κύκλον κθ ἐπακτῶν μενουσῶν σὺ τρίακοντα ὕφειλε ἐκ τῶν μγ, ἐπεὶ |$_{8v}$ οὐκ εὑρίσκεις τὸ[d] ἀσφαλές. εὑρίσκονται δὲ αἱ ἐπακταὶ τῆς σελήνης οὕτως. ἐν μὲν τῷ πρώτῳ κύκλῳ ἔχει ἕνδεκα ἐπακτάς, καὶ ἐν τῷ δευτέρῳ, κβ, ἐν δὲ τῷ γ^{ῳ}, τρεῖς, ἐπειδὴ ἀπήρτισται ὁ τριακοστὸς ἀριθμός· τρεῖς γὰρ ἑνδεκάκις, τριακοντατρεῖς· καὶ ὑφειλόντος σου τὰ λ μένουσιν γ. αὗται εἰσὶν αἱ ἐπακταὶ τοῦ τρίτου κύκλου. ὁμοίως καὶ καθεξῆς ἕως ἐννεακαιδεκάτου[e] κύκλου τῆς σελήνης ἑνδεκαπλασίαζε τὸν οἱονδηποτοῦν[f] κύκλον τῆς σελήνης, καὶ ὕφειλε τριακοστά, καὶ τὰ μένοντα κάτωθεν τῶν τριάκοντα εἰσὶν αἱ ἐπακταὶ τῆς σελήνης.

[a] κράτη [b] μήνουσιν [c] ὁμοίος [d] τω [e] ἐννακαιδεκάτου [f] οἱονδιποτοῦν

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[155] Other occurrences of this algorithm can be found in *Anonymus* 830, sect. 13, in GASTGEBER, Neue texte XXX; *Anonymus* 1256, sect. 15.

Calculation <for> finding Passover

Always keep 43 in your hands, and remove the epacts of the Moon of each year from them, and if, once you removed the epacts of the Moon, there remain more than 30 in your fingers, remove 30 and, if that which <remains> down from 30 is from one up to 19, know that Passover is in April. If there remain 20 or 21 or 22 or 23 and so on, always add one day, and seek for Passover in March, with the <following> exceptions: in the 18th cycle you will not add <anything>, in the seventeenth cycle of the Moon, where 7 epacts remain for you, remove eight from forty-three. Similarly also in the 19th cycle, where 29 epacts remain, remove thirty from 43, for you do not find what is secure. The epacts of the Moon can be found as follows. In the first cycle it has eleven epacts, and in the second, 22, in the 3rd, three, since the thirtieth number really turns out to be completed: for eleven times three, thirty-three; and once you removed 30, there remain 3. These are the epacts of the third cycle. Similarly also, continuously up to the nineteenth cycle of the Moon, undecuple whatever cycle of the Moon, and remove thirtieths, and that which remains down from thirty are the epacts of the Moon.

A computation of Passover. The algorithm is as follows[156]:

$(e_m) \rightarrow 43 - e_m \rightarrow$

$| \; 43 - e_m \leq 19, 43 - e_m \rightarrow (43 - e_m) \bmod 30 = p_m \in A.$

$| \; 43 - e_m > 19, 43 - e_m \rightarrow (43 - e_m) \bmod 30 + 1 = p_m \in M.$

$| \; m = 18, (43 - e_m) \bmod 30 = p_{18}.$

$| \; m = 17, 43 - 8 \rightarrow [43 - 8] \bmod 30 + 1 = p_{17}.$

$| \; m = 19, 43 - 30 \rightarrow [43 - 30] \bmod 30 = p_{19}.$

In the last three branches of the algorithm, the month in which Passover is located is tacitly determined by the inequalities that single out the first two branches. The branching conditions involve both a "norm minus epacts" quantity $43 - e_m$ (see the commentary on sect. **12**) and the lunar cycle m; the latter is the case after the *saltus lunae*. The formulation of the last branch makes it certain that $e_{19} = 29$; consequently, the list of epacts has a discontinuity at the end of cycle year 19, whereas the *saltus lunae* is shifted back by the algorithm to the end of cycle year 16. Thus, in this case the *saltus lunae* is a feature of the algorithm, not of the epact sequence.

To clarify this statement, recall that the epacts are the accumulated advance of 12 lunar months over a calendar year of 365 days; this advance is of 11 days for each lunar cycle year. Whenever this sum exceeds 30 units, these are subtracted to form an embolismic month. Since 11 and 30 are mutually prime, the sequence of epacts is cyclic and runs through all numbers from 1 to 30 before returning to the initial value. The order in which the numbers from 1 to 30 appear is as follows:

30	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26	7	18	29
	10	21	2	13	24	5	16	27	8	19	30								

Choosing the initial point of the sequence amounts to fixing the age of the Moon on the first day of the first cycle. Any initial point can be selected; the list is cyclic, as I have emphasized by repeating number 30. Two "natural" starting points are $30 \equiv 0 \pmod{30}$, so that the first day of the first cycle is *luna I*, and 11, the value of the yearly lunar advance. These values also occupy consecutive places in the list; the former was adopted as a starting point in the Alexandrian Computus, the latter in the Byzantine Computus (see sect. **12**). If returning to the starting point takes an entire cycle, a good approximation of an exact return is after 19 steps, where the difference is 1: $11 \times 19 = 209 \equiv -1 \equiv 29 \pmod{30}$[157]. The epact sequence singled out by this approximation is the sequence of the 19-year cycle, with a "*saltus lunae*"—namely, a "discontinuity" of the epact sequence—at the end: the epact value that follows 29 in this 19-token sequence is again 11, which is obtained from 29 by adding 12, not 11, and then reducing modulo 30. However, one must be careful in distinguishing between the *saltus lunae* and the "discontinuity" of the epact sequence. The point is that any cyclic sequence of epacts that does not coincide with the whole

[156] See George, sect. II.4, in DIEKAMP, Der Mönch 30.5–31.2, *Anonymus* 982, sect. 15, and the discussion in sect. **12** above.

[157] A 30-year cycle might underlie the Paschal canon presented by the Eastern bishops at the Council of Serdica (*ca*. AD 343): see SCHWARTZ, Christliche und jüdische Ostertafeln 121–125, GRUMEL, La Chronologie 41–43, and MOSSHAMMER, The Easter Computus 184–186. This cycle began in AD 328.

30-token sequence must have a "discontinuity" (that is, a disruption of the principle of constant difference 11) somewhere: a *saltus lunae* at the point of discontinuity has exactly the function of making consecutive lunar cycles fit to each other. However, the *saltus lunae* can be placed at any point of the cycle, but this does not entail a displacement of the discontinuity of the epact sequence. For this reason, in Byzantine Computi, actually set out or reconstructed lists of epacts can be found that differ as to the position of the discontinuity (not of the *saltus lunae*) but that nevertheless give rise to the same list of Passover dates, which is impossible if equivalent Passover algorithms are intended: very simply, there is a one-to-one correspondence between epact values and Passover dates for any given algorithm. In the same way, a list of epacts is not significant unless the algorithm in which it is used is also provided. To sum up, one must look at the Passover algorithm actually used in order to ascertain where the *saltus lunae* is really located.

Since 50 – 6 = 44, and on account of the definition of the epacts of the Moon provided later in the text and of the position of the modulo 30 reduction, the norm 43 algorithm for computing Passover expounded in this section is equivalent both to the norm 50 algorithm of sect. **12** and to the norm 44 algorithm of sect. **15**. The latter equivalence is proved by observing that the number 44 (= 43 + 1) figures in the algorithm branch for $43 - e_m > 19$ (namely, if Passover falls in March). If, conversely, $43 - e_m \leq 19$, one unit is absorbed by the shifted origin of day-counting from 1_M to 1_A because of the discrepancy between modulo 30 reduction and March having 31 days. Since most Easter dates fall in April, the norm is set to 43 rather than 44. For $m = 18$, the unit to be added to the epacts because of the *saltus lunae* is offset by the unit to be added for $43 - e_m > 19$. For $m = 17$, $e_{17} = 7$, to which 1 must be added because of the *saltus lunae*, and the second branch of the algorithm applies. For $m = 19$, $e_{19} = 29$, to which 1 must be added because of the *saltus lunae*, and the first branch of the algorithm applies. The number of branches makes this algorithm unwieldy.

The text continues by showing how to find the epacts of the Moon[158]. The rule is:

$e_1 = 11$; $e_m = [e_1 + 11(m - 1)] \bmod 30$ if $m > 1$.

This is a simple variant of the standard rule for finding the epacts. No mention is made of the *saltus lunae*; therefore, the discontinuity is located at the end of the epact sequence. The resulting epacts are set out in the following table; again for the convenience of the reader, the table includes the Passover dates:

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
e_m	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26	7	18	29
p_m	2	22_M	10	30_M	18	7	27_M	15	4	24_M	12	1	21_M	9	29_M	17	5	25_M	13

15

ἕτερος ψῆφος τοῦ νομικοῦ Πάσχα

Γίνωσκε τὰς ἐπακτὰς ἑκάστου ἔτους τῆς σελήνης, καὶ σύναπτε αὐτὰς ἀπὸ Μαρτίου ἕως οὗ ποιή|$_{9r}$σεις ἀριθμὸν μδ· ἐκεῖ ἐστὶν τὸ νομικὸν Πάσχα. ἐὰν δὲ ἔχεις ἐπακτὰς σεληνιακὰς[a] κε ἢ κς ἢ κζ ἢ κη ἢ κθ, πρόσθες ἐν αὐταῖς πάντοτε ἡμέραν μίαν, καὶ ἀρίθμησον ἀπὸ Ἀπριλίου ἕως οὗ καταντήσεις εἰς τὸν τῶν μδ ἀριθμόν, καὶ ἐκεῖ ἐστὶν τὸ νομικὸν Πάσχα. Εἰς δὲ τὸν ιζ^{ον} καὶ ιη^{ον} κύκλον, πρόσθες ἑνὶ ἑκάστῳ ἐπακτὴν μίαν, καὶ ἀρίθμησον ἀπὸ Μαρτίου ἕως μδ. Εἰς δὲ τὸν ἐννεακαιδέκατον[b] κύκλον, ἀπὸ Μαρτίου μόνον ἄρξαι μηδεμίαν ἐπακτὴν συνάπτων αὐτῷ ἕως οὗ καταντήσεις[c] εἰς τὸν τῶν μδ ἀριθμόν· ἐκεῖ ἐστὶν τὸ νομικὸν Πάσχα.

