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# 1 Muons and New Physics

2 Eduardo de Rafael

3 The muon is an elementary particle. It has the same electric charge and spin as the  
4 electron, but it is about two hundred times heavier. It was unexpected when observed  
5 in cosmic ray experiments in 1936.

6 “Who ordered that?” Isidor Rabi asked.<sup>1</sup>

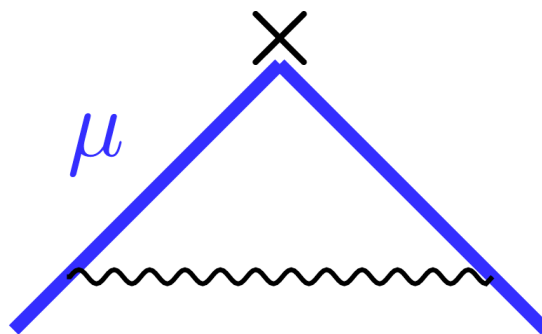
7 Muons and electrons behave as if they are tiny electrically charged magnets. The  
8 magnitude of their magnetic moments is given by the ratio of the electric charge to  
9 twice their mass times a dimensionless parameter  $g$ —the gyromagnetic ratio. Paul  
10 Dirac’s relativistic quantum mechanics equation fixes the value of  $g$  at 2 for both  
11 electrons and muons. The development of quantum electrodynamics imposed radiative  
12 corrections on the value of  $g$ . Its precise value is slightly different for electrons and  
13 muons, not precisely 2 in either case. Dirac’s result is simply the first term in a  
14 perturbative expansion.<sup>2</sup>

15 The quantum field theory formulation of electrodynamics (QED) was developed after  
16 the Second World War by Richard Feynman, Julian Schwinger, and Sin-Itiro  
17 Tomonaga.<sup>3</sup> Results in QED are obtained by series expansion over powers of the  
18 dimensionless fine structure constant  $\alpha$ . The measured reciprocal of  $\alpha$  is  
19 137.035999046(27), one of the very few quantities in nature whose value is known to  
20 ten decimal places.<sup>4</sup> Schwinger used QED to determine the lowest order radiative  
21 correction to Dirac’s value for the gyromagnetic ratio of any charged lepton:

22 
$$a \equiv \frac{1}{2}(g - 2) = \frac{\alpha}{2\pi} \simeq 0.001166 \dots,$$

23 where  $a$  is a measure of the departure of  $g$  from its zeroth-order value of 2.<sup>5</sup>  
24 The prediction was soon confirmed by a measurement of the electron anomaly  $a_e$  to a  
25 level of 4% precision.<sup>6</sup>  
26 Schwinger's calculation represents the emission by a charged lepton in a magnetic  
27 field of an unobserved particle of light—a virtual photon—that is later reabsorbed by  
28 the lepton. The propagation of this virtual photon, illustrated by the Feynman diagram  
29 in Figure 1, generates a correction to the Dirac  $g = 2$  result and hence an anomalous  
30 magnetic moment called the anomaly  $a$ .

31 **Figure 1.**



32  
33 Feynman diagram corresponding to the Schwinger contribution in the equation  
34 above. The X in the muon  $\mu$  (blue line) represents the external magnetic field. The  
35 black wavy line represents the propagating virtual photon.

36 Schwinger's result applies to any spin- $\frac{1}{2}$  charged particle or antiparticle; it is the same  
37 for the electron and the muon as well as the tau lepton, which has a higher mass,  
38 because  $g$  is a dimensionless number.<sup>7</sup> In his calculation, a massless photon is emitted  
39 prior to the particle's interaction with an external magnetic field and is absorbed  
40 afterward. No mass scale is involved other than that of the particle involved. Theorists  
41 have calculated corrections to Schwinger's one-loop order- $\alpha$  result, involving

42 additional loops and powers of  $\alpha$ , as the precision of experimental measurements has  
43 increased. These corrections are now known in terms of exact mathematical quantities  
44 up to order  $\left(\frac{\alpha}{\pi}\right)^3$ , and at good numerical approximations up to  $\left(\frac{\alpha}{\pi}\right)^5$ , which is  
45 extraordinary. The analytic evaluation of the coefficients of this series in  $\left(\frac{\alpha}{\pi}\right)$  powers  
46 involves higher and higher transcendental numbers, such as integral values of the  
47 Riemann zeta function, showing once more the beautiful relationship between  
48 mathematics and physics.

49 The values of the electron and the muon anomalies  $a_e$  and  $a_\mu$  differ at the order of the  
50  $\left(\frac{\alpha}{\pi}\right)^2$  and higher power contributions because of quantum vacuum fluctuations induced  
51 by other particles. Quantum fluctuations induced by charged lepton–antilepton pairs  
52 are calculable using QED techniques.<sup>8</sup> Such fluctuations can be induced by all the  
53 particles of the electroweak theory of the Standard Model—charged leptons with their  
54 associated neutrinos, quarks and gluons, the heavy gauge W and Z bosons, as well as  
55 Higgs particles. They all contribute to the muon anomaly  $a_\mu$ . These quantum  
56 fluctuations can be evaluated in the Standard Model because this theory,<sup>9</sup> when  
57 combined with the Higgs-particle mechanism,<sup>10</sup> is renormalizable.<sup>11</sup> Observables can  
58 be calculated without parameters other than those already present in the initial  
59 formulation of the theory.

