

Partial resource pooling in loss systems with two players^{*}

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1 Introduction

It is well known that resource sharing or resource pooling can help improve resource utilisation of independent entities providing similar for using similar resources. In traditional sharing mechanisms, entities pool either all their resources or do not participate. Binary mechanisms are helpful when the objective is to maximize the system (or overall) utility. However, when each entity is maximizing its own utility, these mechanisms are not always successful at prescribing solutions that are acceptable to all the participating entities.

In this paper, we propose two partial sharing models in which both providers improve their individual utilities by following the prescribed sharing strategy. By partial sharing we mean that each provider possibly pools in only a part of its resources. For example, when providers can be modelled as M/M/N/N queues, a provider may pool in only a certain number out of its N servers. Our partial sharing models encompasses the two extreme cases of full pooling and no pooling that appear in traditional mechanisms.

These models can also be viewed as a cooperative game in which providers decide to pool a limited amount of resources in exchange for improved utility. Such an approach for resource sharing has been studied in [1] which focusses on coalition formation. Their model allows for side payments, that is the overall cost of the coalition can be shared arbitrarily between the partners as long as no partner is worse-off. Our sharing models are more restrictive as they does not allow for side-payments and each provider improves the utility defined as the blocking probability of its users.

Contributions

We propose two partial sharing models : the *bounded overflow* sharing model and the *probabilistic* sharing model. For both these models, the blocking probabilities and their monotonicity properties are derived. Using these results, we show that the *Pareto frontier* of the sharing configurations is non-empty. That is, there are configurations for which both providers are better off than not sharing. Moreover, no provider can improve its utility without decreasing that of the other. The Pareto frontier is shown to have the property that at least one of the providers shares all its servers. We also look at several bargaining solutions (Nash, Kalai-Smorodinsky bargaining, egalitarian sharing, and utilitarian sharing). Since the utility sets over which these bargaining solutions will be computed for our model do not satisfy the usual properties of convexity or comprehensiveness, it is not easy to show uniqueness of the bargaining solution. Nevertheless, using monotonicity properties of the blocking probabilities we are able to show uniqueness of the Kalai-Smorodinsky and egalitarian solutions. Finally, we also obtain simpler asymptotic expressions for blocking probabilities for large loss systems.

^{*}This is an extended abstract of the contents in [2]. A. Nandigam was with IIT Bombay, India, when this work was performed.

2 Partial sharing models and results

Consider two providers : one with N_1 and the other with N_2 servers. Assume that provider P_i can be modelled as an $M/M/N_i/N_i$ queue. The two partial sharing models are defined as follows :

1. Probabilistic sharing model – P_i accepts an overflow call from P_{-i} with probability x_i .
2. Bounded overflow model – P_i accepts an overflow call only if it has fewer than k_i ongoing calls of P_{-i} .

By the share of P_i , we shall mean either x_i or k_i depending on the context.

2.1 Results

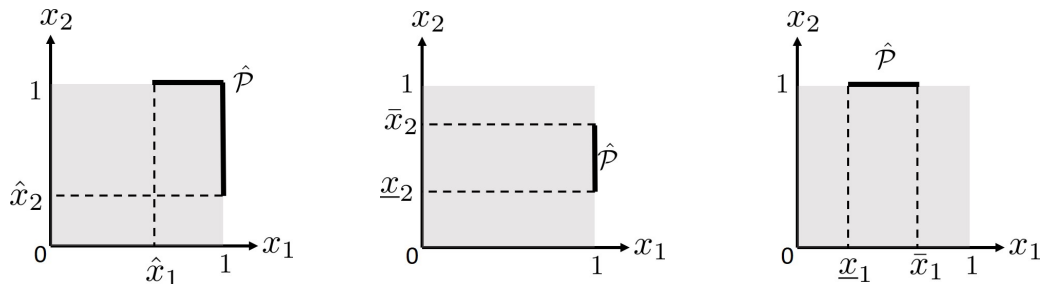
Théorème 1 *Under the probabilistic as well as the bounded overflow partial sharing models, the steady state blocking probability of P_i satisfies the the following properties :*

1. *It is a strictly increasing function of its share.*
2. *It is a strictly decreasing function of the share of P_{-i} .*
3. *If the service time distribution is the same for both the providers, then the overall blocking probability is a strictly decreasing function of individual shares.*

Using the above results, we can show that :

Théorème 2 *Under the probabilistic as well as the bounded overflow partial sharing models, the set of Pareto-efficient sharing configurations is non-empty. Moreover, any Pareto-stable sharing configuration involves at least one of the providers sharing all its resources.*

Figure 1 shows the various possible sharing configurations on the Pareto frontier.



(a) Case 1 : Both providers are strictly better off under full pooling. (b) Case 2 : Only Provider 1 is strictly better off under full pooling. (c) Case 3 : Only Provider 2 is strictly better off under full pooling.

FIG. 1 – The set $\hat{\mathcal{P}}$ of Pareto-efficient partial sharing configurations.

3 Future work

Extensions of these models to more providers as well as to waiting systems are being pursued.

Références

- [1] F. Karsten, M. Slikker, and G. J. van Houtum. Analysis of resource pooling games via a new extension of the erlang loss function. *Operations Research*, 63(2) :476–488, 2015.
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