[a] σελινιακὰς [b] ἐννακαιδέκατον [c] καταντίσεις

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[158] For the epacts of the Moon in Byzantine Computi, see the early and clear expositions by George, sect. II.2, in DIEKAMP, Der Mönch 25.25–35, and by Maximus, *Enarratio* I.7, in *PG* XIX 1223. Early Computi in which the epacts of the Moon are calculated or tabulated include Stephanus-Heraclius' *in Ptolemaei Tabulas Manuales*, sects. 12 (the era Philip and the era Constantine are used) and 30 (the era Maurice is used), in LEMPIRE, Le commentaire 154.2–156.12, and USENER, De Stephano Alexandrino 315–316, respectively; the table in the computistical section of the *Florilegium Coislinianum*, letter Π, nº 166 (with fractional parts generated by accumulating twelfths); *Anonymus* 830, sects. 7 and 34, in GASTGEBER, Neue texte XXX and XXX; *Theophylaktos* 956, sect. 4 (the AD era is used); *Anonymus* 1183, sects. 9 and 12 (with fractional parts generated by accumulating nineteenths: the *saltus lunae* is evenly distributed among the lunar cycles). Epacts with fractional parts are also found in *Anonymus* 1092B, sect. 7, in KARNTHALER, Die chronologischen Abhandlungen 11.207–248 (3 minutes are added for each cycle, plus 1 minute every time six cycles have been completed: the *saltus lunae* is unevenly distributed among the lunar cycles).

A further calculation of Passover

Know the epacts of each year of the Moon, and conjoin them from March until you will make number 44: Passover is there. If you have lunar epacts 25 or 26 or 27 or 28 or 29, always add one day in them, and count from April until you will arrive at number 44, and Passover is there. In the 17th and in the 18th cycle, add one epact to each one, and count from March up to 44. In the nineteenth cycle, just begin from March without conjoining any epact to it, until you will arrive at number 44: Passover is there.

A computation of Passover. The (annotated) algorithm is as follows[159]:

$(e_m) \rightarrow$

$| m = 1\text{–}4, 6\text{–}7, 9\text{–}15, 44 - e_m \text{ –}: 1_M = p_m.$

$| 25 \leq e_m \leq 29, e_m + 1 \rightarrow 44 - (e_m + 1) \text{ –}: 1_A = p_m.$ {$m = 5, 8, 16, 19$}

$| 17 \leq m \leq 18, e_m + 1 \rightarrow 44 - (e_m + 1) \text{ –}: 1_M = p_m.$ {*saltus lunae*}

$| m = 19, 44 \text{ –}: 1_M = p_{19}.$ {$p_{19} = 13_A$}

This norm 44 algorithm for computing Passover is straightforwardly equivalent, after deletion of the modulo 30 reductions, to the norm 43 algorithm of sect. **14**. Adding 1 to large epacts (as in the second branch) derives from counting from 1_A and from March having 31 days. The additional unit in cycle years 17 and 18 (third branch) is the *saltus lunae*. The no-epacts prescription for cycle year 19 (fourth branch) originates from $e_{19} + 1 = 29 + 1 = 30 \equiv 0 \pmod{30}$: the additional unit coming from the *saltus lunae* leads to an epactsless branch if days are counted from 1_M. This shows that the value $e_m = 29$ should be deleted from the second branch. The explicit mention of the epact value $e_m = 29$ shows that the discontinuity of the epact sequence is located after cycle year 19. However, the *saltus lunae* is placed after lunar cycle 16; therefore, it is a feature of the algorithm. As in sect. **14**, the number of branches makes this algorithm unwieldy.

16

ἕτερος ψῆφος τοῦ Πάσχα

Γνῶθι[a] πόσα ἔτη[b] εἰσὶν ἀπὸ Ἀδὰμ ἕως τοῦ ἐνεστῶτος, καὶ ὕφειλε ἐξ αὐτῶν ιγ, καὶ τὰ λοιπὰ πάντα ὕφειλε διὰ τῶν ιθ, καὶ τὰ μένοντα κάτωθεν τῶν ιθ |$_{9v}$ ἐνδεκαπλασίασον, καὶ ὕφειλε διὰ τῶν λ, καὶ τὰ μένοντα κάτωθεν τῶν λ σύναψον ἀπὸ α Μαρτίου ἕως οὗ ποιήσεις ν ἡμέρας, καὶ ἐκεῖ ἐστὶν τὸ νομικὸν Πάσχα.

[a] Γνώθη [b] ἔτι

A further calculation of Passover

Know how many years there are from Adam up to the present one, and remove 13 from them, and the whole of the remainder remove by 19, and undecuple what remains down from 19, and remove by 30, and conjoin what remains down from 30 from March 1 until you will make 50 days, and Passover is there.

A computation of Passover. The algorithm is:

$(y) \rightarrow y - 13 \rightarrow (y - 13) \bmod 19 \rightarrow 11[(y - 13) \bmod 19] \rightarrow \{11[(y - 13) \bmod 19]\} \bmod 30 \rightarrow$

$\rightarrow 50 - \{11[(y - 13) \bmod 19]\} \bmod 30 \text{ –}: 1_M = p_y.$

This algorithm is equivalent to the algorithm of sect. **12** because, by definition, $y \bmod 19 = m$ and because $6 \equiv -13 \pmod{19}$; hence, $11(y - 13) \equiv 11m + 66 \pmod{19}$. Finally, $11m + 66 \equiv 11m + 6 \pmod{30}$. However, the

[159] See *Anonymus* 982, sects. 10 and 14; *Anonymus* 1172, sect. 8 (an incomplete algorithm); *Anonymus* 1273, sect. 7, in BUCHEGGER, Wiener griechische Chronologie 31.58–75; *Meliteniotes* 1352, sect. III.24. These two algorithms do not mention the epacts, replacing them with their definition $e_m = 11m \bmod 30$. See also the discussion in sect. **12** above. An algorithm that is straightforwardly equivalent to this norm 44 algorithm is a norm 45 algorithm in which the epacts are replaced by the "pastoral base" $b_m = e_m + 1$: see *Anonymus* 1079, sects. 3–4, in MENTZ, Beiträge zur Osterfestberechnung 78; *Anonymus* 1172, sect. 7 (an incomplete algorithm), and sect. 18 for the "base"; *Anonymus* 1273, sect. 6, in BUCHEGGER, Wiener griechische Chronologie 30.43–31.57; *Anonymus* 1095. In all these algorithms, the *saltus lunae* is a feature of the sequence of bases.

compiler omitted to include the *saltus lunae* after cycle 16. This algorithm can also be found in *Anonymus* 830, sect. 26; *Anonymus* 982, sect. 11.

17

ἕτερος ψῆφος σύντομος τοῦ Πάσχα

Γίνωσκε ἀκριβῶς τὸ πόσας ἡμέρας ἔχει ἡ σελήνη τὴν α τοῦ Ἰαννουαρίου, καὶ κράτει[a] αὐτάς, καὶ ἀνάβα ψηφίζων[b] ἀπὸ τὴν α τοῦ Ἰαννουαρίου τὰς ἡμέρας τῶν μηνῶν ἕως οὗ ποιήσεις ρε ἡμέρας, καὶ ἐκεῖ ἐστὶν τὸ νομικὸν Πάσχα. εἰ[c] δὲ ἔστιν βίσεκτον, <ρ>ς. ἐπὰν δὲ εὕρῃς τὸ νομικὸν Πάσχα, ψήφισον[d] ἐν ποίᾳ ἡμέρᾳ τῆς ἑβδομάδος ἐστίν, καὶ ἀνελθὼν ἕως τῆς κυριακῆς εὑρήσεις τὸ ἡμέτερον Πάσχα.

[a] κράτη [b] ψιφίζων [c] ἡ [d] ψίφησον

A further, concise, calculation of Passover

Know exactly how many days the Moon has on January 1, and keep them, and calculating from January 1 mount the days of the months until you will make 105 days, and Passover is there. If it is a leap year, <mount up to> <10>6. As soon as you have found Passover, calculate on what weekday it is, and reaching up to Sunday you will find our Easter.

A computation of Passover. The algorithm is:

$(y,a[1_J]) \rightarrow 105 + [\![(y \bmod 4)/4]\!] - a(1_J) -: 1_J = p_y$.

This norm 105 algorithm for computing Passover shifts the origin of day-counting from 1_M to 1_J and thereby introduces the age of the Moon on January 1 (in other Computi, this is the "base" b_m) as a reference. As seen in sect. **12**, the shift adds 59 days to the count; these become 60 days in leap years (summand $[\![(y \bmod 4)/4]\!]$)[160]. Since 105 – 59 = 46 and looking for instance at the norm 44 algorithm of sect. **15**, one gets that, at lunar cycle *m* and including the *saltus lunae* in the sequence of epacts, $a_m(1_J) = b_m = e_m + 2$, instead of $e_m + 1$ as one should expect. If we accept that the latter is correct, the norm of the algorithm should be corrected to 104[161]. The text adds that Easter is the first Sunday after Passover.