60 The contribution to  $a_\mu$  from hadron quantum fluctuations—strongly interacting  
61 particles like pions and protons—is not straightforward. Calculation requires the  
62 underlying theory of the strong interactions, quantum chromodynamics (QCD),  
63 figuring at all energy scales and, in particular, at the low energies and long distances at  
64 which the fundamental quarks and gluon-gauge particles of QCD are confined and

65 condense into the observed hadronic particles. Theorists do not have as yet a full  
66 dynamical understanding of confinement. They resort either to phenomenological  
67 estimates, or to numerical simulations of the underlying dynamics of QCD theory  
68 made in discrete lattices of the space-time continuum. Implemented in some of the  
69 world's most powerful supercomputers, these simulations involve sophisticated  
70 techniques that have been developed under the name of lattice QCD (LQCD).<sup>12</sup> LQCD  
71 simulations in smaller and smaller space-time lattices of larger and larger volumes  
72 have already produced impressive results in hadron physics.<sup>13</sup>

73 Two dedicated experiments have provided precise measurements of the muon  
74 magnetic moment anomaly, one at the Brookhaven National Laboratory (BNL) the  
75 other at Fermilab (FNAL). These labs give the results

76 
$$a_{\mu}^{\text{BNL}} = 116,592,089(63) \times 10^{-11}$$

77 and

78 
$$a_{\mu}^{\text{FNAL}} = 116,592,040(54) \times 10^{-11}.$$
<sup>14</sup>

79 They agree with each other at the level of  $0.6\sigma$  (0.6 standard deviations), and their  
80 combined result,

81 
$$a_{\mu}(2021) = 116,592,040(41) \times 10^{-11} \text{ (0.35 ppm),}$$

82 is accurate to 0.35 parts per million (ppm).

83 The theoretical evaluation of the same observable in the Standard Model, if made at a  
84 comparable level of precision, involves many subtle details about the interaction  
85 dynamics of *all* Standard Model particles, as well as high precision measurements in  
86 atomic physics.

87 It requires a total knowledge of the relevant physics.

88 The failure to reproduce the experimental results would constitute strong evidence for  
89 physics lying beyond the Standard Model.

90 What do physicists know for sure about this confrontation?

## 91 **Measurements of the Muon Anomalous Magnetic** 92 **Moment**

93 The clever experimental determination of the anomalous magnetic moment of the  
94 muon  $a_\mu$  is based on three fundamental physical properties:

- 95 1. Muons produced by the decay of pions are longitudinally polarized: the  
96 positively charged muons in the FNAL experiment have their spins oriented  
97 opposite to their momenta.
- 98 2. The orbit frequency of a muon turning in a horizontal storage ring in the  
99 presence of a uniform vertical magnetic field differs from its spin precession by  
100 a factor precisely proportional to the muon anomalous magnetic moment  $a_\mu$  that  
101 one wants to measure. This difference is called the anomalous precession  
102 frequency.
- 103 3. Muons, like pions, are unstable particles and have a lifetime at rest of about  
104 two microseconds. Their weak decay into an electron and two neutrinos  
105 violates parity, and this provides the trick to measuring the anomalous  
106 precession frequency, hence the muon anomalous magnetic moment.

107 The trick succeeds because the muon spins are strongly correlated with the decay  
108 electron momenta. This spin-energy correlation results in a modulation of the electron

109 decay energy spectrum that occurs at the rate of the anomalous precession frequency,  
110 and this is how the anomaly is measured.

111 In practice, one needs good statistics of a pion source. This is provided by the proton  
112 collisions emerging from a high energy accelerator, and for this reason, the  
113 experiments have been made at places such as CERN, BNL, and FNAL. The polarized  
114 muons from pion decays are then boosted in a storage ring, so as to sufficiently  
115 increase their lifetimes, in the presence of a very precise homogeneous vertical  
116 magnetic field and an inner ring of energy detectors measuring the electron decay from  
117 the stored muons. This approach was pioneered in a series of dedicated experiments at  
118 CERN between 1962 and 1968, which were further pursued by the E821 experiment at  
119 BNL and recently by the FNAL Muon  $g - 2$  experiment, which is still underway.<sup>15</sup>

## 120 **Vacuum Polarization Effects**

121 Vacuum polarization is a characteristic phenomenon arising in the quantum  
122 formulation of electrodynamics. First discussed by Dirac and Werner Heisenberg, it  
123 occurs when a virtual photon propagating between two sources probes a vacuum  
124 fluctuation of electrically charged particle–antiparticle pairs.<sup>16</sup> These particles can be  
125 leptons or quark–antiquark pairs. Observable effects of vacuum polarization in atomic  
126 physics were first evaluated to lowest order in powers of the fine structure constant  $\alpha$   
127 by Robert Serber and Edwin Uehling in 1935.<sup>17</sup> In QED, Feynman, Schwinger,  
128 Tomonaga, and Dyson showed how to do systematic calculations of these effects in  
129 powers of the small coupling constant  $\alpha$  and, in particular, how to evaluate the QED  
130 contributions to the lepton anomalies. The lepton vacuum polarization effects (lepton-  
131 VP) to  $a_\mu$  induced by muon–antimuon pairs give mass-independent contributions, since  
132  $g$  is massless and the only mass in this case is that of the muon. Those induced by tau–

