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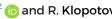
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ABSTRACT

Parametric instability of a standing gravity-capillary wave is observed in conditions of irradiation of a surface of a liquid by a plane ultrasound beam modulated by amplitude. A threshold value of ultrasound intensity for excitation of the parametric oscillations of the surface is determined, and the frequency response of the process is measured. A mechanism of instability based on sound radiation pressure applied to a liquid surface curved with menisci is suggested.

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I. INTRODUCTION

Parametric excitation of gravity-capillary waves (GCWs) at a liquid surface is usually considered in the frames of the Faraday instability arising when the entire liquid volume undergoes vibrations as a whole. 1-4 Alternating acceleration modulates gravity force acting on particles of liquid, which results in modulation of natural frequencies of GCW. The acceleration can be not only vertical as in the classic Faraday mechanism but horizontal as well. 5,6 Parametric excitation of GCW is utilized in particular, for atomization of liquids, ^{7,8} in studies of interphase boundaries in multilayered structures^{9,10} and studies of surfactant dynamics.11 The Faraday instability was also used for manipulating the shape of a liquid lens¹² and as a tool for a phaseconjugating acoustic mirror.¹³ In Ref. 14, parametric excitation of capillary waves was observed in conditions of modulation of a capillary curvature by ultrasound with double frequency. Modulated ultrasound was used for parametric excitation of capillary oscillations in levitating

The mentioned phenomena can be attributed to linear parametric processes when generalized forces of parametric interaction are linear with respect to GCW amplitudes. A possibility of parametric excitation of GCW under radiation pressure of ultrasound (RPU) at a liquid surface was studied in Refs. 17 and 18. A nonlinear three-wave parametric excitation of GCW in the field of RPU modulated by amplitude was studied theoretically and experimentally. Conditions for the development of an explosive instability were considered, and amplitudes of the three-wave interactions were measured. 18 A new type of instability of GCW similar to an inverse energy cascade was

discovered in conditions of permanent ultrasonic impact on the liquid surface.¹⁹ Excitation of a low-frequency GCW by high-frequency ultrasound was considered theoretically in a model of a nondissipative medium.²⁰

The subject of the present paper is an experimentally observed parametric instability of a liquid surface that differs by the principle from the Faraday instability and takes place in a motionless volume under the impact of high-frequency ultrasound. We report results on the experimental realization of a linear parametric excitation of GCW in an amplitude-modulated RPU field. The frequency response of the observed parametric oscillations is obtained, and the threshold value of ultrasound intensity required for the generation of GCW is found. Non-triviality of the mechanism of the discovered instability follows from the fact that an RPU with a uniform distribution along a nonperturbed liquid surface provides no conditions for a linear parametric coupling of GCW. An explanation of the observed instability is proposed which accounts for static distortions of the liquid surface by menisci near walls limiting the excitation area.

II. EXPERIMENTAL SETUP AND METHOD

The geometry of the experiment is shown in Fig. 1. Measurements were performed in a glass cell with the internal chamber of $18 \times 50 \times 30$ mm along X, Y, and Z, respectively, and having walls of 3 mm thickness. The cell was filled to a height of 12 mm by fluorinert FC-72 (Perfluorohexane, C_6F_{14}). The choice of the fluid was conditioned by a requirement of a sufficient cavitation strength for the operating range of ultrasound intensities. In addition, FC-72 has low

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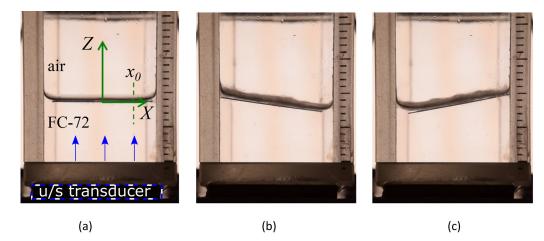


FIG. 1. Photos of the liquid surface in the equilibrium state without ultrasonic pump (a) and in two extremities [(b) and (c)] of parametric oscillations in the observed mode. The geometry of the experiments is shown in (a). When excitation is switched on, the transducer emits an ultrasound wave in the cell, which is schematically displayed by dashed arrows. x_0 shows the plane where displacement is measured. The pump modulation frequency is 12 Hz, and the time interval between the frames in (b) and in (c) is 0.083 s.

viscosity $\eta = 6.4 \times 10^{-4}$ Pa's and relatively high density $\rho = 1860$ kg/m³, which makes it a convenient medium for experiments with floating objects. At a room temperature, sound speed and surface tension values in FC-72 are c = 572 m/s and $\sigma = 10$ dyn/cm correspondingly.