18

β^{α} ψηφοφορία[a] τοῦ Πάσχα

Γίνωσκε ὅτι ἐν τοῖς ιθ κύκλοις τῆς σελήνης εὑρίσκονται τὰ πασχάλια, $|_{10r}$ τουτέστιν ἀπὸ $\alpha^{ου}$ ἕως ἐννεακαιδεκάτου[b], τὰ πάντα γινόμενα[c] ιθ Πάσχα. ταῦτα τὰ ιθ εὑρίσκονται ἐν ἑκάστῳ μηνὶ[d] κατὰ τάξιν οὕτως, τουτέστιν Μαρτίῳ καὶ Ἀπριλίῳ α Πάσχα. ἐν πρώτοις κάτεχε ἐπὶ δακτύλων σου πάντοτε δύο ἡμέρας τοῦ Ἀπριλίου, τουτέστι πρώτην καὶ δευτέραν, καὶ μηδὲν ἄλλο ποιήσεις, ἀλλὰ ζήτησον ποῖος κύκλος ἐστὶν τοῦ ἡλίου, καὶ εὗρε[e] ἐξ αὐτοῦ ἐν ποίᾳ ἡμέρᾳ τῆς ἑβδομάδος κατήντησεν ἡ δευτέρα τοῦ Ἀπριλίου, καὶ εἰ μὲν κατήντησεν ἐν κυριακῇ, [[..]] ἐν ἄλλῃ κυριακῇ ἐστὶν τὸ ἅγιον Πάσχα· εἰ[f] δὲ εὑρέθη β^{α} ἢ $\gamma^{η}$ ἢ $\delta^{η}$ ἢ $\varepsilon^{η}$ ἢ $\varsigma^{η}$ καὶ καθεξῆς ἕως τοῦ σαββάτου, ἄνελθε εἰς κυριακήν, καὶ ἐκεῖ εὑρήσεις τὸ θειότατον Πάσχα. $\beta^{ον}$ Πάσχα. ὁμοίως πάλιν κράτει ἐπὶ δακτύλων σου κβ $|_{10v}$ ἡμέρας τοῦ Μαρτίου, καὶ ποίησον ὡσαύτως[g], τουτέστιν εὗρε τὴν ἡμέραν τοῦ Πάσχα. <$\gamma^{ον}$ Πάσχα.> κράτει[h] ἐπὶ δακτύλων σου δέκα τοῦ Ἀπριλίου. $\delta^{ον}$ Πάσχα λ τοῦ Μαρτίου. $\varepsilon^{ον}$ Πάσχα ιη τοῦ Ἀπριλίου. $\varsigma^{ον}$ Πάσχα ζ τοῦ Ἀπριλίου. $\zeta^{ον}$ Πάσχα κζ τοῦ Μαρτίου. $\eta^{ον}$ Πάσχα ιε τοῦ Ἀπριλίου. $\theta^{ον}$ Πάσχα δ τοῦ Ἀπριλίου. $\iota^{ον}$ Πάσχα κδ τοῦ Μαρτίου. $\iota\alpha^{ον}$ Πάσχα ιβ Ἀπριλίου. $\iota\beta^{ον}$ Πάσχα α

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[160] As leap years must be disregarded in lunar computations, adding this summand constitutes a misunderstanding of the procedure. The supplementary unit in leap years is not mentioned in *Anonymus* 1079, sect. 1, MENTZ, Beiträge zur Osterfestberechnung 76. The mention of leap years in our Computus might be a faulty annotation that was inserted in the text.

[161] This is confirmed by *Anonymus* 1172, sect. 6, and by *Anonymus* 1273 (which do not add the unit of leap years, either). Both computists calls the standard base $b_m = e_m + 1$ "pastoral base" (ποιμενικὸς θεμέλιος): see the text inserted in the wheel in Vat. gr. 432, on f. 144v, and BUCHEGGER, Wiener griechische Chronologie 30–31, respectively; see also MENTZ, Beiträge zur Osterfestberechnung 46–47 n. 11. Unsurprisingly, *Anonymus* 830, sect. 35, in GASTGEBER, Neue texte XXX, uses the incorrect norm 105. On "bases", see footnote 95 above.

Ἀπριλίου. ιγ^{ον} Πάσχα κα τοῦ Μαρτίου. ιδ^{ον} Πάσχα Ἀπριλίῳ θ. ιε^{ον} Πάσχα Μαρτίῳ κθ. ις^{ον} Πάσχα Ἀπριλίῳ ιζ. ιζ^{ον} Πάσχα Ἀπριλίῳ ε. ιη^{ον} Πάσχα Μαρτίῳ κε. ιθ^{ον} Πάσχα Ἀπριλίῳ ιγ.

[a] ψηφοφορίη [b] ἐννεκαιδεκάτου [c] γινώμενα [d] μινὶ [e] εὗραι [f] ἡ [g] ὡσαύτος [h] κράτη

2nd calculation of Passover

Know that the Passover lists can be found in the 19 cycles of the Moon, that is, from the 1st up to the nineteenth, all of them amounting to 19 Passover's. These 19 are found in each month in order as follows, that is, 1 Passover in March or in April. In the first <cycles> always keep two days of April in your fingers, that is, the first and the second, and you will not do anything else than seek for what the cycle of the Sun amounts to, and find from it on what weekday occurred the second of April, and if it occurred on a Sunday, the Holy Easter is on the other Sunday; if it was found a Tuesday or a Wednesday or a Thursday or a Friday, and continuously up to Saturday, reach up to the <following> Sunday, and there you will find the most divine Easter. 2nd Passover. Similarly again keep 22 days of March in your fingers, and do likewise, that is, find the day of Passover. <3rd Passover.> Keep the ten of April in your fingers. 4th Passover 30 of March. 5th Passover 18 of April. 6th Passover 7 of April. 7th Passover 23 of March. 8th Passover 15 of April. 9th Passover 4 of April. 10th Passover 24 of March. 11th Passover 12 of April. 12th Passover 1 of April. 13th Passover 21 of March. 14th Passover April 9. 15th Passover March 29. 16th Passover April 17. 17th Passover April 5. 18th Passover March 25. 19th Passover April 13.

The list of Passover dates. There are 19 different Passover dates, one for each year of the lunar cycle. Easter is the first Sunday after Passover. A complete list of Passover dates is provided, as in the following table:

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
p_m	2	22_M	10	30_M	18	7	27_M	15	4	24_M	12	1	21_M	9	29_M	17	5	25_M	13

19

περὶ ἐμβολίμου[a] χρόνου

Γίνωσκε ἀπὸ πρώτου ἔτους τῆς σελήνης (ἤγουν κύκλου) ἕως ιθ^{ου} κατὰ τρία ἔτη δεκατρεῖς[b] μῆνας ἔχει ἡ σελήνη ἐκ τῆς περισσείας[c] τῶν ια ἡμερῶν ὧν προέλαβεν τὸν χρόνον |$_{11r}$ τοῦ ἡλίου· ὁ γὰρ χρόνος τοῦ ἡλίου τξε ἡμερῶν ἐστὶν ἄνευ τοῦ δ^{ου}, ὁ δὲ τῆς σελήνης τνδ· ἀπὸ γὰρ τῶν τνδ ἕως τῶν τξε ια ἡμέραι εἰσίν· κατέτος[d] γὰρ ια ἡμέραι περισσεύουσιν τῆς σελήνης αἱ ἐπακταὶ λεγόμεναι. οἷον ἐν τῷ α^{ῳ} ἔτη περισσεύουσιν ἕνδεκα, καὶ ἐν τῷ δευτέρῳ ια, ἰδοὺ κβ διὰ τὴν προσθήκην τῶν ια, καὶ ἐν τῷ γ^{ῳ} ια, ἰδοὺ τρὶς[e] ἕνδεκα, λγ, καὶ ἔχει τὸ ἔτος ἐκεῖνο[f] τῆς σελήνης ιγ μῆνας ἐκ τῆς συνάξεως τῶν ἑνδεκάδων· μετὰ γὰρ τὸν ἀπαρτισμὸν τῶν ιβ μηνῶν τῆς σελήνης περισσεύει μὴν εἷς, ὁ καὶ ἐμβόλιμος λεγόμενος· οὕτως γὰρ κατὰ γ ἔτη περισσεύει ἐμβόλιμος μὴν τῆς σελήνης.

[a] ἐμβολήμου [b] δεκατρὶς [c] περισσίας [d] καθέτος [e] τρεῖς [f] ἐκεῖνω

On the embolismic year

Know that from the first year of the Moon (namely, cycle) up to the 19th, the Moon has thirteen months every three years because of the remainder of 11 days by which it was in advance with respect to the year of the Sun; for the year of the Sun is of 365 days disregarding $^1/_4$, the <year> of the Moon is of 354; for from 354 up to 365 there are 11 days; for there remain over 11 days of the Moon per year, the so-called epacts. For instance, in the 1st <year> there remain over 11 days, and in the second, 11; there it is, 22 because of the addition of 11; and in the 3rd, 11; there it is, three eleven, 33, and that year of the Moon has 13 months because of the gathering of the hendecads; for after the completion of the 12 months of the Moon there remains over one month, the one also called embolismic; for in this way there remains over an embolismic month of the Moon every three years.