133 antitau pairs decouple by a factor  $\frac{m_\mu^2}{M_\tau^2}$  because virtual photons coming from the external  
134 muon carry an average momentum too small to probe the heavy tau–antitau pairs from  
135 the vacuum. The lepton-VP effects induced by the vacuum fluctuations of electron–  
136 positron pairs are large because the average momentum carried by the virtual photon  
137 emitted by the external muon is large compared to the electron mass. This produces a  
138  $\log \frac{m_\mu}{m_e} \sim 5$  factor at the  $\left(\frac{\alpha}{\pi}\right)^2$  order of the QED contribution and higher  $\log \frac{m_\mu}{m_e}$  powers  
139 at higher orders. These give rise to large contributions to the total theoretical budget.  
140 Although the calculations are technically difficult, they have been done analytically  
141 and in some cases numerically to the needed accuracy, mostly by Toichiro Kinoshita  
142 and his collaborators.<sup>18</sup>

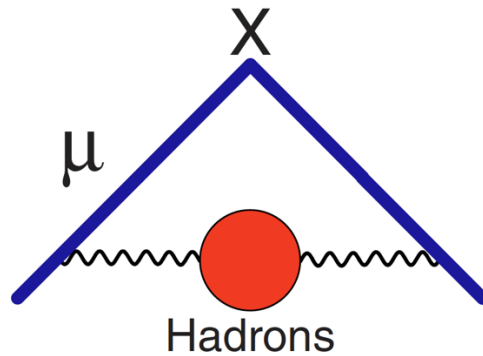
143 Vacuum polarization resulting from lepton–antilepton loops was first considered in the  
144 context of QED. However, every charged particle contributes to vacuum polarization.  
145 Hadronic vacuum polarization (HVP) is due to hadron–antihadron loops. Its evaluation  
146 is not straightforward because hadrons are subject to strong QCD forces, but its  
147 magnitude can easily be estimated: it appears first at the level of the  $\left(\frac{\alpha}{\pi}\right)^2$  contributions  
148 because, as illustrated by the Feynman diagram in Figure 2, at least two virtual photon  
149 propagators, represented by the wavy lines, are needed to connect to the hadron–  
150 antihadron loop indicated by the disc. The virtual photons in this case carry an average  
151 momentum that is small compared to the masses of the hadronic particles that they can  
152 probe. Because of spin symmetry properties, the overall effect must be modulated by a  
153 mass factor  $m_\mu^2$ , which, in order to make  $g$  dimensionless, must be normalized to  
154 another mass scale, which is expected to have the size of the mass of the lowest lying  
155 hadronic resonance to which the photons can couple. This is the  $\rho$  (770 MeV) particle.  
156 The overall  $m_\mu^2$  dependence makes the  $g$  factor of the muon more sensitive to hadronic



157 effects than the electron by a ratio  $\frac{m_\mu^2}{m_e^2} \simeq 43,000$ . The muon anomaly is much more  
 158 sensitive to effects beyond those encoded in the Standard Model, and these may be  
 159 contributing to vacuum polarization. This is the reason for the interest in a precise  
 160 determination of  $a_\mu$ .

161 The estimated order of magnitude,  $\left(\frac{\alpha}{\pi}\right)^2 \frac{m_\mu^2}{M_\rho^2} \sim 10^{-7}$ , of the HVP contribution seems  
 162 rather small, but it is big enough when compared to the 0.35 ppm accuracy reached in  
 163 recent experimental determinations of  $a_\mu$ . It requires more attention than just an order  
 164 of magnitude evaluation.

165 **Figure 2.**



166

167 Feynman diagram of the HVP contribution. The X in the muon line represents the  
 168 external magnetic field.

169 The HVP contribution to  $a_\mu$  is called  $a_\mu(\text{HVP})$ . Its evaluation depends critically on the  
 170 hadronic forces generated by QCD, whose effects cannot be perturbatively calculated,  
 171 but which are described by an analytic function  $\Pi_{\text{HVP}}(q^2)$  of the energy-momentum  
 172 squared of the virtual photon. This function encodes how photons interact via QCD  
 173 with hadrons. The contribution to  $a_\mu(\text{HVP})$  due to hadron loops is given by  $q^2$ -range

174 dependence of  $\Pi_{\text{HVP}}(q^2)$ . Cauchy's integral theorem shows how to evaluate  $\Pi_{\text{HVP}}(q^2)$   
175 everywhere once its shape on the real axis is known from the hadronic threshold at  
176  $q^2 = 4m_\pi^2$ , where  $m_\pi$  is the mass of a charged pion, to the spectral function region at  
177 infinity. It turns out that it is precisely over a large range of this spectral function  
178 region that  $\Pi_{\text{HVP}}(q^2)$  is accessible to experimental determination by means of the total  
179 cross section that emerges when electrons and positrons annihilate, thereby producing  
180 hadrons. Furthermore, the shape of the spectral function in the very high  $q^2$  energy  
181 region, where there is no more experimental information, is well predicted by the  
182 asymptotic freedom property of QCD.<sup>19</sup>

183 The precise wording of  $a_\mu(\text{HVP})$ , what is now called the dispersive HVP evaluation of  
184  $a_\mu$ , was first given by Claude Bouchiat and my thesis advisor Louis Michel in 1961.<sup>20</sup>  
185 In 1969, using the results from experiments at the Linear Accelerator in Orsay, a first  
186 quantitative evaluation was made at the level of 8% accuracy, which was sufficient for  
187 an overall comparison with the experimental value of  $a_\mu$ .<sup>21</sup> With the advent of  
188 increasingly refined experiments on  $e^+e^-$  annihilations into hadrons, the precision of  
189 the HVP contribution to  $a_\mu$  has improved considerably. The most recent determination,  
190 made by Michel Davier and collaborators from dedicated experiments at several  
191 facilities, quotes a precision level of 0.5%.<sup>22</sup> In spite of this accuracy, it still remains  
192 the one that at present has the largest error.

