A Generalized Nash Equilibrium analysis of the interaction between a peer-to-peer financial market and the distribution grid

Ilia Shilov[1,3] , Hélène Le Cadre[2], Ana Bušić[1]

[1] Inria Paris, DI ENS, CNRS, PSL University, France

[2] Inria Lille-Nord Europe, 40 Avenue Halley, 59650 Villeneuve d'Ascq, France

[3] VITO/EnergyVille, Thorpark 8310, Genk, Belgium

Keywords : *Pricing, Generalized Nash Equilibrium, Peer-to-Peer Market*

1 Introduction

Currently, due to the large-scale integration of Distributed Energy Resources (DERs), electricity markets are starting to restructure, from centralized to decentralized local market designs. Considering a peer-to-peer electricity market design, agents (prosumers) negotiate with their peers their energy procurement seeking to minimize their costs with respect to both individual and trading reciprocity coupling constraints while taking into account trading cost preferences.

Practical problem for peer-to-peer energy trading implementation is related to the feasibility of the power flows corresponding to the bilateral trades negotiated on the financial market, that must accommodate the distribution grid network constraints. In case of infeasibility, some trades might be curtailed and the resulting loss are allocated to the agents. Another important aspect which justifies the need for financial and physical level decoupling is the information sharing between prosumers and the DSO, as the latter might be reluctant to share sensitive power grid data with the former.

In our model we focus on the interaction between (i) the financial level, in which the agents minimize the sum of their *generation flexibility* cost and *bilateral trading* costs minus their *usage benefit*, and (ii) the physical level, in which the DSO minimizes the *total generation flexibility cost* taking into account the physics of the distribution network, which we model through a linear DC power-flow approximation. This interaction is modeled as a noncooperative generalized Nash equilibrium problem (GNEP).

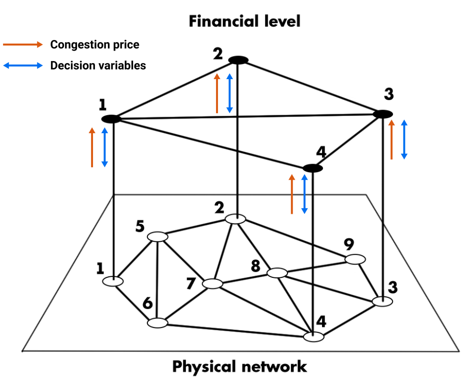

FIG. 1: Example of two-level interaction

We compare two designs of the financial level prosumer market: a centralized design and a peer-to-peer fully distributed design. We consider a two-player GNEP, in which the financial level is operated in a centralized fashion by a Market Operator (MO). We compare the two-player GNEP outcome to a N+1 GNEP outcome, in which the financial level made of N prosumers is operated in a fully distributed peer-to-peer fashion. We prove the Pareto efficiency of the generalized Nash equilibria (GNE) under homogeneity of the trading cost preferences. In addition, we prove that the pricing structure of our noncooperative game does not permit free-lunch behavior.

2 Model

More precisely, we consider a single-settlement market for energy trading made of a set $\mathcal{N}$ of N agents (prosumers) – each one of them being located in a node of the distribution grid. On top of the physical level network, the agents interact through a financial market, with financial flows defined on a (virtual) communication network. We denote Γ_n to be the set of neighbors of n in this communication network, that reflects the agents she wants to trade with. In this financial level, agents make the decisions about their *demand* D_n, *generation flexibility* G_n and bilateral *financial trades* q_{nm} $\forall m \in \Gamma_n \setminus \{n\}$.

On the physical level, we consider a distribution grid, which is represented by an undirected graph. Ω_n is the set of the agents with whom agent n is connected in the distribution grid. Ω_n does not necessary coincide with Γ_n. DSO makes a decision about the *power flows* F_{nm} for $m \in \Omega_n$, *voltage angles* θ_n and coefficient ρ_n for the *fraction of the generation flexibility* to be used.

To model the interaction between the two levels, we assume that the decision variables D_n and G_n of the agents act as parameters in the DSO's optimization problem. DSO's decision variable ρ_n and the Lagrangian multiplier γ_n, which can be interpreted as the *congestion price*, are used as parameters in the agent n's optimization problem. This interaction model implies that each agent n chooses the level of the generation flexibility G_n she is willing to utilize, while the DSO chooses the share ρ_n of this generation flexibility to use. Supply-demand balance constraint should hold both on the financial and physical levels. Moreover, the coupling between the two levels appears explicitly through the *modified* congestion price $f(\gamma_n)$ shared by the DSO and which appears as a component of the bilateral trading costs imposed to the agents.

3 Results

We analyze generalized Nash equilibrium (GNE), and a refinement of it, called variational equilibria (VE), assuming that the shadow variables associated with the shared coupling constraints are aligned among the prosumers. For the GNE of two-level interaction, induced by VE of the financial level, we show some desirable properties.

A strategy is a *Pareto efficient* outcome if no joint strategy is both a weakly better outcome for all players and a strictly better outcome for some player.

Proposition 1 (Pareto-efficiency of GNE) *If the trading coefficients in the trading costs of the agents are homogeneous, then GNE_{VE} of two-level game are Pareto-efficient.*

We characterize the solution of the GNEPs and discuss the effects of the prosumers' pricing mechanism, which captures the interaction between the financial and physical levels. In case the two levels are uncoupled, more precisely, in the absence of $f(\gamma_n)$ in the trading costs, there might be a free-lunch behavior, i.e. agents increasing their trades up to infinity to minimize their costs. On the contrary, we prove that there is no such possibility in our model. The following result states, that adding the term $f(\gamma_n)$ in the trading costs of the agents prohibits the *free-lunch behavior*.

Proposition 2 (No free-lunch behavior) *Free lunch behavior is not possible in the GNE_{VE} if the trading costs of the agents include the term $f(\gamma_n)$.*

In the numerical section we provide additional insights on the efficiency loss with respect to the different levels of agents' flexibility G_n and D_n and amount of renewables in the network. Additionally, we quantify the impact of the prosumers' pricing on the noncooperative game social cost.

References

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