Embolismic years. There are 7 "embolismic lunar years" (they contain 13 lunations instead of 12; the additional lunar month has 30 days)[162] because the solar year of 365 ¼ days exceeds 12 lunar months (= 354 days) by 11 ¼ days. If we neglect fractional parts, these 11 days—the so-called "epacts of the Moon" (τῆς σελήνης ἐπακταί)—complete an "embolismic" lunar month of 30 days every two or three solar years; these days are subtracted from the cumulating epacts. Accordingly, the cumulating epacts must be reduced modulo 30. The text computes the epacts of the first three lunar cycles as an example; the implicit algorithm is $e_m = 11m \bmod 30$. In a whole cycle of 19 years, there are 209 (= 11×19) additional days, which complete 7 embolismic months of 30 days via the trick of the *saltus lunae* (that is, the age of the Moon is increased by one day at some point of its cycle). Hollow years (κοιλοί) in the 19-year cycle are years 1, 2, 4, 6, 7, 9, 10, 12, 13, 15, 17, 18; embolismic years (ἐμβόλιμοι) are years 3, 5, 8, 11, 14, 16, 19[163].

Below is a table of the main numerical data related to the lunar cycle that are explicitly or implicitly assumed in *Anonymus* 892, sects. **12**, **14**, **15**, **17**, **19**: these data are the epacts (the *saltus lunae* is placed after cycle year 19) the location of the embolismic year, and the date of Passover.

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
e_m	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26	7	18	29
emb.			*		*			*			*			*		*			*
p_m	2	22_M	10	30_M	18	7	27_M	15	4	24_M	12	1	21_M	9	29_M	17	5	25_M	13

20

ψῆφος ἕτερος, εἰς τὸ εὑρεῖν τὰς ἐπακτὰς τοῦ ἡλίου

|$_{11v}$ Ἐὰν θέλεις εὑρεῖν τὰς ἐπακτὰς τοῦ ἡλίου ἑκάστου ἔτους, γίνωσκε ὅτι ἔχει ὁ χρόνος ἡμέρας τξε δ^{ov}, καὶ ὕφειλε αὐτὰ διὰ τῶν ζ, καὶ τὰ μένοντα κάτωθεν τῶν ζ εἰσὶν αἱ ἐπακταὶ ἑκάστου ἔτους τοῦ ἡλίου ἤτοι κύκλου. οἷον α^{oς} κύκλος τοῦ ἡλίου (ἤγουν χρόνος) ἔχει ἐπακτὴν μίαν τέταρτον, ἐπειδὴ ὑφείλαμεν τὰς τξε δ^{ov} διὰ τῶν ζ· οἷον ἑπτάκις ν, τν· καὶ μένουσιν ιε δ^{ov} ἡμέραι. πάλιν δὶς ἑπτά, ιδ· καὶ μένει μία δ^{ov}, ἥτις λέγεται ἐπακτὴ^{a} ἡλιακή. καὶ πάλιν ὁ δεύτερος ἔχει δύο ἥμισυ ἐπακτάς, ἐπειδὴ κατέτος περισσεύει μία τέταρτον. καὶ ὁ τρίτος ἔχει τρὶς ἥμισυ^{b} τέταρτον. ὁ δὲ τέταρτος ἔχει ε ἐπακτὰς ἐκ τῆς προσθήκης |$_{12r}$ τῶν τετάρτων. ὁ δὲ πέμπτος ἔχει ἓξ τέταρτον ἐπακτάς. ὁ δὲ

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[162] Other expositions of lunar embolism include George, sect. II.4, in DIEKAMP, Der Mönch 31.3–15; Maximus, *Enarratio* I.7–10, and 12–13, in *PG* XIX 1224–1232; *Anonymus* 830, sect. 32, in GASTGEBER, Neue texte XXX; *Anonymus* 982, sect. 12; *Psellos* 1092, sects. II.24–25 and 27, in REDL, La chronologie appliquée II 261.14–264.9 and 270.21–271.7; *Anonymus* 1172, sect. 14; *Anonymus* 1183, sect. 11.

[163] A prescription for locating all new Moons in a lunar cycle is a "lunar calendar"; such calendars were paid special attention in Latin West; Dionysius Exiguus' table did not include one. Most of the range of variability I shall mention just below is mentioned at appropriate places in HOLFORD-STREVENS, Paschal Lunar Calendars. It must be stressed that some lunar calendars we find in secondary literature are reconstructed, and that, frequently, lunar calendars are not uniquely determined by the available data: additional constraints must be imposed, such as the base principle of alternation of full and hollow months (a principle that is not followed in the *latercus*), or the requirement that full lunar months end in odd-place solar months of the Julian calendar. Consequently, extant or reconstructed lunar calendars display some variability as to: (*a*) the exact mechanism of embolism (this can be the standard insertion of 30-day months, or a clever disposition of lunar months entirely included in a calendar month—the *lunae abortivae*—as in the *De ratione conputandi*); (*b*) the exact position of the embolismic months; (*c*) the exact position of the *saltus lunae*, as in principle any full month in the year that carries the *saltus* can be made hollow; (*d*) how the intercalary days of any Julian-style calendar are to be taken into account; (*e*) where the lunar year begins (Bede canonically has the "lunar years" begin on the first new Moon of Spring; *Psellos* 1092, sect. II.25, in REDL, La chronologie appliquée II 263.1–267.2, has the "lunar years" begin on Passover [the same in Dionysius, KRUSCH, Studien (1938) 85–86]; the first month of a lunar cycle year ends with the first new Moon of January). The matter is complicated by the fact that a lunar cycle year does not coincide with a "lunar year", and it is not obvious whether the embolisms should be keyed to the former or to the latter. Bede's lunar calendar coincides with the one printed in the excellent edition of the Carolingian standard calendars A. BORST, Der karolingische Reichskalender und sein Überlieferung bis ins 12. Jahrhundert (*Monumenta Germaniae Historica. Libri Memoriales* 2). I–III. Hannover 2001, 1647–1727; see also GRUMEL, La Chronologie 303, who corrects a mistake in GINZEL, Handbuch III 136–137, from where he draws the table, but introduces several more by his decision to mark full lunar months in italics; it is better to check HOLFORD-STREVENS, Paschal Lunar Calendars 202, and *passim* for the other lunar calendars. Other schemes of embolismic years in a late Western source are discussed in O. NEUGEBAUER, Astronomical and Calendrical Data in the Très Riches Heures, in: M. MEISS, French Painting in the Times of Jean de Berry: The Limbourgs and Their Contemporaries. Paris 1974, 421–432.

ἕκτος ἔχει ἑπτὰ ἥμισυ· ὕφειλε δὲ τὰς ἑπτά, καὶ μένει ἥμισυ ἐπακτῆς. οὕτως καὶ καθεξῆς ἕως τοῦ κη^{ου} κύκλου τοῦ ἡλίου. πλεονάζοντος ἀριθμοῦ ὑπὲρ τῶν ζ ὕφειλε τὰ ἑπτά, καὶ τὰ κάτωθεν [[τῶν]] κράτει. γίνωσκε δὲ ὅτι [[ἐ]] καθ' ἕκαστον[c] κύκλον τοῦ ἡλίου ἐπακταὶ λέγονται, καὶ θεῶν ἐπακταὶ κατ' Αἰγυπτίους. οἱ γὰρ Αἰγύπτιοι τὴν ἑβδομάδα κατὰ μίμησιν τῶν ζ πλανητῶν ἔλεγον, οὕστινας πλανήτας καὶ θεοὺς ὠνόμαζον[d] οἱ κακῶς φρονοῦντες· οἱ δὲ μὴ οὕτως φρονοῦντες θεοὺς μὲν ἔλεγον αὐτοὺς οὐ φύσει δὲ ἀλλ' ἐνεργείᾳ· διὰ γὰρ τὸ θέειν, ὃ ἐστὶν τρέχειν ἔλεγον αὐτοὺς θεούς, ὁ δὲ |$_{12v}$ φύσει θεὸς διὰ τὸ θεωρεῖν[e] τὰ πάντα λέγεται θεός. γίνωσκε δὲ ὅτι, ὡς λέγουσιν οἱ ἀρχαῖοι σοφισταί, κατὰ μίμησιν τῶν ζ ἀστέρων τῶν καὶ πλανητῶν λεγομένων ἐποίησεν ὁ θεὸς τὰς ἑπτὰ ἡμέρας τῆς ἑβδομάδος, καὶ ἀναλογεῖ ἡ μὲν κυριακὴ τῷ ἡλίῳ, ἡ δὲ δευτέρα τῇ σελήνῃ, ἡ δὲ τρίτη τῷ Ἄρει, ἡ δὲ τετάρτη τῷ Ἑρμῇ[f], ἡ δὲ πέμπτη τῇ Διΐ[g], ἡ δὲ ἕκτη τῇ Ἀφροδίτῃ, ἡ δὲ ἑβδόμη τῷ Κρόνῳ.

♄ α^{ος} ὁ Κρόνος [ἐν] ἐν τῇ πρώτῃ ζώνῃ[h] τοῦ οὐρανοῦ ἐστι, καὶ ὁδεύει τὰ ιβ ζῴδια διὰ ἐτῶν λ.

♃ β^{ος} ὁ δὲ Ζεὺς ἐν τῇ δευτέρᾳ, καὶ ὁδεύει τὰ ιβ ζῴδια διὰ ἐτῶν ιβ.

♂ γ^{ος} Ἄρης[i] ἐν τῇ τρίτῃ, καὶ ὁδεύει τὰ ιβ ζῴδια διὰ ἐτῶν ιε.