193 Alternative and increasingly accurate evaluations of the  $a_\mu(\text{HVP})$  contribution have  
194 been made during the last few years using LQCD techniques, and, in fact, the recent  
195 results published by the BMW collaboration are comparable to the dispersive  
196 evaluations.<sup>23</sup>

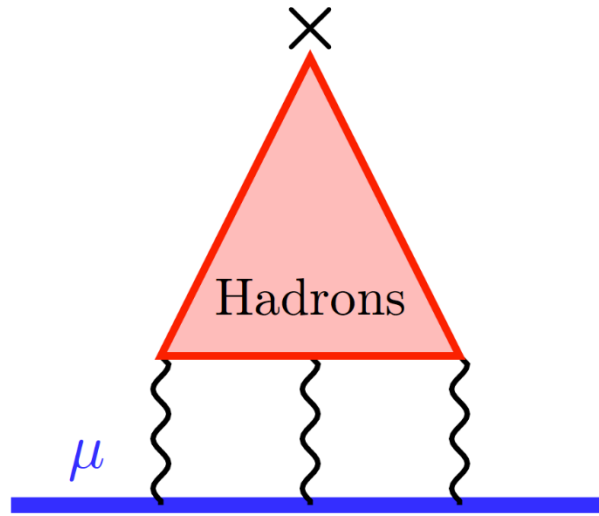
## 197 **Light-by-Light Scattering Effects**

198 The scattering of light off light (LbyLS) is another rare phenomenon that only happens  
199 at the quantum level and is predicted by QED. It has been recently detected at the  
200 Large Hadron Collider at CERN, but it also occurs due to the vacuum fluctuations  
201 induced by the effect of four photons. LbyLS contributes to  $a_\mu$  at the order of  $\left(\frac{\alpha}{\pi}\right)^3$   
202 because, in the presence of an external magnetic field, as seen as one photon, an  
203 incoming external muon can emit and reabsorb three virtual photons that  
204 simultaneously—one + three—probe the vacuum fluctuation of leptons and/or  
205 hadrons. The Feynman diagram in Figure 3 illustrates this process; the red triangle  
206 represents either leptons or hadrons.

207 As for the lepton-VP effects discussed earlier, the LbyLS induced by muon loops  
208 yields mass-independent contributions to  $a_\mu$ . These have been calculated to the needed  
209 accuracy either numerically or analytically.<sup>24</sup> Those arising from tau-lepton loops also  
210 decouple like those of lepton-VP and are very small. Those arising from electron loops  
211 turn out to be very important giving a  $\pi^2 \log \frac{m_\mu}{m_e}$  factor at the leading  $\left(\frac{\alpha}{\pi}\right)^3$  level.<sup>25</sup> This  
212 came as a big surprise in 1969 because in QED, the LbyLS does not have an intrinsic  
213 new coupling constant beyond its electric charge coupling  $e^4$  to the four photons and,  
214 therefore, does not require renormalization. This is in contrast to the vacuum  
215 polarization calculation discussed earlier. The unexpectedly large LbyLS contribution  
216 served to eliminate a possible discrepancy between theory and experiment.<sup>26</sup>

217

**Figure 3.**



218

219

Feynman diagram of the hadronic LbyLS contribution.

220

Hadronic light-by-light scattering (HLbyL) is the corresponding effect induced by

221

hadron loops, and its evaluation is not as straightforward as the one induced by leptons

222

because QCD must be dealt with. Furthermore, in the HLbyL case, Cauchy's integral

223

theorem is unavailing. Contributions of order  $\left(\frac{\alpha}{\pi}\right)^3$  became relevant at the last BNL

224

experiment.<sup>27</sup> the remarkable accuracy of the  $a_\mu$  measurement indicated a deviation

225

from theoretical predictions.<sup>28</sup> Theorists proposed several model estimates of the

226

$a_\mu(\text{HLbyL})$  contribution, which all agreed within the quoted errors. They found that

227

the contribution came with an overall negative sign. Soon after the publication of the

228

BNL result, this theoretical conclusion was questioned.<sup>29</sup> There is a QCD limit at

229

which the  $a_\mu(\text{HLbyL})$  contribution is calculable. It is obtained in a model where the

230

number of quark colors and the mass gap are allowed to increase without limit.<sup>30</sup> In

231

that unphysical limit, the contribution to  $a_\mu(\text{LbyLS})$  turns out to be positive, implying

232

that model calculations compatible with QCD must *also* give a positive result when

233 model parameters are extrapolated accordingly. They all failed the test! The authors  
234 eventually found a convention inconsistency in their calculations which, when  
235 corrected, indeed gave a positive contribution. The significant discrepancy between  
236 theory and experiment was, if not eliminated, then reduced.

237 Conflicting theoretical estimates of  $a_\mu(\text{LbyLS})$  created some confusion. At a workshop  
238 at the University of Glasgow in 2009, Lee Roberts suggested that three theorists of  
239 different groups get together and examine the discrepancies between models. He  
240 appointed Joachim Prades, Arkady Vainshtein, and me to do the job. Our result was  
241  $a_\mu(\text{LbyLS} - \text{Glasgow}) = (10.5 \pm 2.6) \times 10^{-10}$ , the so-called Glasgow consensus.<sup>31</sup>  
242 There have been many evaluations of the  $a_\mu(\text{LbyLS})$  contribution since then, using  
243 ever more sophisticated models. They turn out to be consistent, within errors, with the  
244 Glasgow consensus.

