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Multistage stochastic programs with the entropic risk measure

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1 Introduction

In order to make sequential decisions under uncertainty, it is convenient to consider risk measures. Most works use dynamic programming, and consider the so-called nested risk measures. \cite{2, 3}. When we care about the outcome of our decision at the last stage, end-of-horizon formulations are more appropriate. In this work we study the entropic risk measure, for which the two approaches coincide.

2 Conditional consistency and the entropic risk measure

Consider a sequence of correlated real-valued random variables, $Z = \{Z_t\}_{t=1}^{T}$. We have

\begin{align*}
\text{End – of – Horizon – Risk}(Z) & = E[Z_1 + Z_2 + \cdots + Z_T], \\
\text{Nested – Risk}(Z) & = E\left[Z_1 + F_{Z_t | Z_1} [Z_2 + F_{Z_t | Z_1} [Z_2 + \cdots + F_{Z_t | Z_{T-1}} [Z_T]]]\right].
\end{align*}

Définition 1 Let $(X_1, X_2)$ and $(Y_1, Y_2)$ be two-dimensional vectors for which requisite expectations are finite. A risk measure $F$ is said to be conditionally consistent if :

\[ F[X_1 + X_2] \leq F[Y_1 + Y_2] \iff F[X_1 + F_{X_1 | X_2}[X_2]] \leq F[Y_1 + F_{Y_1 | Y_2}[Y_2]]. \]

An example of a convex risk measure that is conditionally consistent is the entropic $(\gamma > 0)$:

\[ \text{ENT}_\gamma[Z] = \frac{1}{\gamma} \log \left( E[e^{\gamma Z}] \right). \]

2.1 Risk-averse multistage Stochastic Programming (MSSP)

We consider a $T$-stage MSSP given by

\begin{align*}
V_t(x_{t-1}, \omega_t) = \min_{\bar{x}_t, x_t \geq 0} & c_t^\top x_t + F_{\omega_{t+1} \in \Omega_{t+1}} [V_{t+1}(x_t, \omega_{t+1})] \\
\bar{x}_t & = x_{t-1} \quad [\lambda] \\
A_t x_t + B_t \bar{x}_t & = b_t.
\end{align*}

(2)

We can form single-cut master problems that approximate the stage-wise problems as follows:

\begin{align*}
V^K_t(x_{t-1}, \omega_t) = \min_{\bar{x}_t, x_t \geq 0, \theta_{t+1} \geq -M_{t+1}} & c_t^\top x_t + \theta_{t+1} \\
\bar{x}_t & = x_{t-1} \quad [\lambda] \\
A_t x_t + B_t \bar{x}_t & = b_t \\
\theta_{t+1} & \geq \alpha_{t+1,k} + \beta_{t+1,k}^\top x_t, \quad k = 1, \ldots, K - 1,
\end{align*}

(3)
where $M_{t+1}$ is a lower bound on $F[V_{t+1}(\cdot, \omega_{t+1})]$. Explicit values of $\alpha_{t+1}$ and $\beta_{t+1}$ can be computed for the entropic risk measure, and such formulation is amenable to SDDP.

3  Numerical illustration

Consider a road network with three arcs (see [1]) as shown in Figure 1. Travel times are independent, and we must choose between $F[X + Z]$ and $F[Y + Z]$.

![FIG. 1 – Road network with three independent travel times, $X$, $Y$, and $Z$.](image_url)

The results are shown in Figure 2: for the CV@R, when $\gamma \in (0.865, 0.97)$, the optimal decision to the end-of-horizon model is to switch back to $X$. In Figure 2b, since the end-of-horizon and nested formulations are equivalent, there is only one line visible on the graph.

![FIG. 2 – $x = 0$ (road $Y$), $x = 1$ (road $X$) against $\gamma$ using CV@R (a) and entropic (b).](image_url)

4  Conclusions

We studied the entropic risk measure in MSSP. We showed that it can be embedded within decomposition algorithms such as the SDPP, and exemplified its use in a transportation problem.

Références