☉ δ^{ος} ὁ ἥλιος ἐν τῇ τετάρτῃ, καὶ ὁδεύει δι|$_{13r}$ὰ μηνῶν ιβ.

♀ ε^{η} ἡ Ἀφροδίτης[j] ἐν τῇ πέμπτῃ, καὶ ὁδεύει διὰ μηνῶν[k] ὀκτώ.

☿ ς^{ος} ὁ Ἑρμῆς ἐν τῇ ἕκτῃ, καὶ ὁδεύει διὰ μηνῶν[l] γ.

☾ ζ^{η} ἡ σελήνη ἐν τῇ ἑβδόμῃ, καὶ ὁδεύει διὰ ἡμερῶν λ.

ἡ σελήνη κατωτέρα[m] ἐστίν· ὡς γὰρ ἐὰν ποιήσεις πόλον ἔνδον ἄλλου πόλου, ὁ ἔνδον πόλος μικρότερος εὑρεθήσεται, οὕτως καὶ ὁ δρόμος τῆς σελήνης κατωτέρας οὔσης ὀλιγώτερος ἐστὶν καὶ ἀνοί<σ>εται τάχιον. πλανῆται δὲ λέγονται[n] οὐχ ὅτι πλανοῦσιν τινάς, ἀλλ' ὅτι πλαγίως πορεύονται τὸν δρόμον αὐτῶν.

[a] ἐκπακστὴ [b] ἥμησυ [c] καθέκαστον [d] ὀνόμαζον [e] θεορεῖν [f] Ἑρμεῖ [g] Διή [h] ζώνι [i] Ἄρις [j] Ἀφροδήτης [k] μεινῶν [l] μεινῶν [m] κατοτέρα [n] λέγωνται

A further calculation, for finding the epacts of the Sun

If you want to find the epacts of the Sun of each year, know that the year has 365 ¼ days, and remove them by 7, and that which remains down from 7 are the epacts of each year of the Sun, viz. cycle. For instance, the 1st cycle of the Sun (namely, year) has one epact and a quarter, since we really removed 365 ¼ by 7: that is, seven times 50, 350: and there remain 15 ¼ days. Again, twice seven, 14: and there remains 1 ¼, which is called "solar epact". And again, the second <cycle> has two and a half epacts, since there really remains over 1 ¼ per year. And the third has three and a half and a quarter. The fourth has 5 epacts because of the addition of the quarters. The fifth has six and a quarter epacts. The sixth has seven and a half; remove seven, and there remain a half of an epact. <Do> also in this way continuously up to the 28th cycle of the Sun. Whenever a number exceeds 7, remove seven, and keep what <remains> down. Know that they are called "epacts" for every cycle of the Sun, and "epacts of the Gods" by the Egyptians. For the Egyptians were used to refer to the week by taking the 7 planets as model, which planets were also named "gods" by those who think wrongly; those who do not think in this way called them "gods" not because of their nature but because of the kind of their activity; for they called them "gods" because of their "going", that is to say "running", whereas <that which is> god because of its nature is called "god" because of its "begolding" everything[164]. Know, as the ancient wise men say, that God made the seven days of the week by taking the 7 heavenly bodies, also called "planets", as model, and He likens Sunday to the Sun, Monday to the Moon, Tuesday to Mars, Wednesday to Mercury, Thursday to Jupiter, Friday to Venus, Saturday to Saturn.

♄ 1st Saturn is in the first zone of the heaven, and travels over the 12 signs in 30 years.

♃ 2nd Jupiter is in the second one, and travels over the 12 signs in 12 years.

♂ 3rd Mars is in the third one, and travels over the 12 signs in 15 years.

[164] I have tried to keep the paraetymology by forging the misspelled verb "to begold" for "to behold".

☉ 4th the Sun is in the fourth one, and travels over in 12 months.
♀ 5th Venus is in the fifth one, and travels over in 8 months.
☿ 6th Mercury is in the sixth one, and travels over in 3 months.
☾ 7th the Moon is in the seventh one, and travels over in 30 days.

The Moon is the lowest <heavenly body>; for, as when you will make a pole inside another pole, the inner pole will be found to be smaller, so the motion of the Moon, insofar as it is lower, is also smaller and it will be carried up faster. They are called "planets" not because they deceive people, but because they travel transversely across their course[165].

A computation for finding the epacts of the Sun. The algorithm is[166]:

$(s) \rightarrow [(365\ ^1/_4 \bmod 7)s] \bmod 7 = e_s$.

This algorithm, which computes with the crudest fractional approximation of the tropical year, is equivalent to adding $1\ ^1/_4 \equiv 365\ ^1/_4 \pmod 7$ for every year of the solar cycle and to reducing the result modulo 7; see sect. **2**. A complete list of epacts of the Sun is provided in the text, as in the following table; in this section, 7 mod 7 = 0, and fractional remainders are admitted (they "pass through" the modulo reduction).

s	1	2	3	4	5	6	7	8	9	10	11	12	13	14
e_s	$1\ ^1/_4$	$2\ ^1/_2$	$3\ ^1/_2\ ^1/_4$	5	$6\ ^1/_4$	$^1/_2$	$1\ ^1/_2\ ^1/_4$	3	$4\ ^1/_4$	$5\ ^1/_2$	$6\ ^1/_2\ ^1/_4$	1	$2\ ^1/_4$	$3\ ^1/_2$
s	15	16	17	18	19	20	21	22	23	24	25	26	27	28
e_s	$4\ ^1/_2\ ^1/_4$	6	$^1/_4$	$1\ ^1/_2$	$2\ ^1/_2\ ^1/_4$	4	$5\ ^1/_4$	$6\ ^1/_2$	$^1/_2\ ^1/_4$	2	$3\ ^1/_4$	$4\ ^1/_2$	$5\ ^1/_2\ ^1/_4$	/

The text states that these epacts are called "epacts of the gods" (θεῶν ἐπακταί) by the Egyptians, for they named the weekdays after the seven planets, which were also called "gods" because of the names of most of them; a polemical remark on the real etymology of the epithet follows[167]. The text continues by mentioning a well-known piece of astrological lore about the generation of the seven weekdays by imitation of the seven planets, in the order Sun, Moon, Mars, Mercury, Jupiter, Venus, Saturn[168], and by listing location and periods of the seven planets, as follows[169]:

planet	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
period	30*years*	12*y*	15*y*	12*months*	8*m*	3*m*	30*days*

The section ends with a remark on the fact that the Moon is the innermost planet and on the etymology of the word "planet".

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[165] I was unable to render the two Greek paraetymologies (πλανήτης vs. πλανάω and πλανήτης vs. πλαγίως) in English. The former paraetymology plays with the two meanings of πλανάω: "to wander" but also "to deceive".

[166] Other occurrences of this algorithm can be found in the computistical section of the *Florilegium Coislinianum*, letter Π, nº 165 (only a table); *Anonymus* 830, sects. 4 and 9, in GASTGEBER, Neue texte XXX and XXX; *Anonymus* 1092B, sect. 10, in KARNTHALER, Die chronologischen Abhandlungen 11.268–12.279; *Anonymus* 1183, sects. 13–14. A wheel of fractional epacts of the Sun is also found in Maximus, *Enarratio* I.1, in *PG* XIX 1219–1220; the fractional parts are deleted in the *PG* edition, but they are present in all manuscripts.

[167] See Theophilus of Antioch, *ad Aut*. I.4, *PG* VI 1029A, and also Nicomachus, *Ench*. 3, 241.15–18 Jan. The solar epacts are called "days of the Gods" by Paul of Alexandria, *Apotel*. 19–20. For the occurrence of the expression in the Index of Athanasius' Festal Letters (Athanasius died in AD 378), see MOSSHAMMER, The Easter Computus 82–83 and 163–166 (an edition of the text, with a translation and a commentary, is found in A. MARTIN – M. ALBERT, Histoire « acéphale » et Index syriaque des Lettres Festales d'Athanase d'Alexandrie [*Sources Chrétiennes* 317]. Paris 1985). For the transliterated occurrence in Ethiopic Computi, see O. NEUGEBAUER, Ṭentyon. *Orientalia* 44 (1975) 487–488. A list of the names of the planets designated by θεῶν ἡμέραι is found in a Pompei graffito: E. SCHÜRER, Die siebentägige Woche im Gebrauche der christlichen Kirche der ersten Jahrhunderte. *ZNW* 6 (1905) 1–66, 27, relying on [A. MAU], *Bullettino dell'Instituto di Corrispondenza Archeologica* (1881) 30; the list begins with Saturn.

[168] See Dio Cassius XXXVII.18.

[169] The list is also found in *Anonymus* 1092B, sect. 11, in KARNTHALER, Die chronologischen Abhandlungen 13.322–328.

Many ancient and Byzantine sources list the periods of the planets; the following table sets out those found in Geminos, *Isagoge* I.24–30, and in an end ninth-century scholium in Vat. gr. 1291[170]:

planet	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Geminos	30*y*	12*y*	2*y*6*m*	12*m*	12*m*	12*m*	27*d* ⅓
Vat. gr. 1291	30*y*	12*y*	1*y*6*m*	12*m*	11*m*6*d*	8*m*	30*d*

21

τὸ πῶς γίνεται βίσεκτον

Τὰ τοῦ κόσμου ἔτη ἀνάλυσον εἰς τὸ τετράκις, καὶ ἐὰν εἰς τέταρτον ἀπαρτισθῇ, λέγε εἶναι τὸ βίσεκτον.

How a leap year comes to be

Resolve out the years of the world into four-times, and if it gets completed to a fourth <year>, say that it is a leap year.