245 The evaluation of the  $a_\mu(\text{LbyLS})$  contribution has also been undertaken by various  
246 LQCD collaborations. These are QCD first principle, model-independent evaluations,  
247 which little by little have reached a better level of accuracy than the Glasgow  
248 consensus.<sup>32</sup> For all that, no one believes that the  $a_\mu(\text{LbyLS})$  contribution can by itself  
249 explain the tension between theory and experiment.

## 250 **Present Status of the Theoretical Contributions**

251 The following table summarizes the status of the various theoretical contributions to  
252 the muon anomaly in  $10^{-11}$  units and prior to the FNAL result. Several comments are in  
253 order:

- 254 • The results in the third column of Table 1 show the level at which each  
 255 contribution is effective.
- 256 • The  $a_\mu(\text{QED})$  results from the interactions of photons and leptons include the  
 257 contributions from the calculated coefficients of the first five powers in  $\frac{\alpha}{\pi}$ . The  
 258 result in the first line has been evaluated using the value of the fine-structure  
 259 constant  $\alpha$  determined from a cesium atom interferometry experiment:<sup>33</sup>

260 
$$\alpha^{-1}(\text{Cs}) = 137.035999046(27).$$

- 261 • The entry in the second line of the table makes use of the high-precision  
 262 determination of the electron  $g - 2$  by Gerald Gabrielse's group at Harvard,<sup>34</sup>

263 
$$a_e(\text{exp.}) = 1,159,652,180.73(28) \times 10^{-12},$$

264 yielding

265 
$$\alpha^{-1}(a_e) = 137.0359991496(13)(14)(330).$$

266 The uncertainty of  $\pm 330$  results from the error in the experimental  
 267 determination of  $a_e$ ,  $\pm 14$  from the hadronic contribution to  $a_e$  and  $\pm 13$  from the  
 268 numerical evaluation of the  $\left(\frac{\alpha}{\pi}\right)^5$  QED term.<sup>35</sup>

269 One can conclude from these results that the theoretical contribution from the  
 270 QED interactions of photons and leptons to the muon  $g - 2$  is well known.

- 271 • By  $a_\mu(\text{HVP})_{\text{lowest order}}$ , one means the HVP contribution from the Feynman  
 272 graph in Figure 2. The third and fourth lines of Table 1 are the result of adding  
 273 third and fourth leading orders hadronic contribution. These results have been

274 obtained using the dispersive HVP evaluation method described above near  
 275 Figure 2.

- 276 • The contribution  $a_\mu(\text{EW})$  from the electroweak interactions of the Standard  
 277 Model is small because it is proportional to  $G_F m_\mu^2$ , where  $G_F \sim 10^{-5} \text{ GeV}^{-2}$  is  
 278 the Fermi coupling constant that governs the strength of the weak interactions.  
 279 Its evaluation is well understood in the Standard Model.<sup>36</sup>

280 **Table 1.**

Contribution	Comments	Results
$a_\mu(\text{QED})$	Photons and leptons, with $\alpha(\text{Cs})$	116,584,718.931(30)
$a_\mu(\text{QED})$	Photons and leptons, with $\alpha(a_e)$	116,584,718.842(34)
$a_\mu(\text{HVP})_{\text{lowest order}}$	Experimental dispersive evaluations	6,931(40)
$a_\mu(\text{HVP})_{\text{total}}$	Experimental dispersive evaluations	6,845(40)
$a_\mu(\text{EW})$	$W^\pm, Z^0$ , and Higgs with leptons and quarks	153.6(1.0)
$a_\mu(\text{HLbL})_{\text{total}}$	Phenomenology and LQCD	92(18)

281 Theoretical contributions to the muon anomaly in units of  $10^{-11}$ .

282 The sum of the contributions listed in the results column of Table 1 gives

283 
$$a_\mu(\text{Th. WP}) = 116,591,810(43) \times 10^{-11},$$

284 which is the consensus reported in the 2020 white paper (WP).<sup>37</sup> When compared to  
 285 the experimental result  $a_\mu(2021) = 116,592,040(41) \times 10^{-11}$  (0.35 ppm), the result  
 286 indicates a significant  $4.2\sigma$  difference—triggering in the literature, of course,  
 287 speculations about whether new physics might be at work. Do notice that a  
 288 discrepancy of  $4.2\sigma$  is about twice the size of the electroweak contribution. The order  
 289 of magnitude of what might be the contribution of new physics (NP) to the muon

290 anomaly are expected to be:  $a_\mu(\text{NP}) \sim a_\mu(\text{EW}) \times \left(\frac{M_W}{M_{\text{NP}}}\right)^2 \times \text{couplings}$  with  $M_{\text{NP}} \gg$   
291  $M_W$ . This would require that the coupling be fine tuned.

292 Many NP models have been excluded for that reason.

293 The situation is confusing. The same day that the results of the FNAL muon  $g - 2$   
294 collaboration were published,<sup>38</sup> *Nature* published a new result by the Budapest–  
295 Marseille–Wuppertal (BMW) LQCD collaboration. At issue was the lowest-order  
296 HVP contribution to the muon  $g - 2$ . The result:<sup>39</sup>

297 
$$a_\mu(\text{HVP})_{\text{BMW}} = 7,075(55) \times 10^{-11},$$

298 which reduces the total discrepancy in  $a_\mu(2021) = 116,592,040(41) \times 10^{-11}$  (0.35  
299 ppm) from  $4.2\sigma$  to  $1.6\sigma$ .

