Benchmarking QAOA through Maximum Cardinality Matching

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1 Introduction

The QAOA (Quantum Approximate Optimization Algorithm) constitutes one of the most promising quantum algorithm using noisy devices due to its low depth. In this study, we try to perform a comparison between QAOA and SA (Simulated Annealing) on their ability to solve problems. A similar study was done with Quantum Annealing [5]. We choose the maximum cardinality matching problem and solve this problem for bipartite graphs. This problem is empirically easy for SA for most of the bipartite graphs (see table 1). However, it fails to find optimal solutions efficiently for Sasaki-Hajek graphs [4] (a restricted class of bipartite graphs). Fully connected subgraphs of SH graph have edges increasing quadratically compared to edges of the optimal solution. Hence, The difficulty to find the optimal solution with SA increases when SH graph grows. The aim of this study is to use instances of SH graphs (hard for SA) and random bipartite graphs (empirically easy for SA) to find out if the QAOA falls in the same pitfall.

	Is bipartite ?	Best Classical complexity	Is complex for SA ?
GI graph	yes	O(n)	yes
Bipartite graph	yes	$O\left(n^{5/2}\right)$	no (most of them)
Random graphs	no (most of them)	$O\left(\sqrt{\lvert V\rvert}\cdot\lvert E\rvert\right)$	no (most of them)

TAB. 1 – Ability to solve Maximum Cardinality Matching considering SA

2 Problem statement

We transform the maximum cardinality matching problem into a combinatorial problem defined by a cost function to optimize. Given a graph $G = (V, E)$ with V the set of vertices and E the set of edges, a matching M is a set of independant edges $e \in M$. The maximum cardinality matching is the matching that maximizes $|M|$. Mapping this problem to a cost function, we have the statement "maximize $|M|$" stated as :

$$\text{Minimize } -\sum_{e\in E} x_e \text{ with } x_e = \begin{cases} 1, & \text{if } e \in M \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Let us define a parametrized subset $\Gamma(e)$ giving the list of adjacent edges to e, the requirement specifying the notion of independant edges is met when :

$$\text{if } e \in M \text{ then } \forall e' \in \Gamma(e),\ x_e x_{e'} = 0 \tag{2}$$

The set of above constraint is expressed with a penalty term $-\lambda \sum_{e\in E}\sum_{e'\in\Gamma(e)} x_e x_{e'}$ where λ acts as weight factor constant for the constraint. Adding this constraint to the cost function, maximum cardinality matching of arbitrary graphs is expressed in its minimization form as :

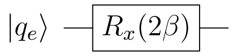
$$\text{Minimize } -\sum_{e\in E} x_e + \lambda \sum_{e\in E}\sum_{e'\in\Gamma(e)} x_e x_{e'} \tag{3}$$

We use the same cost function to perform our experiments on SA and QAOA.

3 Mapping into QAOA with penalty

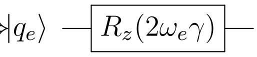
The first implementation of QAOA is based on Farhi et al. paper [2]. Each edge of the graph is encoded with a single qubit. The mixing unitary $U_M(\beta)$ consists of a wall of R_X gates. The phase separation unitary $U_p(\gamma)$ encodes the cost function with the penalty terms. The cost function is turned into the phase separation unitary by replacing each x_e by $(1+Z_e)/2$ where Z_e denotes the operator Z acting on the e^{th} qubit. This transformation leads to a linear part (single Z_e operators with weight ω_e) and a quadratic part (product of $Z_e Z_{e'}$ operators with weight $\omega_{ee'}$).

$U_M(\beta) : X_e$ terms $\Rightarrow$

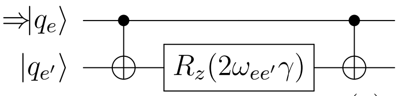


(4)

$U_p(\gamma) :\ \omega_e Z_e$ terms $\Rightarrow$



and $\omega_{ee'} Z_e Z_{e'}$ terms $\Rightarrow$

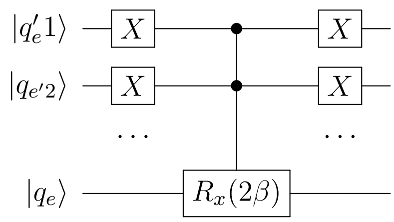


(5)

4 Mapping into H-QAOA

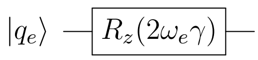
The second implementation is based on articles [1, 3]. The mixing unitary $U_M(\beta)$ changes and restricts the overall search space to valid matchings. The penalty term from $U_p(\gamma)$ operator is removed and encoded into $U_M(\beta)$ with a control clause. R_X rotation is done on qubit e only if the condition in Equation 2 is fulfilled implying multi-control R_X rotation.

$U_M(\beta) : \{q_{e'1}, q_{e'2}, ...\} \in \Gamma(e),\ X_e$ terms $\Rightarrow$



(6)

$U_p(\gamma) : \omega_e Z_e$ terms $\Rightarrow$



(7)

5 Conclusion and expectations

From these implementations, we will demonstrate whether or not QAOA meets the same difficulty to solve SH instance as SA and QA (see. [5]). Even if H-QAOA should provide better results than the basic version of the algorithm, the depth of the circuit highly increases due to multi-control operations especially if the number of $U_M(\beta)$ and $U_p(\gamma)$ is high. The density of the graph also has a huge impact on circuit depth. The benefits provided by the second implementation are still unclear, particularly concerning the tradeoff between the quality of the solution, the increase of the circuit depth and the noise induced by this increase.

Références

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