A criterion for identifying a leap year. If $y \equiv 4 \pmod 4$, then y is a leap year.

22

|13v Γίνωσκε τὸν σεληνιακὸν[a] κύκλον τῆς σελήνης ἐν ᾧ καὶ τὸ νομικὸν εὑρίσκεται Πάσχα ἀρχόμενον ἀπὸ κα Μαρτίου ἕως ιη Ἀπριλίου. γίνωσκε δὲ ὅτι ἔξωθεν τοῦ μηνὸς τούτου νομικὸν Πάσχα οὐ γίνεται, οὔτε ἄνωθεν οὔτε κάτωθεν. νόησον δὲ τὴν μὲν ἀρχὴν ἄνω εἶναι τὸ δὲ τέλος κάτωθεν. οὗτος[b] ὁ σεληνιακὸς μὴν λέγεται καὶ μέσος μὴν τῆς σελήνης.

[a] σελινιακὸν [b] οὗτως

Know that the lunar cycle of the Moon in which Passover is also found begins from March 21 <and reaches> up to April 18. Know that outside this month there cannot be Passover, <viz.,> neither above nor below. Consider that the beginning is above and the end is below. This lunar month is also called the "middle month of the Moon".

Terms for Passover. The terms for Passover are: $21_M \leq p \leq 18_A$ (see sect. **26**). A clarification follows, on the use of the adverbs ἄνωθεν and κάτωθεν as ways to specify the time arrow (see sects. **12** and **26**)[171]. The Passover interval is also called "middle month of the Moon" (μέσος μὴν τῆς σελήνης).

23

ἐρώτησις[a]

Διὰ τί κρατεῖ τὸ φέγγος λ ἡμέρας καὶ ἑβδομάδας τέσσαρες[b];

ἀπόκρισις

οὖν[c] καὶ ἡ σελήνη τέσσαρες[d] τροπὰς ἔχει· γίνεται γὰρ τὴν $\alpha^{\eta\nu}$ ἑβδομάδα θ, καὶ τὴν δευτέραν, ϵ, καὶ τὴν $\gamma^{\eta\nu}$, ο, καὶ τὴν $\delta^{\eta\nu}$, c, καὶ γράφει θεός. διὰ τῆς Ἄρκτου[e] ζ, διὰ τῆς Πλειάδος[f] ω, διὰ τῆς νοτίου ν, καὶ τὸ φέγγος θεός· |14r καὶ γράφει θεὸς ζῶν.

[a] ἐρώτισις [b] τέσσαρις [c] ὧν [d] τέσσαρεις [e] ἄκτρου [f] πλίαδος

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[170] See J. MOGENET, Les scholies astronomiques du *Vat. gr*. 1291. *BIBR* 40 (1069) 69–91, Text 4. For other sources see NEUGEBAUER, HAMA 604–607 (on the "great year", on which Psellos wrote, see also P. TANNERY, Psellus sur la grande année. *REG* 5 [1892] 206–211, repr. ID., Mémoires scientifiques IV. Toulouse – Paris 1920, 261–268) and 782–785. Geminos' value for the Moon is the length of the sidereal month.

[171] The opposite convention can be found in George, sect. II.3, in DIEKAMP, Der Mönch 29.29–30, and in Maximus, *Enarratio* I.14, in *PG* XIX 1232.

Question

Why does the lunar month keep 30 days and four weeks?

Answer

Then, the Moon also has four turning points: for <at the end of> the 1st week it becomes a θ, < at the end of> the second, an ϵ, <of> the 3rd, an o, and <of> the 4th, a c, and it writes θϵoc. Through Ursa Major ζ, through the Pleiads ω, through South ν, and the lunar month θϵoc; and it writes θϵoc ζων [*scil*. "Living God"].

Why does the lunar month (φέγγος) have 30 days and 4 weeks? Because the Moon has 4 turning-points (τροπαί); they bound 4 periods within the lunar month at the end of which the phases of the Moon, if they are put together one after the other, trace the word θϵoc. Moreover, through (?) Ursa Major ζ, through the Pleiads ω, through South ν, yielding θϵoc ζων. I have been unable to understand the connection with Ursa Major, the Pleiads, and South, nor have I found parallel texts.

24

σὺν θεῷ ψῆφος <εἰς> τὸ [[εὐ]] εὑρίσκειν τὴν ἀρχιμηνίαν[a] ἑκάστου μηνὸς ἐν ποίᾳ ἡμέρᾳ τῆς ἑβδομάδος ἐστίν

Γίνωσκε ἐφ'[b] οὗ θέλεις μηνὸς τοῦ ἐνεστῶτος χρόνου τὴν ἀρχιμηνίαν[c] ἐν ποίᾳ ἡμέρᾳ τῆς ἑβδομάδος ἐστίν, καὶ εἰ[d] μὲν ἐστὶν κυριακὴ ἡ τοῦ ἐνεστῶτος χρόνου ἀρχιμηνία[e], τοῦ μέλλοντος χρόνου ἐν δευτέρᾳ εὑρίσκεται, καὶ οὕτως καθεξῆς κατὰ τὴν τάξιν τῆς ἑβδομάδος γίνωσκε κατ' ἔτος[f] ἄρχεσθαι τὸν οἱονδηποτοῦν[g] μῆνα.

Γίνωσκε δὲ καὶ τοῦτο, ὅτι ἐὰν ἐστὶν τοῦ ἐνεστῶτος χρόνου ἀρχιμηνία[h] ἐν οἱῳδήποτε μηνὶ ἐν κυριακῇ καὶ ἐπιφέρεται βίσεκτον[i], τρίτην νόησον τὴν ἀρχιμηνίαν[j] τοῦ μέλλοντος χρόνου διὰ τὴν προσθήκην τῆς ἡμέρας. ὁμοίως καὶ καθεξῆς |$_{14v}$ ὅπου καταντήσει βίσεκτον[k], οὕτως ποιεῖ· τὰς δὲ πρὸ τοῦ βισέκτου καὶ μετὰ τὸ βίσεκτον κατὰ τὴν τάξιν τῆς ἑβδομάδος γίνωσκε εἶναι.

[a] ἀρχημινίαν [b] εἰ [c] ἀρχημινίαν [d] ἡ [e] ἀρχημινία [f] καθ' ἔπ^{τ}ος [g] οἱονδιποτοῦν [h] ἀρχημηνία [i] ἐπιφέριται βήσεκτον [j] ἀρχημηνίαν [k] βήσεκτον

God willing, calculation for finding on which weekday the starting-day of each month falls

Know on what weekday the starting-day falls of the month you want of the present year, and if the starting-day of the month of the present year is a Sunday, next year is found on Monday, and in this way know that whatever month of every year coninuously begins according to the weekly ordering.

Know also this, that, if the starting-day of whatever month of the present year falls on a Sunday and a bissextile <day> is impending, consider that the starting-day of next year will be a Tuesday because of the addition of a day. Similarly also, whenever a leap year will be arrived at, continuously do in this way; know that the <days> before a leap year and after a leap year fall according to the weekly ordering.

A computation of the weekday of the first day of month X in year y + 1, if the weekday of the first day of month X in year y is given. The implicit algorithm is (I have been charitable with our text, translating βίσεκτον both as "bissextile <day>" and as "leap year"):

$(y,X) \rightarrow$

$| \; 1_X > 28_F, w_y(1_X) \rightarrow w_y(1_X) + 1 + [\![(y + 1) \bmod 4]/4]\!] \rightarrow [w_y(1_X) + 1 + [\![(y \bmod 4)/4]\!]] \bmod 7 = w_{y+1}(1_X).$

$| \; 1_X < 28_F, w_y(1_X) \rightarrow w_y(1_X) + 1 + [\![(y \bmod 4)/4]\!] \rightarrow \{w_y(1_X) + 1 + [\![(y \bmod 4)/4]\!]\} \bmod 7 = w_{y+1}(1_X).$

Since 365 ≡ 1 (mod 7), the weekdays of the same dates in consecutive years are consecutive, unless the interval between the two dates includes February 29, in which case the difference is of two weekdays. I could not find this algorithm in other Computi.

25

ἕτερος ψῆφος, εἰς τὸ γνῶναι πόσας[a] ὥρας λάμπει ἡ σελήνη καὶ αὖθις[b] πότε ἀνέρχεται

Ἀπὸ α^{ου} ἕως ιε^{ου} τῆς σελήνης, πεντάπλου[c] ἅπερ ἔχεις, καὶ ἀνάλυε εἰς τὸ τετράκις[d], καὶ τὰ περισσεύοντα σημαίνουσιν πόσας ὥρας λάμπει. ἀπὸ δὲ ις^{ου} γίνωσκε· τὴν μὲν ἑξκαιδεκαταίαν νοῆ λάμπειν ὥρας τῆς σελήνης[e] ὅσας ἔλαμπεν τῇ τεσσαρεσκαιδεκαταίᾳ[f], καὶ τὴν ἑπτὰ καὶ δεκάτην ὅσας ἔλαμπεν τῇ τρεῖς καὶ δεκάτῃ, καὶ καθεξῆς οὕτως νόησον ἕως τέλους τοῦ ἀριθμοῦ τῆς σελήνης.

[a] πώσας [b] αὖθης [c] πεντάπλοι : expect. τετράπλοι [d] expect. πεντάκις [e] expect. τὴν σελήνην [f] τεσσαρισκαιδεκατέᾳ

A further calculation, for knowing how many hours the Moon shines and when it reaches anew

From the 1st to the 15th <day> of the Moon, quadruple what you have, and resolve out into five-times[172], and that which remains over signifies how many hours it shines. From the 16th know <as follows>; consider that in the sixteenth day the Moon shines as many hours as it shone in the fourteenth day, and in the seventeenth <day> as many as it shone in the thirteenth, and continuously consider in this way up to the end of the number of the Moon.