300 The discrepancy is still there, but it is not significant enough to suggest that new  
301 physics is at work.

302 The BMW result is under detailed examination by other LQCD collaborations. They  
303 are expected to produce their own results. If the disagreement between LQCD and the  
304 experimental dispersive evaluation of the HVP persists, researchers will have to find  
305 the explanation for that. Because they involve integrals of different quantities,  
306 comparison of the two methods is difficult but not impossible.

307 As new statistics accumulate, the FNAL muon  $g - 2$  experiment is expected to reduce  
308 their error. There is also a new experiment at the Japan Proton Accelerator Research  
309 Complex in Tokai. This J-PARC experiment E34 will employ a new and different  
310 technique to measure the muon anomaly.<sup>40</sup> All this gives more time for the theorists to  
311 check and improve their calculations before the next awaited confrontation.



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<sup>1</sup> See endnote 20 in Sheldon Lee Glashow, “[The Standard Model](#),” *Inference: International Review of Science* 4, no. 1 (2018), doi:10.37282/991819.18.15. Glashow’s article is a good introduction to the physics discussed here.

<sup>2</sup> Glashow, “[The Standard Model](#).”

<sup>3</sup> Richard Feynman, “[Nobel Lecture: The Development of the Space-Time View of Quantum Electrodynamics](#),” December 11, 1965, *NobelPrize.org*; Julian Schwinger, “[Nobel Lecture: Relativistic Quantum Field Theory](#),” December 11, 1965, *NobelPrize.org*; and Sin-Itiro Tomonaga, “[Nobel Lecture: Development of Quantum Electrodynamics](#),” May 6, 1966, *NobelPrize.org*.

<sup>4</sup> The numbers in parenthesis, here and subsequently, indicate the experimental uncertainty of the final two decimal digits.

<sup>5</sup> Schwinger’s  $\frac{\alpha}{2\pi}$  is inscribed on the memorial marker near his grave in the Mount Auburn Cemetery in Cambridge, Massachusetts. Julian Schwinger, “[On Quantum-Electrodynamics and the Magnetic Moment of the Electron](#),” *Physical Review* 73, no. 4 (1948): 416–17, doi:10.1103/physrev.73.416; and Julian Schwinger, “[Quantum Electrodynamics. III. The Electromagnetic Properties of the Electron—Radiative Corrections to Scattering](#),” *Physical Review* 76, no. 6 (1949): 790–817, doi:10.1103/physrev.76.790.

<sup>6</sup> Polykarp Kusch and Henry Foley, “[Precision Measurement of the Ratio of the Atomic ‘g Values’ in the  \$^2P\_{3/2}\$  and  \$^2P\_{1/2}\$  States of Gallium](#),” *Physical Review* 72 (1947): 1,256, doi:10.1103/PhysRev.72.1256.2; and Polykarp Kusch and Henry Foley, “[The Magnetic Moment of the Electron](#),” *Physical Review* 74, no. 3 (1948): 250–63, doi:10.1103/physrev.74.250.

<sup>7</sup> Negatively charged electrons, muons, and tau leptons, as well as their associated neutrinos, are called leptons. Their antiparticles are called antileptons.

<sup>8</sup> Freeman Dyson, “[The Radiation Theories of Tomonaga, Schwinger, and Feynman](#),” *Physical Review* 75, no. 3 (1949): 486–502, doi:10.1103/physrev.75.486. See also Feynman, “[Nobel Lecture: Development of the Space-Time View](#)”; Schwinger, “[Nobel Lecture: Relativistic Quantum Field Theory](#)”; and Tomonaga, “[Nobel Lecture: Development of Quantum Electrodynamics](#).”

<sup>9</sup> Sheldon Lee Glashow, “[Nobel Lecture: Towards a Unified Theory—Threads in a Tapestry](#),” December 8, 1979, *NobelPrize.org*; Abdus Salam, “[Nobel Lecture: Gauge Unification of Fundamental Forces](#),” December 8, 1979, *NobelPrize.org*; and Steven Weinberg, “[Nobel Lecture: Conceptual Foundations of the Unified Theory of Weak and Electromagnetic Interactions](#),” December 8, 1979, *NobelPrize.org*.

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<sup>10</sup> Peter Higgs, “[Nobel Lecture: Evading the Goldstone Theorem](#),” December 8, 2013, *NobelPrize.org*; and François Englert, “[Nobel Lecture: The BEH Mechanism and Its Scalar Boson](#),” December 8, 2013, *NobelPrize.org*.

<sup>11</sup> Gerard 't Hooft, “[Nobel Lecture: A Confrontation with Infinity](#),” December 8, 1999, *NobelPrize.org*; and Martin Veltman, “[Nobel Lecture: From Weak Interactions to Gravitation](#),” December 8, 1999, *NobelPrize.org*.

<sup>12</sup> One year’s work on one of these supercomputers is equivalent to 100,000 years on a laptop (information from Laurent Lellouch, the spokesman of the BMW collaboration).

<sup>13</sup> For example, the calculation of the proton mass from QCD first principles made by the Budapest–Marseille–Wuppertal Collaboration. Stephan Dürr et al., “[Ab Initio Determination of Light Hadron Masses](#),” *Science* 322, no. 5,905 (2008): 1,224–27, doi:10.1126/science.1163233.

<sup>14</sup> The numbers in parentheses indicate the error in the last two significant figures.

