Primal and dual decision rules for multi-stage robust optimization

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1 Introduction

In this work, we consider multi-stage adaptive robust optimization (MSARO) problems formalized as :

$$\min \quad \max_{\boldsymbol{\xi}^T \in \Xi} \sum_{t\in[T]} c_t^\top x_t(\boldsymbol{\xi}^t) + h_t^\top s_t(\boldsymbol{\xi}^t) \tag{1}$$

$$\text{s.t.} \quad A_t s_t(\boldsymbol{\xi}^t) + B_t s_{t-1}(\boldsymbol{\xi}^{t-1}) + C_t x_t(\boldsymbol{\xi}^t) \le b_t(\boldsymbol{\xi}^t) \qquad t \in [T],\ \boldsymbol{\xi}^t \in \Xi^t \tag{2}$$

$$(s_t(\boldsymbol{\xi}^t), x_t(\boldsymbol{\xi}^t)) \in X_t(\boldsymbol{\xi}^t) \qquad t \in [T],\ \boldsymbol{\xi}^t \in \Xi^t \tag{3}$$

where T denotes the number of decision stages, $c_t, h_t, A_t, B_t, C_t, b_t$ are of conforming dimensions, and uncertain parameters are represented by $\{\boldsymbol{\xi}_t\}_{t=1}^T$ with support Ξ. We denote by $\boldsymbol{\xi}^t = (\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_t)$ the history of realizations at stage t, where $\boldsymbol{\xi}_t$ is the vector of uncertain parameters at stage t with elements ξ_t (with $\xi_1 = 1$). Here, $x_t(\boldsymbol{\xi}^t)$ and $s_t(\boldsymbol{\xi}^t)$ are the decision variables, i.e., nonanticipative policies that only depend on the history of uncertain parameters. Variables $(s_t(\boldsymbol{\xi}^t), x_t(\boldsymbol{\xi}^t))$ are restricted to be in the set $X_t(\boldsymbol{\xi}^t)$ for $t \in [T]$, which includes integrality restrictions, if any. Throughout, we use $[a] := \{1, 2, \ldots, a\}$ for $a \in \mathbb{Z}_+$. The objective function (1) minimizes the worst outcome. Constraints (2) and (3) are *state* and *recourse* constraints, respectively : while the former link different stages, the latter are stage-wise restrictions. In a similar manner, we will call variables $s_t(\boldsymbol{\xi}^t)$ and $x_t(\boldsymbol{\xi}^t)$, *state* and *recourse* variables, respectively. When the set $X_t(\boldsymbol{\xi}^t)$ for $t \in [T],\ \boldsymbol{\xi}^t \in \Xi^t$ does not have integrality restrictions, we will talk of *continuous recourse*, otherwise we will talk of *integer recourse*.

2 Literature review

Although, exact solution methods for (MSARO) with continuous recourse have been recently proposed under the assumption that the uncertainty set is stage-wise rectangular, (MSARO) remains extremely difficult to solve, especially with integer recourse. To alleviate this difficulty, approximations restricting the form of $x_t(\boldsymbol{\xi}^t)$ have been proposed. These are known in the literature as "decision rules". Ben-Tal et al. ([1]) propose an affine decision rule where continuous recourse decisions are expressed as affine functions of uncertain parameters where the parameters of this function are to be optimized. More elaborate decision rule schemes such as deflected and segregated affine ([3]), extended affine ([4]), and piecewise affine ([6]) rules were also proposed. Kuhn et al. ([7]) applied linear decision rules in both the primal and dual spaces to evaluate the optimality gap resulting from using decision rule approximations. Recently, two-stage ([2]) and Lagrangian dual decision rules ([5]), are introduced for multistage stochastic programming with mixed-integer recourse and shown to be numerically promising.

3 Methodological development and algorithms

The purpose of this work is to adapt these new decision rules to (MSARO) and assess their quality with respect to existing decision rules. To this end, let $s_t(\boldsymbol{\xi}^t) \subseteq \mathbb{R}^{q_t}$ be characterized by a linear decision rule, i.e., $s_t(\boldsymbol{\xi}^t) = \Lambda_t \boldsymbol{\xi}^t$ such that $\Lambda_t \subseteq \mathbb{R}^{q_t \times \ell_t}$ where $\ell_t = \sum_{t'=1}^{t} dim(\boldsymbol{\xi}'_t)$. We then obtain :

$$\min \quad \max_{\boldsymbol{\xi}^T \in \Xi} \sum_{t \in [T]} c_t^\top x_t(\boldsymbol{\xi}^t) + h_t^\top \Lambda_t \boldsymbol{\xi}^t \tag{4}$$

$$\text{s.t.} \quad C_t x_t(\boldsymbol{\xi}^t) \leq b_t(\boldsymbol{\xi}^t) - (A_t \Lambda_t \boldsymbol{\xi}^t + B_t \Lambda_{t-1} \boldsymbol{\xi}^{t-1}) \qquad t \in [T],\ \boldsymbol{\xi}^t \in \Xi^t \tag{5}$$

$$(\Lambda_t \boldsymbol{\xi}^t, x_t(\boldsymbol{\xi}^t)) \in X_t(\boldsymbol{\xi}^t) \qquad t \in [T],\ \boldsymbol{\xi}^t \in \Xi^t \tag{6}$$

Note that the decision rule is only applied to the state variables whereas recourse variables continue to remain fully adjustable to the uncertain parameter. Problem (4)-(6) is equivalent to a two-stage robust optimization problem. Indeed, since the temporal dependency between stages is removed thanks to the decision rule, all decisions $x_t(\boldsymbol{\xi}^t)$ can be taken simultaneously. The resulting problem can be solved using the constraint-and-column generation algorithm in the continuous recourse case, and can be approximated using $K-$adaptability in the integer recourse case. In a similar manner, linear decisions rules can also be applied to a dual of (MSARO), obtained by applying the Lagrangian relaxation to its non-anticipative reformulation, and imposing decision rules on the Lagrangian multipliers. The resulting problem provides an upper bound on the optimal value of (MSARO) for any probability distribution that is assigned over Ξ. In order to obtain the best upper bound possible we optimize over the assigned probability distribution as well. These decision rules allow us to propose primal and dual bounds on the optimal value of (MSARO), and are well suited to handle non-linearities including integrality restrictions on $x_t(\boldsymbol{\xi}^t)$.

4 Conclusions et perspectives

In this work, we adapt the recent decision rules introduced in the stochastic programming literature to robust optimization both from the primal and the dual perspective. The resulting problems are challenging and require advanced techniques in their solution. Our methodology is illustrated with preliminary results on production planning and transportation problems.

Références

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