Duration of visibility v_a of the waxing and waning Moon. The implicit algorithm is as follows, where a = age of the Moon:

$(a) \rightarrow$

$| 1 \leq a \leq 15, 4a \rightarrow 4a/5 = v_a.$

$| 16 \leq a, v_a = v_{30-a}.$

The duration of visibility of the waxing (waning) Moon[173] is supposed to increase (decrease) stepwise every day of the lunar month. Since the full Moon is supposed to "shine" for the length of the interval between sunset and moonset, the step is $^4/_5$ an hour, which is the scaling factor between 12 hours and 15 days. This, and comparison with other sources[174], shows that the algorithm in our text must be corrected by inverting its implied scaling factor $^5/_4$. Seasonal hours are intended[175].

26

[a]σὺν θεῷ ψῆφος σύντομος τοῦ νομικοῦ Πάσχα

$|_{15r}$ Γίνωσκε ὅτι ἐν τῷ α^{ῳ} κύκλῳ τῆς σελήνης εἰς τὰς β τοῦ Ἀπριλίου εὑρίσκεται πάντοτε τὸ νομικὸν Πάσχα, καὶ ἐὰν θέλεις εὑρεῖν εὐκόλως τὰ λοιπὰ Πάσχα ἀπὸ α^{ου} κύκλου ἕως ιθ^{ου}, πρόσθες κ, καὶ ὕφειλε ιβ, καὶ εὑρίσκεις ἑκάστου ἔτους τὸ νομικὸν Πάσχα. οἷον πρόσθες εἰς τὰς β τοῦ Ἀπριλίου κ· καὶ γίνονται κβ. καὶ ἰδοὺ εἰς τὰς κβ τοῦ Μαρτίου ἐστὶν τὸ νομικὸν Πάσχα. καὶ πάλιν ὕφειλον ἐκ τῶν κβ τοῦ Μαρτίου ιβ· καὶ μένουσιν ι. ἰδοὺ εἰς τὰς ι τοῦ Ἀπριλίου ἐστὶν τὸ νομικὸν Πάσχα. τῷ[b] αὐτῷ τρόπῳ ἀπὸ πρώτου ἕως ἐννεακαιδεκάτου[c] κύκλου ψήφιζε· προστιθέντος σου κ καὶ ὑφειλόντος[d] ιβ εὑρήσεις ἑκάστου ἔτους τὸ νομικὸν Πάσχα, σεσημειωμένων γ κύκλων μὴ προσδεχομένων τὴν προσθήκην τῶν κ $|_{15v}$ ἀλλὰ ὑφειλμὸν[e] ια καὶ οὐχὶ ιβ (εἰσὶν δὲ οὗτοι ς θ ιβ), ἀλλὰ καὶ ὁ[f] ιζ μὴ προσδεχόμενος τὴν προσθήκην τῶν κ προσδέχεται ὑφειλμὸν[g] ιβ καὶ οὐχὶ ια. γίνωσκε δὲ καὶ τοῦτο, ὅτι ὅταν ἔχεις πλεῖον τῶν κ ἡμερῶν ἀριθμόν, ἐν τῷ Μαρτίῳ ἐστὶν τὸ νομικὸν Πάσχα· ἀπὸ δὲ ιη καὶ τὴν ἄνω (ἄνω δὲ λέγω τὴν ἀρχὴν τοῦ μηνός) ζήτει αὐτὰς ἐν τῷ Ἀπριλίῳ.

[a] α marg. [b] τὸ [c] ἐννακαιδεκάτου [d] ὑφηλόντος [e] ὑφιλμὸν [f] οἱ [g] ὑφιλμὸν

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[172] The text wrongly reads "quintuple" and "four-times"; see the commentary.

[173] This quantity coincides with the illuminated part of the lunar disk measured as a fraction of its diameter.

[174] For this algorithm, see also *Anonymus* 830, sects. 24 and 33, in GASTGEBER, Neue texte XXX and XXX; *Anonymus* 982, sects. 20, 23, and 27; *Anonymus* 1041, sect. 19; *Anonymus* 1092B, sect. 5, in KARNTHALER, Die chronologischen Abhandlungen 9.159–170; *Anonymus* 1172, sect. 21; *Rhabdas* 1342, sect. 9; *Anonymus* 1377, sect. 7, in *PG* XIX 1324–1328, where the procedure is described in detail. Latin computistical treatises include Bede, *De Temporum Ratione* XXIV and the Computus printed in *PL* CXXIX 1305. The connection with Western sources is also made explicit in *Theophylaktos* 956 (μάθημα τοῦ ψήφου τῶ<ν> Λατίνω<ν> ἑρμινευθὲν [*sic*] παρὰ τοῦ ἐλαχίστου Θεοφυλάκτου), whose sect. 6 expounds the same algorithm, and, in the same manuscript, in *Nicholas* 916, sects. 5–8 (which, however, does not present this algorithm). See also NEUGEBAUER, HAMA 830, and NEUGEBAUER, Ethiopic Astronomy 164–165.

[175] The conversion from seasonal hours to equinoctial hours is carried out in *Rhabdas* 1342, sect. 9, and in *Anonymus* 1377, sect. 7, in *PG* XIX 1325–1328.

God willing, a concise calculation of Passover

Know that, in the 1st cycle of the Moon, Passover is always found on April 2, and if you want to easily find the remaining Passover's from the 1st cycle up to the 19th, add 20, and remove 12, and you find the Passover of each year. For instance, add 20 to April 2: and they yield 22. And there it is, Passover is on March 22. And again remove 12 from March 22: and there remain 10. There it is, Passover is on April 10. Calculate in the same way from the first up to the nineteenth cycle; adding 20 and removing 12 you will find the Passover of each year. There are 3 remarkable cycles that do not admit of the addition of 20 but a removal of 11 and not of 12 (these are <cycles> 6, 9, 12), and also <cycle> 17, while not admitting of the addition of 20, does admit of a removal of 12 and not of 11. Know also this, that whenever you have a number of days greater than 20, Passover is in March; from <number> 18 and above (I mean by "above" the beginning of the month), seek for them [*scil*. the days] in April.

A computation of Passover. To follow the prescription more easily, recall the sequence of Passover dates:

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
p_m	2	22_M	10	30_M	18	7	27_M	15	4	24_M	12	1	21_M	9	29_M	17	5	25_M	13

The algorithm is[176]:

$(p_m) \rightarrow$

$| p_m \in A, p_m \leq 11, p_m + 20 –: 1_M = p_{m+1}.$

$| p_m \in A, p_m > 11, p_m - 11 –: 1_A = p_{m+1}.$

$| p_m \in M, p_m - 12 –: 1_A = p_{m+1}.$

This algorithm formalizes the following data: since each year the epacts increase by 11 units, the date of Passover shifts backwards by 11 days from an assigned year to the next (second branch of the algorithm). However, Passover cannot fall earlier than March 21; therefore, such early dates are replaced by a day falling one lunar month later (this lunar month lasts 30 days, as we shall see in a moment); to this day does not correspond the same date in April, but the numerically precedent date because March has 31 days. Therefore, whenever the Passover date falls outside the lower bound, March 21, of the Passover interval $21_M \leq p \leq 18_A$[177], it enters again this interval from its upper bound numerically lowered by 12 units instead of 11 (third branch of the algorithm). Finally (first branch of the algorithm), adding 20 comes from $20 \equiv -11 \pmod{31}$. Thus, the prescription is as follows: from the date of Passover of cycle year 1 alternately add 20 and subtract 12, with the exceptions of cycles 6, 9, 12 (resp. 17), which are reached by subtracting 11 (resp. 12) from the previous cycle instead of adding 20. Of course, the special case of cycle 17 corresponds to the *saltus lunae*. The text has a final remark (see sects. **12** and **22** for the clarification about the adverb ἄνω): if $p_m > 20$, then $p_m \in M$; if $p_m \leq 18$, then $p_m \in A$.

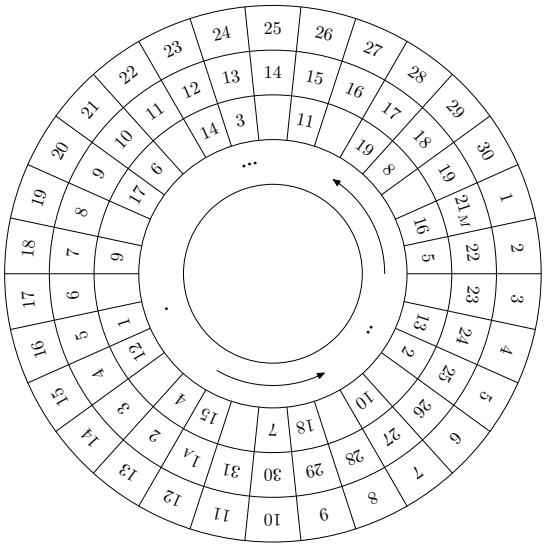
Both in primary sources and in secondary literature[178], the Passover terms are the 29-day time interval $21_M \leq p \leq 18_A$, but the principle of 11-day backward shift of lunar dates entails that the real terms are the boundaries of the 30-day time interval $21_M \leq p \leq 19_A$: April 19 is discarded because this date coincides with a gap in the Passover sequence and it is located at the end of the interval. This can be seen in the "wheel" (τροχός) below, which is the standard representation of many cyclic structures in early Computi. The days in the time interval $21_M \leq p \leq 19_A$ (middle ring) are numbered in succession (outer ring, clockwise); the inner ring carries the se-

[176] For this algorithm, see George, sect. II.3, in DIEKAMP, Der Mönch 29.7–30.2; *Anonymus* 686, sect. 3; *Anonymus* 830, sect. 27; *Anonymus* 951, sect. 18; *Psellos* 1092, sect. I.4, in REDL, La chronologie appliquée I 213–215; an isolated procedure in Par. suppl. gr. 387, f. 149v, in ACERBI, Struttura e concezione, Appendice A.6; *Blastares* 1335, in RHALLES – POTLES, Σύνταγμα VI 417 n. 1; *Rhabdas* 1342, sect. 15.