<sup>15</sup> See, e.g., Francis Farley, “Muon  $g - 2$  and Tests of Relativity,” in *60 Years of CERN Experiments and Discoveries*, ed. Herwig Schopper and Luigi Di Lella (Singapore: World Scientific, 2015), 371–96. Bennett et al., “[Final Report of the E821 Muon Anomalous Magnetic Moment Measurement at BNL](#)”; Babak Abi et al., “[Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm](#),” *Physical Review Letters* 126 (2021): 141801, doi:10.1103/PhysRevLett.126.141801; and Talal Albahri et al., “[Measurement of the Anomalous Precession Frequency](#),” *Physical Review D* 103 (2021): 072002, doi:10.1103/PhysRevD.103.072002.

<sup>16</sup> Paul Dirac, “[Discussion of the Infinite Distribution of Electrons in the Theory of the Positron](#),” *Mathematical Proceedings of the Cambridge Philosophical Society* 30 (1934): 150–63, doi:10.1017/s030500410001656x; and Werner Heisenberg, “[Bemerkungen zur Diracschen Theorie des Positrons](#),” *Zeitschrift für Physik* 90, no. 3–4 (1934): 209–31, doi:10.1007/bf01333516.

<sup>17</sup> Robert Serber, “[Linear Modifications in the Maxwell Field Equations](#),” *Physical Review* 48, no. 1 (1935): 49–54, doi:10.1103/physrev.48.49; and Edwin Uehling, “[Polarization Effects in the Positron Theory](#),” *Physical Review* 48, no. 1 (1935): 55–63, doi:10.1103/physrev.48.55.

<sup>18</sup> Tatsumi Aoyama, Toichiro Kinoshita, and Makiko Nio, “[Theory of the Anomalous Magnetic Moment of the Electron](#),” *Atoms* 7 (2019): 28, doi:10.3390/atoms7010028.

<sup>19</sup> At very high energies, the short-distance interactions between quarks and gluons in QCD become weaker and weaker and thus calculable. The quarks become less and less confined, and this is why one says that the QCD theory is asymptotically free. See David Gross, “[Nobel Lecture: The Discovery of Asymptotic Freedom and the Emergence of QCD](#),” December 8, 2004, *NobelPrize.org*; David Politzer, “[Nobel Lecture: The Dilemma of Attribution](#),”

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December 8, 2004, *NobelPrize.org*; and Frank Wilczek, “[Nobel Lecture: Asymptotic Freedom: From Paradox to Paradigm](#),” December 8, 2004, *NobelPrize.org*.

<sup>20</sup> I remember Claude Bouchiat discussing this on the blackboard with Louis Michel in Louis’s office sixty years ago, but I could not understand a word of it at that time! Claude Bouchiat and Louis Michel, “[La Résonance dans la diffusion méson  \$\pi\$ — méson  \$\pi\$  et le moment magnétique anormal du méson  \$\mu\$](#) ,” *Journal de Physique et le Radium* 22, no. 2 (1961): 121–21, doi:10.1051/jphysrad:01961002202012101.

<sup>21</sup> Michel Gourdin and Eduardo de Rafael, “[Hadronic Contributions to the Muon  \$g\$ -Factor](#),” *Nuclear Physics B* 10 (1969): 667–74, doi:10.1016/0550-3213(69)90333-2. The 8% accuracy of this first estimate only reflected the uncertainty in the Orsay data used as an input. I was then a postdoc at CERN and gave a seminar there on my work with Gourdin. John Bell was in the audience and asked: “What do you know about the behavior of the spectral function at infinity?” I answered that I assumed that it must be like the one in QED for leptons. That did not satisfy John, for the good reason that there was no theory, like QCD nowadays, which answered that question. It gave me, however, the privilege to work with him on an effective field theory approach to HVP. John Bell and Eduardo de Rafael, “[Hadronic Vacuum Polarization and  \$g\_{\mu}-2\$](#) ,” *Nuclear Physics B* 10, no. 4 (1969): 611–20, doi:10.1016/0550-3213(69)90250-8.

<sup>22</sup> Michel Davier et al., “[A New Evaluation of the Hadronic Vacuum Polarisation Contributions to the Muon Anomalous Magnetic Moment and to  \$\alpha\(m^2\_Z\)\$](#) ,” *European Physical Journal C* 80, no. 3 (2020): 241, doi:10.1140/epjc/s10052-020-7792-2; and Michel Davier et al., “[Erratum to: A New Evaluation of the Hadronic Vacuum Polarisation Contributions to the Muon Anomalous Magnetic Moment and to  \$\alpha\(m^2\_Z\)\$](#) ,” *European Physical Journal C* 80, no. 5 (2020): 410, doi:10.1140/epjc/s10052-020-7857-2. For a detailed description of the various HVP contribution evaluations, see Tatsumi Aoyama et al., “[The Anomalous Magnetic Moment of the Muon in the Standard Model](#),” *Physics Reports* 887 (2020): 1–166, doi:10.1016/j.physrep.2020.07.006.

<sup>23</sup> Szabolcs Borsanyi et al. (BMW Collaboration), “[Leading Hadronic Contribution to the Muon Magnetic Moment from Lattice QCD](#),” *Nature* 593, no. 7,857 (2021): 51–55, doi:10.1038/s41586-021-03418-1.