A plane rectangular piezoceramic transducer with dimensions of $20 \times 60 \times 2$ mm on X, Y, and Z attached from the outside to the bottom of the cell through a contact gel was utilized as an acoustic source. The central frequency of the operating band of the source was 1 MHz. The transducer was excited by amplitude-modulated voltage with the carrier frequency of 970 kHz. The modulation frequency was either constant or linearly swept. The excitation signal was applied to the transducer from a Sony Tektronix AFG 3102 generator through a KL-400 RF power amplifier (RM Italy).

In order to suppress small-scale perturbations of the liquid–air interface, a polypropylene film with dimensions $15.5 \times 47.5 \times 0.2$ mm along X, Y, and Z correspondingly was put floating on the surface. The values of density and speed of longitudinal waves in polypropylene are $\rho_f = 890 \text{ kg/m}^3$ and $c_f = 2260 \text{ m/s}$. From the acoustical point of view at a frequency of 1 MHz, the film could be considered as a thin layer, much smaller than the wavelength. Thus, distortions of the acoustic field introduced by the film were negligible compared to the case of a free surface and this was confirmed in auxiliary numerical simulations.

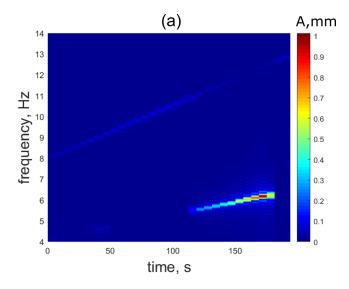
The cell was placed between a light source and a photo camera. The camera took images of the oscillating liquid surface in the plane-parallel to XZ with the size 1920×1080 pixels and at a frame rate of 60 fps. The lens of the camera was focused on the nearest edge of the film, and the resolution in the measurement mode was $5 \mu m/pixel$. The time dependence of vertical displacement of a point of the surface was extracted from the stored frame sequence. To do this, vertical cross sections of the frames were accumulated and stacked for a given coordinate x_0 , and in each cross section the position of the surface was determined and the function $\zeta(t,x_0)$ was found as the deviation of the surface from the equilibrium. The bottom edge of the film at $x_0=6.5 \, \mathrm{mm}$ was taken as the surface point. The described procedure was performed using the ImageJ software.

To estimate the active electrical power P_e at the acoustic emitter, we measured the electric impedance of the transducer in its combination with the filled cell in the frequency band 500–1500 kHz. In these measurements, multiple acoustic resonances of the liquid volume were observed on a background of a dependence known normally for a solitary piezoelectric transducer. The resonances were associated with the thickness of the liquid layer in the cell that has been aligned horizontally along X and Y. As a consequence, routine temperature dependence of sound speed in the liquid without thermostating brought instability in the impedance value and therefore in the radiated acoustic power. To suppress this effect, in the main experiments the cell was tilted in the YZ plane by a small angle \approx 5° with respect to the horizontal. In this way, the acoustic resonances were eliminated, and at the same time, the effect of the tilt on the behavior of GCW was negligible due to the relatively large depth of the layer.

III. RESULTS OF EXPERIMENTS

Figures 1(b) and 1(c) show typical views of the surface in a mode of observed parametric oscillations. The resonant frequency of the mode is close to 6 Hz. The photos taken for the case of the active power at the transducer $P_e=6\,\mathrm{W}$ and the modulation frequency of the ultrasonic pump 12 Hz display opposite positions with maximum deviation from the equilibrium state.

Frequency properties of the observed oscillations were studied by sweeping the pump modulation period, and the result of these measurements is shown in Fig. 2. Figure 2(a) displays evolution of the Fourier spectrum of the displacement $\zeta(t,x_0)$ at a linear sweep of the modulation frequency of pump from 8 to 13 Hz during the interval $T_0=200$ s. The applied power in the experiments was $P_e=3.5$ W. The figure depicts spectral amplitudes of the displacement in millimeters as a brightness graph in time–frequency coordinates. The graph data are obtained by splitting the time interval T_0 into 32 subintervals where the spectra are calculated separately. This method permits to determine what spectral components are generated at a certain time point at a given frequency of the pump.