[177] For this interval, see also sect. **22** and the computistical glossary above.

[178] Almost all Computi list the Passover and Paschal terms. Early explicit statements are in Stephanus-Heraclius' *in Ptolemaei Tabulas Manuales*, sect. 30, in USENER, De Stephano Alexandrino 314; Maximus, *Enarratio* I.14, in *PG* XIX 1232; *Anonymus* 951, sect. 16; *Anonymus* 982, sects. 14 and 29; the Computus contained in Par. suppl. gr. 690, sect. 4; *Anonymus* 1090, sect. 1 (with an explanation); *Psellos* 1092, sect. I.4, in REDL, La chronologie appliquée I 213.11–21, *Anonymus* 1183, sect. 8; *Anonymus* 1204.

quence of Passover dates, identified by the associated lunar cycle year[179]. The direction of the 11-day cyclic backward shift is exemplified inside the wheel, by means of dots (first three steps) and of two arrows. The shift is cyclic in the sense that March 21 is adjacent to April 19 and the shift crosses this border. In this diagram, the *saltus lunae* occurs after lunar cycle year 19: the Passover date of lunar cycle year 1 is found 12 cells (and not 11 cells) counterclockwise to the Passover date of lunar cycle year 19.



27

χρὴ γινώσκειν τὸ πῶς ὑφείλονται τὰ ἔτη ἀπὸ κτίσεως κόσμου εἰς τὸ εὑρίσκειν τὴν ἴνδικτον καὶ τοὺς λοιποὺς[a] κύκλους

ιε υ, ˏς[b]· ιε κ, τ· ιε ς, ϙ.

τὸ πῶς δεῖ εὑρίσκειν τὸν κύκλον τοῦ ἡλίου. κ σ, ˏδ· η σ, ˏαχ· κ κ, υ· η κ, ρξ. ἰδοὺ ὑφείλαμεν ς χιλιάδας καὶ ρξ· καὶ ἐμείνασιν σμ ἐκ τῶν <ˏς>υ· κ ε, ρ· η ε, μ· λοιπὸν ἐμείνασιν ρ· κ γ, ξ· η γ, κδ· λοιπὸν ἐμείνασιν ις ἐκ τῶν ρ, ἥτις ἐστὶν ις^{ος} κύκλος τοῦ ἡλίου.

|16r τῆς σελήνης ὑφειλμὸς[c] τὸ πῶς εὑρίσκεται. ιθ τ, ˏεψ· ιθ λ, φο. ἰδοὺ ὑφείλαμεν ς χιλιάδας καὶ σο ἔτη· λοιπὸν ἐμείνασιν ρλ· ιθ ς, ριδ· ἰδοὺ ἐμείνασιν ις, ἥτις ἐστὶν ὁ κύκλος τῆς σελήνης τοῦ ἐνεστῶτος χρόνου.

[a] λυποὺς [b] ˏς e corr. m.2 [c] ὑφιλμὸς

One has to know how the years from the foundation of the world are removed for finding the indiction and the remaining cycles

15 <times> 400, 6000; 15 <times> 20, 300; 15 <times> 6, 90.

How one must find the cycle of the Sun. 20 <times> 200, 4000; 8 <times> 200, 1600; 20 <times> 20, 400; 8 <times> 20, 160. There it is, we removed 6 thousand and 160: and there remain 240 from <6>400; 20 <times> 5, 100; 8 <times> 5, 40: there remain 100 as a remainder; 20 <times> 3, 60; 8 <times> 3, 24: there remain 16 as a remainder from 100, which is the 16th cycle of the Sun.

How the removal of the Moon can be found. 19 <times> 300, 5700; 19 <times> 30, 570. There it is, we removed 6 thousand and 270 years: there remain 130 as a remainder; 19 <times> 6, 114: there it is, there remain 16, which is the cycle of the Moon of the present year.

A computation of indiction, solar, and lunar cycle years. The computation is carried out, for current year AM 6400 [= AD 891/2], by subtracting suitable multiples of 15, 28, and 19, respectively. The results are $i = 10$, $s = 16$, and $m = 16$ (see sect. **7**).

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[179] For instance, the cell in the middle ring carrying number 13 corresponds to the cell in the inner ring carrying number 3. This means that, in lunar cycle year 3, Passover falls on April 13.

APPENDIX. A CONCORDANCE OF *ANONYMUS* 830 AND *ANONYMUS* 892

Anonymus 830 is edited in GASTGEBER, Neue texte XXX–XXX; the division into sections is mine.

|$_{36r}$ 1. An algorithm for computing the epacts of the months and the weekday of the beginning of a month {see footnote 153}
2. A compendium of computistical parameters: subdivisions of the year and length of the solar, lunar, and indiction cycles {≈ *Anon*. 892, sect. **1**}
3. A (meaningless) computation of solar cycle years
|$_{36v}$ 4. An algorithm for computing the epacts of the sun (with fractional parts), {= *Anon*. 892, sect. **20**} and their use to find the weekday of the beginning of a month
5. About the period of 532 years {≈ *Anon*. 892, sect. **7**}
6. An algorithm for computing the epacts of the sun {= *Anon*. 892, sect. **2**}
7. An algorithm for computing the epacts of the Moon; features of the lunar cycle {= *Anon*. 892, sect. **14**}
|$_{37r}$ 8. Lunar wheel, featuring lunar cycles – epacts of the Moon – Passover, Byzantine epact sequence
|$_{37v}$ 9. Solar wheel, featuring solar cycles – epacts of the Sun – leap years, with fractional epacts {= *Anon*. 892, sect. **20**}
|$_{38r–v}$ 10–12. Interpretation (διήγησις) of the three wheels (πόλοι); the wheel for indiction is missing {= *George*, sect. III}
|$_{39r}$ 13. A computation of solar, lunar, indiction, and leap-year cycle years for AM 6321 [= AD 812/3] {= *Anon*. 892, sects. **3**–**7** and **9**–**11**, **27**, **21**}
14. An algorithm for computing the epacts of the sun {= *Anon*. 892, sect. **2**}
15. An algorithm for computing the weekday |$_{39v}$ of a given day {= *Anon*. 892, sect. **12**}
16. About the Period of 532 years; the role of Hero and Athanasius; the number of ἄλφα {≈ *Anon*. 892, sect. **7**}
17. An algorithm for computing the weekday using September 30 as epacts {= *Anon*. 892, sect. **13**}
18. The norm 50 algorithm for Passover; Easter is the Sunday next thereafter {see *Anon*. 892, sect. **12**}
19. Age of the Moon computed for April 4, leap-year, m = 12, |$_{40r}$ using the algorithm of the πενταπλοῦντες καὶ ἑξαπλοῦντες {see *Anon*. 892, sect. **8**}
20. A computation of the solar cycle year of indiction 8, AM 6338 [= AD 830]; features of the solar cycle {see *Anon*. 892, sects. **4** and **10**}
21. A computation of the lunar cycle year of indiction 8, AM 6338 [= AD 830]; features of the lunar cycle {see *Anon*. 892, sects. **5** and **11**}
|$_{40v}$ 22. An algorithm for computing the lunar cycle years by adding 11 [end-of-century 6300 is intended] and removing the result by 19 {= *Anon*. 892, sect. **7**}
23. An algorithm for computing the indiction; a computation of the indiction of indiction 8 (*sic*) AM 6338 [= AD 830]; features of the indiction cycle {see *Anon*. 892, sects. **6** and **9**}
24. An algorithm for computing the duration of visibility of the waxing and waning Moon {see *Anon*. 892, sect. **25**}
25. The norm 50 algorithm for Passover; |$_{41r}$ an algorithm for computing Meat-Fare Sunday {= *Anon*. 892, sect. **12**}
26. A concise computation of Passover {= *Anon*. 892, sect. **16**}
27. A concise |$_{41v}$ computation of Passover {= *Anon*. 892, sect. **26**}
28. The norm 50 algorithm for Passover; |$_{42r–v}$ an algorithm for computing the weekday of an assigned day; an algorithm for computing Meat-Fare Sunday {see *Anon*. 892, sect. **12**}
29. An algorithm for computing the weekday of the beginning of a month by using the solar epacts {= *Anon*. 1092B, sect. 16}
30. Partition of the base epacts of the Moon as 11 = 5 + 6 {= *Anon*. 1092B, sect. 2}
31. How the bissextile day comes to be {= *Anon*. 892, sect. **3**}
32. About the embolismic years |$_{43r}$ and months {= *Anon*. 892, sect. **19**}
33. An algorithm for computing the duration of visibility of the waxing and waning Moon {see *Anon*. 892, sect. **25**}
34. An algorithm for computing the epacts of the Moon {see Maximus, *Enarratio* III.10, in *PG* XIX 1272}
35. The norm 105 algorithm for |$_{43v}$ Passover {= *Anon*. 892, sect. **17**}

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