<sup>24</sup> For details, see Toichiro Kinoshita, “Lepton  $g - 2$  from 1947 to Present,” in *Advance Series on Directions in High Energy Physics, Vol. 20, Lepton Dipole Moments*, ed. B. Lee Roberts and William Marciano (Singapore: World Scientific Publishing, 2010). Aoyama et al., “[Anomalous Magnetic Moment](#).”

<sup>25</sup> Janis Aldins et al., “[Photon-Photon Scattering Contribution to the Sixth-Order Magnetic Moment of the Muon](#),” *Physical Review Letters* 23, no. 8 (1969): 441–43, doi:10.1103/physrevlett.23.441; and Janis Aldins et al., “[Photon-Photon Scattering Contribution to the Sixth-Order Magnetic Moments of the Muon and Electron](#),” *Physical Review D* 1, no. 8 (1970): 2,378–95, doi:10.1103/physrevd.1.2378.

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<sup>26</sup> The unexpected  $\pi^2 \log \frac{m_\mu}{m_e}$  factor emerges because of a particular singular mass behavior, specific to certain configurations which happen in the case of the LbyLS in the limit where the internal  $m_e$  mass becomes small compared to the external  $m_\mu$  mass.

<sup>27</sup> Bennett et al., “[Final Report](#).”

<sup>28</sup> I remember a paper by illustrious theorists claiming that the beyond the Standard Model theory of supersymmetry fit the discrepancy like a glove fits a hand.

<sup>29</sup> Marc Knecht et al., “[Hadronic Light-By-Light Scattering Contribution to the Muon  \$g - 2\$ : An Effective Field Theory Approach](#),” *Physical Review Letters* 88, no. 7 (2002): 071802, doi:10.1103/physrevlett.88.071802.

<sup>30</sup> The number of colors ( $N_c$ ) denotes the charge-like degrees of freedom of the quarks. The mass gap is the one between the Goldstone-like pion mass  $m_\pi$  and the larger mass scale  $\sim M_\rho$ , where quarks and gluons condensate into hadrons. The large- $N_c$  limit selects the effect of all the contributing hadronic resonances in the limit where their widths vanish. In this limit  $a_\mu(\text{HLbyL}) = \left(\frac{\alpha}{\pi}\right)^3 N_c^2 \frac{m_\mu^2}{48\pi^2 f_\pi^2} \log^2 \frac{M_\rho}{m_\pi} \simeq 95 \times 10^{-11}$ , where  $f_\pi \simeq 92$  MeV is the coupling constant associated to the  $\pi^0$  decay into two photons (Stephen Adler, “[Axial-Vector Vertex in Spinor Electrodynamics](#),” *Physical Review* 177, no. 5 [1969]: 2,426–38, doi:10.1103/physrev.177.2426; and John Bell and Roman Jackiw, “[A PCAC Puzzle:  \$\pi^0 \rightarrow \gamma\gamma\$  in the  \$\sigma\$ -Model](#),” *Il Nuovo Cimento A* 60, no. 1 [1969]: 47–61, doi:10.1007/bf02823296). The first numerical check of this limit with a phenomenological model was made in Marc Knecht and Andreas Nyffeler, “[Hadronic Light-by-Light Corrections to the Muon  \$g - 2\$ : The Pion-Pole Contribution](#),” *Physical Review D* 65, no. 7 (2002): 073034, doi:10.1103/physrevd.65.073034.

<sup>31</sup> Joachim Prades, Eduardo de Rafael, and Arkady Vainshtein, “[The Hadronic Light-by-Light Scattering Contributions to the Muon and Electron Anomalous Magnetic Moments](#),” in *Advance Series on Directions in High Energy Physics, Vol. 20, Lepton Dipole Moments*, ed. B. Lee Roberts and William Marciano (Singapore: World Scientific Publishing, 2010), 303–17, doi:10.1142/9789814271844\_0009.

<sup>32</sup> Details can be found in Aoyama et al., “[Anomalous Magnetic Moment](#),” and more recently in En-Hung Chao et al., “[Hadronic Light-by-Light Contribution to  \$\(g - 2\)\_\mu\$  from Lattice QCD: A Complete Calculation](#),” arXiv:2104.02632v1[hep-lat] (2021).

<sup>33</sup> Richard Parker et al., “[Measurement of the Fine-Structure Constant as a Test of the Standard Model](#),” *Science* 360, no. 3,685 (2018): 191–95, doi:10.1126/science.aap7706.

<sup>34</sup> David Hanneke, Shannon Fogwell, and Gerald Gabrielse, “[New Measurement of the Electron Magnetic Moment and the Fine Structure Constant](#),” *Physical Review Letters* 100, no. 12 (2008): 120801, doi:10.1103/physrevlett.100.120801.

<sup>35</sup> Aoyama, Kinoshita, and Nio, “[Theory of the Anomalous Magnetic Moment](#).”

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<sup>36</sup> See, e.g., Aoyama et al., “[Anomalous Magnetic Moment](#).”

<sup>37</sup> Aoyama et al., “[Anomalous Magnetic Moment](#).”

<sup>38</sup> Abi et al., “[Measurement of the Positive Muon Anomalous Magnetic Moment](#)”; and Albahri et al., “[Measurement of the Anomalous Precession Frequency](#).”

<sup>39</sup> Borsanyi et al., “[Leading Hadronic Contribution](#).”

<sup>40</sup> M. Abe et al., “[A New Approach for Measuring the Muon Anomalous Magnetic Moment and Electric Dipole Moment](#),” *Progress of Theoretical and Experimental Physics* 2,019, no. 5 (2019): O53C02, doi:10.1093/ptep/ptz030.