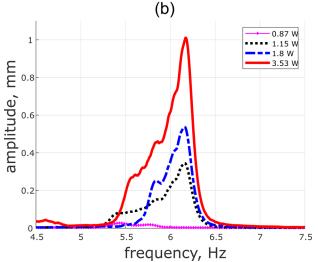


FIG. 2. Amplitude–frequency response of the observed oscillations of the surface. The modulation frequency of pump is linearly increased from 8 to 13 Hz during 200 s. (a) Evolution of the spectrum in time, $P_{\rm e}=3.5\,{\rm W}$; and (b) amplitude of parametric oscillations vs frequency at different pump power.

Due to non-perfect homogeneity of the radiation force of the ultrasound, small adverse oscillations of the liquid surface are always present at a frequency of the pump. These components manifest themselves as the tilted bright line in the upper part of Fig. 2(a). After the time $t\approx 110$ s in parallel to this component, oscillations also appear at 1/2 of the current frequency value. With the frequency growth, the amplitude of this process at first increases and then goes down rapidly. The oscillations display a clearly expressed resonant behavior, and their amplitude is many times greater than the adverse component at the frequency of the pump. Frequency dependencies of amplitude of these parametric oscillations for different output power of the pump amplifier are shown in Fig. 2(b). The dependencies were obtained as the displacement spectra on the total time interval T_0 of the frequency sweep.

Figure 3 shows the amplitude of the parametric oscillations of the liquid surface as a function of the pump power at the modulation frequency of 12 Hz. The critical power that was still sufficient for the presence of the oscillations in these conditions was P_{ec} = 1.15 W, which, taking into account the square of the cell's bottom, corresponded to the critical intensity I_c = 0.128 W/cm².

Thus, the dependencies presented in Figs. 2 and 3 demonstrate frequency selectivity and the existence of a threshold for the observed process of parametric generation of the GCW mode.

IV. MECHANISM OF THE PARAMETRIC INSTABILITY OF GCW

We consider the parametric excitation of GCW on a basis of the dynamic boundary condition for the vertical displacement $\zeta(t,x)$ of the free liquid surface at z=0 taking into account the ultrasound (US) radiation pressure.²¹

$$\rho \left[-\frac{\partial \phi}{\partial t} + \frac{1}{2} \left(\vec{\nabla} \phi \right)^{2} \right] + \rho g \zeta - \sigma \frac{\partial^{2} \zeta}{\partial x^{2}} = \langle E_{a} \rangle \bigg|_{z=\zeta}, \tag{1}$$

where ϕ is velocity potential of particles of liquid, $\rho-$ density of the liquid, g is gravity acceleration, σ is the surface tension coefficient, and $\langle E_a \rangle|_{z=\zeta}$ is a kinetic energy density of the acoustic field averaged over the US wave period, which is much shorter than the GCW period. We suppose, however, that this energy can vary during a time interval comparable to the period of the GCW. Equation (1) is supplemented by the kinematic boundary condition

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial \phi}{\partial z}\bigg|_{z=\zeta} + \frac{\partial \phi}{\partial x}\bigg|_{z=\zeta} \frac{\partial \zeta}{\partial x}.$$
 (2)

The bottom z=-h and the lateral surfaces $x=\pm d$ are supposed to be rigid. In order to reveal the mechanism of parametric instability induced by the radiation pressure, we ignore an intrinsic nonlinearity of the GCW and present solution of the problem as a superposition of

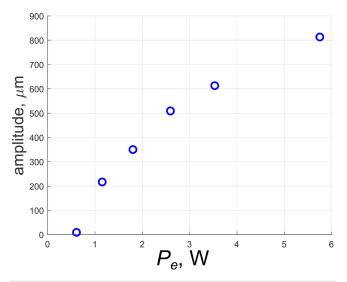


FIG. 3. Amplitude of the parametric oscillations vs pump power. The modulation frequency is 12 Hz.

static displacement and normal GCW modes with amplitudes equal to ε and $\zeta_n(t)$, respectively,

$$\zeta(x,t) = \varepsilon \cdot \cosh(\gamma x) + \sum_{n} \zeta_{n}(t) \sin k_{n} x,$$
 (3)

where $\gamma^2 = \rho g/\sigma$ and *n* is a mode number.

There are two reasons to neglect the intrinsic nonlinearity. Since we consider a mechanism of parametric excitation of GCW, we are interested in conditions near the threshold of instability where the amplitudes of GCW are low and the nonlinearity is weak. Nevertheless, the quadratic terms in Eqs. (1) and (2) could be a source of instability via excitation of surface displacements with a frequency of modulation of the radiation pressure. However, homogeneous displacement of the surface by the radiation pressure is prohibited by the incompressibility of the liquid. Inhomogeneous fluctuations observed near the threshold due to near-field interference of ultrasound do not exceed 0.05 mm, which is one order of value smaller than the threshold of instability expected in the nonlinear mechanism. The static displacement is conditioned by wetting the side walls with the liquid. The parameter ε can be determined from experimental measurements of the menisci height near the walls. Note that for a case of the Faraday instability, the role of meniscus was studied in Ref. 22. According to the series expansion (3), the velocity potential could be found from the Laplacian equation as

$$\phi(x,z,t) = \sum_{n} \phi_n(t) \cosh[k_n(z+h)] \sin k_n x. \tag{4}$$

Taking into account the linearized boundary condition (2), the potential amplitudes are expressed via velocity amplitudes

$$\phi_n(t) = -\dot{\zeta}_n(t)/k_n \sinh(k_n h). \tag{5}$$

Combination of Eqs. (3), (4), and (1) results in the equation of motion for amplitudes of GCW modes excited by the US radiation pressure

$$\ddot{\zeta}_n + 2\delta_n \dot{\zeta}_n + \omega_n^2 \zeta_n$$

$$= \rho^{-1} k_n \tanh(k_n h) \frac{1}{d} \int_{-d}^d dx \, \langle E_a \rangle \Big|_{z=\zeta} \sin(k_n x), \tag{6}$$

where ω_n and δ_n are the eigenfrequency and attenuation coefficient of the mode correspondingly. The radiation pressure depends on the angle of incidence θ of the acoustic wave on the liquid surface. For high-frequency ultrasound with a wavelength, which is much smaller compared to that of the GCW, the acoustic field energy on the curved surface of the liquid depends on the tangent of the angle of incidence $\tan \theta = \partial \zeta / \partial x$ (Ref. 23, Chap. 8)

$$\langle E_a \rangle \Big|_{z=\zeta} = E_0 (1 + \cos \omega_p t) \frac{1}{1 + (\partial \zeta / \partial x)^2}.$$
 (7)

Here, we took into account a single-tone modulation of the ultrasound intensity at a pump frequency ω_p . As the displacement (3) is the superposition of the meniscus profile and the GCW, for small amplitudes of wave $k_n\zeta(t)\ll 1$, the denominator in (7) contains the term

$$\left(\frac{\partial \zeta}{\partial x}\right)^2 \cong \varepsilon^2 \gamma^2 \sinh^2(\gamma x) + 2\varepsilon \cdot \gamma \cdot \sinh(\gamma x) \sum_n k_n \zeta_n(t) \cos k_n x. \tag{8}$$

Thus, the pumping pressure part in the energy density (7) can be found as following:

$$\delta \langle E_a \rangle |_{z=\zeta} = -2E_0 \cos \omega_p t \cdot \varepsilon \cdot \gamma \cdot \sinh(\gamma x) \times \left(1 + \varepsilon^2 \gamma^2 \sinh^2(\gamma x) \right)^{-2} \sum_{x} k_n \zeta_n(t) \cos k_n x.$$
 (9)

Deformation of the liquid surface due to meniscus plays a fundamental role in the considered mechanism of the parametric action of the radiation pressure. First, it linearizes the radiation pressure (7) with respect to alternating displacement $\zeta_n(t)$ of a GCW. Second, it ensures compatibility of symmetry of the radiation pressure and a GCW mode relatively to the x=0 position. The function $\delta\langle E_a\rangle|_{z=\zeta}$ is proportional to the derivative of displacement in a GCW and, therefore, has the symmetry inverse to that of the displacement. The antisymmetric multiplier $\sinh(\gamma x)$, which is conditioned by a meniscus, brings the symmetry of the distribution of the alternating radiation force along the liquid surface in agreement with the symmetry of the distribution of displacement in the GCW mode.

At a single-mode parametric interaction, Eq. (6) can be transformed using (9) into the Mathieu equation

$$\ddot{\zeta}_n + 2\delta_n \dot{\zeta}_n + \omega_n^2 (1 + \Psi_n \cos \omega_p t) \zeta_n = 0, \tag{10}$$

where the natural frequency of GCW is modulated with an amplitude

$$\Psi_n = \varepsilon E_0 \rho^{-1} \omega_n^{-2} \gamma k_n^2 \tanh(k_n h)$$

$$\times \frac{1}{d} \int_{-d}^d dx \cdot \sinh(\gamma x) \sin 2k_n x \left(1 + \varepsilon^2 \gamma^2 \sinh^2(\gamma x)\right)^{-2}.$$
 (11)

A threshold of the parametric instability of GCW is conditioned by the equality: $Q_n^{-1} \equiv 2\delta_n/\omega_n = |\Psi_n|/2$, where Q_n is the quality factor of the mode n.

By integrating the relationship (11), it is possible to estimate the critical density of energy E_{0c} at the liquid surface. The following parameters of the system were taken for the estimations: d=9 mm, h=12 mm, $\rho=1.68$ g/m³, $\sigma=10$ dyn/cm, and $\gamma=13.5$ cm $^{-1}$. The measured height of the meniscus 2 mm corresponds to $\varepsilon=2.1\times10^{-5}$ mm. Implementing to the fundamental mode with the wave number $k_n=\pi/2d$ and the quality factor measured by relaxation time of the oscillations $Q_n=21$, the predicted critical density of energy is $E_{0c}=160$ J/m³.

To compare the obtained result with the measured critical power P_{ec} applied to the transducer in the experiments, the ratio I/E_0 was numerically calculated between the average intensity of the acoustic field at the emitting surface of the cell's bottom and the average energy density at the liquid surface. The simulation using the COMSOL Multiphysics software was carried out in the frames of acoustics for a simplest two-dimensional model having the geometry correspondent to the XZ plane of Fig. 1(a). The model included a $20 \times 24 \,\mathrm{mm}$ (H × W) glass cell with 3-mm-thick walls and with a plane 2 mm ultrasound source attached to the bottom. The cell contained 12-mm layer of FC-72 with a flat motionless surface. The manufacturer's data for density and speed of sound (see Sec. II) were used for the liquid. The parameter of sound absorption in FC-72 $\alpha = 1.3 \text{ m}^{-1}$ utilized in the calculations was measured at a frequency of 1 MHz using the variable distance method.²⁴ For the material of the cell, standard parameters of the glass were taken.

The upper face of the source provided vertical harmonic oscillations to the bottom of the cell at a set of frequencies with a prescribed velocity amplitude, thus generating elastic waves in the cell's walls and ultrasound in the liquid. The oscillation frequencies were taken in the range from 960 to 972 kHz with a step of 0.5 kHz in order to average the final results through the frequency dependence of the acoustic field in the cell. The simulations demonstrated that the application of the film to the liquid surface had a negligibly small effect on the field. Several cases with a small inclination of the liquid surface in the XZ plane with respect to the bottom were studied. The solution for particle velocity vectors was obtained in the frequency domain, and the average values of E_0 and I were calculated along the liquid surface and the bottom correspondingly for the set of the frequencies. For a small tilt of the surface equal to 1° , the ratio was found to be $I/E_0 \approx 3.7$ m/s. This corresponds to a critical value $I_c \approx 59 \text{ mW/cm}^2$ and does not exceed the measured power density applied to the transducer in the experiments.

V. CONCLUSION

The presented results demonstrate the possibility of a supercritical parametric excitation of gravity-capillary waves at the surface of a liquid by alternating radiation pressure of ultrasound. Despite the low value of the quality factor of the oscillations, the critical intensity of the ultrasound pump is found to be relatively small and is about tenths of a watt per square centimeter. In a framework of the proposed theoretical model, a phenomenon of wetting the side borders that leads to the static deformation of the surface by menisci plays a fundamental role in the mechanism of the linear parametric resonance.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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